

## 1) Heat loss through a composite wall

A 3 m high and 5 m wide wall consists of long 32 cm 22 cm cross section horizontal bricks ( $k = 0.72 \text{ W/m} \cdot ^\circ\text{C}$ ) separated by 3 cm thick plaster layers ( $k = 0.22 \text{ W/m} \cdot ^\circ\text{C}$ ). There are also 2 cm thick plaster layers on each side of the brick and a 3-cm-thick rigid foam ( $k = 0.026 \text{ W/m} \cdot ^\circ\text{C}$ ) on the inner side of the wall. The indoor and the outdoor temperatures are  $20^\circ\text{C}$  and  $10^\circ\text{C}$ , and the convection heat transfer coefficients on the inner and the outer sides are  $h_1 = 10 \text{ W/m}^2 \cdot ^\circ\text{C}$  and  $h_2 = 25 \text{ W/m}^2 \cdot ^\circ\text{C}$ , respectively. Assuming one-dimensional heat transfer and disregarding radiation, determine the rate of heat transfer through the wall.

The answer:

$$A = (0.015 + 0.22 + 0.015) \cdot 1 = 0.25 \text{ m}^2$$

$$R_{1, \text{conv.}} = \frac{1}{h_1 \cdot A_1 - \text{dimen}} = \frac{1}{10 \cdot 0.25} = 0.4 \frac{\text{C}}{\text{W}}$$

$$R_{\text{foam}} = \frac{L_{\text{foam}}}{K_{\text{foam}} \cdot A_1 - \text{dimen}} = \frac{0.03}{0.026 \cdot 0.25} = 4.615 \frac{\text{C}}{\text{W}}$$

$$R_{\text{plaster, up}} = R_{\text{plaster, down}} = \frac{L_{\text{fp, up or down}}}{K_p \cdot A_{p, \text{up or down}}} = \frac{0.32}{0.22 \cdot 0.15 \cdot 1} = 96.97 \frac{\text{C}}{\text{W}}$$

$$R_{\text{brick}} = \frac{L_{\text{brick}}}{K_{\text{brick}} \cdot A_{\text{brick}}} = \frac{0.32}{0.72 \cdot 0.22 \cdot 1} = 2.02 \frac{\text{C}}{\text{W}}$$

$$\frac{1}{R_{\text{total, parallel}}} = \frac{1}{96.97} + \frac{1}{2.02} + \frac{1}{96.97} = 0.516 \frac{\text{C}}{\text{W}}$$

$$R_{\text{total, parallel}} = \frac{1}{0.516} = 1.94 \frac{\text{C}}{\text{W}}$$

$$R_{\text{plaster, left}} = R_{\text{plaster, right}} = \frac{L_{p, \text{left or right}}}{K_p \cdot A_{p, \text{left or right}}} = \frac{0.02}{0.22 \cdot 0.25 \cdot 1} = 0.363 \frac{\text{C}}{\text{W}}$$

$$R_{2, \text{conv.}} = \frac{1}{h_2 \cdot A_1 - \text{dimen}} = \frac{1}{40 \cdot 0.25} = 0.1 \frac{\text{C}}{\text{W}}$$

$$R_{\text{wall, total}} = 0.4 + 4.615 + 0.363 + 1.94 + 0.363 + 0.1 = 7.781 \frac{\text{C}}{\text{W}}$$

**The rate of heat transfer loss:**

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{\text{wall, total}}} = \frac{20 - (-10)}{7.781} = 3.86 \text{ W}$$

**AND**

$$R_{\text{wall, total}} \quad \text{Thickness of brick} = 16 \text{ mm} \\ = 6.81 \frac{\text{C}}{\text{W}}$$

**SO, the heat transfer rate is:**

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{\text{wall, total}}} = \frac{20 - (-10)}{6.81} = 4.41 \text{ W}$$

By comparing the two results, we found that: double the thickness of a brick inside a composite wall doesn't significantly increase the thermal resistance of a whole wall, so the rate of heat transfer doesn't significantly decrease.

## 2. R-values

	A wood	B Insulation
Outside Air	0.03	0.03
Wood Bevel	0.14	0.14
13mm Plywood	0.11	0.11
Urethane Rigid Foam	NA	$0.98 \times (90/25) = 3.528$
Wood Studs	0.63	NA
Gypsum Board	0.79	0.79
Inside Surface	0.12	0.12

$$R'_{with\ wood} = 0.03 + 0.14 + 0.11 + 0.63 + 0.79 + 0.12 = 1.82\ m^2 \cdot \frac{^{\circ}C}{W}$$

$$R'_{with\ insulation} = 0.03 + 0.14 + 0.11 + 3.528 + 0.79 + 0.12 = 4.718\ m^2 \cdot \frac{^{\circ}C}{W}$$