

- | | | |
|-----------------------|----------|-------|
| Laser λ & f | ϕ m | cm |
| 20 | 200 mks | 10 mm |
- (9d)

९८

(1)

(1)

(2)

▽

6

(3)

9

(4)

△

$\phi_{\bar{E}}$

$$dF = \frac{dF}{dx} dx$$

$$T(x, y, z) \quad dT = \frac{\partial T}{\partial x} dx + \frac{\partial T}{\partial y} dy + \frac{\partial T}{\partial z} dz$$

$$= \left(\frac{\partial T}{\partial x} \mathbf{i} + \frac{\partial T}{\partial y} \mathbf{j} + \frac{\partial T}{\partial z} \mathbf{k} \right) \cdot (dx \mathbf{i} + dy \mathbf{j} + dz \mathbf{k})$$

$$dT = \nabla T \cdot (dl) \rightarrow \text{line element}$$

$$\textcircled{1} \quad \nabla \rightarrow \text{Del} = \left(\mathbf{i} \frac{\partial}{\partial x} + \mathbf{j} \frac{\partial}{\partial y} + \mathbf{k} \frac{\partial}{\partial z} \right)$$

$\Delta \rightarrow$ operator \rightarrow does not simplify multiply
 \rightarrow ordering to differentiate

$\textcircled{2}$ Max^m value ($\nabla T = 0$)
 \rightarrow differential along line element

$$\textcircled{1} \quad \Delta T = 0 \rightarrow \text{at max^m / min^m}$$

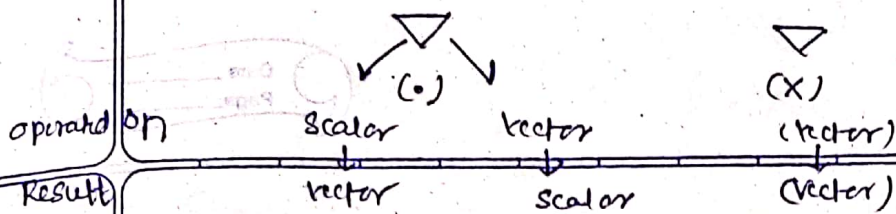
operator on [scalar qty]

$$\textcircled{1} \quad \nabla \cdot T = \text{"Gradient"} \leftarrow \text{called result (vector)}$$

$$\textcircled{2} \quad \nabla \cdot (\mathbf{v}) = \text{"Divergence"} \leftarrow \text{resulted into scalar}$$

vector via dot product.

Eg: $\left(\mathbf{i} \frac{\partial}{\partial x} + \mathbf{j} \frac{\partial}{\partial y} + \mathbf{k} \frac{\partial}{\partial z} \right) \cdot (i v_x + j v_y + k v_z)$



$$= \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} \quad \left. \vphantom{\frac{\partial V_x}{\partial x}} \right\} \text{Scalar qty}$$

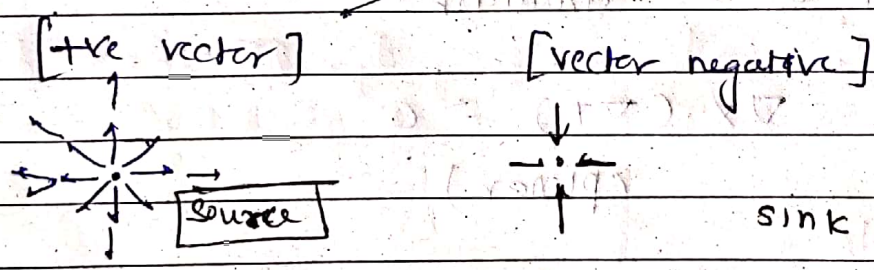
③ $\nabla \times \vec{v} \rightarrow$ "curl" result into vector

Operator Vector via cross product

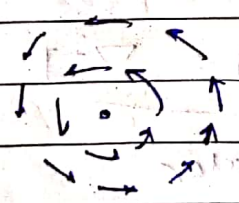
i	j	k
$\frac{\partial}{\partial x}$	$\frac{\partial}{\partial y}$	$\frac{\partial}{\partial z}$
V_x	V_y	V_z

असल में है क्या?

① Divergence = how the fcn is spreading out in space
(Vectorial 3D effect)



② Curl = How the vector is squirreling about a point
(Planner effect)



Gradient

$\nabla(\nabla T)$
NOT possible

(1) Divergence of a Gradient

$$\nabla \cdot (\nabla T) = \nabla^2 T \quad \text{[Laplacian operator]}$$

$$\left(i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z} \right) \cdot \left(i \frac{\partial T}{\partial x} + j \frac{\partial T}{\partial y} + k \frac{\partial T}{\partial z} \right)$$

$$= \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}$$

(2) Curl of a Gradient

$$\nabla \times (\nabla T) = 0$$

(plenary)

(3) Gradient of Divergence

$$\underbrace{\nabla(\nabla \cdot \mathbf{v})}_{\text{vector}} \neq \underbrace{\nabla \cdot (\nabla T)}_{\text{scalar}} = \nabla^2 T$$

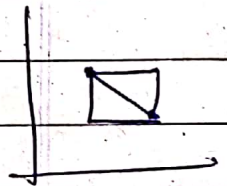
Cur

① $\nabla \cdot (\nabla \times \mathbf{v}) = 0$

② $\nabla \times (\nabla \times \mathbf{v}) = \nabla (\nabla \cdot \mathbf{v}) - (\nabla \cdot \nabla) \mathbf{v}$
 $= \nabla (\nabla \cdot \mathbf{v}) - \nabla^2 (\mathbf{v})$

ordinary integral $\int_a^b dF dx \neq F(b) - F(a)$

depends on path



$\oint \mathbf{F} \cdot d\mathbf{l} \neq 0$

Non-conservative

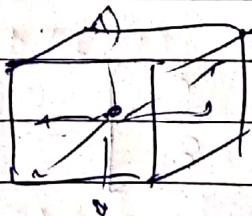
Ⓐ depends on initial & final points - Not path dependent

$\int_a^b \nabla T \cdot d\mathbf{l} = T(b) - T(a)$

(ii) $\oint \nabla T \cdot d\mathbf{l} = 0$

[v.v. imp]

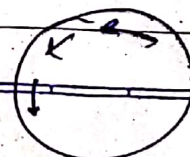
$\int (\nabla \cdot \mathbf{v}) d\tau = \oint \mathbf{v} \cdot d\mathbf{a}$



Greens theorem /
Gauss theorem

$\int (\nabla \times \mathbf{v}) d\mathbf{a} = \oint \mathbf{v} \cdot d\mathbf{l}$

[Stokes theorem]



$$\int (\nabla \cdot \mathbf{v}) d\tau = \oint \mathbf{v} \cdot d\mathbf{a}$$

$$\int (\nabla \times \mathbf{v}) d\mathbf{a} = \oint \mathbf{v} \cdot d\mathbf{l}$$

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \frac{q_0}{r^2}$$

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \sum \frac{q_i}{r_i^2} = \frac{1}{4\pi\epsilon_0} \frac{\hat{r} (dq)}{r^2}$$

\downarrow
 $\rho d\tau$

σdA

$\rho d\tau$

Spherical coordinates

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \int dq \cdot r^2 \sin\theta d\theta d\phi$$

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\epsilon_0}$$

$$\int (\nabla \cdot \mathbf{E}) d\tau = \frac{1}{\epsilon_0} \int \rho d\tau$$

$$\left[\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \right]$$

$$\nabla \cdot \mathbf{v} = \nabla \cdot (\nabla \times \mathbf{A})$$

$$\oint \vec{E} \cdot d\vec{l} = 0 \quad \text{for } \frac{1}{r^2}$$

$$\frac{q}{4\pi\epsilon_0} \int \frac{1}{r^2} dl = \frac{q}{4\pi\epsilon_0} \left. \frac{1}{r} \right|_{r_a}^{r_b} = (0) \quad \text{loop}$$

Earlier

Now

$$\oint \vec{E} \cdot d\vec{l} = \text{emf} = - \frac{\partial(\phi_b)}{\partial t}$$

$$\left[\oint \vec{E} \cdot d\vec{l} = - \frac{\partial(\phi_b)}{\partial t} \right]$$

Stokes

$$\int (\nabla \times \vec{E}) \cdot d\vec{A} = - \frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{A}$$

$$\left[\nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t} \right]$$

$$\left[\oint \mathbf{B} \cdot d\mathbf{a} = 0 \right]$$

integral form

$$\int (\nabla \cdot \mathbf{B}) d\tau = 0$$

$$[\nabla \cdot \mathbf{B} = 0]$$

Differential form

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I \quad \left(\begin{array}{l} \text{[Integral form]} \\ \text{(Steady current)} \end{array} \right)$$

$$\int (\nabla \times \mathbf{B}) d\mathbf{a} = \mu_0 \int \mathbf{J} \cdot d\mathbf{a}$$

$$[\nabla \times \mathbf{B} = \mu_0 \mathbf{J}]$$

Diff

*

$$\nabla \cdot (\nabla \times \mathbf{E}) = 0 = \nabla \cdot \left(-\frac{\partial \mathbf{B}}{\partial t} \right) = -\frac{\partial}{\partial t} (\nabla \cdot \mathbf{B})$$

$$\nabla \cdot (\nabla \times \mathbf{B}) = 0 = \nabla \cdot (\mu_0 \mathbf{J}) = \mu_0 (\nabla \cdot \mathbf{J})$$

$$\frac{dq}{dt} = I = \int \mathbf{J} \cdot d\mathbf{a}$$

$$\frac{d}{dt} \int \rho d\tau = \int (\nabla \cdot \mathbf{J}) d\tau$$

$$\int \nabla \cdot \mathbf{J} d\tau = -\frac{d}{dt} \int \rho d\tau = -\frac{d}{dt} (\epsilon_0 \nabla \cdot \mathbf{E}) d\tau$$

See -
continuity eqn

conservation of
charge inside

$$\nabla \cdot \mathbf{J} = - \nabla \cdot \left(\epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$$

$$\left[\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \underbrace{\left(\mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)}_{\text{additional term}} \right]$$

additional term

$$\left[\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I + \frac{1}{c^2} \int \frac{d\mathbf{E}}{dt} \cdot d\mathbf{a} \right]$$

* Fibre optics = ~~all~~ Roden collen
(Electronic communication) *