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Unit - 2 → Probability

Method - 1 → Counting

Example of Method-1: Counting

| A | 1 | How many four digit numbers are there with no digit repeated? |
|---|---|---|
| | | Answer: 4536 |
| A | 2 | How many 3-digit numbers can be formed by using the digits 1 to 9 if no digit |
| | | is repeated? What if repetition is allowed? |
| | | Answer: 504 (without repeating), 729 (with repeating) |
| Α | 3 | How many words, with or without meaning, can be formed using all letters |
| | | of the word EQUATION, using each letter exactly once? |
| | | Answer: 40320 |
| В | 4 | Find the number of permutations of the letters of the word ALLAHABAD. |
| | | Answer: 7560 |
| A | 5 | How many ways to select 2 students from 5 students? |
| | | Answer: 10 |
| | | |
| Α | 6 | In how many different ways can 4 of 15 laboratory assistants be chosen to |
| | | assist with an experiment? |
| | | Answer: 1365 |
| В | 7 | Out of 6 boys and 4 girls in how many ways a committee of five members can |
| | | be formed in which there are at most 2 girls are included? |
| | | Answer: 186 |
| | | |





Method - 2 → Basic Terminologies and results on probability

Example of Method-2.1: Sample Space and Event

| Α | 1 | Describe the sample space for the indicated random experiments. |
|---|---|--|
| | | (1) A coin is tossed 3 times. |
| | | (2) A coin and die are tossed together. |
| | | |
| | | Answer: (1) $S = \{ HHH, HHT, HTH, HTT, THH, THT, TTH, TTT \}$ |
| | | (2) S = { H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6 } |
| Α | 2 | Find the sample space associated with the experiment of rolling a pair of dice |
| | | once. Also, find the number of elements of this sample space. |
| | | |
| | | Answer: S = $\begin{cases} (1, 1), & (1, 2), & (1, 3), & (1, 4), & (1, 5), & (1, 6) \\ (2, 1), & (2, 2), & (2, 3), & (2, 4), & (2, 5), & (2, 6) \\ (3, 1), & (3, 2), & (3, 3), & (3, 4), & (3, 5), & (3, 6) \\ (4, 1), & (4, 2), & (4, 3), & (4, 4), & (4, 5), & (4, 6) \\ (5, 1), & (5, 2), & (5, 3), & (5, 4), & (5, 5), & (5, 6) \\ (6, 1), & (6, 2), & (6, 3), & (6, 4), & (6, 5), & (6, 6) \end{cases}$ |
| | | (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6) |
| | | (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6) |
| | | Answer: $S = \{ (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6) \}$ |
| | | (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6) |
| | | (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) |
| | | (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) |
| | | |
| | | Number of elements in sample space $= 6 \times 6 = 6^2 = 36$ |
| В | 3 | A coin is tossed. If it shows head, we draw a ball from a bag which contains 3 |
| | | blue and 4 white balls; if it shows tail, we throw a die. |
| | | Describe the sample space of this experiment. |
| | | |
| | | |
| | | Answer: $S = \begin{cases} HB_1, & HB_2, & HB_3, & HW_1, & HW_2, & HW_3, & HW_4, \\ & & T1, & T2, & T3, & T4, & T5, & T6 \end{cases}$ |
| | | T1, T2, T3, T4, T5, T6 |
| | |) |



A One die of red color, one of white color and one of blue color are placed in a bag. One die is selected at random and rolled, its color and the number on its uppermost face is noted. Describe the sample space.

Answer:

$$S = \left\{ \begin{aligned} &R1, &R2, &R3, &R4, &R5, &R6\\ &W1, &W2, &W3, &W4, &W5, &W6\\ &B1, &B2, &B3, &B4, &B5, &B6 \end{aligned} \right\}$$

B 6 A balanced coin is tossed thrice. If three tails are obtained, a balance die is rolled. Otherwise, the experiment is terminated. Write down elements of the sample space.

$$Answer: S = \begin{cases} HHH, & HHT, & HTH, & HTT, & THH, & THT, & TTH, \\ TTT1, & TTT2, & TTT3, & TTT4, & TTT5, & TTT6 \end{cases}$$



Example of Method-2.2: Probability of an Event

| A | 1 | If probability of event A is $\frac{9}{10}$, what is the probability of the event "not A"? |
|---|---|---|
| | | Answer: 0. 1 |
| В | 2 | A single die is tossed once. Find the probability of a 2 or 5 turning up. |
| | | Answer: $\frac{1}{3}$ |
| | | |
| Α | 3 | Two unbiased dice are thrown. Find the probability that: |
| | | (1) Both the dice show the same number. |
| | | (2) The first die shows 6. |
| | | (3) The total of the numbers on the dice is 8. |
| | | (4) The total of the numbers on the dice is divisible by 2 or 3. |
| | | Answer: (1) $\frac{1}{6}$, (2) $\frac{1}{6}$, (3) $\frac{5}{36}$, (4) $\frac{2}{3}$ |
| A | 4 | Three coins are tossed. Find the probability of |
| | | (1) Getting at least 2 heads. |
| | | (2) Getting exactly 2 heads. |
| | | (2) Getting exactly 2 neads. |
| | | Answer: (1) 0.5, (2) 0.375, |
| В | 5 | A card is drawn from a pack of 52 cards. Find the probability of getting a king |
| | | or a heart or a red card. |
| | | _ |
| | | Answer: $\frac{7}{13}$ |
| A | 6 | One card is drawn at random from a well shuffled pack of 52 cards. Find |
| | | probability that the card will be an ace, a card of black color, a diamond, and |
| | | not an ace. |
| | | |
| | | Answer: 0.0769, 0.5, 0.25, 0.9231 |



| В | 7 | Four cards are drawn from the pack of cards. Find the probability that |
|---|----|---|
| | | (1) All are diamonds |
| | | (2) There is one card of each suit |
| | | (3) There are two spades and two hearts |
| | | (4) All are red or all are picture(face) cards. |
| | | 11 2197 468 3086 |
| | | Answer: (1) $\frac{11}{4165}$, (2) $\frac{2197}{20825}$, (3) $\frac{468}{20825}$, (4) $\frac{3086}{54145}$ |
| В | 8 | 4 cards are drawn at random from a pack of 52 cards. Find probability that |
| | | (1) They are a king, a queen, a jack and an ace. |
| | | (2) Two are kings and two are queens. |
| | | (3) Two are black and two are red. |
| | | (4) Two cards of hearts and two cards of diamonds. |
| | | Answer: (1) $\frac{256}{270725}$, (2) $\frac{36}{270725}$ |
| | | |
| | | $(3) \frac{325}{833}, \qquad (4) \frac{468}{20825}$ |
| В | 9 | A box contains 5 red, 6 white and 2 black balls. The balls are identical in all |
| | | aspects other than color. |
| | | (1) One ball is drawn at random from the box. Find the probability that the |
| | | selected ball is black. |
| | | (2) Two balls are drawn at random from the box. Find the probability that |
| | | one ball is white and one is red. |
| | | 2 5 |
| | | Answer: (1) $\frac{2}{13}$, (2) $\frac{5}{13}$ |
| В | 10 | If 3 balls are "randomly drawn" from a bowl containing 6 white and 5 black |
| | | balls. What is the probability that one of the balls is white and the other two |
| | | black? |
| | | Answer: 0.3636 |



| С | 11 | There are 5 yellow, 2 red, and 3 white balls in the box. Three balls are |
|---|----|--|
| | | randomly selected from the box. Find the probability of the following events. |
| | | (1) All balls are of different color |
| | | (2) 2 yellow and 1 red color ball |
| | | (3) all balls are of same color. |
| | | Answer: (1) 0.25, (2) 0.1667, (3) 0.0917 |
| В | 12 | A box contains 6 red balls, 4 white balls, 5 black balls. A person draws 4 balls |
| | | from the box at random. Find the probability that among the balls drawn |
| | | there is at least one ball of each color. |
| | | |
| | | Answer: 0. 5275 |
| В | 13 | A machine produces a total of 12000 bolts a day, which are on the average |
| | | 3% defective. Find the probability that out 600 bolts chosen at random, 12 |
| | | will be defective. |
| | | Answer: $\frac{\binom{360}{12}\binom{11640}{588}}{\binom{12000}{600}}$ |
| В | 14 | If 5 of 20 tyres in storage are defective and 5 of them are randomly chosen |
| | | for inspection (that is, each tire has the same chance of being selected), what |
| | | is the probability that the two of the defective tires will be included? |
| | | Answer: 0. 2935 |
| С | 15 | In a group of 1000 persons, there are 650 who can speak Hindi, 400 can |
| | | speak English, and 150 can speak both Hindi and English. If a person selected |
| | | at random, what is the probability that a person speaks |
| | | (1) Hindi only, |
| | | (2) English only, |
| | | (3) Only one of two languages, |
| | | (4) At least one of the two languages. |
| | | Answer: (1) 0.5, (2) 0.25, (3) 0.75, (4) 0.9 |



| С | 16 | A person applies for a job in two firms A and B, the probability of his being |
|---|----|---|
| | | selected in firm A is 0.7 and being rejected in firm B is 0.5. The probability of |
| | | at least one of the applications being rejected is 0.6. What is the probability |
| | | that person will be selected in one of the two firms? |
| | | Answer: 0.8 |
| | | |
| Α | 17 | Three newspapers A, B, C are published in a certain city. It is estimated from |
| | | a survey that of the adult population: 20% read A, 16% read B, 14% read C, |
| | | 8% read both A and B, 5% read both A and C, 4% read B and C, 2% read all |
| | | three. Find what percentage read at least one of the papers? |
| | | Answer: 35% |
| В | 18 | A room has three lamp sockets. From a collection of 10 light bulbs of which |
| | | only 6 are good. A person selects 3 at random and puts them in the socket. |
| | | What is the probability that the room will have light? |
| | | Answer: $\frac{29}{30}$ |
| | | 30 |
| С | 19 | A class has 10 boys and 5 girls. Three students are selected at random one |
| | | after the other. Find the probability that |
| | | (1) First two are boys and third is girl. |
| | | (2) First and third of same gender and second is of opposite gender. |
| | | Answer: (1) $\frac{15}{91}$, (2) $\frac{5}{21}$ |



Method - 3 → Conditional Probability and Independent Events

Example of Method-3: Conditional Probability and Independent Events

| A | 1 | If $P(A) = \frac{1}{3}$, $P(B) = \frac{3}{4}$ and $P(A \cup B) = \frac{11}{12}$. Find $P(A \mid B)$. |
|---|---|---|
| | | Answer: $\frac{2}{9}$ |
| В | 2 | For two independent events A & B if $P(A) = 0.3$ and $P(A \cup B) = 0.6$. |
| | | Find P(B). |
| | | Answer: 0.4286 |
| A | 3 | |
| A | 3 | If A, B are independent events and $P(A) = \frac{1}{4}$, $P(B) = \frac{2}{3}$. Find $P(A \cup B)$. |
| | | Answer: 0.75 |
| | 4 | |
| В | 4 | Check weather events A and B are independent or not if $P(A) = 0.20$, |
| | | $P(B) = 0.40 \text{ and } P(A \cup B) = 0.50.$ |
| | | Answer: Not Independent |
| Α | 5 | If A and B are independent events with $P(A) = 0.26$, $P(B) = 0.45$, find |
| | | $P(A \cap B), P(A \cap B'), P(A' \cap B').$ |
| | | Answer: 0. 117, 0. 143, 0. 407 |
| В | 6 | In producing screws, let A mean "screw is too slim" and B "screw is too short". |
| | | Let $P(A) = 0.1$ and $P(B \cap A) = 0.02$. A screw, selected randomly, is of type A, |
| | | what is probability that a screw is of type B? |
| | | |
| | | Answer: 0.2 |
| С | 7 | A problem in statistics is given to three students A, B, C whose chances of |
| | | solving it are 0.5, 0.75 and 0.25 respectively. What is the probability that the |
| | | problem will be solved if all of them try independently? |
| | | Answer: 0. 90625 |



| С | 8 | In a box, 100 bulbs are supplied out of which 10 bulbs have defects of type A, |
|---|----|---|
| | | 5 bulbs have defects of type B and 2 bulbs have defects of both the type. Find |
| | | the probability that, |
| | | (1) A bulb to be drawn at random has a B – type defect under the condition |
| | | that it has an A – type defect. |
| | | (2) A bulb to be drawn at random has no B – type defect under the |
| | | condition that it has no A – type defect. |
| | | Answer: (1) 0.2, (2) 0.9667 |
| В | 9 | In a certain college 25% of the students failed in probability and 15% of the |
| | | student failed in statistics and 10% of the students failed in both. A student |
| | | is selected at random, if he failed in probability, what is probability that he |
| | | failed in statistics? |
| | | Answer: 0.4 |
| С | 10 | Two integers are selected at random from 1 to 11. If the sum is even, find the |
| | | probability that both the integers are odd. |
| | | |
| | | Answer: 0.6 |
| В | 11 | A card is drawn from a well-shuffled deck of 52 cards and then second card |
| | | is drawn, find the probability that one card is a spade and then second card |
| | | is club if the first card is not replaced. |
| | | 13 |
| | | Answer: 13/204 |
| В | 12 | From a bag containing 4 white and 6 black balls, two balls are drawn at |
| | | random. If the balls are drawn one after the other without replacements, find |
| | | the probability that one is white and one is black. |
| | | 8 |
| | | Answer: $\frac{3}{15}$ |
| | | |



Method - 4 → Bayes' Theorem

Example of Method-4: Total Probability and Bayes' Theorem

| Α | 1 | There are three boxes, Box - I contains 10 light bulbs of which 4 are defective, |
|---|---|--|
| | | Box - II contains 6 light bulbs of which 1 is defective and Box - III contains 8 |
| | | light bulbs of which 3 are defective. A box is chosen and a bulb is drawn. Find |
| | | the probability that the bulb is defective. |
| | | |
| | | Answer: 0. 3139 |
| Α | 2 | Suppose that the population of a certain city is 40% male & 60% female. |
| | | Suppose also that 50% of male & 30% of female smokes. Find the probability |
| | | that a smoker is male. |
| | | 10 |
| | | Answer: $\frac{10}{19}$ |
| С | 3 | A card from a pack of 52 cards is lost. From the remaining cards of pack, two |
| | | cards are drawn and are found to be hearts. Find the probability of the |
| | | missing card to be a heart. |
| | | S . |
| | | Answer: 11 |
| | | 50 |
| Α | 4 | A microchip company has two machines that produce the chips. Machine-I |
| | | produces 65% of the chips, but 5% of its chips are defective. Machine-II |
| | | produces 35% of the chips, but 15% of its chips are defective. A chip is |
| | | selected at random and found to be defective. What is the probability that it |
| | | came from Machine-I? |
| | | Answer: 0.3824 |
| Α | 5 | There are two boxes A and B containing 4 white, 3 red and 3 white, 7 red |
| | | balls respectively. A box is chosen at random and a ball is drawn from it, if |
| | | the ball is white, find the probability that it is from box A. |
| | | |
| | | Answer: $\frac{40}{61}$ |
| | | VI |



| В | 6 | In a computer engineering class, 5% of the boys and 10% of the girls have an |
|---|----|---|
| | | IQ of more than 150. In this class, 60% of student are boys. If a student is |
| | | selected random and found to have IQ more than 150, find the probability |
| | | that the student is a boy. |
| | | |
| | | Answer: $\frac{3}{7}$ |
| Α | 7 | Three hospitals contain 10%, 20% and 30% of diabetes patients. A Patient is |
| | | selected at random who is diabetes patient. Determine the probability that |
| | | this patient comes from first hospital. |
| | | Answer: 0. 1667 |
| С | 8 | Suppose there are three chests each having two drawers. The first chest has |
| | | a gold coin in each drawer, the second chest has a gold coin in one drawer |
| | | and a silver coin in the other drawer and the third chest has a silver coin in |
| | | each drawer. A chest is chosen at random and a drawer opened. If the drawer |
| | | contains a gold coin, what is the probability that the other drawer also |
| | | contains a gold coin? |
| | | Answer: $\frac{2}{3}$ |
| В | 9 | State Bayes' theorem. In a bolt factory, three machines A, B and C |
| | | manufacture 25%, 35% and 40% of the total product respectively. Out of |
| | | these outputs 5%, 4% and 2% respectively, are defective bolts. A bolt is |
| | | picked up at random and found to be defective. What are the probabilities |
| | | that it was manufactured by machine A, B and C? |
| | | that it was manufactured by machine 11, b and G: |
| | | Answer: 0.3623, 0.4058, 0.2319 |
| В | 10 | A factory has three machines X, Y, Z producing 1000, 2000, 3000 bolts per |
| | | day respectively. Machine X produces 1% defective bolts, Y produces 1.5%, |
| | | Z produces 2% defective bolts. At end of the day, a bolt is drawn at random |
| | | and it is found to be defective. What is the probability that this defective bolt |
| | | has been produced by the machine X? |
| | | Answer: 0.1 |





| В | 11 | Urn A contain 1 white, 2 black, 3 red balls; Urn B contain 2 white, 1 black, 1 |
|---|----|--|
| | | red balls; Urn C contain 4 white, 5 black, 3 red balls. One urn is chosen at |
| | | random & two balls are drawn. These happen to be one white & one red. |
| | | What is probability that they come from urn A? |
| | | Amorrow, 0, 2707 |
| | | Answer: 0. 2797 |
| В | 12 | An insurance company insured 2000 bike drivers, 4000 car drivers and 6000 |
| | | truck drivers. The probability of an accident involving a bike driver, a car |
| | | driver and a truck driver is 0.10, 0.03 and 0.15 respectively. One of the |
| | | insured persons meets with an accident. What is the probability that he is a |
| | | bike driver? |
| | | |
| | | Answer: 0. 1639 |



Method - 5 → Random Variable and Probability Function

Example of Method-5: Random Variable and probability Function

| <u>Exan</u> | <u>ipie o</u> | <u>f Method-5: R</u> | kandom ' | <u>variable ar</u> | <u>ıa probabı</u> | <u>lity Functio</u> | <u>on</u> | | | | | |
|-------------|---------------|--|--|--------------------|-------------------|------------------------|------------------|--|--|--|--|--|
| A | 1 | Is $P(X = x) =$ | Is $P(X = x) = \left(-\frac{1}{2}\right)^x$; $x = 0$, 1, 2 a probability function? | | | | | | | | | |
| | | Answer: No | Answer: No | | | | | | | | | |
| A | 2 | If $P(x) = \frac{2x}{4}$ | $\frac{+1}{8}$, x = | = 1, 2, 3, | 4, 5, 6. | | | | | | | |
| | | Verify wheth | er P(x) is | probabilit | y function o | or not. | | | | | | |
| | | Answer: Yes | 5 | | | | | | | | | |
| В | 3 | Probability d | istributio | on of a rand | om variable | e X is given | below. | | | | | |
| | | Find $P(2 \le x)$ | \leq 4), P(| (x > 2), P(| (x is odd) ar | nd P(x is eve | en) | | | | | |
| | | X | 1 | 2 | 3 | 4 | | | | | | |
| | | P(X = x) | 0.1 | 0.2 | 0.5 | 0.2 | | | | | | |
| | | | | | | | | | | | | |
| | | Answer: $P(2 \le x \le 4) = 0.9$, $P(x > 2) = 0.7$, | | | | | | | | | | |
| | | $P(x \text{ is odd}) = 0.6, \qquad P(x \text{ is even}) = 0.4$ | | | | | | | | | | |
| В | 4 | Find k for the | e probabi | lity distrib | ution p(x) = | $= k {4 \choose x}, x$ | = 0, 1, 2, 3, 4. | | | | | |
| | | | | | | | | | | | | |
| | | Answer: k = | = 1 16 | | | | | | | | | |
| В | 5 | _ | Verify that the following function is P.D.F or not? | | | | | | | | | |
| | | $\left(\frac{x}{8} ; 0 \le x < 2 \right)$ | | | | | | | | | | |
| | | $f(x) = \begin{cases} \frac{1}{2} \end{cases}$ | $f(x) = \begin{cases} \frac{1}{4} & \text{; } 2 \le x < 4 \end{cases}$ | | | | | | | | | |
| | | $f(x) = \begin{cases} \frac{x}{8} & ; \ 0 \le x < 2 \\ \frac{1}{4} & ; \ 2 \le x < 4 \\ \frac{6-x}{8} & ; \ 4 \le x < 6 \end{cases}$ | | | | | | | | | | |
| | | 8 | ; 4 <u>\</u> | ≤ X < 0 | | | | | | | | |



A 6 Check whether the following function

$$f(x) = \begin{cases} \frac{3+2x}{18} & \text{is a probability density function?} \\ 0 & \text{otherwise} \end{cases}$$

If yes, then find $P(3 \le X \le 4)$.

Answer: Yes, $\frac{5}{9}$

B 7 Find the constant c such that the function

$$f(x) = \begin{cases} cx^2 \; ; \; 0 < x < 3 \\ 0 \qquad ; elsewhere \end{cases}$$
 is a probability density function and

Compute P(1 < X < 2), $P(X \le 2)$, $P(X \ge 2)$.

Answer:
$$c = \frac{1}{9}$$
, $P(1 < X < 2) = \frac{7}{27}$,

$$P(X \le 2) = \frac{8}{27}, \qquad P(X \ge 2) \frac{19}{27}$$



Method - 6 - Various Measures of Statistics

Example of Method-6: Various Measures of Statistics

| В | 1 | The probability distribution of a random variable X is given below. |
|---|---|---|
| | | Find a, E(X), E(2X + 3), E(X^2 + 2), V(X), V(3X + 2). |

| X | -2 | -1 | 0 | 1 | 2 |
|------|------|---------------|---|-----|---------------|
| P(X) | 1 12 | $\frac{1}{3}$ | a | 1 4 | $\frac{1}{6}$ |

Answer:
$$a = \frac{1}{6}$$
,

$$E(X)=\frac{1}{12},$$

$$E(X) = \frac{1}{12}, \qquad E(2X+3) = \frac{19}{6},$$

$$E(X^2+2)=rac{43}{12}, \qquad V(X)=rac{227}{144}, \qquad V(3X+2)=rac{227}{16}$$

$$V(X) = \frac{227}{144}$$

$$V(3X+2) = \frac{227}{16}$$

A 2 The probability distribution of a random variable X is given below.

| X | -1 | 0 | 1 | 2 | 3 |
|------|------|------|---|------|------|
| P(X) | 3 10 | 1 10 | k | 3 10 | 1 10 |

Find k, E(X), E (4X + 3), E(X²), V(X), V(2X + 3).

Answer:
$$k = \frac{1}{5}$$
, $E(X) = \frac{4}{5}$, $E(4X + 3) = \frac{31}{5}$,

$$E(X)=\frac{4}{5},$$

$$E(4X+3) = \frac{31}{5}$$

$$E(X^2) = \frac{13}{5},$$

$$V(X) = \frac{49}{25}$$

$$E(X^2) = \frac{13}{5}$$
, $V(X) = \frac{49}{25}$, $V(2X+3) = \frac{196}{25}$

В 3 Three balanced coins are tossed, find the mathematical expectation of tails.

Answer: $\frac{3}{2}$

- Α
- A contestant tosses a coin and receives \$5 if head appears and \$1 if tail **(1)** appears. What is the expected value of a trial?
- A contestant receives \$4.00 if a coin turns up heads and pays \$3.00 if it **(2)** turns tails. What is the expected value of a trail?

Answer: \$ 3.00, \$0.50



| D | _ | A |
|---|----|--|
| В | 5 | A machine produces on average of 500 items during first week of the month |
| | | & average of 400 items during the last week of the month. The probability |
| | | for these being 0.68 and 0.32. Determine the expected value of the |
| | | production. |
| | | |
| | | Answer: 468 |
| В | 6 | In a business, the probability that a trader can get profit of Rs. 5000 is 0.4 |
| | | and probability for loss of Rs. 2000 is 0.6. Find his expected gain or loss. |
| | | |
| | | Answer: 800 |
| Α | 7 | There are 8 apples in a box, of which 2 are rotten. A person selects 3 Apples |
| | | at random from it. Find the expected value of the rotten apples. |
| | | |
| | | Answer: 0.75 |
| Α | 8 | There are 10 bulbs in a box, out of which 4 are defectives. If 3 bulbs are taken |
| | | at random, find the expected number of defective bulbs. |
| | | |
| | | Answer: 1.2 |
| В | 9 | A random variable X has pdf $f(x) = kx^2(4 - x)$; $0 < x < 4$. Find the value |
| | | of k and hence find its mean and standard deviation. |
| | | |
| | | Answer: $k = \frac{3}{64}$, Mean = 2.4, SD = 0.8 |
| В | 10 | A random variable X has pdf $f(x) = kx^2(1-x^3)$; $0 < x < 1$. Find the value |
| | | of k and hence find its mean and variance. |
| | | |
| | | Answer: $k = 6$, Mean $= \frac{9}{14}$, $SD = \frac{9}{245}$ |



Method - 7 ---> Cumulative Distribution Function

Example of Method-7: Cumulative Distribution Function

| A | 1 | Two dies are rolled. Let X denotes the random variable which counts the total |
|---|---|--|
| | | number of points on the upturned faces, construct a table giving the non – |
| | | zero values of the probability mass function. Also find the distribution of X. |

Answer:

| X | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|------|---|----------------|------|---------|-------------|----------|----------|----------|-------------|------|----------|-------------|
| P(X) | 0 | <u>1</u> 36 | 36 | 3 36 | 4 36 | 5 36 | 6 36 | 5 36 | 4 36 | 3 36 | 36 | 1 36 |
| F(x) | 0 | <u>1</u> 36 | 3 36 | 6 36 | 10 36 | 15 36 | 21 36 | 26 36 | | | 35 36 | |

B 2 The following is the distribution function F(x) of a discrete random variable X. Find probability distribution of X, $P(-2 \le X \le 1)$ and $P(X \ge 1)$.

| X | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|------|------|-----|-----|------|-----|-----|---|
| F(x) | 0.08 | 0.2 | 0.4 | 0.65 | 0.8 | 0.9 | 1 |

Answer:

| X | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|------|------|------|-----|------|------|-----|-----|
| P(X) | 0.08 | 0.12 | 0.2 | 0.25 | 0.15 | 0.1 | 0.1 |

$$P(-2 \le X \le 1) = 0.72 \,, \qquad P(X \ge 1) = 0.35 \,$$

B 3 Find the value of k and the distribution function F(x) given the probability density function of a random variable X as, $f(x) = \frac{k}{1 + x^2}$; $-\infty < x < \infty$.

Answer:
$$k = \frac{1}{\pi}$$
, $F(x) = \frac{1}{\pi} \left[tan^{-1} x + \frac{\pi}{2} \right]$



C 4 The probability density function of a continuous random variable X is given

$$f(x) = \begin{cases} ax & ; 0 \le x < 1 \\ a & ; 1 \le x < 2 \\ 3a - ax ; 2 \le x < 3 \end{cases}$$

Find the value of a and also find C.D.F of X.

Answer:
$$a = \frac{1}{2}$$
, $F(x) = \begin{cases} \frac{ax^2}{2} & ; \ 0 \le x < 1 \\ ax - \frac{a}{2} & ; \ 1 \le x < 2 \\ 3ax - \frac{ax^2}{2} - \frac{5a}{2} ; \ 2 \le x < 3 \\ 0 & ; \ otherwise \end{cases}$

* * * * * End of the Unit * * * *

