THEOREM I *The area of a surface of revolution is equal to the length of the generating curve times the distance traveled by the centroid of the curve while the surface is being generated.*

Proof. Consider an element dL of the line L (Fig. 5.15) which is revolved about the x axis. The area dA generated by the element dL is equal to $2\pi y\, dL$. Thus, the entire area generated by L is $A = \int 2\pi y\, dL$. But we saw in Sec. 5.2 that the integral $\int y\, dL$ is equal to $\bar{y}L$. We have therefore

$$A = 2\pi\bar{y}L \tag{5.10}$$

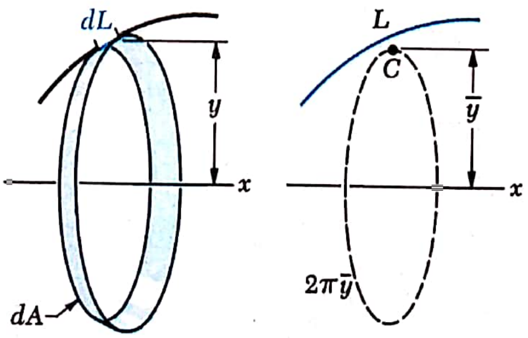


Fig. 5.15

where $2\pi\bar{y}$ is the distance traveled by the centroid of L. It should be noted that the generating curve should not cross the axis about which it is rotated; if it did, the two sections on either side of the axis would generate areas of opposite signs and the theorem would not apply.

THEOREM II *The volume of a body of revolution is equal to the generating area times the distance traveled by the centroid of the area while the body is being generated.*

Proof. Consider an element dA of the area A which is revolved about the x axis (Fig. 5.16). The volume dV generated by the element dA is equal to $2\pi y\, dA$. Thus, the entire volume generated by A is $V = \int 2\pi y\, dA$. But since the integral $\int y\, dA$ is equal to $\bar{y}A$ (Sec. 5.2), we have

$$V = 2\pi\bar{y}A \tag{5.11}$$

where $2\pi\bar{y}$ is the distance traveled by the centroid of A. Again, it should be noted that the theorem does not apply if the axis of rotation intersects the generating area.

The theorems of Pappus-Guldinus offer a simple way for computing the area of surfaces of revolution and the volume of bodies of revolution. They may also be used conversely to determine the centroid of a plane curve when the area of the surface generated by the curve is known or to determine the centroid of a plane area when the volume of the body generated by the area is known (see Sample Prob. 5.8).

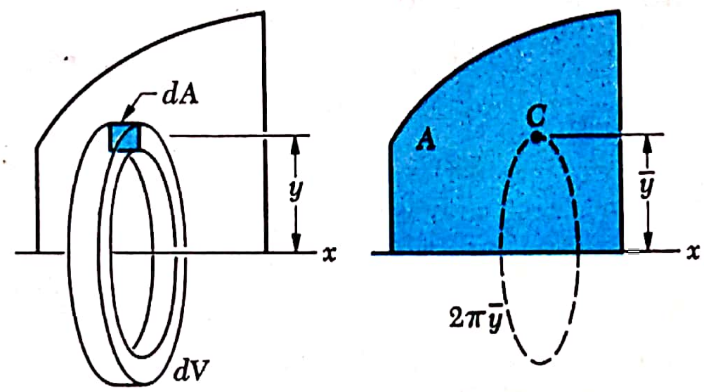
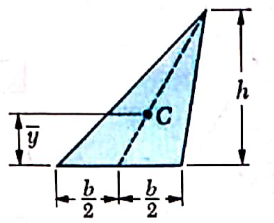
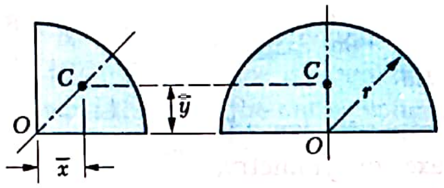
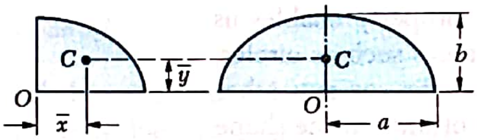
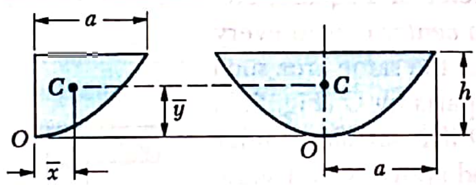
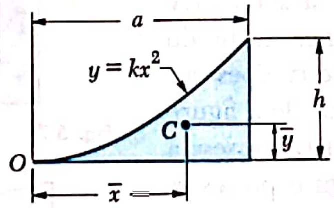
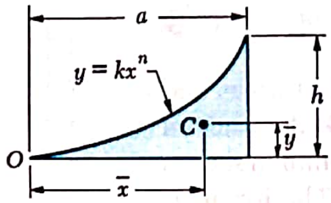
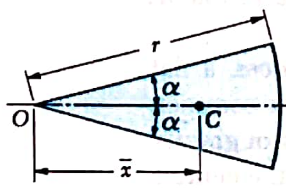


Fig. 5.16

Shape		$\bar{x}$	$\bar{y}$	Area
Triangular area			$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
General spandrel		$\frac{n+1}{n+2}a$	$\frac{n+1}{4n+2}h$	$\frac{ah}{n+1}$
Circular sector		$\frac{2r\sin\alpha}{3\alpha}$	0	αr^2

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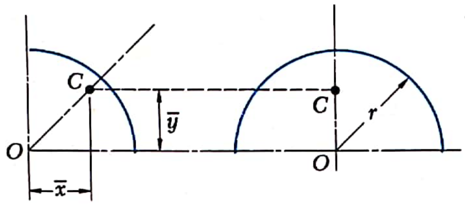
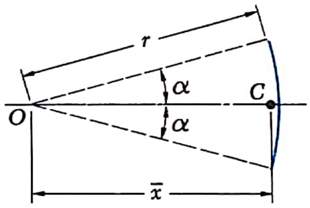
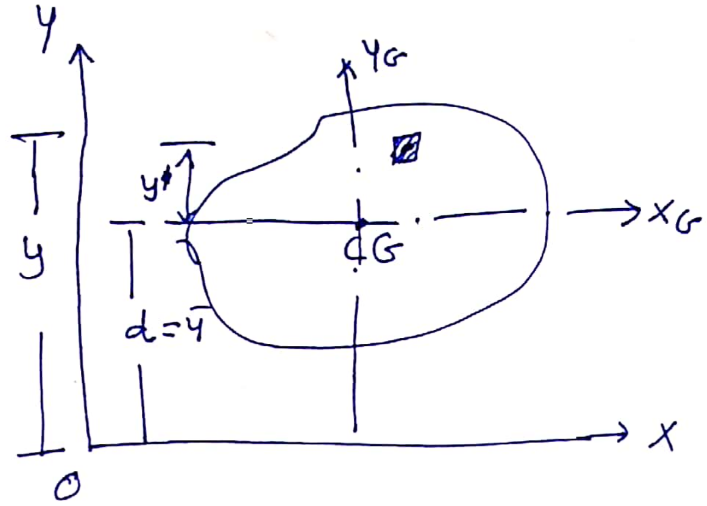
Shape		$\bar{x}$	$\bar{y}$	Length
Quarter-circular arc		$\frac{2r}{\pi}$	$\frac{2r}{\pi}$	$\frac{\pi r}{2}$
Semicircular arc		0	$\frac{2r}{\pi}$	πr
Arc of circle		$\frac{r \sin \alpha}{\alpha}$	0	$2\alpha r$

Parallel axis theorm :



Consider the moment of Inertia I of an area A with respect to an axis XX. Denoting by 'y' the distance from an element of area dA to XX axis. We write

$$I = \int y^2 dA$$

Now XG axis parallel to XX through the centroid CG of the area; this axis is called a centroidal axis. Denoting by y' the distance from element dA to CG axis. We write $y = y' + \bar{y}$ where $\bar{y}$ is the distance between the axes XX and XG.

$$I_{XX} = \int y^2 dA = \int (y' + \bar{y})^2 dA$$

$$= \underset{①}{\int y'^2 dA} + \underset{②}{2\bar{y}\int y' dA} + \underset{③}{\bar{y}^2 \int dA}$$

① represent moment of inertia I_{XG} of the area w.r.t centroidal axis CG. X axis. The second integral represents the first moment of the area with respect to XX CG axis; so, it is zero. Finally we write.

$$\boxed{I_{XX} = I_{XG} + A \cdot \bar{y}^2}$$

parallel axis theorm.