Analyse2 | CM: 5

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Démonstration 0.1.

Comme g ne s'annule pas sur $(I \cap V) \backslash \{a\}$, par la proposition 3.1.1, on a:

$$\begin{aligned} f(x) \underset{x \to a}{\sim} g(x) &\iff f(x) - g(x) \underset{x \to a}{\sim} o(g(x)) \\ &\iff \lim_{x \to a} \frac{f(x) - g(x)}{g(x)} = 0 \\ &\iff \lim_{x \to a} \frac{f(x)}{g(x)} - 1 = 0 \\ &\iff \lim_{x \to a} \frac{f(x)}{g(x)} = 1 \end{aligned}$$

$\square$

Example 0.1. *1. On veut calculer* $\lim_{x \to 0} \frac{\sin(x \ln(x))}{x}$

Comme $\lim_{x \to 0} x \ln(x) = 0$ *et* $\sin(x) \underset{x \to 0}{\sim} x$*, on a:* $\sin(x \ln(x)) \underset{x \to 0}{\sim} x \ln(x)$

Ainsi

$$\frac{\sin(x \ln(x))}{x} \underset{x \to 0}{\sim} \ln(x)$$

D'où:

$$\lim_{x \to 0} \frac{\sin(x \ln(x))}{x} = \lim_{x \to 0} \ln(x) = -\infty$$

2. On veut calculer $\lim_{x \to 0} \frac{\ln(1+2\tan(x))}{\sin(x)}$

Comme $\lim_{x \to 0} \tan(x) = 0$*,* $\ln(1+x) \underset{x \to 0}{\sim} x$*,* $\sin(x) \underset{x \to 0}{\sim} x$ *et* $\tan(x) \underset{x \to 0}{\sim} x$*, on a:*

$$\begin{aligned} \frac{\ln(1 + 2\tan(x))}{\sin(x)} &\underset{x \to 0}{\sim} \frac{2\tan(x)}{\sin(x)} \\ &\underset{x \to 0}{\sim} \frac{2x}{x} \\ &\underset{x \to 0}{\sim} 2 \end{aligned}$$

D'où $\lim_{x \to 0} \dfrac{1 + 2\tan(x)}{\sin(x)} = 2$