# Index

# Unit – 3 ⇝ Probability Distribution

## Method 1 ⇝ Binomial Distribution

Example of Method-1: Binomial Distribution

<table>
<tr><td>A</td><td>1</td><td>Find the binomial distribution for n = 4 and p = 0.3.<br><br>Answer:<br><br>
<table>
<tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>P(X)</td><td>0.2401</td><td>0.4116</td><td>0.2646</td><td>0.0756</td><td>0.0081</td></tr>
</table>
</td></tr>
<tr><td>A</td><td>2</td><td>If the probability of a defective bolt is 0.1,Find mean and standard deviation of the distribution of defective bolts in a total of 400.<br><br>Answer: Mean = 40, Standard Deviation = 6</td></tr>
<tr><td>B</td><td>3</td><td>If 3 of 12 car drivers do not carry driving license, what is the probability that a traffic inspector who randomly checks 3 car drivers, will catch 1 for not carrying driving license.<br><br>Answer: $\frac{27}{64}$</td></tr>
<tr><td>B</td><td>4</td><td>Find the probability that in a family of 4 children there will be at least 1 boy. Assume that the probability of a male birth is 0.5.<br><br>Answer: 0.9375</td></tr>
<tr><td>B</td><td>5</td><td>20% of the bulbs produced are defective. Find probability that at most 2 bulbs out of 4 bulbs are defective.<br><br>Answer: 0.9728</td></tr>
</table>

| | | |
|---|---|---|
| B | 6 | A university warehouse has received a shipment of 25 printers, of which 10 are laser printers and 15 are inkjet models. If 6 of these 25 are selected at random to be checked by a particular technician, what is the probability that exactly 3 of those selected are laser printers (so that the other 3 are inkjets)?<br><br>**Answer: 0.2765** |
| B | 7 | What are the properties of Binomial Distribution? The average percentage of failure in a certain examination is 40. What is the probability that out of a group of 6 candidates, at least 4 passed in examination?<br><br>**Answer: 0.5443** |
| B | 8 | Each sample of water has a 10% chance of containing a particular organic pollutant. Assume that the samples are independent with regard to the presence of the pollutant. Find the probability that in the next 18 samples, at least 4 samples contain the pollutant.<br><br>**Answer: 0.0982** |
| B | 9 | Ten coins are thrown simultaneously. Find the probability of getting at least seven heads.<br><br>**Answer: 0.1719** |
| B | 10 | Assume that on average one telephone number out of fifteen called between 1 p.m. and 2 p.m. on weekdays is busy. What is the probability that, if 6 randomly selected telephone numbers were called,<br>(a) not more than three?<br>(b) at least three, of them would be busy?<br><br>**Answer: (a) 0.9997, (b) 0.0051** |
| B | 11 | The probability that one of the ten telephone lines is busy at an instant is 0.6.<br>(a) What is the probability that 5 of the lines are busy?<br>(b) What is the probability that all the lines are busy?<br><br>**Answer: (a) 0.2006, (b) 0.0060** |

| | | |
|---|---|---|
| B | 12 | If 20% of the bolts produced by a machine are defective, determine the probability that out of 4 bolts chosen at random, one, zero and less than 2, bolts will be defective.<br><br>**Answer: 0.4096, 0.4096, 0.8192** |
| B | 13 | Probability of man hitting a target is $\frac{1}{3}$. If he fires 6 times, what is the probab of hitting at most 5 times, at least 5 times, and exactly one?<br><br>**Answer: 0.9986, 0.0179, 0.2634** |
| B | 14 | A dice is thrown 6 times getting an odd number of successes. Find probability of five successes, at least five successes and at most five successes.<br><br>**Answer: $\frac{3}{32}$, $\frac{7}{64}$, $\frac{63}{64}$** |
| C | 15 | Out of 2000 families with 4 children each, how many would you expect to have at least 1 boy, 2 boys, 1 OR 2 girls, and no girls? Assume equal probabilities for boys and girls.<br><br>**Answer: 1875, 750, 1250, 125** |
| C | 16 | A multiple-choice test consists of 8 questions with 3 answers to each question (of which only one is correct). A student answers each question by rolling a balanced dice & checking the first answer if he gets 1 or 2, the second answer if he gets 3 or 4 & the third answer if he gets 5 or 6. To get a distinction, the student must secure at least 75% correct answers. If there is no negative marking, what is the probability that the student secures a distinction?<br><br>**Answer: $P(X \geq 6) = 0.0197$** |

# Method 2 ⇝ Poisson Distribution

## Example of Method-2: Poisson Distribution

| | | |
|---|---|---|
| A | 1 | For Poisson variant X, if $P(X = 1) = P(X = 2)$, then find mean and standard deviation of this distribution. Also, find $P(X = 3)$.<br><br>**Answer: Mean $= 2$ , SD $= \sqrt{2}$ , $P(X = 3) = 0.1804$** |
| B | 2 | Assume that the probability that a wafer contains a large particle of contamination is 0.01 and that the wafers are independent; that is, the probability that a wafer contains a large particle is not dependent on the characteristics of any of the other wafers. If 15 wafers are analyzed, what is the probability that no large particles are found?<br><br>**Answer: 0.8607** |
| B | 3 | Suppose a book of 585 pages contains 43 typographical errors. If these errors are randomly distributed throughout the book, what is the probability that 10 pages, selected at random, will be free from error?<br><br>**Answer: 0.4795** |
| B | 4 | A book contains 100 misprints distributed randomly throughout its 100 pages. What is the probability that a page observed at random contains at least two misprints?<br><br>**Answer: 0.2642** |
| B | 5 | 100 Electric bulbs are found to be defective in a lot of 5000 bulbs. Find the probability that at the most 3 bulbs are defective in a box of 100 bulbs.<br><br>**Answer: $P(X \leq 3) = 0.8571$** |

| | | |
|---|---|---|
| B | 6 | The number of monthly breakdowns of a computer is a random variable having Poisson distribution with mean 1.8. Find the probability that the computer will function for a month<br>(a) without a breakdown.<br>(b) with at least one breakdown.<br><br>**Answer: (a) 0.1653, (b) 0.8347** |
| C | 7 | In a bolt manufacturing company, it is found that there is a small chance of $\frac{1}{500}$ for any bolt to be defective. The bolts are supplied in a packed of 20 bolts. Use Poisson distribution to find approximate number of packets containing no defective bolt, containing two defective bolts, in the consignment of 10000 packets.<br><br>**Answer: 9608, 8** |
| B | 8 | The number of flaws in a fiber optic cable follows a Poisson process with an average of 0.6 per 100 feet.<br>(a) Find the probability of exactly 2 flaws in a 200 feet cable.<br>(b) Find the probability of exactly 1 flaw in the first 100 feet and exactly 1 flaw in the second 100 feet.<br><br>**Answer: (a) 0.2169, (b) 0.1084** |
| B | 9 | If a publisher of nontechnical books takes great pains to ensure that its books are free of typographical errors, so that the probability of any given page containing at least one such error is 0.005 and errors are independent from page to page. What is the probability that one of its 400-page novels will contain<br>(a) exactly one page with errors?<br>(b) at most three pages with errors?<br><br>**Answer: (a) 0.2707, (b) 0.8571** |

| | | |
|---|---|---|
| B | 10 | If the probability that an individual suffers a bad reaction from a certain injection is 0.001. Find the probability that out of 2000 individuals<br>(a) more than 2 individuals<br>(b) exactly 3 individuals will suffer a bad reaction.<br><br>**Answer: (a) 0.3233, (b) 0.1804** |
| B | 11 | The probability that a person catch corona virus is 0.001. Find the probability that out of 3000 persons exactly 3, more than 2 persons will catch the virus.<br><br>**Answer: 0.2240, 0.5768** |
| C | 12 | In a certain factory turning out razor blades, there is a small chance of 0.002 for any blade to be defective. The blades are supplied in packet of 10. Calculate the approximate number of packets containing no defective, one defective, two defective blades in a consignment of 10000 packets.<br><br>**Answer: 9802, 196, 2** |
| C | 13 | A car hire firm has two cars, which are hires out day by day. The number of demands for a car on each day is distributed on a Poisson distribution with mean 1.5. Calculate the proportion of days on which neither car is used and proportion of days on which some demand is refused. ($e^{-1.5} = 0.2231$).<br><br>**Answer: $P(X = 0) = 0.2231$, $1 - P(X \leq 2) = 0.1912$** |

# Method 3 ⇝ Exponential Distribution

## Example of Method-3: Exponential Distribution

| | | |
|---|---|---|
| A | 1 | The lifetime T of an alkaline battery is exponentially distributed with $\theta = 0.02$ per hour. What are mean and variance of batteries lifetime?<br><br>**Answer: 50, 2500** |
| A | 2 | The time required to repair a machine is exponentially distributed with parameter $\theta = 0.5$ per hour. What is the probability that the repair time exceeds 3 hours?<br><br>**Answer: 0.2231** |
| B | 3 | Suppose the length of life of an appliance has an exponential distribution with mean 10 years. What is the probability that the average life time of a random sample of the appliances is at least 10.5?<br><br>**Answer: 0.3499** |
| B | 4 | The time intervals between successive barges passing a certain point on a busy waterway have an exponential distribution with mean 8 minutes. Find the probability that the time interval between two successive barges is less than 5 minutes.<br><br>**Answer: 0.4647** |
| B | 5 | On the average, a certain computer part lasts ten years. The length of time the computer part lasts is exponentially distributed.<br>(a) What is the probability that a computer part lasts more than 7 years?<br>(b) What is the probability that a computer part lasts between 9 and 11 years?<br><br>**Answer: (a) 0.4966, (b) 0.0737** |

| | | |
|---|---|---|
| B | 6 | The mileage which car owners get with a certain kind of radial tire is a random variable having exponential distribution with mean 4000 km. What is the probability that one of these tiers will last (a) at least 2000 km, (b) at most 3000 km.<br><br>**Answer: (a) 0.6065, (b) 0.5270** |
| C | 7 | Accidents occur with Poisson distribution at an average of 4 per week.<br>(a) Calculate the probability of more than 5 accidents in any one week<br>(b) What is probability that at least two weeks will elapse between accidents?<br><br>**Answer: (a) 0.2148, (b) 0.0003** |
| C | 8 | The arrival rate of cars at a gas station is 40 customers per hour.<br>(a) What is the probability of having no arrivals in 5 min. interval?<br>(b) What is the probability of having 3 arrivals in 5 min. interval?<br><br>**Answer: (a) 0.0356, (b)0.2202** |
| B | 9 | The length of time X to complete a job is exponentially distributed with $E(X) = \mu = \frac{1}{\lambda} = 10$ hours.<br>(a) Compute the probability of job completion between two consecutives jobs exceeding 20 hours.<br>(b) The cost of job completion is given by $C = 4 + 2X + 2X^2$, then find the expected value of C.<br><br>**Answer: (a) 0.1353, (b) 424** |
| C | 10 | The life of an electronic component follows exponential distribution with a mean of 4 years. The manufacturer of this component gives a replacement warranty of 3 years.<br>(a) What proportion of components will be replaced in the period of warranty?<br>(b) What is the probability that a randomly selected component will have life within two standard deviations of the mean life?<br><br>**Answer: (a) 0.5276, (b) 0.9502** |

| | | |
|---|---|---|
| B | 11 | The income tax of a man is exponentially distributed with<br>$f(x) = \frac{1}{3}e^{-\left(\frac{x}{3}\right)}; x > 0.$<br>What is the probability that his income will exceed Rs. 17000? Assume that the income tax is levied at the rate of 15% on the income above Rs. 15000.<br>**Answer: $e^{-100}$** |
| B | 12 | A random variable has an exponential distribution with probability density<br>$f(x) = \begin{cases} ce^{-2x} & ; x > 0 \\ 0 & ; \text{otherwise} \end{cases}$<br>Find value of the constant c and $P(X > 5)$.<br>**Answer: 2, 0.0183** |

# Method 4 ⇝ Normal Distribution

## Example of Method-4: Normal Distribution

| | | |
|---|---|---|
| A | 1 | If mean is 50 and standard deviation is 10, then find $P(50 \leq X \leq 80)$. $[P(z = 3) = 0.4987]$<br><br>**Answer: 0.4987** |
| A | 2 | If X is a normal variate with a mean of 30 and an SD of 5, find the probability that (i) $26 \leq X \leq 40$, (ii) $X \geq 45$.<br><br>**Answer: (i) 0.7653, (ii) 0.0013** |
| B | 3 | The breaking strength of cotton fabric is normally distributed with $E(x) = 16$ and $\sigma(x) = 1$. The fabric is said to be good if $x \geq 14$. What is the probability that a fabric chosen at random is good? $[P(z = 2) = 0.4772]$<br><br>**Answer: 0.9773** |
| B | 4 | A manufacture knows from his expenditure that the resistance of resistor he produces is normal with $\mu = 100$ ohms and $SD = \sigma = 2$ ohms. What percentage of resistors will have resistance between 98 ohms and 102 ohms?<br><br>**Answer: 0.6826** |
| B | 5 | The average seasonal rainfall in a place is 16 inches with an SD of 4 inches. What is the probability that the rainfall in that place will be between 20 and 24 inches in a year?<br><br>**Answer: 0.1359** |
| C | 6 | Weights of 500 students of college is normally distributed with $\mu$=95 lbs. & $\sigma$ =7.5 lbs. Find how many students will have the weight between 100 and 110 lbs.<br>$[P(z = 2) = 0.4772, \quad P(z = 0.67) = 0.2486]$<br><br>**Answer: 114** |

| | | |
|---|---|---|
| C | 7 | Distribution of height of 1000 soldiers is normal with mean 165 cm & standard deviation 15 cm. How many soldiers are of height<br>(a) less than 138 cm?<br>(b) more than 198 cm?<br>(c) between 138 & 198 cm?<br>[P(z = 1.8) = 0.4641, P(z = 2.2) = 0.4861]<br><br>**Answer: (a) 36, (b) 14, (c) 950** |
| B | 8 | The customer accounts of certain department store have an average balance of 120 Rs. and standard deviation of 40 Rs. Assume that account balances are normally distributed.<br>(a) What proportion of the account is over 150 Rs.?<br>(b) What proportion of account is between 100 & 150 Rs.?<br>(c) What proportion of account is between 60 & 90 Rs.?<br>[P(z = 0.75) = 0.2734, P(z = 0.5) = 0.1915, P(z = 1.5) = 0.4332]<br><br>**Answer: (a) 0.2266, (b) 0.4649 , (c) 0.1598** |
| B | 9 | What is the probability that a standard normal variate Z will be<br>(a) greater than 1.09? (b) less than -1. 65?<br>(c) lying between -1 and 1.96? (d) lying between 1.25 and 2.75?<br><br>**Answer: (a) 0.1379, (b) 0.0495 , (c) 0.8163, (d) 0.1026** |
| C | 10 | In a company, amount of light bills follows normal distribution with σ = 60. 11.31% of customers pay bill less than 260. Find average amount of light bill.<br>[P(z = 1.21) = 0.3869]<br><br>**Answer: 332.60** |

*** * * * * End of the Unit * * * * ***