ADMISSION No: U19CS012

DIVISION No: D-12

Email : u19cs012@coed.svnit.ac.in

bhagyarana2001@gmail.com

Q1.)

(i) $\boxed{(a)\ z = ax + by + 2\sqrt{ab}}$

form IV of non-linear partial eq$^n$ of first eq$^n$

$z = px + qy + f(p,q)$ ⇒ using Clairaut's eq$^n$

replace $p \to a$ $q \to b$ to get required sol$^n$ (A)

(ii) $\boxed{(b)\ \text{Bessel's equation}}$

Eq$^n$ of form $x^2y'' + xy' + (x^2-n^2)y = 0$ is called Bessel's eq$^n$ of order 'n'

(iii) $\boxed{(c)\ 3}$

The Number of non-zero rows in diagonal matrix

= (3)

[Rank(A) = 3]

$$A = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 \end{bmatrix}$$

(iv) $\boxed{(C)\ -2}$

$$\vec{V} = \underbrace{(2x+y)}_{V_1}\hat{i} + \underbrace{(3x-2z)}_{V_2}\hat{j} + \underbrace{(x+pz)}_{V_3}\hat{k} \text{ is solenoidal}$$

[div $\vec{V}$ = 0] cond$^n$ for $\vec{V}$ to be solenoidal

$$\vec{\nabla}.V = 0$$

$$= \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)(V_1\hat{i} + V_2\hat{j} + V_3\hat{k})$$

$$= \frac{\partial V_1}{\partial x} + \frac{\partial V_2}{\partial y} + \frac{\partial V_3}{\partial z} = (2(1)+0) + 0 + (p) = 0$$

$2 + p = 0$ $\boxed{p = -2}$ option (C)

(V) $\boxed{(c)\ \frac{1}{2}}$

$$I.F = e^{\alpha y^2}$$

Diff eq$^n$: $\left(e^{-\frac{y^2}{2}} - xy\right) dy - dx = 0$

$$\frac{dx}{dy} = e^{-y^2/2} - xy$$

$$\frac{dx}{dy} + x\underbrace{y}_{P(y)} = e^{-y^2/2}$$

→ First order Linear Differential Eq$^n$ of form $\frac{dx}{dy} + P(x) = Q$ [P, Q ⇒ f(y)]

$$I.F = (e)^{\int P(y)\,dy}$$

$$= (e)^{\int y\,dy} = e^{\frac{y^2}{2}}$$

comparing $\overset{2}{\underset{=}{e}}$ $\alpha y^2 = \frac{y^2}{2}$ $\left[\alpha = \frac{1}{2}\right]$ (c)

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Q.-2.)

(i) Solve $2xz - px^2 - 2qxy + pq = 0$

(Since it does not fit in first 4 forms)

Let $f(x, y, z, p, q) = 2xz - px^2 - 2qxy + pq$ — ①

Using Charpits method, the subsidary eq$^n$ are:

$$\frac{dx}{-\frac{\partial f}{\partial p}} = \frac{dy}{-\frac{\partial f}{\partial q}} = \frac{dz}{-p\frac{\partial f}{\partial p} - q\frac{\partial f}{\partial q}} = \frac{dp}{\frac{\partial f}{\partial x} + p\frac{\partial f}{\partial z}} = \frac{dq}{\frac{\partial f}{\partial y} + pq\frac{\partial f}{\partial z}}$$

$$\left[\frac{dx}{(-1)(-x^2+q)} = \frac{dy}{(-1)(-2xy+p)} = \frac{dz}{px^2 - 2pq + 2qxy} = \frac{dp}{(2z - \cancel{2px} - 2qy) + p(\cancel{2x})} = \frac{dq}{-\cancel{2qx} + \cancel{2qx}}\right]$$

for dz → $p(x^2 - q) + q(2xy - p)$

⇒ $px^2 - 2pq + 2qxy$

$$\frac{dp}{(2z - 2qy)} = \frac{dq}{0}$$

$\because$ $dq = 0$ (integrating)

$\boxed{q = a}$ — ②

substituting Eq ② $q=a$, in $f(x,y,z,p,q)$

$$2xz - px^2 - 2(a)xy + p(a) = 0$$

$$p(x^2 - a) = 2x(z - ay)$$

$$\left[ p = \frac{2x(z-ay)}{(x^2-a)} \right] \quad -③$$

We know $dz = p\,dx + q\,dy$

$$dz = \left(\frac{2x(z-ay)}{(x^2-a)}\right) dx + a\,dy$$

term

$f'(x)$ consider $\frac{dt}{t}$ $f(x)$

$$\int \frac{(dz - a\,dy)}{(z-ay)} = \int \frac{2x\,dx \rightarrow dt}{(x^2-a) \rightarrow t} \quad \text{(variable separable)}$$

Integrating both sides

$$\log(z-ay) = \log(x^2-a) + \log(b)$$

$$(z-ay) = b(x^2-a)$$

ANS: $$\boxed{\begin{aligned} &[z = ay + b(x^2-a)] \\ &[z = bx^2 + ay - ab] \end{aligned}}$$

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(ii) $\oint_C (y - \sin x)dx + (\cos x)\,dy$

Using Green's theorem

$$\iint \left[\frac{\partial}{\partial x}(\cos x) - \frac{\partial}{\partial y}(y - \sin x)\right] dx\,dy$$

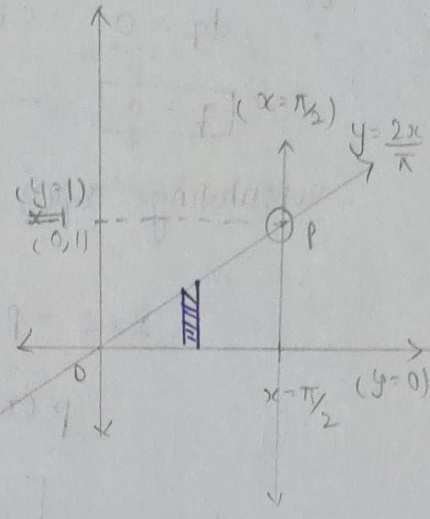
$$= \iint (-\sin x - 1)\,dx\,dy$$

At $x = \pi/2$

$y = \frac{2}{\pi} \times \frac{\pi}{2} = (1)$

$x = 0$ to $\pi/2$

$y = 0$ to $\left(\frac{2x}{\pi}\right)$




$\therefore \int_0^{\pi/2}\int_0^{2x/\pi} (-\sin x - 1)\, dy\, dx$

$= \int_0^{\pi/2} [-y]_0^{2x/\pi} (1+\sin x)\,dx$

$= \left(\frac{-2}{\pi}\right)\int_0^{\pi/2} (x\sin x + x)\,dx$

$= \left(\frac{-2}{\pi}\right)\left[\int_0^{\pi/2}(x\sin x)\,dx + \left[\frac{x^2}{2}\right]_0^{\pi/2}\right]$

$= \left(\frac{-2}{\pi}\right)\left[\frac{\pi^2}{8} + x\int \sin x\,dx - \int\left(\frac{dx}{dx}\int \sin x\,dx\right)dx\right]$

$= \left(\frac{-2}{\pi}\right)\left[\frac{\pi^2}{8} + (-x\cos x + \sin x)_0^{\pi/2}\right]$

$= \left(\frac{-2}{\pi}\right)\left[\frac{\pi^2}{8} + \left(-\frac{\pi}{2}(0) + 1 - 0\right)\right] = \frac{-2}{\pi}\left(\frac{\pi^2}{8} + 1\right)$

$\cancel{\frac{+2}{\pi}\left[\left[\frac{\pi^2}{8} - \frac{4\pi}{8}\right] + 1\right]} \Rightarrow \cancel{\left(\frac{-2}{\pi}\right)\left(\frac{\pi^2 - 4\pi + 8}{8}\right)}$

$\cancel{\frac{(-2)}{\pi}\frac{\pi(\pi-4)}{(8)}} = \qquad = \cancel{\pi} - \frac{\pi^2}{4\pi} - \frac{2}{\pi}$

**ANS:** $= \boxed{\left[\frac{(-1)(\pi^2+8)}{4\pi}\right]}$

(iii) Gauss Jordan method to solve eq^n

$x + y + z = 9$
$2x - 3y + 4z = 13$
$3x + 4y + 5z = 40$

Matrix form :

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -3 & 4 \\ 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 13 \\ 40 \end{bmatrix}$$

make '0'

Step 1 : $R_2 \to R_2 - 2(R_1)$
$R_3 \to R_3 - 3(R_1)$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -5 & 2 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ -5 \\ 13 \end{bmatrix}$$

Step 2 : $R_3 \Rightarrow R_3 + R_2 \times \left(\frac{1}{5}\right)$ ; $R_1 \to R_1 + \frac{R_2}{(5)}$

$$\begin{bmatrix} 1 & 0 & 1+2/5 \\ 0 & -5 & 2 \\ 0 & 0 & 12/5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -5 \\ 12 \end{bmatrix}$$

Rough

$\frac{7}{5} = (X) \frac{12}{5}$

$2 = (Y) \frac{12}{5}$

Step 3 : $R_1 \to R_1 - \frac{7}{12}(R_3)$ $R_2 \to R_2 - \frac{5}{6}(R_3)$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 12/5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -15 \\ 12 \end{bmatrix}$$

Step 4 : $R_2 \to R_2 \times \left(\frac{-1}{5}\right)$ $R_3 \to \frac{5}{12} R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} \Rightarrow$$

$\therefore$ ANS :

$x = 1$

$y = 3$

$z = 5$

---x---