



Ambiguity in CFGs

- A CFG is said to be *ambiguous* if there exists a string which has more than one left-most derivation

Example:

$S \Rightarrow AS \mid \epsilon$

$A \Rightarrow A1 \mid 0A1 \mid 01$

Input string: 00111

Can be derived in two ways

LM derivation #1:

$S \Rightarrow AS$
 $\Rightarrow 0A1S$
 $\Rightarrow 00A11S$
 $\Rightarrow 00111S$
 $\Rightarrow 00111$

LM derivation #2:

$S \Rightarrow AS$
 $\Rightarrow A1S$
 $\Rightarrow 0A11S$
 $\Rightarrow 00111S$
 $\Rightarrow 00111$

Why does ambiguity matter?

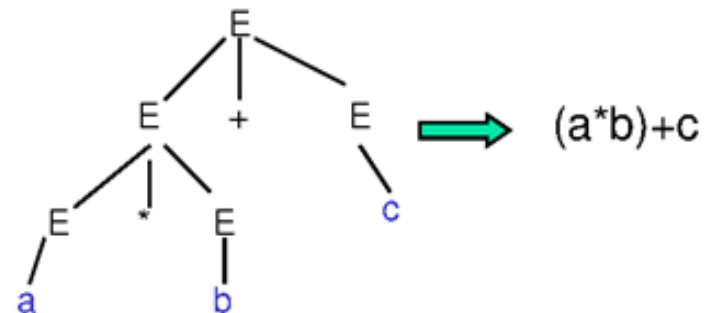
$E \Rightarrow E + E \mid E * E \mid (E) \mid a \mid b \mid c \mid 0 \mid 1$

Values are
different !!!

string = $a * b + c$

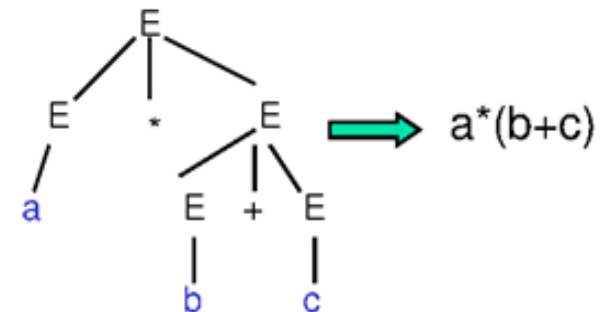
• LM derivation #1:

• $E \Rightarrow E + E \Rightarrow E * E + E \Rightarrow^* a * b + c$



• LM derivation #2

• $E \Rightarrow E * E \Rightarrow a * E \Rightarrow a * E + E \Rightarrow^* a * b + c$



The calculated value depends on which of the two parse trees is actually used.



Removing Ambiguity in Expression Evaluations

- It MAY be possible to remove ambiguity for some CFLs
 - E.g., in a CFG for expression evaluation by imposing rules & restrictions such as precedence
 - This would imply rewrite of the grammar

Modified unambiguous version:

- Precedence: $()$, $*$, $+$

$$\begin{aligned} E &\Rightarrow E + T \mid T \\ T &\Rightarrow T * F \mid F \\ F &\Rightarrow I \mid (E) \\ I &\Rightarrow a \mid b \mid c \mid 0 \mid 1 \end{aligned}$$

Ambiguous version:

$$E \Rightarrow E + E \mid E * E \mid (E) \mid a \mid b \mid c \mid 0 \mid 1$$

How will this avoid ambiguity?



Inherently Ambiguous CFLs

- However, for some languages, it may not be possible to remove ambiguity
- A CFL is said to be *inherently ambiguous* if every CFG that describes it is ambiguous

Example:

- $L = \{ a^n b^n c^m d^m \mid n, m \geq 1 \} \cup \{ a^n b^m c^m d^n \mid n, m \geq 1 \}$
- L is inherently ambiguous
- Why?

Input string: $a^n b^n c^n d^n$