

Formal Language and Automate Theory

Properties of Regular Languages

Closure under simple set operations- union, intersection, concatenation, complementation and star-closure.

Closure properties on regular languages are defined as certain operations on regular language which are guaranteed to produce regular language. Closure refers to some operation on a language, resulting in a new language that is of same “type” as originally operated on i.e., regular.

Regular languages are closed under following operations.

In an automata theory, there are different closure properties for regular languages. They are as follows –

- Union
- Intersection
- concatenation
- Kleene closure
- Complement

Union

If L_1 and L_2 are two regular languages, their union $L_1 \cup L_2$ will also be regular.

Example

$L_1 = \{a^n \mid n > 0\}$ and $L_2 = \{b^n \mid n > 0\}$

$L_3 = L_1 \cup L_2 = \{a^n \cup b^n \mid n > 0\}$ is also regular.

Intersection

If L_1 and L_2 are two regular languages, their intersection $L_1 \cap L_2$ will also be regular.

Example

$L_1 = \{a^m b^n \mid n > 0 \text{ and } m > 0\}$ and

$L_2 = \{a^m b^n \cup b^n a^m \mid n > 0 \text{ and } m > 0\}$

$L_3 = L_1 \cap L_2 = \{a^m b^n \mid n > 0 \text{ and } m > 0\}$ are also regular.

Concatenation

If L_1 and L_2 are two regular languages, their concatenation $L_1.L_2$ will also be regular.

Example

$L_1 = \{a^n \mid n > 0\}$ and $L_2 = \{b^n \mid n > 0\}$

$L_3 = L_1.L_2 = \{a^m . b^n \mid m > 0 \text{ and } n > 0\}$ is also regular.

Kleene Closure

If L_1 is a regular language, its Kleene closure L_1^* will also be regular.

Example

$L_1 = (a \cup b)$

$L_1^* = (a \cup b)^*$

Complement

If $L(G)$ is a regular language, its complement $L'(G)$ will also be regular. Complement of a language can be found by subtracting strings which are in $L(G)$ from all possible strings.

Example

$L(G) = \{a^n \mid n > 3\}$ $L'(G) = \{a^n \mid n \leq 3\}$

Note – Two regular expressions are equivalent, if languages generated by them are the same. For example, $(a+b)^$ and $(a+b)^*$ generate the same language. Every string which is generated by $(a+b)^*$ is also generated by $(a+b)^*$ and vice versa.*