5

Superconductivity

5.1 INTRODUCTION

KEY WORDS

resistivity, thermal vibrations, impurity scattering, residual resistivity, critical temperature, phase diagram, Kamerlingh Onnes, electrical resistance and superconductivity, refrigeration, cryostatic apparatus, critical temperature, phenomenon of superconductivity, ceramic materials, critical magnetic field, magnetic induction, Meissner effect, thermal conductivity, bound system, infinite conductivity, neutron scattering, critical temperature, Faraday's law of induction, flux, transition temperature, critical field, phase diagram, free energy, condensation energy, Type I and Type II superconductors, perfect diamagnetism, hysteresis, flux pinning, hard superconductors, flux jumping, quench, critical field and transition temperature, London penetration depth, current density, characteristic length, Ginsberg-Landu theory, characteristic length, intrinsic coherence length, mean free path, normal conduction electrons, penetration depth, fluxoid, lower critical field, upper critical field, specific heat, Sommerfeld constant, Fermi surface, lattice contribution, lattice specific heat, Boltzmann factor, isotopic effect, ionic mass, electron-electron interaction, cooper-pairs, thermodynamic aspects, reversible transition, free energy density, internal energy, entropy, pressure, volume, external field, magnetic moment, first law of thermodynamics, BCS microscopic theory, correlated system, third law of thermodynamics, second order phase transition, Ruter's formula, mechanical properties, thermal conductivity, superelectrons, exchange energy, thermal switches, thermoelectric effects, Thomson relations, Peltier and Thomson coefficients, London's theory, superelectrons and normal electrons, conduction electron density, Meissner effect, scattering and lattice vibrations, density of charge carriers, Ohm's law, current density, magnetic induction vector, London equation, London's penetration depth, coherence length, condensed state, microscopic theory, phonon interactions, cooper-pairs, Bardeen, Cooper and Schrieffer, de Broglie waves, zero electrical resistivity, phonons (lattice waves/vibrations), scattering centres, isotopic effect, Fröhlich, the electron-phonon interaction, virtual phonon, Arrhenius plot, electron-lattice interaction energy, electron density of states, Fermi level, dynamic process, collective state, attenuation, copper oxide perorskites, ceramic oxide, antiferromagnetism, 1-2-3 compounds, YBCO, joule heating, memory element, ductility, Josephson effect, SQUID.

5.1 INTRODUCTION

The electrical resistivity of many metals and alloys at low temperatures is nearly constant. For a perfectly pure metal, where the electron motion is impeded only by the thermal vibrations of the lattice, the resistivity should approach zero as the temperature is reduced to 0 K. Any real specimen of a metal cannot be perfectly pure and will contain some impurities. Therefore the electrons, in addition to being scattered by *thermal vibrations* of the *lattice atoms*, are scattered by the *impurities*, and this *impurity scattering* is more or less independent of temperature. As a result, there is a certain "*residual resistivity*" (ρ_0) which remains even at the lowest temperatures. The more impure the metal, the larger will be its residual resistivity.

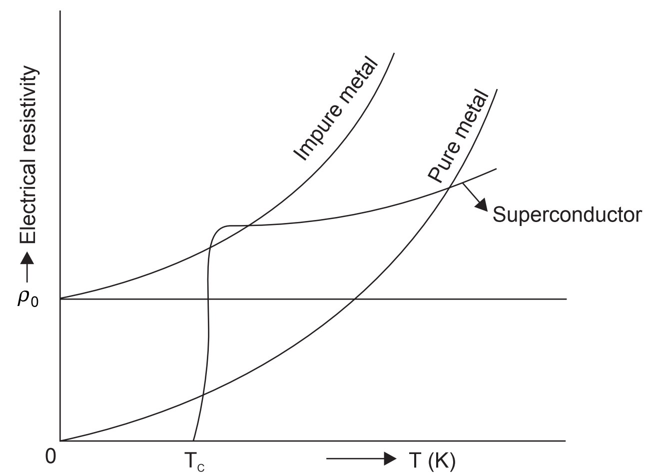


Fig. 5.1.1 *Variation of resistance of a metal and a superconductor with temperature*

However a good number of metals and alloys behave differently at low temperatures near absolute zero. As the temperature decreases, the resistivity at first decreases regularly, like that of any metal. At a particular temperature (*known as critical temperature*), *a phase transition* occurs and the resistivity suddenly drops to zero as shown in Fig. 5.1.1. The transition from normal conductivity occurring over a very narrow range of temperature of the order of 0.05 K (presence of small trace of impurity may be the cause for the range of temperature over which it drops to zero). This phenomenon was first observed by *Kamerlingh Onnes* in 1911 while studying the behaviour of mercury at liquid helium temperature. He observed that at 4.2 K, the resistance of mercury suddenly vanished. This phenomenon of disappearance of *electrical resistance* below a certain temperature is called *superconductivity*. This zero resistivity was observed in other metals such as Al, Pb, Sn, Nb, etc.

Refrigeration is cheap when the difference between the working temperature and that of the *heat sink* is not too great. A domestic refrigerator requires about 0.2 J to remove 1 J of heat from its interior. However, liquid helium boils at 4.2 K and 300 J must be expended to remove 1 J from a body at 4.2 K to be rejected to a heat sink at 300 K, when the *efficiency* of the cooling plant is taken into account. So using liquid helium means keeping down heat leaks as much as possible: usually liquid nitrogen is used to cool radiation shields surrounding the liquid helium, and the *cryostatic apparatus* is bulky and costly- so much so that superconducting magnets are the only large-scale application of superconductivity.

Though some bubble chamber and accelerator magnets are enormous, there are not many of them and the tonnage of superconducting material used word-wide is relatively small. The economic picture will be vastly changed, however, when superconducting materials capable of carrying reasonable current densities are able to operate at 77 K. In such a case large *generators, motors*, *transformers* and *transmission lines* will be a practical.

Q 5.1 Give an elementary account of superconductivity.

Certain substances completely lose their electrical resistance below a *certain critical temperature*. This critical *temperature* is different for different substances. When a substance loses its electrical resistance, a current set up in it unaltered for ever. This phenomenon is known as *superconductivity*. The *characteristic transition temperature* T_c varies form 0.15 K to 20 K for pure metals. However *ceramic materials* have transition temperature around 90 K. Superconducting elements have at room temperature, greater electrical resistivity than others. When impurities are added to superconducting elements, the superconducting properties are not lost, but the transition temperature is lowered. The transition temperature of an element differs for different *isotopes*.

Below the transition temperature, when a substance is a superconductor, the superconducting property may be destroyed by the application of a sufficiently strong magnetic field. At any given temperature below T_c, there is a *critical magnetic field* H_c such that the superconducting property is destroyed by the application of a magnetic field of intensity $H \geq H_c$. The value of H_c decreases as the temperature (which is less than T_c) increases. See Fig. Q 5.2.2 (*a*).

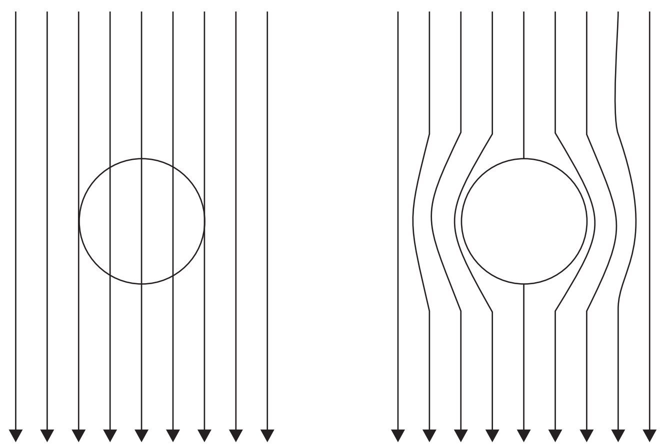


Fig. Q 5.1.1 *The Meissner effect (a) Normal $T > T_c$ or $H > H_c$ (b) Superconducting $T < T_c$ or $H < H_c$*

If a superconducting substance is placed in a magnetic field H_c such that $H < H_c$, at a temperature $T \leq T_c$, it is found that no lines of magnetic induction exist inside the substance. The substance, therefore, pushes out the lines, of *magnetic induction*, so that B = 0 inside the substance. See Fig. Q 5.1.1. This is known as *Meissner effect*. The thermal properties such as *specific heat* and *thermal conductivity* of a substance change abruptly when it passes over into the superconducting state. The phenomenon of superconductivity can be explained satisfactorily on the basis of wave mechanics. In ordinary metal, the electrical resistance is the result of the collisions of the conduction electrons with the vibrating ions in the crystal lattice. In the superconducting state, the electrons tend to scatter in pairs rather than individually. This gives rise to an exchange force (similar to the force between the atoms in a hydrogen molecule and the forces between *nucleans* in a nucleus) between the electrons. The force is attractive, and is very strong if the electrons have opposite spins and momenta.

In the superconducting state, the forces of attraction between the conduction electrons exceed the forces of electrostatic repulsion. The entire system of conduction electrons then becomes a *bound system*. No transfer of energy takes place from this system to the lattice ions. If an electric field is established inside the substance, the electrons gain additional kinetic energy and give rise to a current. But they do not transfer this energy to the lattice, so that they do not get slowed down. As a consequence of this, substance does not possess any electrical resistivity.

Q 5.2 Discuss the important experimental results in the study of superconductivity using simple models and suitable diagrams.

Answer*: 1. *Infinite conductivity

When a superconducting specimen is cooled to a temperature below the transition temperature, the resistivity of the sample becomes zero, and not just small. Experimental measurements conclude that the resistance of a superconductor is about 10^{18} times smaller than that of a metal; or it is zero for all practical purposes. Other conclusions are: there is no change in crystal structure at this temperature; similarly, the transition is not a *magnetic transition*, as can be inferred from *neutron scattering* experiments. The state that is produced is clearly a completely new thermodynamic state associated with an electronic transition. Assuming that the relationship between current and electric field is given by *Ohm's law*,

$$\vec{J} = \sigma \vec{E} \tag{Q 5.2.1}$$

in order that the current density is infinite, the electric field E inside a superconductor must be zero.

2. *Meissner effect*

Although the infinite conductivity characterizes a superconducting state, the truly different nature of the superconducting state is manifested in its magnetic property. Let us first consider a normal metal say aluminium or lead in a uniform magnetic field.

Refer Fig. Q 5.2.1*a*. The given metal is first cooled to a temperature, below the *critical temperature*, making it superconducting. This cooled sample is now placed under the influence of a uniform external field. The flux lines will not penetrate into the specimen or they are excluded. This is because (according to *Faraday's law of induction*), currents induced in the sample will oppose any change of flux through the specimen. Since the sample is a superconductor, such currents, once induced, will persist in the *absence of any resistance* will keep the *magnetic flux* out. The worth mentioning point is that, not only does a superconductor oppose the entry of *flux* into a superconducting specimen, it expels any flux that might be there in the specimen before it became a superconductor. This behaviour is best illustrated in Fig. Q 5.2.1 (b), where a normal metal in a magnetic field is shown. Since the metal is in a normal state, magnetic flux lines will penetrate the sample. When such a sample is cooled to a temperature below its transition temperature, the magnetic flux, which is already inside the specimen, will be expelled. This is contrary to the expectation from *Faraday's law* which would tend to trap the flux which is already inside the specimen. Thus, a superconductor is not just a perfect conductor, it is also a *perfect diamagnetic*

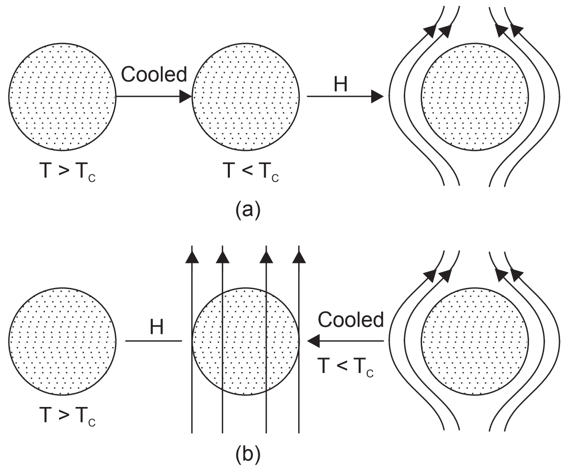


Fig. Q 5.2.1 *Meissner effect. In Fig. Q 5.2.1 (a), the sample is first cooled and then subjected to the field, while in (b) a normal metal is first subjected to a magnetic field and then cooled below critical temperature*

3. *Critical field*

Prof. Onnes important finding is that the application of suitable magnetic fields destroy superconductivity. The minimum magnetic field necessary to destroy superconductivity is called *critical field* (generally denoted as H_c). It is also observed that this critical magnetic field is a function of temperature. Fig. Q 5.2.2 shows that the critical magnetic field H_0 near absolute zero falls to zero at the *superconducting transition temperature*. This figure can also be called *phase diagram* of a superconductor. The metal will be a superconducting one for any combination of the applied magnetic field and temperature. This combination gives a point P. The arrows indicate, the metal can be driven into the normal state by increasing either the field or temperature.

Metallic elements with *low transition temperatures* have low electric fields at zero degree kelvin. Hence every superconductor has a different *Phase diagram*. See Fig. Q 5.2.2 (b). As the magnetic field strength is increased, flux lines start penetrating the sample, destroying superconductivity.

If $H_c(T)$ is the strength of the magnetic field required to destroy the superconductivity in a specimen at a temperature T, the difference in the *free energy* per unit volume between the superconducting state and the normal state at this temperature is given by the energy density of the magnetic field.

$$\boxed{A_n(T) - A_s(T) = \frac{H_c^2(T)}{8\pi}} \quad \text{(Q 5.2.1)}$$

This energy difference is called the *condensation energy*. Experimental results guide that the critical fields fall almost as the square of the absolute temperature, so the curves can be closely approximated by the parabola of the form:

$$\boxed{H_c = H_0\left[1 - \left(\frac{T}{T_c}\right)^2\right]} \quad \text{(Q 5.2.2)}$$

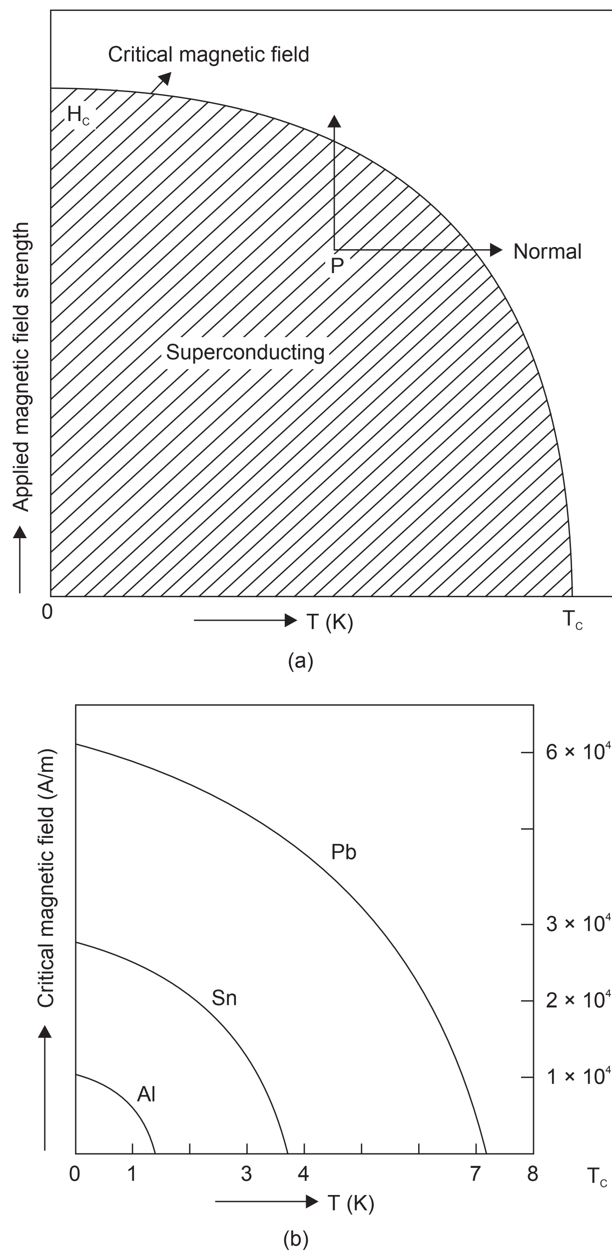
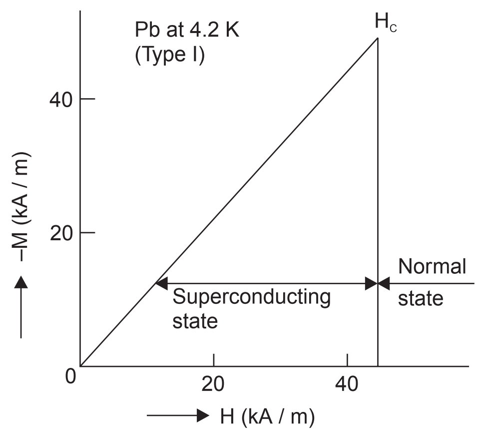


Fig. Q 5.2.2 *(a) Phase diagram of a superconductor (b) Critical fields of superconductors*

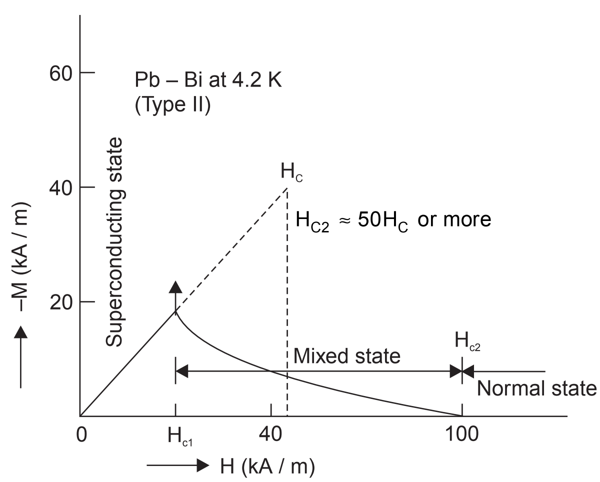
Here H_o and T_c are the critical field at 0 K and the critical temperature respectively for a given specimen. Thus the critical field at any temperature can be computed using this equation.

Q 5.3 Explain Type I and II superconductors. Also briefly discuss the important property changes during the transition.

***Answer*:** Both type I and type II materials up to H_{c1} exhibit *perfect diamagnetism*, since $\mathbf{M} = -\mathbf{H}$, $\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) = 0$, and $\chi = \dfrac{M}{H} = -1$. But on reducing the magnetic field from H_c or H_{c2}, we find that it does not trace the same path in reverse as when we increase the field from zero. In fact, in type II superconductors, the magnetization follows the path shown in Fig. Q 5.3.1 (c) below the field axis, while in type I materials it follows a path a little below the perfectly diamagnetic plot. Thus both samples show hysteresis, but it is much more pronounced for the type II material. Indeed, if a type I superconductor is made from very pure material, and it is well annealed so that it contains very few dislocations, then the *hysteresis* is slight. The hysteresis indicates that the flux which penetrated the sample while the field was increasing is not all expelled when the field is reduced. Cold working a type I material increases the *hysteresis* considerably, so that dislocations and other defects must cause *flux pinning*: this is a clue to making hard superconductors that will keep their magnetic flux in place-make them 'dirty'. Movement of magnetic flux (known as *flux-jumping*) is not desirable in a superconducting wire because it causes heating and reversion of superconducting regions to normal conduction, leading sometimes to a runway condition known as a *quench* in which the whole solenoid may revert suddenly to normal conduction.



(a)



(b)

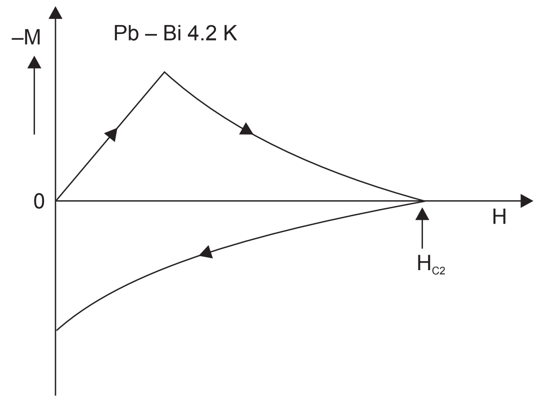


Fig. Q 5.3.1 *(a) Magnetization curve for type I superconductors*
(b) Magnetization curve for type II superconductors
(c) Hysteresis in a type II superconductors

The critical fields are found to be highly dependent on the temperature, and in type I materials we can write,

$$\boxed{H_c(T) = H_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right]} \qquad \text{(Q 5.3.1)}$$

where $H_c(T)$ is the *critical field* at T K and T_c is the *transition temperature*. For lead, H_c (4.2) = 42 kA/m, T_c = 7.2 K, so $H_c(0)$ 64 *kA*/m. This relation is approximately true for upper critical field of type II materials.

It was believed that Maxwell's equations be supplemented in the case of superconductors as per F. London and H. London around the year 1935. This leads to

$$\boxed{\nabla \times \vec{J} = \frac{\vec{H}}{\lambda_L^2}} \qquad \text{(Q 5.3.2)}$$

where λ_L is the *London penetration depth*. Now $\nabla \times \mathbf{H} = \mathbf{J}$ by Maxwell's equations, where **J** is the current density, so $\nabla \times \nabla \times \mathbf{H} = \nabla \times \mathbf{J}$, which is $-\frac{\vec{H}}{\lambda_L^2}$ from Eqn. (Q 5.3.2). It can be shown that $\nabla \times \nabla \times \mathbf{H} = -\nabla^2\ \mathbf{H}$, so that $\nabla^2\,\mathbf{H} = \frac{\vec{H}}{\lambda_L^2}$. Considering a superconductor with the field parallel to its surface, whose normal is the x–axis, we can see that

$$\boxed{H(x) = H_0 \exp(-x/\lambda_L)} \qquad \text{(Q 5.3.3)}$$