Growing networks: develop from a bunch of neurons to a well-connected network

There was no data

But we have found a great dataset.

There is no existing model

Thanks to the observation in the science paper, we know the neurons have different "cell types" --- they play different "roles" in the development of the network. So we try to model the "cell types".

The "cell types" are not well-defined

Define the "cell types" by classifying the neurons according to their connecting patterns, where the connecting pattern of node i is defined as the point processes of connection between node i and each of other nodes, i.e. $N_{i,1}, ... , N_{i,n}$.

The naive clustering based on the connecting patterns is not flexible enough when we want to combine multiple networks in different individuals, because they are determined by all the specific neurons. (The reasons for combining multiple networks include (i) for each individual there is only one sample, which is not enough for estimating the patterns (ii) we are interested in the common connecting patterns among individuals.)

The neurons should be clustered based on their connecting pattern with each cell type rather than each single neuron.
So the common connecting pattern of node i should be defined as the point processes between node i and nodes from each cell type.

The model should be able to identify the cell types as well as the typical connecting patterns simultaneously

Introduce the stochastic block model.

> ❓ So introducing SBM is actually providing potential extension for the model to be able to combine multiple networks?

**In practice, the connections are constrained by spatial location and time delay of neurons.
Also, there is at most one event in each point process.**

Introduce assumptions:
1. For spatial location, assume the neurons from the same cluster have similar constitution of neighborhood.
2. For time delay, assume $\tau_{ij} = \tau_i$.

For the small number of events, integrate the point processes.
According to the previous two assumptions, the integrated point processes of the same type neurons are identically distributed.

> ❓ How to justify the assumption for time delay?

Contribution

Propose a model for the growing network based on the stochastic block model.

Propose the least square estimator for the problem and apply the k-means algorithm equipped with the shape invariant method in order to solve for the estimator.

Show the theoretical property of the estimator.