FA with ε-Transitions

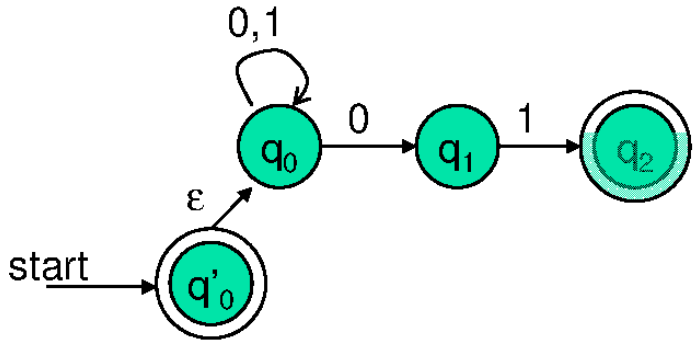
- We can allow <u>explicit</u> ε-transitions in finite automata
  - i.e., a transition from one state to another state without consuming any additional input symbol
  - Makes it easier sometimes to construct NFAs

<u>Definition:</u> ε -NFAs are those NFAs with at least one explicit ε-transition defined.

- ε -NFAs have one more column in their transition table

Example of an ε-NFA

L = {w | w is empty, or if non-empty will end in 01}



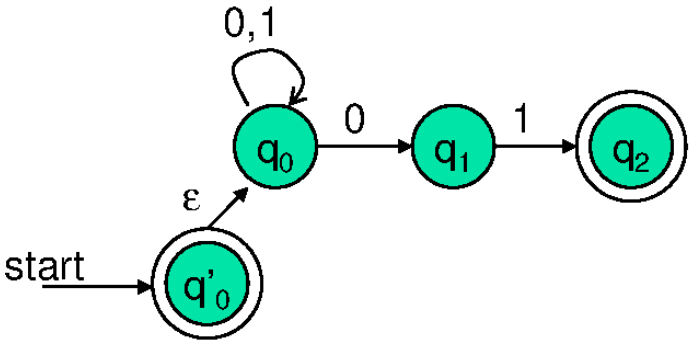
δ_E	0	1	ε	
→ $*q'_0$	∅	∅	$\{q'_0, q_0\}$	ECLOSE(q'_0)
q_0	$\{q_0, q_1\}$	$\{q_0\}$	$\{q_0\}$	ECLOSE(q_0)
q_1	∅	$\{q_2\}$	$\{q_1\}$	ECLOSE(q_1)
$*q_2$	∅	∅	$\{q_2\}$	ECLOSE(q_2)

- ε-closure of a state q, ***ECLOSE(q)***, is the set of all states (including itself) that can be *reached* from q by repeatedly making an arbitrary number of ε-transitions.

To simulate any transition:
Step 1) Go to all immediate destination states.
Step 2) From there go to all their ε-closure states as well.

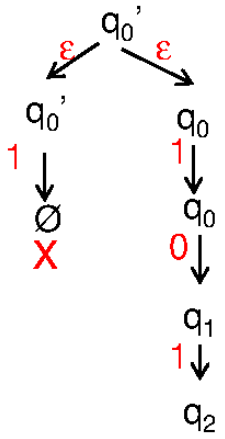
Example of an ε-NFA

L = {w | w is empty, or if non-empty will end in 01}



δ_E	0	1	ε	
→ *q'_0	Ø	Ø	$\{q'_0, q_0\}$	ECLOSE(q'_0)
q_0	$\{q_0, q_1\}$	$\{q_0\}$	$\{q_0\}$	ECLOSE(q_0)
q_1	Ø	$\{q_2\}$	$\{q_1\}$	
*q_2	Ø	Ø	$\{q_2\}$	

Simulate for w=101:

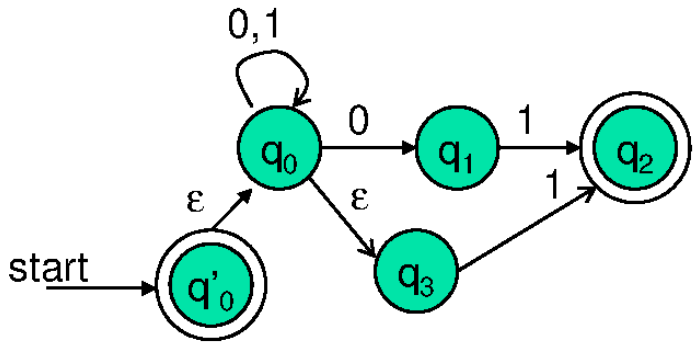


To simulate any transition:

Step 1) Go to all immediate destination states.

Step 2) From there go to all their ε-closure states as well.

Example of another ε-NFA



δ_E	0	1	ε
→ *q'_0	Ø	Ø	$\{q'_0,q_0,q_3\}$
q_0	$\{q_0,q_1\}$	$\{q_0\}$	$\{q_0,q_3\}$
q_1	Ø	$\{q_2\}$	$\{q_1\}$
*q_2	Ø	Ø	$\{q_2\}$
q_3	Ø	$\{q_2\}$	$\{q_3\}$

Simulate for w=101:

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