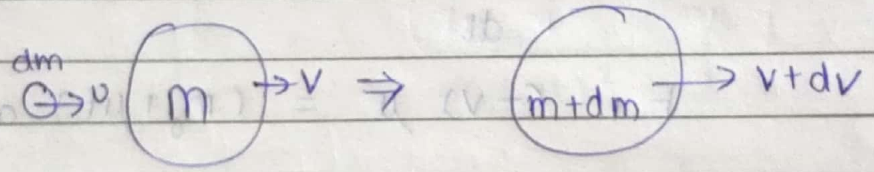
$$V_2 = \frac{m_1}{m_2} u \tan\theta$$

Velocity of wedge $\left[V_2 = \frac{m_1 u \tan\theta}{m_2} \right]$

$$e = \frac{V_1 \sin\theta + V_2 \sin\theta}{u \cos\theta}$$

$$e = \frac{\tan\theta}{u} (V_1 + V_2)$$

* Variable Mass System



$$F_{ext}\, dt = (m+dm)(v+dv) - dm\,u - mv$$

$$= \cancel{mv} + m\,dv + v\,dm + \underbrace{dm\,dv}_{\text{neglect}} - dm\,u - \cancel{mv}$$

$$= m\,dv + v\,dm - u\,dm$$

$$F_{ext}\, dt = m\,dv + dm(v-u)$$

$$\left[F_{ext}\, dt + dm(u-v) = m\,dv \right]$$

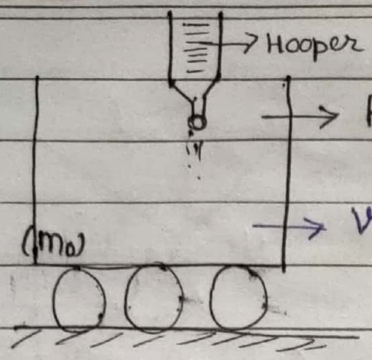
$$F_{ext} + \frac{dm\,(u-v)}{dt} = \frac{m\,dv}{dt}$$

$$\left[F_{ext} + \left(\frac{dm}{dt}\right) V_{rel} = ma \right]$$

(Total thrust)

mass flow rate $= F_{thrust}$

DATE: / /



1) ~~Sand~~

Flat car of mass (m_0), starts moving to right due to a constant force F, sand spills on the flat car from a stationary hooper. The rate of loading is constant (μ)

Find the velocity and acceleration after time 't'.

$$m = m_0 + \mu t$$

$$\frac{dm}{dt} = \mu$$

$$F_{ext} + \frac{dm}{dt}(u - v) = m a$$ instantaneous mass

$$F + (-v)\mu = (m_0 + \mu t)\, a$$

$$(F - \mu v) = (m_0 + \mu t)\frac{dv}{dt}$$

$$\int_0^t \frac{dt}{(m_0 + \mu t)} = \int_0^v \frac{dv}{F - \mu v}$$

$$\frac{[\ln(m_0 + \mu t)]_0^t}{\cancel{\mu}} = \frac{[\ln(F - \mu v)]_0^v}{\cancel{-\mu}}$$

Apply Limit ~~$\ln(m_0 + \mu t) = \ln(F - \mu v$~~

$$-[\ln(m_0 + \mu t) - \ln(m_0)] = \ln[F - \mu v] - \ln(F)$$

$$\cancel{m_0}\ \cancel{\ln}\ \frac{m_0}{m_0 + \mu t} = \cancel{\ln}\left(\frac{F - \mu v}{F}\right)$$

$$\frac{m_0}{m_0 + \mu t} = \frac{F - \mu v}{F}$$

$$\frac{F m_0}{m_0 + \mu t} - F = -\mu v$$

$$\frac{F}{\mu} - \frac{F m_0}{\mu(m_0+\mu t)} = v$$

$$v = \frac{F}{\mu}\left(1 - \frac{m_0}{m_0+\mu t}\right)$$

$$= \frac{F}{\mu}\left[\frac{m_0+\mu t - m_0}{m_0+\mu t}\right]$$

$$= \frac{F}{\mu}\left[\frac{\mu t}{m_0+\mu t}\right]$$

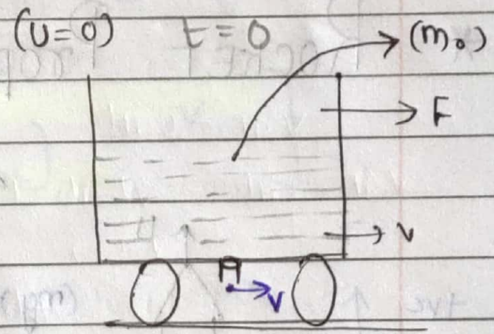
$$v = \left[\frac{Ft}{m_0+\mu t}\right]$$

$$\frac{u}{v} \qquad \frac{vu' - uv'}{(v)^2}$$

$$\frac{dv}{dt} = \frac{(m_0+\mu t)F - Ft(\mu)}{(m_0+\mu t)^2}$$

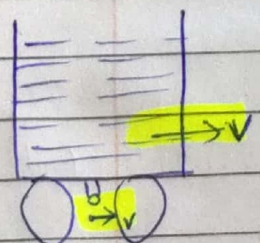
$$\left[a = \frac{F m_0}{(m_0+\mu t)^2}\right]$$

2) A cart loaded with sand moves along a horizontal force 'F' in the direction of velocity



Mass is ejected with a constant rate μ kg/sec from the hole. Calculate the velocity after time t & acceleration.

After time t



$m = m_0 - \mu t$

$$\frac{dm}{dt} = -\mu$$

$$F_{ext} + \frac{dm}{dt}(u-v) = ma$$

$$F_{ext} + \cancel{(-\mu)(-v)} = (m_0-\mu t)\,a$$

$u = v$

$v_{rel} = u - v = 0$

$$F_{ext} + 0 = (m_0 - \mu t)\,a$$

$$F + 0 = (m_0 - \mu t)\frac{dv}{dt}$$

$$\int_0^t \frac{F}{m_0 - \mu t}\,dt = \int_0^v dv$$

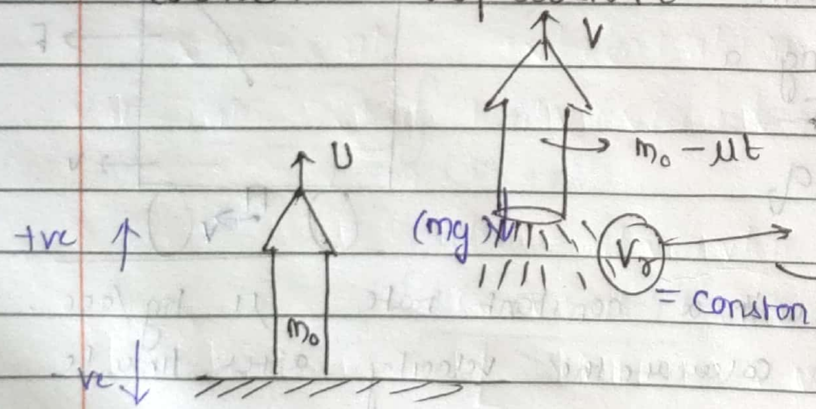
$$F \ln\left(\frac{m_0 - \mu t}{m_0}\right) = v - 0$$

$$F \ln\left(\frac{m_0 - \mu t}{(-\mu) m_0}\right) = v$$

$$\frac{F}{-\mu}\left[\ln(m_0 - \mu t)\right]_0^t = v$$

$$\boxed{\frac{F}{\mu}\ln\left(\frac{m_0}{m_0 - \mu t}\right) = v}$$

* Rocket Propulsion



v_r = constant — If given, then use. If not, then $v_r = v_g - v_{rocket}$

wrt to rocket

From the rocket

$$F_{ext} + v_{rel}\left(\frac{dm}{dt}\right) = m a_T$$

$$-mg + (-v_r)(-\mu) = m a_T$$

$$\boxed{-mg + v_r \mu = F_T}$$

(instantaneous mass) (given or to be found) (net thrust)

PAGE:
DATE: / /

'm' is instaneous mass

Calculate velocity after time 't'

$$\frac{-mg + V_r\mu}{m} = a_t$$

$$-g + \frac{V_r\mu}{m} = \frac{dV}{dt}$$

$$\int_0^t \left(-g + \frac{V_r\mu}{m}\right) dt = \int_0^V dV$$

$$\left[-gt\right]_0^t + \int \frac{V_r\mu \, dt}{(m_0 - \mu t)} = [V]_0^V$$

$$-gt + \frac{V_r\mu}{-\mu}\left(\ln\frac{(m_0-\mu t)}{m_0}\right) = V - U$$

$$U - gt - V_r \ln\left(\frac{m_0 - \mu t}{m_0}\right) = V$$

$$\boxed{U - gt + V_r \ln\left(\frac{m_0}{m_0 - \mu t}\right) = V}$$