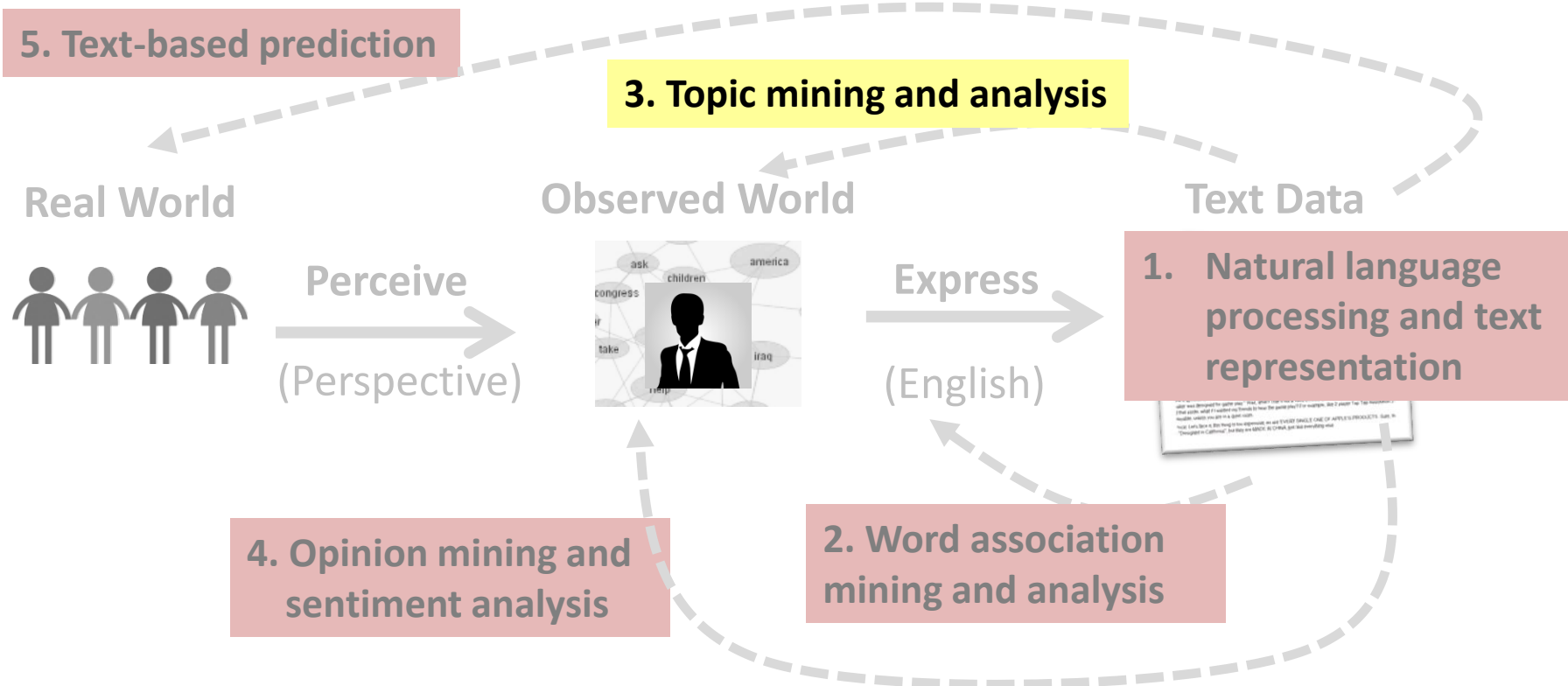




# Topic Mining and Analysis: Mining One Topic

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# Probabilistic Topic Models: Mining One Topic



# Simplest Case of Topic Model: Mining One Topic

INPUT:  $C=\{d\}, V$

OUTPUT:  $\{\theta\}$

Text Data



$P(w|\theta)$

$\theta$

text ?  
mining ?  
association ?  
database ?  
...  
query ?  
...

Doc d

100%



# Language Model Setup

- **Data:** Document  $d = x_1 x_2 \dots x_{|d|}$ ,  $x_i \in V = \{w_1, \dots, w_M\}$  is a word
- **Model:** Unigram LM  $\theta$ (=topic) :  $\{\theta_i = p(w_i | \theta)\}$ ,  $i=1, \dots, M$ ;  
 $\theta_1 + \dots + \theta_M = 1$
- **Likelihood function:**  $p(d | \theta) = p(x_1 | \theta) \times \dots \times p(x_{|d|} | \theta)$   
$$= p(w_1 | \theta)^{c(w_1, d)} \times \dots \times p(w_M | \theta)^{c(w_M, d)}$$
$$= \prod_{i=1}^M p(w_i | \theta)^{c(w_i, d)} = \prod_{i=1}^M \theta_i^{c(w_i, d)}$$
- **ML estimate:**  $(\hat{\theta}_1, \dots, \hat{\theta}_M) = \arg \max_{\theta_1, \dots, \theta_M} p(d | \theta) = \arg \max_{\theta_1, \dots, \theta_M} \prod_{i=1}^M \theta_i^{c(w_i, d)}$

# Computation of Maximum Likelihood Estimate

**Maximize  $p(d | \theta)$**   $(\hat{\theta}_1, \dots, \hat{\theta}_M) = \arg \max_{\theta_1, \dots, \theta_M} p(d | \theta) = \arg \max_{\theta_1, \dots, \theta_M} \prod_{i=1}^M \theta_i^{c(w_i, d)}$

**Max. Log-Likelihood**  $(\hat{\theta}_1, \dots, \hat{\theta}_M) = \arg \max_{\theta_1, \dots, \theta_M} \log[p(d | \theta)] = \arg \max_{\theta_1, \dots, \theta_M} \sum_{i=1}^M c(w_i, d) \log \theta_i$

**Subject to constraint:**

$$\sum_{i=1}^M \theta_i = 1$$

Use Lagrange multiplier approach

Lagrange function:  $f(q | d) = \sum_{i=1}^M c(w_i, d) \log q_i + \lambda (\sum_{i=1}^M q_i - 1)$

$$\frac{\partial f(q | d)}{\partial q_i} = \frac{c(w_i, d)}{q_i} + \lambda = 0 \rightarrow q_i = -\frac{c(w_i, d)}{\lambda}$$

$$\sum_{i=1}^M -\frac{c(w_i, d)}{\lambda} = 1 \rightarrow \lambda = -\sum_{i=1}^M c(w_i, d) \rightarrow \hat{q}_i = p(w_i | \hat{q}) = \frac{c(w_i, d)}{\sum_{i=1}^M c(w_i, d)} = \frac{c(w_i, d)}{|d|}$$

**Normalized  
Counts**



# What Does the Topic Look Like?

d

Text mining  
paper

$p(w | \theta)$

the 0.031  
a 0.018  
...  
**text 0.04**  
**mining 0.035**  
**association 0.03**  
**clustering 0.005**  
**computer 0.0009**  
...  
**food 0.000001**  
...

Can we get rid of  
these common words?