Turboelectric Propulsion
Worksheet CW1

Abhinandan Thour
32453515
Meng Mechanical & Aerospace Engineering
University of Southampton

2023-2024

The coursework commences with the definition of the provided data for the aircraft, resembling the configuration of STARC-ABL as elucidated by Welstead and Felder in their 2016 study.

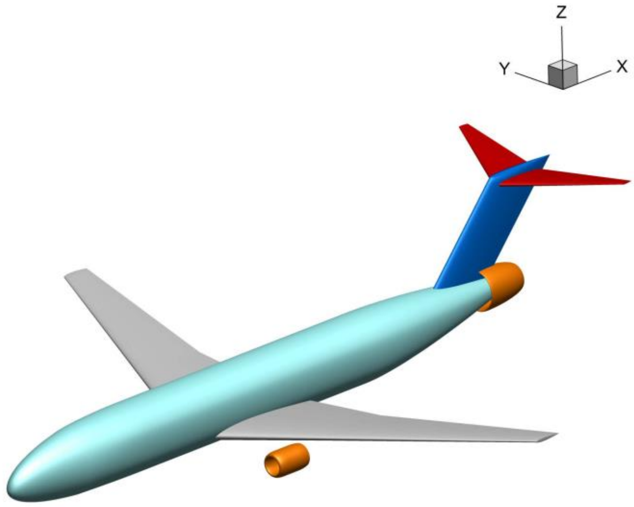

Figure 1 - STARC-ABL isometric view. Key features include the rear fuselage BLI fan, the T-tail empennage, and the reduced size turbofans allowed by the turboelectric propulsion system (Welstead and Felder, 2016).

Assumptions were made during the analysis of aircraft, air and combustion products were treated as flows of perfect gases with properties given in Table 1. The addition of fuel mass was neglected, all nozzles were assumed fully expanded, and any pressure drop in intakes or ducts were neglected.

Cruise	Mach 0.7 at 35,000 ft, International Standard Atmosphere
Fan	Pressure ratio: 1.45; Isentropic efficiency 94%
LPC	Pressure ratio: 1.45; Isentropic efficiency 92%
HPC	Pressure ratio: 27.9; Isentropic efficiency 92%
HPT	Isentropic efficiency: 92%
LPT	Isentropic efficiency: 94%
Tail cone Fan	Pressure ratio: 1.25; Isentropic efficiency 96%
Overall electrical efficiency	90%
T_{04}	1540 K
Air properties	c_p: 1005 J.kg^{-1}.K^{-1}; c_p/c_v = 1.4
Combustion products	c_p: 1100 J.kg^{-1}.K^{-1}; c_p/c_v = 1.3
Number of fan-generators	$n_{FG} = 2$
Overall Bypass Ratio	$(n_{FG} \times \dot{m}_{Bypass} + \dot{m}_{TCF})/(n_{FG} \times \dot{m}_{Core}) = 14.4$
Fan-Gen Bypass Ratio	$\dot{m}_{Bypass}/\dot{m}_{Core} = 6.4$
R_1	0.35 m
R_2	1.0 m
Pressure loss in combustor	5% of stagnation pressure
Boundary layer thickness	δ: 1.0 m

Table 1 - Parameters for coursework analysis

4.1.

Flight velocity	$V_\infty = 207.5519\ m/s^2$	
Fan inlet	$T_{02} = 240.2424\ K$	$P_{02} = 33.0130\ KPa$
Fan outlet	$T_{013} = 268.8672\ K$	$P_{013} = 47.8689\ KPa$
LPC outlet	$T_{023} = 299.5232\ K$	$P_{023} = 69.4098\ KPa$
HPC outlet	$T_{03} = 816.6535\ K$	$P_{03} = 1936.5350\ KPa$
HPT outlet	$T_{045} = 1067.5310\ K$	$P_{045} = 317.1638\ KPa$
Bypass jet velocity	$V_{jB} = 312.7424\ m/s^2$	

Table 2 - 4.1 Table for the fanGen

As presented in Table 2, the calculation of the aircraft's flight velocity is the initial step, given its necessity for subsequent calculations.

To determine V_∞, it is necessary to obtain both M_∞ and a_∞. The former is provided in Table 1, while the latter can be computed based on atmospheric conditions.

$$T_{35,000ft} = 218.8\ K$$

$$P_{35,000ft} = 23.8\ KPa$$

Thus:

$$V_\infty = \frac{M_\infty}{\sqrt{\gamma_{air} * R_{air} * T_{35,000ft}}}$$

$$V_\infty = \frac{0.7}{\sqrt{1.4*287*218.8}} = \mathbf{207.5519\ m/s^2}$$

For a more comprehensive understanding of the engine's design, an engine layout and station numbering schematic has been designed to visualize the distribution of various properties:

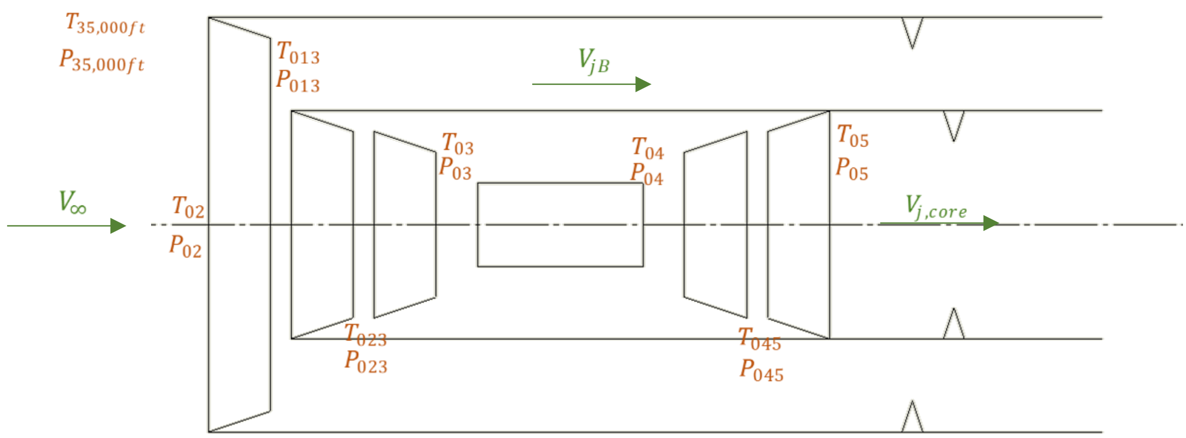

Figure 3 - Engine Layout and Station Numbering

We follow a procedural approach to compute the fan inlet conditions T_{02} and P_{02}:

$$T_{02} = T_{35,000ft} * \left(1 + \frac{\gamma_{air} - 1}{2} * M_\infty^2\right) = 218.8 * \left(1 + \frac{1.4 - 1}{2} * 0.7^2\right) = \mathbf{240.2424\ K}$$

$$P_{02} = P_{35,000ft} * \left(\frac{T_{02}}{T_{35,000ft}}\right)^{\frac{\gamma_{air}}{\gamma_{air}-1}} = 23.8 * \left(\frac{240.2424}{218.8}\right)^{\frac{1.4}{1.4-1}} = \mathbf{33.0130\ KPa}$$

With the aforementioned conditions in place, we proceed to calculate the exit conditions of the fan, T_{013} and P_{013}:

The temperature ratio across the fan is:

$$\frac{T_{013}}{T_{02}} = 1 + \frac{pr_{fan}^{\frac{\gamma_{air-1}}{\gamma_{air}}} - 1}{\eta_{fan}}$$

Arranging the formula for T_{013}:

$$T_{013} = T_{02} + \frac{T_{02}}{\eta_{fan}} * \left(pr_{fan}^{\frac{\gamma_{air-1}}{\gamma_{air}}} - 1\right) = 240.2424 + \frac{240.2424}{0.94} * \left(1.45^{\frac{1.4-1}{1.4}} - 1\right) = \mathbf{268.8672\ K}$$

$$P_{013} = P_{02} * pr_{fan} = 33.0130 * 1.45 = \mathbf{47.8688\ KPa}$$

Upon obtaining the outlet values of the fan, we can now proceed to calculate the exit parameters T_{023} and P_{023} of the LPC. It is worth noting that the outlet temperature and pressure of the fan pass through the LPC individually, as they are not interconnected. In this context, we introduce a new variable, $pr_{booster}$, which serves to define the pressure ratio between the fan inlet and the LPC outlet.

$$pr_{booster} = pr_{fan} * pr_{LPC}$$

As, the temperatures resulting from isentropic compression and expansions can be evaluated using the following relation for isentropic processes in perfect gases, $P/T^{\frac{\gamma_{air-1}}{\gamma_{air}}}$ which leads to $\frac{T_{023s}}{T_{02}} = \left(\frac{P_{03}}{P_{35000ft}}\right)^{\frac{\gamma_{air-1}}{\gamma_{air}}}$, therefore:

$$T_{023s} = pr_{booster}^{\frac{\gamma_{air-1}}{\gamma_{air}}} * T_{02} = (1.45 * 1.45)^{\frac{1.4-1}{1.4}} * 240.2424 = 297.0707\ K$$

$$T_{023} = (T_{023s} - T_{013})/\eta_{LPC} + T_{02} = (297.0707 - 268.8672)/0.92 + 240.2424 = \mathbf{299.5232\ K}$$

$$P_{03} = P_{02} * pr_{booster} = 33.0130 * (1.45 * 1.45) = \mathbf{69.4098\ KPa}$$

Subsequently, the outlet of the HPC, T_{03} and P_{03}:

$$T_{03s} = pr_{HPC}^{\frac{\gamma_{air-1}}{\gamma_{air}}} * T_{023} = 27.9^{\frac{1.4-1}{1.4}} * 299.5232 = 775.2831\ K$$

$$T_{03} = (T_{03s} - T_{023})/\eta_{HPC} + T_{023} = (775.2831 - 299.5232)/0.92 + 299.5232 = \mathbf{816.6535\ K}$$

$$P_{03} = P_{023} * pr_{HPC} = 69.4098 * 27.9 = \mathbf{1936.5350\ kPa}$$

During the combustion process, there is a Pressure drop of 5% therefore the outlet pressure is 95% of the inlet pressure, as follows:

$$P_{04} = P_{03} * (1 - Pressure\ loss) = 1936.5350 * 0.95 = 1839.7082\ kPa$$

To determine the T_{045} ,we apply the following condition by equating the inlet and outlet enthalpy conditions of the combustor.

$$m_{air} * C_{p,air} * (T_{03} - T_{023}) = (m_{air} + m_{fuel}) * C_{p,product} * (T_{04} - T_{045})$$

Since $m_{air} \gg m_{fuel}$, we can simplify the equation by disregarding m_{fuel}, resulting in:

$$T_{045} = T_{04} - \frac{C_{p,air}}{C_{p,comb}} * (T_{03} - T_{023}) = 1540 - \frac{1005}{1100} * (816.6535 - 299.5232) = \mathbf{1067.5310\ K}$$

To determine the outlet P_{045}, T_{045s} is required:

$$T_{045s} = T_{04} - \frac{T_{04} - T_{045}}{\eta_{HPT}} = 1540 - \frac{1540 - 1067.5310}{0.92} = 1026.4466\ K$$

$$P_{045} = P_{04} * \left(\frac{T_{045s}}{T_{04}}\right)^{\frac{\gamma_{comb-1}}{\gamma_{comb}}} = 1839.7082 * \left(\frac{1067.5310}{1540}\right)^{\frac{1.3-1}{1.3}} = \mathbf{317.1637\ kPa}$$

Following the table, V_{jB}, the velocity in the bypass is to be calculated:

$$V_{jB} = \sqrt{2 * C_{p,air} * T_{013} * \left(1 - \left(\frac{P_{35,000ft}}{P_{013}}\right)^{\frac{\gamma_{air-1}}{\gamma_{air}}}\right)} = \sqrt{2 * 1005 * 268.8672 * \left(1 - \left(\frac{23.8}{47.8688}\right)^{\frac{1.4-1}{1.4}}\right)}$$
$$= \mathbf{312.7424\ m/s^2}$$

4.2:

Inlet *static* state	$T_{BLI} = 224.5486\ K$	$P_{BLI} = 23.8\ kPa$
Inlet velocity and area	$V_{BLI} = 178.2001\ m/s^2$	$A_{in} = 2.7567\ m^2$
Density and mass flow	$\rho_{BLI} = 0.3690\ kg/m^2$	$\dot{m}_{TCF} = 181.4219\ kg/s$
Inlet *stagnation* state	$T_{02,TCF} = 240.3550\ K$	$P_{02,TCF} = 30.1979\ kPa$
Tail Cone Fan outlet	$T_{013,TCF} = 256.8373\ K$	$P_{013,TCF} = 37.7474\ kPa$
Tail Cone Fan jet velocity	$V_{jTCF} = 252.4655\ m/s^2$	
Net thrust (kN)	$F_{N,TCF} = 13.4733\ kN$	
Shaft power (kW)	$\dot{W}_{x,TCF} = 3005.1903\ kW$	
Propulsive efficiency of tail cone fan	$\eta_{p,TCF} = 58.2918\ \%$	

Table 3 - 4.2 Table for the Tail Cone Fan

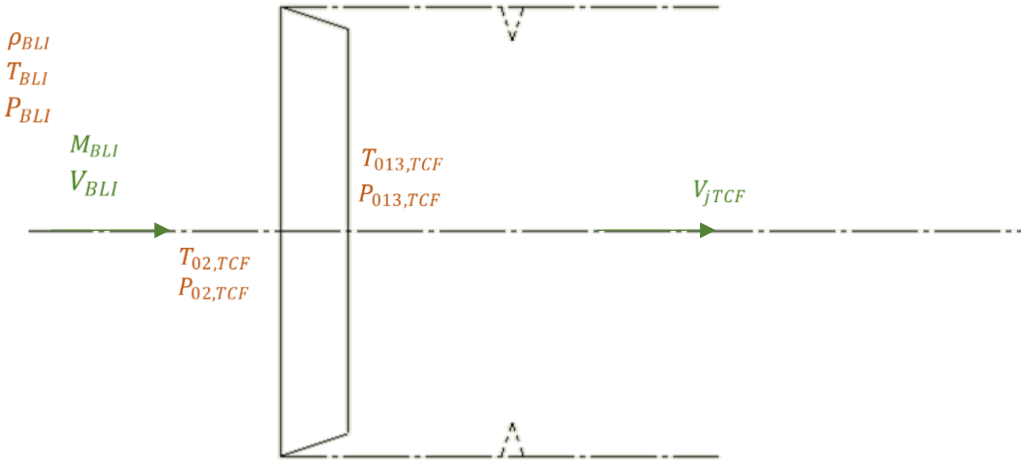

Figure 4 - Tail cone fan Layout and Station Numbering

To calculate T_{BLI} and P_{BLI} The flow through the aircraft boundary layer as adiabatic, implying constant specific stagnation enthalpy, giving, $h_{02,BLI} = h_{02}$ (where stations 2 and 2BLI denote the entries to the genFan and the tail cone fan respectively) which implies:

$$T_{02,BLI} = T_{02} = 240.2424 \text{ K}$$

For the calculation of T_{BLI}, we can utilize the inverse formula of the Mach number, M_{BLI}. This Mach number can be determined using a Mach number relation provided in the Thermofluid data book, which, in turn, necessitates the value of V_{BLI} :

The formula for V_{BLI} was given in the description of the assignment:

$$V_{BLI} = V_\infty \delta^{-1/7} \frac{\left[\frac{7}{16}(R_2 - R_1)^{\frac{16}{7}} + \frac{7}{9}R_1(R_2 - R_1)^{\frac{9}{7}}\right]}{\left[\frac{7}{15}(R_2 - R_1)^{\frac{15}{7}} + \frac{7}{8}R_1(R_2 - R_1)^{\frac{8}{7}}\right]} = 207.5518 * 1^{-\frac{1}{7}} \frac{\left[\frac{7}{16}(1 - 0.35)^{\frac{16}{7}} + \frac{7}{9}0.35(1 - 0.35)^{\frac{9}{7}}\right]}{\left[\frac{7}{15}(1 - 0.35)^{\frac{15}{7}} + \frac{7}{8}0.35(1 - 0.35)^{\frac{8}{7}}\right]}$$
$$= \mathbf{178.2001}\, m/s^2$$

$$\frac{V_{BLI}}{\sqrt{C_{p,air} * T_{02,BLI}}} = \frac{178.2001}{\sqrt{1005 * 240.2424}} = 0.3626$$

Upon interpolating this value in the data book, we find that $M_{BLI} = 0.5933$, resulting in:

$$T_{BLI} = \frac{V_{BLI}^2}{M_{BLI}^2 * \gamma_{air} * R_{air}} = \frac{178.2001^{\ 2}}{0.5933^2 * 1.4 * 287} = \mathbf{224.5485\ K}$$

The pressure in the boundary layer, P_{BLI} is distributed uniformly and assumed to be the equal to Atmospheric pressure, $P_{35,000ft}$

$$P_{BLI} = P_{35,000ft} = \mathbf{23.8\ kPa}$$

Having both inner and outer radius, R_1 and R_2, Area, A_{in} is:

$$A_{in} = \pi * R_2^2 - \pi * R_1^2 = \pi * 1^2 - \pi * 0.35^2 = \mathbf{2.7568\ m^2}$$

Density, ρ_{BLI}, can be determined with Equation of State in Ideal Gas Relationship:

$$\rho_{BLI} = \frac{P_{BLI}}{R_{air} * T_{BLI}} = \frac{23.8}{287 * 224.5485} = \mathbf{0.3693\,\frac{kg}{m^3}}$$

Having, V_{BLI}, A_{in} and ρ_{BLI}, $\dot{m}_{TCF}$ can be determined by the following formula:

$$\dot{m}_{TCF} = V_{BLI} * A_{in} * \rho_{BLI} = 178.2001 * 2.7568 * 0.3693 = \mathbf{181.4219\,\frac{kg}{s}}$$

With the environmental conditions surrounding the Tail Cone Fan, it is now possible to calculate both the inlet and outlet states of this component:

- Inlet Stagnation State:

$$T_{02,TCF} = T_{BLI} * \left(1 + \frac{\gamma_{air} - 1}{2} * M_{BLI}^2\right) = \mathbf{240.3550\ K}$$

Or

$$T_{02,TCF} = T_{02,BLI} = \mathbf{240.2424\ K}$$

$$P_{02,BLI} = P_{BLI} * \left(1 + \frac{\gamma_{air} - 1}{2} * M_{BLI}^2\right)^{\frac{\gamma_{air}}{\gamma_{air}-1}} = 23.8 * \left(1 + \frac{1.4 - 1}{2} * 0.5933^2\right)^{\frac{1.4}{1.4-1}} = 3\mathbf{0.1979\ kPa}$$

$$P_{02,TCF} = P_{02,BLI} = \mathbf{30.1979\ kPa}$$

- Tail Cone Fan outlet:

$$T_{013,TCF} = T_{02,TCF} + \frac{T_{02,TCF}}{\eta_{TCF}} * \left(pr_{TCF}^{\frac{\gamma_{air-1}}{\gamma_{air}}} - 1\right) = 240.3550 + \frac{240.3550}{0.96} * \left(1.25^{\frac{1.4-1}{1.4}} - 1\right) = \mathbf{256.8373\ K}$$

$$P_{013,TCF} = P_{02,TCF} * pr_{TCF} = 30.1979 * 1.25 = \mathbf{37.7474\ kPa}$$

These values allow for the calculation of the Tail Cone Fan jet velocity, V_{jTCF}:

$$V_{jTCF} = \sqrt{2 * C_{p,air} * T_{013,TCF} * \left(1 - \left(\frac{P_{BLI}}{P_{013,TCF}}\right)^{\frac{\gamma_{air-1}}{\gamma_{air}}}\right)} = \sqrt{2 * 1005 * 256.8373 * \left(1 - \left(\frac{23.8}{37.7474}\right)^{\frac{1.4-1}{1.4}}\right)}$$
$$= \mathbf{252.4655\ m/s^2}$$

Which can be used to calculate the Net Trust produced by the Tail Cone Fan, $F_{N,TCF}$ which is the Flux of momentum entering the TCF minus the Flux of momentum leaving the TCF:

$$F_{N,TCF} = \dot{m}_{TCF} * \left(V_{jTCF} - V_{BLI}\right) = 181.4219 * (252.4655 - 178.2001) = \mathbf{134733.687\ N}$$

$$F_{N,TCF} = \mathbf{13.4733\ kN}$$

Shaft Power $\dot{W}_{x,TCF}$ can also be computed, as it will be utilized in the calculation of the LPT power, $\dot{W}_{xLPT}$, as specified in table 4.5. This can be determined using the Steady-Flow Energy Equation which is an expression of the First Law of Thermodynamics applied to a steady-flow device. $\triangle\, h_0$ is the stagnation enthalpy difference between the exhaust flow and the inlet air.

$$\dot{Q}_{net} - \dot{W}_{x,net} = \dot{m}_{net} * \triangle\, h_0$$

As there is there is no addition of heat, $\dot{Q}_{net} = 0$

$$\dot{W}_{x,net} = \dot{m}_{net} * C_{p,air} * \left(T_{013,TCF} - T_{02,TCF}\right)$$

$$\dot{W}_{x,TCF} = \dot{m}_{TCF} * C_{p,air} * \left(T_{013,TCF} - T_{02,TCF}\right) = 181.4219 * 1005 * (256.8373 - 240.3550) = \mathbf{30051903\ W}$$

$$\dot{W}_{x,TCF} = \mathbf{3005.1903\ kW}$$

Therefore, the Propulsive efficiency of the Tail Cone Fan, $\eta_{p,TCF}$ can vary due to various factors, including the intake area. Adjusting R_2 can be employed to achieve a particular Propulsive efficiency, as illustrated in the graph in Section 4.3.

$$\eta_{p,TCF} = \frac{Power\ to\ the\ aircraft}{Power\ TCF\ jet}$$

$$\eta_{p,TCF} = \frac{V_\infty * \left(V_{jTCF} - V_\infty\right)}{\frac{1}{2}\left(V_{jTCF}^2 + V_{BLI}^2\right)} = \frac{207.5518 * (252.4655 - 207.5518)}{\frac{1}{2}(252.4655^2 * 178.2001^2)} = \mathbf{0.5829}$$

$$\eta_{p,TCF} = \mathbf{58.2918\%}$$

4.3

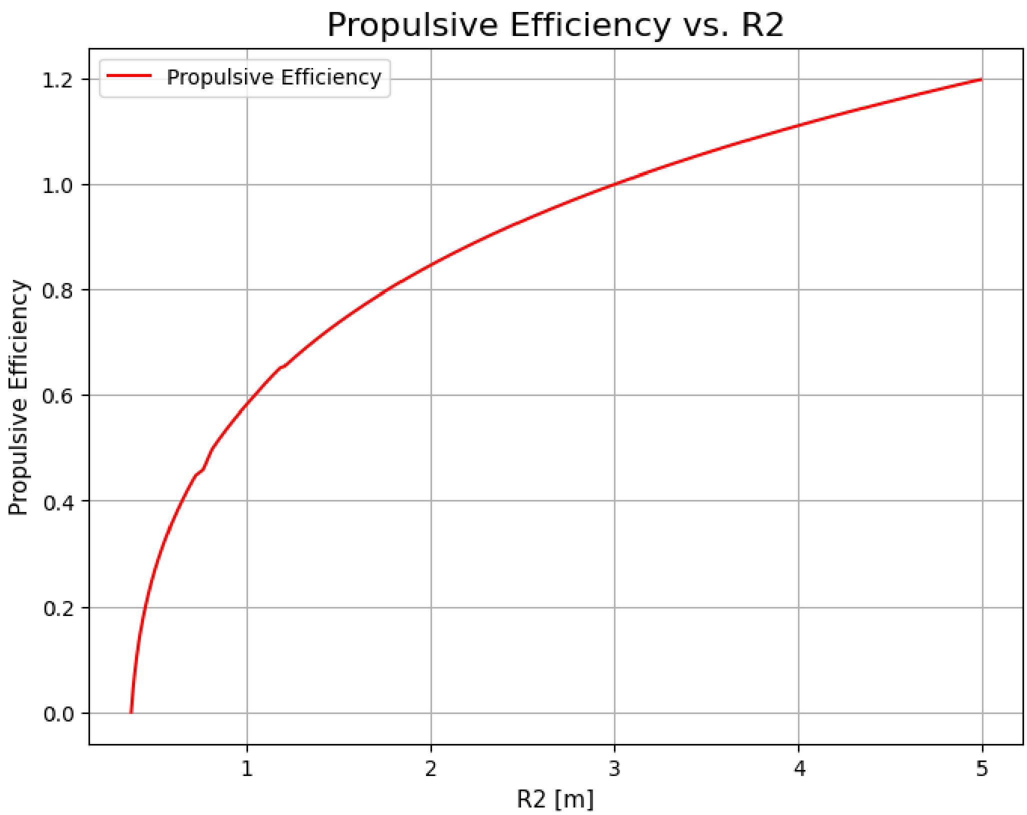

Figure 5 - Propulsive Efficiency change with Outer Radius

4.4

The η_p of a jet engine, particularly the tail cone fan in a turbofan engine, is influenced by various factors, with R_2 playing a central role:

Inlet Velocity and Outlet Velocity Ratio: η_p is maximized when the velocities V_{jTCF} and V_{BLI} are equal, increasing the outer radius, R_2 of the tail cone fan leads to a higher Inlet/Outlet velocity ratio. Thus, a larger R_2 generally leads to higher efficiency, aligning with the observed trend in the graph. There is a limit to how much R_2 can increased due to additional weight, impracticality in manufacturing and logistics.

Intake Area: Increasing R_2 results in a larger intake area, allowing more air to be ingested into the engine. This can enhance overall engine performance and efficiency.

Fan Pressure Ratio: A larger R_2 tends to reduce the pr_{fan}. This reduction contributes to an increased bpr which, leads to higher gross thrust and improved sfc.

Trade-offs: It's important to note that while a larger fan can boost efficiency, it may also introduce increased drag and weight. Therefore, engineering and design considerations are necessary to strike the right balance between fan size and trade-offs.

4.5

Core mass flow in one fanGen	$\dot{m}_{core} = 11.3388\ kg/s$	
Bypass mass flow in one fanGen	$\dot{m}_{bypass} = 72.5687\ kg/s$	
LPT power in one fanGen (kW)	$\dot{W}_{xLPT} = 4432.7461\ kW$	
LPT outlet	$T_{05} = 712.1367\ K$	$P_{05} = 47.6964\ kPa$
Core jet velocity	$V_{j,core} = 481.8840\ m/s^2$	
Overall net thrust	$F_N = 34.9616\ kN$	

Table 4 - 4.5 Table for Overall Turboelectric Propulsion System

In order to calculate the Overall net thrust, it is necessary to determine the values of $\dot{\mathrm{m}}_{\mathrm{core}}$ and $\dot{\mathrm{m}}_{\mathrm{bypass}}$.

The Fan-Gen Bypass Ratio is given: $\dot{\mathrm{m}}_{\mathrm{Bypass}}/\dot{\mathrm{m}}_{\mathrm{Core}} = 6.4$

This can be used to get $\dot{\mathrm{m}}_{\mathrm{Bypass}} = 6.4 * \dot{\mathrm{m}}_{\mathrm{Core}}$ which substituted in $(\mathrm{n_{FG}} \times \dot{\mathrm{m}}_{\mathrm{Bypass}} + \dot{\mathrm{m}}_{\mathrm{TCF}})/(\mathrm{n_{FG}} \times \dot{\mathrm{m}}_{\mathrm{Core}}) = 14.4$ and solved for $\dot{\mathrm{m}}_{\mathrm{Core}}$:

$$\dot{m}_{Core} = \frac{\dot{m}_{TCF}}{(Overall\ Bypass\ Ratio - FanGen\ Bypass\ Ratio) * n_{FG}} = \frac{181.4219}{(14.4 - 6.4) * 2} = \mathbf{11.3389} \frac{\boldsymbol{kg}}{\boldsymbol{s}}$$

Thus:

$$\dot{m}_{Bypass} = 6.4 * \dot{m}_{Core} = 6.4 * 11.3389 = \mathbf{72.5687} \frac{\boldsymbol{kg}}{\boldsymbol{s}}$$

These values enable the calculation of the LPT power in one fanGen, $\dot{\mathrm{W}}_{\mathrm{xLPT}}$ which is equivalent to the power distributed among the LPC, Fan, and TCF.

$$\dot{W}_{xLPC} = \dot{m}_{Core} * C_{p,air} * (T_{023} - T_{02}) = 11.3389 * 1005 * (299.5232 - 240.2424) * 1e^{-3} = 675.5388\ kW$$

$$\dot{W}_{xFan} = FanGenbpr * \dot{m}_{Core} * C_{p,air} * (T_{013} - T_{02}) = 6.4 * 11.3389 * 1005 * (268.8672 - 240.2424) * 1e^{-3} = 2087.6570\ kW$$

Thus:

$$\dot{W}_{xLPT} = \dot{W}_{xLPC} + \dot{W}_{xFan} + \frac{\dot{W}_{xTCF}}{2 * \eta_{electrical}}$$

$$\dot{W}_{xLPT} = 675.5388 + 2087.6570 + \frac{3005.1903}{2 * 0.9} = \mathbf{4432.7461\ kW}$$

By utilizing the equation $\dot{W}_{xLPT}$ we can calculate T_{05} and P_{05} as $\dot{W}_{xLPT} = \dot{m}_{core} * C_{p,comb} * (T_{045} - T_{05})$, To do this, we can rearrange the equation and solve for T_{05}, using the inverse formula as follows::

$$T_{05} = T_{045} - \frac{\dot{W}_{xLPT}}{\dot{m}_{core} * C_{p,comb}} = 1067.5310 - \frac{4432.7461}{11.3389 * 1100} = \mathbf{712.1367\ K}$$

$$T_{05s} = T_{045} - \frac{T_{045} - T_{05}}{\eta_{LPT}} = 1067.5310 - \frac{1067.5310 - 712.1367}{0.94} = 689.4519\ K$$

$$P_{05} = P_{045} * \left(\frac{T_{05s}}{T_{045}}\right)^{\frac{\gamma_{comb}}{\gamma_{comb}-1}} = 317.1638 * \left(\frac{689.4519}{1067.5309}\right)^{\frac{1.3}{1.3-1}} = \mathbf{47.6964\ kPa}$$

Having the outlet pressure of the LPT, P_{05} we can proceed to calculate the core velocity, V_{jcore}:

$$P_9 = P_{35,000ft} = 23.8\ KPa$$

$$V_{jcore} = \sqrt{2 * C_{p,comb} * T_{05} * \left(1 - \left(\frac{P_9}{P_5}\right)^{\frac{\gamma_{comb-1}}{\gamma_{comb}}}\right)} = \sqrt{2 * 1100 * 712.1367 * \left(1 - \left(\frac{23.8}{47.6964}\right)^{\frac{1.3-1}{1.3}}\right)}$$
$$= \mathbf{481.8840\ m/s^2}$$

With all the necessary velocity values determined, we can now proceed to calculate the Overall Net Thrust, F_N:

$$F_{N,Bypass} = \dot{m}_{bypass} * (V_{jB} - V_\infty) = (72.5687 * (312.7424 - 207.5518)) * 1e^{-3} = 7.6335\ kN$$

$$F_{N,Core} = \dot{m}_{core} * (V_{j,core} - V_\infty) = (11.3388 * (481.8840 - 207.5518)) * 1e^{-3} = 3.1106\ kN$$

$$F_N = 2 * F_{N,Bypass} + 2 * F_{N,Core} + F_{N,Fan}$$

$$F_N = 15.2670 + 6.2212 + 13.4733 = \mathbf{34.9617\ kN}$$

The completion of this coursework involved both analytical calculations and the utilization of Python programming.

- *4.1 – Code*

```python
#4.1
# Find Velocity Vinf
vinf = Mcruise * np.sqrt(gammaAir*Rair*Tatm)
print(f'Flight velocity is: {vinf} m/s^2\n')

#Fan Inlet T02, P013
T02 = Tatm*(1+0.5*(gammaAir-1)*Mcruise**2)
P02 = Patm*((T02/Tatm)**(gammaAir/(gammaAir-1)))
print(f'T02 is: {T02} K')
print(f'P02 is: {P02} KPa\n')

#Fan Outlet T013, P013
T013 = T02 + (T02/Fann)*((FanPr)**((gammaAir-1)/gammaAir)-1)
P013 = P02 * FanPr
print(f'T013 is: {T013} K')
print(f'P013 is: {P013} KPa\n')

#LPC T023, P023
T023s = (BoosterPr**((gammaAir-1)/gammaAir))*T02
T023 = ((T023s - T013)/LPCn) + T013
P023 = P02*BoosterPr
print(f'T023 is: {T023} K')
print(f'P023 is: {P023} KPa\n')

#HPC T03, P03
T03s = (HPCPr**((gammaAir-1)/gammaAir))*T023
T03 = ((T03s - T023)/HPCn) + T023
P03 = P023*HPCPr
print(f'T03 is: {T03} K')
print(f'P03 is: {P03} KPa\n')

#HPT T045, P045
T045 = T04 - (cpAir/cpComb)*(T03 - T023)
T045s = T04 - ((T04 - T045)/HPTn)
P04 = P03*0.95
P045 = P04*((T045s/T04)**(gammaComb/(gammaComb-1)))
print(f'T045 is: {T045} K')
print(f'P045 is: {P045} KPa\n')

#Velocity at the Bypass -> Vjb
Vjb = np.sqrt(2*cpAir*T013*(1-((Patm/P013)**((gammaAir-1)/gammaAir))))
print(f'Vjb is: {Vjb} m/s^2')
```

Figure 6 - Code for Table 4.1

```
Flight velocity is: 207.5518768886468 m/s^2

T02 is: 240.24239999999998 K
P02 is: 33.01301182514138 KPa

T013 is: 268.86725951231483 K
P013 is: 47.868867146455 KPa

T023 is: 299.5232661692171 K
P023 is: 69.40985736235976 KPa

T03 is: 816.6535555091601 K
P03 is: 1936.5350204098372 KPa

T045 is: 1067.5309629212338 K
P045 is: 317.16377636887336 KPa

Vjb is: 312.74241491669216 m/s^2
```

Figure 7 - Results for 4.1

- *4.2 – Code*

```
#4.2
# Calculating VBLI with the given formula
VBLI = vinf * (h**(-1/7))*(((7/16)*((R2-R1)**(16/7)))+(7/9)*R1*((R2-
R1)**(9/7)))/(((7/15)*((R2-R1)**(15/7)))+(7/8)*R1*((R2-R1)**(8/7)))
print(f'VBLI is {VBLI} m/s^2\n')

# Calculating the Area
Area = np.pi*((R2)**2) - np.pi*((R1)**2)
print(f'Area is {Area} m^2\n')

# Inlet Static State TBLI, TBLI
T02BLI = T02
ndMach = VBLI/(np.sqrt(cpAir*T02BLI))
print(f'ndMach: {ndMach}')

Minter = [0.57, 0.58, 0.59, 0.60, 0.61, 0.62, 0.63]
Tinter = [0.9390, 0.9370, 0.9349, 0.9328, 0.9307, 0.9286, 0.9265]
Pinter = [0.8022, 0.7962, 0.7901, 0.7840, 0.7778, 0.7716, 0.7654]
rhoInter = [0.8544, 0.8498, 0.8451, 0.8405, 0.08357, 0.8310, 0.8262]
ndInter = [0.3493, 0.3551, 0.3608, 0.3665, 0.3772, 0.3779, 0.3835]

MBLI = np.interp(ndMach, ndInter, Minter)
TBLI = ((VBLI**2)/((MBLI**2)*gammaAir*Rair))
print(f'TBLI is: {TBLI} K')

# Pressure BLI - PBLI
PBLIratio = np.interp(ndMach, ndInter, Pinter)
PBLI = Patm
print(f'PBLI is: {PBLI} KPa \n')

# rhoBLI
rhoBLI = ((PBLI)/(Rair*TBLI))*1e3
print(f'rhoBLI is: {rhoBLI} kg/m^3')

# mass flow
mTCF = Area*rhoBLI*VBLI
print(f'mTCF is: {mTCF} Kg/s\n')

# Inlet stagnation state
T02TCF = TBLI*(1+0.5*(gammaAir-1)*MBLI**2)

# T02TCF = T02BLI
P02BLI = PBLI*((1+0.5*(gammaAir-1)*(MBLI**2))**(gammaAir/(gammaAir-1)))
P02TCF = P02BLI
print(f'T02TCF is: {T02TCF} K')
print(f'P02TCF is: {P02TCF} KPa\n')

# Tail cone Fan outlet
T013TCF = T02TCF + (T02TCF/TailConen)*((TailConePr)**((gammaAir-1)/gammaAir)-
1)
P013TCF = P02TCF * TailConePr
print(f'T013TCF is: {T013TCF} K')
print(f'P013TCF is: {P013TCF} KPa\n')

# Tail cone Fan jet velocity
VjTCF = np.sqrt(2*cpAir*T013TCF*(1-((Patm/P013TCF)**((gammaAir-1)/gammaAir))))
print(f'VjTCF is: {VjTCF} m/s^2')
# print(f'Inlet/Outlet ratio: {VBLI/VjTCF}')

# Net trust
FnFan = (mTCF * (VjTCF - VBLI))*1e-3
print(f'Net Trust is {FnFan} kN \n')

# Shaft Power
WxTCF = (mTCF*cpAir*(T013TCF-T02TCF))
print(f'Wx is: {WxTCF*1e-3} KW \n')

# Propulsive efficiency of the tail cone fan
nProp = (2*VBLI)/(VjTCF + VBLI)
print(f'Propulsive efficiency is: {nProp}')
```

Figure 8 – Code for Table 4.2

```
VBLI is 178.20010348072125 m/s^2

Area is 2.7567475535250434 m^2

ndMach: 0.3626603631782497
TBLI is: 224.54856304910945 K
PBLI is: 23.8 KPa

rhoBLI is: 0.3693046534889486 kg/m^2
mTCF is: 181.42190789359483 Kg/s

T02TCF is: 240.3550721982084 K
P02TCF is: 30.197952407246447 KPa

T013TCF is: 256.8373106775483 K
P013TCF is: 37.74744050905806 KPa

VjTCF is: 252.46549345710878 m/s^2

Net Trust is 13.47336873997808 kN

Wx is: 3005.1903470354655 KW

Propulsive efficiency is: 0.5829187724365271
```

Figure 9 - Results for 4.2

- *4.3 – Code*

```
#4.3 - Creating a graph
# Define a range of values for "r"
r_values = np.arange(0.4, 10, 0.05)

# Calculate propulsive efficiency for each "r" value
prop_efficiency_values = []
for R2 in r_values:
    VBLI = vinf * (h**(-1/7))*(((7/16)*((R2-R1)**(16/7)))+(7/9)*R1*((R2-
R1)**(9/7)))/(((7/15)*((R2-R1)**(15/7)))+(7/8)*R1*((R2-R1)**(8/7)))
    VjTCF = np.sqrt(2*cpAir*T013TCF*(1-((Patm/P013TCF)**((gammaAir-
1)/gammaAir))))
    nProp = (2 * VBLI) / (VjTCF + VBLI)
    prop_efficiency_values.append(nProp)

print(prop_efficiency_values)

# Create the graph
plt.figure(figsize=(8, 6))
plt.plot(r_values, prop_efficiency_values,color='red', label="Propulsive
Efficiency")
plt.xlabel("R2 [m]", fontsize = 11)
plt.ylabel("Propulsive Efficiency", fontsize = 11)
plt.title("Propulsive Efficiency vs. R2", fontsize = 16)
x_ticks = range(0, 11)  # Adjust the range as needed
plt.xticks(x_ticks) # Set the positions and labels for the x-axis ticks
plt.grid(True)
plt.legend()
plt.show()
```

Figure 10 - Code to Plot Efficiency/R2

- *4.5 Code*

```
# 4.5
# Core mass flow in one fanGen
mCore = (mTCF/((BypassRatio-FanGenBypass)*nfg))
print(f'mCore is {mCore} kg/s \n')

# Bypass mass flow in one fanGen
mBypass = mCore*FanGenBypass
print(f'mBypass is {mBypass} kg/s \n')

# LPT power in one fanGen (kW)
WLPC = (mCore*cpAir*(T023-T02))
WFan = (FanGenBypass*mCore*cpAir*(T013-T02))
WxLPT = (mCore*cpAir*(T023-T02)) + (FanGenBypass*mCore*cpAir*(T013-T02)) +
((WxTCF/nfg)/0.9)
print(f'WLPC is {WLPC*1e-3} KW\nWFan is {WFan*1e-3} KW\nWxTCF is {WxTCF*1e-3}
KW \n')
print(f'Turbine Work is {WxLPT*1e-3} KW \n')

# LPT outlet -> T05, P05
T05 = T045 - WxLPT/((mCore*cpComb))
T05s = T045 - ((T045 - T05)/LPTn)
print(f'T05 is: {T05} K \n')
P05 = P045*((T05s/T045)**(gammaComb/(gammaComb-1)))
print(f'P05 is: {P05} KPa \n')

# Core jet velocity
P9 = Patm
Vjcore = np.sqrt(2*cpComb*T05*(1-(P9/P05)**((gammaComb-1)/gammaComb)))
print(f'Vjcore is: {Vjcore} m/s^2 \n')

# Overall net trust
FnBypass = 2*((mBypass * (Vjb - vinf))*1e-3)
FnCore = 2*((mCore * (Vjcore - vinf))*1e-3)
FnOverall = 1*FnBypass + 1*FnCore + FnFan
print(f'FnByass is: {FnBypass/2} kN \nFnCore is: {FnCore/2} kN \nFnFan is:
{FnFan}\n')
print(f'FnOverall is: {FnOverall} KN \n')
```

Figure 12 - Code for Table 4.5

```
mCore is 11.338869243349677 kg/s

mBypass is 72.56876315743794 kg/s

WLPC is 675.5388800758907 KW
WFan is 2087.657003615932 KW
WxTCF is 3005.1903470354655 KW

Turbine Work is 4432.746076489303 KW

T05 is: 712.1366926496803 K

P05 is: 47.69639145077003 KPa

Vjcore is: 481.88403262967677 m/s^2

FnByass is: 7.633547240560692 kN
FnCore is: 3.1106164431937784 kN
FnFan is: 13.47336873997808
```

Figure 11 - Results for 4.5