Bohr Atom, Photoelectric Effect and Compton Effect

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Recapitulate

Planck's Black body radiation formula

Einstein's theory for specific heats of solids

Specific heats of gases

What we learn

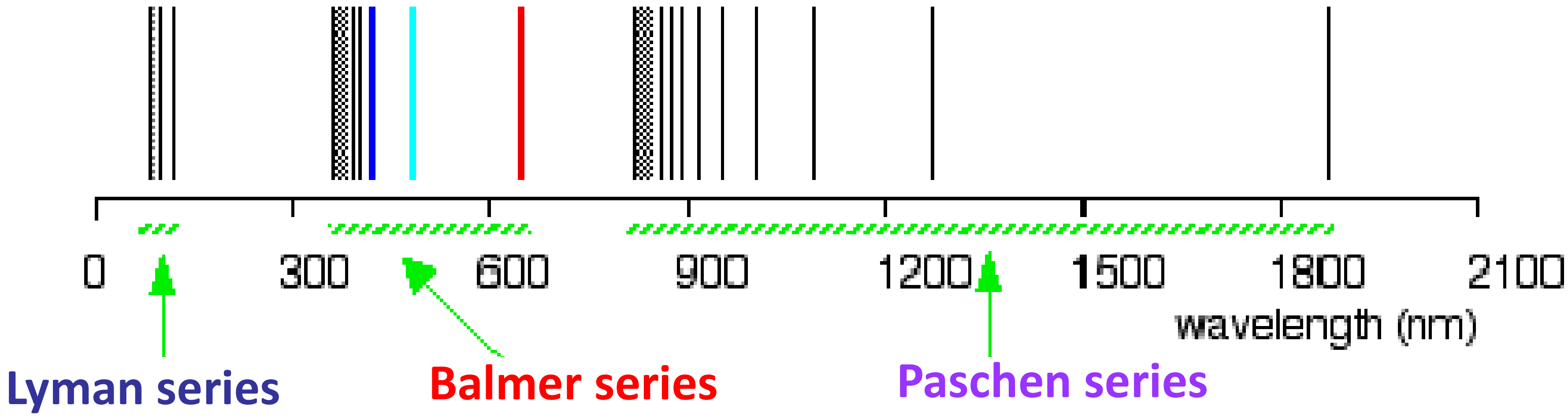
Failure of classical equipartition theorem

Need for 'quantization' to explain experimental results

'Quantum' results lead to classical results under some specific conditions

Appearance of Planck's constant *h*

Spectrum of hydrogen atom (1859 -   )



Wavelengths of lines can be fitted to a general formula (Rydberg formula) with ***R = Rydberg constant = 1.0973732x10^7 m^{-1}***

$$\frac{1}{\lambda} = \frac{\nu}{c} = R\left(\frac{1}{n^2} - \frac{1}{m^2}\right) \qquad m > n$$

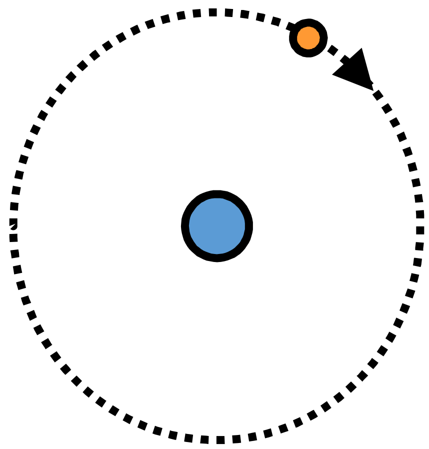
Lyman series:
n=1, m=2, 3, 4, ……

Balmer series:
n=2, m=3, 4, 5…….

Paschen series:
n=3, m=4, 5, 6…….

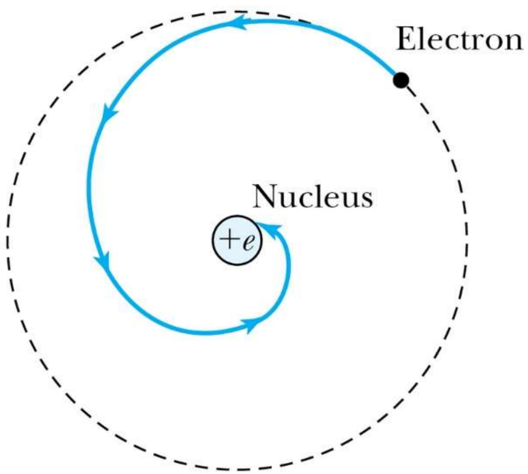
One more case which gives a hint of quantization!

Planetary Model of Atom



Electrons orbit the nucleus, much the same way planets orbit the Sun.

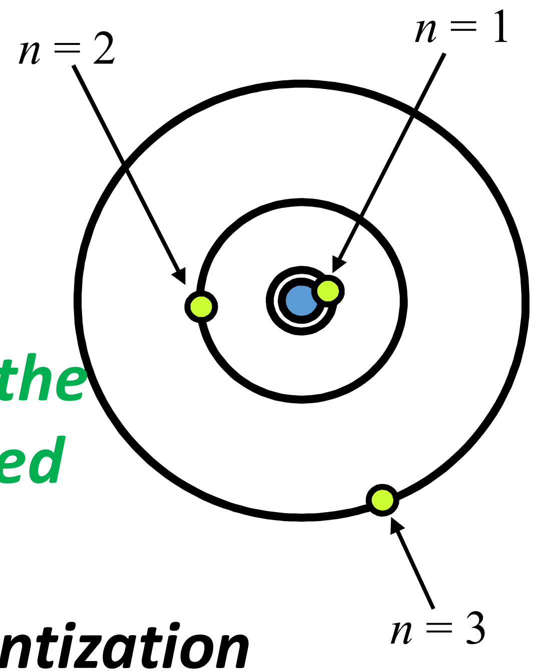
As per the classical electromagnetic theory, an accelerated electric charge radiates energy (EM radiation)



Total energy must decrease. So the **radius r must decrease!**

Electron should crash into the nucleus in ~10^{-22} s!!

Bohr's Model (1913)



Quantization condition

- *Atoms contain stable orbits in which the electrons can stay indefinitely if undisturbed*

- ***These orbits are identified by Bohr's quantization condition: electron angular momentum is an integral multiple of Planck's constant:*** $m_e vr = n(h/2\pi) = n\hbar$

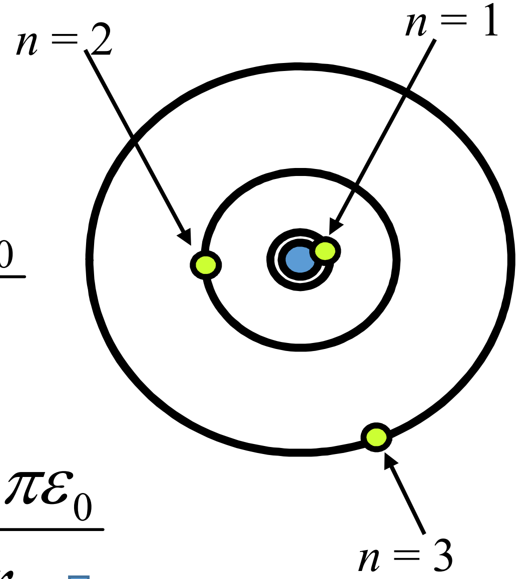
- ***Stable orbits are defined by quantum number n***

- ***Energy is emitted/absorbed when electron jumps from one stationary state to the other.***

$$\Delta E = E_n - E_m$$

h cross or h cut $\hbar = \dfrac{h}{2\pi}$

Bohr's Model (Z=1)



$$\frac{e^2/4\pi\varepsilon_0}{r^2} = \frac{m_e \mathrm{v}^2}{r}$$

$$KE = \frac{1}{2} m_e \mathrm{v}^2 = \frac{e^2/4\pi\varepsilon_0}{2r}$$

$$PE = -\frac{e^2/4\pi\varepsilon_0}{r}$$

$$E = KE + PE = -\frac{e^2/4\pi\varepsilon_0}{2r}$$

$$m_e \mathrm{v} r = n\hbar$$

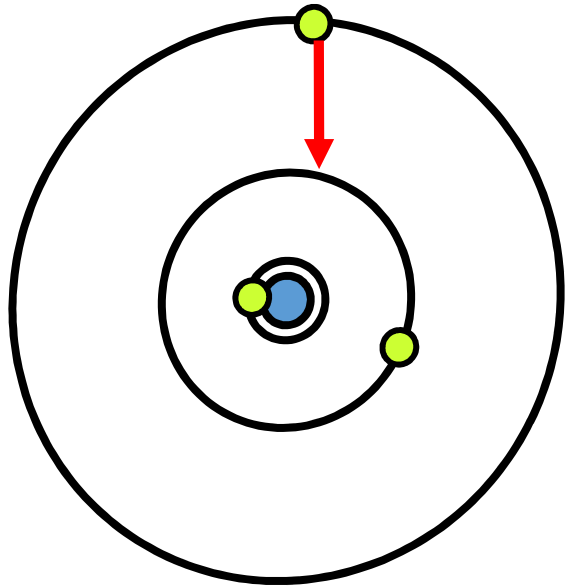
$$r = \frac{n^2\hbar^2}{(e^2/4\pi\varepsilon_0) m_e}$$

$$E_n = -\frac{(e^2/4\pi\varepsilon_0)^2 m_e}{2\hbar^2 n^2}$$

Bohr radius

$$r(n=1) = a_0 = \frac{\hbar^2}{(e^2/4\pi\varepsilon_0) m}$$

$$E_n = -\frac{(e^2/4\pi\varepsilon_0)^2 m_e}{2\hbar^2 n^2}$$



Transition m $\rightarrow$ n (m > n)

$$\Delta E = \frac{(e^2/4\pi\varepsilon_0)^2 m_e}{2\hbar^2}\left(\frac{1}{n^2}-\frac{1}{m^2}\right)$$

$$\Delta E = h\nu = hc/\lambda$$

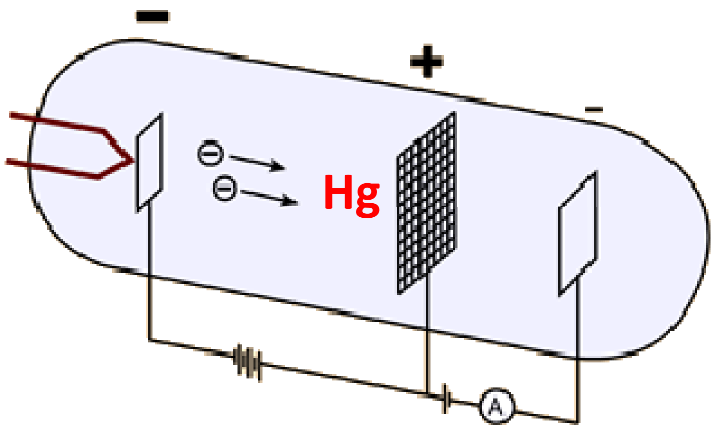
$$\frac{1}{\lambda} = \left(\frac{1}{hc}\right)\frac{(e^2/4\pi\varepsilon_0)^2 m_e}{2\hbar^2 n^2}\left(\frac{1}{n^2}-\frac{1}{m^2}\right) = R\left(\frac{1}{n^2}-\frac{1}{m^2}\right)$$

R = Rydberg constant = 1.0973732x10^{7} m^{-1}

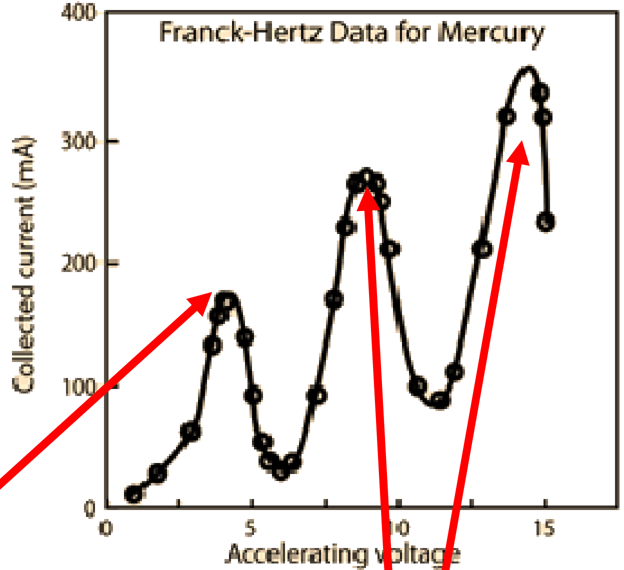
Rydberg constant in terms of fundamental constants!

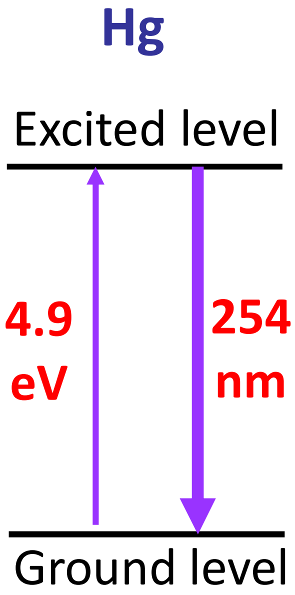
Franck-Hertz Experiment

Direct confirmation of atomic energy levels



Accelerating Apparatus





At accelerating voltage of 4.9 V, the current sharply drops, due to inelastic collisions between the accelerated electrons and atomic electrons in Hg atoms.

Drops in the current occur at multiples of 4.9 V, since an accelerated electron which has 4.9 eV of energy removed in a collision can be re-accelerated to produce other such collisions at multiples of 4.9 volts.

Expressions derived so far are for nucleus with infinite mass

$$E_n = -\frac{(e^2/4\pi\varepsilon_0)^2 m_e}{2\hbar^2 n^2} = -\frac{R_\infty}{n^2}$$

$$r = \frac{n^2\hbar^2}{(e^2/4\pi\varepsilon_0)m_e}$$

Nucleus with finite mass

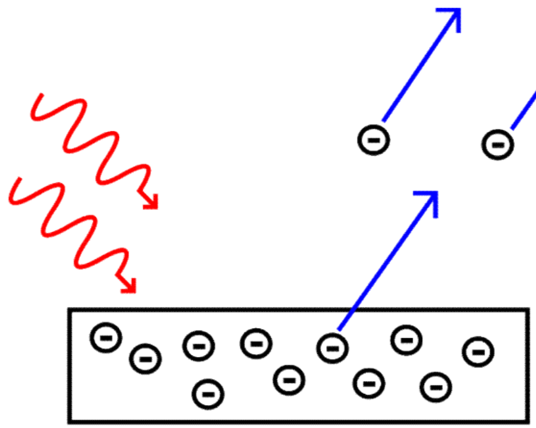
$$m_e \rightarrow \mu = \frac{m_e m_N}{m_e + m_N}$$

$\mu =$ ***Reduced mass***

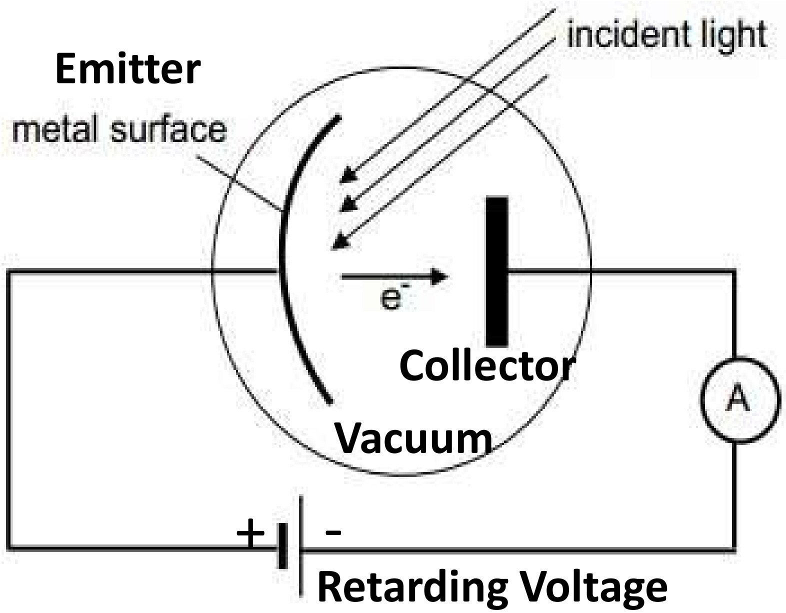
$$\mu_{Hydrogen} = \frac{m_e m_{proton}}{m_e + m_{proton}}$$

$$\mu_{Deuterium} = \frac{m_e (2m_{proton})}{m_e + 2m_{proton}}$$

Photoelectric Effect



Metal surfaces when exposed to light emit electrons **(photoelectrons)**



V_s = Stopping voltage

(when photocurrent becomes 0)

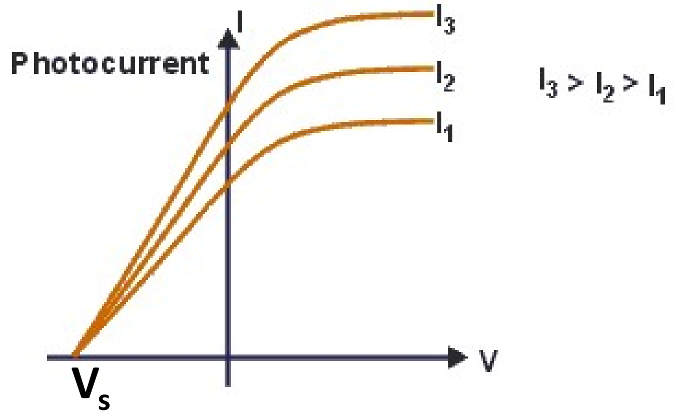
K_{max} = Maximum kinetic energy of photoelectron

$$K_{\max} = \frac{1}{2} m_e v^2 = eV_s$$

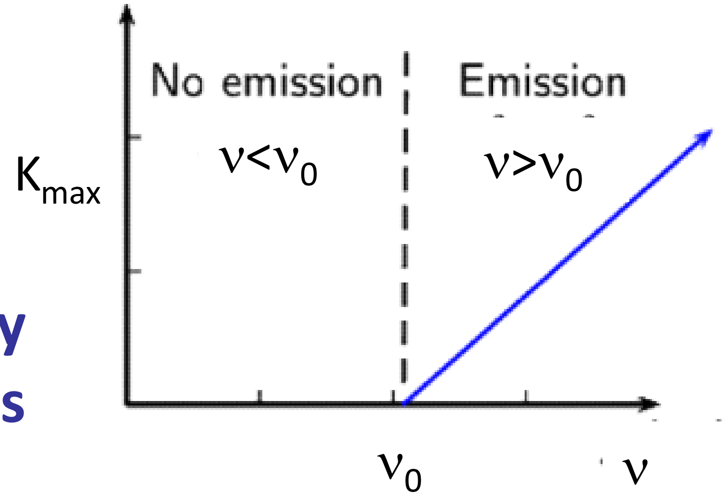
Observations

K_{max} = Maximum KE of photoelectron

- K_{max} **does not depend on intensity (I) of light**



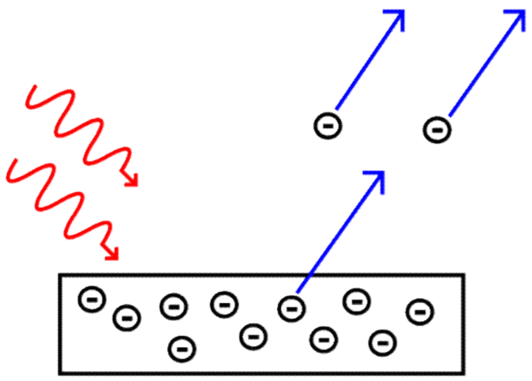
- K_{max} **increases with frequency of light**



- **Existence of a threshold frequency (ν_0) below which no photoelectrons are detected.**

- **Photoelectrons are detected instantaneously.**

Photoelectric effect: Classical Theory



I : Intensity of incident light

I_a : Absorbed light intensity = cI

K : Kinetic energy of electron

A : Cross sectional area of atom

t : Time of exposure to radiation

φ : Work function

$$K = I_a A t$$

$$\boxed{K_{\max} = K - \varphi = I_a A t - \varphi}$$

- **For** $I_a A t > \varphi$ **electron should be emitted with higher KE**

- Photoelectron detection time = **Time to acquire sufficient energy to overcome work function**

$$\boxed{t = \frac{\varphi}{I_a A}}$$

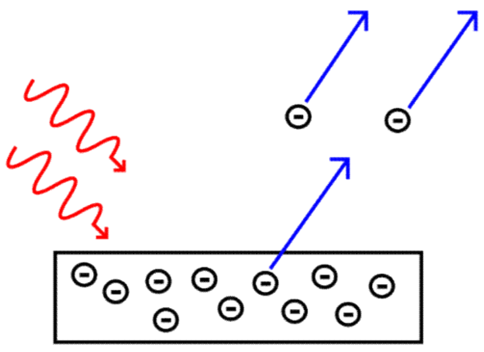
$\varphi = 2.28 eV$ $\quad I_a = 1 \times 10^{-7} mW/cm^2$

$A = \pi \times 10^{-16} cm^2$ $\quad t = 1.2 \times 10^7 s$

(Not consistent with experimental observations)

Einstein's Theory of Photoelectric Effect (1905)

Einstein extended Planck's hypothesis:
Radiation is not just emitted in 'quanta'
but ***is always in the form of 'quanta'***



Light of frequency ν consist of quanta of energy hν

When a quantum of energy E= hν hits a bound electron in a metal, that electron is freed and emitted as a photoelectron

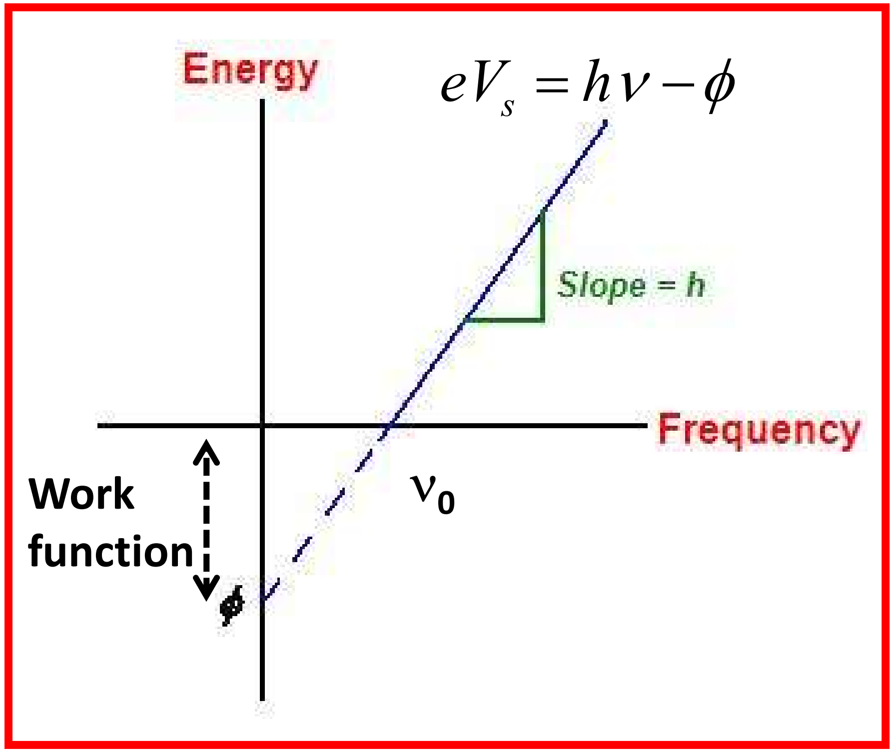
Energy conservation:

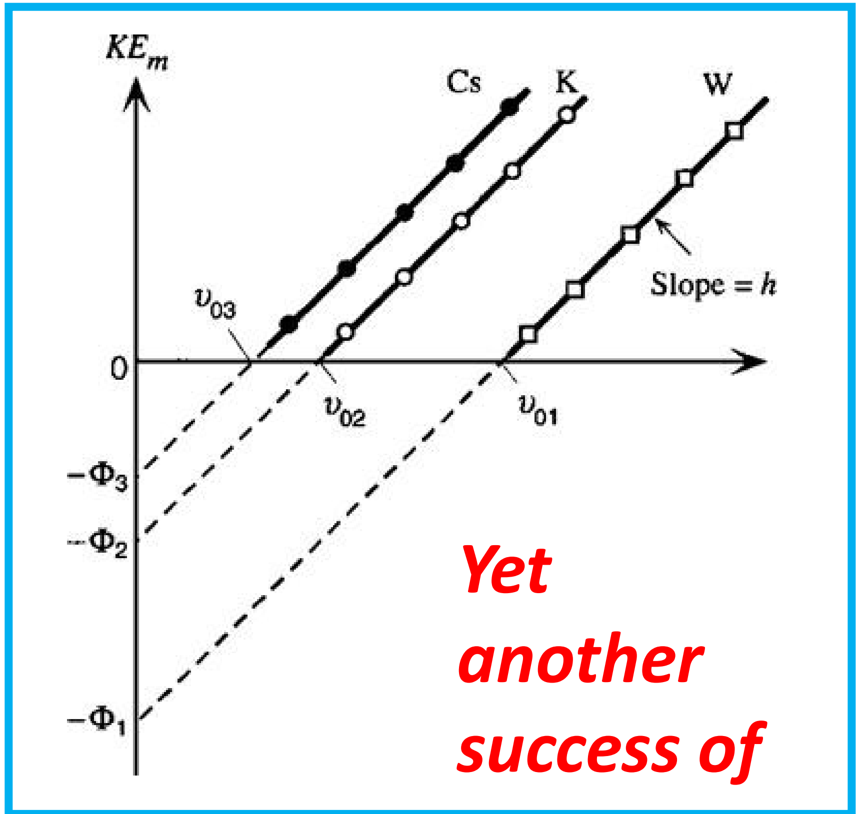
$$K_{\max} = h\nu - \phi$$

Work function of a metal
(Energy with which an electron is bound in a metal)

$$K_{\max} = h\nu - \phi \qquad eV_s = h\nu - \phi \qquad K_{\max} = \frac{1}{2} m_e v^2 = eV_s$$

For fixed ν, increase in intensity (I=nhν) causes more photoelectron emission per second, but K_{max} remains unchanged.





Yet another success of quantum hypothesis

Complete explanation of photoelectric effect

Method to obtain value of *h* (Mulliken, 1916)

Photoelectric Effect

- Provides strength to the particle picture of photon

- Demonstrates that photon has energy $E=h\nu$, thus combining wave and particle aspects

- Demonstrates energy conservation in photon interacting with matter

- *Can we think of momentum of photon?*

A bit of Special Relativity

*Modifying our notion of **energy (E)** and **momentum (p)***

An object has intrinsic (or rest) mass m_0

Such an object has rest mass energy $E = m_0 c^2$

Mass $m = \dfrac{m_0}{\sqrt{1-(\mathrm{v}^2/c^2)}} = \gamma m_0$

Momentum $\vec{p} = m\vec{\mathrm{v}} = \gamma m_0 \vec{\mathrm{v}}$

*Energy of an object with momentum **p** is*

$$E = \sqrt{p^2c^2 + m_0^2c^4} = \gamma m_0 c^2$$

Particles with zero rest mass $m_0 = 0$

$$E = \sqrt{p^2c^2 + m_0^2c^4} \Rightarrow E = pc$$

Consider the case of photon

$$E = h\nu$$ **Photon energy** $$pc = h\nu$$

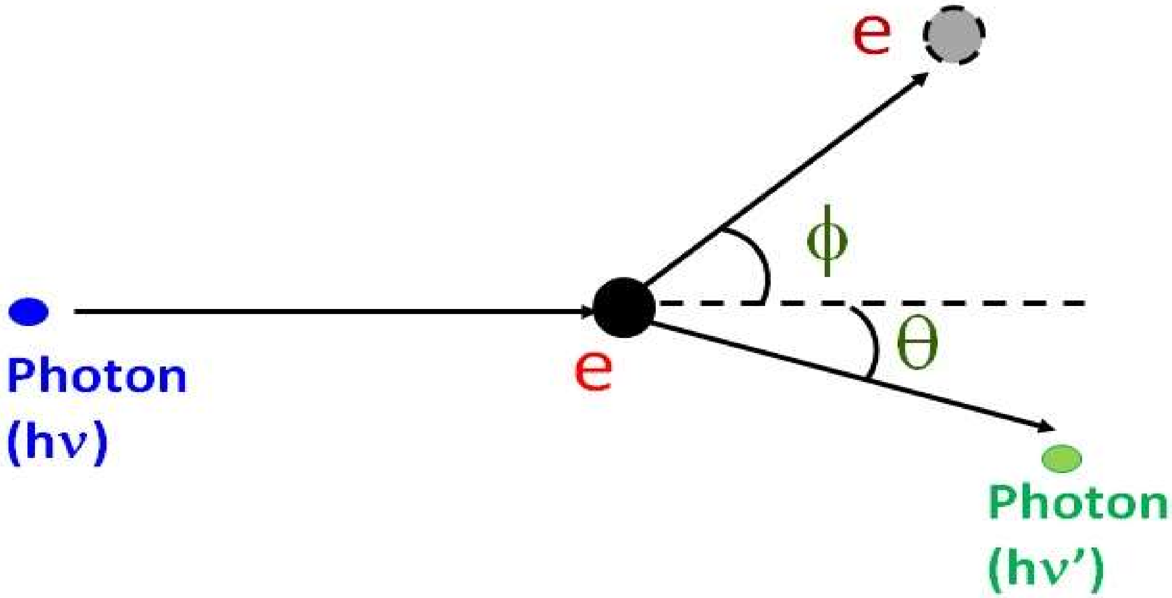
$$p = \frac{h\nu}{c} = \frac{h}{\lambda}$$ ***Photon momentum***

Can we demonstrate conservation of momentum when photon interacts with matter?

Compton Effect

Recoil of a photon by a free electron.

Going ahead with an idea that light shows a particle nature



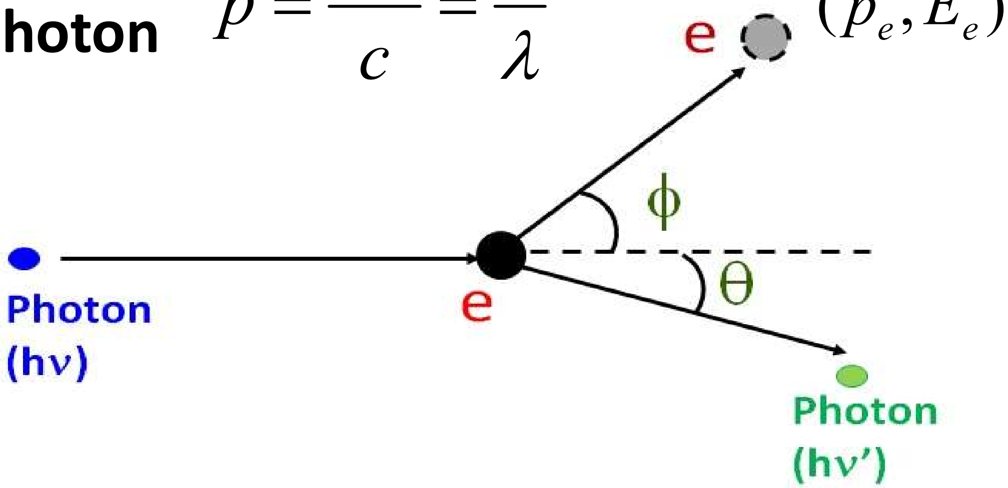
Need to know Relativistic Energy

$$E^2 = p^2c^2 + (m_0c^2)^2$$

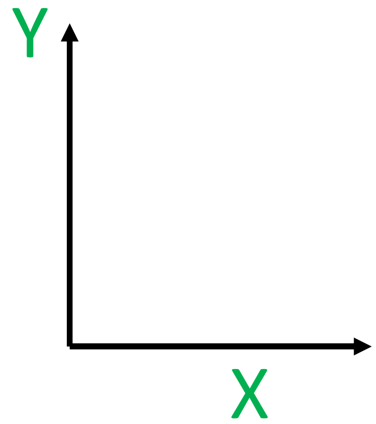
For photons,
$m_0=0$

$$E = pc$$
$$= (h/\lambda)c$$
$$= h\nu$$

For photon $p = \frac{h\nu}{c} = \frac{h}{\lambda}$



$$E_e^2 = p_e^2 c^2 + m_0^2 c^4$$



$$\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\theta + p_e \cos\varphi$$

$$\frac{h\nu'}{c}\sin\theta = p_e \sin\varphi$$

Momentum conservation (x- and y-components)

$$m_0 c^2 + h\nu = h\nu' + E_e$$

Energy conservation

$$\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\theta + p_e\cos\varphi \qquad \frac{h\nu'}{c}\sin\theta = p_e\sin\varphi$$

Eliminate φ

$$p_e^2(\cos^2\varphi + \sin^2\varphi) = p_e^2 = \left(\frac{h\nu}{c} - \frac{h\nu'}{c}\cos\theta\right)^2 + \left(\frac{h\nu'}{c}\sin\theta\right)^2$$

$$p_e^2 = \left(\frac{h\nu}{c}\right)^2 + \left(\frac{h\nu'}{c}\right)^2 - 2\left(\frac{h\nu}{c}\right)\left(\frac{h\nu'}{c}\right)\cos\theta$$

Energy equation….

$$m_0c^2 + h\nu = h\nu' + E_e \quad \Rightarrow \quad E_e = h\nu - h\nu' + m_0c^2$$

Relativistic energy

$$E_e^2 = p_e^2c^2 + (m_0c^2)^2$$

Expression for p_e is available from momentum equations

$$p_e^2 = \left(\frac{h\nu}{c}\right)^2 + \left(\frac{h\nu'}{c}\right)^2 - 2\left(\frac{h\nu}{c}\right)\left(\frac{h\nu'}{c}\right)\cos\theta$$

$$\Downarrow$$

$$(h\nu - h\nu' + m_0c^2)^2$$

$$= (h\nu)^2 + (h\nu')^2 - 2(h\nu)(h\nu')\cos\theta + (m_0c^2)^2$$

$$(h\nu - h\nu' + m_0c^2)^2$$
$$= (h\nu)^2 + (h\nu')^2 - 2(h\nu)(h\nu')\cos\theta + (m_0c^2)^2$$

$$(h\nu)^2 + (h\nu')^2 + (m_0c^2)^2 - 2(h\nu)(h\nu') - 2(h\nu')(m_0c^2) + 2(h\nu)(m_0c^2)$$
$$= (h\nu)^2 + (h\nu')^2 - 2(h\nu)(h\nu')\cos\theta + (m_0c^2)^2$$

$$(h\nu - h\nu')m_0c^2 = (h\nu)(h\nu')(1-\cos\theta)$$

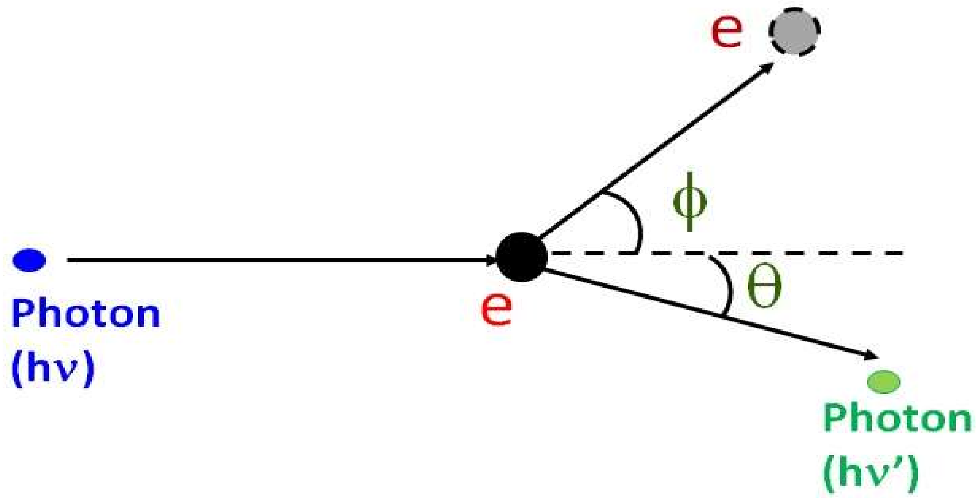
$$\left(\frac{hc}{\lambda} - \frac{hc}{\lambda'}\right)m_0c^2 = \frac{hc}{\lambda}\frac{hc}{\lambda'}(1-\cos\theta)$$

$$\boxed{\lambda' - \lambda = \frac{h}{m_0c}(1-\cos\theta)}$$

Compton Wavelength

$$\lambda_c = \frac{h}{m_0c}$$

= 2.43 x 10^{-3} nm



$$\lambda_c = \frac{h}{m_0 c}$$

$= 2.43 \times 10^{-3}$ nm

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}(1 - \cos\theta)$$

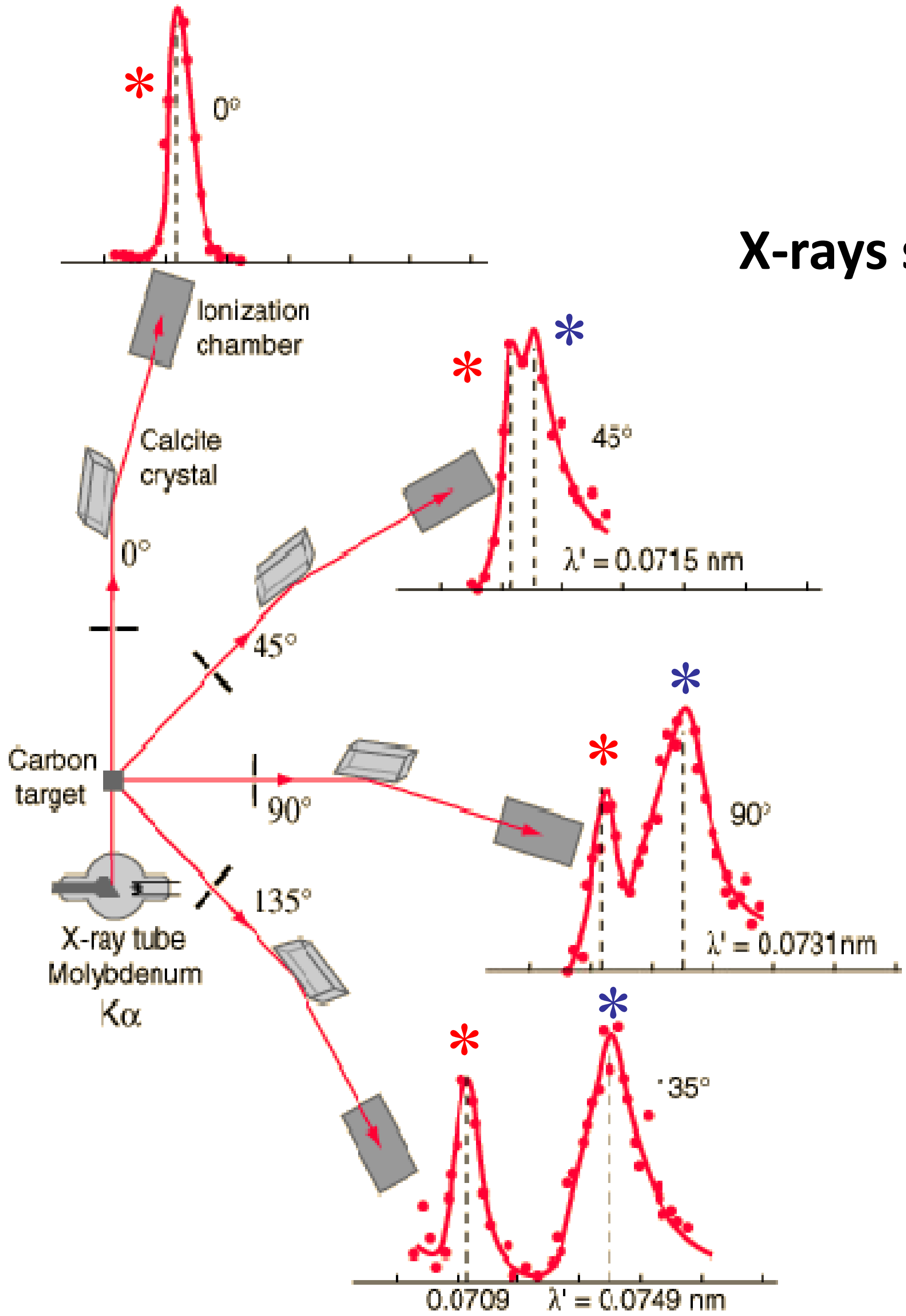
$\Delta\lambda=0$ for $\theta=0$ and

$\Delta\lambda = 2\lambda_c$ for $\theta=180^o$.

Need high frequency photon to observe Compton Effect.

Max. change in wavelength: $2\lambda_c=4.86 \times 10^{-12}$ m. This is insignificant for visible light (10^{-7} m) but not for x-ray or γ-ray ($< 10^{-10}$ m)

The equation is valid even if we use particle other than electron to scatter photon. One has to, however, use appropriate mass.



Compton scattering

X-rays scattered from carbon target

*** Scattering from free electrons in carbon target**

*** Scattering from inner electrons in carbon target (*this produces very little wavelength shift because they are tightly bound*)**

Compton Effect

- Demonstrates momentum conservation in photon interacting with matter

- Provides strength to the particle picture of photon