Title

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May 28, 2020

Networks

- Pictures of social networks
- Pictures of neural networks
- Pictures of static and dynamic networks

Dynamic networks

Pictures of:

- Networks with slowly-changing communities & connecting probabilities
  - Include related literatures
- <u>Growing networks</u> in nervous system
  - However, not many literatures

Growing networks

- Picture of real growing neural network (Cell paper)
- Pictures of their results
  - Prior expectation: roles of nodes
- Weakness of their method

Outline

- Model growing networks via stochastic block model
- Inference procedure
- Simulation and application to real data

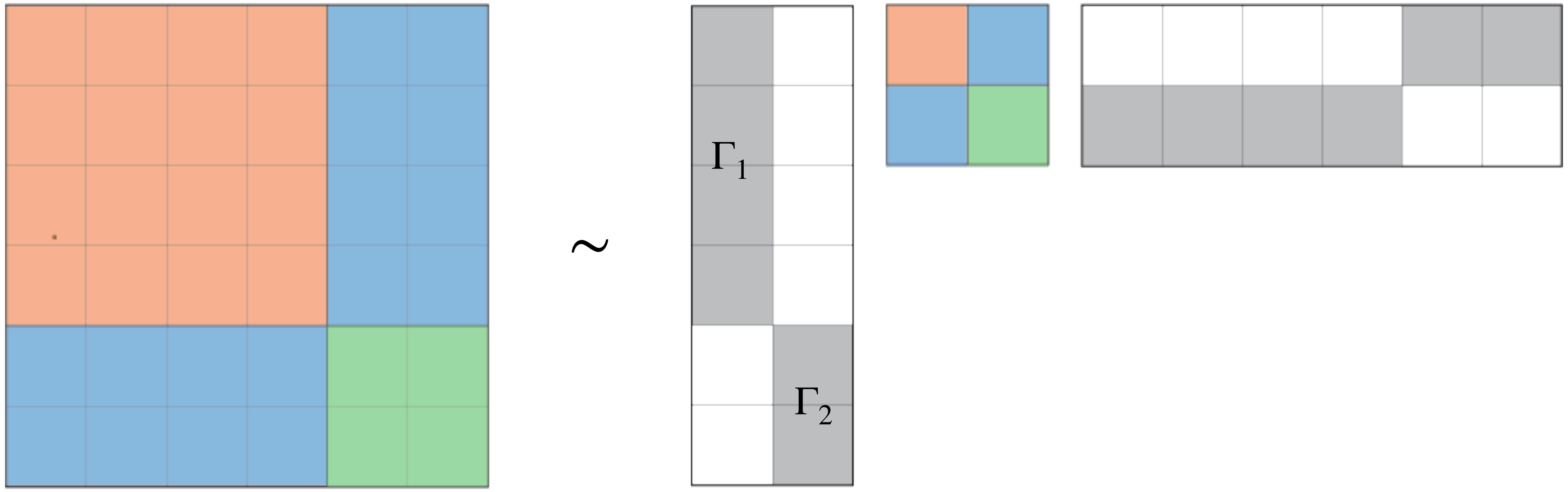
Modeling the growing networks

Introduce terms and notations through pictures:

- Observed neural activity: N_{i,j}
- Roles of nodes: z_i
- “Bridge” between multiple networks and common cell types — — connecting pattern: \lambda_{q,l}

Graphical representation of the model

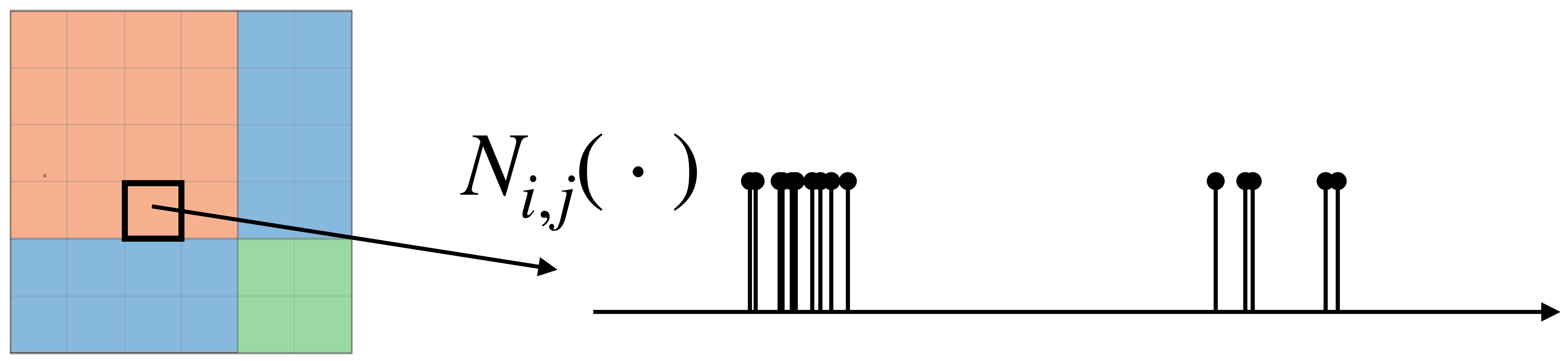
Observed $N_{i,j}(\cdot),\ i, j \in [n]$ **Membership** z_i **Connecting pattern** $\lambda_{q,l}(\cdot)$

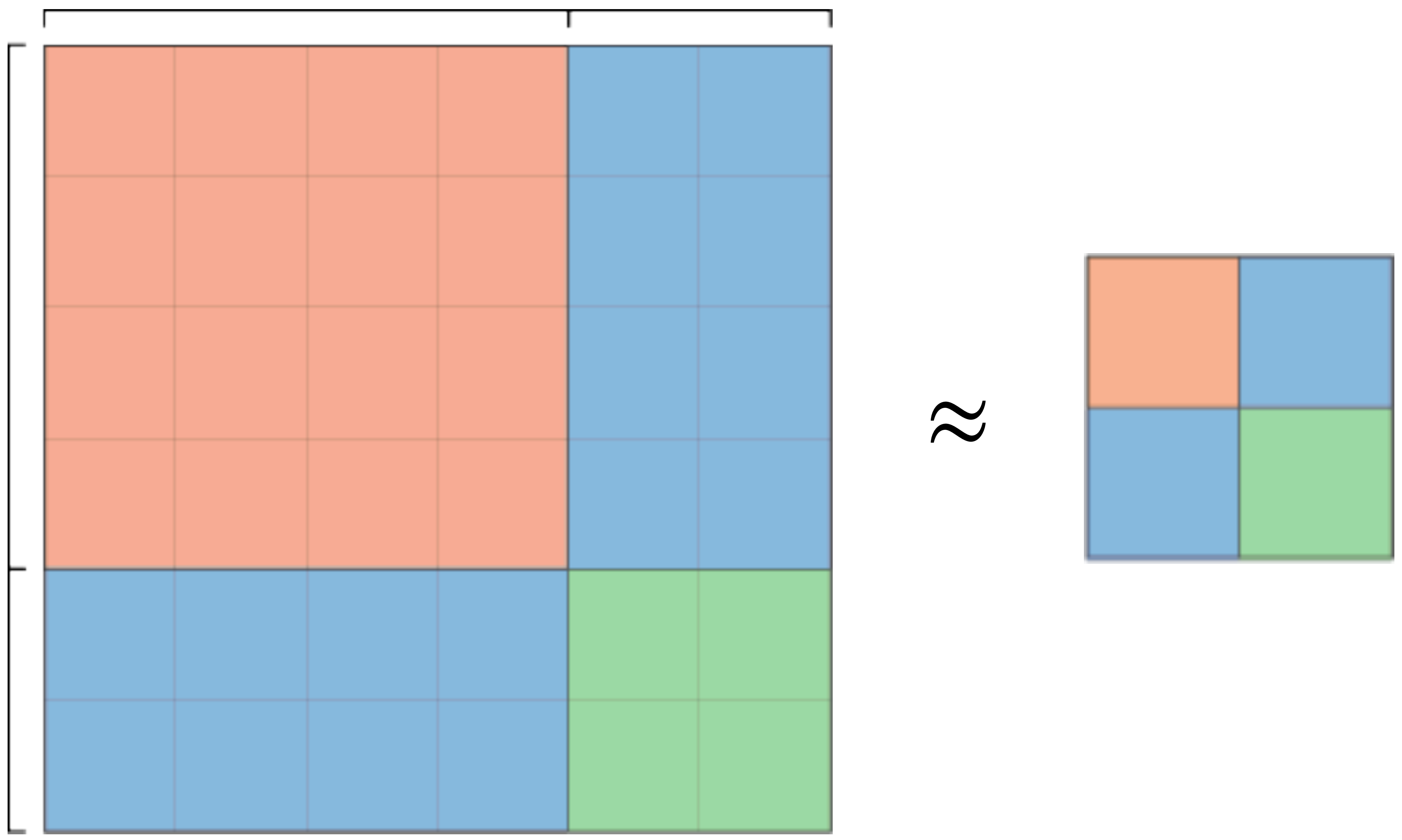


The conditional density of $N_{i,j}(\cdot)$ **given** $z_i = q$ **and** $z_j = l$ **is** $\lambda_{q,l}(\cdot)$

(Matias et al, 2018)

There is at most one event in each point process





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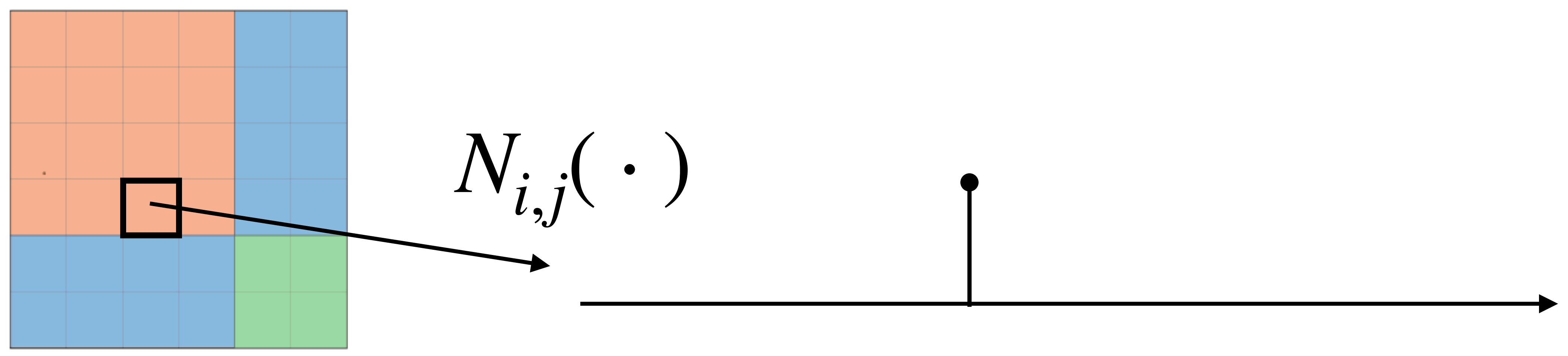

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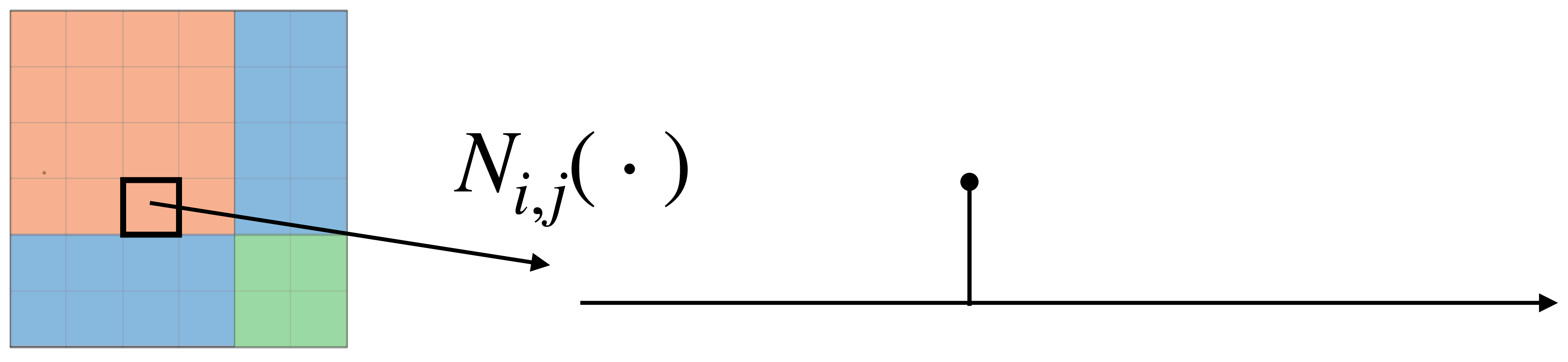
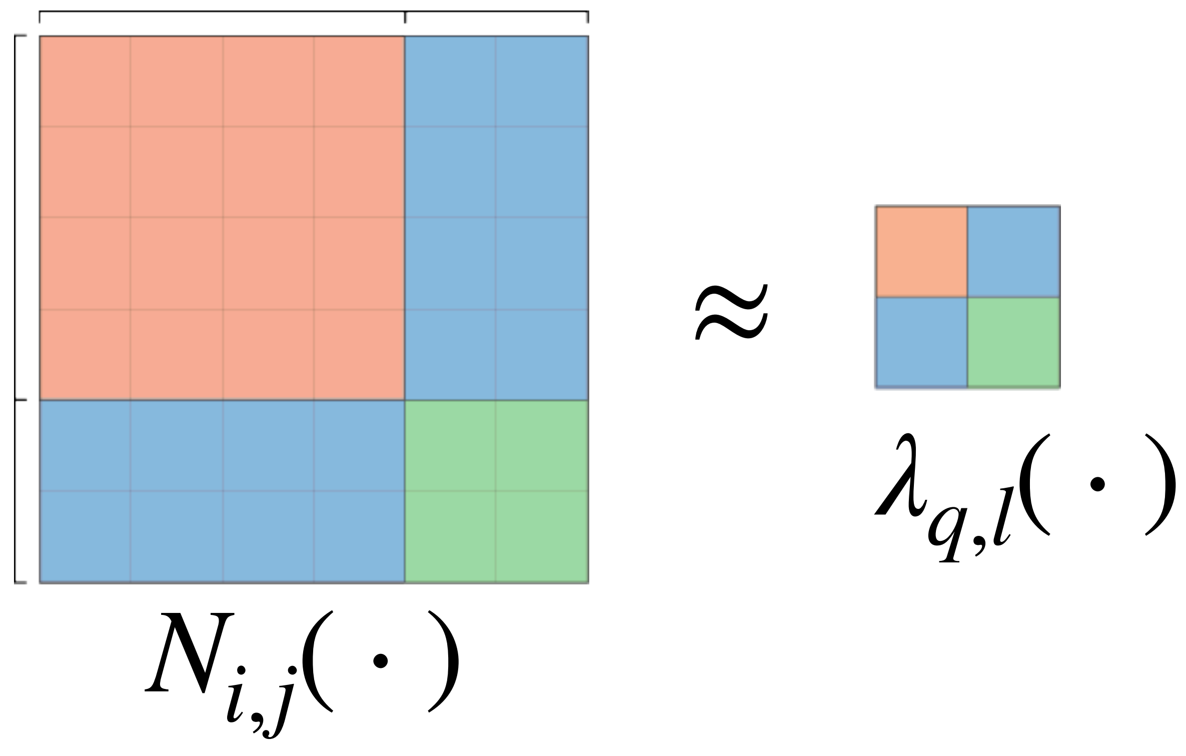
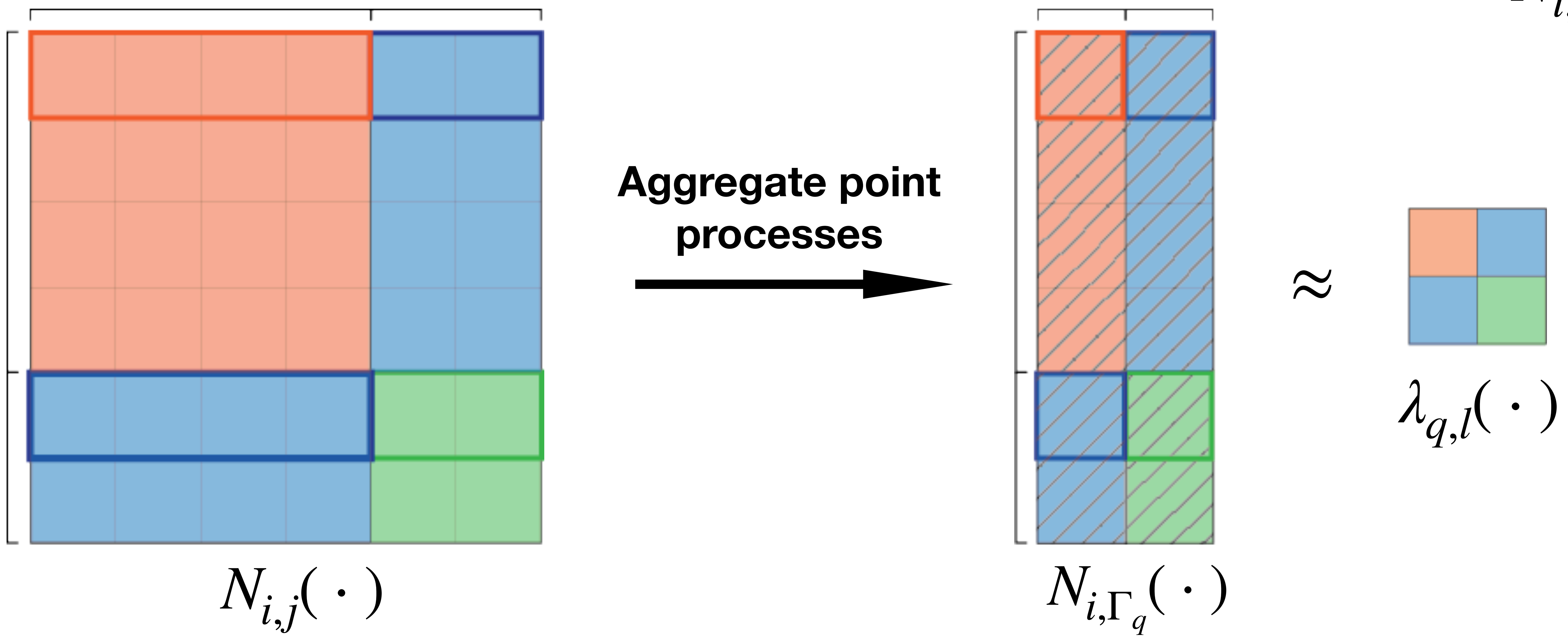


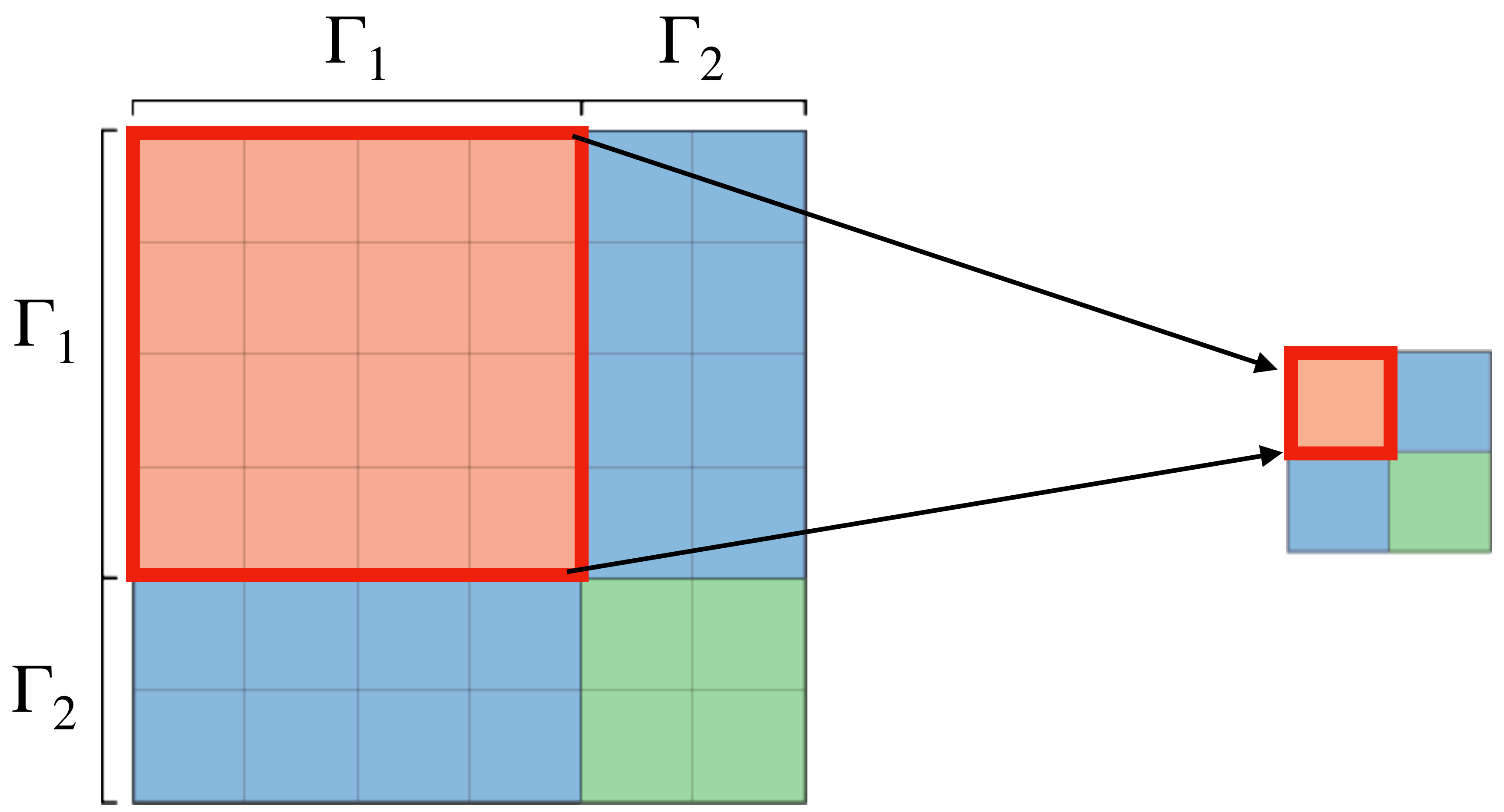




Inference

Approximate k-means method

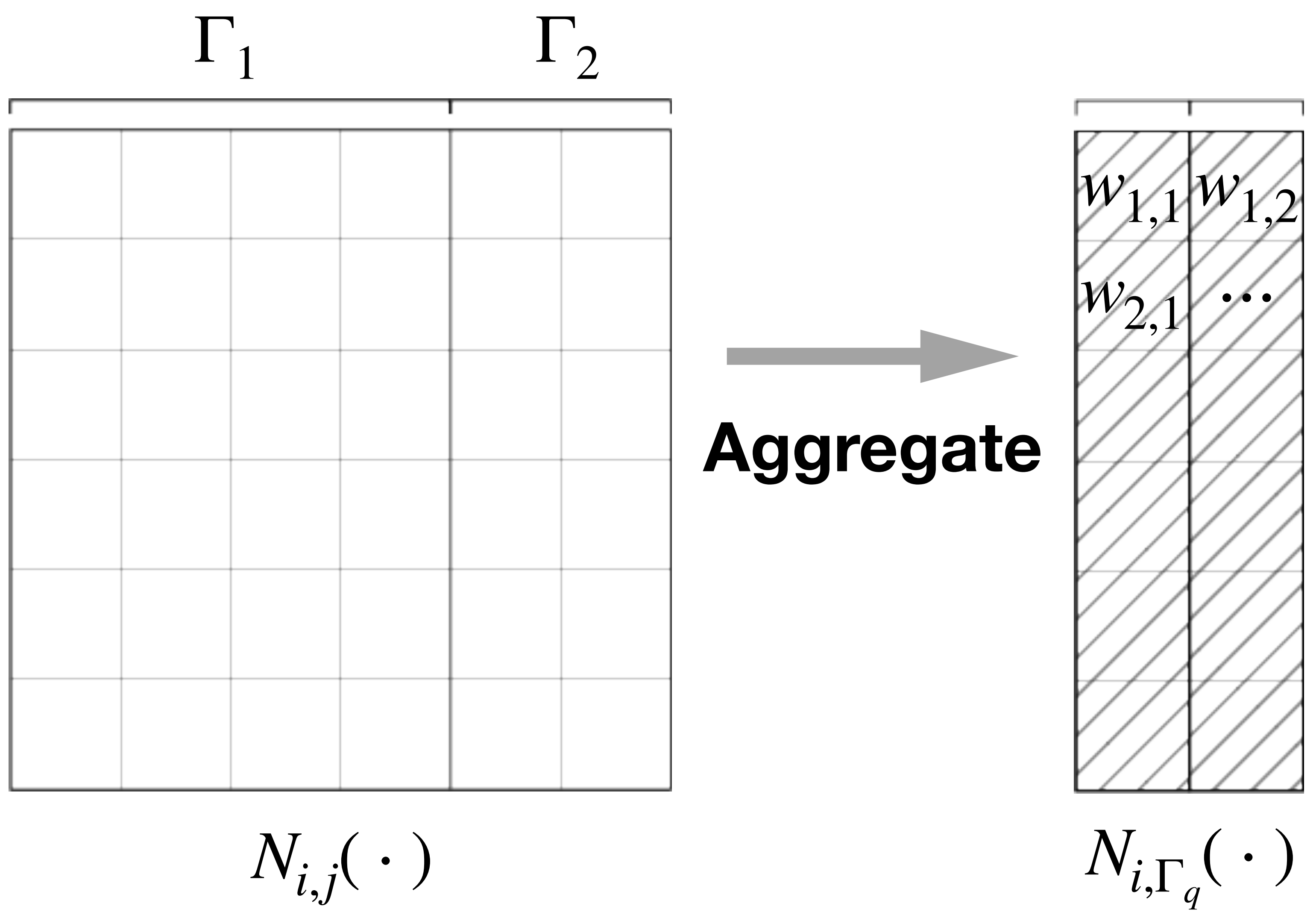
Re-center based on current clustering



Kernel density estimation

$$\hat{f}_h(x) = \frac{1}{nh}\sum_{i=1}^{n} K(\frac{x - x_i}{h})$$

Re-cluster based on estimated connecting patterns and current clustering

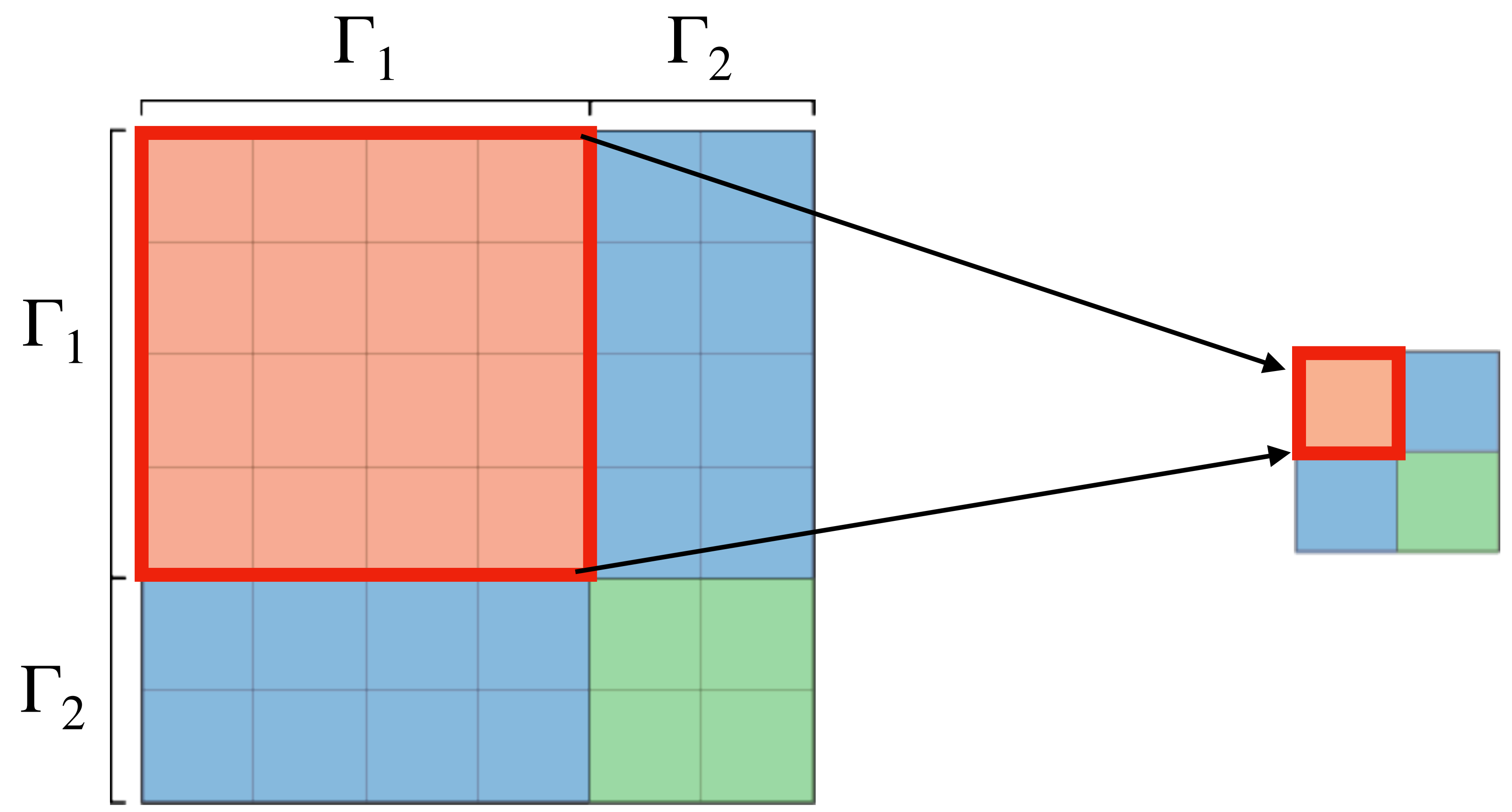


$$\sum_q w_{1,q} = 1$$

Inference

Approximate k-means method

Re-center based on current clustering



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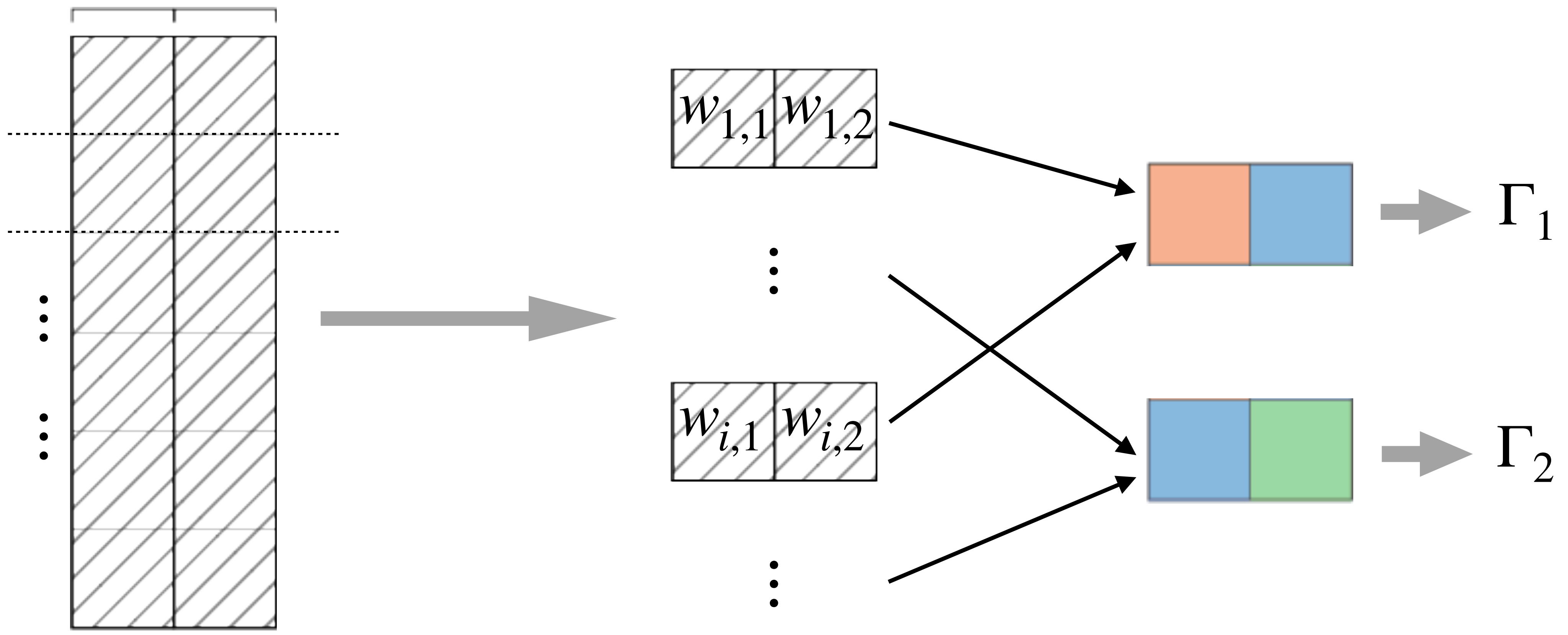
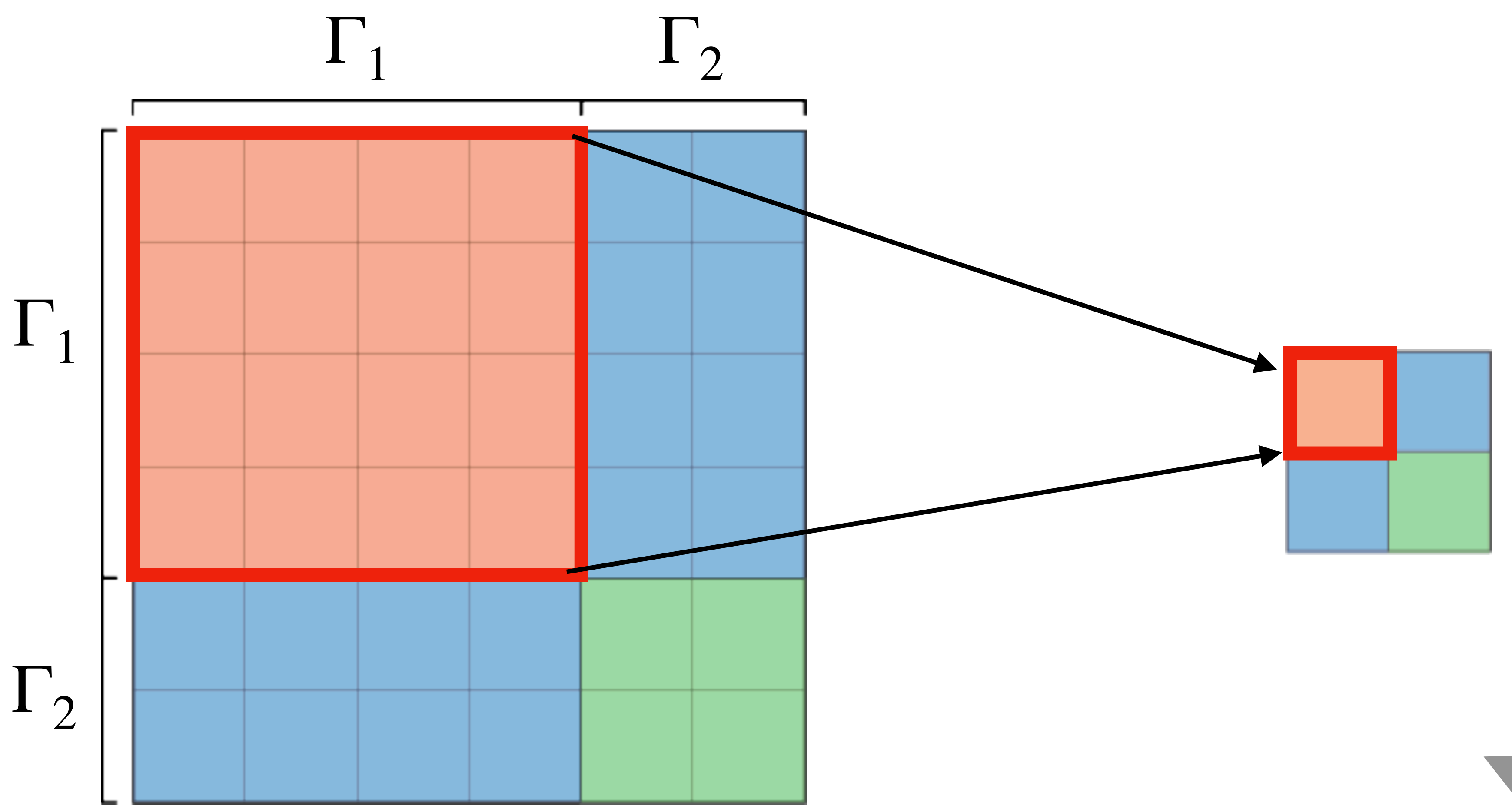


$$dist(i, \Gamma_q) = \sum_l w_{i,l} \cdot d(N_{i,\Gamma_l}, \lambda_{q,l}) = \sum_l w_{i,l} \cdot \|f_{N_{i,\Gamma_l}} - \lambda_{q,l}\|_2$$

Inference

Approximate k-means method

Re-center based on current clustering



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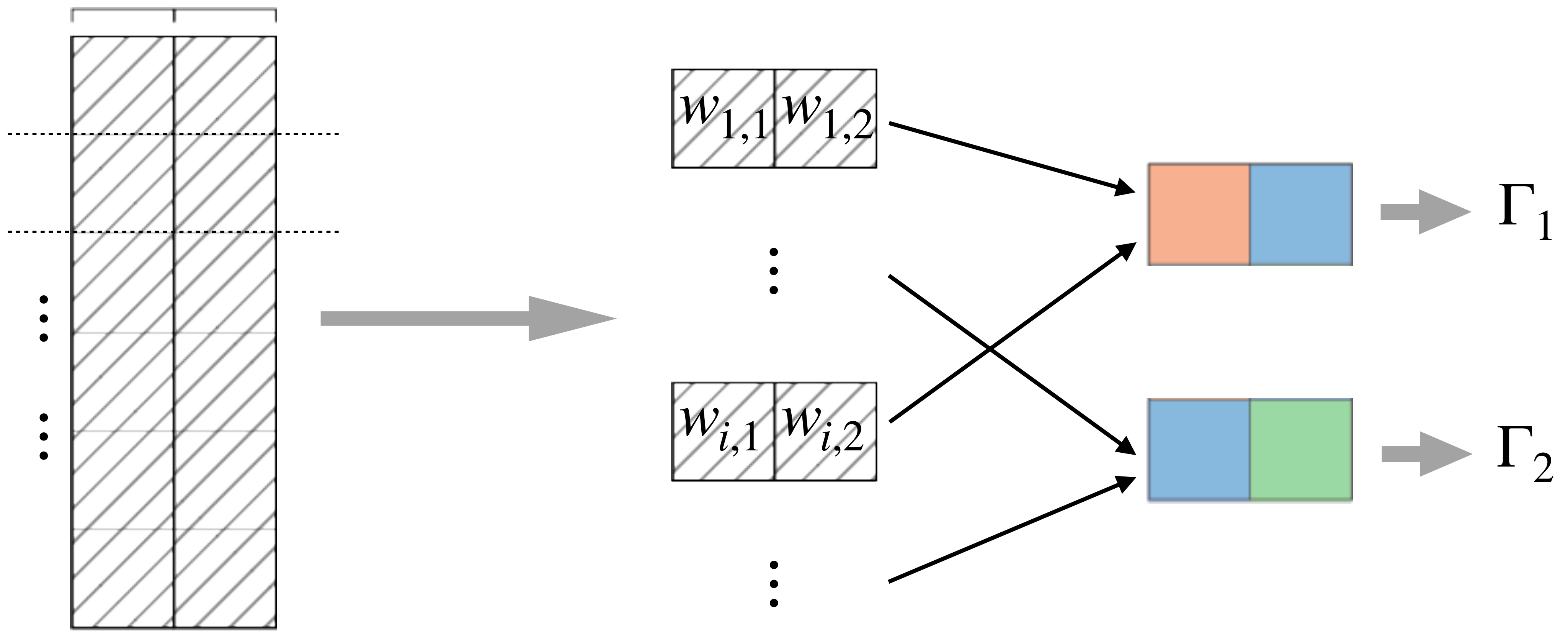

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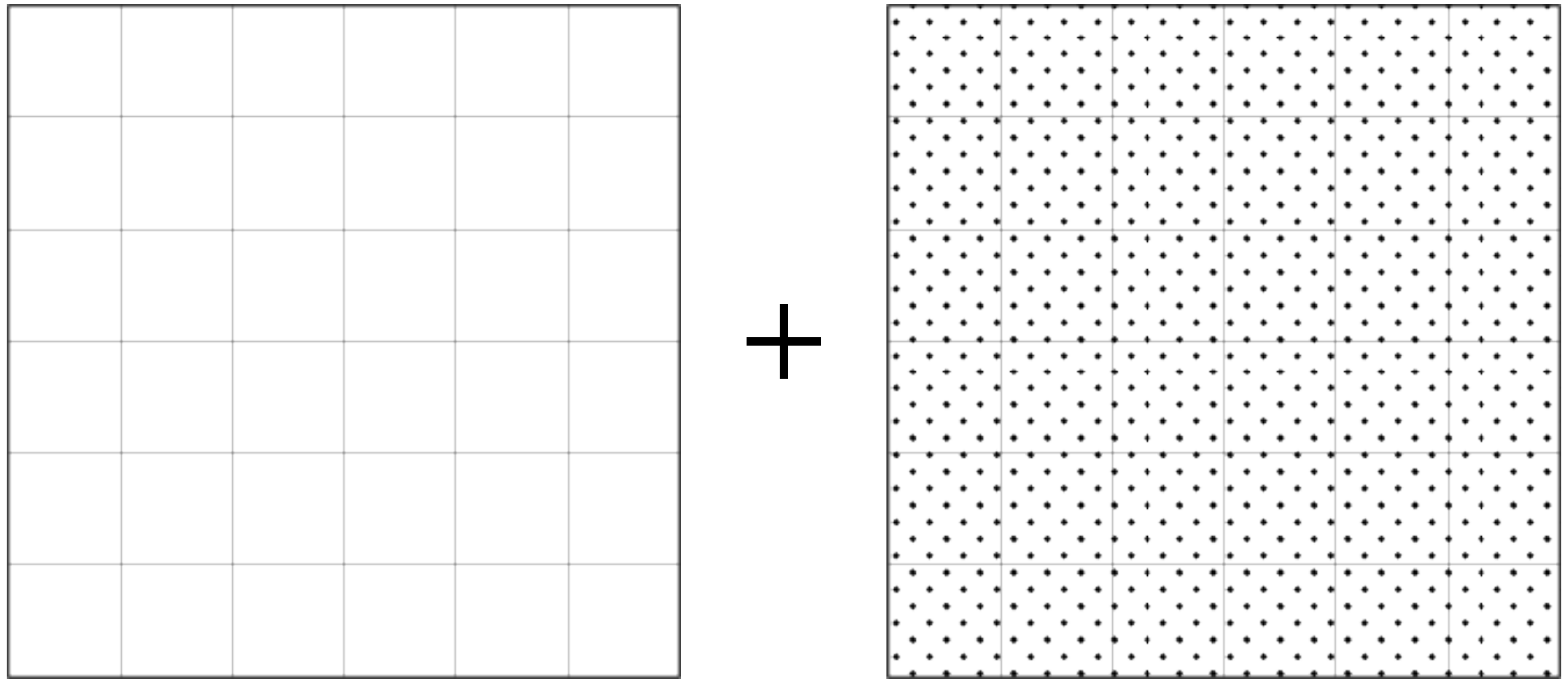
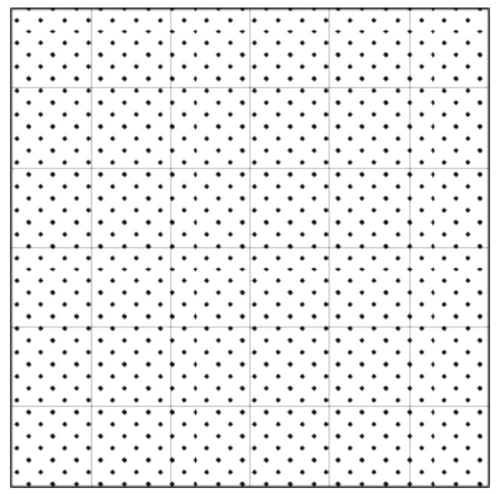
Time shifts associated with each edge

- Pictures from real data showing the existence of time shifts



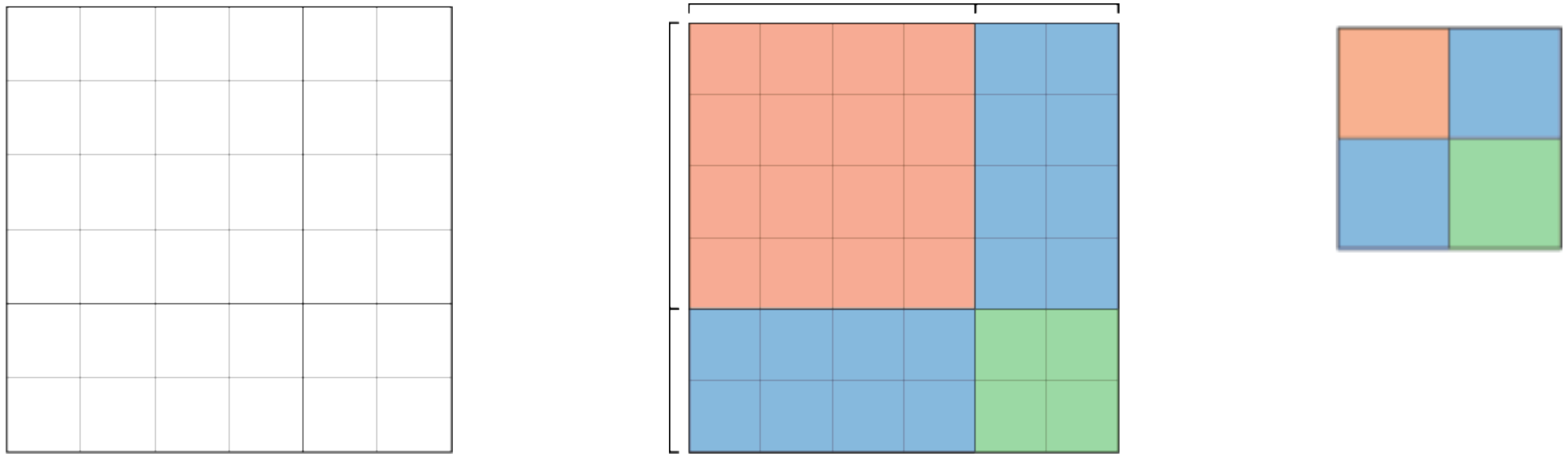
$$N_{i,j}(\,\cdot - \tau_{i,j}),\ i,j \in [n]$$

Problem



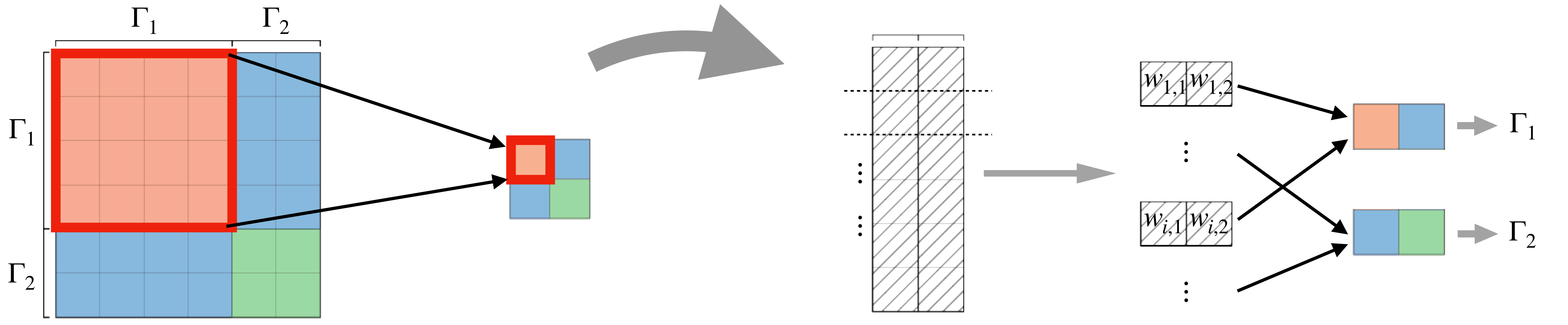
$$N_{i,j}(\cdot - \tau_{i,j}),\ i, j \in [n]$$

Align back **Find clusters** **Connecting patterns**



Incorporating time shifts

Given aligned point processes:

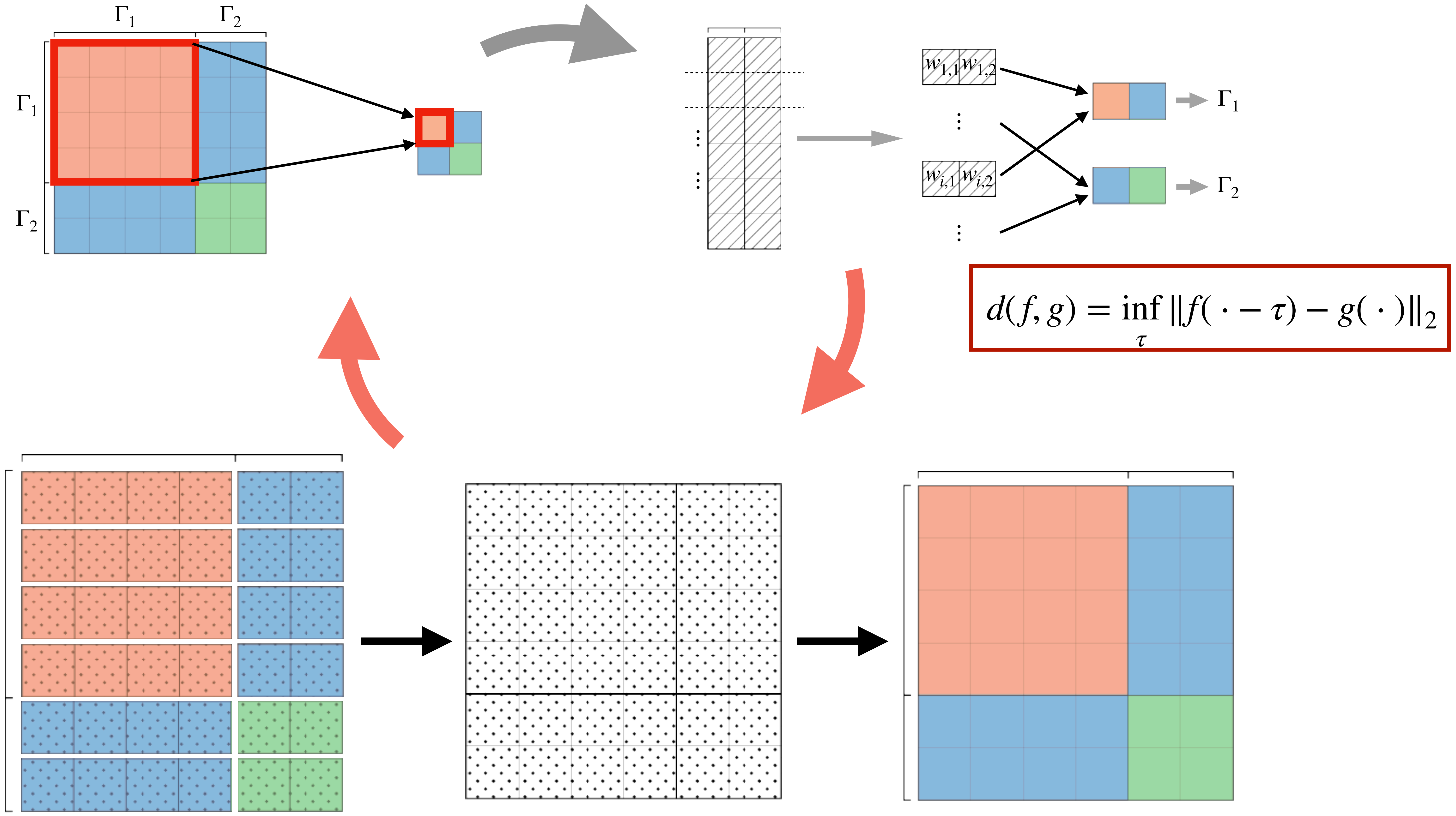


$$\boxed{d(f, g) = \inf_{\tau} \|f(\cdot - \tau) - g(\cdot)\|_2}$$

Incorporating time shifts

Given aligned point processes:

Re-align point processes based on current clustering



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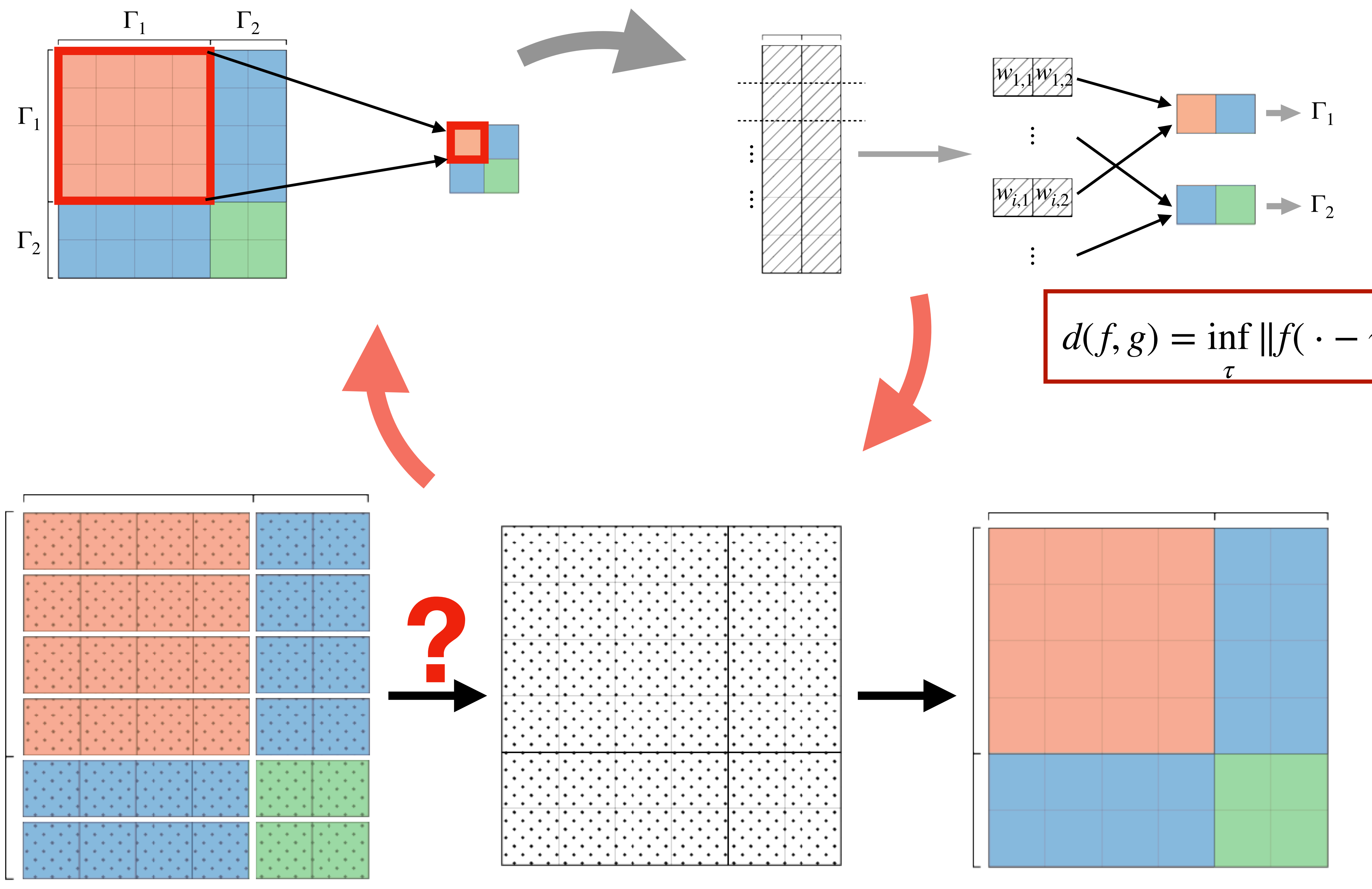
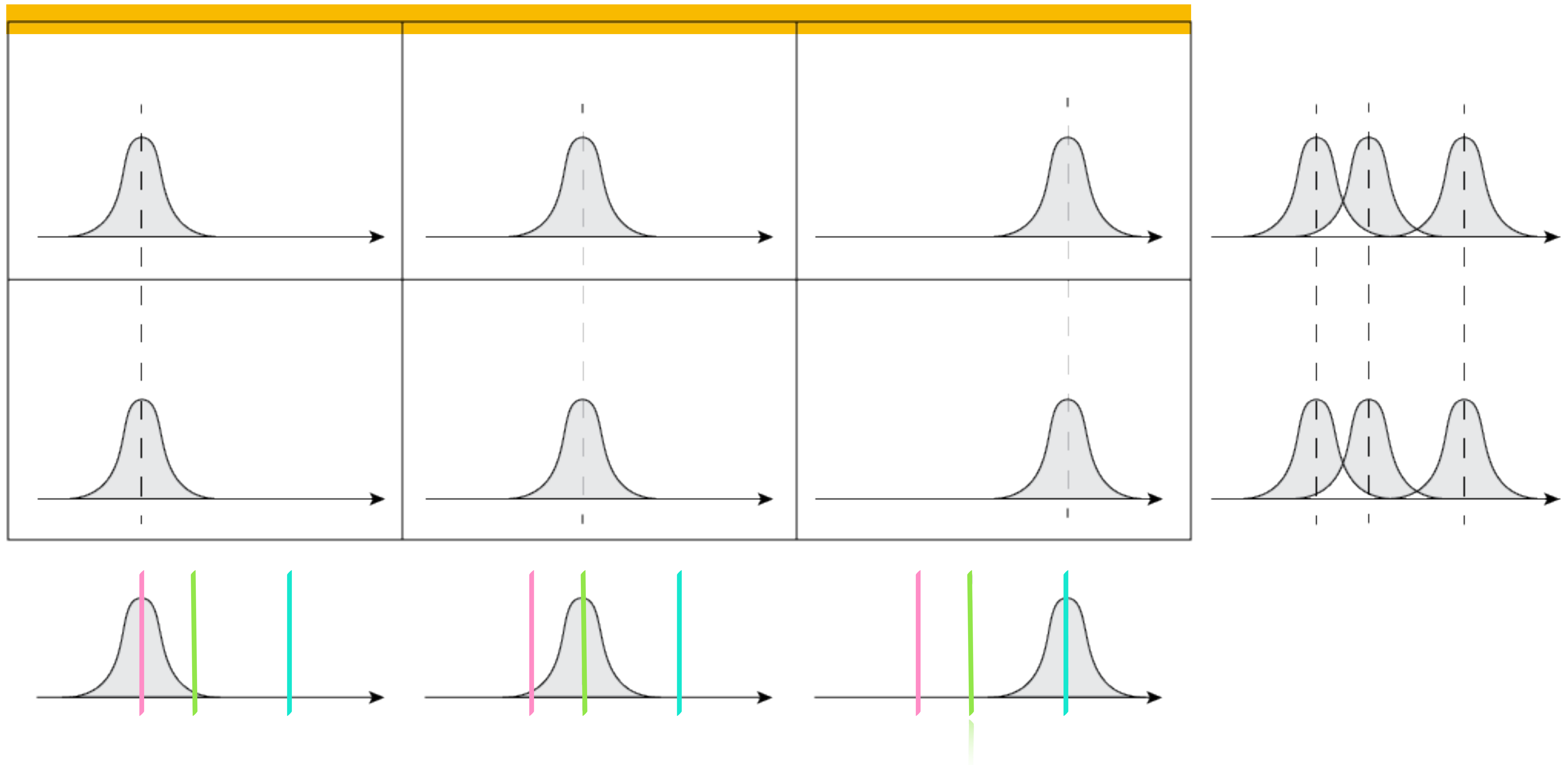


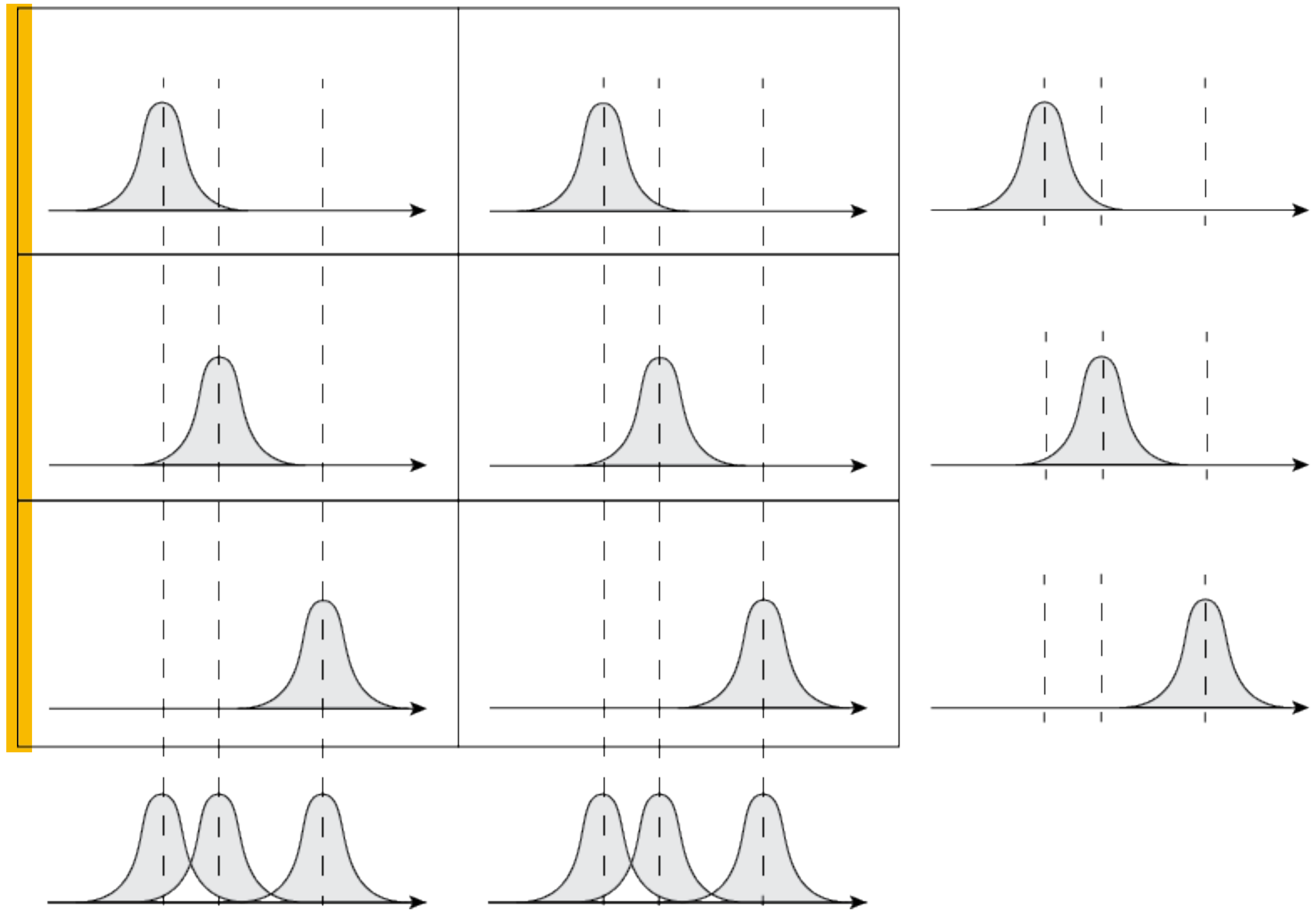
$$d(f, g) = \inf_{\tau} \|f(\cdot - \tau) - g(\cdot)\|_2$$

Assumption on time shifts

Align columns

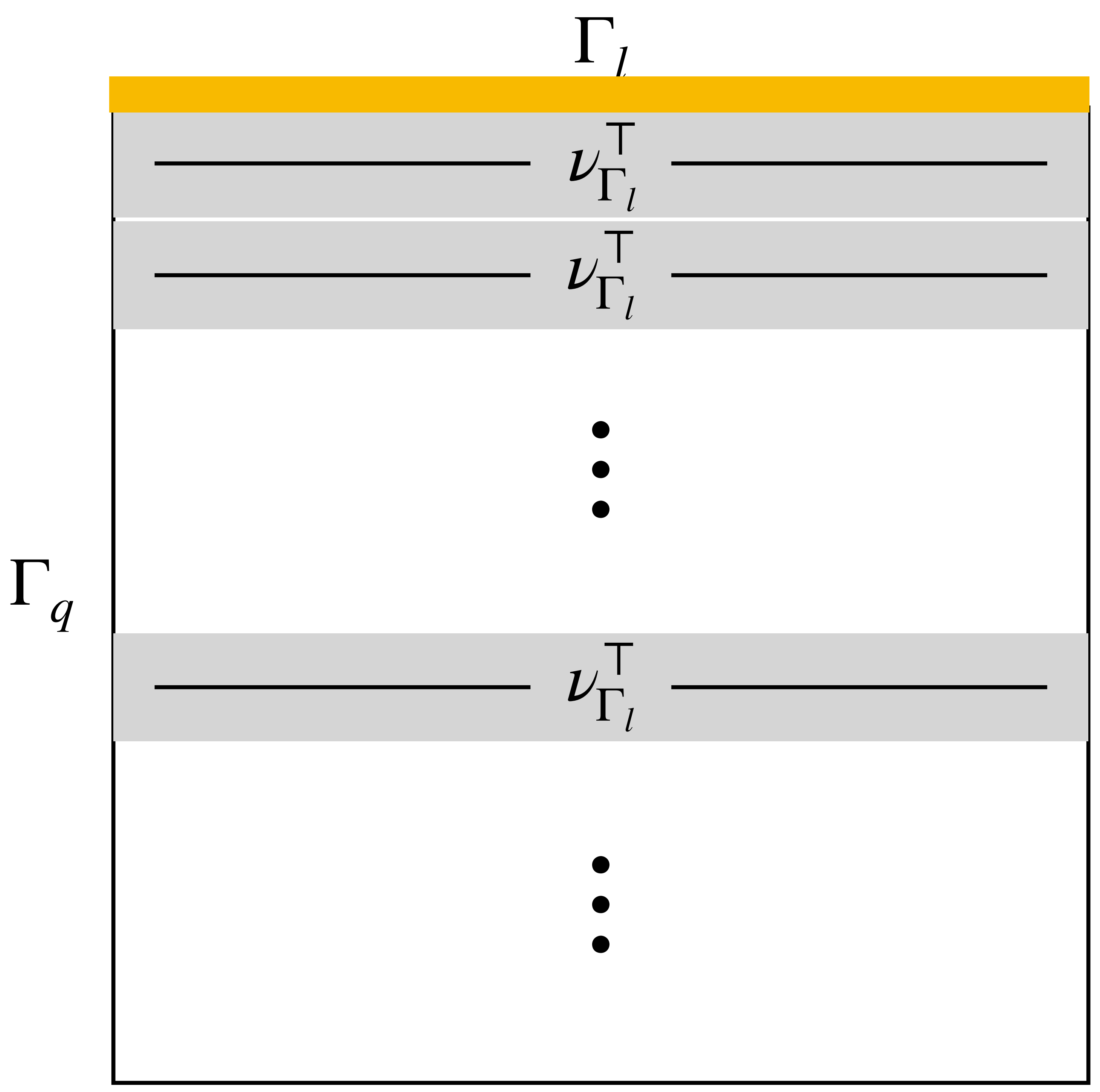


Align rows



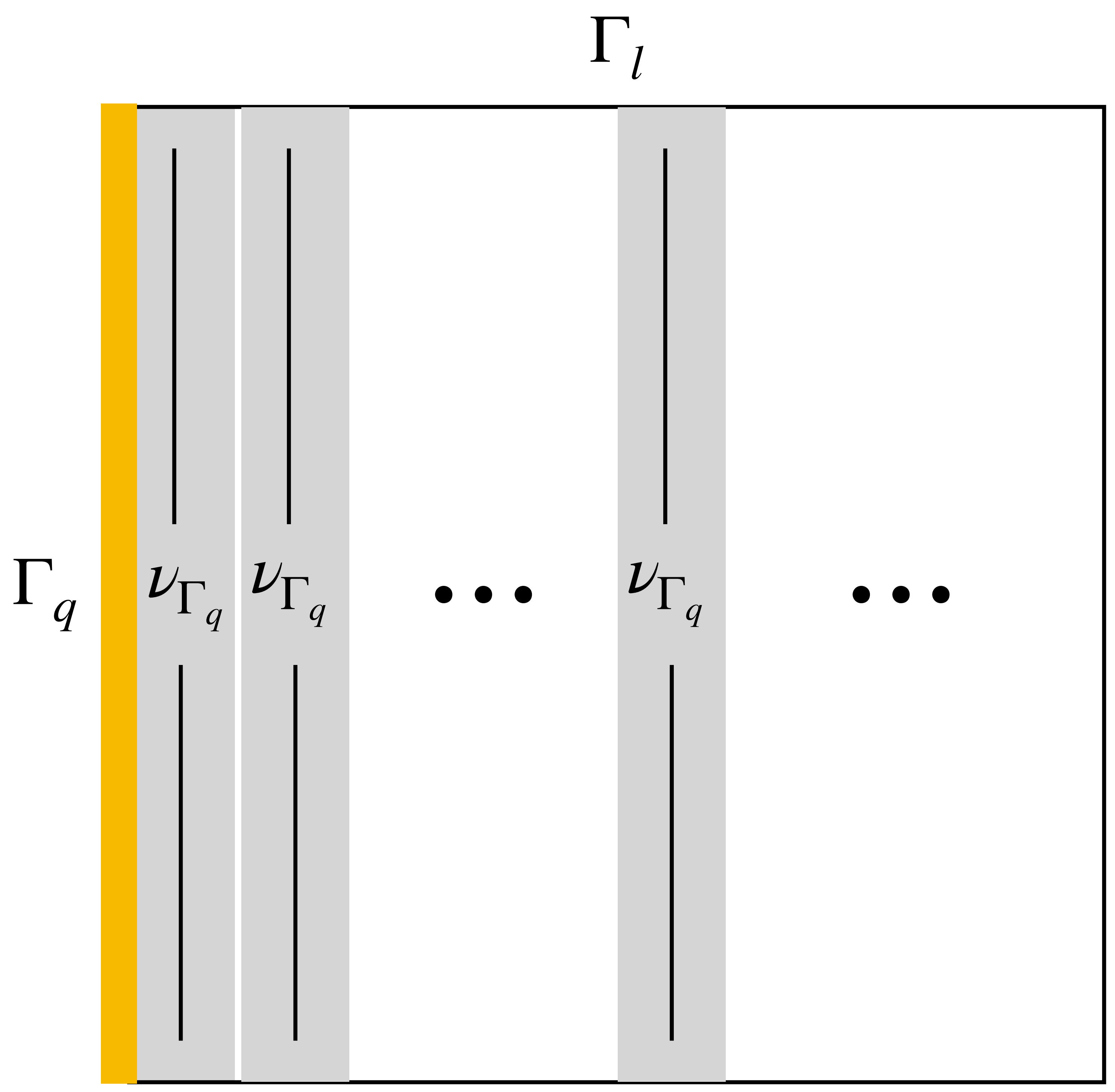
Assumption on time shifts

Align columns



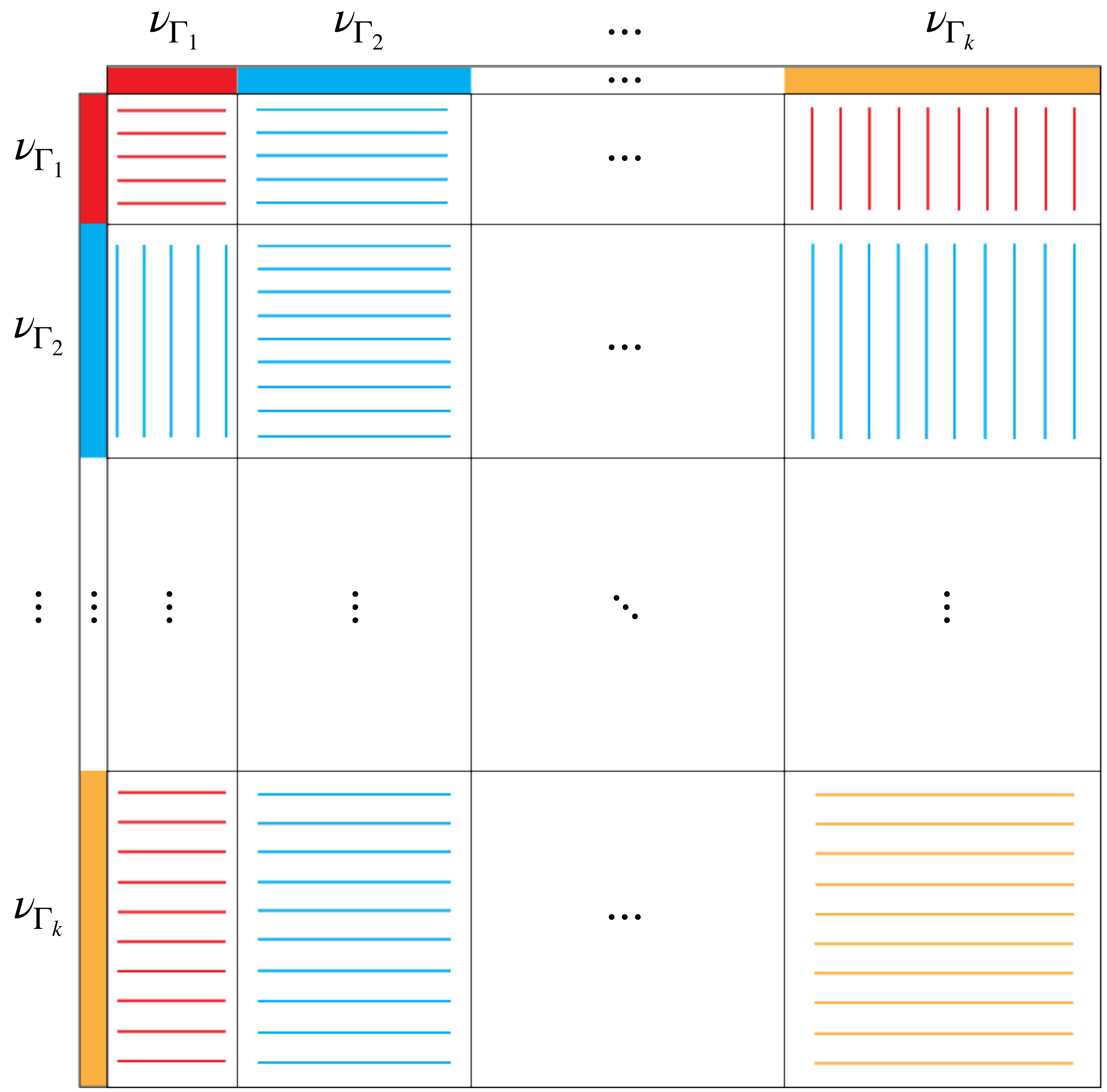
$$\tau_{i,\cdot} - \tau_{j,\cdot} = 0$$

Align rows



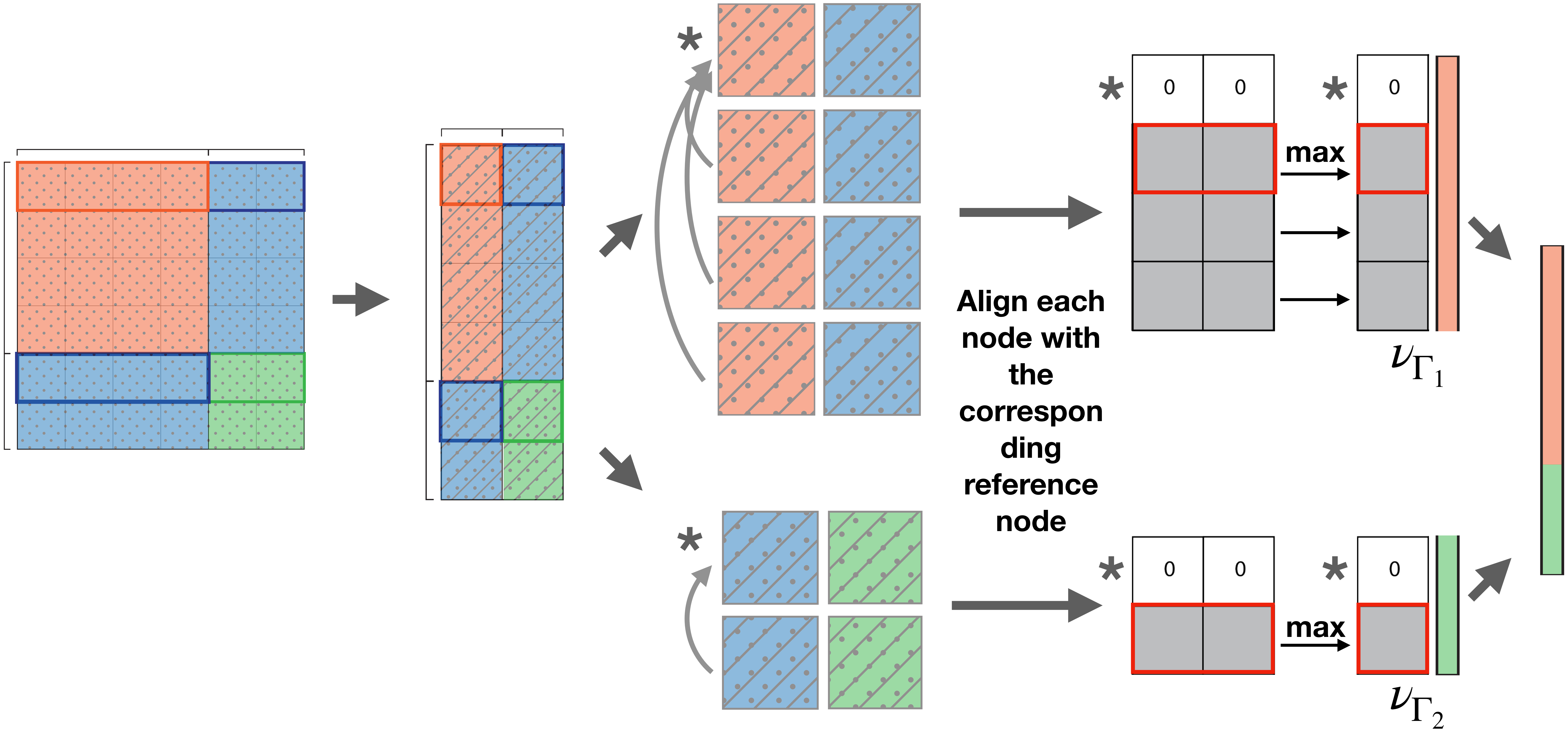
$$\tau_{i,\cdot} - \tau_{j,\cdot} = \nu_i - \nu_j$$

Assumption on time shifts

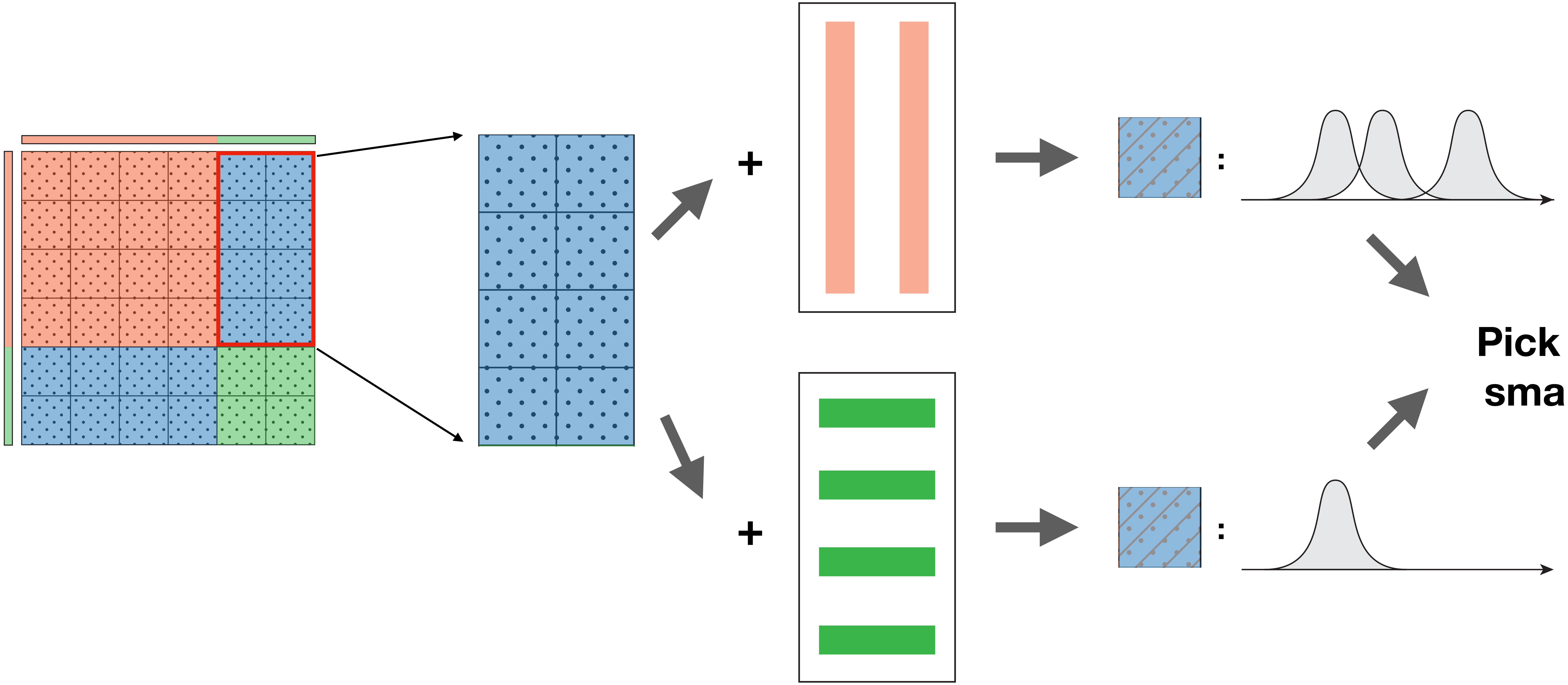


Matrix of time shifts

Estimate time shifts

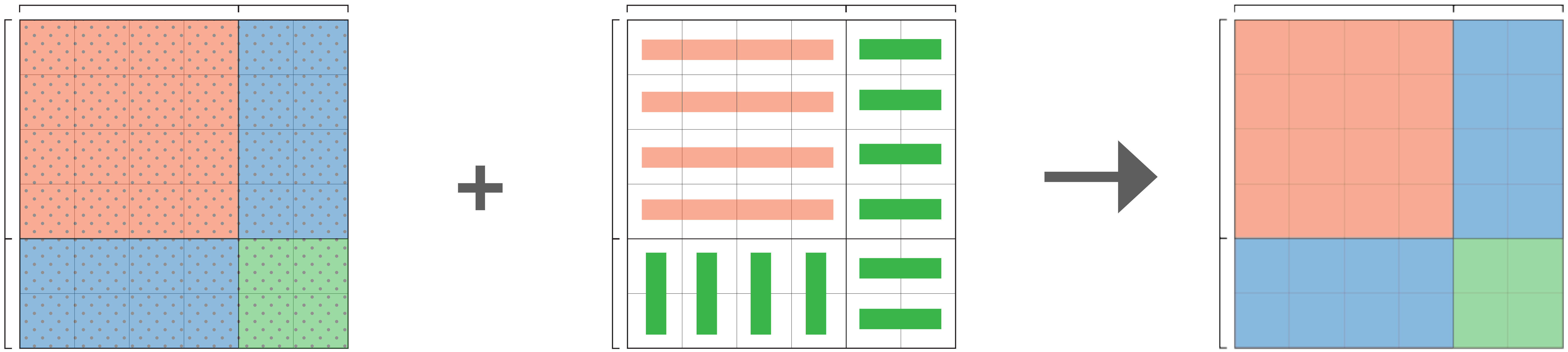


Estimate time shifts

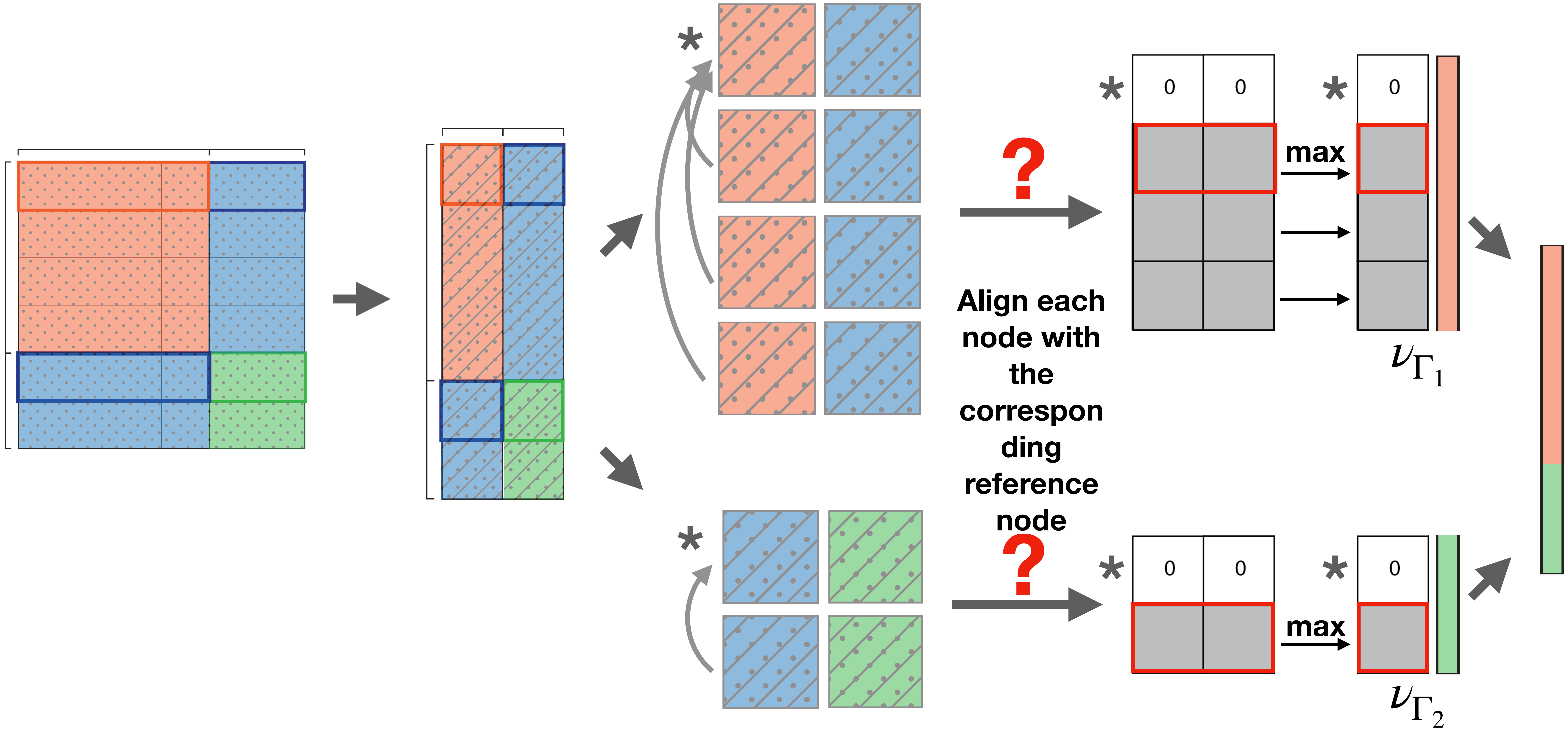


Pick the one with smaller variance

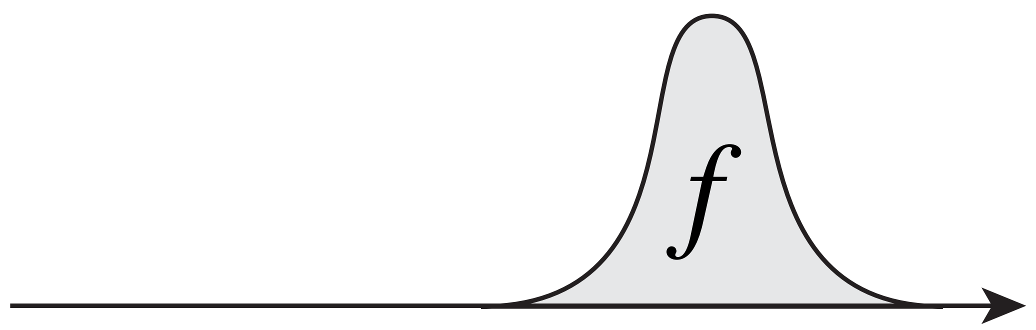
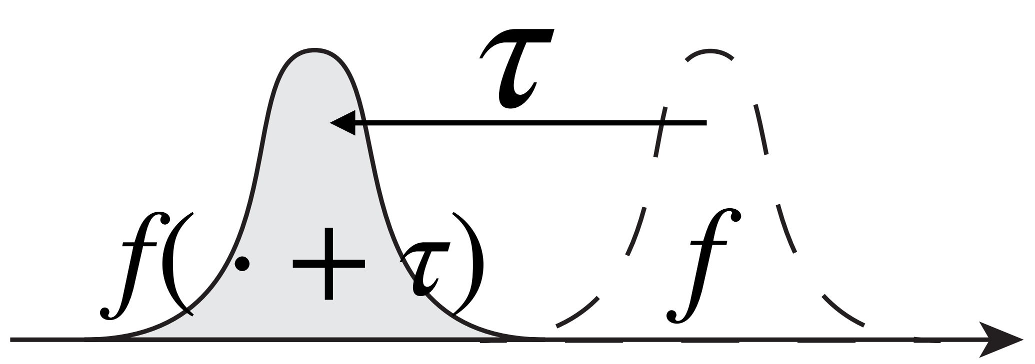
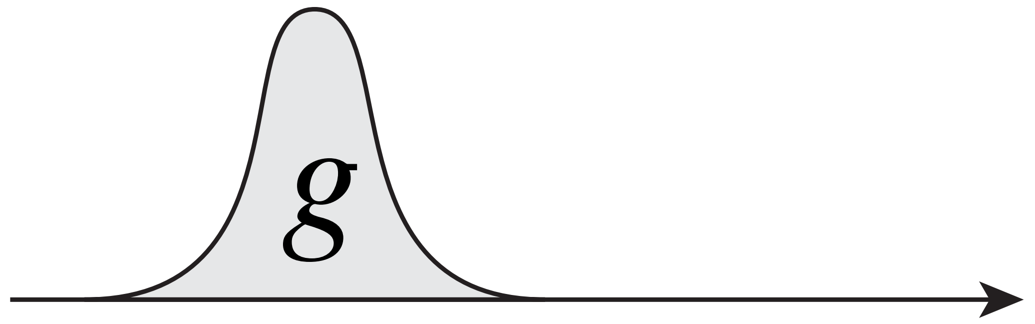
Re-align point processes



Estimate time shifts



Aligning curves and evaluating their distance

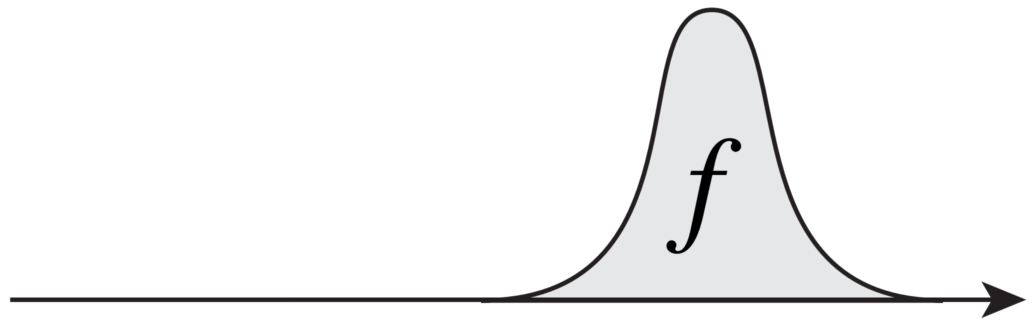
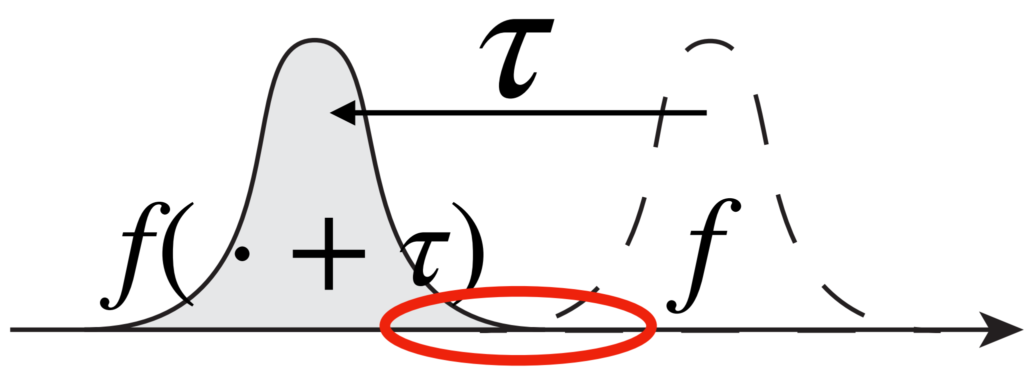
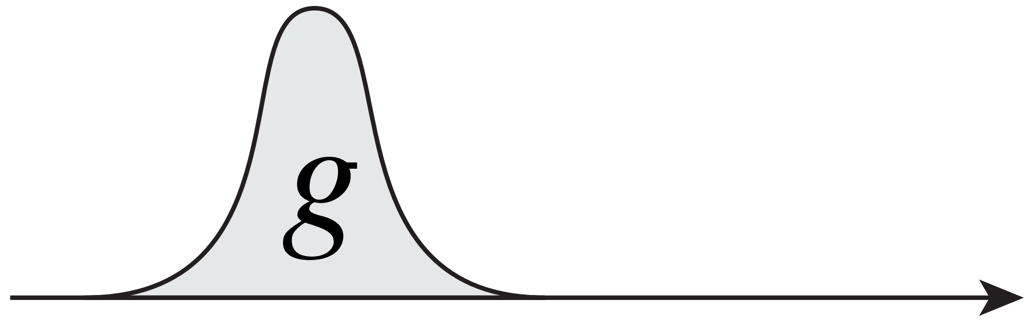
Curves	Fourier coefficients
	$\theta_j \quad {\scriptstyle j=-\frac{N-1}{2},\dots,\frac{N-1}{2}}$
	$\theta'_j = \theta_j e^{i2\pi j(n_0/N)} \quad {\scriptstyle n_0=\frac{\tau}{T}N}$
	$\gamma_j \quad {\scriptstyle j=-\frac{N-1}{2},\dots,\frac{N-1}{2}}$

$$d(f,g) = \min_{\tau} \|f-g\|_2$$

$$= \min_{n_0} \left[\frac{T}{N^2} \boxed{\sum_j |\theta'_j - \gamma_j|^2} \right]^{1/2}$$

Apply gradient descent algorithm

Aligning curves and evaluating their distance

Curves	Fourier coefficients
	$\theta_j \quad {}_{j=-\frac{N-1}{2},\dots,\frac{N-1}{2}}$
	$\theta'_j = \theta_j e^{i2\pi j(n_0/N)} \quad n_0 = \frac{\tau}{T}N$
	$\gamma_j \quad {}_{j=-\frac{N-1}{2},\dots,\frac{N-1}{2}}$

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$$= \min_{n_0} \left[\frac{T}{N^2} \boxed{\sum_j |\theta'_j - \gamma_j|^2} \right]^{1/2}$$

Apply gradient descent algorithm

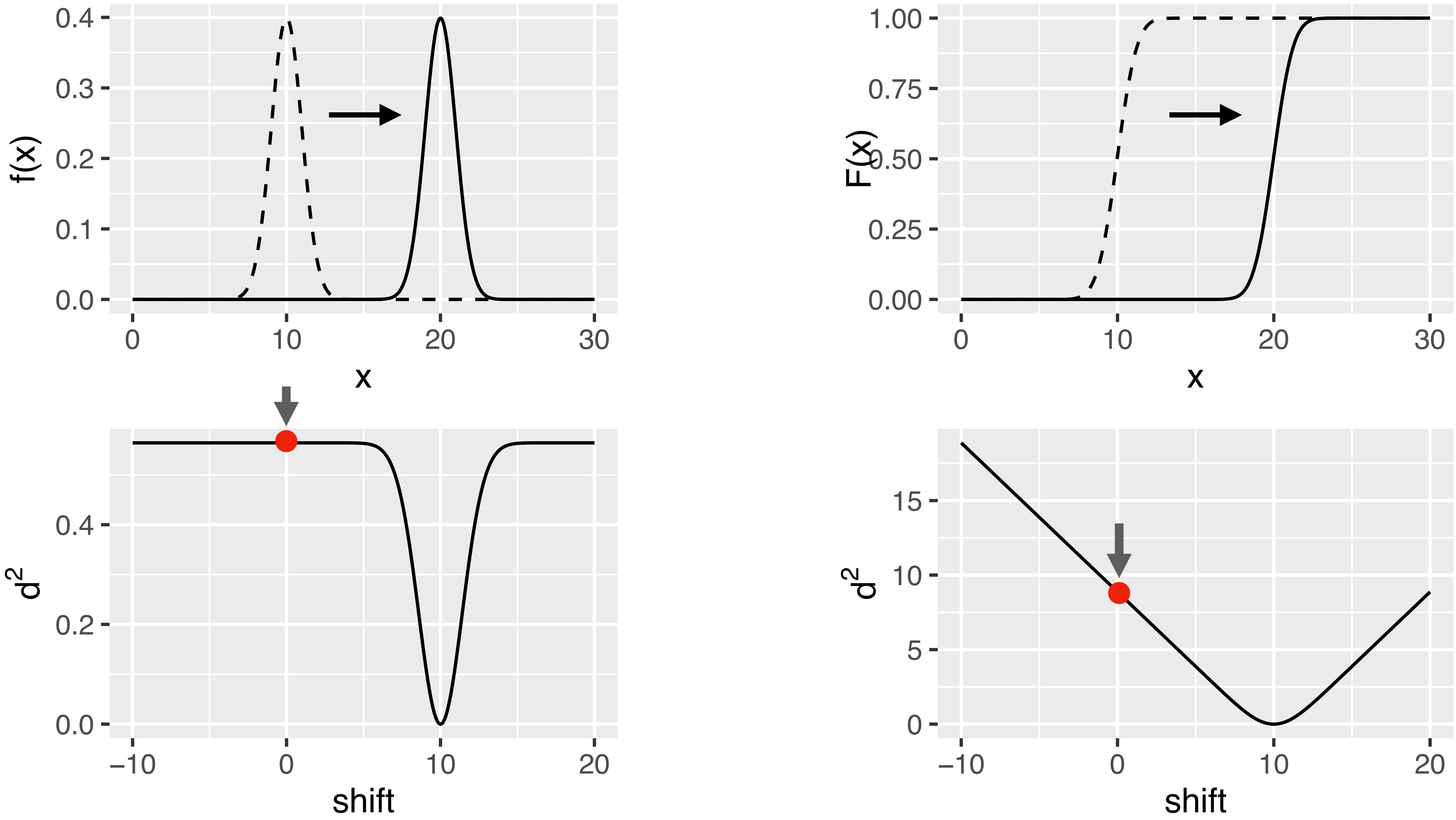
Good initialization?

Initializing gradient descent by aligning c.d.f.

Align p.d.f. | **Align c.d.f.**

Curves

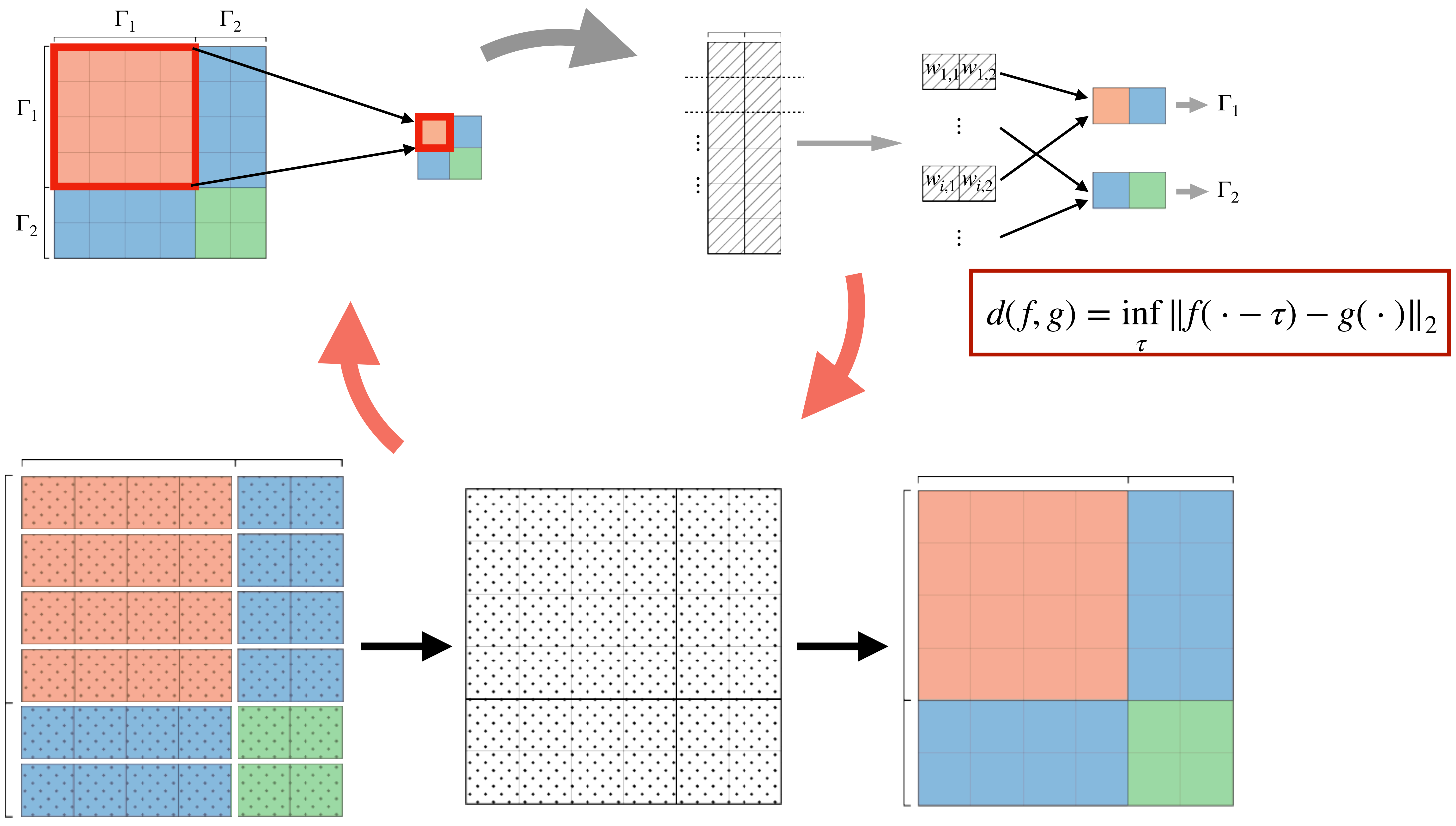
Loss function



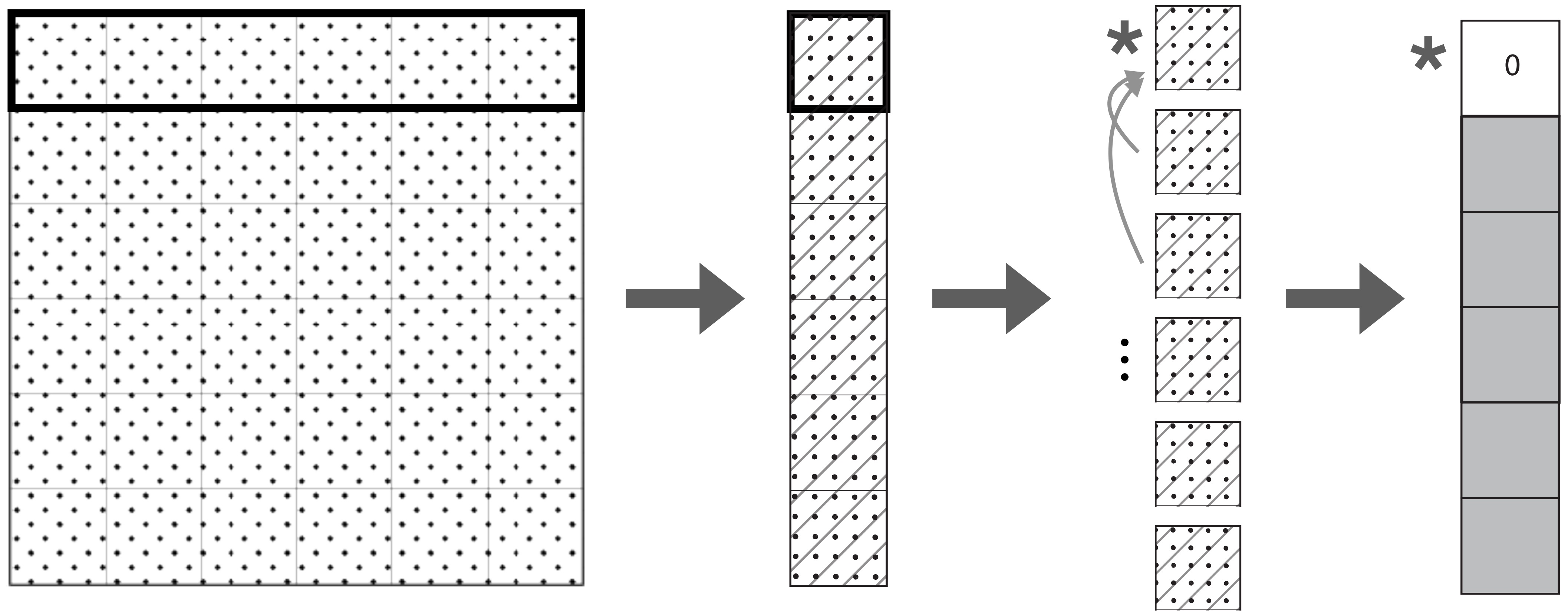
Algorithm

Given aligned point processes:

Re-align point processes based on current clustering



Initialization of clustering and time shifts



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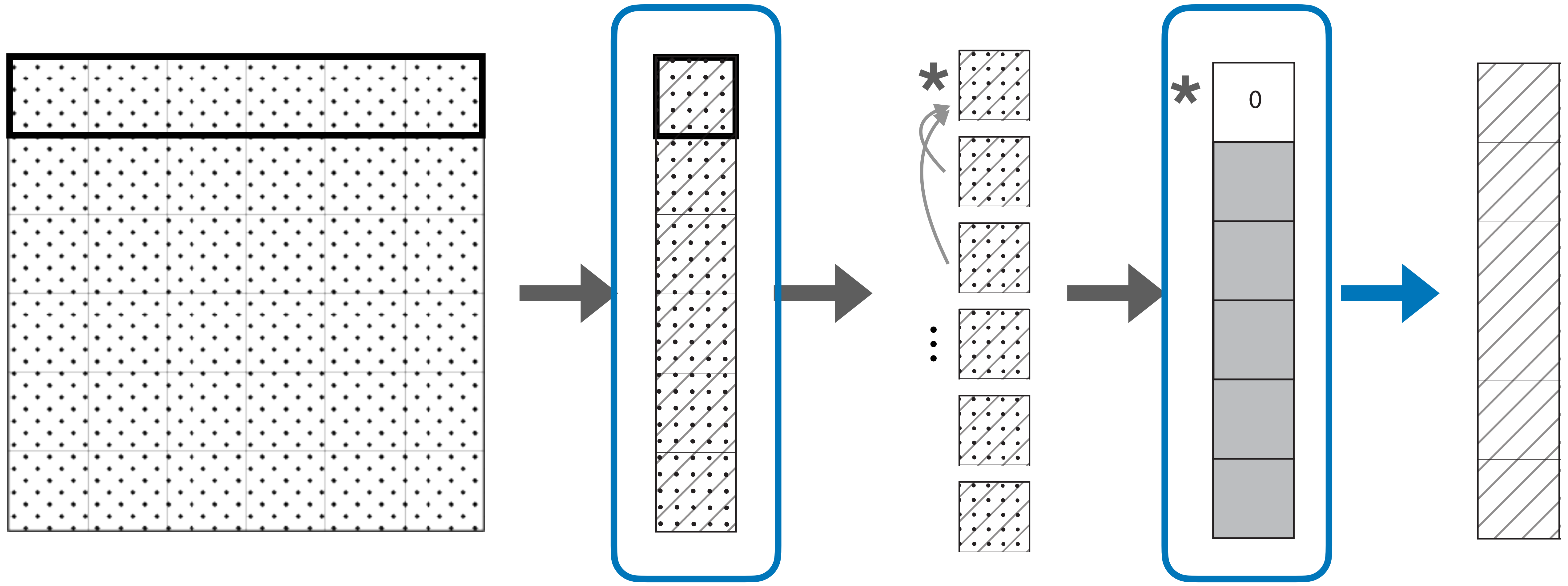
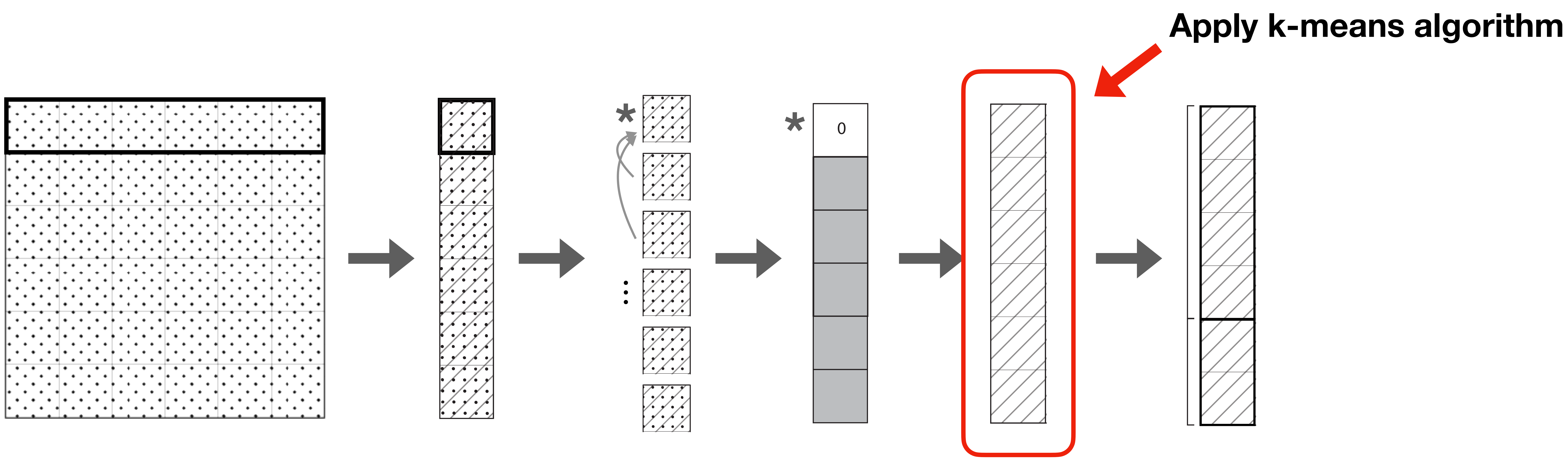


Initialization of clustering and time shifts



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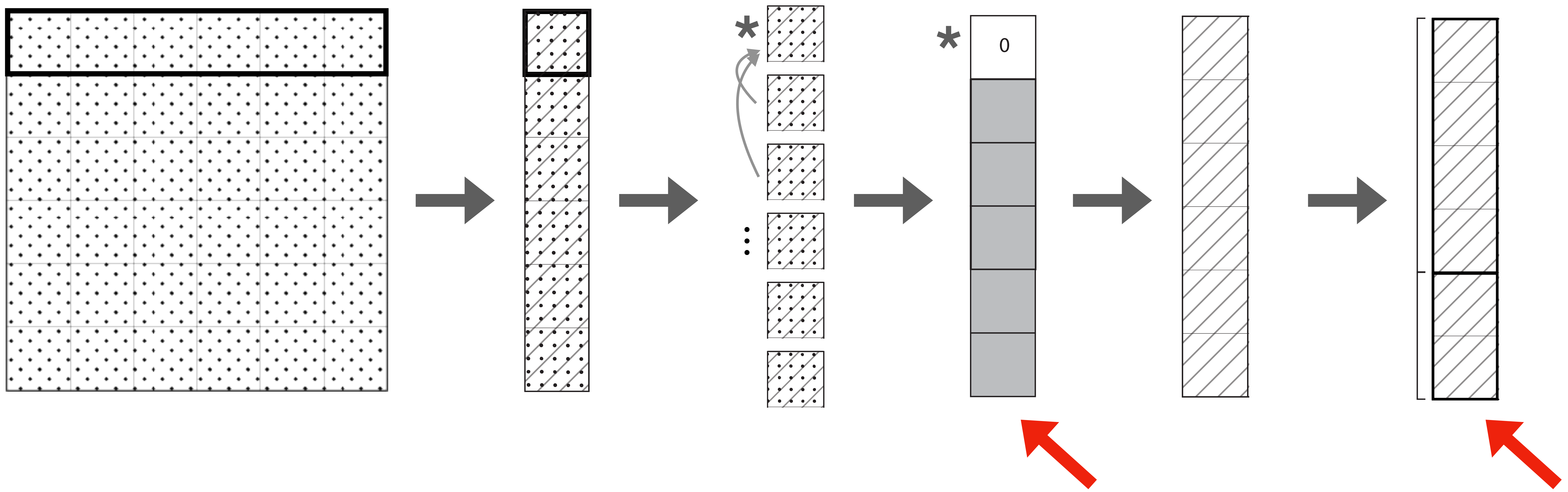

Simulation

Over-clustering can help to find the correct cluster numbers

Real data

Future work

Thank you

Appendix

- Derivation of aligning pdf and cdf
- Details about simulation set up, tuning parameters in algorithm
- Details about dealing with negative event time