Cables with Point Loads

Cables are flexible elements such that they cannot take compressive forces and also bending moments. They carry only tensile force.

Examples → suspension bridges, transmission lines, aerial tramways

Primary concern in design of cables is that to find out shape of cable when loaded with external loads.

External load can be concentrated load or distributed load.

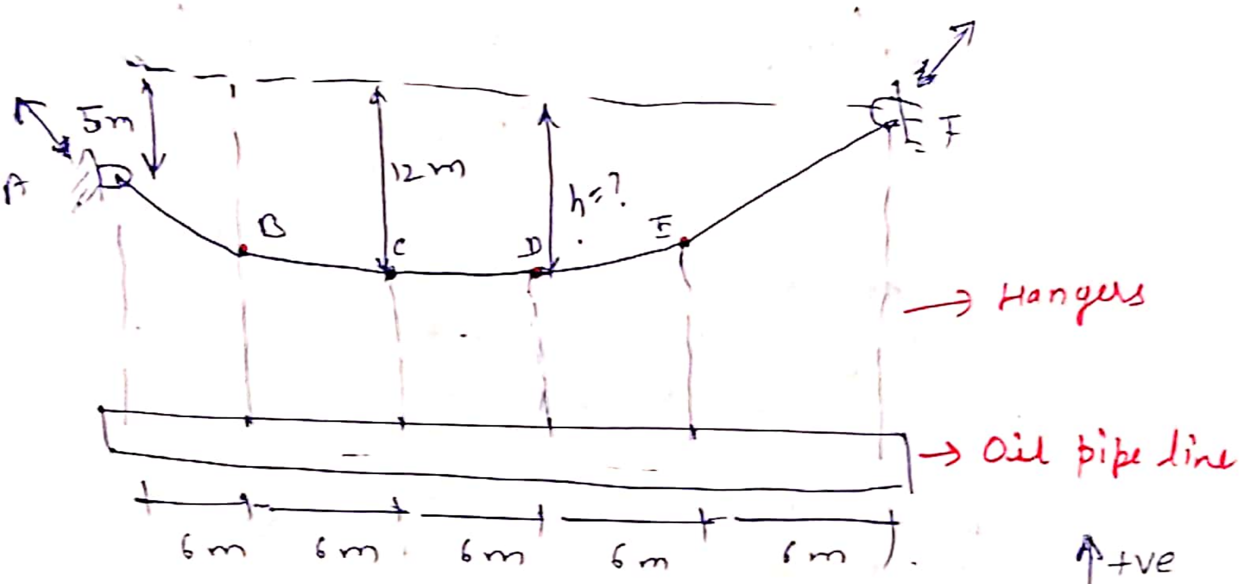
Assumption:- Weight of cable is neglected. ~~As such~~

Steps to Solve Cable Problems

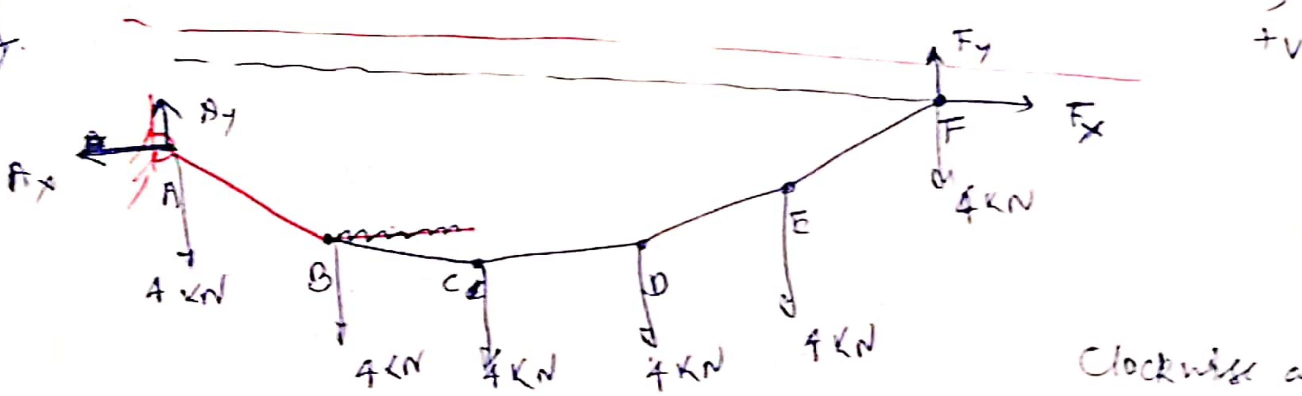
1. Write down equilibrium equations using $\Sigma F_x = 0$; $\Sigma F_y = 0$ and $\Sigma M = 0$
2. Using the concept that B.M. in cable is zero, use method of section and write down additional equations.
3. Solve four equations simultaneously to obtain end reactions.
4. The reactions at ~~first and last part~~ end points are equal to tension in first and last part of the cable. To find tension in other parts draw FBD of points where loads are attached and use equilibrium equations.
5. That part of cable has maximum tension which makes maximum angle with horizontal.

Note that for only vertical loads, only the first and last part of cable can have maximum tension.

Example :- An oil pipe line is supported at 6m intervals by vertical hangers attached to the cable shown in figure. Due to weight of pipe and its contents, the tension in each hanger is 4 kN. Determine (a) the maximum tension in cable and (b) height (h)



FBD



Clockwise as -ve

Step (1)

Writing equilibrium equation

$\Sigma F_x = 0 \Rightarrow -A_x + F_x = 0$

$\boxed{A_x = F_x}$ — (1)

$\Sigma F_y = 0 \Rightarrow A_y + F_y - 4\times 6 = -24 \Rightarrow \boxed{A_y + F_y = 24}$ — (2)

Now taking moment about A, we get,

$-4\times6 - 4\times12 - 4\times18 - 4\times24 - 4\times30 - F_x\times5 + F_y\times30 = 0$

Solving we get,

$\cancel{5F_x + }$ $30F_y - 5F_x = 4 \times 90 = 360.$

$\Rightarrow \boxed{6F_y - F_x = 72}$ — (3)

Step (2),

Consider a section X-X at C, Taking right hand side

$\sum M_c = 0$

$\Rightarrow -4\times6 - 4\times12 - 4\times18 - F_x \cancel{\$}2 + F_y(18) = 0$

$\Rightarrow 18F_y - 12F_x = 4\times36 = 144$

$\Rightarrow \boxed{3F_y - \cancel{4}2F_x = 24}$ — (4)

Solving (3) and (4) we get

$F_x = 8\ KN$; $F_y = 13.333\ KN$

Then $A_x = 8\ KN$; $A_y = 10.667\ KN$

As $F_y > A_y$

$\Rightarrow$ Last part has maximum tension

$\Rightarrow$ $T_{max} = \sqrt{F_x^2 + (F_y - 4)^2} = \underline{12.29\ KN}$

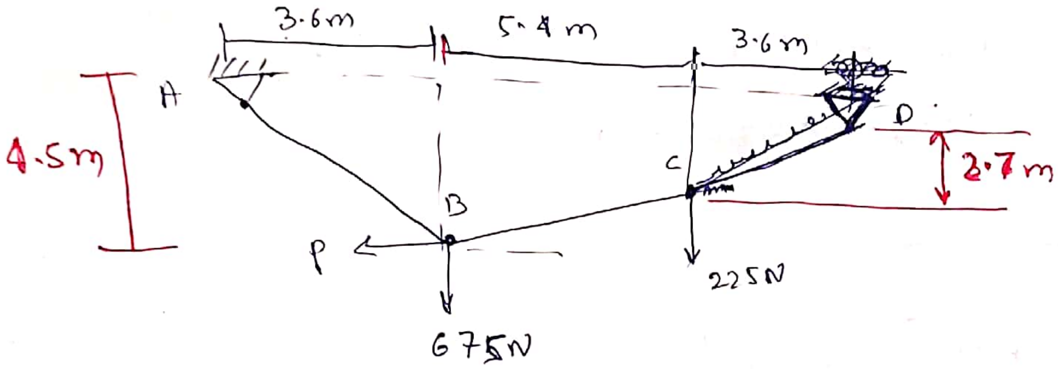
b) Cutting section at D and Taking moment about D, we get,

$-4\times6 - 4\times12 - F_x\times\cancel{h} + F_y\times12 = 0$

$-F_x\times h$ $= 72 - 13.33\times12 =$

$\boxed{h = 11\ m}$

Q.2) A cable ABCD is supported at A and D, 12.6 m apart horizontally. The cable carries load of 675 N and 225 N at B and C. If cable also carries a force P acting horizontally at B, find its value for equilibrium.



Lami's Theorem #: [forces are nonlinear, coplanar and concurrent)

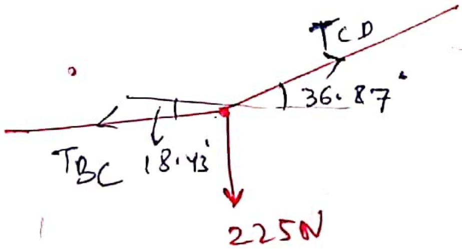
If three forces acting at a point are in equilibrium, each force is proportional to the sine of angle between the other two forces.

First determine angle of AB with horizontal $= \tan^{-1}\left(\frac{4.5}{3.6}\right) = 51.34°$

Similarly, angle of BC with horizontal $= \tan^{-1}\left(\frac{4.5-2.7}{5.4}\right) = 18.43°$

~~3rd~~ Similarly angle of CD with horizontal $= \tan^{-1}\left(\frac{2.7}{3.6}\right) = 36.87°$

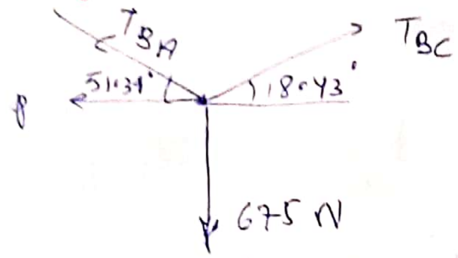
FBD of C



$$\frac{T_{BC}}{\sin(90+36.87°)} = \frac{225}{\sin(180-36.87+18.43)}$$

$T_{BC} = 569.06\,N$

Drawing FBD of B,



$\Sigma F_y = 0$

$\Sigma F_x = 0$

$\Sigma F_y = 0$

$T_{BA} \sin(51.34°) + T_{BC} \sin 18.43° = 675\ N$

$T_{AB} = 634.03\ N$

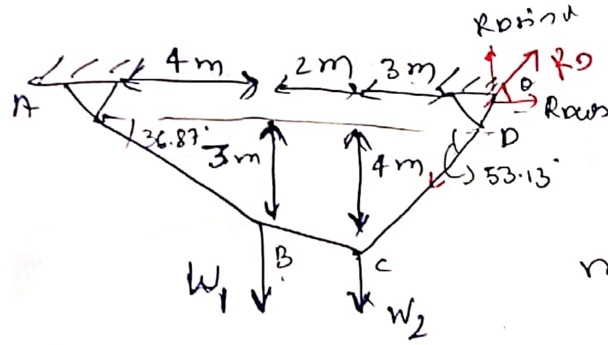
Now, $\Sigma F_x = 0$

$-P - T_{BA} \cos 51.34° + T_{BC} \cos 18.43° = 0 \Rightarrow \boxed{P = 1430.8\ N}$

Practice Problem for HW

Q-3) If ~~the cylinder at A in Fig has a weight of 700 N,~~

~~det~~ The maximum allowable tension in the cable as shown in Fig. is 10 kN. ~~Detrm~~ Determine the magnitude of load W_1 and W_2 for the equilibrium. Also find reactions at A and D.



First of all we will find what part of cable will have maximum tension. As here only vertical loads exist, therefore either first or ~~and~~ last part of cable will have maximum tension. Therefore, finding θ_A and θ_D

$\theta_A = \tan^{-1}\left(\frac{3}{4}\right)$ $\quad \theta_D = \tan^{-1}\left(\frac{4}{3}\right)$

$\Rightarrow \theta_D > \theta_A \Rightarrow$ CD will have ~~darg~~ more tension

Assigning ~~max~~ permissible tension to CD = 10 KN, we get,

$R_D = 10\ KN$ $\theta_D = 53.13°$

Now, taking $\Sigma M_B = 0$ for Right hand side of B, we get,

$$-W_2(2) - R_D \cos 53.13 \times 3 + R_D \sin 53.13° \times 5 = 0$$

$$\Rightarrow \boxed{W_2 = 11\ KN}$$

Taking moment about A, we get,

$$-W_1 \times 4 - W_2 \times 6 - R_D \cos 53.13 \times 0 + R_D \sin 53.13 \times 9 = 0$$

$$\boxed{W_1 = 1.5\ KN}$$

Now, using $\Sigma F_x = 0$

$$A_x = 10 \cos 53.13° = 6 KN (\leftarrow)$$

$$\Sigma F_y = 0$$

$$A_y - W_1 - W_2 + 10 \sin 53.13 = 0$$

$$A_y = 4.5\ KN \uparrow$$