(U19CS012) **MATHS ASSIGNMENT** (29 May 11:59 pm)

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1.) Form the following partial differential eq^n's by eliminating arbitrary constants and functions:

(i) $z = (x^2+a)(y^2+b)$

$p = \frac{\partial z}{\partial x} = (2x)(y^2+b) \qquad (y^2+b) = \frac{(p)}{(2x)}$ — ①

$q = \frac{\partial z}{\partial y} = (2y)(x^2+a) \qquad (x^2+a) = \frac{(q)}{(2y)}$ — ②

From ① and ②

$$\frac{p}{2x} = \frac{q}{2y}$$

ANS: $(py - qx) = 0$

(ii) $2z = (ax+y)^2 + b$

$2\frac{\partial z}{\partial x} \Rightarrow 2p = 2(ax+y)(a)$ — ①

$(a \ne 0)$

$2\frac{\partial z}{\partial y} = 2q = 2(ax+y)(1)$ — ②

$[ax+y = q]$ — ②

$a = \frac{(q-y)}{x}$ — ③

substituting Eq^n ③ in ①

$\cancel{p = ((q-y) + y)}$

$p = (ax+y)(a)$

$p = (q)\frac{(q-y)}{x}$

ANS: $[px = q(q-y)]$

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iii) $z = F(x^2-y^2)$

$$\frac{\partial z}{\partial x} = F'(x^2-y^2)\,(2x)$$

$$\left[\frac{p}{2x} = F'(x^2-y^2)\right] \quad -①$$

$$\frac{\partial z}{\partial y} = F'(x^2-y^2)\,(-2y)$$

$$\left[\frac{q}{(-2y)} = F'(x^2-y^2)\right] \quad -②$$

$$\frac{p}{2x} = \frac{q}{(-2y)}$$

ANS: $[py + qx = 0]$

(iv) $z = x + y + f(xy)$

$$\frac{\partial z}{\partial x} = 1 + 0 + f'(xy)\,(1y)$$

$$[(p-1) = y\,f'(xy)] \quad -①$$

$$\frac{\partial z}{\partial y} = 0 + 1 + f'(xy)\,(x)$$

$$[(q-1) = f'(xy)\,(x)] \quad -②$$

Dividing we get

$$\frac{(p-1)}{(q-1)} = \frac{y}{(x)}$$

$$px - x = qy - y$$

ANS: $[px - qy = x - y]$

2.7 Find Directional derivative of $f(x,y,z) = xyz$ in direction of outer normal to surface $z = xy$ at point $(3,1,3)$

2.7 $f(x,y,z) = \phi_1 = xyz$

$$\text{grad } \phi_1 = \nabla \phi_1$$

$$= \frac{\partial}{\partial x}(\phi_1)\,\hat{i} + \frac{\partial}{\partial y}(\phi_1)\,\hat{j} + \frac{\partial}{\partial z}(\phi_1)\,\hat{k}$$

$$= yz\,\hat{i} + xz\,\hat{j} + xy\,\hat{k}$$

grad (ϕ_1) at $P(3,1,3)$

$$= 3\hat{i} + 9\hat{j} + 3\hat{k}$$

$\phi_2 = z - xy$

normal $= \nabla \phi_2$

$$= \frac{\partial}{\partial x}(\phi_2).\hat{i} + \frac{\partial}{\partial y}(\phi_2)\hat{j} + \frac{\partial}{\partial z}(\phi_2)\hat{k}$$

$$= [-y\,\hat{i} + (-x)\,\hat{j} + (1)\hat{k}]$$

At $P(3,1,3)$ $= -\hat{i} + -3\hat{j} + \hat{k}$

$$\hat{n} = \frac{-\hat{i} + 3\hat{j} + \hat{k}}{\sqrt{1^2+3^2+1^2}} = \frac{-\hat{i} - 3\hat{j} + \hat{k}}{(\sqrt{11})}$$

Directional Derivative $= (\text{grad}(\phi_1))\,\hat{n}$

$$= 3(\hat{i} + 3\hat{j} + \hat{k})\,.\frac{(-\hat{i} - 3\hat{j} + \hat{k})}{(\sqrt{11})}$$

ANS: $= \left|\frac{-27}{\sqrt{11}}\right| = \left(\frac{27}{\sqrt{11}}\right)$

3.) Work Done = (?) $\vec{F} = (x^2 - y^2 + x)\,\hat{i} - (2xy + y)\,\hat{j}$ moves particle from origin (1,1) along parabola $y^2 = x$.

$\vec{F} = (x^2 - y^2 + x)\,\hat{i} - (2xy + y)\,\hat{j}$

$\vec{dr} = dx\,\hat{i} + dy\,\hat{j}$

work $= \int_s \vec{F}.\vec{dr} = \int_s (\quad)(dx\,i + dy\,\hat{j})$

$$= \int (x^2 - y^2 + x)\,dx - (2xy + y)\,dy$$

Using $y^2 = x$ $\quad 2y(dy) = dx$

$$\left[y\,dy = \left(\frac{dx}{2}\right)\right]$$

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$$= \int_0^1 (x^2 - \cancel{x}(1+\cancel{x})\,dx - (2x+1)\,\frac{dx}{2}$$

$$= \int_0^1 \left(x^2\,dx - x\,\cancel{\phi}\,\frac{1}{2}\right) dx$$

$$= \frac{1}{3} - \frac{1}{2}\,\cancel{\phi}\left(\frac{(1)}{2}\right) = \left[\left(\frac{-2}{3}\right) \text{ ANS}\right]$$

4.) $\vec{F} = 2y\,\hat{i} - 3\,\hat{j} + x^2\,\hat{k}$ and

S : surface of parabolic cylinder in first octant

$(y^2 = 8x)$ by planes $y=4$ & $z=6$

$\iint_S \vec{F}\cdot\hat{n}\,ds = (?)$

$\vec{F} = 2y\,\hat{i} - 3\,\hat{j} + x^2\,\hat{k}$

$(y^2 = 8x) \Rightarrow [\phi = y^2 - 8x]$

$$\text{grad } \phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k} = 2y\,\hat{j} - 8\,\hat{i}$$

$$|\text{grad } \phi| = 2\sqrt{y^2+16}.$$

$$\vec{F}\cdot\hat{n} = \cdot\,(2y\,\hat{i} - 3\,\hat{j} + x^2\,\hat{k})\,\frac{(2y\,\hat{j} - 8\,\hat{i})}{2\sqrt{(y^2+16)}}$$

$$= \frac{-16y - 6y}{2\sqrt{y^2+16}}$$

$$= \left(\frac{-11y}{\sqrt{y^2+16}}\right)$$

$$\iint_S \vec{F}\cdot\hat{n}\,dS = \int_{z=0}^{z=6}\int_{y=0}^{y=4} \frac{(-11y)}{(\sqrt{y^2+16})}\,\frac{dx\,dz}{(\hat{j}\cdot\hat{n})}$$

$$= \int_{z=0}^{z=6}\int_0^4 \left(\frac{(-11y)}{\sqrt{y^2+16}}\;\frac{2\sqrt{y^2+16}}{(2y)}\right) dx\,dz.$$

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$$= \int_0^6 \int_0^4 \left(\frac{-11}{2}\right) dx\,dz$$

$$= \frac{-11}{2} \times 6 \times \cancel{y}(4) = |-132|$$

Ans: (132)

5.) Evaluate $\iiint_V (2x+y)\,dV$, where V is closed region bounded by cylinder $z = 4-x^2$ and the planes $x=0, y=0, y=2$ and $z=0$.

5.) $\iiint_V (2x+y)\,dV$ $\quad x=0$ to $x=\sqrt{4}=(2)$ $\quad y=0$ to 2 $\quad z=0$ to $(4-x^2)$ $\quad (x=\sqrt{4-x})$

$$= \int_{x=0}^{2} \int_{y=0}^{2} \int_{z=0}^{4-x^2} (2x+y)\,dz\,dy\,dx$$

$$= \int_0^2 \int_0^2 (2x+y)[(4-x^2)]\,dy\,dx$$

$$= \int_0^2 \left[\int_0^2 ((2x+y)4)\,dy - \int_0^2 (2x+y)(x^2)\,dy \right] dx$$

$$= \int_0^2 \left[\left[16x + \frac{(\cancel{4})^2(4)}{\cancel{2}}\right] - \left[4x^3 + x^2(2)\right] \right] dx$$

$$= \int_0^2 (-4x^3 - 2x^2 + 16x + 8)\,dx$$

$$= \cancel{\int_0} (-\cancel{4})\frac{(2)^4}{(\cancel{4})} - \frac{2(2)^3}{(3)} + \frac{16(2)^{\cancel{2}}}{(\cancel{2})} + (8)(2)$$

$$= \cancel{-(2)^4} - \frac{(2)^4}{3} + 32 + \cancel{(16)}$$

ANS: $= 32 - \frac{16}{3} = \frac{96-16}{(3)} = \left[\frac{80}{3}\right]$

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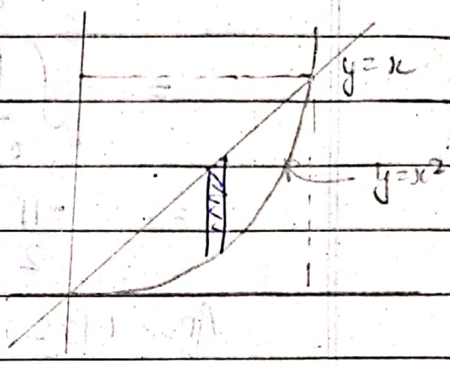
6.) $y = x$ $\qquad y = x^2$

$\therefore x = x^2$

$x = 0, \quad x = 1$



$\int_C [(xy+y^2)\,dx + x^2\,dy]$ [from green's theorem]

$\Rightarrow \iint_C \left(\frac{\partial}{\partial x}(x^2) - \frac{\partial}{\partial y}(xy+y^2)\right) dy\,dx$

$= \int_{x=0}^{x=1}\int_{y=x^2}^{y=x} (\underbrace{2x - x}_{x} - 2y)\, dy\, dx$

$= \int_{x=0}^{x=1} \left[x(x-x^2) - \cancel{2}\frac{(x^2-x^4)}{\cancel{2}}\right] dx$

$= \int_0^1 [\cancel{x^2} - x^3 - \cancel{x^2} + x^4]\, dx$

$= \left[\frac{1}{5} - \frac{1}{4}\right] = \left[\frac{-1}{20}\right]$ ANSWER (1)

7.) ~~Using Divergence theorem,~~

$\oint_C (xy+y^2)\,dx + x^2\,dy = \int_{c_1} (xy+y^2)\,dx + x^2\,dy$

$c_1 = (y = x) \qquad - \int_{c_2} (xy+y^3)\,dx + x^2\,dy$

$c_2 = (y = x^2)$

For $c_1 \Rightarrow$ $[y = x]$ $\quad dy = dx$ $\quad [x=0 \text{ to } x=1]$

$\int_0^1 (x^2+x^2+x^2)\,dx = \int_0^1 3x^2\,dx$

$= [x^3]_0^1 = (1)$

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For C_2; $y = x^2$. $dy = 2x\,dx$

$x = 0$ to $x = 1$

$$= \int_0^1 (x(x^2) + (x^2)^2)\,dx + x^2(2x\,dx)$$

$$= \int_0^1 (3x^3 + x^4)\,dx$$

$$= \left[\frac{3x^4}{4} + \frac{x^5}{5}\right]_0^1 = \left[\frac{3}{4} + \frac{1}{5}\right] = \left(\frac{19}{20}\right)$$

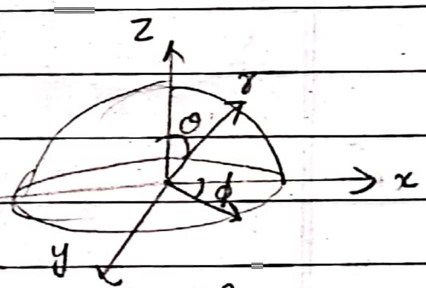
$$\left[C = \frac{19}{20} - 1 = \frac{-1}{20}\right]$$ ANSWER (2)

From Answer (1) & (2), Green's theorem is Verified.

7.) $I = \iint_S \left[xz^2\,dy\,dz + (x^2y - z^3)\,dz\,dx + (2xy + y^2z)\,dx\,dy\right]$

$$= \iiint_V \left(\frac{\partial}{\partial x}(xz^2) + \frac{\partial}{\partial y}(x^2y - z^3) + \frac{\partial}{\partial z}(2xy + y^2z)\right) dv$$

$$= \iiint_V (z^2 + x^2 + y^2)\,dv$$



let $z = r\cos\theta$
$x = r\sin\theta\cos\phi$
$y = r\sin\theta\sin\phi$ $\Big\}$ $[x^2 + y^2 + z^2 = r^2]$

$r = 0$ to $r = 2$ $\phi = 0$ to 2π $\theta = 0$ to $(\pi/2)$

$$I = \int_{\theta=0}^{\pi/2}\int_{\phi=0}^{2\pi}\int_0^2 (r^2)\,r^2\sin\theta\,d\theta\,d\phi\,dr$$

$$= \left[\frac{r^5}{5}\right]_0^2 \left[-\cos\theta\right]_0^{\pi/2} \left[\phi\right]_0^{2\pi}$$

ANS: $= \left[\frac{64\pi}{5}\right]$

8.7 $\vec{F} = yz\,\hat{i} - xz\,\hat{j} + xy\,\hat{k}$

$\oint_C \vec{F}\cdot d\vec{r} = \iint_S \text{curl } \vec{F}\cdot\hat{n}\, ds$

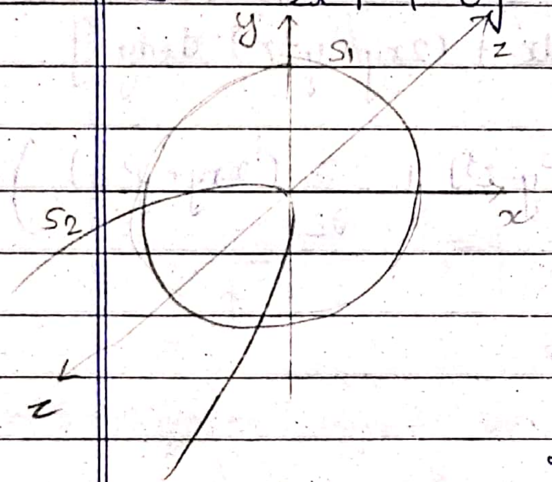
(from stokes theorem)

$\text{curl } \vec{F} = \nabla \times \vec{F}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & -xz & xy \end{vmatrix}$$

$$= \hat{i}\left[\frac{\partial(xy)}{\partial y} + \frac{\partial(xz)}{\partial z}\right] - \hat{j}\left[\frac{\partial(xy)}{\partial x} - \frac{\partial(yz)}{\partial z}\right] + \hat{k}\left[-\frac{\partial(xz)}{\partial x} - \frac{\partial(yz)}{\partial y}\right]$$

$$= 2x\,\hat{i} + 0\,\hat{j} - 2z\,\hat{k}$$



For surface 1 S_1

$\hat{n} = \hat{k}$ $(S_1 = (x^2+y^2=1))$

$\iint_{S_1} (2x\hat{i} - 2z\hat{k})\cdot\hat{k}\; dx\,dy$

$= \iint (-2z)\, dx\,dy$ but $z=0$

$\therefore \boxed{S_1 = 0}$

for $S_2 =$ $\hat{n} = \hat{i}$ $\iint_{S_2} (2x\hat{i} - 2y\hat{j})\,\hat{i}\; dy\,dz$

[but $(x=0)$] $\Rightarrow \iint_S 2x\; dy\,dz$

$\therefore \boxed{S_2 = 0}$

Ans: $\boxed{S_1 + S_2 = 0}$

9.) (i) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 10 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ $R_2 \to R_2 - (2 \times R_1)$, $R_3 \to R_3 - (3 \times R_1)$

$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ $R_3 \to R_3 - R_2$

[Rank of matrix = 2]

(ii) $\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \begin{bmatrix} 0 & 1 & 2 & -2 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{bmatrix}$

$\sim \begin{bmatrix} 2 & 1 & 3 & 1 \\ 4 & 0 & 2 & 6 \\ 0 & 1 & 2 & -2 \end{bmatrix}$ $R_1 \leftrightarrow R_3$

$\sim \begin{bmatrix} 1 & 1/2 & 3/2 & 1/2 \\ 4 & 0 & 2 & 6 \\ 0 & 1 & 2 & -2 \end{bmatrix}$ $R_1 \to R_1/2$

$\sim \begin{bmatrix} 1 & 1/2 & 3/2 & 1/2 \\ 0 & -2 & -4 & 4 \\ 0 & 1 & 2 & -2 \end{bmatrix}$ $R_2 \to R_2 - (R_1 \times 4)$

$\sim \begin{bmatrix} 1 & 1/2 & 3/2 & 1/2 \\ 0 & -2 & -4 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_3 \to R_3 - \frac{R_2}{2}$

Ans. Rank of matrix is '2'.

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10.) $2x_1 + 4x_2 + x_3 = 3$
$3x_1 + 2x_2 - 2x_3 = -2$
$x_1 - x_2 + x_3 = 3$

The Augmented Matrix $C = [A : B] = \left[\begin{array}{ccc|c} 2 & 4 & 1 & 3 \\ 3 & 2 & -2 & -2 \\ 1 & -1 & 1 & 3 \end{array}\right]$

1) Interchanging Row 1 & Row 3 we get

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ 3 & 2 & -2 & -2 \\ 2 & 4 & 1 & 3 \end{array}\right]$$

2) $R_2 \to R_2 - ((3) R_1)$ & $R_3 \to R_3 - (2 \times R_1)$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ 0 & 5 & -5 & -11 \\ 0 & 6 & -1 & -3 \end{array}\right]$$

3) $R_2 \to R_2 + (-5 \times R_3)$

$x_1 - x_2 + x_3 = 3$ —(1)
$-25x_2 = 4$ —(2)
$6x_2 - x_3 = -3$ —(3)

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ 0 & -25 & 0 & 4 \\ 0 & 6 & -1 & -3 \end{array}\right]$$

$\left[x_2 = \frac{-4}{25}\right]$ —(1) $\left[x_3 = 6\left(\frac{-4}{25}\right) + 3 = \frac{51}{25}\right]$ —(2)

Substituting in eqn (1) $\left[x_1 = 3 + \left(\frac{-4}{25}\right) - \left(\frac{51}{25}\right) = \left(\frac{4}{5}\right)\right]$ (3)

UNIQUE Solution $(x_1, x_2, x_3) = \left[\frac{4}{5}, \frac{-4}{25}, \frac{51}{25}\right]$

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