The Pumping Lemma for CFLs

Let L be a CFL.

Then there exists a constant N, s.t.,

- if $z \in L$ s.t. $|z| \geq N$, then we can write $z = uvwxy$, such that:
  1. $|vwx| \leq N$
  2. $vx \neq \varepsilon$
  3. For all $k \geq 0$: $uv^kwx^ky \in L$

Note: we are pumping in two places (v & x)

Application of Pumping Lemma for CFLs

Example 1: $L = \{a^m b^m c^m \mid m>0\}$

Claim: L is not a CFL

Proof:

- Let N <== P/L constant
- Pick $z = a^N b^N c^N$
- Apply pumping lemma to z and show that there exists at least one other string constructed from z (obtained by pumping up or down) that is $\notin$ L

Proof contd…

- $z = uvwxy$
- As $z = a^N b^N c^N$ *and* $|vwx| \leq N$ *and* $vx \neq \varepsilon$
  - ==> v, x cannot contain all three symbols (a,b,c)
  - ==> we can pump up or pump down to build another string which is $\notin$ L

Example #2 for P/L application

- $L = \{ ww \mid w \text{ is in } \{0,1\}^* \}$
- Show that L is not a CFL
  - Try string $z = 0^N 0^N$
    - what happens?
  - Try string $z = 0^N 1^N 0^N 1^N$
    - what happens?

Example 3

- L = { 0^{k^2} | k is any integer)
- Prove L is not a CFL using Pumping Lemma

Example 4

- $L = \{a^i b^j c^k \mid i<j<k\}$

- Prove that L is not a CFL