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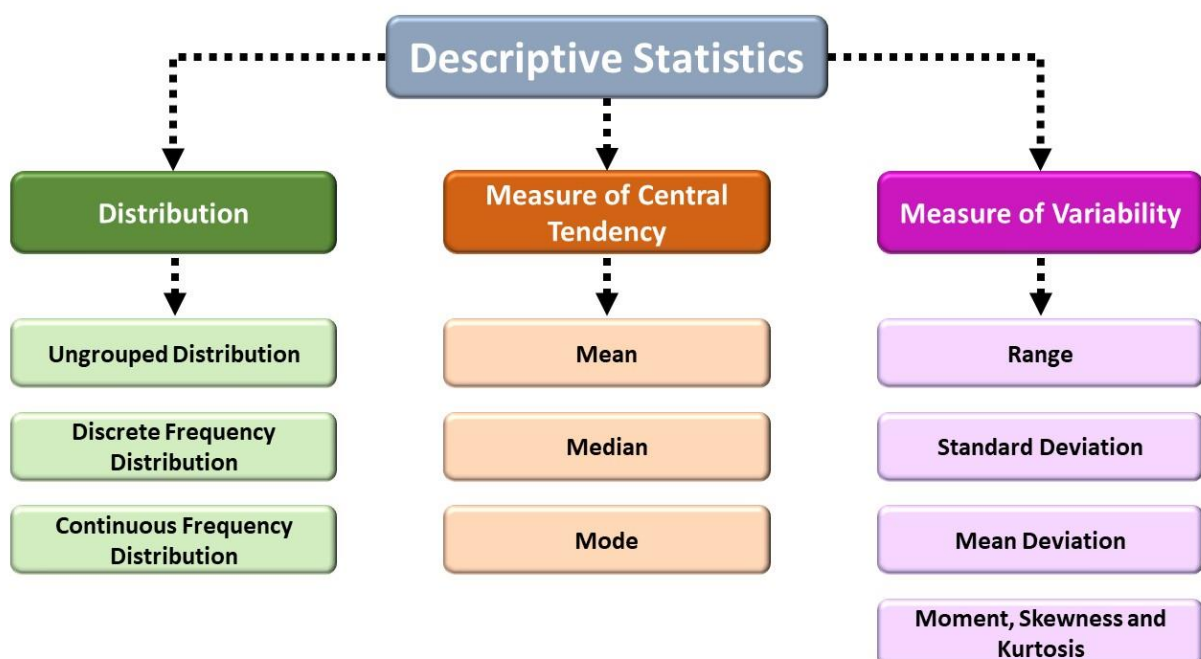
Unit – 1 \rightsquigarrow Descriptive Statistics

Introduction

- Statistics is the branch of science where we plan, gather and analyze information about a particular collection of objects under investigation.
- Statistics techniques are used in every other field of science, engineering and humanity, ranging from computer science to industrial engineering to sociology and psychology.
- For any statistical problem the initial information collection from the sample may look messy, and hence confusing. This initial information needs to be organized first before we make any sense out of it.
- There are two types of statistics:
 - (1) Descriptive Statistics (Unit – 1)
 - (2) Inferential Statistics (Unit – 4 & 5)

Descriptive Statistics and Inferential Statistics

- A data set is a collection of observations from a sample or entire population. This data set is summarized and organized by **Descriptive Statistics**.



Method 1 \rightarrow Frequency Distributions

Frequency Distribution

\rightarrow The distribution is a summary of the frequency of individual values or ranges of values for a variable.

Types of Frequency Distribution

\rightarrow There are two types of frequency distribution which is

(1) Frequency distribution of **ungrouped data**

(2) Frequency distribution of **grouped data**

Frequency Distribution of Ungrouped Data

\rightarrow The ungrouped frequency distribution is a type of frequency distribution that displays the frequency of each **individual** data value.

\rightarrow For Example:

Marks of 10 students are 10, 25, 26, 35, 03, 08, 19, 29, 30, 18.

Frequency Distribution of Grouped Data

(1) Discrete Frequency Distribution

- A discrete frequency distribution is a type of frequency distribution that shows each number and the number of times it appears in a list.

- For Example:

Data of students using library during exam time.

No. reading hours (x_i)	1	2	3	4	5	6
No. of students (f_i)	4	7	8	9	10	2

(2) Continuous Frequency Distribution

- A continuous frequency distribution is a series in which the data are classified into different **class intervals**.

- For Example:

Data of students using library during exam time.

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No. reading hours (x_i)	0 – 2	3 – 5	6 – 8	9 – 11
No. of students (f_i)	11	7	8	0

Exclusive Class

- If classes of frequency distributions are 0 – 2, 2 – 4, 4 – 6, ... such classes are known as exclusive classes.

Inclusive Class

- If classes of frequency distributions are 0 – 2, 3 – 5, 6 – 8, ... such classes are known as inclusive classes.

Lower Boundary & Upper Boundary

- For the class $x_i - x_{i+1}$,
lower boundary is x_i and upper boundary is x_{i+1} .
- For Example:
For the class 3 – 5, Lower boundary is 3 and upper boundary is 5.

Class Length

- A difference between Upper boundary and Lower boundary is known as class length.
- It is denoted by c and defined as
$$c = \text{Upper Boundary} - \text{Lower Boundary} = x_{i+1} - x_i$$
- For Example:
For the class 3 – 5, class length is $5 - 3 = 2$.

Mid-Point of Class

- Mid point of class is used to find x_i in case of continuous frequency distribution.
- It is defined as follow:

$$\text{Mid point of class} = x_i = \frac{\text{Lower Boundary} + \text{Upper Boundary}}{2}$$

- For Example:

$$\text{For the class 3 – 5, mid point is } \frac{3 + 5}{2} = 4.$$

Method 2 \rightarrow Measure of Central Tendency

Central Tendency

\rightarrow The central tendency of a distribution is an estimate of the **center** of a distribution of values.

\rightarrow There are three measures to estimate central tendency which is

(1) Mean(\bar{x})

(2) Median(M)

(3) Mode(Z)

2.1 Mean

\rightarrow The mean means **average**.

\rightarrow Mean is denoted by " \bar{x} " and read as x bar.

\rightarrow Table of different formulae of mean.

Method	Ungrouped Data	Discrete Grouped Data	Continuous Grouped Data
Direct Method	$\frac{\sum x_i}{n}$	$\frac{\sum f_i x_i}{\sum f_i}$	
Assumed Mean Method	$\frac{\sum d_i}{n}$	$A + \frac{\sum f_i d_i}{\sum f_i}$	
Step Deviation Method	-----	-----	$A + \frac{\sum f_i u_i}{\sum f_i} \times c$

\rightarrow n = total number of observations

\rightarrow In case of **continuous frequency distribution**,

x_i = mid value of the respective class.

\rightarrow In case of **assumed mean method**, A can be any value from x_i .

\rightarrow Use below formula to calculate d_i & u_i

$$d_i = x_i - A ; u_i = \frac{x_i - A}{c}$$

Example of Method-2.1: Examples of Mean

C	1	Find the mean of data 10.2, 9.5, 8.3, 9.7, 9.5, 11.1, 7.8, 8.8, 9.5, 10. Answer: 9.44														
C	2	Find the mean for following data: <table><tr><td>Marks obtained</td><td>20</td><td>9</td><td>25</td><td>50</td><td>40</td><td>80</td></tr><tr><td>Number of students</td><td>6</td><td>4</td><td>16</td><td>7</td><td>8</td><td>2</td></tr></table> Answer: 32.23	Marks obtained	20	9	25	50	40	80	Number of students	6	4	16	7	8	2
Marks obtained	20	9	25	50	40	80										
Number of students	6	4	16	7	8	2										
C	3	Find the mean using direct method, assumed mean method and step deviation method: <table><tr><td>Marks</td><td>0 – 10</td><td>10 – 20</td><td>20 – 30</td><td>30 – 40</td><td>40 – 50</td></tr><tr><td>No. of students</td><td>5</td><td>10</td><td>40</td><td>20</td><td>25</td></tr></table> Answer: 30	Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	No. of students	5	10	40	20	25		
Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50											
No. of students	5	10	40	20	25											
C	4	Find the missing frequency f_1 and f_2 in the table given below, it is being given that the mean of the given frequency distribution is 50. <table><tr><td>Class</td><td>0 – 20</td><td>20 – 40</td><td>40 – 60</td><td>60 – 80</td><td>80 – 100</td><td>Total</td></tr><tr><td>f</td><td>17</td><td>f_1</td><td>32</td><td>f_2</td><td>19</td><td>100</td></tr></table> Answer: $f_1 = 18$, $f_2 = 14$	Class	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	Total	f	17	f_1	32	f_2	19	100
Class	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	Total										
f	17	f_1	32	f_2	19	100										
C	5	A co-operative bank has two branches employing 50 and 70 workers respectively. The average salaries paid by two respective branches are 360 and 390 rupees per month. Calculate the mean of the salaries of all the employees. Answer: 377.5														

2.2 Median

- The median is the value found at the **exact middle** of the set of values.
- Median is denoted by capital letter "**M**".
- To compute the median, list all observations in ascending order and then locate the value in the center of the sample.
- Table of formula of median.

Data	Formula
Ungrouped Data	If n is odd , then $M = \left(\frac{n + 1}{2} \right)^{\text{th}} \text{ observation}$
Discrete Grouped Data	If n is even , then $M = \frac{\left(\frac{n}{2} \right)^{\text{th}} \text{ observation} + \left(\frac{n}{2} + 1 \right)^{\text{th}} \text{ observation}}{2}$
Continuous Grouped Data	$M = L + \left(\frac{\frac{n}{2} - F}{f} \right) \times c$

- Where,

Median class = Class whose cumulative frequency with property $\min \left\{ cf \mid cf \geq \frac{n}{2} \right\}$

L = Lower boundary point of the median class

n = Total number of observation (sum of the frequencies)

F = Cumulative frequency of the class preceding the median class

f = The frequency of the median class

Example of Method-2.2: Median

C	1	Find the median of following data: 20, 25, 30, 15, 17, 35, 26, 18, 40, 45, 50. Answer: 26														
C	2	The given observations have been arranged in ascending order. If the median of the data is 63, find the value of x for the following data: 29, 32, 48, 50, x, x + 2, 72, 78, 84, 95. Answer: x = 62														
C	3	Calculate the median for the following data: <table border="1"><tr><td>Marks</td><td>20</td><td>9</td><td>25</td><td>50</td><td>40</td><td>80</td></tr><tr><td>No. of students</td><td>6</td><td>4</td><td>16</td><td>7</td><td>8</td><td>2</td></tr></table> Answer: 25	Marks	20	9	25	50	40	80	No. of students	6	4	16	7	8	2
Marks	20	9	25	50	40	80										
No. of students	6	4	16	7	8	2										
C	4	The following table gives marks obtained by 50 students in statistics. Find the median. <table border="1"><tr><td>Marks</td><td>0 – 10</td><td>10 – 20</td><td>20 – 30</td><td>30 – 40</td><td>40 – 50</td></tr><tr><td>No. of students</td><td>16</td><td>12</td><td>18</td><td>3</td><td>1</td></tr></table> Answer: 17.5	Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	No. of students	16	12	18	3	1		
Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50											
No. of students	16	12	18	3	1											
C	5	The median of 60 observations (following data) is 28.5. Find x and y. <table border="1"><tr><td>Marks</td><td>0 – 10</td><td>10 – 20</td><td>20 – 30</td><td>30 – 40</td><td>40 – 50</td><td>50 – 60</td></tr><tr><td>No. of students</td><td>5</td><td>x</td><td>20</td><td>15</td><td>y</td><td>5</td></tr></table> Answer: x = 8, y = 7	Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	No. of students	5	x	20	15	y	5
Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60										
No. of students	5	x	20	15	y	5										

2.3 Mode

- The mode is the **most frequently** occurring value in the set.
- Mode is denoted by capital letter "**Z**".
- The mode is not necessarily unique, like mean and median. we can have data with two modes (bi-modal) or more than two modes (multi-modal).
- Table of formula of mode.

Data	Formula
Ungrouped Data	Most repeated observation among given data
Discrete Grouped Data	Highest frequency among given data
Continuous Grouped Data	$Z = L + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times c$

- Where,
- Modal class = A class with highest frequency
- L = Lower boundary of modal class
- c = Class length
- f_1 = Frequency of the modal class
- f_0 = Frequency of the class before the modal class
- f_2 = Frequency of the class after the modal class

Relation Between Mean, Median and Mode

- $Z = 3M - 2\bar{x}$; where \bar{x} = Mean, M = Median, Z = Mode

Example of Method-2.3: Mode

C	1	If mean is 16 and median is 20. Calculate the mode. Answer: 28																												
C	2	Find the mode of following data: (a) 2, 4, 2, 5, 7, 2, 8, 9. (b) 2, 8, 4, 6, 10, 12, 4, 8, 14, 16. Answer: (a) 2, (b) 4 & 8																												
C	3	Find the mode of following data: <table border="1"><tr><td>x</td><td>11</td><td>22</td><td>33</td><td>44</td></tr><tr><td>f</td><td>15</td><td>20</td><td>19</td><td>10</td></tr></table> Answer: 22					x	11	22	33	44	f	15	20	19	10														
x	11	22	33	44																										
f	15	20	19	10																										
C	4	Find the mode of following data: <table border="1"><tr><td>Class</td><td>0 – 10</td><td>10 – 20</td><td>20 – 30</td><td>30 – 40</td><td>40 – 50</td></tr><tr><td>f</td><td>3</td><td>5</td><td>7</td><td>10</td><td>12</td></tr><tr><td></td><td>50 – 60</td><td>60 – 70</td><td>70 – 80</td><td>80 – 90</td><td>90 – 100</td></tr><tr><td></td><td>15</td><td>12</td><td>6</td><td>2</td><td>8</td></tr></table> Answer: 55					Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	f	3	5	7	10	12		50 – 60	60 – 70	70 – 80	80 – 90	90 – 100		15	12	6	2	8
Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50																									
f	3	5	7	10	12																									
	50 – 60	60 – 70	70 – 80	80 – 90	90 – 100																									
	15	12	6	2	8																									
C	5	Obtain the mean, mode and median for the following information: <table border="1"><tr><td>x</td><td>< 10</td><td>< 20</td><td>< 30</td><td>< 40</td><td>< 50</td><td>< 60</td></tr><tr><td>f</td><td>12</td><td>30</td><td>57</td><td>77</td><td>94</td><td>100</td></tr></table> Answer: \bar{x} = 28, M = 27.407, Z = 25.625					x	< 10	< 20	< 30	< 40	< 50	< 60	f	12	30	57	77	94	100										
x	< 10	< 20	< 30	< 40	< 50	< 60																								
f	12	30	57	77	94	100																								

Method 3 \rightsquigarrow Measure of Variability

Measure of Variability

- It refers to the **spread** of the values around the central tendency.
- It is known as Measure of Dispersion.
- For Example:
–5, 0, 5 and – 50, 0, 50 both have the same mean 0 but clearly the data given in the second case much more widely dispersed than those in the first case.
- So, measures of central tendency are not sufficient for having some idea about dispersion.
- Measures of dispersion gives the idea about the degree to which numerical data tend to spread about an average life.
- There are certain measures of variability,
 - (1) Range
 - (2) Standard Deviation
 - (3) Mean Deviation

Range

- Range is simply the highest value **minus** the lowest value of a set of data values.
- For Example:
Range of –5, 0, 5 is 10

Reason: Range = Highest value – lowest value

$$= 5 - (-5)$$
$$= 10$$

Standard Deviation

- Standard deviation is a measure that is used to quantify the amount of variation or dispersion of a set of data values.
- It is denoted by " σ " and read as "sigma".

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→ Table of different formulae of standard deviation.

Method	Ungrouped Data	Discrete Grouped Data	Continuous Grouped Data
Direct Method	$\sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$	$\sqrt{\frac{\sum f_i x_i^2}{\sum f_i} - \left(\frac{\sum f_i x_i}{\sum f_i}\right)^2}$	
Assumed Mean Method	$\sqrt{\frac{\sum d_i^2}{n} - \left(\frac{\sum d_i}{n}\right)^2}$	$\sqrt{\frac{\sum f_i d_i^2}{\sum f_i} - \left(\frac{\sum f_i d_i}{\sum f_i}\right)^2}$	
Step Deviation Method	-----	-----	$\sqrt{\frac{\sum f_i u_i^2}{\sum f_i} - \left(\frac{\sum f_i u_i}{\sum f_i}\right)^2} \times c$

→ n = total number of observations

→ In case of **continuous frequency distribution**,

x_i = mid value of the respective class.

→ In case of **assumed mean method**, A can be any value from x_i .

→ Use below formula to calculate d_i & u_i

$$d_i = x_i - A ; u_i = \frac{x_i - A}{c}$$

Variance

→ Variance is **expectation** of the squared deviation.

→ It informally measures how far a set of (random) numbers are spread out from their mean.

→ It is denoted by capital letter "**V**" and defined as $V = \sigma^2$.

Coefficient of Variation

→ The Coefficient of Variation is the **ratio** of the standard deviation to the mean and shows the extent of variability in relation to the mean of the population.

→ Coefficient of Variance is defined as

$$C.V. = \frac{\sigma}{\bar{x}} \times 100$$

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- If C.V. is high, then it is less consistent. Similarly, if C.V. is less, then it is more consistent.
- The higher the Coefficient of Variation, the greater the dispersion.

Mean Deviation

- The mean deviation is defined as a statistical measure that is used to calculate the average deviation from the mean value of the given data set.
- In a simple word, the mean deviation is used to calculate how far the values fall from the middle of the data set.
- Table of different formulae of mean deviation:

Method	Ungrouped Data	Grouped Data
M.D. about Mean	$\frac{\sum x_i - \bar{x} }{n}$	$\frac{\sum f_i x_i - \bar{x} }{\sum f_i}$
M.D. about Median	$\frac{\sum x_i - M }{n}$	$\frac{\sum f_i x_i - M }{\sum f_i}$
M.D. about Mode	$\frac{\sum x_i - Z }{n}$	$\frac{\sum f_i x_i - Z }{\sum f_i}$

Example of Method-3: Dispersion

C	1	Find the standard deviation for the following data: 6, 7, 10, 12, 13, 4, 8, 12. Answer: 3.0414																						
C	2	Find the standard deviation for the following distribution: <table><tr><td>x</td><td>2</td><td>4</td><td>6</td><td>8</td><td>10</td></tr><tr><td>f</td><td>3</td><td>4</td><td>2</td><td>1</td><td>6</td></tr></table> Answer: 10.1094	x	2	4	6	8	10	f	3	4	2	1	6										
x	2	4	6	8	10																			
f	3	4	2	1	6																			
C	3	Find the standard deviation and variance for the following distribution: <table><tr><td>x</td><td>0 – 10</td><td>10 – 20</td><td>20 – 30</td><td>30 – 40</td><td>40 – 50</td><td>50 – 60</td><td>60 – 70</td></tr><tr><td>f</td><td>6</td><td>14</td><td>10</td><td>8</td><td>1</td><td>3</td><td>8</td></tr></table> Answer: $\sigma = 19.6214$, $V = 384.9993$	x	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	f	6	14	10	8	1	3	8						
x	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70																	
f	6	14	10	8	1	3	8																	
C	4	The arithmetic means of runs scored by three batsmen A,B and C, in the same series of 10 innings, are 50,48 and 12 respectively. The standard deviations of their runs are 15,12 and 2 respectively. Who is the most consistent of the three? Answer: Batsman C is more consistent.																						
C	5	Two machines A, B are used to fill a mixture of cement concrete in a beam. Find the standard deviation of each machine & comment on the performances of two machines. <table><tr><td>A</td><td>32</td><td>28</td><td>47</td><td>63</td><td>71</td><td>39</td><td>10</td><td>60</td><td>96</td><td>14</td></tr><tr><td>B</td><td>19</td><td>31</td><td>48</td><td>53</td><td>67</td><td>90</td><td>10</td><td>62</td><td>40</td><td>80</td></tr></table> Answer: $\sigma_A = 25.4950$, $\sigma_B = 24.4290$ There is less variability in the performance of the machine B.	A	32	28	47	63	71	39	10	60	96	14	B	19	31	48	53	67	90	10	62	40	80
A	32	28	47	63	71	39	10	60	96	14														
B	19	31	48	53	67	90	10	62	40	80														

C	6	<p>An analysis of monthly wages paid to the workers of two firms A and B belonging to the same industry gives the following results:</p> <table><tr><td></td><td>Firm A</td><td>Firm B</td></tr><tr><td>Number of workers</td><td>500</td><td>600</td></tr><tr><td>Average daily wage</td><td>186</td><td>175</td></tr><tr><td>Variance of distribution of wages</td><td>81</td><td>100</td></tr></table> <p>(1) Which firm has a larger wage bill? (2) In which firm, is there greater variability in individual wages? (3) Calculate average daily wages of all the workers in the firms A & B taken together.</p> <p>Answer: (1) Firm B, (2) Firm B, (3) 180</p>		Firm A	Firm B	Number of workers	500	600	Average daily wage	186	175	Variance of distribution of wages	81	100
	Firm A	Firm B												
Number of workers	500	600												
Average daily wage	186	175												
Variance of distribution of wages	81	100												
C	7	<p>Find the mean deviation about the mean and median for the following data: 2, 4, 7, 8, 9.</p> <p>Answer: $MD(\bar{x}) = 2.4$, $MD(M) = 2.2$</p>												
C	8	<p>Find mean deviation about the mean, median and mode for the following data:</p> <table><tr><td>x</td><td>5</td><td>10</td><td>15</td><td>20</td><td>25</td></tr><tr><td>f</td><td>7</td><td>4</td><td>6</td><td>3</td><td>5</td></tr></table> <p>Answer: $MD(\bar{x}) = 6.32$, $MD(M) = 6.2$, $MD(Z) = 9$</p>	x	5	10	15	20	25	f	7	4	6	3	5
x	5	10	15	20	25									
f	7	4	6	3	5									
C	9	<p>Find mean deviation about the mean, median and mode for the following data:</p> <table><tr><td>Class</td><td>5 – 25</td><td>25 – 45</td><td>45 – 65</td><td>65 – 85</td><td>85 – 105</td></tr><tr><td>f</td><td>12</td><td>8</td><td>14</td><td>20</td><td>6</td></tr></table> <p>Answer: $MD(\bar{x}) = 21.33$, $MD(M) = 21.904$, $MD(Z) = 23.466$</p>	Class	5 – 25	25 – 45	45 – 65	65 – 85	85 – 105	f	12	8	14	20	6
Class	5 – 25	25 – 45	45 – 65	65 – 85	85 – 105									
f	12	8	14	20	6									

Method 4 \rightsquigarrow Moments

Moments

- Moment is the arithmetic **mean** of the various powers of the deviations of items from their assumed mean or actual mean.
- There are three types of moments which is
 - (1) Moment about mean
 - (2) Moment about assumed mean
 - (3) Moment about zero

Moment about mean

- If deviations of data are taken from mean, then it is known as moment about mean.
- It is also known as central moment.
- It is denoted by " μ_r ".

Moment about assumed mean

- If deviations of data are taken about any assumed value(a) from x_i , then it is known as moment about assumed mean.
- It is denoted by " μ'_r ".

Moment about zero

- If deviations of data are taken about any zero, it is known as moment about zero.
- It is denoted by " v_r ".
- Table of different formulae of moment.

Moment about	Ungrouped Data	Grouped Data
Mean (μ_r)	$\frac{\sum (x_i - \bar{x})^r}{n}$	$\frac{\sum f_i (x_i - \bar{x})^r}{\sum f_i}$
Assumed mean (μ'_r)	$\frac{\sum (x_i - a)^r}{n}$	$\frac{\sum f_i (x_i - a)^r}{\sum f_i}$
Zero (v_r)	$\frac{\sum x_i^r}{n}$	$\frac{\sum f_i x_i^r}{\sum f_i}$

Where, $r = 1, 2, 3, 4, \dots$

Example of Method-4: Moments

C	1	<p>Find the first four moments about assumed mean 5, actual mean and zero for the data 1, 3, 7, 9, 10.</p> <p>Answer: $\mu = 0, \quad 12, \quad -12, \quad 208.8$</p> <p>$\mu' = 1, \quad 13, \quad 25, \quad 233.8$</p> <p>$v = 6, \quad 48, \quad 420, \quad 3808.8$</p>												
C	2	<p>Calculate the four moments about assumed mean 5, actual mean and zero for following distribution.</p> <table><tr><td>x</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>f</td><td>1</td><td>3</td><td>7</td><td>3</td><td>1</td></tr></table> <p>Answer: $\mu = 0, \quad 0.9333, \quad 0, \quad 2.5333$</p> <p>$\mu' = -1, \quad 1.9333, \quad -3.8, \quad 9.1333$</p> <p>$v = 4, \quad 16.9333, \quad 75.2, \quad 348.1333$</p>	x	2	3	4	5	6	f	1	3	7	3	1
x	2	3	4	5	6									
f	1	3	7	3	1									
C	3	<p>Calculate the moments about assumed mean 25, actual mean and zero for following distribution:</p> <table><tr><td>Class</td><td>0 – 10</td><td>10 – 20</td><td>20 – 30</td><td>30 – 40</td></tr><tr><td>f</td><td>1</td><td>3</td><td>4</td><td>2</td></tr></table> <p>Answer: $\mu = 0, \quad 81, \quad -144, \quad 14817$</p> <p>$\mu' = -3, \quad 90, \quad -900, \quad 21000$</p> <p>$v = 22, \quad 565, \quad 15850, \quad 471625$</p>	Class	0 – 10	10 – 20	20 – 30	30 – 40	f	1	3	4	2		
Class	0 – 10	10 – 20	20 – 30	30 – 40										
f	1	3	4	2										

Method 5 \rightsquigarrow Measures of Skewness

Skewness

- In symmetrical distribution, the mean, median, and mode is equal.
- When a frequency distribution is not symmetrical, it is known as asymmetrical or skewed.
- Skewness means **lake** of symmetry.

Measures of Skewness

- Measure of skewness give us an idea about the extent of lopsidedness in a data.
- The various measures of skewness are
 - (1) Karl Pearson's method
 - (2) Method of moments

Karl Pearson's Method

- Karl Pearson's **coefficient of skewness** is defined as

$$\text{Skewness} = \frac{\text{Mean} - \text{Mode}}{\text{Standard Deviation}}$$

Method of Moments

- **Measure of skewness** is obtained from moment about mean and defined as

$$\text{Skewness} = \beta_1 = \frac{(\mu_3)^2}{(\mu_2)^3}$$

- The skewness value can be either positive or negative or zero or even undefined.
- If skewness value is zero, then the distribution is known as symmetric.
- If standard deviation is zero, then the skewness is not defined.

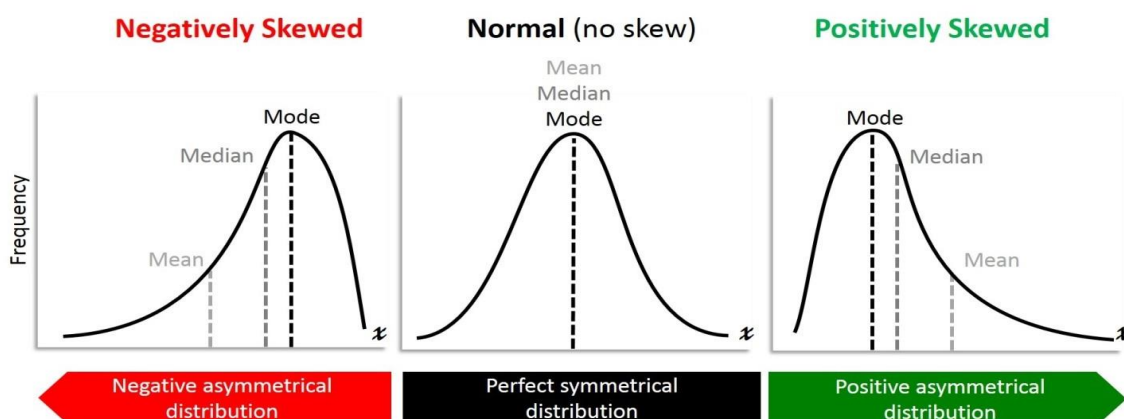
Types of Skewness

- **Positive skewness**

The right tail is longer; the mass of the distribution is concentrated on the left.

- **Negative skewness**

The left tail is longer; the mass of the distribution is concentrated on the right.



Example of Method-5: Skewness

C	1	Compute the Karl Pearson's coefficient of skewness for data: 25, 15, 23, 40, 27, 25, 23, 25, 20 Answer: -0.03								
C	2	Karl Pearson's coefficient of skewness of a distribution is 0.32, its standard deviation is 6.5 and mean is 29.6. Find the mode for the distribution. Answer: 27.52								
C	3	For a group of 10 items, $\sum x = 452$, $\sum x^2 = 24270$ and mode is 43.7 then, find Karl Pearson's coefficient of skewness. Answer: 0.077								
C	4	From the marks scored by 100 students in section A and 100 students in section B of a class, the following measures were obtained <table border="1"><tr><td>Section A</td><td>$\mu_A = 55$</td><td>$\sigma_A = 15.4$</td><td>Mode = 58.72</td></tr><tr><td>Section B</td><td>$\mu_B = 53$</td><td>$\sigma_B = 15.4$</td><td>Mode = 48.83</td></tr></table> Determine which distribution of marks is more skewed. Answer: Section B is more skewed	Section A	$\mu_A = 55$	$\sigma_A = 15.4$	Mode = 58.72	Section B	$\mu_B = 53$	$\sigma_B = 15.4$	Mode = 48.83
Section A	$\mu_A = 55$	$\sigma_A = 15.4$	Mode = 58.72							
Section B	$\mu_B = 53$	$\sigma_B = 15.4$	Mode = 48.83							

C	10	Prove that the skewness of the following data is 0.0390 using method of moment.				
		Class	0 – 10	10 – 20	20 – 30	30 – 40
		Frequency	1	3	4	2

Method 6 \rightsquigarrow Kurtosis

Introduction

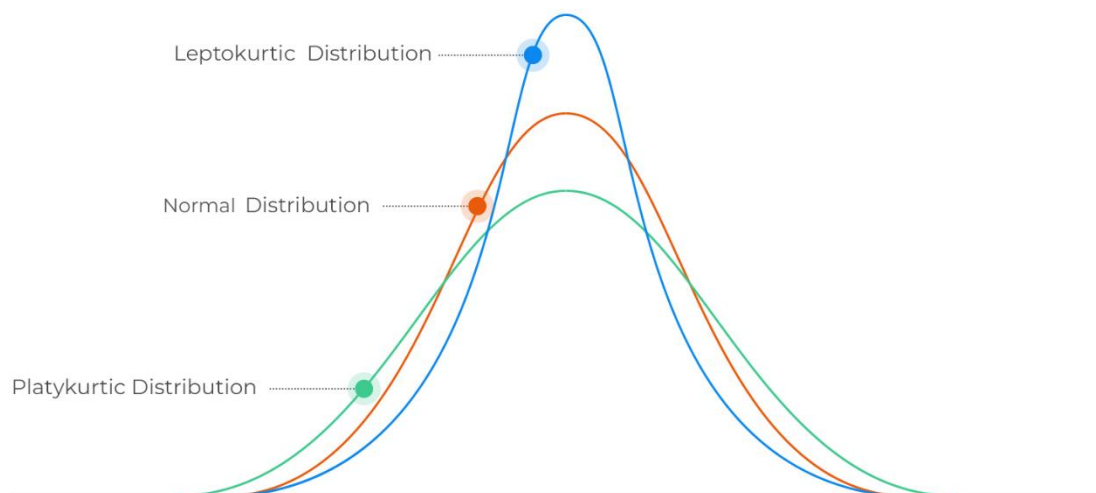
- Measures of central tendency, dispersion and skewness of a random variable cannot give a complete idea about the probability distribution.
- So, another characteristic, Kurtosis is required.

Kurtosis

- Kurtosis measures the flatness/peakedness of a distribution.
- It is denoted by " β_2 " defined as

$$\beta_2 = \frac{\mu_4}{(\mu_2)^2}$$

- The greater the value of β_2 , the more peaked is the distribution.
- When the value of $\beta_2 = 3$, the curve is normal curve and the distribution is known as **mesokurtic**.
- When the value of $\beta_2 > 3$, the curve is more peaked than normal curve and the distribution is known as **leptokurtic**.
- When the value of $\beta_2 < 3$, the curve is less peaked than normal curve and the distribution is known as **platykurtic**.



Example of Method-6: Kurtosis

C	1	Find the Kurtosis for the following data. Also comment on type of distribution.																	
		<table><tr><td>x</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>f</td><td>1</td><td>3</td><td>7</td><td>3</td><td>1</td></tr></table>						x	2	3	4	5	6	f	1	3	7	3	1
		x	2	3	4	5	6												
		f	1	3	7	3	1												
Answer: Kurtosis = 2.9082, Distribution is platykurtic																			

C	2	Find out the kurtosis of the following data:																	
		<table><tr><td>Class</td><td>0 – 10</td><td>10 – 20</td><td>20 – 30</td><td>30 – 40</td><td>40 – 50</td></tr><tr><td>f</td><td>10</td><td>20</td><td>40</td><td>20</td><td>10</td></tr></table>						Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	f	10	20	40	20	10
		Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50												
		f	10	20	40	20	10												
Answer: 2.5																			

***** End of the Unit *****