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Unit – 5 \rightsquigarrow Inferential Statistics – II

Hypothesis Testing for Large Sample – II

Method – 1 \rightsquigarrow Test for Single Mean

Example of Method-1: Test for Single Mean

A	1	<p>The mean weight obtained from a random sample of size 100 is 64 gm. The S.D. of the weight distribution of the population is 3 gm. Test the statement that the mean weight of the population is 67 gms. at 5% level of significance. ($Z_{0.05} = 1.96$)</p> <p>Answer: The mean weight of the population is not 67 gm.</p>
A	2	<p>Sugar is packed in bags by an automation machine with mean contents of bags as 1.000 kg. A random sample of 36 bags is selected and mean mass has been found to be 1.003 kg. If a S.D. of 0.01 kg is acceptable on all the bags being packed, determine on the basis of sample test whether the machine requires adjustment. ($Z_{0.05} = 1.96$)</p> <p>Answer: The machine does not require any adjustment.</p>
A	3	<p>A random sample of 100 Indians has an average life span of 71.8 years with standard deviation of 8.9 years. Can it be concluded that the average life span of an Indian is 70 years? ($Z_{0.05} = 1.96$)</p> <p>Answer: The average life span of an Indian is not 70 years.</p>
A	4	<p>A random sample of 50 items gives the mean 6.2 and variance 10.24. Can it be regarded as drawn from a normal population with mean 5.4 at 5% level of significance? ($Z_{0.05} = 1.96$)</p> <p>Answer: The sample is drawn from a normal population with mean 5.4.</p>

Unit 5 Inferential Statistics - II

A	5	<p>It is claimed that a random sample of 49 tyre has a mean life of 15200 km. This sample was drawn from a population whose mean is 15150 km and a standard deviation of 1200 km. Test the significance at 0.05 level. ($Z_{0.05} = 1.96$)</p> <p>Answer: The null hypothesis is accepted</p>
B	6	<p>15.5 % of a random sample of 1600 undergraduate smokers, whereas 20% of a random sample of 900 postgraduate smokers in a state. Can we conclude that less number of undergraduates are smokers than postgraduates? ($Z_{0.05} = -1.645$)</p> <p>Answer: Yes, less number of undergraduates are smokers than postgraduates.</p>
A	7	<p>In a random sample of 60 workers, the average time taken by them to get to work is 33.8 minutes with a standard deviation of 6.1 minutes. Can we reject the null hypothesis $\mu > 32.6$ at $\alpha = 0.025$ level of significance? ($Z_{0.05} = 1.645$)</p> <p>Answer: The null hypothesis is accepted.</p>
B	8	<p>A tyre company claims that the lives of tyre have mean 42000 km with S.D. of 4000 km. A change in the production process is believed to result in better product. A test sample of 81 new tyre has a mean life of 42500 km. Test at 5% level of significance that the new product is significantly better than the old one. ($Z_{0.05} = 1.645$)</p> <p>Answer: The new product is not significantly better than the old one.</p>
C	9	<p>An ambulance service claims that it takes on the average 10 minutes to reach its destination in emergency calls. A sample of 36 calls has a mean of 11 minutes and the variance of 16 minutes. Test the claim at 0.05 level of significance. ($Z_{0.05} = 1.645$)</p> <p>Answer: The ambulance service takes on the average 10 minutes to reach its destination.</p>

Method – 2 \Rightarrow Test for Difference of Means

Example of Method-2: Test for Difference of Means

A	1	<p>Random samples drawn from two places gave the following data relating to the heights of children:</p> <table><tr><td></td><td>Mean height in cm</td><td>SD in cm</td><td>No. of samples</td></tr><tr><td>Place A</td><td>68.50</td><td>2.5</td><td>1200</td></tr><tr><td>Place B</td><td>68.58</td><td>3.0</td><td>1500</td></tr></table> <p>Test at 5% level of significance that the mean height is the same for children at two places. ($Z_{0.05} = 1.96$)</p> <p>Answer: The mean height is same for children at two places.</p>		Mean height in cm	SD in cm	No. of samples	Place A	68.50	2.5	1200	Place B	68.58	3.0	1500
	Mean height in cm	SD in cm	No. of samples											
Place A	68.50	2.5	1200											
Place B	68.58	3.0	1500											
A	2	<p>Samples of students were drawn from two universities and from their weights in kilograms, the mean and standard deviations are calculated. Make a large sample test to test the significance of the difference between the means. ($Z_{0.05} = 1.96$)</p> <table><tr><td></td><td>Mean</td><td>SD</td><td>Size of the Sample</td></tr><tr><td>University A</td><td>55</td><td>10</td><td>400</td></tr><tr><td>University B</td><td>57</td><td>15</td><td>100</td></tr></table> <p>Answer: There is no significant difference between the means.</p>		Mean	SD	Size of the Sample	University A	55	10	400	University B	57	15	100
	Mean	SD	Size of the Sample											
University A	55	10	400											
University B	57	15	100											
A	3	<p>In a certain factory there are two different processes of manufacturing the same item. The average weight in a sample of 250 items produced from one process is found to be 120 gm with a SD of 12 gm; the corresponding figures in a sample of 400 items from the other process are 124 gm and 14 gm. Is this difference between the two sample means significant? ($Z_{0.05} = 1.96$)</p> <p>Answer: There is significant difference between the means.</p>												
A	4	<p>The mean life of a sample of 10 electric bulbs was found to be 1456 hours with SD of 423 hours. A second sample of 17 bulbs chosen from a different batch showed a mean life of 1280 with SD of 398 hours. Is there a significant difference between the means of two batches? ($Z_{0.05} = 1.96$)</p> <p>Answer: There is no difference between the mean of two batches.</p>												

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C	5	<p>For sample I, $n_1 = 1000$, $\sum x = 49,000$, $\sum (x - \bar{x})^2 = 7,84,000$. For sample II, $n_2 = 1500$, $\sum x = 70,500$, $\sum (x - \bar{x})^2 = 24,00,000$. Discuss the significance of the difference of the sample means. ($Z_{0.05} = 1.96$)</p> <p>Answer: No significant difference between the sample means.</p>									
B	6	<p>A company claims that alloying reduces resistance of electric wire by more than 0.050 ohm. To test this claim samples of 32 standard wire and alloyed wire are tested yielding the following results. ($Z_{0.05} = 1.645$).</p> <table border="1"> <thead> <tr> <th>Type of wire</th><th>Mean resistance (ohms)</th><th>S.D. (ohms)</th></tr> </thead> <tbody> <tr> <td>Standard</td><td>0.136</td><td>0.004</td></tr> <tr> <td>Alloyed</td><td>0.083</td><td>0.005</td></tr> </tbody> </table> <p>At the 0.05 level of significance, does this support the claim?</p> <p>Answer: The data supports the claim.</p>	Type of wire	Mean resistance (ohms)	S.D. (ohms)	Standard	0.136	0.004	Alloyed	0.083	0.005
Type of wire	Mean resistance (ohms)	S.D. (ohms)									
Standard	0.136	0.004									
Alloyed	0.083	0.005									
B	7	<p>A simple sample of heights of 6400 English men has a mean of 170 cm and a S.D. of 6.4 cm, while a simple sample of heights of 1600 Americans has a mean of 172 cm and a S.D. of 6.3 cm. Do the data indicate that American are, on the average, taller than the English men? ($Z_{0.01} = 2.33$)</p> <p>Answer: Yes, American are, on the average, taller than the English men.</p>									

Method – 3 \Rightarrow Test for Difference of Standard Deviation

Example of Method-3: Test for Difference of Standard Deviations

A	1	<p>The SD of a random sample of 1000 is found to be 2.6 and the SD of another random sample of 500 is 2.7. Assuming the samples to be independent, find whether the two samples could have come from populations with the same SD. ($Z_{0.05} = 1.96$)</p> <p>Answer: Two samples could have come from populations with the same SD.</p>									
A	2	<p>Intelligence test of two groups of boys and girls gives the following results:</p> <table border="1"> <thead> <tr> <th></th><th>n</th><th>S. D.</th></tr> </thead> <tbody> <tr> <td>Girls</td><td>121</td><td>10</td></tr> <tr> <td>Boys</td><td>81</td><td>12</td></tr> </tbody> </table> <p>Is the difference between the standard deviations significant? ($Z_{0.05} = 1.96$)</p> <p>Answer: There is no significant difference between sample SDs.</p>		n	S. D.	Girls	121	10	Boys	81	12
	n	S. D.									
Girls	121	10									
Boys	81	12									
A	3	<p>The mean yield of two plots and their variability are as given below: No. of plot = 40 ; SD = 34 and No. of plot = 60 ; SD = 28. Check whether the difference in the variability in yields is significant. ($Z_{0.05} = 1.96$)</p> <p>Answer: There is no significant difference between sample SDs.</p>									
A	4	<p>Examine whether the two samples for which the data are given in the following table could have been drawn from populations with the same SD.</p> <table border="1"> <thead> <tr> <th></th><th>Size</th><th>SD</th></tr> </thead> <tbody> <tr> <td>Sample I</td><td>100</td><td>5</td></tr> <tr> <td>Sample II</td><td>200</td><td>7</td></tr> </tbody> </table> <p>Is the difference between the standard deviation significant? ($Z_{0.05} = 1.96$)</p> <p>Answer: The sample standard deviations do not differ significantly.</p>		Size	SD	Sample I	100	5	Sample II	200	7
	Size	SD									
Sample I	100	5									
Sample II	200	7									

Hypothesis Testing for Small Sample

Method – 4 \Rightarrow t - Test for Single Mean

Example of Method-4: t - Test for Single Mean

A	1	<p>A machine is designed to produce insulating washers for electrical devices of average thickness of 0.025 cm. A random sample of 10 washers was found to have an average thickness of 0.024 cm with S.D. of 0.002 cm. Test the significance of the deviation.</p> <p>($t_{0.05,9} = 2.2622$)</p> <p>Answer: There is no significant difference between population mean and sample mean.</p>
A	2	<p>A random sample of six steel beams has a mean compressive strength of 58392 psi (pounds per square inch) with a SD of 648 psi. Use this information and level of significance $\alpha = 0.05$ to test whether the true average compressive strength of the steel from which this sample came is 58000 psi. Assume normality. ($t_{0.05,5} = 2.5706$)</p> <p>Answer: The average compressive strength of the steel beam is not equal to 58000 psi.</p>
A	3	<p>A random sample of size 16 from a normal population showed a mean of 103.75 cm and sum of squares of deviations from the mean 843.75 cm² can we say that the population has a mean of 108.75 cm? ($t_{0.05,15} = 2.1314$)</p> <p>Answer: No, We cannot say that the population mean is 108.75 cm.</p>
B	4	<p>A manufacturer of external hard drives claims that only 10% of his drives require repairs within the warranty period of 12 months. If 5 of 20 of his drives required repairs within the first year, does this tend to support or refute the claim? ($t_{0.05,19} = 1.7291$)</p> <p>Answer: The claim should be refuted.</p>

Unit 5 Inferential Statistics - II

A	5	<p>A random sample of 10 boys had the following IQs: 70, 120, 110, 101, 88, 83, 95, 98, 107 and 100. Do these data support the assumption of a population mean IQ of 100? Find 95% confidence limits for the mean IQ.</p> <p>($t_{0.05,9} = 2.2622$)</p> <p>Answer: 86.9892 and 107.4108</p>
A	6	<p>The 9 items of a sample have the values 45, 47, 50, 52, 48, 47, 49, 53, 51. Does the mean of these values differ significantly from assumed mean 47.5? ($t_{0.05,9} = 2.2622$)</p> <p>Answer: The mean of given values does not differ significantly from assumed mean 47.5.</p>
B	7	<p>Producer of gutkha claims that the nicotine content in his gutkha on the average is 1.83 mg. Can this claim be accepted if a random sample of 8 gutkha of this type have the nicotine contents of 2, 1.7, 2.1, 1.9, 2.2, 2.1, 2, 1.6 mg? Use a 0.05 level of significance. ($t_{0.05,7} = 2.3646$)</p> <p>Answer: Yes, The claim is accepted.</p>
C	8	<p>The mean weekly sales of soap bars in departmental stores was 146.3 bars per store. After an advertising campaign the mean weekly sales in 22 stores for a typical week increased to 153.7 and showed a standard deviation of 17.2. Was the advertising campaign successful? ($t_{0.05,21} = 1.7207$)</p> <p>Answer: The advertisement campaign is successful.</p>
B	9	<p>A random sample from a company's very extensive files shows that the orders for a certain kind of machinery were filled respectively in 10, 12, 19, 14, 15, 18, 11 and 13 days. Use the level of significance $\alpha = 0.01$ to test the claim that on the average such orders are filled in 10.5 days. Choose and Test the alternative hypothesis so that rejection of null hypothesis $\mu = 10.5$ days implies that it takes longer than indicated. ($t_{0.01,7} = 2.9980$)</p> <p>Answer: The orders on average are filled in more than 10.5 days.</p>

Method – 5 \Rightarrow t - Test for Difference of Means

Example of Method-5: t - Test for Difference of Means

A	1	<p>Two sample of 6 and 5 items, respectively, gave the following data.</p> <table border="1" data-bbox="316 461 884 633"> <thead> <tr> <th></th><th>1st sample</th><th>2nd sample</th></tr> </thead> <tbody> <tr> <td>Mean</td><td>40</td><td>50</td></tr> <tr> <td>S.D.</td><td>8</td><td>10</td></tr> </tbody> </table> <p>Is the difference of the means significant? (Test at 5% level of significance) $(t_{0.05,9} = 2.2622)$</p> <p>Answer: There is no significant difference between two population means.</p>		1st sample	2nd sample	Mean	40	50	S.D.	8	10
	1st sample	2nd sample									
Mean	40	50									
S.D.	8	10									
A	2	<p>Two sample of 10 and 14 items, respectively, gave the following data.</p> <table border="1" data-bbox="316 947 884 1120"> <thead> <tr> <th></th><th>1st sample</th><th>2nd sample</th></tr> </thead> <tbody> <tr> <td>Mean</td><td>20.3</td><td>18.6</td></tr> <tr> <td>S.D.</td><td>3.5</td><td>5.2</td></tr> </tbody> </table> <p>Is the difference of the means significant? $(t_{0.05,22} = 2.0739)$</p> <p>Answer: There is no significant difference between two population means.</p>		1st sample	2nd sample	Mean	20.3	18.6	S.D.	3.5	5.2
	1st sample	2nd sample									
Mean	20.3	18.6									
S.D.	3.5	5.2									
B	3	<p>A mechanist is making engine parts with axle diameter of 0.7 cm. A random sample of 10 parts shows a mean diameter of 0.742 cm with a standard deviation of 0.04 cm. Compute the statistic you would use to test whether work is meeting the specification at 0.05 level of significance. $(t_{0.05,9} = 2.2622)$</p> <p>Answer: The sample are not drawn from the same population.</p>									

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A	4	<p>The following figures refer to observations in live independent samples:</p> <table><tr><td>Sample I</td><td>25</td><td>30</td><td>28</td><td>34</td><td>24</td><td>20</td><td>13</td><td>32</td><td>22</td><td>38</td></tr><tr><td>Sample II</td><td>40</td><td>34</td><td>22</td><td>20</td><td>31</td><td>40</td><td>30</td><td>23</td><td>36</td><td>17</td></tr></table> <p>Analyze whether the samples have been drawn from the population of equal means. Test whether the means of two populations are same at 5% level. ($t_{0.05,18} = 2.1009$)</p> <p>Answer: Samples have been drawn from population with equal mean.</p>	Sample I	25	30	28	34	24	20	13	32	22	38	Sample II	40	34	22	20	31	40	30	23	36	17
Sample I	25	30	28	34	24	20	13	32	22	38														
Sample II	40	34	22	20	31	40	30	23	36	17														
A	5	<p>Random samples of specimens of coal from two mines A & B are drawn and their heat producing capacity (in millions of calories per ton) were measured yielding the following result:</p> <table><tr><td>Mine A</td><td>8260</td><td>8130</td><td>8350</td><td>8070</td><td>8340</td><td>—</td></tr><tr><td>Mine B</td><td>7950</td><td>7890</td><td>7900</td><td>8140</td><td>7920</td><td>7840</td></tr></table> <p>Test whether the difference between the means of these two samples is significant. ($t_{0.05,9} = 2.2622$)</p> <p>Answer: There is significant difference in average heat producing capacity of coal from mines.</p>	Mine A	8260	8130	8350	8070	8340	—	Mine B	7950	7890	7900	8140	7920	7840								
Mine A	8260	8130	8350	8070	8340	—																		
Mine B	7950	7890	7900	8140	7920	7840																		
B	6	<p>A group of 5 patients treated with medicine A weight 42, 39, 48, 60 and 41 kg. Second group of 7 patients from the same hospitals treated with medicine B weigh 38, 42, 56, 64, 68, 69 and 62 kg. do you agree with the claim that medicine B increases the weight significantly? ($t_{0.05,10} = -1.8125$)</p> <p>Answer: The medicine B does not increase in weight.</p>																						
B	7	<p>A large group of teachers are trained, where some are trained by institution A and some are trained by institution B. In a random sample of 10 teachers taken from a large group; the following marks are obtained in an appropriate achievement test.</p> <table><tr><td>Institution A</td><td>65</td><td>69</td><td>73</td><td>71</td><td>75</td><td>66</td><td>71</td><td>68</td><td>68</td><td>74</td></tr><tr><td>Institution B</td><td>78</td><td>69</td><td>72</td><td>77</td><td>84</td><td>70</td><td>73</td><td>77</td><td>75</td><td>65</td></tr></table> <p>Test the claim that institute B is more effective. ($t_{0.05,18} = 1.7341$)</p> <p>Answer: The claim is valid.</p>	Institution A	65	69	73	71	75	66	71	68	68	74	Institution B	78	69	72	77	84	70	73	77	75	65
Institution A	65	69	73	71	75	66	71	68	68	74														
Institution B	78	69	72	77	84	70	73	77	75	65														

Method – 6 \Rightarrow t - Test for Correlation Coefficient

Example of Method-6: t - Test for Correlation Coefficient

A	1	<p>A random sample of fifteen paired observations from a bivariate population gives a correlation coefficient of -0.5. Does this signify the existence of correlation in the sample population? ($t_{0.05,13} = 2.1604$)</p> <p>Answer: The sample population is uncorrelated.</p>
A	2	<p>A coefficient of correlation of 0.2 is derived from a random sample of 625 pairs of observations. Is this value of r significant? ($t_{0.05,623} = 1.9600$)</p> <p>Answer: It is highly significant.</p>

Method – 7 \Rightarrow F – Test for Ratio of Variances

Example of Method-7: F - Test for Ratio of Variances

A	1	<p>In two independent samples of sizes 8 and 10 the sum of squares of derivations of the sample's values from the respective sample means were 84.4 and 102.6. Test whether the difference of variances of the populations is significant or not. ($F_{0.05}(7,9) = 3.29$)</p> <p>Answer: There is no significant difference between the variances of two populations.</p>																
A	2	<p>Two samples of size 9 and 8 give the sum of squares of deviations from their respective means equal 160 inches and 91 inches respectively. Can they be regarded as drawn from two normal populations with the same variance? ($F_{0.05}(8,7) = 3.73$)</p> <p>Answer: The samples can be regarded as drawn from normal population with same SD.</p>																
B	3	<p>Two independent sample of size 7 and 6 had the following values:</p> <table border="1"><tr><td>A</td><td>28</td><td>30</td><td>32</td><td>33</td><td>31</td><td>29</td><td>34</td></tr><tr><td>B</td><td>29</td><td>30</td><td>30</td><td>24</td><td>27</td><td>28</td><td>-</td></tr></table> <p>Examine whether the samples have been drawn from normal populations having the same variance. ($F_{0.05}(5,6) = 4.39$)</p> <p>Answer: Samples have been drawn from the normal populations with same variance.</p>	A	28	30	32	33	31	29	34	B	29	30	30	24	27	28	-
A	28	30	32	33	31	29	34											
B	29	30	30	24	27	28	-											

Unit 5 Inferential Statistics - II

B	4	<p>Two independent samples of 8 and 7 items respectively had the following values of the variable (weight in kg):</p> <table><tr><td>Sample I</td><td>9</td><td>11</td><td>13</td><td>11</td><td>15</td><td>9</td><td>12</td><td>14</td></tr><tr><td>Sample II</td><td>10</td><td>12</td><td>10</td><td>14</td><td>9</td><td>8</td><td>10</td><td>-</td></tr></table> <p>Do the two estimates of population variance differ significantly? ($F_{0.05}(7,6) = 4.21$)</p> <p>Answer: There is no significant difference between two estimates of population variances.</p>	Sample I	9	11	13	11	15	9	12	14	Sample II	10	12	10	14	9	8	10	-				
Sample I	9	11	13	11	15	9	12	14																
Sample II	10	12	10	14	9	8	10	-																
B	5	<p>Two samples are drawn from two normal populations. From the following data test whether the two samples have the same variance at 5 % level? ($F_{0.05}(9,7) = 3.68$)</p> <table><tr><td>Sample I</td><td>60</td><td>65</td><td>71</td><td>74</td><td>76</td><td>82</td><td>85</td><td>87</td><td>-</td><td>-</td></tr><tr><td>Sample II</td><td>61</td><td>66</td><td>67</td><td>85</td><td>78</td><td>63</td><td>85</td><td>86</td><td>88</td><td>91</td></tr></table> <p>Answer: Two samples have the same variances.</p>	Sample I	60	65	71	74	76	82	85	87	-	-	Sample II	61	66	67	85	78	63	85	86	88	91
Sample I	60	65	71	74	76	82	85	87	-	-														
Sample II	61	66	67	85	78	63	85	86	88	91														
C	6	<p>The standard deviations calculated from two random samples of sizes 9 and 13 are 2.1 and 1.8 respectively. Can the samples be regarded as drawn from normal populations with the same SD? ($F_{0.05}(8,12) = 2.85$; $t_{0.05,20} = 2.0860$)</p> <p>Answer: The samples can be regarded as drawn from normal population with same SD.</p>																						
C	7	<p>Two random samples drawn from 2 normal populations are as follows:</p> <table><tr><td>A</td><td>17</td><td>27</td><td>18</td><td>25</td><td>27</td><td>29</td><td>13</td><td>17</td></tr><tr><td>B</td><td>16</td><td>16</td><td>20</td><td>27</td><td>26</td><td>25</td><td>21</td><td>-</td></tr></table> <p>Test whether the samples are drawn from the same normal population. ($F_{0.05}(7,6) = 4.21$; $t_{0.05,13} = 2.1604$)</p> <p>Answer: The samples are drawn from same normal population.</p>	A	17	27	18	25	27	29	13	17	B	16	16	20	27	26	25	21	-				
A	17	27	18	25	27	29	13	17																
B	16	16	20	27	26	25	21	-																

Unit 5 Inferential Statistics - II

Chi – Square Test: Introduction

- The chi-square (χ^2) test is a useful measure of comparing experimentally obtained results with those expected theoretically and based on hypothesis.
- Symbol χ^2 is read as “**Ky Square**”.
- It is used as a test statistic in testing a hypothesis that provides a set of theoretical frequencies with which observed frequencies are compared.
- The magnitude of discrepancy between observed and theoretical frequencies is given by the quantity χ^2 .
- If $\chi^2 = 0$, the observed and expected frequencies completely coincides. As the value of χ^2 increases, the discrepancy between the observed and theoretical frequency decreases.
- If $f_{o_1}, f_{o_2}, \dots, f_{o_n}$ be a set of observed frequencies and $f_{e_1}, f_{e_2}, \dots, f_{e_n}$ be the corresponding set of expected frequencies χ^2 then it is defined by,

$$\chi^2 = \frac{(f_{o_1} - f_{e_1})^2}{f_{e_1}} + \frac{(f_{o_2} - f_{e_2})^2}{f_{e_2}} + \dots + \frac{(f_{o_n} - f_{e_n})^2}{f_{e_n}} = \sum_{i=1}^n \frac{(f_{o_i} - f_{e_i})^2}{f_{e_i}}$$

- **Conditions for validity of χ^2 test:**
 - The sample observations should be independent.
 - The total frequency $N = \sum f_i$ should be reasonably large, say, greater than 50.
 - Each expected frequency $f_{e_i} \geq 5$. If not, then it is pooled with preceding or succeeding frequency so that the pooled frequency is more than 5. In this case, we need to adjust degree of freedom lost in pooling.
- **Applications of χ^2 test:**
 - Goodness of Fit
 - Independence of Attributes

Unit 5 Inferential Statistics - II

Working Rule for Chi- Square Test

- **Step 1:** Set up null hypothesis H_0 .
- **Step 2:** Set up alternative hypothesis H_1 .
- **Step 3:** Set up level of significance α .
- **Step 4:** Find appropriate f_e and apply test statistic.
- **Step 5:** Find appropriate degree of freedom and set up critical region. (Given in data OR to find from statistical tabular table).
- **Step 6:** Conclusion.
 - Compare the computed value of χ^2 with critical value $\chi_{\alpha, v}$.
 - If $\chi^2 > \chi_{\alpha, v}$, we **reject H_0** and conclude that there is significant difference.
 - If $\chi^2 < \chi_{\alpha, v}$, we **accept H_0** and conclude that there is no significant difference.

Method – 8 \rightsquigarrow Chi – Square Test: Goodness of Fit

Example of Method-8: Chi – Square Test: Goodness of Fit

A	1	<p>A die is thrown 276 times and the results of these throws are given below:</p> <table><tr><td>Number appeared on the die</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>Frequency</td><td>40</td><td>32</td><td>29</td><td>59</td><td>57</td><td>59</td></tr></table> <p>Test whether the die is biased or not. ($\chi^2_{0.05,5} = 11.070$)</p> <p>Answer: The die is biased.</p>	Number appeared on the die	1	2	3	4	5	6	Frequency	40	32	29	59	57	59
Number appeared on the die	1	2	3	4	5	6										
Frequency	40	32	29	59	57	59										
A	2	<p>The following table gives the number of accidents that took place in an industry during various days of the week. Test if accidents are uniformly distributed over the week.</p> <table><tr><td>Day</td><td>Mon</td><td>Tue</td><td>Wed</td><td>Thus</td><td>Fri</td><td>Sat</td></tr><tr><td>No. of accidents</td><td>14</td><td>18</td><td>12</td><td>11</td><td>15</td><td>14</td></tr></table> <p>($\chi^2_{0.05,5} = 11.070$)</p> <p>Answer: The accidents are uniformly distributed over the week.</p>	Day	Mon	Tue	Wed	Thus	Fri	Sat	No. of accidents	14	18	12	11	15	14
Day	Mon	Tue	Wed	Thus	Fri	Sat										
No. of accidents	14	18	12	11	15	14										
C	3	<p>A sample analysis of examination results of 200 Computer Engineer was made. It was found that 46 students had failed, 68 secured a third division, 62 secured a second division and the rest were placed in first division. Are these figures commensurate with the general examination result which is in the ratio of 4 : 3 : 2 : 1 for various categories respectively?</p> <p>($\chi^2_{0.05,3} = 7.815$)</p> <p>Answer: The data are not commensurate with the general examination result.</p>														

C	4	<p>A set of five similar coins is tossed 320 times and result is obtained as follows:</p> <table><tr><td>No. of male heads</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>Frequency</td><td>6</td><td>27</td><td>72</td><td>112</td><td>71</td><td>32</td></tr></table> <p>Test the hypothesis that the data follow a binomial distribution.</p> <p>$(\chi^2_{0.05,5} = 11.070)$</p> <p>Answer: The data don't follow binomial distribution.</p>	No. of male heads	0	1	2	3	4	5	Frequency	6	27	72	112	71	32
No. of male heads	0	1	2	3	4	5										
Frequency	6	27	72	112	71	32										
C	5	<p>The following mistakes per page were observed in a book:</p> <table><tr><td>No. of mistakes per page</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>No. of pages</td><td>211</td><td>90</td><td>19</td><td>5</td><td>0</td></tr></table> <p>Fit a Poisson distribution and test the goodness of fit. $(\chi^2_{0.05,4} = 9.488)$</p> <p>Answer: Mistakes follow Poisson's distribution.</p>	No. of mistakes per page	0	1	2	3	4	No. of pages	211	90	19	5	0		
No. of mistakes per page	0	1	2	3	4											
No. of pages	211	90	19	5	0											

Method – 9 \rightsquigarrow Chi – Square Test: Independence of Attributes

Example of Method-9: Chi – Square Test: Independence of Attributes

B

1

A random sample of 500 students were classified according to economic condition of their family and also according to merit as shown below:

Merit	Economic Condition			Total
	Rich	Middleclass	Poor	
Meritorious	42	137	61	240
Not – Meritorious	58	113	89	260
Total	100	250	150	500

Test whether the two attributes merit and economic condition are associated or not. ($\chi^2_{0.05,2} = 5.991$)

Answer: Two attributes are associated.

B

2

From the following data, find whether there is any significant linking in the habit of taking soft drinks among the categories of employees.

($\chi^2_{0.05,4} = 9.488$)

Soft drinks	Employees		
	Clerks	Teachers	Officers
Pepsi	10	25	65
Thumsup	15	30	65
Fanta	50	60	30

Answer: Two attributes are not independent.

Unit 5 Inferential Statistics - II

B	3	<p>A company operates three machines on three different shifts daily. The following table presents the data of the machine breakdowns resulted during a 6-month time period.</p> <table><tr><th>Shift</th><th>Machine A</th><th>Machine B</th><th>Machine C</th></tr><tr><td>1</td><td>12</td><td>12</td><td>11</td></tr><tr><td>2</td><td>15</td><td>25</td><td>13</td></tr><tr><td>3</td><td>17</td><td>23</td><td>10</td></tr></table> <p>Test hypothesis that for an arbiter breakdown machine causing breakdown & the shift on which the breakdown occurs are independent.</p> <p>$(\chi^2_{0.05,4} = 9.488)$</p> <p>Answer: Machine causing breakdown and the shift are independent.</p>	Shift	Machine A	Machine B	Machine C	1	12	12	11	2	15	25	13	3	17	23	10																		
Shift	Machine A	Machine B	Machine C																																	
1	12	12	11																																	
2	15	25	13																																	
3	17	23	10																																	
B	4	<p>From the following data, find whether hair color and gender are associated.</p> <table><tr><th>Color</th><th>Fair</th><th>Red</th><th>Medium</th><th>Dark</th><th>Black</th><th>Total</th></tr><tr><td>Boys</td><td>592</td><td>849</td><td>504</td><td>119</td><td>36</td><td>2100</td></tr><tr><td>Girls</td><td>544</td><td>677</td><td>451</td><td>97</td><td>14</td><td>1783</td></tr><tr><td>Total</td><td>1136</td><td>1526</td><td>955</td><td>216</td><td>50</td><td>3883</td></tr></table> <p>$(\chi^2_{0.05,4} = 9.488)$</p> <p>Answer: The hair color and gender are associated.</p>	Color	Fair	Red	Medium	Dark	Black	Total	Boys	592	849	504	119	36	2100	Girls	544	677	451	97	14	1783	Total	1136	1526	955	216	50	3883						
Color	Fair	Red	Medium	Dark	Black	Total																														
Boys	592	849	504	119	36	2100																														
Girls	544	677	451	97	14	1783																														
Total	1136	1526	955	216	50	3883																														
C	5	<p>The following table gives the level of education and the marriage adjustment score for a sample of married women:</p> <table><tr><th rowspan="2">Level of Education</th><th colspan="4">Marriage Adjustment</th><th rowspan="2">Total</th></tr><tr><th>Very Low</th><th>Low</th><th>High</th><th>Very High</th></tr><tr><td>College</td><td>24</td><td>97</td><td>62</td><td>58</td><td>241</td></tr><tr><td>High School</td><td>22</td><td>28</td><td>30</td><td>41</td><td>121</td></tr><tr><td>Middle School</td><td>32</td><td>10</td><td>11</td><td>20</td><td>73</td></tr><tr><td>Total</td><td>78</td><td>135</td><td>103</td><td>119</td><td>435</td></tr></table> <p>Can you conclude from the above data the higher the level of education, the greater is the degree of adjustment in marriage? $(\chi^2_{0.05,6} = 12.592)$</p> <p>Answer: Level of education and adjustment in marriage are related.</p>	Level of Education	Marriage Adjustment				Total	Very Low	Low	High	Very High	College	24	97	62	58	241	High School	22	28	30	41	121	Middle School	32	10	11	20	73	Total	78	135	103	119	435
Level of Education	Marriage Adjustment				Total																															
	Very Low	Low	High	Very High																																
College	24	97	62	58	241																															
High School	22	28	30	41	121																															
Middle School	32	10	11	20	73																															
Total	78	135	103	119	435																															

***** End of the Unit *****