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Discounting under disagreement

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Abstract

A group of time consistent agents has access to a common productive resource stock whose output provides their consumption needs. The agents disagree about the appropriate pure rate of time preference to use when choosing a consumption policy, and thus delegate the management of the resource to a social planner who allocates consumption efficiently across individuals and over time. We show that the planner's optimal policy is equivalent to that of a representative agent with a time varying rate of impatience. The representative agent's time preferences depend on the distribution of time preferences in the group, on the agents' tolerance for consumption fluctuations, and on the productivity of the resource. The representative agent's rate of impatience coincides with that of the individual with the lowest rate of impatience in the long run, and under plausible conditions, is monotonically declining. In the work-horse case of iso-elastic felicity functions, and Gamma distributed rates of impatience, analytic solutions are possible, and the representative agent has hyperbolic time preferences. We thus provide a normative justification for the use of declining rates of time preference in dynamic welfare analysis.

Key words: Pure rate of time preference, heterogeneity, time consistency, hyperbolic discounting, dynamic welfare analysis
JEL Classification: D61, D99

1 Introduction

Ever since the seminal contributions of Koopmans (1960), we have known that two plausible desiderata of dynamic choice – independence and stationarity (see e.g. Heal, 2005) – imply the

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standard exponential discounting model that forms the basis of dynamic welfare economics[1]. Decision makers with constant discount rates are time consistent, and this, in combination with the appeal of Koopmans' axioms, has provided the basis for the dominance of the exponential discounting model in normative applications.

In this paper we take the normative appeal of the exponential discounting model for granted, and assume that agents have time consistent preferences. However, like all preference representation results, Koopmans provides us with a functional form for agents' preferences, but leaves the parameter values that enter their preferences – the actual value of the discount rate – unspecified. In order to operationalize the preference representation, we must look to the real world, and ask what agents' preferences actually are, or motivate a choice of discount rate by other normative considerations (Dasgupta & Heal, 1979; Broome, 1992; Arrow, 1999). Once we realize this, things become more murky. Whose discount rate should we choose? Are some discount rates more legitimate than others? What counts as a good argument for one value, rather than another?

We view an individual's choice of discount rate as an ethical primitive – it is a fundamental judgement about how she values realizations of welfare that are distant in time. Many things may inform this choice: ethical considerations of inter temporal equity, as well as judgements of whether the savings rate implied by her choice is desirable (Dasgupta, 2008). Although others may *disagree* with her choice, they cannot claim it is 'incorrect' – this is much the same as claiming that a preference for whisky over wine is incorrect. In this view, the problem of social discounting acquires a social choice dimension – there are many discount rates in the population, none is privileged over the others, and each should play a role in social decision making. There is abundant evidence for heterogeneous time preferences in the empirical literature (Frederick et al., 2002). There is also substantial disagreement about the appropriate discount rate for normative applications, as demonstrated by the debate that followed the Stern Review of the Economics of Climate Change (Stern, 2007; Nordhaus, 2007; Weitzman, 2007; Dasgupta, 2008), where the choice of discount rate was a crucial determinant of policy recommendations. Given this heterogeneity, and assuming the social choice approach has some democratic appeal, how should different discount rates be aggregated?

The standard dynamic welfare model deals with this question, if it deals with it at all, by *fait accompli*. We assume a representative agent (RA) whose preferences are themselves time consistent, and the problem boils down to motivating a choice for this agent's discount rate. This simplification, while undoubtedly attractive due to its tractability dividends, is often misleading. It has long been recognized that heterogeneous time preferences may lead the RA to have time preferences that differ substantially from any individual's (Marglin, 1963). In general, the RA will not be time consistent. This has been explored in models in which agents have common consumption streams (Li & Löfgren, 2000; Jackson & Yariv, 2012),

[1] A few additional technical conditions are also needed, but these are not controversial.

and also in models where agents receive an efficient share of an exogenous income stream that cannot be saved at each point in time (Gollier & Zeckhauser, 2005). In the former case, individual consumption shares are determined (the consumption good is public), and aggregate consumption is endogenous, while in the latter consumption shares are endogenous and aggregate consumption is exogenously given. Our contribution is to generalize these results to the more realistic case in which *both* consumption shares and aggregate consumption are endogenously determined, and derived from a common resource stock. This context is, we feel, much better suited to many of the interesting dynamic economic problems that require collective public choices to be made over policies. Examples include applications to Ramsey-style optimal savings problems (Ramsey, 1928), and renewable and exhaustible resource management.

Following Li & Löfgren (2000) and Gollier & Zeckhauser (2005), we assume a heterogenous population of agents with idiosyncratic pure rates of time preference. A social planner wishes to maximize a weighted sum of each agent's dynamic welfare integral. Thus our social choice rule is utilitarian, and by varying the weights on agent's welfare, we sketch out the efficient Pareto frontier of dynamic consumption allocations. The group's income in our model is derived from a productive resource stock, which the planner exploits optimally over time. By endogenizing both aggregate income and agents' consumption shares, we are able to derive a general expression for the representative agent's discount rate that depends on the distribution of time preferences in the population, agents' tolerance for consumption fluctuations, and the productivity of the income generating resource stock. The evolution of the group's discount rate depends on the trajectories of a dynamical system, so is in general quite complex.

Despite this complexity, our result allows us to make connections between previously disparate literatures. We begin by showing that the expression for the group's discount rate obtained by Gollier & Zeckhauser (2005) holds only in the infinitely distant future when agents face an endogenous resource management decision problem. We then make a conceptual link between the analysis of long-run equilibria in economies with heterogeneous agents by Becker (1980), and the analysis of uncertain real discount rates by Weitzman (1998, 2001). Through reasoning related to Becker's, we show that there is an analogue of Weitzman's results when we disagree about the pure rate of time preference – the group's rate of impatience approaches the lowest rate in the population in the long run. We analyze the term structure of the group's discount rate, and show that under plausible conditions on agents' felicity functions it is monotonically declining. Under these conditions, the exogenous income collective rate of impatience obtained by Gollier & Zeckhauser (2005) underestimates the group's discount rate when the resource stock is renewable, and overestimates it when the resource stock is exhaustible.

Finally, we demonstrate the application of our results in two quantitative worked examples. First we show that in the work horse case in which agents have iso-elastic felicity functions it is possible to solve analytically for the group's discount rate for all stock dynamics. If we further assume that the distribution of rates of impatience in the population is described by a Gamma distribution, the group's discount rate is a hyperbolic function of time. This allows us to reinterpret the classic behavioral models of hyperbolic discounting (e.g Laibson, 1997; Barro, 1999) in a normative light. Second, we demonstrate the new effects endogenous resource management introduces into the group's discount rate in simulations with a simple neoclassical growth model. We show that, unlike in the exogenous income case, endogenous income effects can make groups more impatient than any of their constituent members.

2 Collective impatience and stock dynamics

2.1 The social planner's problem

Consider a set of agents, indexed by i. Each of the agents is assumed to have additive, time consistent, preferences. We assume that the agents have common felicity functions over consumption[2] $U(c)$, but distinct heterogeneous pure rates of time preference, ρ_i. Thus, the utility of agent i from consumption c_{it} realized at time t in the future is $U(c_{it})e^{-\rho_i t}$. The group of agents has access to a common managed 'resource' S, which is the source of their income. We can think of S as physical or natural capital. The resource has a natural growth rate $F(S)$, which we assume to be a concave function with $F'(0) \geq 0$. Otherwise, we keep the form of $F(S)$ unspecified at this stage. This model accommodates a wide class of stock problems, including neoclassical growth models, and models of renewable and exhaustible resource management.

We assume that a social planner (SP) wishes to exploit the resource S so that allocations of consumption to individuals over time maximize social welfare, which is taken to be a weighted sum of individuals' welfare. The social planner's allocation problem is thus:

$$\max_{c_{it}} \sum_i w_i \left(\int_0^\infty U(c_{it}) e^{-\rho_i t} dt \right) \quad \text{s.t.} \quad \dot{S} = F(S) - \sum_i c_{it}, \tag{1}$$

where the welfare weights $w_i > 0$ satisfy $\sum_i w_i = 1$, and the initial value of the stock is $S(0) = S_0$.

In addition to the standard assumptions $U' > 0, U'' < 0$ on the felicity function, we also assume that $\lim_{c \to 0} U'(c) = \infty$. This ensures that the solution to the planner's control problem is interior. The curvature of U measures agents' tolerance for consumption fluctuations. It will prove useful in what follows to have a way of measuring this important aspect of agents' inter temporal preferences. To this end, following Gollier & Zeckhauser (2005), we define the

[2] We assume common felicity functions, as we wish to focus purely on the effects of heterogeneity in pure time preference, and not on heterogeneity in attitudes to consumption smoothing. Accounting for heterogenous felicity functions would be a natural extension of our analysis.

Absolute Tolerance for Consumption Fluctuations:

$$T(c) := -\frac{U'(c)}{U''(c)} \tag{2}$$

This quantity tells us about agents' consumption smoothing preferences. The smaller is T, the more agents care about smoothing their consumption over time. $T(c)$ is of course just the inverse of the coefficient of absolute risk aversion, though uncertainty will play no role in our analysis. The familiar assumption of decreasing absolute risk aversion corresponds to the assumption that $T(c)$ is an increasing function. This has an equally direct interpretation in our context: the higher is c, the more tolerant agents are towards small additive consumption fluctuations. We will assume this condition in what follows. The requirement that marginal utility be unbounded at $c = 0$ places restrictions on the behavior of $T(c)$ near zero. We show in Appendix A that it implies

$$\lim_{c \to 0} T(c) = 0. \tag{3}$$

This condition will be useful later on.
The Hamiltonian for the social planner's problem is:

$$H = \sum_i w_i U(c_{it}) e^{-\rho_i t} + \lambda(F(S) - \sum_i c_{it}), \tag{4}$$

where λ is the shadow price of the resource. Since the Hamiltonian is concave, the necessary conditions of the Pontryagin Maximum Principle are sufficient for an optimum, and the optimum is unique. The necessary conditions are:

$$\frac{\partial H}{\partial c_{it}} = 0 \Rightarrow \lambda = e^{-\rho_i t} U'(c_{it}) w_i \tag{5}$$

$$\frac{\partial H}{\partial S} = -\dot{\lambda} \Rightarrow -\dot{\lambda} = F'(S)\lambda \tag{6}$$

$$\lim_{t \to \infty} \lambda S = 0 \tag{7}$$

Hence along an optimal trajectory, the stock evolves according to the dynamical system

$$\dot{\lambda} = -\lambda F'(S) \tag{8}$$

$$\dot{S} = F(S) - \sum_i (U')^{-1} \left(e^{\rho_i t} \lambda / w_i \right) \tag{9}$$

and at each point in time consumption is allocated so that

$$c_{ti} = (U')^{-1} \left(e^{\rho_i t} \lambda / w_i \right) \tag{10}$$

The initial condition on the shadow price will be part of the solution.

2.2 The representative agent and the group's discount rate

Our task is to find a representative agent (RA) whose time preferences coincide with those of the SP. In order to achieve this we rely on a simple observation: the RA's optimum must be *observationally equivalent* to that of the SP. We now show that this is sufficient to determine the RA's discount rate.

Consider a RA who shares the agents' felicity function $U(c)$, but has some unspecified discount factor $\beta(t)$. She will solve the following problem:

$$\max_{C_t} \int_0^\infty U(C_t)\beta(t)dt \;\; \text{s.t.} \;\; \dot{S} = F(S) - C_t \tag{11}$$

The Hamiltonian for this problem is:

$$\tilde{H} = U(C_t)\beta(t) + \mu(t)(F(S) - C_t) \tag{12}$$

and applying the Maximum Principle leads to:

$$\mu(t) = \beta(t)U'(C_t) \tag{13}$$

$$-\dot{\mu} = F'(S)\mu \tag{14}$$

$$\lim_{t\to\infty} S\mu = 0 \tag{15}$$

Observational equivalence requires the evolution of the stock for the RA to be identical to that for the SP, and hence we require

$$\sum_i c_{it} = C_t \tag{16}$$

for all time. Substituting this relationship into (13), we see that a necessary condition on β is that

$$\mu = \beta(t)U'(\sum_i c_{ti}) \tag{17}$$

Differentiating this equation with respect to t, and substituting into (14), we find that the RA's implied rate of time preference $\rho^*(t)$ satisfies

$$\rho^*(t) := -\frac{\dot{\beta}}{\beta} = F'(S) - \eta(\sum c_{ti})\frac{\sum \dot{c}_{ti}}{\sum c_{ti}}, \tag{18}$$

where $\eta(c) := -cU''(c)/U'(c)$ is the elasticity of marginal utility. That is, the RA's discount rate $\rho^*(t)$ is just given by the Ramsey formula, where the terms on the right hand side (i.e. S and c_{it}) are determined from the solution to the SP's problem. All this says is that the representative agent must allocate aggregate consumption efficiently over time, and we have chosen the discount rate $\rho^*(t)$ so that the RA's consumption decision coincides with the SP's aggregate consumption decision. It is straightforward to see that choosing $\rho^*(t)$ according to (18) also ensures that the RA discounts aggregate consumption at the same rate as the group[3]. The group's consumption discount rate is the crucial input to cost benefit analysis of marginal projects. Thus $\rho^*(t)$ is an operationally meaningful quantity for real cost benefit analysis.

We now put the expression (18) into a more illuminating form which separates out two effects that contribute to the RA's time preferences: an 'exogenous income' distributive effect, and an 'endogenous income' collective consumption smoothing effect. The former originates from the time dependence of the problem of allocating an exogenously given stream of perishable income between individuals with different discount rates, and the latter from the problem of choosing the group's aggregate income level optimally over time.

To begin, define

$$\rho_{GZ} := \frac{\sum_i \rho_i T(c_{ti})}{\sum_i T(c_{ti})}. \tag{19}$$

Gollier & Zeckhauser (2005) (henceforth GZ) show that when the group has access only to an exogenous, perishable, income source at each time t, the RA's time preferences are given by (19). The key driver of this result is the distributive motive induced by the heterogeneity in the utility function $U(c)e^{-\rho_i t}$. Each agent's marginal utility from consumption varies idiosyncratically with time, and hence the allocation decision has a temporal dependence. What GZ are not picking up however, is how the heterogeneity in ρ_i will affect agent's preferences for the timing of consumption. This arises because in their model current consumption choices have no effect on future consumption possibilities, as income is exogenous and cannot be saved. Put simply, agents decide how to divide an exogenous and perishable cake, but not how much cake to bake over time.

Now differentiate (5) with respect to t, and substitute into (6), to find

$$\dot{c}_{ti} = (F'(S) - \rho_i)T(c_{ti}). \tag{20}$$

This is nothing more than another set of Ramsey equations – at the SP's optimum allocations of consumption to each agent i are efficient over time. Use the definition (19), and sum (20)

[3] By definition, the RA's consumption discount rate is $\rho^*(t) + \eta(C_t)\frac{\dot{C}_t}{C_t} = F'(S)$, where the equality follows from (18), and $C_t = \sum_i c_{ti}$. The value to the group of a marginal unit of aggregate consumption received at time t in the future is just $\lambda(t)/\lambda(0)$ in today's consumption units. The group's consumption discount rate is minus the growth rate of this value: $-\frac{d}{dt}\log\lambda(t) = F'(S)$ by (8). Thus the two consumption discount rates agree.

over i to find

$$\sum_i \dot{c}_{ti} = (F'(S) - \rho_{GZ}) \sum_i T(c_{ti}). \tag{21}$$

Now define

$$X := \frac{\sum_i T(c_{ti})}{T(\sum_i c_{ti})}. \tag{22}$$

Using the fact that $\eta(c) = c/T(c)$, the definition (22), and the relationship (21), we find that (18) is equivalent to

$$\rho^*(t) = (1 - X)F'(S) + X\rho_{GZ}. \tag{23}$$

This expression is our central result. It says that the RA's discount rate is a linear combination of the distributive term ρ_{GZ}, and the endogenous income term $F'(S)$, the rate of return on the resource S. The relative importance of these two factors is controlled by the endogenous, time varying, quantity X. To interpret X, note that Wilson (1968) and GZ show that the group's tolerance for consumption fluctuations is given by $\sum_i T(c_{ti})$. Thus X is a factor that converts between the group's tolerance for consumption fluctuations at the SP's optimum ($\sum_i T(c_{ti})$), and the representative agent's tolerance for consumption fluctuations at the optimum ($T(\sum_i c_{ti})$). The evolution of the group's discount rate is entirely determined by the dynamical system (8–9).

2.3 The steady state and the long-run discount rate

We now show that at a steady state S_∞ of the dynamical system (8–9), the expression for the discount rate $\rho^*(t)$ reduces to the expression GZ obtained in the exogenous income case. This is to be expected: at a steady state of the system the group's income is constrained to be constant, and hence current consumption decisions do not affect future consumption possibilities. This is precisely the case GZ examine.

To see this notice that when the resource stock is stationary, i.e. $\dot{S} = 0$, we must have

$$F(S) - \sum_i c_{ti} = 0. \tag{24}$$

Since this equation must hold for all times at the steady state, we can differentiate it with respect to time, and set the result to zero:

$$F'(S)\dot{S} - \sum_i \dot{c}_{ti} = -(F'(S) - \rho_{GZ}) \sum_i T(c_{ti}) = 0, \tag{25}$$

where we've used (21). There are two possibilities, either $F'(S) = \rho_{GZ}$, or $\sum_i T(c_{ti}) = 0$. Assuming a non-trivial steady state (i.e. $S \neq 0$) at least one agent's consumption will be positive, thus we must have $F'(S) = \rho_{GZ}$, and hence from (23), we conclude that

$$\rho^*|_{S=S_\infty} = \rho_{GZ}. \tag{26}$$

Thus we see that in general we can expect GZ's results to be accurate only in the infinite future, when the system settles down to its steady state.

The result (26) says that, even before we know how consumption is allocated in the steady state, we know that the group's discount rate must be in agreement with GZ's prediction in the long run. The following proposition determines how consumption is allocated at the steady state, and hence allows us to infer the group's long run discount rate in general.

Proposition 1. *Let L be the index of the agent with the lowest discount rate, ρ_L. Assume that the equation $F'(S) = \rho_L$ has a solution*[4] *in S. Then the only nontrivial steady state S_∞ of our model is given by*

$$S_\infty = (F')^{-1}(\rho_L) \tag{27}$$

$$c_{\infty i} = \begin{cases} 0 & i \neq L \\ F(S_\infty) & i = L \end{cases} \tag{28}$$

and the steady state is a saddle point of the dynamical system (8–9). Moreover,

$$\lim_{t\to\infty} \rho^*(t) = \rho_L \tag{29}$$

Proof. Equations (10) and (8) imply that in the steady state

$$U'(c_{ti}) \propto \frac{e^{-(F'(S_\infty)-\rho_i)t}}{w_i}, \tag{30}$$

where the proportionality constant is positive (since the shadow price is positive). Since U' is decreasing by assumption, this implies that in the steady state c_{ti} is growing indefinitely if $\rho_i < F'(S_\infty)$, declining if $\rho_i > F'(S_\infty)$, and constant if $\rho_i = F'(S_\infty)$. Now in the steady state we require $\dot{S} = 0$, i.e. $\sum_i c_{ti} = F(S_\infty)$. This cannot be the case if any of the c_{ti} are growing indefinitely. Since the ρ_i are distinct, this means that at most one of the c_{ti} can be constant, and the rest must all decline to zero. Clearly, this can only be so if $F'(S_\infty) = \rho_L$, which implies $\lim_{t\to\infty} c_{ti} = 0$ for $i \neq L$. By the concavity of $F(S)$ there is only one value of S that satisfies $F'(S) = \rho_L$. Thus this is the unique nontrivial steady state.

To prove that the steady state is a saddle point, from (8–9), the Jacobian of the system is:

$$J = \begin{pmatrix} \frac{\partial\dot{\lambda}}{\partial\lambda} & \frac{\partial\dot{\lambda}}{\partial S} \\ \frac{\partial\dot{S}}{\partial\lambda} & \frac{\partial\dot{S}}{\partial S} \end{pmatrix} = \begin{pmatrix} -F'(S) & -\lambda F''(S) \\ \sum_i T(c_{ti}) & F'(S) \end{pmatrix} \tag{31}$$

[4] This is guaranteed by continuity of $F'(S)$ if $\lim_{S\to 0} F'(S) > \rho_L, \lim_{S\to\infty} F'(S) < \rho_L$. The solution must be unique by our assumption that F is concave.

where in the bottom left element we've used the fact that

$$\frac{\partial}{\partial \lambda}(U')^{-1}\left(e^{\rho_i t}\lambda/w_i\right) = -\frac{e^{\rho_i t}/w_i}{U''((U')^{-1}(e^{\rho_i t}\lambda/w_i))} = T(c_{ti}). \tag{32}$$

Evaluated at the steady state values of λ, S, c_{ti}, this becomes

$$J|_{S=S_\infty} = \begin{pmatrix} -\rho_L & 0 \\ T(F(\rho_L)) & \rho_L \end{pmatrix} \tag{33}$$

The eigenvalues of the Jacobian are $\pm\rho_L$, and hence the steady state is a saddle point. Finally, by (26) the steady state discount rate is given by the formula (19). Substituting the steady state consumption values into this formula, and making use of (3), yields (29). □

The fact that the steady state is a saddle point, and the only non-trivial steady state of the system (8–9), means that we know that the optimal trajectory of the system must be a saddle path. In general, the saddle path will be monotonic, i.e. the state variable S will approach its steady state value S_∞ monotonically from its initial value S_0 (see e.g. Kamien & Schwartz, 1991).

Our result parallels that obtained by Becker (1980) in the case of a decentralized competitive equilibrium. The endogeneity of the group's consumption path in our model has thus allowed us to obtain an analogue of the results of Weitzman (1998) for the pure rate of time preference: the group's rate of time preference approaches the lowest rate in the population asymptotically.

2.4 Term structure of $\rho^*(t)$

A natural further question to ask is whether we can find conditions that determine the qualitative properties of the term structure of the group's discount rate, and its relationship to the exogenous income discount rate $\rho_{GZ}(t)$. In general, the derivative of $\rho^*(t)$ is a very complex function. We have however obtained the following proposition:

Proposition 2. *Suppose that*

1. *T is increasing and convex.*
2. *T/T' is convex.*
3. *If there is a non-trivial steady state: $S_0 < S_\infty$.*

Then $\rho^*(t)$ is monotonically declining in time. $\rho^*(t) \geq \rho_{GZ}(t)$ if there is a non-trivial steady state, and $\rho^*(t) \leq \rho_{GZ}(t)$ if S is an exhaustible resource (i.e. $F(S) = 0$).

Proof. See Appendix B. □

As we show in Appendix A, the conditions on T in this proposition must hold locally at $c = 0$, we have simply converted them to global conditions. An example of a class of functions that satisfies these conditions is $T(c) = Kc^{\alpha}$, where $K > 0, \alpha \geq 1$. The familiar case of iso-elastic felicity functions corresponds to $\alpha = 1$, and will be examined in detail in the following section. The condition on S_0, i.e. that the initial value of S be below its steady state value, is natural in most applications where the resource S is 'renewable', including neoclassical growth models and renewable resource management problems. When the resource is exhaustible, i.e. $F(S) = 0$, the only steady state is $S = 0$. We show in Appendix B that the conditions on T in the proposition still guarantee a monotonically declining discount rate in this case.

3 Parametric examples

In this section we specialize to specific parametric cases for the felicity function $U(c)$ and the resource's growth function $F(S)$. This will allow us to investigate quantitative properties of the group's discount rate, thus moving beyond the qualitative results of the previous section.

3.1 Iso-elastic felicity and Gamma discounting

The evolution of the discount rate (23) is in general complex, as it depends on the evolution of the dynamical system (8–9). In this section we focus on the special case of iso-elastic felicity functions. This serves two purposes: iso-elastic felicity is a widely used functional form in applications, and analytic solutions for $\rho^*(t)$ are possible in this case.
It's clear from the definition of X in (22), that $X = 1$ if $T(c)$ is a linear function. This case corresponds to the work-horse case of an iso-elastic felicity function. Specifically,

$$U(c) = \frac{c^{1-\eta}}{1-\eta} \Rightarrow T(c) = \frac{c}{\eta}. \tag{34}$$

We can use this explicit functional form to calculate the inverse of marginal utility, and hence (10) determines the values of the agents' consumption:

$$c_{ti} = \left(\frac{w_i e^{-\rho_i t}}{\lambda(t)}\right)^{\frac{1}{\eta}} \tag{35}$$

Substituting (34–35) and $X = 1$ into (23) shows that in this special case:

$$\rho^*_\eta(t) = \frac{\sum_i \rho_i (w_i e^{-\rho_i t})^{\frac{1}{\eta}}}{\sum_i (w_i e^{-\rho_i t})^{\frac{1}{\eta}}}. \tag{36}$$

This result holds for *any* stock dynamics $F(S)$ – the shadow price has fallen out of the expression for $\rho^*(t)$. Thus, we have shown that the group's discount rate reduces to the GZ formula (19). This is the only felicity function for which their expression remains valid for problems where the group's income is endogenous.

The group's discount rate in this case is a weighted sum of the individuals' discount rates, with time dependent weights $y_{it} := (w_i e^{-\rho_i t})^{\frac{1}{\eta}}$. Defining the expectation operator $\mathsf{E}x_i := \sum_i x_i y_{it} / \sum_i y_{it}$, and differentiating (36) we find

$$\frac{d}{dt}\rho^*_\eta = -\frac{1}{\eta}\left(\mathsf{E}\rho_i^2 - (\mathsf{E}\rho_i)^2\right) < 0. \tag{37}$$

Recalling that $i = L$ indexes the agent with the lowest discount rate, we have

$$\lim_{t\to\infty} \rho^*_\eta(t) = \lim_{t\to\infty} \frac{\rho_L + \sum_{i\neq L} \rho_i (w_i/w_L)^{\frac{1}{\eta}} e^{-(\rho_i-\rho_L)t/\eta}}{1 + \sum_{i\neq L} (w_i/w_L)^{\frac{1}{\eta}} e^{-(\rho_i-\rho_L)t/\eta}} = \rho_L. \tag{38}$$

Thus for iso-elastic utility, the RA's discount rate declines monotonically to the lowest rate. This is in agreement with Propositions 1 and 2.

We can move beyond these qualitative features to find an analytic expression for $\rho^*(t)$ in a special case for the weights w_i. Take the continuum limit of (36), and let $w_i \to w(\rho)$ be the probability density of discount rate ρ. Assume, following Weitzman (2001), that $w(\rho)$ is Gamma distributed with shape and scale parameters $k, \theta > 0$. This implies:

$$w(\rho) \propto \rho^{k-1} e^{-\frac{\rho}{\theta}}. \tag{39}$$

Thus the time dependent weights y_{it} become

$$y_{it} \to y_t(\rho) \propto \rho^{(\frac{k-1}{\eta}+1)-1} e^{-\frac{1}{\eta}(\frac{1}{\theta}+t)\rho}. \tag{40}$$

These are also Gamma distributed, with modified parameters $(k', \theta') = (\frac{k-1}{\eta} + 1, \eta/(\theta^{-1} + t))$, provided that $k' > 0$. Using the fact that the mean of the Gamma distribution with parameters (k', θ') is just $k'\theta'$, we have

$$\rho^*_\eta(t) = \frac{k - 1 + \eta}{t + \frac{1}{\theta}}. \tag{41}$$

For a Gamma distribution with mean μ and variance σ^2, $k = \frac{\mu^2}{\sigma^2}$, and $\theta = \frac{\sigma^2}{\mu}$, and so we can rewrite this as

$$\rho^*_\eta(t) = \frac{\mu + \frac{\sigma^2}{\mu}(\eta - 1)}{1 + \frac{\sigma^2}{\mu} t}. \tag{42}$$

The constraint $k' > 0$ ensures that the numerator is non-negative. To find the value of $\rho^*_\eta(t)$ when $k' < 0$, note that by continuity of ρ^*_η in η, and the fact that $\rho^*_\eta \geq 0$, we must have $\rho^*_\eta = 0$

when the numerator of the previous expression is negative[5]. Hence we have,[6]

$$\rho^*_\eta(t) = \begin{cases} \frac{\mu+(\sigma^2/\mu)(\eta-1)}{1+\sigma^2 t/\mu} & \eta \geq 1 - \frac{\mu^2}{\sigma^2} \\ 0 & \eta < 1 - \frac{\mu^2}{\sigma^2} \end{cases} \tag{43}$$

Note that ρ^*_η is non-decreasing in η. We expect this since the larger is η, the less tolerant agents are of consumption fluctuations – this makes the group's consumption decisions more myopic. ρ^*_η is also non-decreasing in μ – the larger is the average pure rate of time preference, the larger is the group's rate of time preference. Interestingly,

$$\mathrm{sgn}\left(\frac{\partial \rho^*_\eta}{\partial \sigma^2}\right) = \mathrm{sgn}\left(\frac{\eta-1}{\mu} - t\right) \tag{44}$$

Thus the direction of the effect of an increase in the variance of time preferences varies with time when $\eta > 1$. For $t < (\eta-1)/\mu$, the group's discount rate increases, and for $t > (\eta-1)/\mu$ it decreases if we increase σ^2. Also, $\lim_{\sigma^2\to 0} \rho^*_\eta(t) = \mu$ and $\lim_{\sigma^2\to\infty} \rho^*_\eta(t) = (\eta-1)/t$ (for $\eta > 1$). The time preferences of high variance groups look more 'hyperbolic', while those of low variance groups look more like the constant discount rate model. Figure 1 illustrates this dependence.

The functional form for $\rho^*_\eta(t)$ in (43) corresponds to the familiar hyperbolic discounting model introduced by Laibson (1997), and applied to neoclassical growth models by Barro (1999). Our analysis suggests an alternative interpretation for their work. Both Laibson and Barro interpret declining pure rates of time preference behaviorally. In our interpretation however, these models can be seen as a description of the behavior of a heterogeneous group of time consistent agents, whose consumption decisions are made by a social planner who distributes consumption between individuals and over time in an efficient manner. This places a normative spin on models with declining discount rates. In addition, unlike the single agent interpretation of declining discount rate models, our interpretation does not require the RA to solve a dynamic game against 'future selves' (e.g. Phelps & Pollak, 1968; Laibson, 1997) in order for her policy choices to be viewed as time consistent. Rather, solving the RA's control problem with time preferences $\rho^*(t)$, and assuming full commitment to future policy choices, yields the group's optimal consumption policy. This policy looks time-inconsistent from the

[5] This also follows from the fact that for $k' < 0$ the denominator of (36) tends to infinity, while the numerator is finite.

[6] Comparing (43) to the results obtained in Weitzman (2001), we see that this formula reduces to his expression when $\eta = 1$. Note however that Weitzman was interested in real discount rates r, and not pure rates of time preference. Real discount rates depend in part on agents' rates of impatience through the Ramsey formula: $r = \rho + \eta g + h.o.t.$, where g is the mean growth rate of consumption, and $h.o.t.$ denotes higher order terms that depend on the higher moments of agent's subjective probability distributions for g. Thus, heterogeneity in r arises from two quite different sources – disagreements about ρ (and η), and agents' different predictions about the value of about g, i.e. heterogeneous beliefs. While Weitzman (2010) has examined the consequences of empirical uncertainties in the consumption growth rate for real discount rates for a time consistent RA, his original work (Weitzman, 2001) is more appropriately seen as a problem in preference and belief aggregation than one of uncertainty. See also Freeman & Groom (2010).

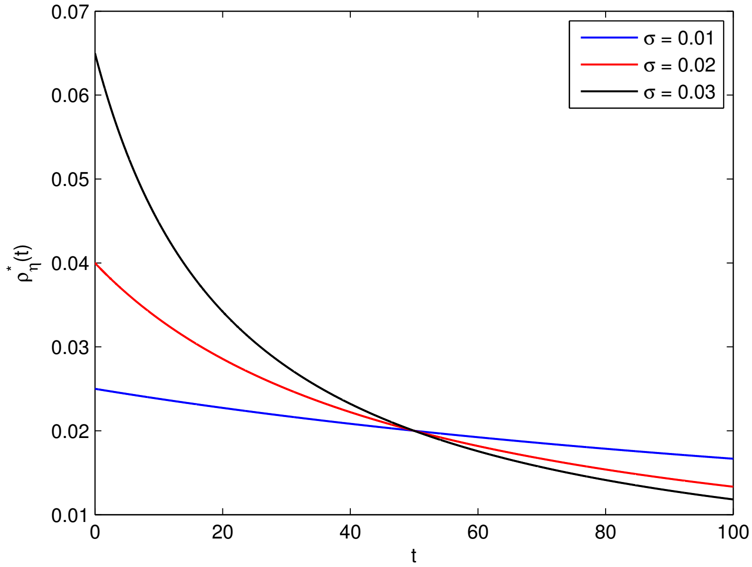

Figure 1: Dependence of the group's discount rate on the standard deviation (σ) of the distribution of individual rates in the population, for iso-elastic felicity functions. $\mu = 0.02, \eta = 2$ in this example.

perspective of the RA, but is in fact time-consistent from the perspective of the group.

3.2 Endogenous income effects in a simple growth model

In order to understand the quantitative dynamics of the group's discount rate $\rho^*(t)$ for non iso-elastic felicity functions, it is necessary to find the optimal saddle path trajectory of the dynamical system (8–9). This requires numerical methods in general. In order to demonstrate the additional insights that are possible from this exercise, we present the results of such an analysis for a simple model.

We consider a neoclassical growth theory interpretation for our model, in which S is a stock of capital, and $F(S)$ is a Cobb-Douglas production function, minus depreciation. We assume that labour supply is constant (normalized to 1), and no technical progress. Thus,

$$F(S) = S^{\gamma} - \delta S. \tag{45}$$

We pick $\gamma = 0.3$ for the capital share of production, and $\delta = 0.1$ for the annual depreciation rate. As we have shown, $T(c)$ must be nonlinear to pick up the additional effects of the endogenous income term $F'(S)$ in (23). We pick $T(c) = c^2$ – this choice obeys the conditions in Proposition 2. Integrating this choice of T using the relationship (46) in Appendix A yields the marginal utility function and its inverse, which enter the dynamical equation (9).

We assume that there are two groups of agents in the economy – an impatient group with

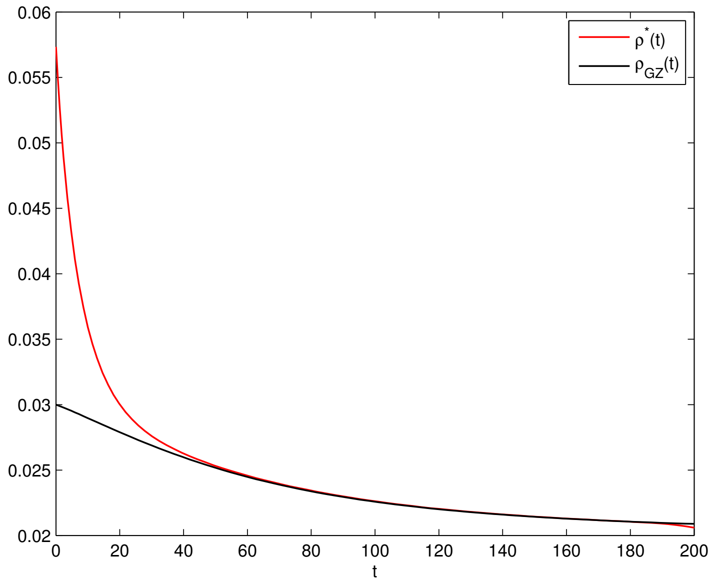

Figure 2: The group's discount rate for our simple growth model

$\rho = 0.04$, and a more patient group with $\rho = 0.02$, each with weight $w_i = 0.5$. We pick an initial condition of $S_0 = 2$, use a shooting method to solve for the initial value of the shadow price on the saddle path, and use (23) to compute the representative agent's discount rate. Figure 2 contains our results.

By Proposition 2, we know that we must have $\rho^* > \rho_{GZ}$, and that $d\rho^*/dt < 0$ for this model specification. This is born out in Figure 2. What the simulation results show however is that the GZ formula, which neglects the endogenous income effects, dramatically underestimates the group's discount rate at early times. The contribution of the endogenous income term in (23) to the RA's time preferences is large at early times, as $F'(S)$ is a declining function. Figure 2 shows that at early times the group's discount rate is larger than the discount rate of either group of agents. We thus conclude that endogenous income effects can make groups more impatient than any of their individual members. This result is not possible when income is exogenous and perishable, as the exogenous income discount rate ρ_{GZ} is a weighted sum of individual rates, and is thus bounded by the highest and lowest discount rates in the population.

4 Conclusions

While there are many arguments for the use of declining real discount rates for the evaluation of marginal projects (Groom et al., 2005), the assumption of a constant pure rate of time preference has remained largely unchallenged in dynamic welfare economics. There are very good reasons for this – time consistency is a very appealing property of rational dynamic choice, and the Koopmans axioms make a strong case of the standard exponential discounting model. Despite this, models in which agents have declining rates of time preference have been applied to welfare analysis of long run stock problems by some authors (e.g. Cropper & Laibson, 1999; Karp, 2005). Whereas exponential discounting renders the distant future all but irrelevant in these problems, declining discount rates place increased emphasis on the long-run, making analysis of policy options more sensitive to persistent effects. Despite the intuitive appeal of these methods, and the argument that exponential discounting neglects the long-run, there is a temptation to see them as an inappropriate application of behavioral models to normative questions.

Building on the work of others, our analysis has shown that there is another way to interpret representative agent models with declining pure rates of time preference. Declining pure rates of time preference arise naturally when aggregating the preferences of heterogeneous agents with idiosyncratic, time consistent, preferences. We feel that this places the use of declining discount rates on firm normative footing.

The exact choice of a discount rate schedule for empirical applications is a complex task – we have shown that in general it depends on agents' preferences, as well as the productivity of the stock that generates their income. While analytic results are only possible for iso-elastic felicity functions, we have argued that under plausible conditions the group's discount rate is declining, and approaches the lowest rate in the population asymptotically. These qualitative properties are shared by the hyperbolic discounting models that have thus far largely been the province of behavioral work. Our analysis suggests that these models may be just as useful for normative welfare applications.

A Conditions on T(c)

The condition $\lim_{c\to 0} U'(c) = \infty$ ensures that solutions to optimal consumption allocation problems are interior. This condition places restrictions on the behavior of $T(c)$, defined in (2), at the origin. The results in this appendix make use of the following simple lemma:

Lemma 1. Suppose $A(c)$ is a twice differentiable function. If $\lim_{c\to 0^+} A(c) = -\infty$, then $\lim_{c\to 0^+} A'(c) = \infty$.

The proof of this lemma is a simple application of the mean value theorem.

Treating (2) as a differential equation in U', we have

$$U'(c) = M \exp\left(-\int_0^c [T(x)]^{-1} dx\right) \tag{46}$$

for some constant $M > 0$. Define $G(c) := \int_0^c [T(x)]^{-1} dx$. $\lim_{c\to 0} U'(c) = \infty$ implies that we require $\lim_{c\to 0} G(c) = -\infty$. This in turn implies that we must have $\lim_{c\to 0} G'(c) =$

$\lim_{c\to\infty}[T(c)]^{-1} = \infty$, and hence we conclude that

$$\lim_{c\to 0} T(c) = 0. \tag{47}$$

The fact that T approaches zero at the origin means that for c small enough $T(c) \sim Kc^{\alpha}$, for some constants $K, \alpha > 0$, where α is the exponent of the dominant term in $T(c)$ as $c \to 0$, i.e. all other terms approach zero faster than c^{α}. Thus for small c, $G(c) \sim \frac{1}{K(1-\alpha)} c^{1-\alpha}$. Since we need $\lim_{c\to 0} G(c) = -\infty$, we must have $\alpha \geq 1$. Hence $T(c)$ must be locally convex at the origin, and approach zero as fast as, or faster than c. This in turn implies that $T/T' \sim c$ as $c \to 0$, and thus T/T' is also locally convex at the origin.

B Term structure of $\rho^*(t)$

We consider the case in which a non-trivial steady state S_∞ exists first, i.e. we assume that $F'(S) = \rho_L$ has a solution. From (23), we have

$$\dot{\rho}^* = (1-X)F''(S)\dot{S} - \dot{X}(F'(S) - \rho_{GZ}) + X\dot{\rho}_{GZ}. \tag{48}$$

By the assumption that T is convex, and (3), we know that T must be a super additive function, and hence $0 \leq X \leq 1$. Thus $\operatorname{sgn}(1-X)F''(S)\dot{S} = -\operatorname{sgn}(\dot{S}) = -\operatorname{sgn}(S_\infty - S_0)$, where the last equality follows from the monotonicity of the saddle path. Similarly, from (21) we have $\operatorname{sgn}(F'(S) - \rho_{GZ}) = \operatorname{sgn}(\sum_i \dot{c}_{ti}) = \operatorname{sgn}(S_\infty - S_0)$. Again, the last equality follows from the properties of the saddle path. If $S_0 < S_\infty$, the group's income $F(S)$ is growing along the saddle path. It thus cannot be optimal for aggregate consumption to be falling, and we conclude that $\sum_i \dot{c}_{ti} > 0$. Thus the crux of the problem is to show that $\dot{X} > 0$ and $\dot{\rho}_{GZ} < 0$.

Consider $\dot{\rho}_{GZ}$. From the definition (19), we have

$$\dot{\rho}_{GZ} = \frac{(\sum \rho_i T_i' \dot{c}_{ti}) \sum T_i - (\sum \rho_i T_i) \sum T_i' \dot{c}_{ti}}{(\sum T_i)^2} \tag{49}$$

$$= (F'(S) + \rho_{GZ}) \frac{\sum \rho_i T_i T_i'}{\sum T_i} - \frac{\sum \rho_i^2 T_i T_i'}{\sum T_i} - F'(S)\rho_{GZ} \frac{\sum T_i' T_i}{\sum T_i} \tag{50}$$

where we've used (20–21) and simplified. If we set $F'(S) = \rho_{GZ}$ in this equation, we recover the expression for $\dot{\rho}_{GZ}$ that Gollier & Zeckhauser (2005) obtain in Proposition 5 of their paper. They prove that at this value of $F'(S)$, $\dot{\rho}_{GZ} < 0$ if $T' > 0$. So if we can show that $\dot{\rho}_{GZ}|_{F'(S) > \rho_{GZ}} < \dot{\rho}_{GZ}|_{F'(S) = \rho_{GZ}}$, we will be done. Partially differentiate the expression for $\dot{\rho}_{GZ}$ above with respect to $F'(S)$ to find:

$$\frac{\partial \dot{\rho}_{GZ}}{\partial F'(S)} = \frac{\sum \rho_i T_i T_i'}{\sum T_i} - \frac{\sum \rho_i T_i}{\sum T_i} \frac{\sum T_i T_i'}{\sum T_i} \tag{51}$$

where we've used $\rho_{GZ} = (\sum \rho_i T_i)/(\sum T_i)$. Define the expectation operator, $\langle x_i \rangle := \frac{\sum x_i T_i}{\sum T_i}$. Then the expression above is,

$$\frac{\partial \dot{\rho}_{GZ}}{\partial F'(S)} = \langle \rho_i T_i' \rangle - \langle \rho_i \rangle \langle T_i' \rangle \tag{52}$$

$$= \mathrm{Cov}(\rho_i, T'(c_{ti})) \tag{53}$$

where $\mathrm{Cov}(x,y)$ is the covariance of the two random variables x, y. Now from (10) it's clear that

$$\frac{\partial c_{ti}}{\partial \rho_i} = -tT(c_{ti}) < 0. \tag{54}$$

Hence ρ_i and c_{ti} are anti-correlated, and when T' is an increasing function, $\mathrm{Cov}(\rho_i, T'(c_{ti})) < 0 \Rightarrow \frac{\partial \dot{\rho}_{GZ}}{\partial F'(S)} < 0$. Hence for $F'(S) > \rho_{GZ}$ this implies that $\dot{\rho}_{GZ} < \dot{\rho}_{GZ}|_{F'(S)=\rho_{GZ}} < 0$.

Now consider $\dot{X}$. Differentiating X directly using the definition (22), we see that $\dot{X} > 0$ iff

$$\begin{aligned}
&\left(\sum_i T_i' \dot{c}_{ti}\right) T(\sum_i c_{ti}) > (\sum T_i) T'(\sum_i c_{ti}) \sum_i \dot{c}_{ti} \\
\iff &\left(\sum_i T_i' T_i (F'(S) - \rho_i)\right) T(\sum_i c_{ti}) > T'(\sum_i c_{ti}) (\sum T_i)^2 (F'(S) - \rho_{GZ}) \\
\iff &\frac{(\sum_i T_i' T_i (F'(S) - \rho_i))}{\sum T_i} - \frac{T'(\sum c_{ti})}{T(\sum c_{ti})} (\sum T_i)(F'(S) - \rho_{GZ}) > 0
\end{aligned}$$

where in the second line we've used the expressions for $\dot{c}_{ti}$ and $\sum_i \dot{c}_{ti}$ in (20–21). Consider the first term:

$$\begin{aligned}
\frac{(\sum_i T_i' T_i (F'(S) - \rho_i))}{\sum T_i} &= F'(S) \langle T_i' \rangle - \langle \rho_i T_i' \rangle \\
&= F'(S) \langle T_i' \rangle - [\mathrm{Cov}(\rho_i, T_i') + \langle T_i' \rangle \langle \rho_i \rangle] \\
&= -\mathrm{Cov}(\rho_i, T_i') + (F'(s) - \rho_{GZ}) \langle T_i' \rangle
\end{aligned}$$

Hence $\dot{X} > 0$ iff

$$-\mathrm{Cov}(\rho_i, T_i') + (F'(S) - \rho_{GZ}) \left[\frac{\sum_i T_i' T_i}{\sum T_i} - \frac{T'(\sum c_{ti})}{T(\sum c_{ti})} (\sum T_i) \right] > 0 \tag{55}$$

We have shown that when T is convex, $-\mathrm{Cov}(\rho_i, T_i') > 0$, and we also have $F'(S) - \rho_{GZ} > 0$ on the saddle path, so the task is to find conditions under which the square bracket is positive. Rewrite this condition as:

$$\frac{T(\sum c_{ti})}{T'(\sum c_{ti})} \sum_i T_i' T_i > (\sum T_i)^2 \tag{56}$$

Now assume that T/T' is a convex function. This, in combination with the fact that

$\lim_{c\to 0} T(c) = 0$, implies that $\lim_{c\to 0} T/T' = 0$, and hence T/T' is super additive. Thus,

$$\frac{T(\sum c_{ti})}{T'(\sum c_{ti})} \geq \sum_j \frac{T_j}{T'_j} \tag{57}$$

Under this condition, it is sufficient to show that

$$\left(\sum_i T'_i T_i\right)\left(\sum_j \frac{T_j}{T'_j}\right) > (\sum T_i)^2 = \sum_i T_i \sum_j T_j$$
$$\iff \sum_i \sum_j T_i T_j \left(\frac{T'_i}{T'_j} - 1\right) > 0$$

Notice that when $i = j$ in this double sum the factor in brackets is zero. Also, the factor $T_i T_j$ is symmetric under $i \leftrightarrow j$, so we can factorize pairs of terms with symmetric indices to find that we require

$$\sum_i \sum_{j>i} T_i T_j \left(\frac{T'_i}{T'_j} + \frac{T'_j}{T'_i} - 2\right) > 0.$$

The factors in the round brackets are all of the form $x + x^{-1} - 2$, where $x \geq 0$. It is easy to show that this function has a global minimum at $x = 1$, at which its value is zero. Hence the double sum is non-negative, and $\dot{X} > 0$. Combining these pieces yields $\dot{\rho}^* < 0$.

Now consider the case of exhaustible resources, in which $F(S) = 0$. Clearly the equation $F'(S) = \rho_L$ has no solution in this case, so there is no non-trivial steady state, and this case is not covered by the derivation above. Since $F'(S) = 0$ in this case, we have from our expression (23) that

$$\rho^*(t) = \frac{\sum_i \rho_i T(c_{ti})}{T(\sum_i c_{ti})}. \tag{58}$$

Differentiate $\rho^*(t)$ explicitly with respect to time to find that $\operatorname{sgn} \dot{\rho}^*(t)$ is given by the sign of

$$\left(\sum_i \rho_i T'(c_{ti})\dot{c}_{ti}\right) T(\sum_i c_{ti}) - \left(\sum_i \rho_i T(c_{ti})\right) T'(\sum_i c_{ti}) \sum_i \dot{c}_{ti} \tag{59}$$

$$= \sum_i \left[\rho_i T'(c_{ti}) T(\sum_i c_{ti}) - \left(\sum_i \rho_i T(c_{ti})\right) T'(\sum_i c_{ti})\right] \dot{c}_{ti} \tag{60}$$

$$= -T'(\sum_i c_{it}) \left[\left(\sum_i \rho_i^2 T_i T'_i\right) \frac{T(\sum_i c_{ti})}{T'(\sum c_{ti})} - (\sum_i \rho_i T_i)^2\right] \tag{61}$$

where we've used $\dot{c}_{ti} = -\rho_i T(c_{ti})$ in the third line. Now consider the factor in the square bracket, and assume T/T' is convex, so that (57) holds. Then to show that this factor is non-negative it is sufficient to show that

$$\left(\sum_i \rho_i^2 T_i T_i'\right)\left(\sum_j \frac{T_j}{T_j'}\right) - (\sum_i \rho_i T_i)^2 > 0$$
$$\iff \sum_i \sum_j \left(\rho_i^2 T_i T_j \frac{T_i'}{T_j'} - \rho_i \rho_j T_i T_j\right) > 0$$
$$\iff \sum_i \sum_j \rho_i \rho_j T_i T_j \left(\frac{\rho_i}{\rho_j}\frac{T_i'}{T_j'} - 1\right) > 0$$
$$\iff \sum_i \sum_{j>i} \rho_i \rho_j T_i T_j \left(\frac{\rho_i}{\rho_j}\frac{T_i'}{T_j'} + \frac{\rho_j}{\rho_i}\frac{T_j'}{T_i'} - 2\right) > 0$$

Again, the factors in brackets are of the form $x + x^{-1} - 2$, and are thus positive. Hence $\dot{\rho}^* < 0$ in the exhaustible resources case as well.
Finally, from (23) and (21), we have

$$\rho^* - \rho_{GZ} = (1 - X)(F'(S) - \rho_{GZ}) \tag{62}$$
$$= (1 - X)\frac{\sum_i \dot{c}_{ti}}{\sum_i T(c_{ti})} \tag{63}$$

Recall that $0 < X < 1$ when T is convex. If there is a non-trivial steady state, $\mathrm{sgn}(\sum_i \dot{c}_{ti}) = \mathrm{sgn}(S_\infty - S_0)$, and we conclude that $\rho^* \geq \rho_{GZ}$. For exhaustible resources $F'(S) = 0$ and we find that $\rho_{GZ} \geq \rho^*$.

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