Gamifying homework

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ΨΨStand amongst the ashes of a trillion dead souls and ask the ghosts if honor matters. The silence is your answer.
Ψ

Ψ— Javik, Mass Effect 3

§1 Introduction

Have you ever played a video game? Yes, right? Well, how was it? Was it fun? Was it addictive? Did you feel happy?
If all of the answers to the above questions are yes, read on.
Gamifying the learning of the bio101 course in the University of the Waterloo as a simple experiment has produced a great result in the Dunning-Kruger graph of learning.

Exercise 1.1. What is a Dunning-Kruger graph?

Well, quoting wikipedia, "A graph of how confidence to speak on a subject develops depending on experience and knowledge. According to the Dunning–Kruger effect, people tend to overestimate their cognitive ability until/unless their competence increases to the point where they become aware of their shortcomings."

But what is it really? It is basically a graph of **confidence vs knowledge.** To get to know more about it, click here. It made the first curve of being simply stupid with almost no competency faster to pass by. To understand this, introduce a 3rd dimension of time; the difference between the slope of the time dimension before and after the experiment was too significant to ignore.

Exercise 1.2. What is gamification?

Gamification is basically to introduce goals, interaction, feedback, problem solving, competition, narrative, and fun learning environments, elements that can increase learner engagement and sustain motivation. We basically introduce a point-system or as such to speed-up learning, or rather just make it not-so-boring. To see how it works with utmost details, please see the fourth section.
Worth an estimated 2.3 trillion, the global entertainment and media market is big business. While this comprises of television, radio and film, it also includes video games. Currently, global revenue of this booming industry is 101.62 billion, as average consumer spending comes in at just over 90 each year. But it isn't just the financial side of things that raises eyebrows... an average gamer spends at least 8 hours per week playing video games.

A Fact. Let's be honest: the current way of teaching is old fashioned and boring. Yes, it is traditional but this is no reason for it to be so inefficient.

The job of a teacher, in what I believe, is to make learning the state(or nation)-provided-material as less painless and less boring as possible.

§1.1 Gamification vs Game based learning

After reading the solution to exercise 1.2, this thought of how is it any different than, say, role-playing the players of a story? Does it not produce the same effect?
Well, it does. But there are a few differences.
Quoting the University of Waterloo's website, **Gamification** is the integration of game elements like point systems, leaderboards, badges, or other elements related to games into "conventional" learning activities in order to increase engagement and motivation. For example, an online discussion forum for a Physics course might be gamified via a badge system: students might be awarded a "Ptolemy" badge after they have made 10 postings, a "Galileo" badge after 20 postings, "Kepler" after 30, "Einstein" after 40, and so on. In ideal gamified learning environments, students can see the online badges that their peers have earned to create a sense of comradery or competition.
But, **Game-based learning**, in contrast, involves designing learning activities so that game characteristics and game principles inhere within the learning activities themselves. For example, in an Economics course, students might compete in a virtual stock-trading competition; in a Political Science course, students might role-play as they engage in mock negotiations involving a labour dispute. In an English literature class, the students might be assigned to play the roles of the players of, say, Shakespeare's drama cum poem The Seven Ages' important characters.

To sum it up, **Gamification** is introducing the game-like structure in studies like levels, points, life, wisdom etc whereas **Game-based learning** is making students part-take in activities that *will* be fun.

§2 The Inspiration and Theory

§2.1 Inspiration

This idea is highly inspired. I observed that the current way of teaching students the syllabus is boring, even while acknowledging the current attempt of NCERT to change it; indeed, the current textbook of maths we have is very good and involved. It is enough in itself to self-teach a student maths with great examples and problems. But, sometimes, the teacher's teaching style does not match the book's. What do we do then? We force the teacher's teaching style into the book's. It will be easier to understand with an example. Same thing is true for the student's learning style.

Example 2.1

Imagine this: you have a teacher. He is a very good physics teacher. Now imagine that the book the school forces you to use is the NCERT, but the teacher's teaching style is different; he is more focused on teaching the theory than actually solving problems, not blaming his teaching style but it will simply not work.

I have personally been a victim to this. I really love mathematics, but the current teacher's teaching style is just not my thing. I will not target anybody here since it's

nobody's fault. I am pretty sure there are more such students who face the same problem. At the same time, on the other end of the spectrum, there are students who really like to memorise the theory more than trying problems. There are some who enjoy diagrams and then there are some who are able to retain the information better when told to (aka given a lecture to in real life.)

Exercise 2.2. But that can be changed easily by self-studying, what's your point?

My point is that not every student likes to self-study or has even enough time to self-study. Most of the students I know spend a lot of their time scrolling social media or watching content off youtube and mostly playing video games. How about we use that to our advantage? Basically, we may introduce a video-game like platform from which they will gain points.

Problem 2.3. There is a problem: the homework the teachers give is monotonous and boring and boring.

How do we solve this? Well, I have an idea. I will write more about it in the next section but for now: we give points to the solutions of the homework problems.

§2.2 Theory

Research has found that adopting game thinking and mechanics in non-game contexts can help improve engagement levels, assist in completing certain tasks, improve individual learning and encourage personal development. By providing rewards or injecting some fun into every chores and routine tasks, the individual is more likely to get it done in a quick, efficient and successful way.
There are a lot of examples of this thing being true but let's just take the current trending social media among the gen-z: instagaram.

Exercise 2.4. How is instagram using gamification?

It is simple: instagram uses numbers for the number of followers, following, likes, shares etc. It has a lot more than just that to it but that is basically it. While there is a clear incentive or reward, our natural human instinct of competition and ambition also kicks in. Gamification proposes a challenge, helps you overcome it and provides gratification at the end. Gamification is all about motivation, the desire and willingness to do something. This feeling of drive and ambition is fuelled by dopamine, the chemical signal that gets passed from one neuron in your brain to another. Essentially, your body releases dopamine when you experience something pleasurable or satisfying. While these can be all sorts of things, receiving a reward is one of the biggest. This is the same chemical which is released during reproduction-an essential human-ly process to help them thrive. While we can not mimic oxytonin with gamification, we sure can mimic dopamine. This gives a sense of accomplishment.
At the heart of gamification is motivation, but where does our enthusiasm specifically come from? There are several theories relating to this subject, but I believe in these below 3 factors:

- **Autonomy.** If you are given the authority to choose whichever problems you want to do, you are more prone to success. Experiments indicate that students given the opportunity and authority to select a course on their own persisted longer in problem solving activities.

- **Levels.** This is basically giving a sense of past accomplishment and a motivation to work. Students can look at their current level and motivate themselves to work even harder and hence solve even more problems. If an individual develops a proficiency or skill for something, they're more likely to continue doing it.
- **Competition.** This, in a healthy way, is very useful. The current competition of the exams is good but it only tests the retention capacity of the student. Students can compare their current levels to others' in the leaderboard.

§3 Example

Given below is an example of the RPG-system game called OTIS, created by Evan Chen. And the leaderboard's example is in the next page.

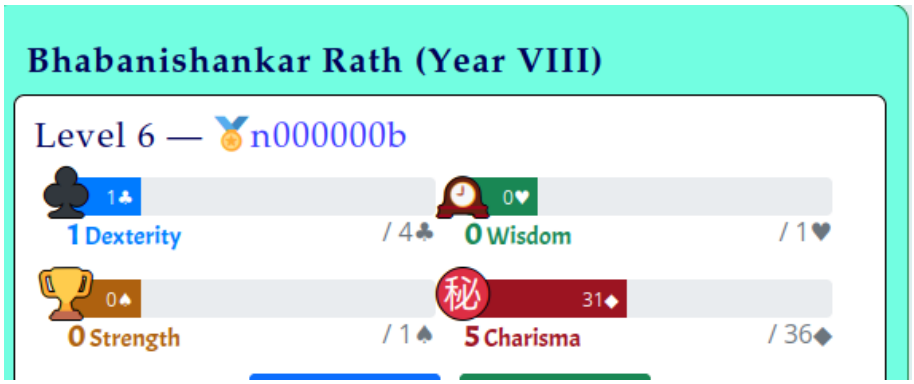

Figure 1: Levels

§4 Working

I want the system work in the following way:

- **Step 1.** The teacher should create a handout (example handout is attached in this pdf only)
- **Step 2.** The handout contains some practice problems or homework problems with some spades next to it, indicating the instructiveness of the problem; 9 is the highest 2 is the lowest.
- **Step 3.** The students are supposed to download the handout pdf from the website or the app where they can also see their spades worth and their level and so their score in the leaderboard.
- **Step 4.** The students are supposed to choose the problems they want to do after reading the theory and then write the problem number and the spades' worth of the problem in their homework copy.
- **Step 5.** The teacher checks off the solutions and gives them points with the maximum point being the spades' worth and the minimum being 0.
- **Step 6.** The teacher uploads the score to the website or app that will be developed later on or ask some it expert to do it for them. **Step 7.** The solutions would be discussed in school after the time limit.

#	Guess	Score	Player
1	108.108108	+0.86♠	Lincoln Liu
2	105.000000	+0.91♠	
3	103.000000	+0.94♠	
4	89.000000	+1.26♠	
5	84.000000	+1.42♠	Benjamin Wang-Tie
6	80.000000	+1.56♠	
7	70.000000	+2.04♠	Mahdi Mahi
8	69.000000	+2.10♠	Aditya Chandrasekhar
9	69.000000	+2.10♠	Eldar Iskanderov
10	69.000000	+2.10♠	William Zhao
11	69.000000	+2.10♠	Abdullahil Kafi
12	59.000000	+2.87♠	Arjun Arunachalam
13	55.000000	+3.31♠	Qiao Zhang
14	55.000000	+3.31♠	
15	55.000000	+3.31♠	Alexander Wang
16	55.000000	+3.31♠	Maximus Lu
17	54.000000	+3.43♠	
18	54.000000	+3.43♠	
19	54.000000	+3.43♠	
20	53.000000	+3.56♠	
21	53.000000	+3.56♠	Lerchen Zhong
22	52.000000	+3.70♠	Aaron Huang
23	50.000000	+4.00♠	Krishna Pothapragada
24	50.000000	+4.00♠	
25	50.000000	+4.00♠	
26	50.000000	+4.00♠	
27	50.000000	+4.00♠	Haozhe Yang
28	50.000000	+4.00♠	Vivian Loh
29	49.000000	+3.84♠	
30	49.000000	+3.84♠	Cordelia Hu
31	49.000000	+3.84♠	
32	48.000000	+3.69♠	
33	48.000000	+3.69♠	Carlos Villagordo Espinosa
34	47.000000	+3.53♠	
35	47.000000	+3.53♠	Jason Mao
36	47.000000	+3.53♠	Pranav Choudhary
37	46.000000	+3.39♠	Luke Robitaille
38	45.000000	+3.24♠	
39	45.000000	+3.24♠	Advaith Avadhanam
40	44.000000	+3.10♠	Elijah Liu
41	44.000000	+3.10♠	
42	43.000000	+2.96♠	Lin Liu

Figure 2: Leaderboard

- **Time limit.** The teacher is supposed to make a handout of a chapter or a section of the chapter if they wish and then upload it and ask the students to complete it in a week.
- **Types of problems.** The problems should range from various difficulties, and different kinds; like some based on memorization, some based on critical thinking, some based on problem solving etc.
- **Levels.** The levels work in the following way: level n consists of n^2 number of spades. For example, you complete level 1 when you have 1 spade, level 2 when you have 4 spades, level 3 when you have 9 spades, etc...
- **Grading.** I suggest using these levels for internal grading instead of the internal projects since this ensures participation.

Problem 4.1. Is it perfect?

Well, no. It is not perfect but it, in my opinion, is good enough to give a head start to the students who are slow at learning. I welcome suggestions ot this system.

§5 Conclusion

Introducing gamification has a lot of advantages and the current way of teaching is old-fashioned boring; turning the homework into a RPG-like game system would benefit the students in a lot of ways. This won't be more work for the teacher since the teachers are already used to creating Power-point slides for teaching. Gamification has shown to increase the speed at which one passes the first stage of the Dunning-Kruger graph. We can try learning from the big social media apps that the youth nowadays is oh-so-addicted to; instagram and facebook both use gamification in a certain way. If students are given autonomy to choose the problems they want to do, they will feel more like "I solved this and that" instead of the typical "I missed this problem and that problem." I noticed that the way the half-full glass of water looked was quite different between them. In the first case, I was freaking out about the other half of the problems I hadn't solved yet. In the second case, I was trying to decide which of the problems would be the most fun to do next and basically which ones I did in the end. It might be more psychological than anything else: Every problem you solve makes you better. Every problem you try and don't solve makes you better, too. But a problem you didn't have time to try doesn't make you worse.I'll admit to being mildly pissed off at high school for having built this particular mindset into all of their students. The straight-A students sitting in a maths class aren't counting how many questions they've answered correctly when checking grades. They're counting how many points they lost, we must change that! Therefore, let us try to change the place where the students spend their time while using the phone rather than trying to force them to get off it; it simply does not work the other way round as most of us might already know.

§6 Credits

I thank Evan Chen for providing me with such a beautiful Latex sty. This project was prepared by Bhabanishankar Rath. He studies in the Class IX 'A' of Kendriya Vidyalaya, Boudh, Odisha. He thanks you for having completed reading his presentation.

§7 Other Useful Details

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Polynomials

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> There's something about Algebra, I just can't figure it out. Polynomials, derivatives, quadratic equations, I see no absolute value in them. A bunch of irrational numbers with square roots and exponential functions. I'm still trying to see through the horizontal and vertical blurred lines. This all reminds me of Y I left my X.
>
> — Charmaine J Forde

§1 Introduction

Note. This unit is only an example of what I meant by "producing handouts." In no way is a teacher forced to use Latex like I am using here and nor is he/she expected to cover it in this format only. They have their freedom.

Polynomials are expressions with one or more terms with a non-zero coefficient. A polynomial can have more than one term. An algebraic expression

$$P(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ... + a_0 x^0.$$

Example 1.1

Some of the examples of polynomials are given below:

- $x^2 + 3x + 9$
- x+y
- $x^4 + 98x + 9$
- 69
- $x^2 + x + 1$

Some terms relating to the polynomials:

- **Term.** Each expression in a polynomial is called a term. To identify the number of terms, the trick I use is this: count how many number of "things" are there before each plus or minus sign or simply add 1 to how many *plus* + *minus* are there.

Example 1.2
$x^3 + 3x^2 + 5x - 8 = 0$.
Using our first trick, there is a x^3 before a plus, a $3x^2$ before the next plus, a $5x$ before the next plus, and finally a 8 before the last minus and therefore, this polynomial has 4 terms.

- **Constant Number vs Variable.** A variable is something that changes its value while a constant number does not.

Example 1.3
4 is a constant number since its value is always 4 but x is not since we can substitute *anything* in the place of x.

- **Coefficient.** This is the constant number before a variable in a polynomial.

Example 1.4
In the polynomial $2x^2 + 3x + 6$, the coefficient of x^2 is 2 since 2 is the constant number before x^2 and similarly 3 for x etc.

- **Like and Unlike terms.** The terms with the same power and the same variable are called like terms and the terms with not the same power and not the same variable are called unlike terms.

Example 1.5
x and $2x$ are like terms but $2x$ and $3z$ are not.

- **Types of Polynomial.** These are divided accordingly to the number of terms. A **monomial** is the one with 1 term; a **binomial** is the one with 2 terms; a **trinomial** is the one with 3 terms.

Example 1.6
$x + 2$ is a binomial;
x is a monomial;
$x + 3x^2 + 98$ is a trinomial.

- **Power/degree of a Polynomial.** The highest power of any given polynomial is called the power/degree of the polynomial.

Example 1.7
The power of $2x^2 + 4x^3 + 3x^7 + 68$ is 7. A polynomial of the degree/power 0 is called a **Constant Polynomial.**

Example 1.8
3 is a constant polynomial since it is equivalent to $3x^0$.

- **Roots or Zeroes of a Polynomial.** Simply put, the zeroes or the roots of a polynomial are those numbers after substituiting which in the place of x makes the polynomial 0.

Example 1.9

The zero of $P(x) = 4x + 4$ is -1 since $P(-1) = 4(-1) + 4 = 0$.

The zeros of $P(x) = x^2 - 3x + 2$ are 1 and 2 since $P(1) = 1^2 - 3(1) + 2 = 2^2 - 3(2) + 2 = 0$

§2 Theorems

This section is about some of the most important theorems in the field of polynomials.

§2.1 Remainder Theorem

Theorem 2.1

If p(x) is any polynomial having degree greater than or equal to 1 and if it is divided by the linear polynomial x – a, then the remainder is p(a).

Proof of this theorem is beyond the scope of this handout, however you can contact me if you are interested.

Example 2.2

Find the remainder when $x^3 - 4x^2 - 7x + 10$ is divided by $x - 2$.

We will simply use the remainder theorem. According to that, the answer should be $P(a = 2) = 2^3 - 4(2^2) - 7(2) + 10 = -12$.

Exercise 2.3. Verify that this is indeed the reminder by long polynomial division.

Theorem 2.4

x – c is a factor of the polynomial p(x), if p(c) = 0. Also, conversely, if x – c is a factor of p(x), then p(c) = 0.

This comes straight from the factor theorem. Proving this is one of your homework problems.

Example 2.5

Find the factors of the polynomial $P(x) = x^2 + 2x - 15$.

We simply use the factor theorem. According to that, the numbers at which our polynomial becomes zero should be our solution. We use **middle term splitting** here. This is also an example of middle-term splitting.
Observe that $x^2 + 2x - 15$ is equivalent to

$$x^2 + 5x - 3x - 15$$

which could be written as

$$x(x+5) - x(x+5)$$

or

$$(x+5)(x-3)$$

which means those two are our factors since $P(-5) = 0$ and $P(3) = 0$.

Exercise 2.6. Verify the solution above.

§3 Algebraic Identities

Algebraic Identities are something which are true for all values of the variables used. We will not prove any of them but some of their proofs are homework. We will be using them in the homework problems too.

- $(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$
- $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$
- $(x–y)^3 = x^3 – y^3 – 3xy(x–y)$
- $x^3 + y^3 + z^3 – 3xyz = (x+y+z)(x^2 + y^2 + z^2 – xy – yz – zx)$
- $(x+a)(x+b) = x^2 + (a+b)x + ab$
- $(x+y)^2 = x^2 + y^2 + 2xy$
- $(x-y)^2 = x^2 + y^2 - 2xy$
- $a^2 - b^2 = (a-b)(a+b)$

Example 3.1

Evaluate the following:

- $(399)^2$
- $(0.98)^2$
- Expand $(a-b)^3$
- Find $(2^3 + 3^3 + 5^3 - 90)$

Not complete solutions. 1 is really easy, and so is 2. The third bit is literally copy pasting the formula. The fourth bit is realizing that $x^3 + y^3 + z^3 – 3xyz = (x+y+z)(x^2 + y^2 + z^2 – xy – yz – zx)$.

§4 Practice Problems

The Law speaks: you are cast out. You are un-dwarf. I AM A WITNESS! Angarthing in *The Hammer of Thursagan*,
from *The Battle for Wesnoth*

Problem 4.1. 9♣ Factor the polynomial

$$a(b-c)^3 + b(c-a)^3 + c(a-b)^3.$$

Problem 4.2. 5♣ Prove all of the identities.

Problem 4.3. 9♣ Let a, b, c be real numbers. Prove that

$$a^3 + b^3 + c^3 = (a+b+c)^3 \quad \text{if and only if} \quad a^5 + b^5 + c^5 = (a+b+c)^5.$$

Problem 4.4. 2♣ Evaluate $(10099)^2$.

Problem 4.5. 3♣ Compute the value of $9x^2 + 4y^2$ if $xy = 6$ and $3x + 2y = 12$.

Problem 4.6. 2♣ Find the value of the polynomial $P(x) = 5x–4x^2 + 3$ at x = 2 and x = –1.

Problem 4.7. 3♣ Calculate the perimeter of a rectangle whose area is $25x^2–35x + 12$.

Problem 4.8. 3♣ Find the value of $x^3+y^3+z^3–3xyz$ if $x^2+y^2+z^2 = 83$ and $x+y+z = 15$

Problem 4.9. 2♣ If $a+b+c = 15$ and $a^2+b^2+c^2 = 83$, find the value of $a^3+b^3+c^3–3abc$.

Problem 4.10. 5♣ If $x - \frac{1}{x} = 4$, then find the values of $x^2 + \frac{1}{x^2}$ and $x^4 + \frac{1}{x^4}$.