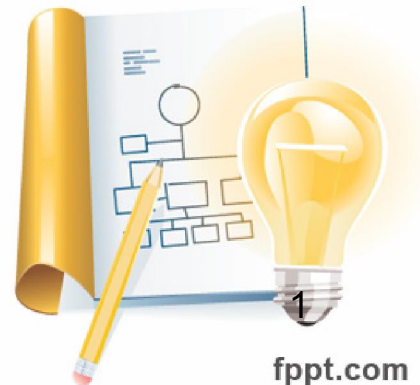


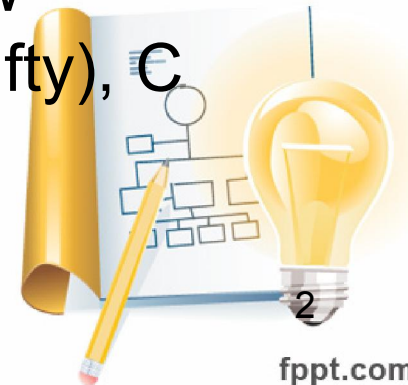
## 2. Number Systems



# Number Systems

- A number system defines a set of values used to represent quantity.
- It can be categorized in two broad categories:
  - 1. Non – Positional Number Systems**
    - Stones or sticks were used to indicate values
    - Difficult to perform arithmetic because it has no symbol for zero.

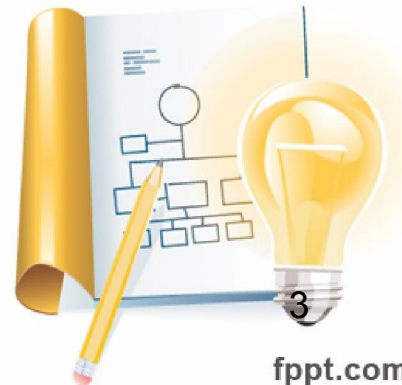
Example: Roman number systems – Few characters are used such as I, V, X, L (fifty), C (hundred)



# Number System

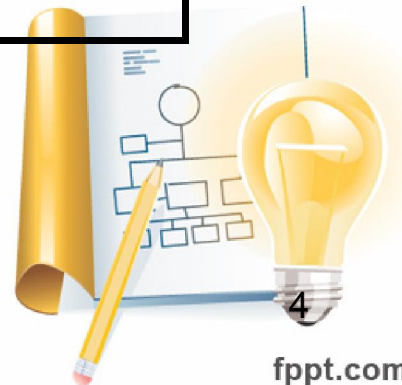
## 2. Positional Number Systems

- The value of each digit in number is defined not only by the symbol but also by symbol's position.
- Positional number systems have base or radix.



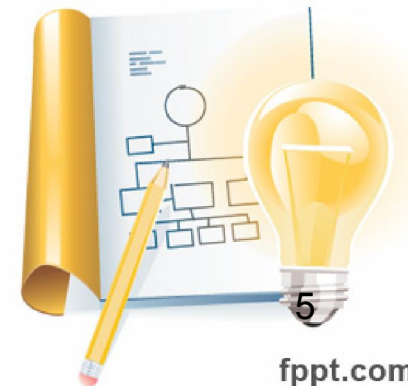
# Common Number Systems

System	Base	Symbols	Used by humans?	Used in computers?
Decimal	10	0, 1, ... 9	Yes	No
Binary	2	0, 1	No	Yes
Octal	8	0, 1, ... 7	No	No
Hexa-decimal	16	0, 1, ... 9, A, B, ... F	No	No



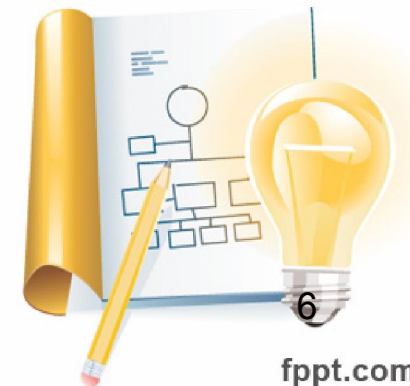
# Quantities/Counting (1 of 3)

Decimal	Binary	Octal	Hexa- decimal
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7



# Quantities/Counting (2 of 3)

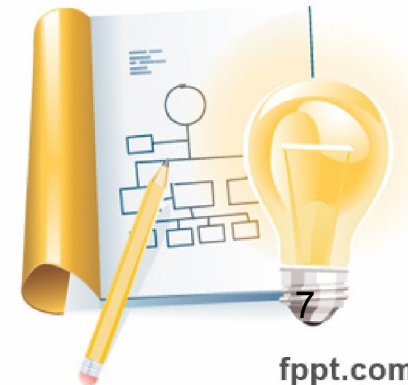
Decimal	Binary	Octal	Hexa- decimal
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F



# Quantities/Counting (3 of 3)

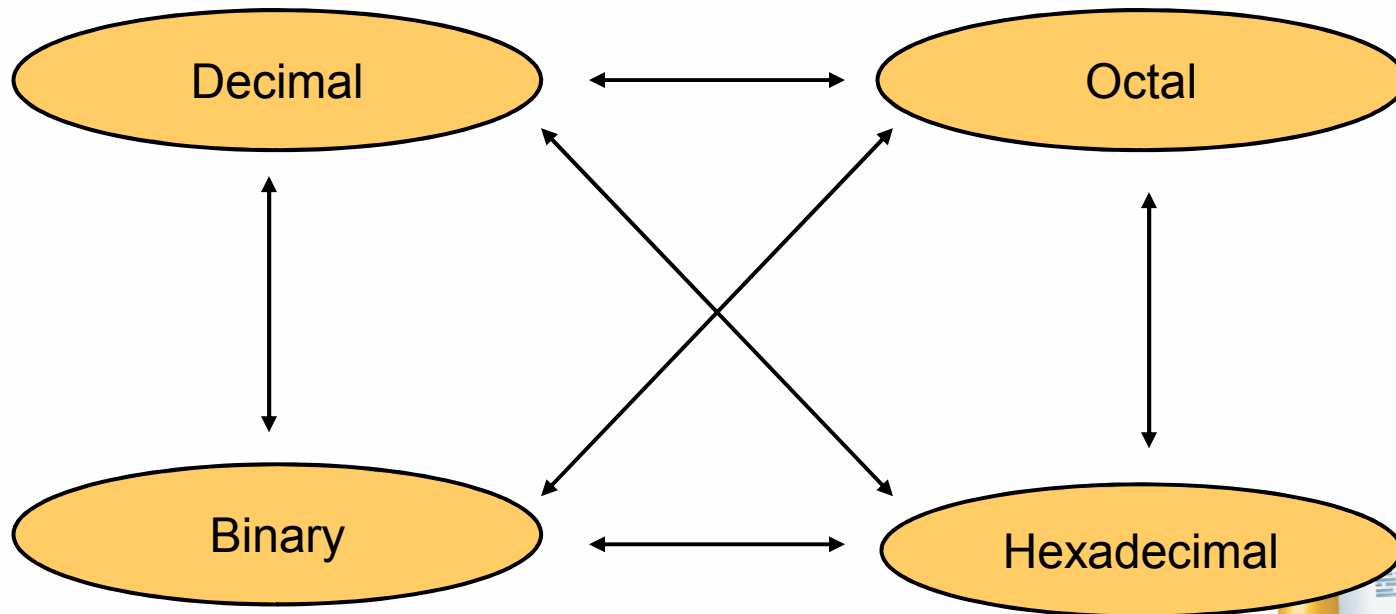
Decimal	Binary	Octal	Hexa- decimal
16	10000	20	10
17	10001	21	11
18	10010	22	12
19	10011	23	13
20	10100	24	14
21	10101	25	15
22	10110	26	16
23	10111	27	17

Etc.



# Conversion Among Bases

- The possibilities:

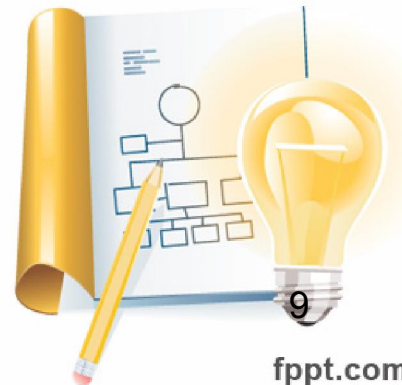




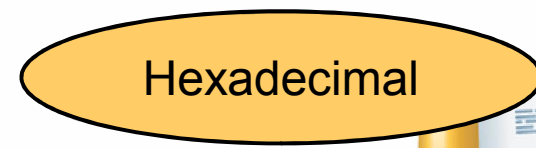
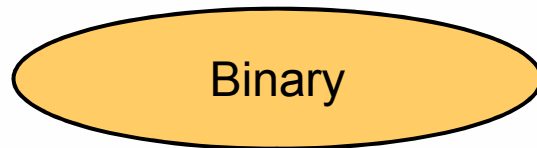
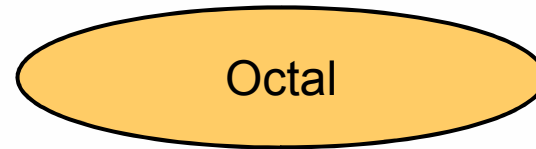
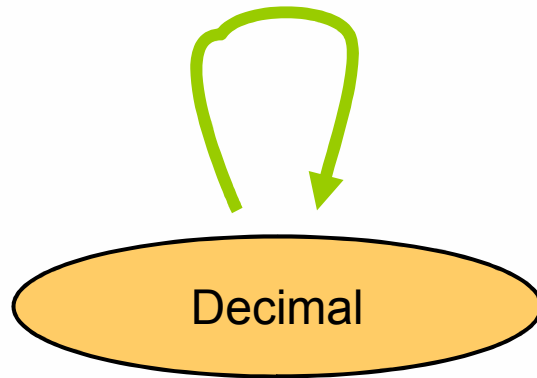
# Quick Example

$$25_{10} = 11001_2 = 31_8 = 19_{16}$$

Base



# Decimal to Decimal (just for fun)

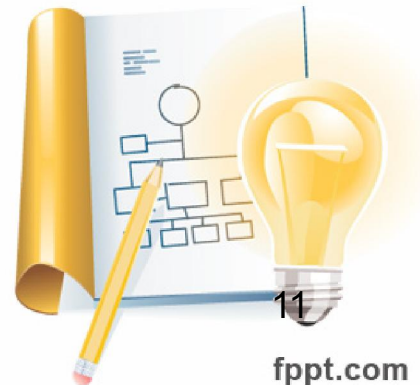


$125_{10} \Rightarrow$

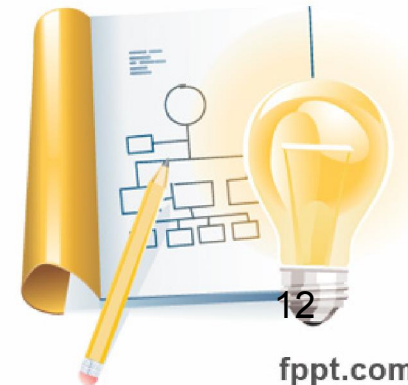
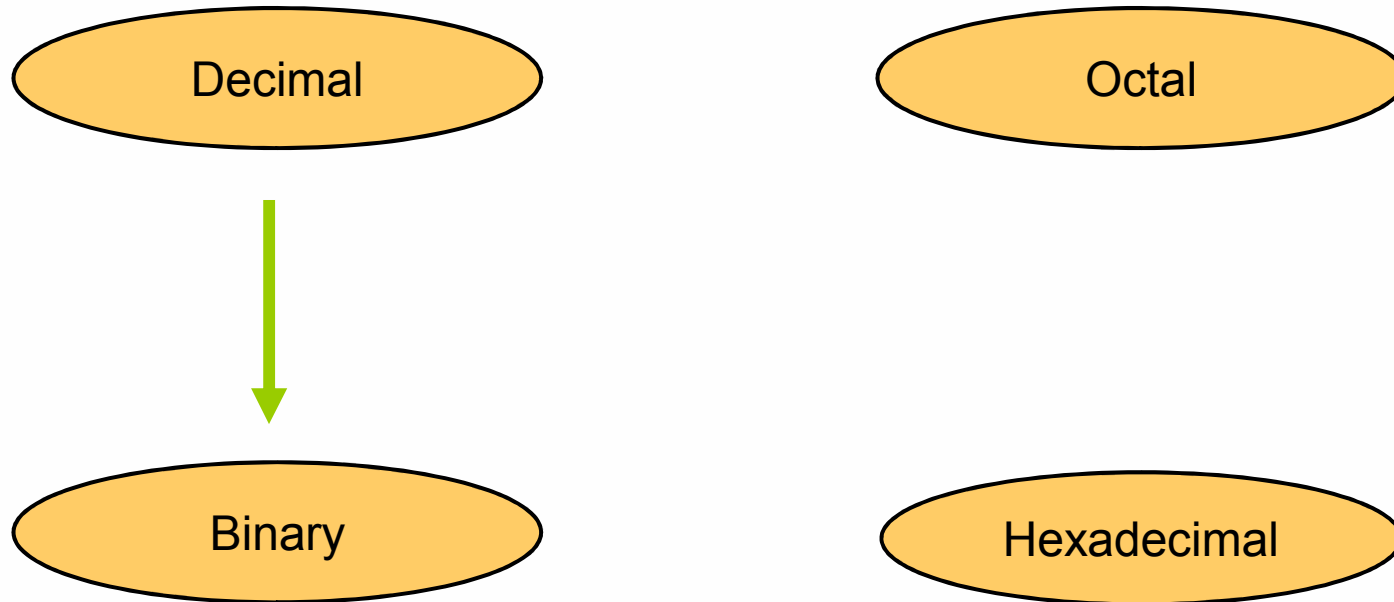
$$\begin{array}{r} 5 \times 10^0 = 5 \\ 2 \times 10^1 = 20 \\ 1 \times 10^2 = 100 \\ \hline 125 \end{array}$$

Weight

Base

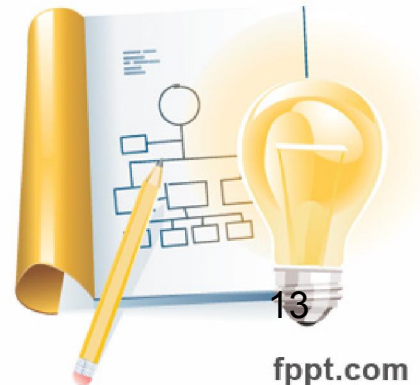


# Decimal to Binary



# Decimal to Binary

- Technique
  - Divide by two, keep track of the remainder
  - First remainder is bit 0 (LSB, least-significant bit)
  - Second remainder is bit 1
  - Etc.

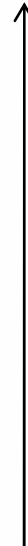


# Conversion of Decimal to Binary

1. Determine the Binary equivalent of  $(125)_{10}$

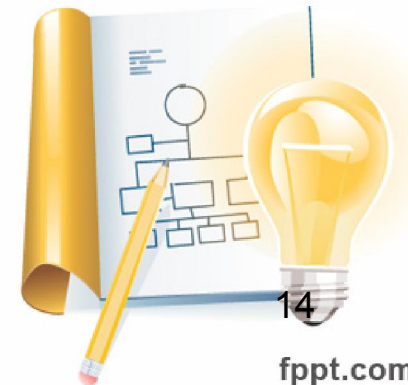
2	125	Remainder
2	62	1
2	31	0
2	15	1
2	7	1
2	3	1
2	1	1
	0	1

Least Significant Bit (LSB)



Most Significant Bit (MSB)

$$125_{10} = 1111101_2$$

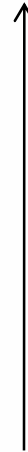


# Conversion of Decimal to Binary

2. Determine the Binary equivalent of  $(36)_{10}$

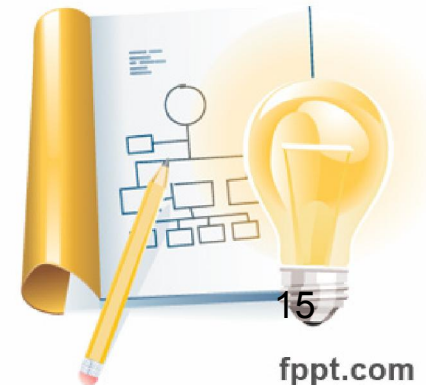
2	36	Remainder
2	18	0
2	9	0
2	4	1
2	2	0
2	1	0
	0	1

Least Significant Bit (LSB)



Most Significant Bit (MSB)

$$36_{10} = 100100_2$$

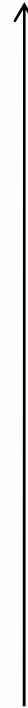


# Conversion of Decimal to Binary

3. Determine the Binary equivalent of  $(671)_{10}$

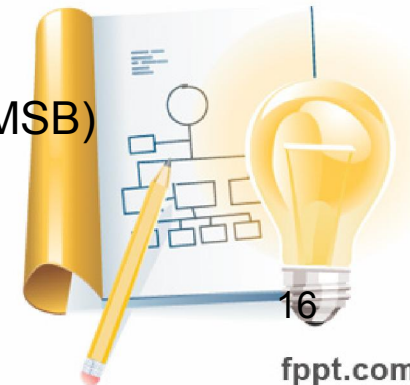
2	671	Remainder
2	335	1
2	167	1
2	83	1
2	41	1
2	20	1
2	10	0
2	5	0
2	2	1
2	1	0
	0	1

Least Significant Bit (LSB)



Most Significant Bit (MSB)

$$671_{10} = 1010011111_2$$





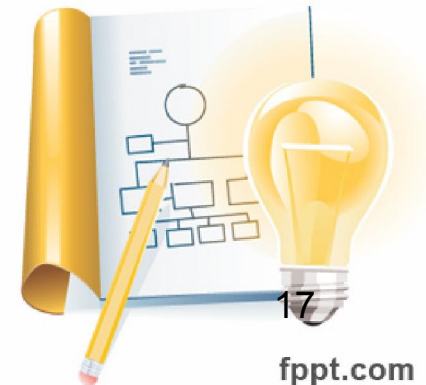
# Conversion of Decimal Fraction to Binary Fraction

1. Determine the Binary equivalent of  $(0.375)_{10}$

<b>0.375</b>	<b>X</b>	<b>2</b>	<b>= 0.750</b>	<b>0</b>
0.750	X	2	= 1.500	1
0.500	X	2	= 1.000	1



$$0.375_{10} = 011_2$$



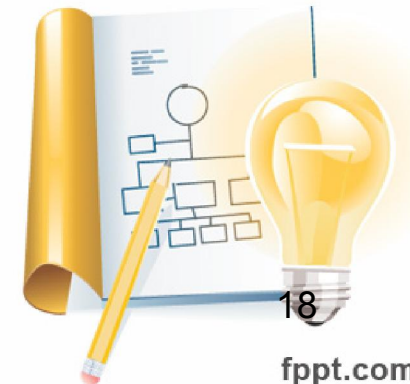
# Conversion of Decimal Fraction to Binary Fraction

2. Determine the Binary equivalent of  $(0.29)_{10}$

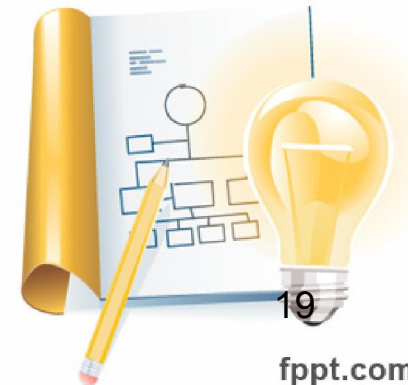
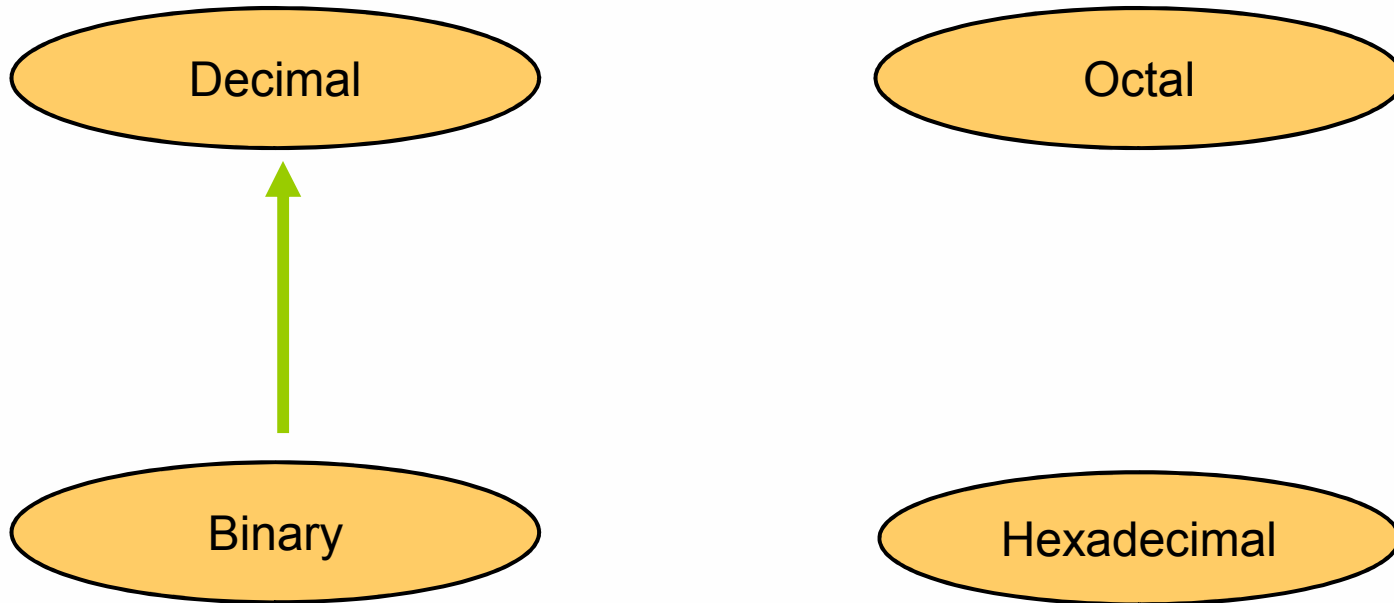
<b>0.29</b>	<b>X</b>	<b>2</b>	<b>= 0.58</b>	<b>0</b>
0.58	X	2	= 1.16	1
0.16	X	2	= 0.32	0
0.32	X	2	= 0.64	0
0.64	X	2	= 1.28	1
0.28	X	2	= 0.56	0
.				
.				
.				
∞				



$$0.29_{10} = (0.010010)_2$$

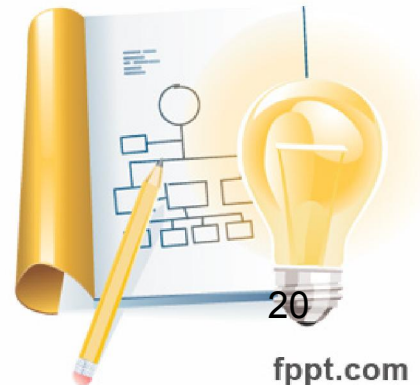


# Binary to Decimal



# Binary to Decimal

- Technique
  - Multiply each bit by  $2^n$ , where  $n$  is the “weight” of the bit
  - The weight is the position of the bit, starting from 0 on the right
  - Add the results

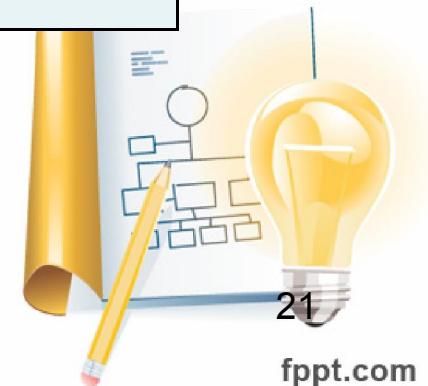


# Binary to Decimal

1. Determine the Decimal equivalent of  $(101011)_2$

Binary Number	1	0	1	0	1	1
Weight of Each Bit	$2^5$	$2^4$	$2^3$	$2^2$	$2^1$	$2^0$
Weighted Value	$1 \times 2^5$	$0 \times 2^4$	$1 \times 2^3$	$0 \times 2^2$	$1 \times 2^1$	$1 \times 2^0$
Solved Multiplication	32	0	8	0	2	1

$$\begin{aligned}\text{Sum of weight of all bits} &= 32 + 0 + 8 + 2 + 1 \\ &= 43\end{aligned}$$

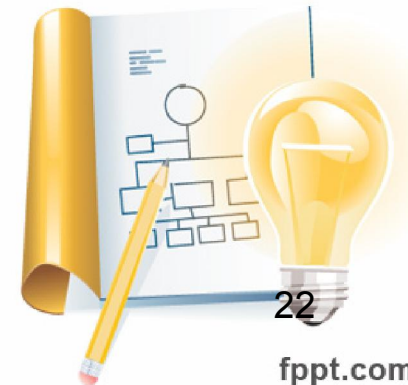


# Binary to Decimal

2. Determine the Decimal equivalent of  $(11010)_2$

Binary Number	1	1	0	1	0
Weight of Each Bit	$2^4$	$2^3$	$2^2$	$2^1$	$2^0$
Weighted Value	$1 \times 2^4$	$1 \times 2^3$	$0 \times 2^2$	$1 \times 2^1$	$0 \times 2^0$
Solved Multiplication	16	8	0	2	0

$$\begin{aligned}\text{Sum of weight of all bits} &= 16 + 8 + 0 + 2 + 0 \\ &= 26\end{aligned}$$

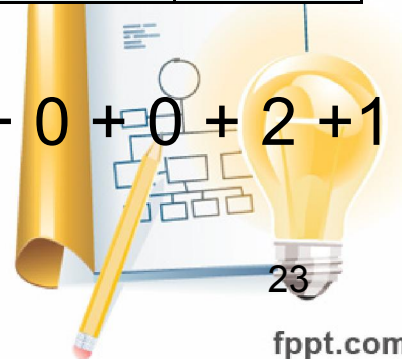


# Binary to Decimal

3. Determine the Decimal equivalent of  $(10110011)_2$

Binary Number	1	0	1	1	0	0	1	1
Weight of Each Bit	$2^7$	$2^6$	$2^5$	$2^4$	$2^3$	$2^2$	$2^1$	$2^0$
Weighted Value	$1 \times 2^7$	$0 \times 2^6$	$1 \times 2^5$	$1 \times 2^4$	$0 \times 2^3$	$0 \times 2^2$	$1 \times 2^1$	$1 \times 2^0$
Solved Multiplication	128	0	32	16	0	0	2	1

Sum of weight of all bits =  $128 + 0 + 32 + 16 + 0 + 0 + 2 + 1$   
= 179

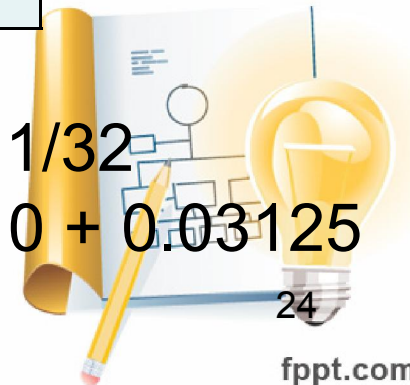


# Conversion of Binary Fraction to Decimal Fractions

1. Determine the Decimal equivalent of  $(0.01101)_2$

Binary Number	0	1	1	0	1
Weight of Each Bit	$2^{-1}$	$2^{-2}$	$2^{-3}$	$2^{-4}$	$2^{-5}$
Weighted Value	$0 \times 2^{-1}$	$1 \times 2^{-2}$	$1 \times 2^{-3}$	$0 \times 2^{-4}$	$1 \times 2^{-5}$
Solved Multiplication	0	$1/4$	$1/8$	0	$1/32$

$$\begin{aligned}\text{Sum of weight of all bits} &= 0 + 1/4 + 1/8 + 0 + 1/32 \\ &= 0 + 0.25 + 0.125 + 0 + 0.03125 \\ &= 0.40625\end{aligned}$$



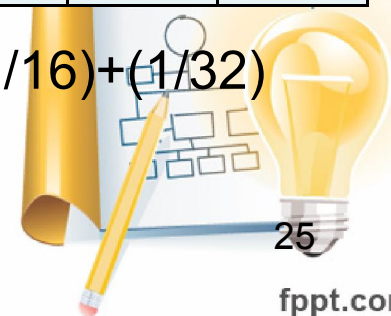


# Conversion of Binary Fraction to Decimal Fractions

2. Determine the Decimal equivalent of  $(11101.10111)_2$

Binary Number	1	1	1	0	1	1	0	1	1	1
Weight of Each Bit	$2^4$	$2^3$	$2^2$	$2^1$	$2^0$	$2^{-1}$	$2^{-2}$	$2^{-3}$	$2^{-4}$	$2^{-5}$
Weighted Value	$1 \times 2^4$	$1 \times 2^3$	$1 \times 2^2$	$0 \times 2^1$	$1 \times 1$	$1 \times 2^{-1}$	$0 \times 2^{-2}$	$1 \times 2^{-3}$	$1 \times 2^{-4}$	$1 \times 2^{-5}$
Solved Multiplication	16	8	4	0	1	1/2	0	1/8	1/16	1/32

Sum of weight of all bits =  $16+8+4+0+1+(1/2)+0+(1/8)+(1/16)+(1/32)$   
=  $16+8+4+0+1+0.5+0+0.125+0.0625+0.03125$   
= 29.71875

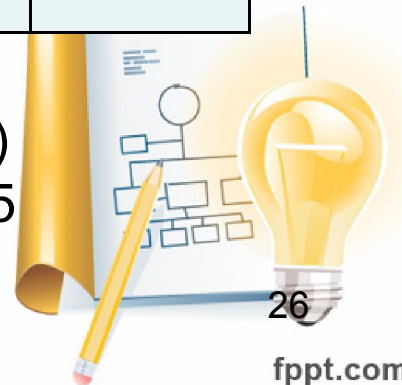


# Conversion of Binary Fraction to Decimal Fractions

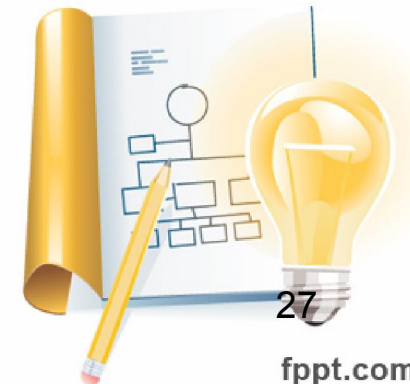
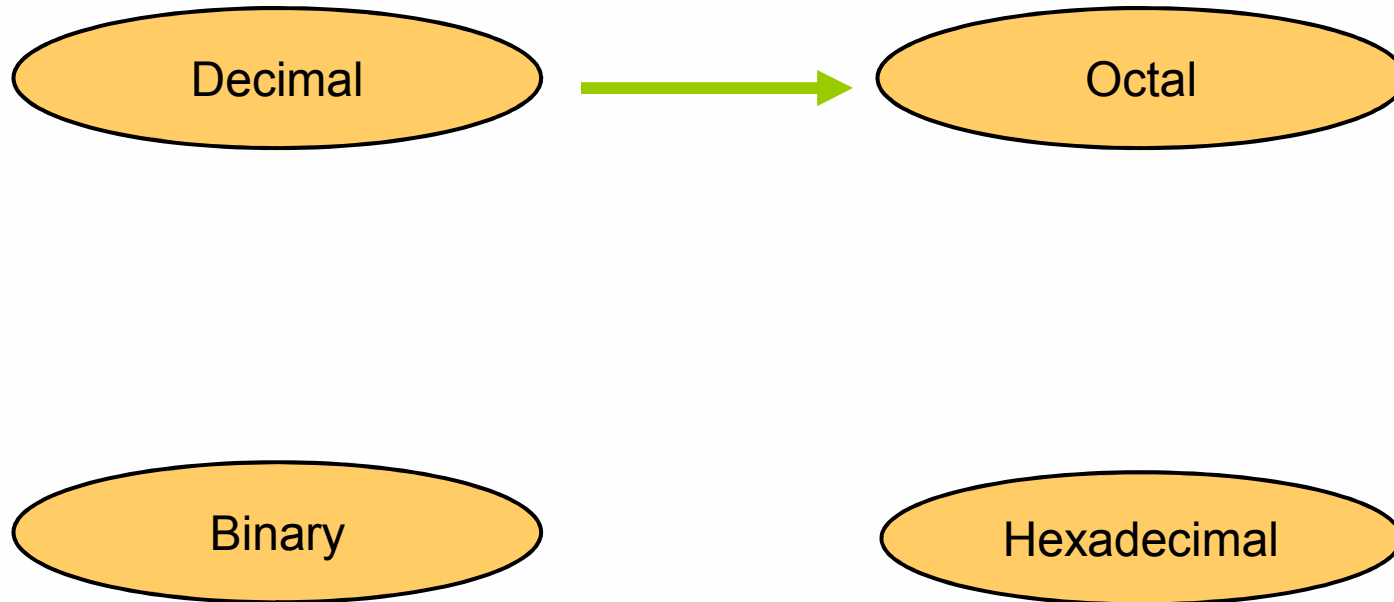
3. Determine the Decimal equivalent of  $(10.1011)_2$

Binary Number	1	0	1	0	1	1
Weight of Each Bit	$2^1$	$2^0$	$2^{-1}$	$2^{-2}$	$2^{-3}$	$2^{-4}$
Weighted Value	$1 \times 2$	$1 \times 2^0$	$1 \times 2^{-1}$	$0 \times 2^{-2}$	$1 \times 2^{-3}$	$1 \times 2^{-4}$
Solved Multiplication	2	1	1/2	0	1/8	1/16

$$\begin{aligned}\text{Sum of weight of all bits} &= 2 + 1 + (1/2) + 0 + (1/8) + (1/16) \\ &= 2 + 1 + 0.5 + 0 + 0.125 + 0.0625 \\ &= 2.6875\end{aligned}$$

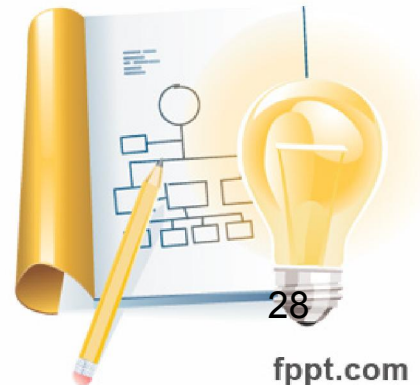


# Decimal to Octal



# Decimal to Octal

- Technique
  - Divide by 8
  - Keep track of the remainder



# Conversion of Decimal to Octal

1. Determine the Octal equivalent of  $(1234)_{10}$

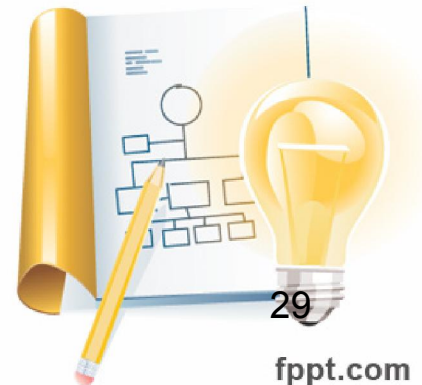
8	1234	Remainder
8	154	2
8	19	2
8	2	3
	0	2

Least Significant Bit (LSB)



Most Significant Bit (MSB)

$$1234_{10} = 2322_8$$

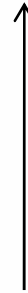


# Conversion of Decimal to Octal

2. Determine the Octal equivalent of  $(359)_{10}$

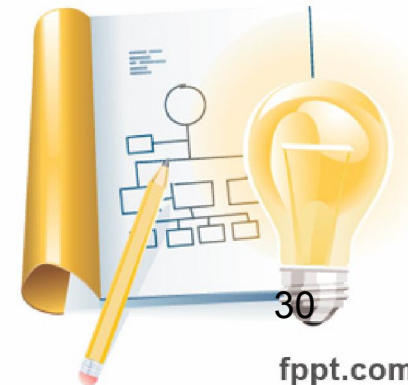
8	359	Remainder
8	44	7
8	5	4
	0	5

Least Significant Bit (LSB)



Most Significant Bit (MSB)

$$359_{10} = 547_8$$



# Conversion of Decimal to Octal

3. Determine the Octal equivalent of  $(432267)_{10}$

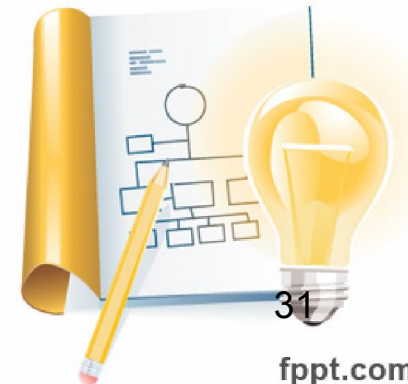
8	432267	Remainder
8	54033	3
8	6754	1
8	844	2
8	105	4
8	13	1
8	1	5
	0	1

Least Significant Bit ( LSB)



Most Significant Bit ( MSB)

$$432267_{10} = 1514213_8$$



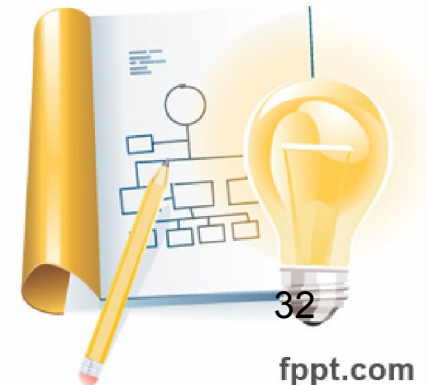
# Conversion of Decimal Fraction to Octal Fraction

1. Determine the Octal equivalent of  $(0.3125)_{10}$

<b>0.3125</b>	<b>X</b>	<b>8</b>	<b>= 2.5</b>	<b>2</b>
0.50	X	8	= 4.0	4



$$0.375_{10} = 0.24_8$$





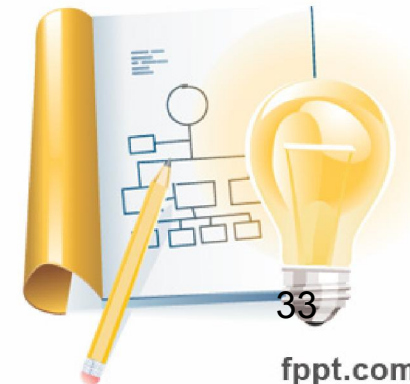
# Conversion of Decimal Fraction to Octal Fraction

2. Determine the Octal equivalent of  $(0.1325)_{10}$

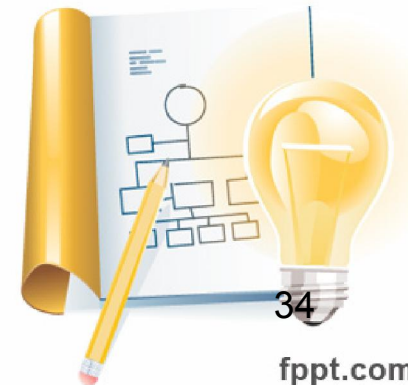
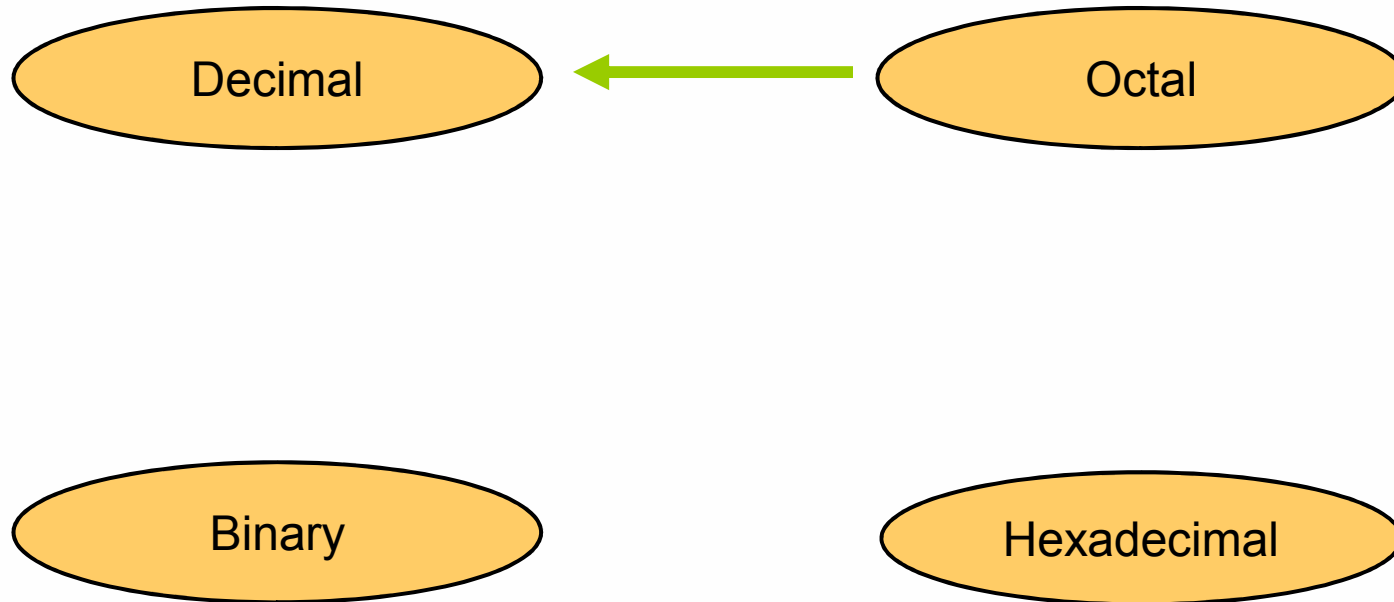
<b>0.1325</b>	<b>X</b>	<b>8</b>	<b>=</b>	<b>1.0600</b>	<b>1</b>
0.0600	X	8	=	0.4800	0
0.4800	X	8	=	3.8400	3
0.8400	X	8	=	6.7200	6
6.7200	X	8	=	5.7600	5
.					
.					
.					
∞					



$$0.1325_{10} = 0.10365_8$$

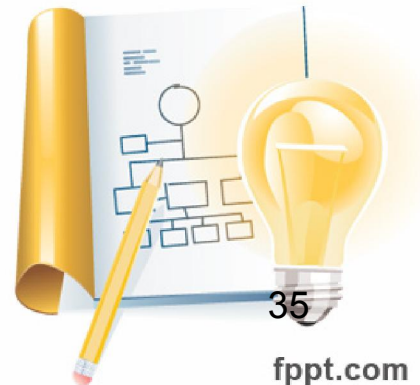


# Octal to Decimal



# Octal to Decimal

- Technique
  - Multiply each bit by  $8^n$ , where  $n$  is the “weight” of the bit
  - The weight is the position of the bit, starting from 0 on the right
  - Add the results

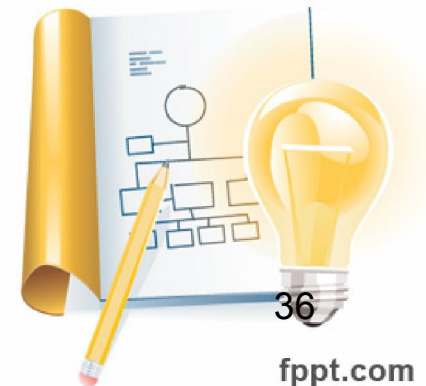


# Conversion of Octal to Decimal

1. Determine the Decimal equivalent of (724)<sub>8</sub>

Octal Number	7	2	4
Weight of Each Bit	$8^2$	$8^1$	$8^0$
Weighted Value	$7 \times 8^2$	$2 \times 8^1$	$4 \times 8^0$
Solved Multiplication	448	16	4

Sum of weight of all bits =  $4 + 16 + 448$   
= 468

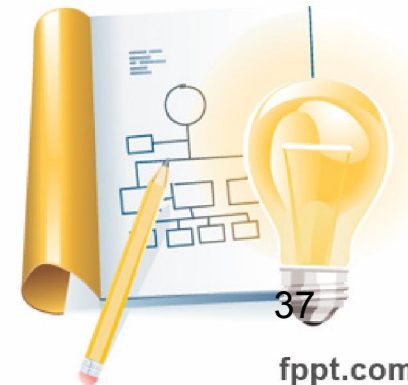


# Conversion of Octal to Decimal

2. Determine the Decimal equivalent of  $(456)_8$

Octal Number	4	5	6
Weight of Each Bit	$8^2$	$8^1$	$8^0$
Weighted Value	$4 \times 8^2$	$5 \times 8^1$	$6 \times 8^0$
Solved Multiplication	256	40	6

$$\begin{aligned}\text{Sum of weight of all bits} &= 256 + 40 + 6 \\ &= 302\end{aligned}$$

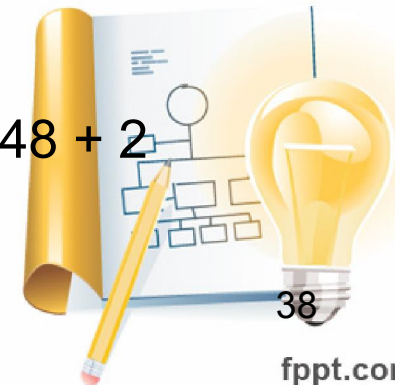


# Conversion of Octal to Decimal

3. Determine the Decimal equivalent of  $(127662)_8$

Octal Number	1	2	7	6	6	2
Weight of Each Bit	$8^5$	$8^4$	$8^3$	$8^2$	$8^1$	$8^0$
Weighted Value	$1 \times 8^5$	$2 \times 8^4$	$7 \times 8^3$	$6 \times 8^2$	$6 \times 8^1$	$2 \times 8^0$
Solved Multiplication	32768	8192	3584	384	48	2

$$\begin{aligned}\text{Sum of weight of all bits} &= 32768 + 8192 + 3584 + 384 + 48 + 2 \\ &= 44978\end{aligned}$$



# Conversion of Octal Fractions to Decimal Fractions

1. Determine the Decimal equivalent of  $(237.04)_8$

Octal Number	2	3	7	0	4
Weight of Each Bit	$8^2$	$8^1$	$8^0$	$8^{-1}$	$8^{-2}$
Weighted Value	$2 \times 8^2$	$3 \times 8^1$	$7 \times 8^0$	$0 \times 8^{-1}$	$4 \times 8^{-2}$
Solved Multiplication	128	24	7	0	0.0625

$$\begin{aligned}\text{Sum of weight of all bits} &= 128 + 24 + 7 + 0 + 0.0625 \\ &= 159.0625\end{aligned}$$

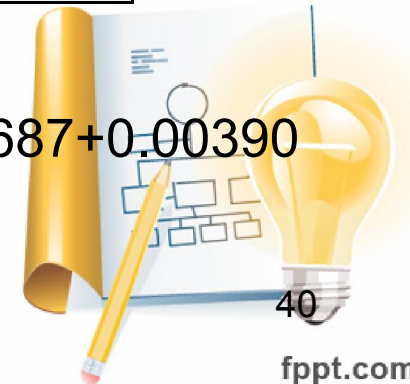


# Conversion of Octal Fractions to Decimal Fractions

2. Determine the Decimal equivalent of  $(6732.032)_8$

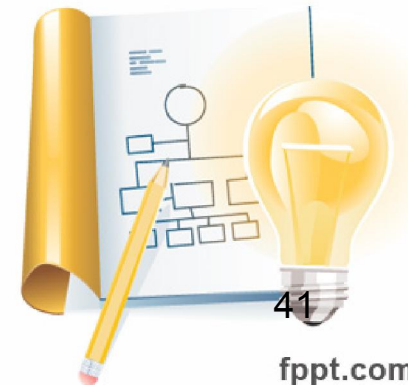
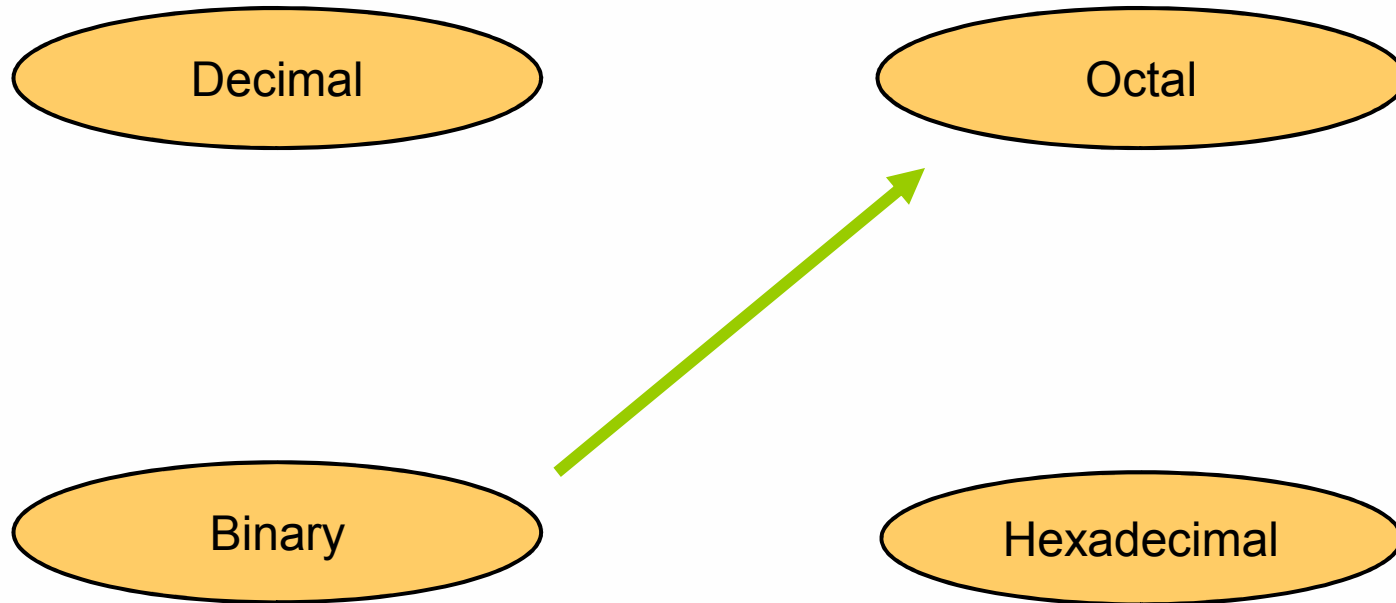
Octal Number	6	7	3	2	0	3	2
Weight of Each Bit	$8^3$	$8^2$	$8^1$	$8^0$	$8^{-1}$	$8^{-2}$	$8^{-3}$
Weighted Value	$6 \times 8^3$	$7 \times 8^2$	$3 \times 8^1$	$2 \times 8^0$	$0 \times 8^{-1}$	$3 \times 8^{-2}$	$2 \times 8^{-3}$
Solved Multiplication	3072	448	24	2	0	0.046875	0.003906

$$\begin{aligned}\text{Sum of weight of all bits} &= 3072 + 448 + 24 + 2 + 0 + 0.046875 + 0.003906 \\ &= 3546.05077\end{aligned}$$



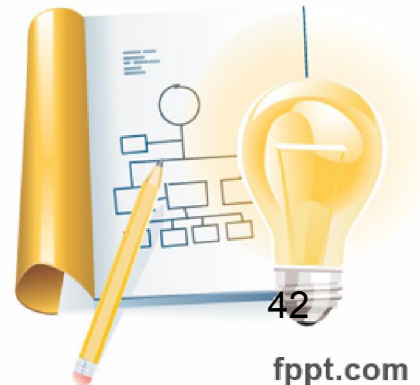


# Binary to Octal



# Binary to Octal

- Technique
  - Break the binary number into 3 sections starting from LSB to MSB (starting on right)
  - Convert to octal digits

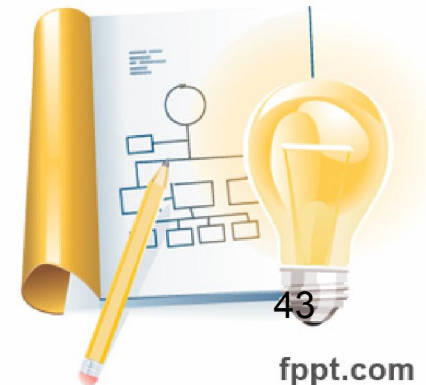


# Conversion of Binary to Octal

1. Determine the Octal equivalent of  $(1011010111)_2$

Binary Number	001 (MSB)	011	010	111 (LSB)
Octal Number	1	3	2	7

$$1011010111_2 = (1327)_8$$

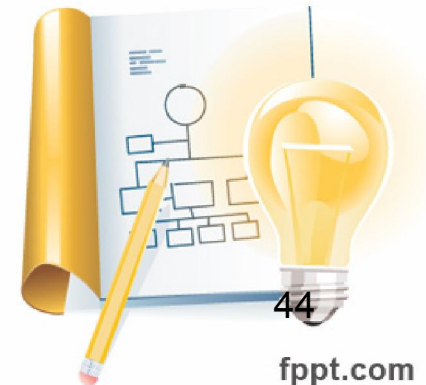


# Conversion of Binary to Octal

2. Determine the Octal equivalent of  $(010111)_2$

Binary Number	010(MSB)	111 (LSB)
Octal Number	2	7

$$010111_2 = (27)_8$$

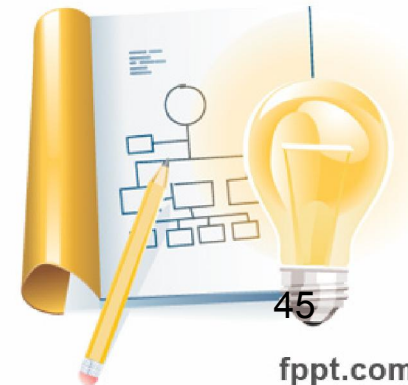


# Conversion of Binary to Octal

3. Determine the Octal equivalent of  $(1010111110110010)_2$

Binary Number	001(MS B)	010	111	110	110	010(LS B)
Octal Number	1	2	7	6	6	2

$$1010111110110010_2 = (127662)_8$$

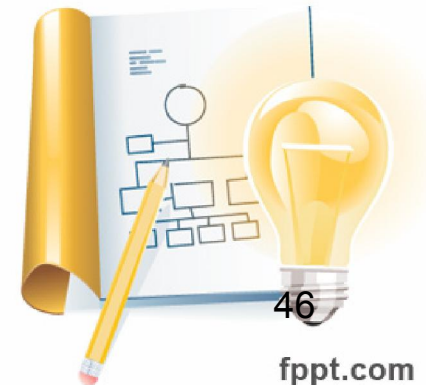


# Conversion of Binary Fractions to Octal Fractions

1. Determine the Octal equivalent of  $(0.110101)_2$

Binary Number	000	110(MSB)	101 (LSB)
Octal Number	0	6	5

$$0.110101_2 = (0.65)_8$$

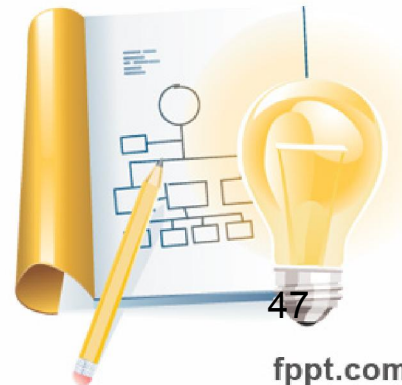


# Conversion of Binary Fraction to Octal Fraction

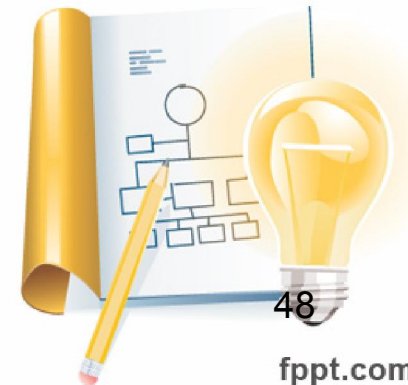
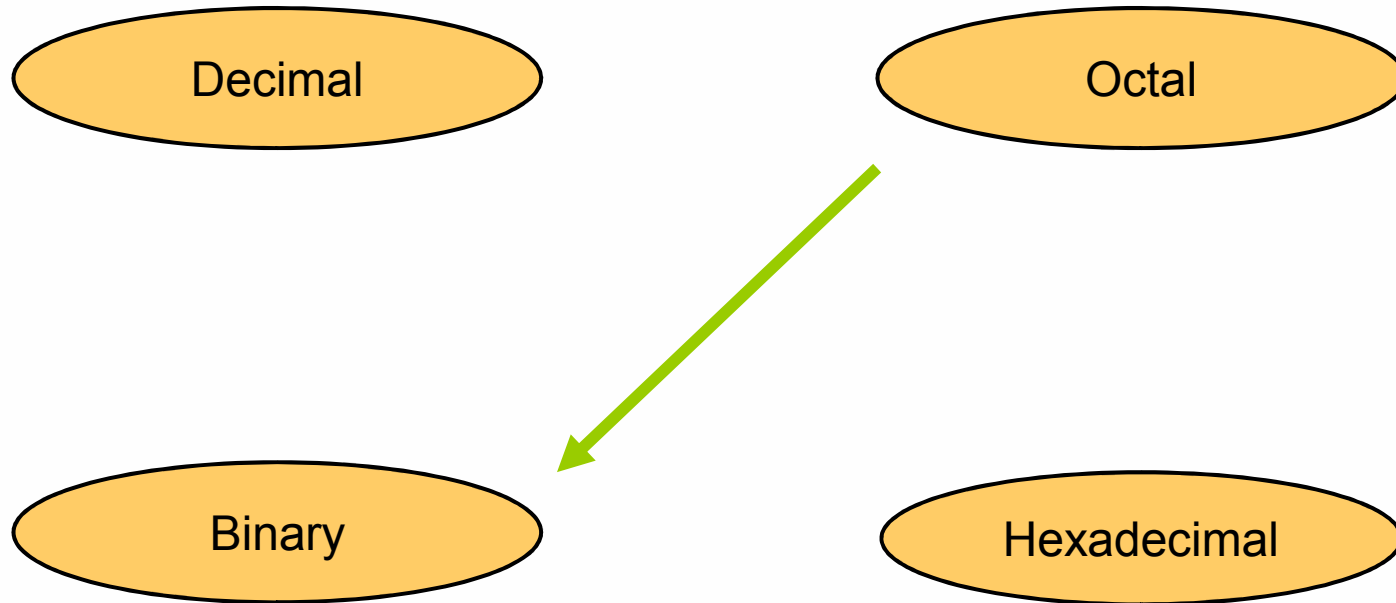
2. Determine the Octal equivalent of  $(1100010.1110110)_2$

Binary Number	001	100	010	111	011	000
Octal Number	1	4	2	7	3	0

$$1100010.1110110_2 = (142.730)_8$$



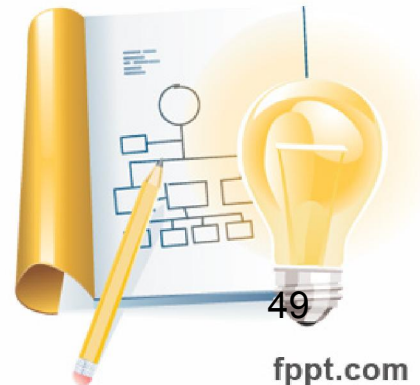
# Octal to Binary





# Octal to Binary

- Technique
  - Convert each octal digit to a 3-bit equivalent binary representation

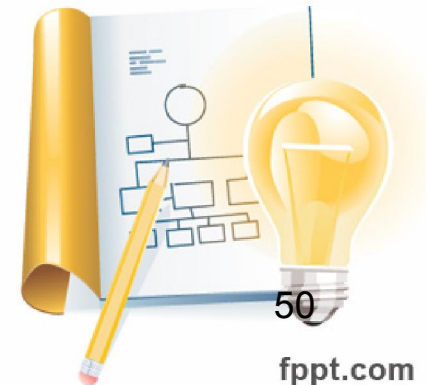


# Conversion of Octal to Binary

1. Determine the Binary equivalent of  $(705)_8$

Octal Number	7	0	5
Binary Coded Value	111	000	101

$$705_8 = (111000101)_2$$

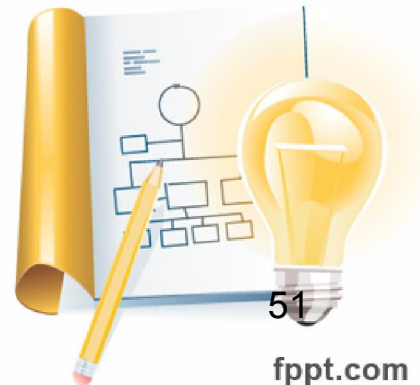


# Conversion of Octal to Binary

2. Determine the Binary equivalent of  $(231)_8$

Octal Number	2	3	1
Binary Coded Value	010	011	001

$$231_8 = (010011001)_2$$

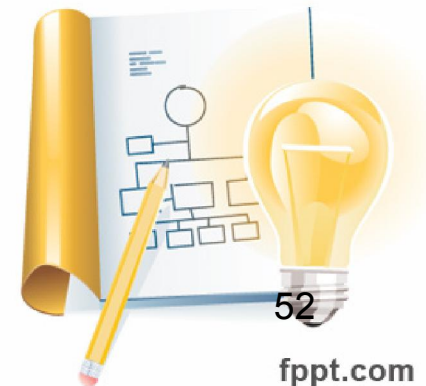


# Conversion of Octal to Binary

3. Determine the Binary equivalent of  $(453267)_8$

Octal Number	4	5	3	2	6	7
Binary Coded Value	100	101	011	010	110	111

$$453267_8 = (100101011010110111)_2$$

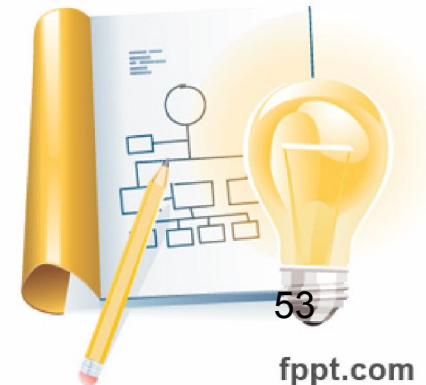


# Conversion of Octal Fractions to Binary Fractions

1. Determine the Binary equivalent of  $(2.335)_8$

Octal Number	2	3	3	5
Binary Coded Value	010	011	011	101

$$2.335_8 = (010.011011101)_2$$

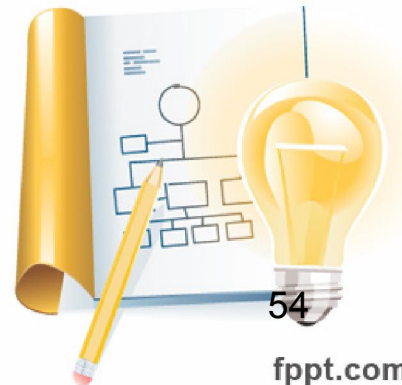


# Conversion of Octal Fractions to Binary Fractions

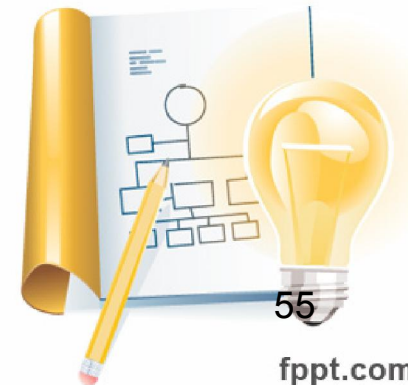
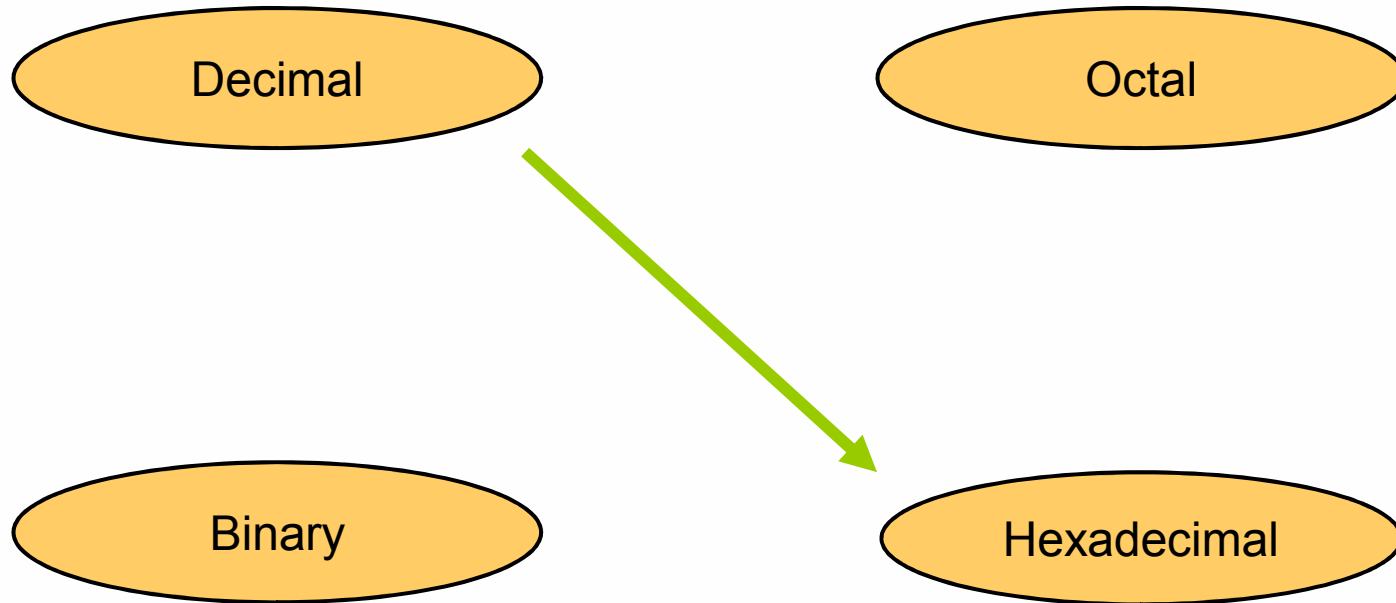
2. Determine the Binary equivalent of  $(5667.2411)_8$

Octal Number	5	6	6	7	2	4	1	1
Binary Coded Value	101	110	110	111	010	100	001	001

$$5667.2411_8 = (101110110111.010100001001)_2$$

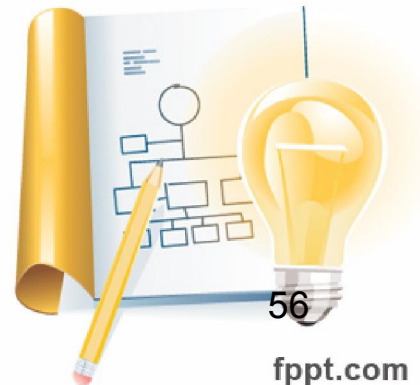


# Decimal to Hexadecimal



# Decimal to Hexadecimal

- Technique
  - Divide by 16
  - Keep track of the remainder



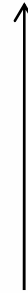


# Conversion of Decimal to Hexadecimal

1. Determine the Hexadecimal equivalent of  $(1234)_{10}$

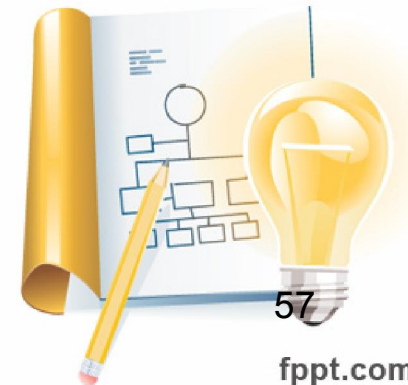
16	1234	Remainder
16	77	2
16	4	13
	0	4

Least Significant Bit (LSB)



Most Significant Bit (MSB)

$$1234_{10} = 4D2_{16}$$

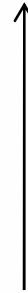


# Conversion of Decimal to Hexadecimal

2. Determine the Hexadecimal equivalent of  $(5112)_{10}$

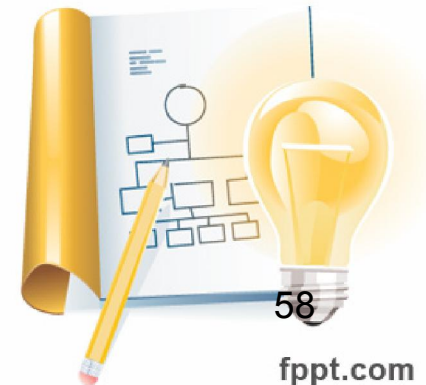
16	511 2	Remainder
16	319	8
16	19	15 = F
16	1	3
	0	1

Least Significant Bit (LSB)



Most Significant Bit (MSB)

$$5112_{10} = 13F8_{16}$$

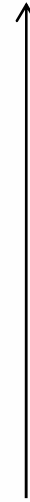


# Conversion of Decimal to Hexadecimal

3. Determine the Hexadecimal equivalent of  $(584666)_{10}$

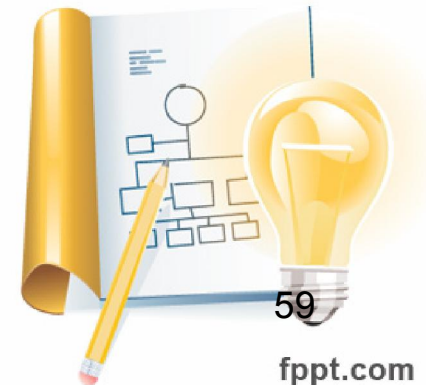
16	584666	Remainder
16	36541	10=A
16	2283	13 = D
16	142	11=B
16	8	14=E
	0	8

Least Significant Bit ( LSB)



Most Significant Bit ( MSB)

$$584666_{10} = 8EBDA_{16}$$



# Conversion of Decimal Fraction to Hexadecimal Fraction

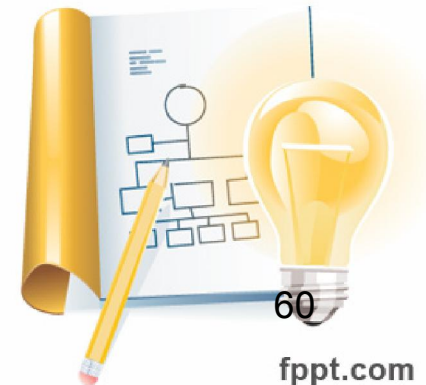
1. Determine the Hexadecimal equivalent of  $(0.625)_{10}$

<b>0.625</b>	<b>X</b>	<b>16</b>	<b>=</b>	<b>10.000</b>	<b>10 = A</b>
--------------	----------	-----------	----------	---------------	---------------

0.000	X	16	=	0.000	0
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$$0.1325_{10} = 0.A0_{16}$$



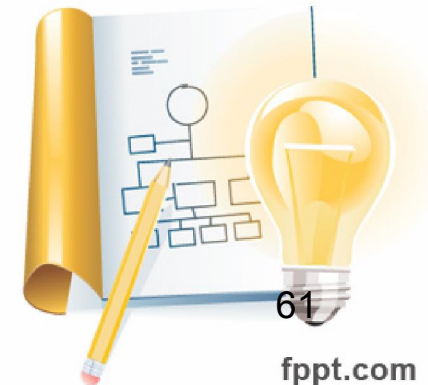
# Conversion of Decimal Fraction to Hexadecimal Fraction

2. Determine the Hexadecimal equivalent of  $(0.2715)_{10}$

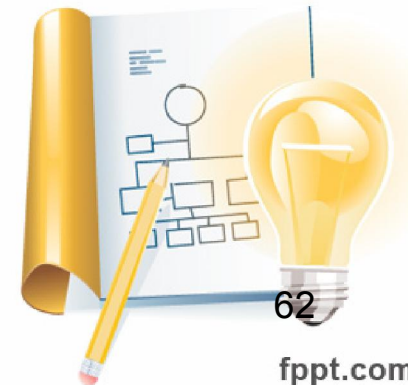
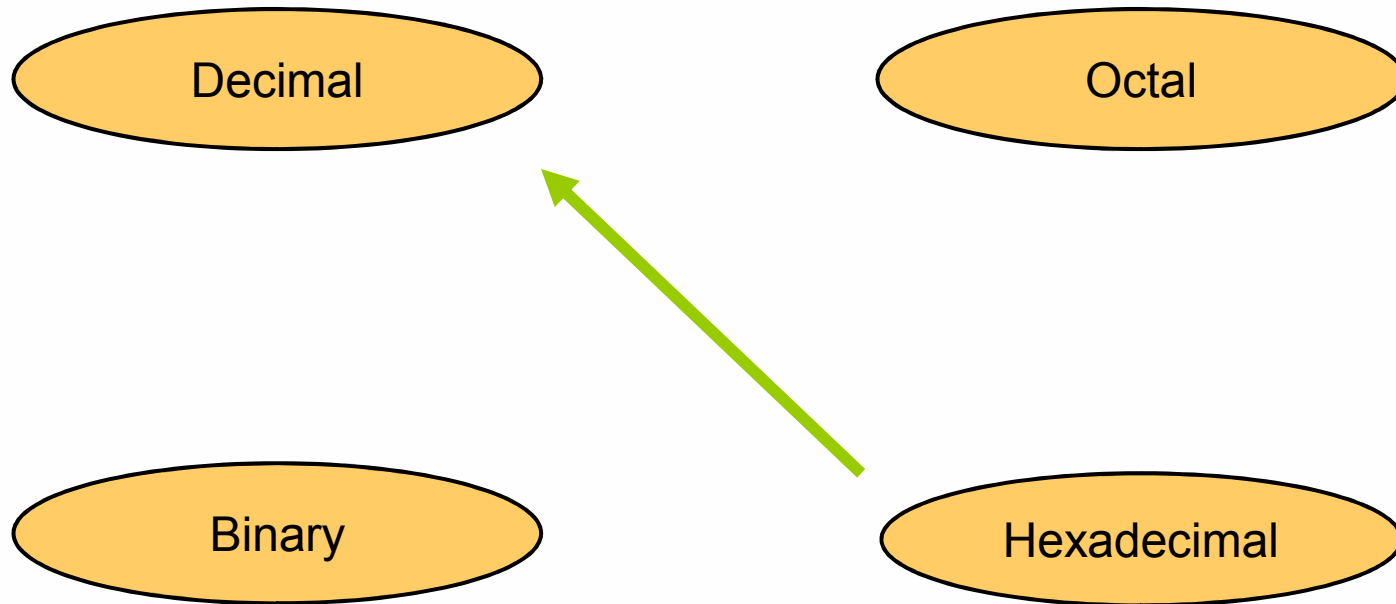
<b>0.2715</b>	<b>X</b>	<b>16</b>	<b>=</b>	<b>4.3440</b>	<b>4</b>
0.3440	X	16	=	5.5040	5
0.5040	X	16	=	8.0640	8
0.0640	X	16	=	1.0240	1
0.0240	X	16	=	0.3840	0
.					
.					
∞					



$$0.2715_{10} = 0.45810_{16}$$

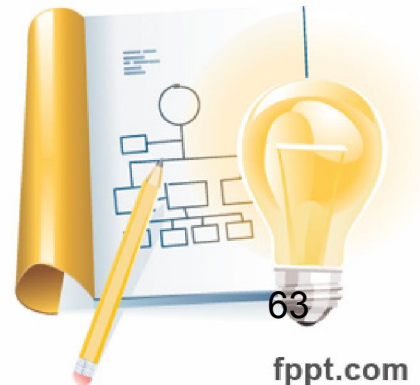


# Hexadecimal to Decimal



# Hexadecimal to Decimal

- Technique
  - Multiply each bit by  $16^n$ , where  $n$  is the “weight” of the bit
  - The weight is the position of the bit, starting from 0 on the right
  - Add the results

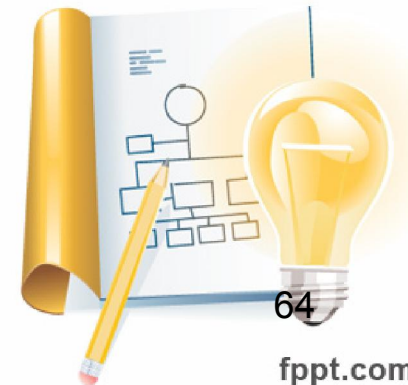


# Conversion of Hexadecimal to Decimal

1. Determine the Decimal equivalent of (B14)<sub>16</sub>

Hexadecimal Number	B=11	1	4
Weight of Each Bit	$16^2$	$16^1$	$16^0$
Weighted Value	$11 \times 16^2$	$1 \times 16^1$	$4 \times 16^0$
Solved Multiplication	2816	16	4

$$\begin{aligned}\text{Sum of weight of all bits} &= 2816 + 16 + 4 \\ &= 2836\end{aligned}$$



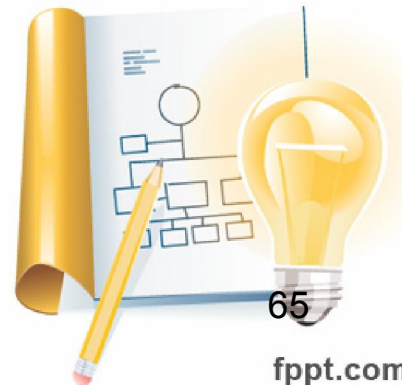


# Conversion of Hexadecimal to Decimal

2. Determine the Decimal equivalent of (ABC)<sub>16</sub>

Hexadecimal Number	A	B	C
Weight of Each Bit	$16^2$	$16^1$	$16^0$
Weighted Value	$10 \times 16^2$	$11 \times 16^1$	$12 \times 16^0$
Solved Multiplication	2560	176	12

Sum of weight of all bits =  $2560 + 176 + 12$   
= 2748

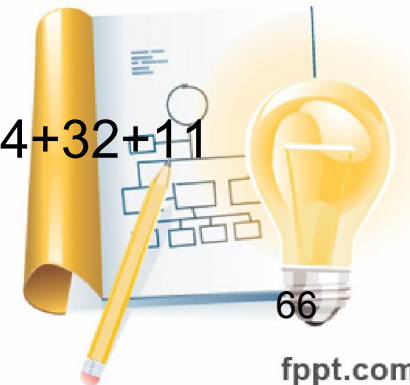


# Conversion of Hexadecimal to Decimal

3. Determine the Decimal equivalent of (8AFE2B)<sub>16</sub>

Hexadecimal Number	8	A	F	E	2	B
Weight of Each Bit	$16^5$	$16^4$	$16^3$	$16^2$	$16^1$	$16^0$
Weighted Value	$8 \times 16^5$	$10 \times 16^4$	$15 \times 16^3$	$14 \times 16^2$	$2 \times 16^1$	$11 \times 16^0$
Solved Multiplication	8388608	655360	61440	3584	32	11

Sum of weight of all bits =  $8388608 + 655360 + 61440 + 3584 + 32 + 11$   
= 9109035

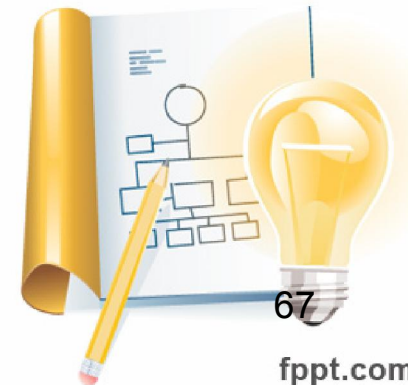


# Conversion of Hexadecimal Fractions to Decimal Fractions

1. Determine the Decimal equivalent of (A.23)<sub>16</sub>

Octal Number	A	2	3
Weight of Each Bit	$16^0$	$16^{-1}$	$16^{-2}$
Weighted Value	$10 \times 1$	$2 \times 16^{-1}$	$3 \times 16^{-2}$
Solved Multiplication	10	0.125	0.01171875

Sum of weight of all bits =  $10 + 0.125 + 0.01171875$   
= 10.13671875

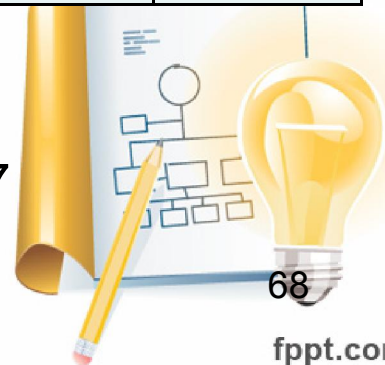


# Conversion of Hexadecimal Fractions to Decimal Fractions

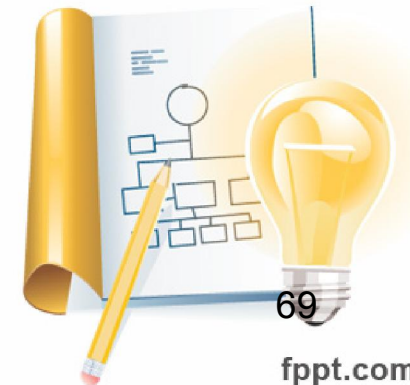
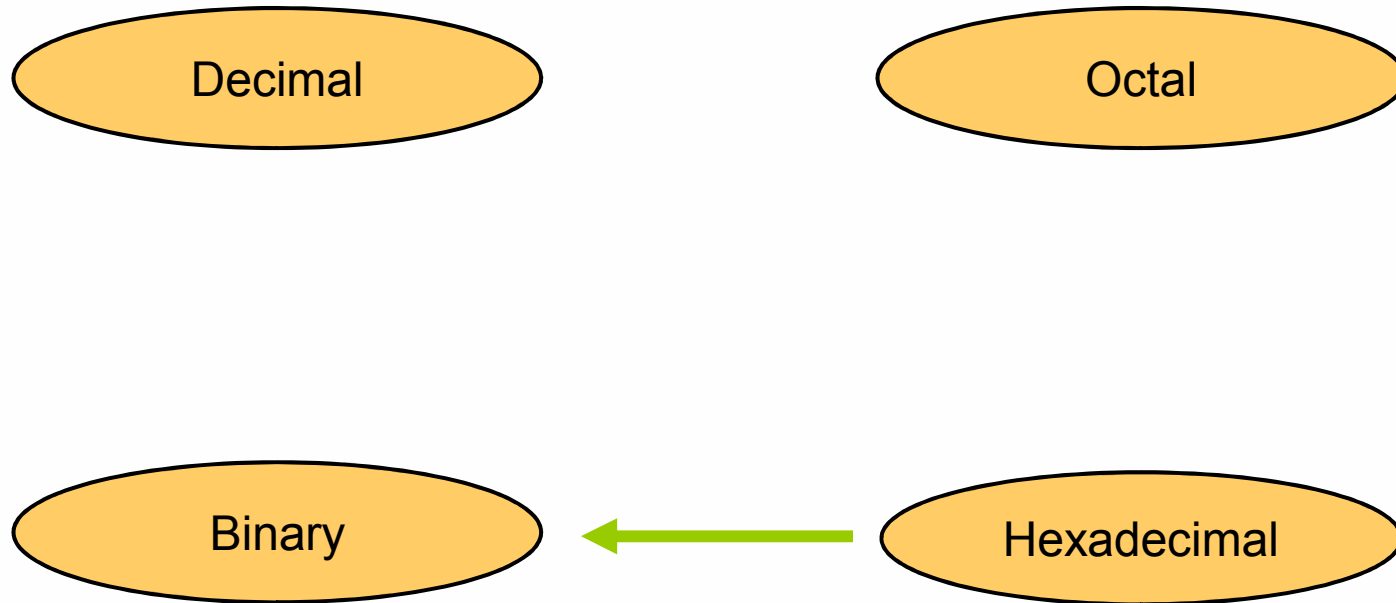
2. Determine the Decimal equivalent of (45C.8BE3)<sub>16</sub>

Octal Number	4	5	C=12	8	B=11	E=14	3
Weight of Each Bit	$16^2$	$16^1$	$16^0$	$16^{-1}$	$16^{-2}$	$16^{-3}$	$16^{-4}$
Weighted Value	4 X 256	5 X 16	12 X 1	8 X $16^{-1}$	11 X $16^{-2}$	14 X $16^{-3}$	3 X $16^{-4}$
Solved Multiplication	1024	80	12	0.5	0.0429687	0.0034179	0.0000457

Sum of weight of all bits =  
= 1024 + 80 + 12 + 0.5 + 0.0429687 + 0.0034179 + 0.0000457  
= 1116.5464323

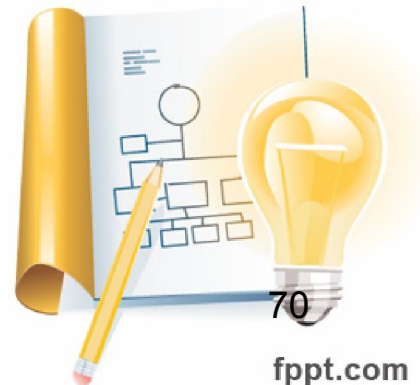


# Hexadecimal to Binary



# Hexadecimal to Binary

- Technique
  - Convert each hexadecimal digit to a 4-bit equivalent binary representation

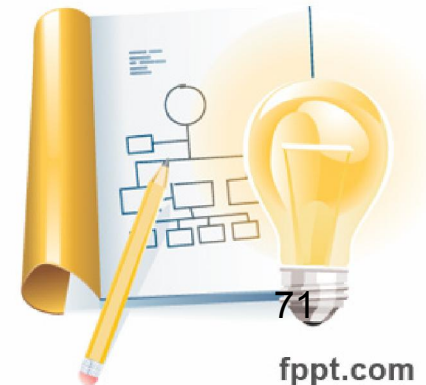


# Conversion of Hexadecimal to Binary

1. Determine the Binary equivalent of (10AF)<sub>16</sub>

Hexadecimal Number	1	0	A=10	F=15
Binary Coded Value	0001	0000	1010	1111

$$10AF_{16} = (0001000010101111)_2$$

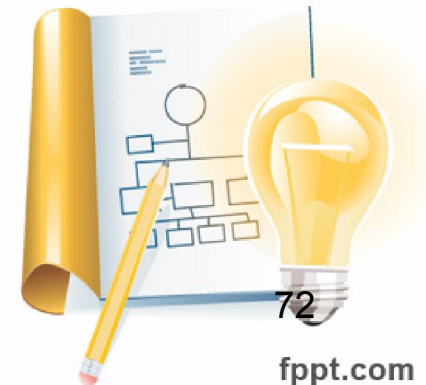


# Conversion of Hexadecimal to Binary

1. Determine the Binary equivalent of (5AF)<sub>16</sub>

Hexadecimal Number	5	A=10	F=15
Binary Coded Value	0101	1010	1111

$$5AF_{16} = (010110101111)_2$$



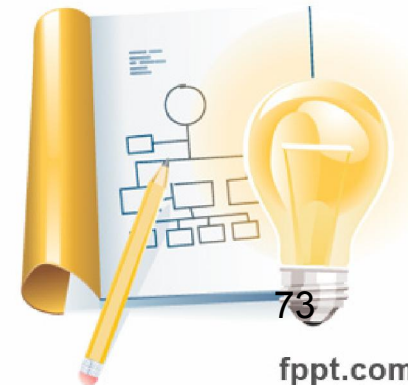


# Conversion of Hexadecimal to Binary

1. Determine the Binary equivalent of (86DB45C)<sub>16</sub>

Hexadecimal Number	8	6	D=13	B=11	4	5	C=12
Binary Coded Value	1000	0110	1101	1011	0100	0101	1100

$$86DB45C_{16} = (1000011011011011010001011100)_2$$

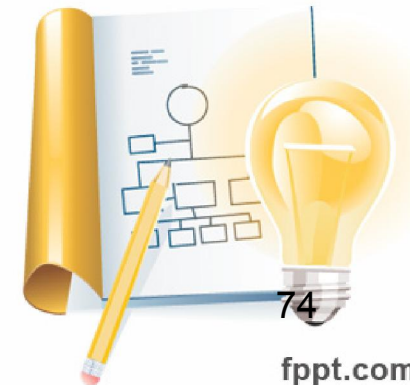


# Conversion of Hexadecimal Fractions to Binary Fractions

1. Determine the Binary equivalent of (2B.6C)<sub>16</sub>

Hexadecimal Number	2	B=11	6	C=12
Binary Coded Value	0010	1011	0110	1100

$$2B.6C_{16} = (00101011.01101100)_2$$

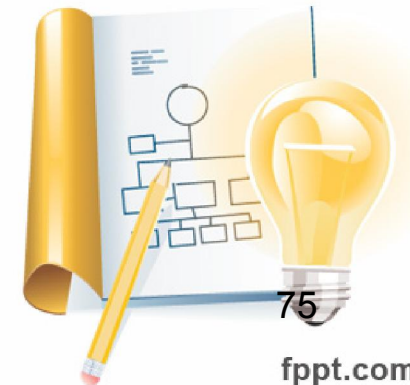


# Conversion of Hexadecimal Fractions to Binary Fractions

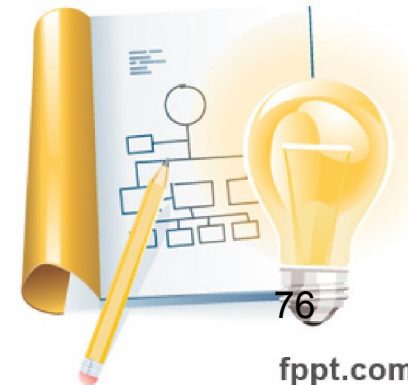
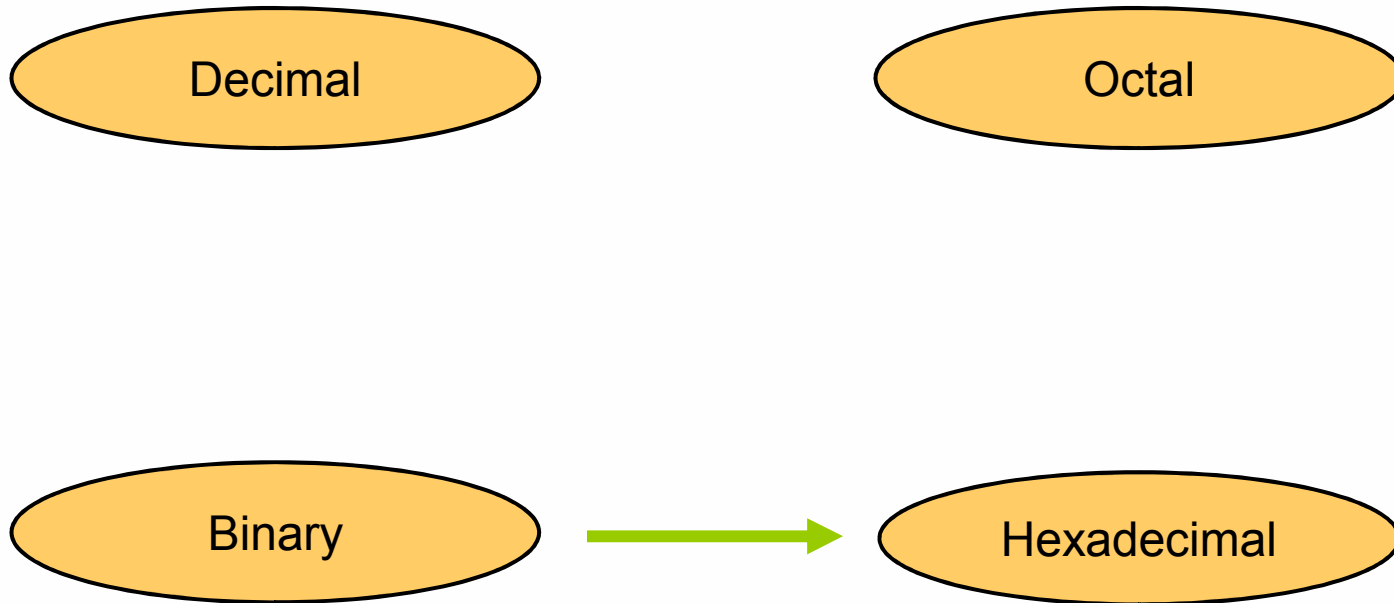
2. Determine the Binary equivalent of  $(576E.34DF)_{16}$

Hexadecimal Number	5	7	6	E=14	3	4	D=13	F=15
Binary Coded Value	0101	0111	0110	1110	0011	0100	1101	1111

$$576E.34DF_{16} = (0101011101101110.0011010011011111)_2$$

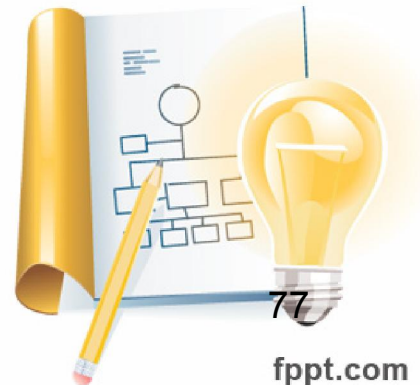


# Binary to Hexadecimal



# Binary to Hexadecimal

- Technique
  - Group bits in fours, starting on right
  - Convert to hexadecimal digits

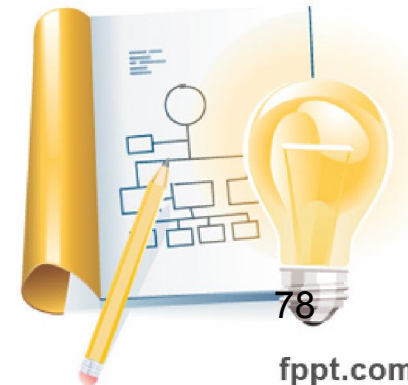


# Conversion of Binary to Hexadecimal

1. Determine the Hexadecimal equivalent of  $(1010111011)_2$

Binary Number	0010	1011	1011
Decimal Number	2	11	11
Hexadecimal Number	2	B	B

$$1010111011_2 = (2BB)_{16}$$

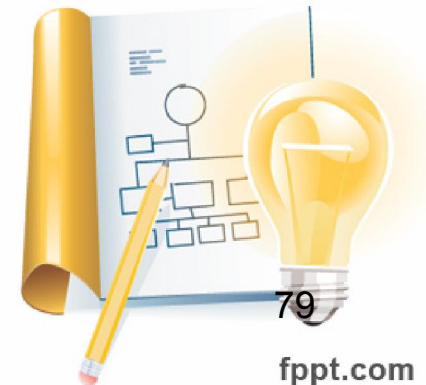


# Conversion of Binary to Hexadecimal

2. Determine the Hexadecimal equivalent of  $(11001011)_2$

Binary Number	1100	1011
Decimal Number	12	11
Hexadecimal Number	C	B

$$11001011_2 = (CB)_{16}$$

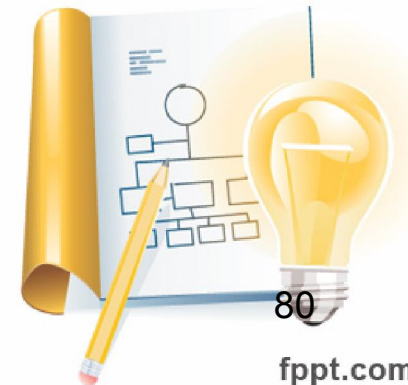


# Conversion of Binary to Hexadecimal

3. Determine the Hexadecimal equivalent of  $(101011110011011001)_2$

Binary Number	0010	1011	1100	1101	1001
Decimal Number	2	11	12	13	9
Hexadecimal Number	2	B	C	D	9

$$11001011_2 = (2BCD9)_{16}$$



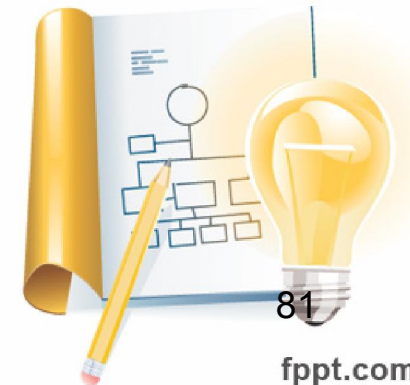


# Conversion of Binary Fraction to Hexadecimal Fraction

1. Determine the Hexadecimal equivalent of  $(0.11101000)_2$

Binary Number	0000	1110	1000
Decimal Number	0	14	8
Hexadecimal Number	0	E	8

$$0.11101000_2 = (0.E8)_{16}$$

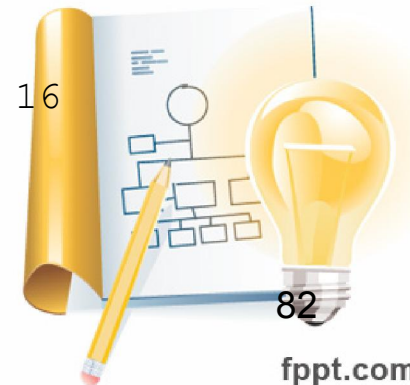


# Conversion of Binary Fraction to Hexadecimal Fraction

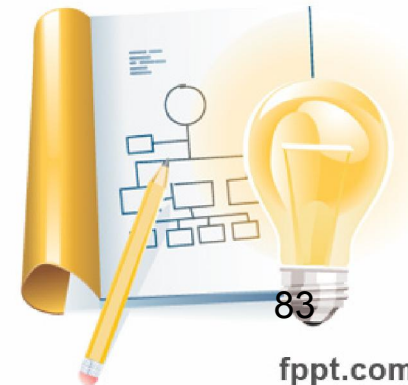
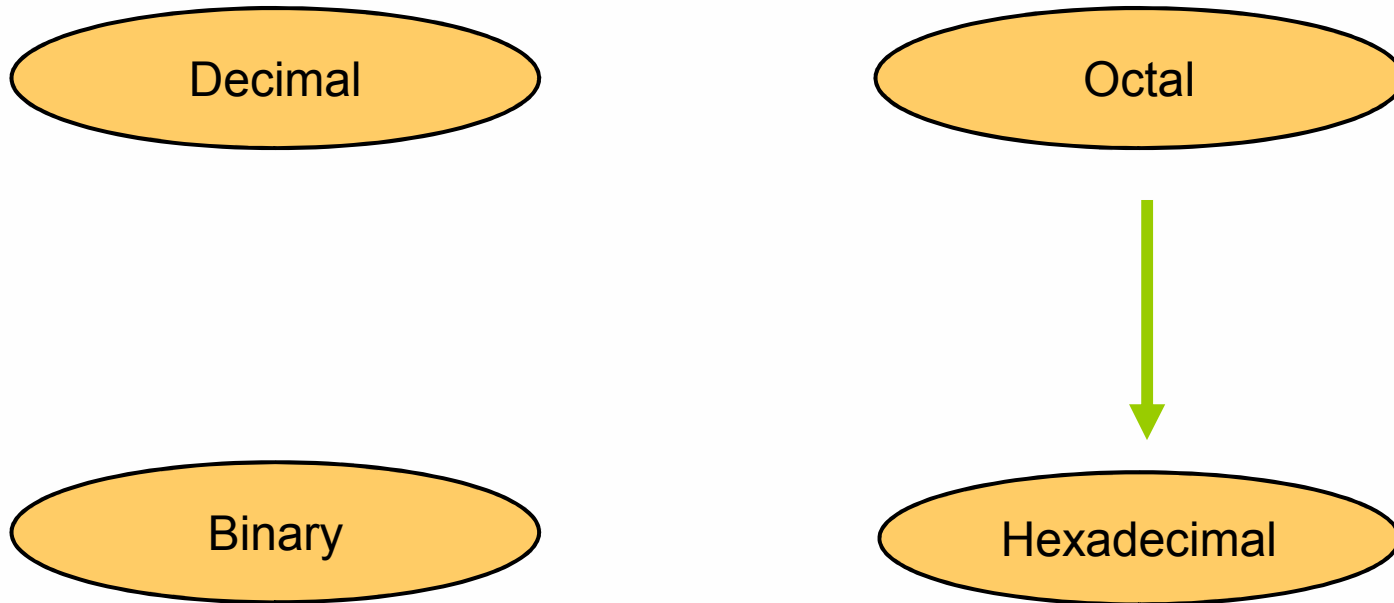
2. Determine the Hexadecimal equivalent of  $(1100001.101011110011)_2$

Binary Number	0110	0001	1010	1111	0011
Decimal Number	6	1	10	15	3
Hexadecimal Number	6	1	A	F	3

$$1100001.101011110011_2 = (61.AF3)_{16}$$

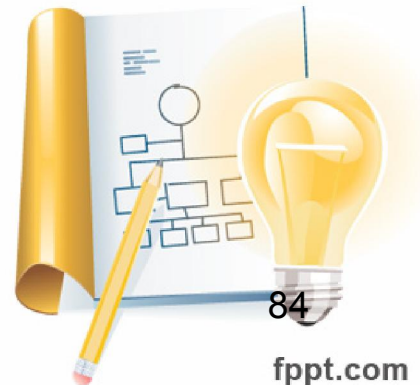


# Octal to Hexadecimal



# Octal to Hexadecimal

- Technique
  - Use binary as an intermediary



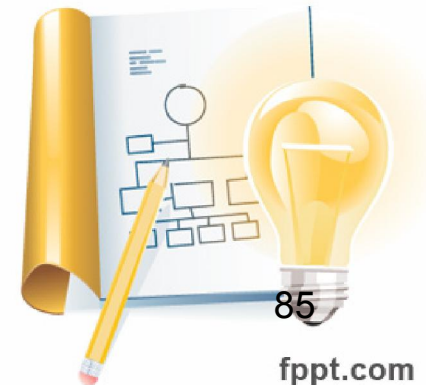
# Conversion of Octal to Hexadecimal

1. Determine the Hexadecimal equivalent of  $(2327)_8$

Octal Number	2	3	2	7
Binary Coded Value	010	011	010	111

$$2327_8 = (\underbrace{0100}_4 \underbrace{1101}_{13} \underbrace{0111}_7)_2$$

$$2327_8 = (4D7)_{16}$$



# Conversion of Octal to Hexadecimal

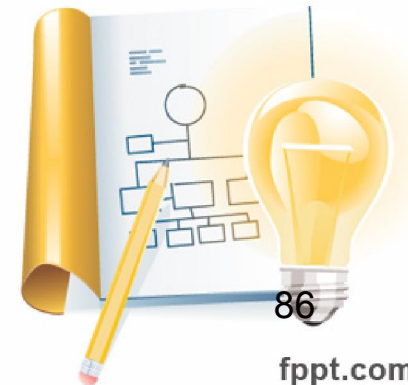
2. Determine the Hexadecimal equivalent of  $(1076)_8$

Octal Number	1	0	7	6
Binary Coded Value	001	000	111	110

$$1076_8 = (\mathbf{0010} \ \mathbf{0011} \ \mathbf{1110})_2$$

2                      3                      14

$$1076_8 = (23E)_{16}$$



# Conversion of Octal Fraction to Hexadecimal Fraction

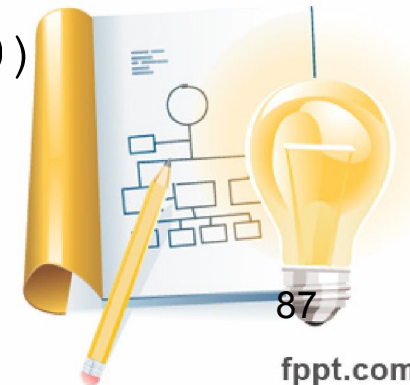
1. Determine the Hexadecimal equivalent of  $(31.57)_8$

Octal Number	3	1	5	7
Binary Coded Value	011	001	101	111

$$\begin{aligned} 31.57_8 &= (011001.101111)_2 \\ &= (0001\ 1001.1011\ 1100) \end{aligned}$$

1            9            .    11            12

$$31.57_8 = (19.BC)_{16}$$



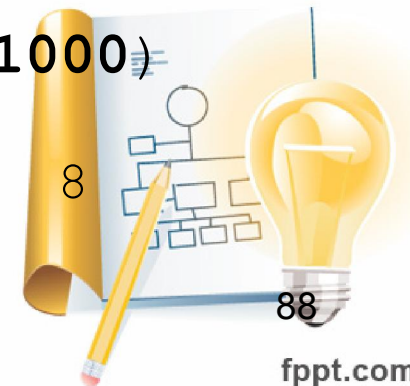
# Conversion of Octal Fraction to Hexadecimal Fraction

2. Determine the Hexadecimal equivalent of  $(76.665)_8$

Octal Number	7	6	6	6	5
Binary Coded Value	111	110	110	110	101

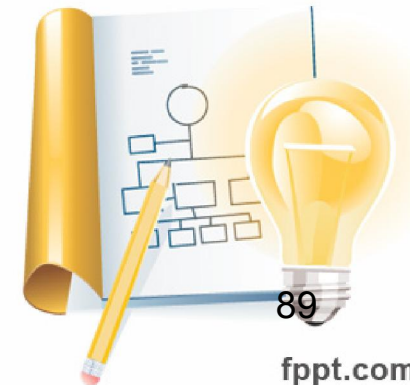
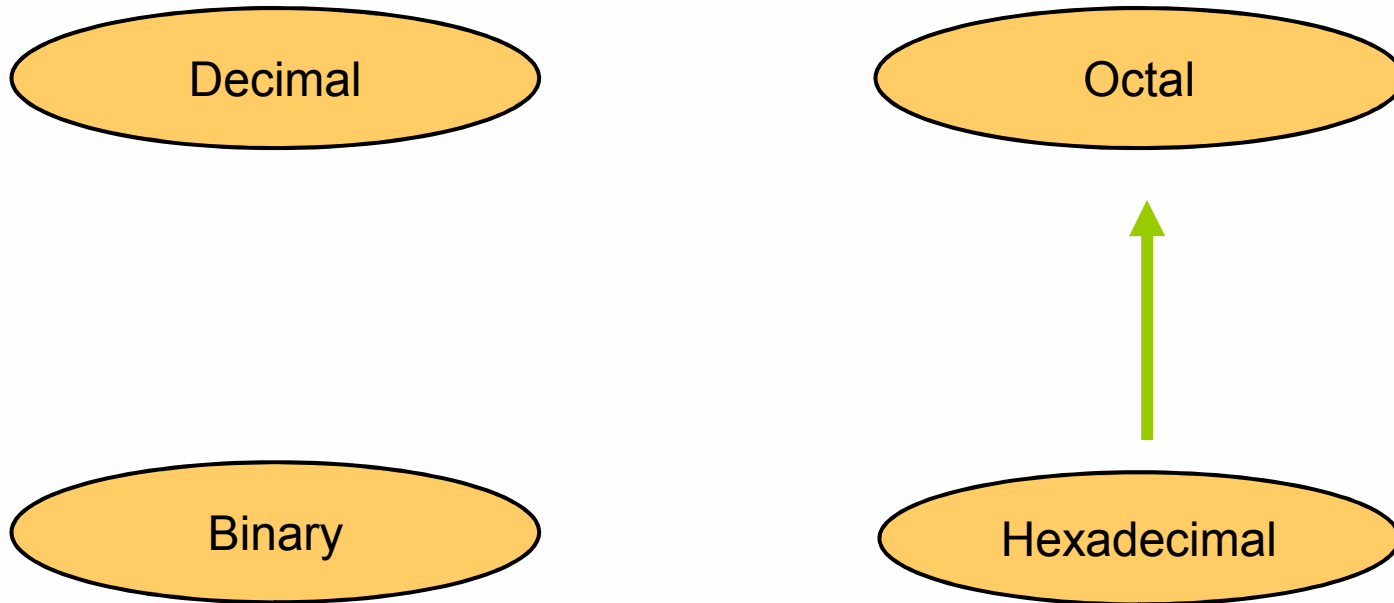
$$\begin{aligned} 76.665_8 &= (111110.110110101)_2 \\ &= (0011 \ 1110 \ . \ 1101 \ 1010 \ 1000)_2 \end{aligned}$$

$$\begin{array}{ccccccc} & 3 & & 14 & & & 13 & & 10 \\ & & & & & & & & \\ 76.665_8 &= & (3E.DA8)_{16} \end{array}$$



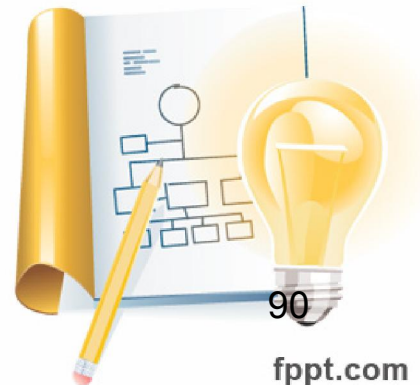


# Hexadecimal to Octal



# Hexadecimal to Octal

- Technique
  - Use binary as an intermediary



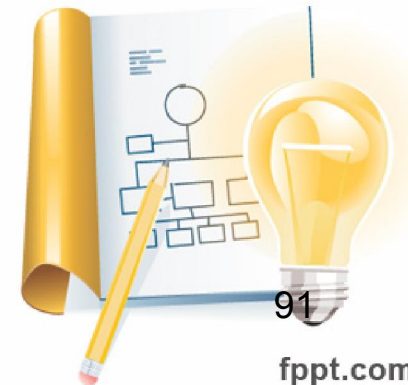
# Conversion of Hexadecimal to Octal

1. Determine the Octal equivalent of  $(2B6)_{16}$

Hexa decimal Number	2	B=11	6
Binary Coded Value	0010	1011	0110

$$\begin{aligned} 2B6_{16} &= (001010110110)_2 \\ &= (001 \ 010 \ 110 \ 110) \end{aligned}$$

$$\begin{array}{ccccccc} & & 1 & & 2 & & 6 & & 6 \\ 2B6_{16} &= & (1266)_8 \end{array}$$



# Conversion of Hexadecimal to Octal

2. Determine the Octal equivalent of  $(1F0C)_{16}$

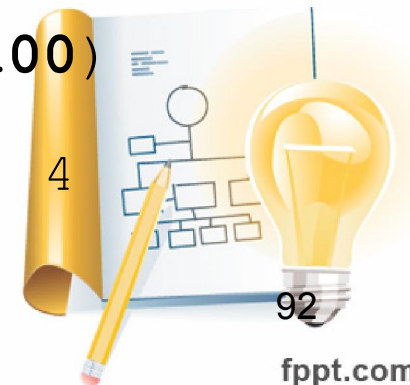
Hexa decimal Number	1	F=15	0	C=12
Binary Coded Value	0001	1111	0000	1100

$$1F0C_{16} = (0001111100001100)_2$$

$$= (000 \ 001 \ 111 \ 100 \ 001 \ 100)$$

0      1      7      4      1      4

$$1F0C_{16} = (017414)_8$$



# Conversion of Hexadecimal to Octal

3. Determine the Octal equivalent of  $(5DE247)_{16}$

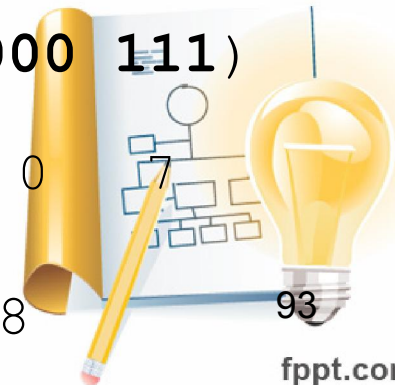
Hexa decimal Number	5	D=13	E=14	2	4	7
Binary Coded Value	0101	1101	1110	0010	0100	0111

$$5DE247_{16} = (010111011110001001000111)_2$$

$$= (010 \ 111 \ 011 \ 110 \ 001 \ 001 \ 000 \ 111)$$

2            7            3            6            1            1            0

$$5DE247_{16} = (27361107)_8$$



# Conversion of Hexadecimal Fractions to Octal Fractions

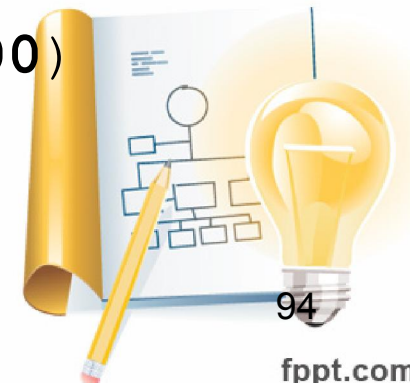
1. Determine the Octal equivalent of  $(4.3C)_{16}$

Hexa decimal Number	4	3	C=12
Binary Coded Value	0100	0011	1100

$$\begin{aligned} 4.3C_{16} &= (0100.00111100)_2 \\ &= (000\ 100\ .\ 001\ 111\ 000) \end{aligned}$$

0      4      .      1      7      0

$$4.3C_{16} = (04.170)_8$$



# Conversion of Hexadecimal Fractions to Octal Fractions

2. Determine the Octal equivalent of  $(7B.64D)_{16}$

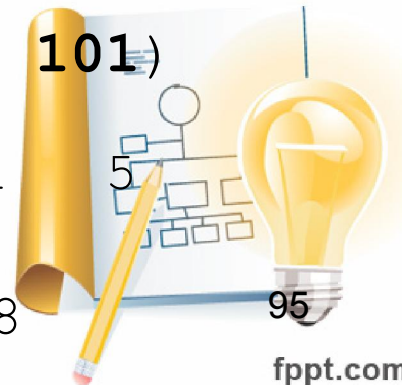
Hexa decimal Number	7	B=11	6	4	D=13
Binary Coded Value	0111	1011	0110	0100	1101

$$7B.64D_{16} = (01111011.011001001101)_2$$

$$= (001\ 111\ 011.011\ 001\ 001\ 101)$$

$$\begin{matrix} 1 & 7 & 3 & . & 3 & 1 & 1 & 5 \end{matrix}$$

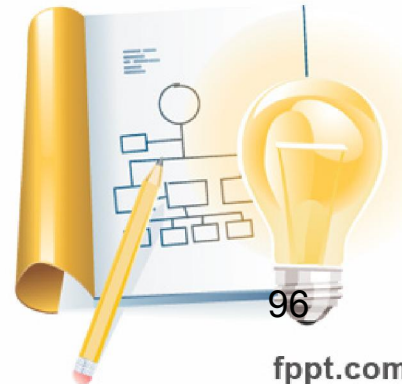
$$7B.64D_{16} = (173.3115)_8$$



# Exercise – Convert ...

Decimal	Binary	Octal	Hexa-decimal
33			
	1110101		
		703	
			1AF

Don't use a calculator!

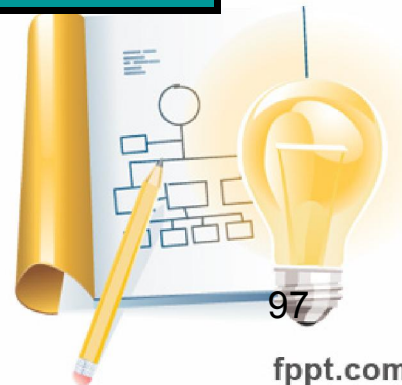




# Exercise – Convert ...

Answer

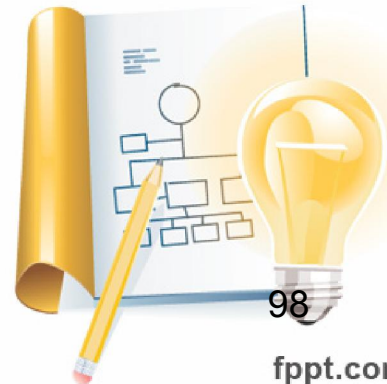
Decimal	Binary	Octal	Hexa- decimal
33	100001	41	21
117	1110101	165	75
451	111000011	703	1C3
431	110101111	657	1AF



# Exercise – Convert ...

Decimal	Binary	Octal	Hexa- decimal
29.8			
	101.1101		
		3.07	
			C.82

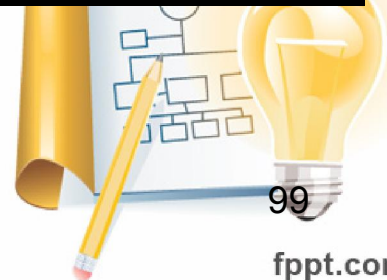
Don't use a calculator!



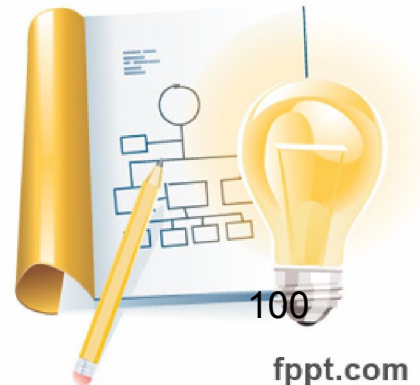
# Exercise – Convert ...

Answer

Decimal	Binary	Octal	Hexa- decimal
29.8	11101.110011...	35.63...	1D.CC...
5.8125	101.1101	5.64	5.D
3.109375	11.000111	3.07	3.1C
12.5078125	1100.10000010	14.404	C.82



# Arithmetic System



# Binary Addition

- Addition of Binary Numbers

INPUT		OUTPUT	
X	Y	SUM(S)	CARRY(C)
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1



# Binary Addition

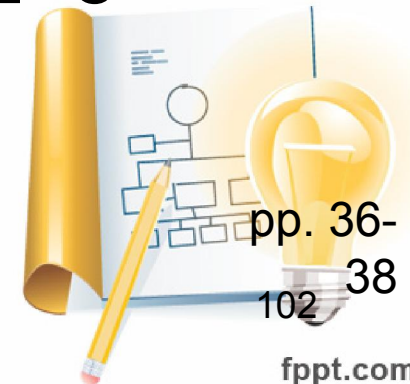
- Add binary numbers 1111 and 1010

**Binary**

$$\begin{array}{r} 111 \\ + 1111 \\ + 1010 \\ \hline 11001 \end{array}$$

**Decimal**

$$\begin{array}{r} 15 \\ + 10 \\ \hline 25 \end{array}$$



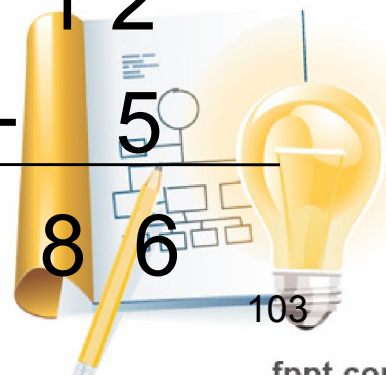
# Binary Addition

- Add binary numbers 110011, 10010, 1100 and 101

**Binary**

$$\begin{array}{r} 111111 \\ 110011 \\ 10010 \\ 1100 \\ + 101 \\ \hline 1010110 \end{array}$$

**Decimal**

$$\begin{array}{r} 1 \\ 51 \\ 18 \\ 12 \\ + 5 \\ \hline 86 \end{array}$$


103

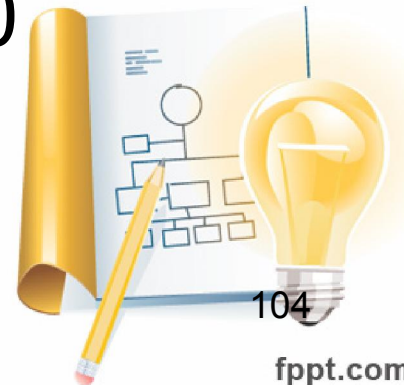
# Binary Addition

- Add binary numbers 11.10, 10.10

**Binary**

$$\begin{array}{r} 111 \\ 11.10 \\ + 10.10 \\ \hline 110.00 \end{array}$$

$$\begin{array}{r} 1 \\ 3.5 \\ 2.5 \\ \hline 6.0 \end{array}$$



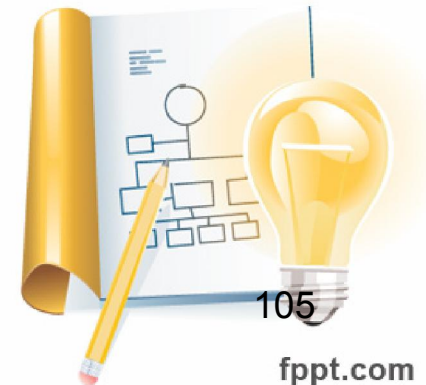


# Binary Addition

- Add binary numbers 11010.0100, 1001.01, 001.11 and 10.1010

**Binary**

$$\begin{array}{r} 11111 \\ 11010.0100 \\ 1001.01 \\ 001.11 \\ + \quad 10.1010 \\ \hline 100111.1110 \end{array}$$



# Binary Subtraction

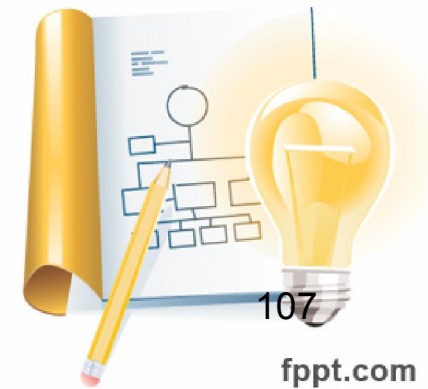
- Subtraction of Binary Numbers

INPUT		OUTPUT	
X	Y	Difference(D)	Borrow(B)
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0



# Example

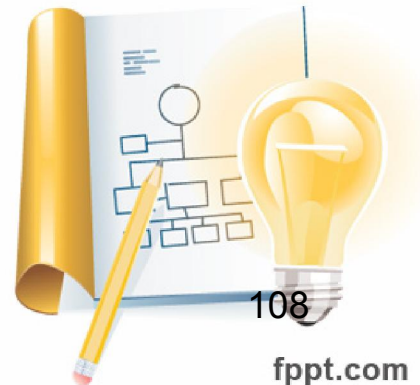
	1				
	<del>10</del>	10	0	10	
<del>1</del>	<del>0</del>	<del>0</del>	<del>1</del>	<del>0</del>	1
0	0	1	0	1	1
<hr/>					
0	1	1	0	1	0



# Example

Find the binary difference of 1101 - 10110

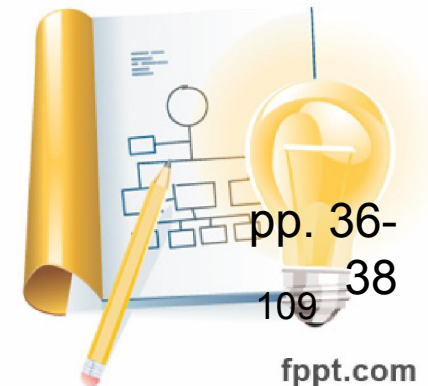
$$\begin{array}{rcccc} & & & 10 & \\ & & & \cancel{0} & \\ 1 & \cancel{1} & & 1 & \\ 1 & 0 & 1 & 1 & \\ \hline 0 & 0 & 1 & 0 & \end{array}$$



# Example

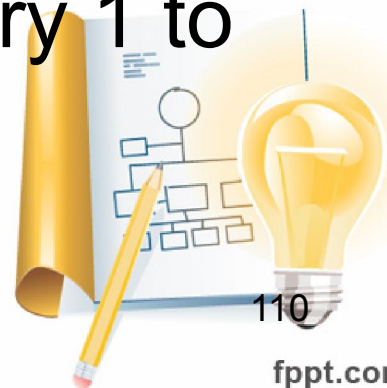
Calculate the binary difference of  
**11100011 - 10101000**

			10	1				
			<del>0</del>	<del>10</del>	10			
1	<del>1</del>	<del>1</del>	<del>0</del>	<del>0</del>	0	1	1	
1	0	1	0	1	0	0	0	
<hr/>								
0	0	1	1	1	0	1	1	



# Octal Addition

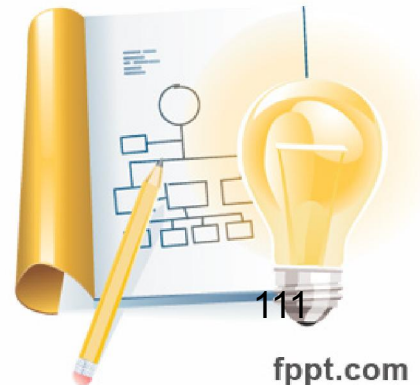
1. First, add the two digits of the unit column of the octal number in decimal.
2. During the process of addition, if the sum is less than or equal to 7, then it can be directly written as octal digit.
3. If the sum is greater than 7, then subtract 8 from that particular digit and carry 1 to the next digit position.



# Octal Addition

- Add the octal numbers 26 and 17.

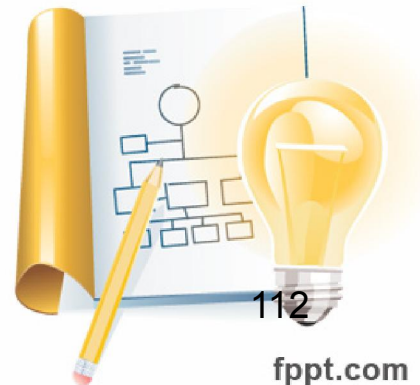
$$\begin{array}{r} 1 \\ 26 \\ + 17 \\ \hline 413 \\ - 8 \\ \hline 45 \end{array}$$



# Octal Addition

- Add the octal numbers 5647 and 1425

$$\begin{array}{r} \phantom{00}1 \phantom{000}1 \\ 5 \ 6 \phantom{00}4 \ 7 \\ + 1 \ 4 \phantom{00}2 \ 5 \\ \hline 7 \ 10 \ 7 \ 12 \\ \phantom{00}-8 \phantom{000}-8 \\ \hline 7 \ 2 \phantom{00}7 \ 4 \end{array}$$

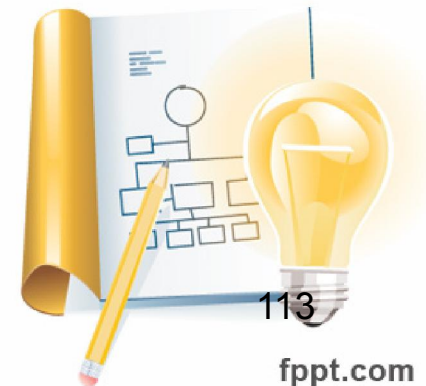




# Octal Subtraction

Subtract the octal numbers 677 from 770

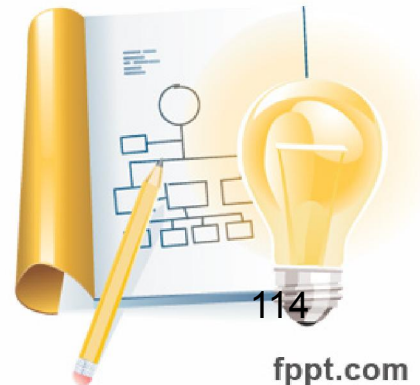
$$\begin{array}{r} \phantom{0}6 \\ \phantom{0}\cancel{7} \\ \hline \phantom{0}6 \\ 0 \end{array} \qquad \begin{array}{r} \phantom{0}8+6=14 \\ \phantom{0}\cancel{6} \\ \phantom{0}\cancel{7} \\ \phantom{0}7 \\ 7 \end{array} \qquad \begin{array}{r} \phantom{0}8 \\ \phantom{0}\cancel{0} \\ \phantom{0}7 \\ 1 \end{array}$$



# Octal Subtraction

Subtract the octal numbers 2761 from 6357

5	8+2=10	8+5=13	
<del>6</del>	<del>3</del>	<del>5</del>	7
2	7	6	1
<hr/>			
3	3	7	6



# Hexadecimal Addition

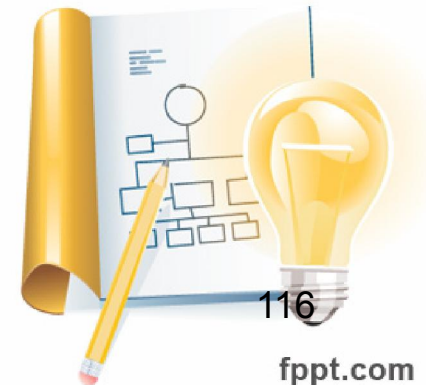
1. First, add the two digits of the unit column of the octal number in decimal.
2. During the process of addition, if the sum is less than or equal to 15, then it can be directly written as a hexadecimal digit.
3. If the sum is greater than 15, then subtract 16 from that particular digit and carry 1 to the next digit position.



# Hexadecimal Addition

- Add the hexadecimal numbers 76 and 45.

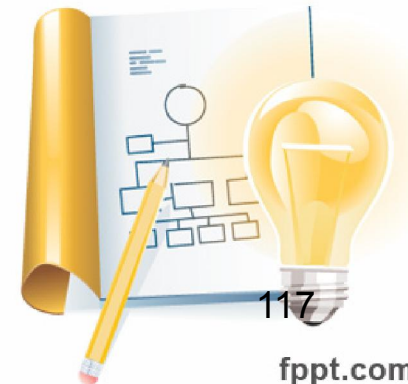
$$\begin{array}{r} 76 \\ + 45 \\ \hline 1111 \\ - - \\ \hline B B \end{array}$$



# Hexadecimal Addition

- Add the hexadecimal numbers  
A27E9 and 6FB43

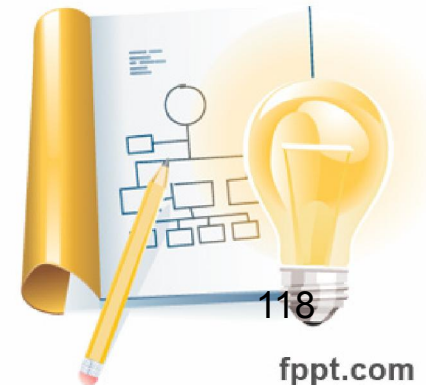
		1	1	1		
		A	2	7	E	9
	+	6	F	B	4	3
1		17	18	19	18	12
		-16	-16	-16	-16	
1		1	2	3	2	12
1		1	2	3	2	C



# Hexadecimal Subtraction

Subtract the hexadecimal numbers 75 from 527

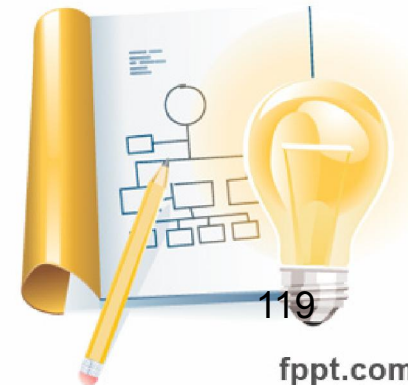
4	16+2=18	
<del>5</del>	<del>2</del>	7
	7	5
<hr/>		
4	11	2
4	B	2



# Hexadecimal Subtraction

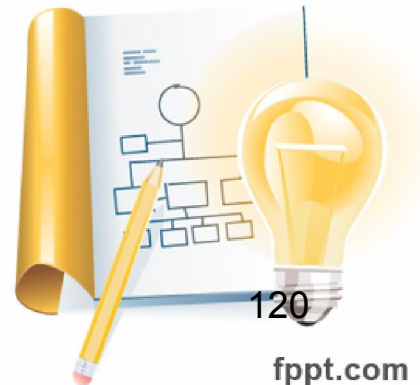
Subtract the hexadecimal numbers 1F65 from 7E2CA

	13	16+2=18		
7	<del>E</del>	<del>2</del>	C	A
	1	F	6	5
<hr/>				
7	12	3	6	5
7	C	3	6	5



# Binary Addition: Example

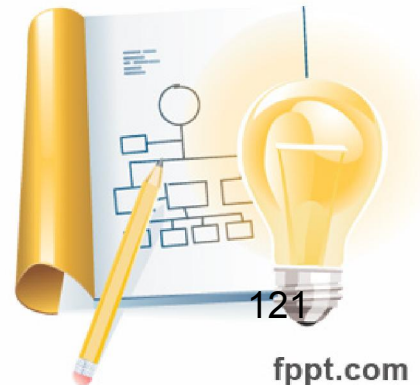
- $10001 + 11101 = ?$
- $101101 + 11001 = ?$
- $1011001 + 111010 = ?$
- $0011010 + 0001100 = ?$





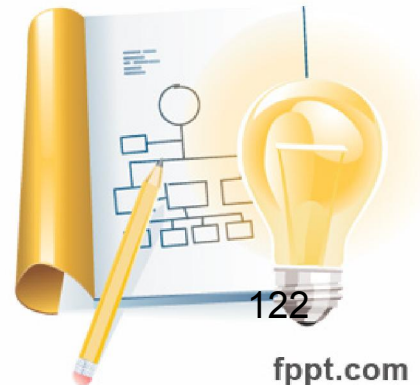
# Binary Addition: Example

- $10001 + 11101 = 101110$
- $101101 + 11001 = 1000110$
- $1011001 + 111010 = 10010011$
- $0011010 + 0001100 = 0100110$



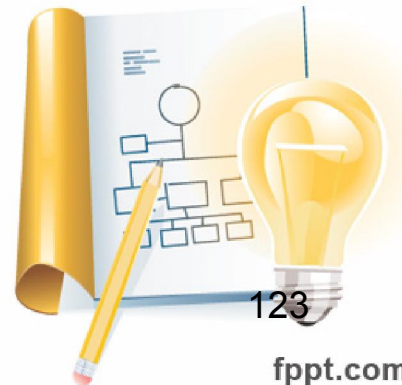
# Binary Subtraction: Example

- $1011011 - 10010 = ?$
- $1010110 - 101010 = ?$
- $100010110 - 1111010 = ?$
- $1110110 - 1010111 = ?$



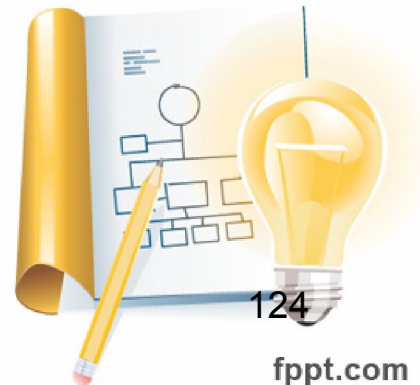
# Binary Subtraction: Example

- $1011011 - 10010 = 1001001$
- $1010110 - 101010 = 0101100$
- $100010110 - 1111010 = 010011100$
- $1110110 - 1010111 = 0011111$



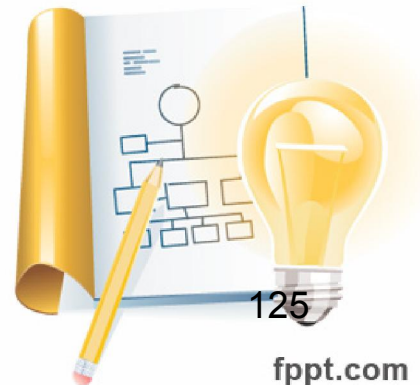
# Octal Addition: Example

- $45667 + 2341 = ?$
- $77542 + 16423 = ?$
- $211345 + 456771 = ?$



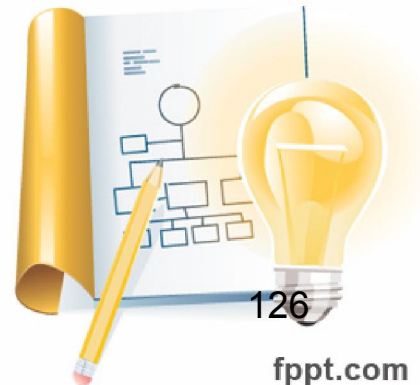
# Octal Addition: Example

- $45667 + 2341 = 50230$
- $77542 + 16423 = 116165$
- $211345 + 456771 = 670336$



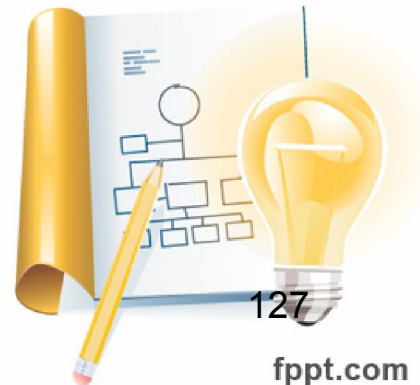
# Octal Subtraction: Example

- $76542 - 44367 = ?$
- $123457 - 44663 = ?$
- $456771 - 211345 = ?$



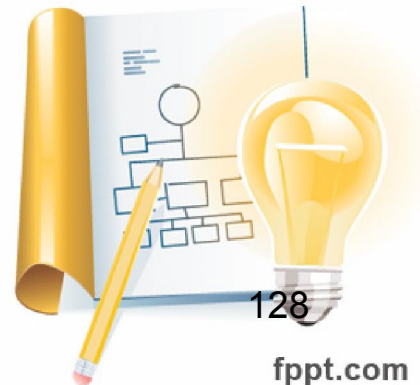
# Octal Subtraction: Example

- $76542 - 44367 = 32153$
- $123457 - 44663 = 56574$
- $456771 - 211345 = 245424$



# Hexadecimal Addition: Example

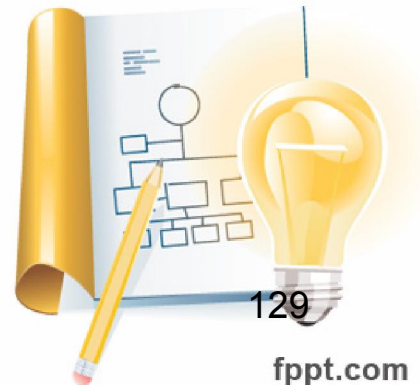
- $A89EF + 347C = ?$
- $2467 + 895A = ?$
- $1B59A + 2E3FD = ?$





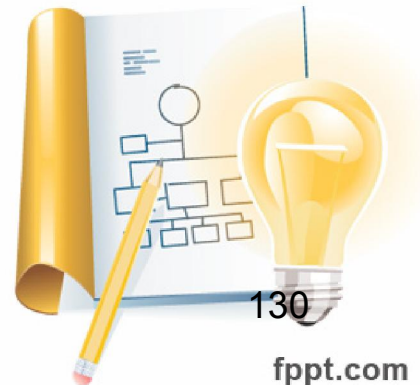
# Hexadecimal Addition: Example

- $A89EF + 347C = ABE6B$
- $2467 + 895A = ADC1$
- $1B59A + 2E3FD = 49997$



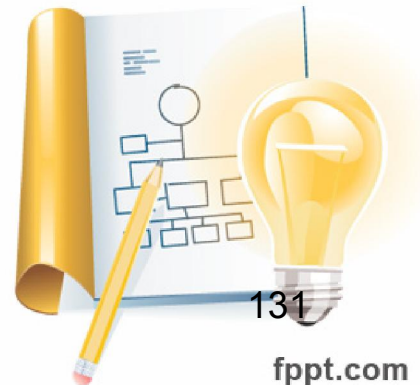
# Hexadecimal Subtraction: Example

- $E89B5 - 1FA27 = ?$
- $6B432 - 59876 = ?$
- $1B59A - 2E3D = ?$
- $ABCDEF - FEDCB = ?$



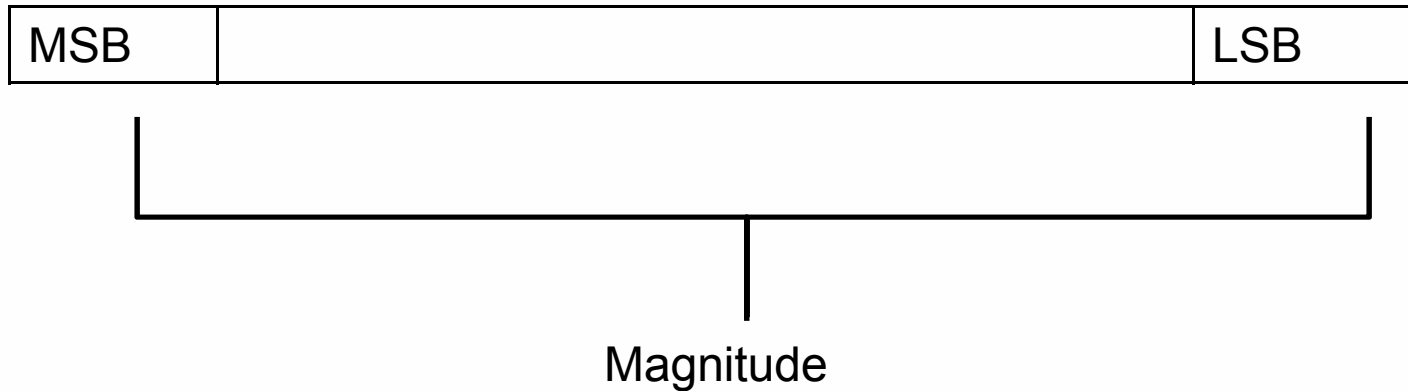
# Hexadecimal Subtraction: Example

- $E89B5 - 1FA27 = C8F8E$
- $6B432 - 59876 = 11BBC$
- $1B59A - 2E3D = 1875D$
- $ABCDEF - FEDCB = 9BE024$

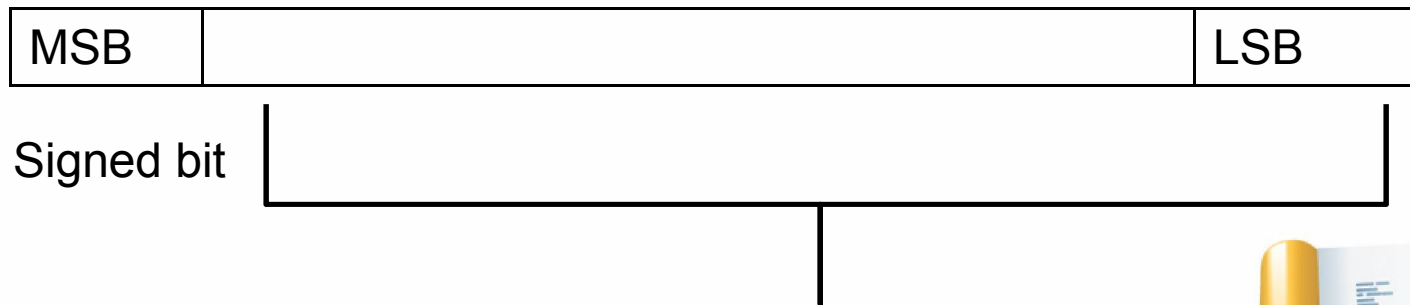


# Signed And Unsigned Numbers

UnSigned Number



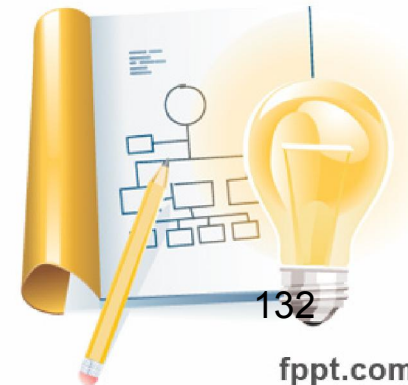
Signed Number



If MSB = 0 → Positive number

Magnitude

If MSB = 1 → Negative number



# COMPLEMENT OF NUMBERS

Two types of complements for base R number system:

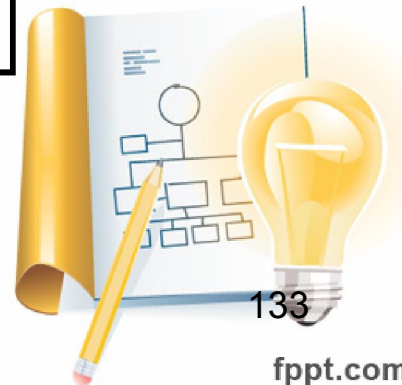
- R's complement and (R-1)'s complement

Example

- Decimal  $r=10$ , 9's complement and 10's complement
- Binary  $r=2$ , 1's and 2's complement
- Octal  $r=8$ , 7's and 8's complement

*The R's Complement*

Add 1 to the low-order digit of its (R-1)'s complement



# SIGNED NUMBERS

Need to be able to represent both *positive* and *negative* numbers

- Following 3 representations

Signed magnitude representation
Signed 1's complement representation
Signed 2's complement representation

Example: Represent +9 and -9 in 7 bit-binary number

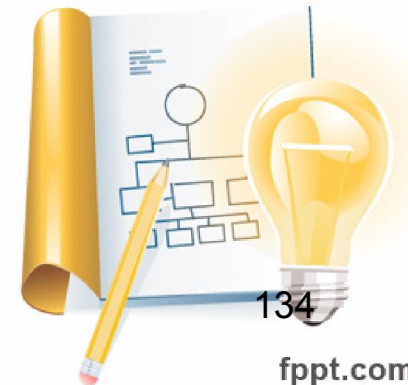
Only one way to represent +9 ==> 0 001001

Three different ways to represent -9:

In signed-magnitude: 1 001001

In signed-1's complement: 1 110110

In signed-2's complement: 1 110111



Given number →



1's complement →



Add 1 +

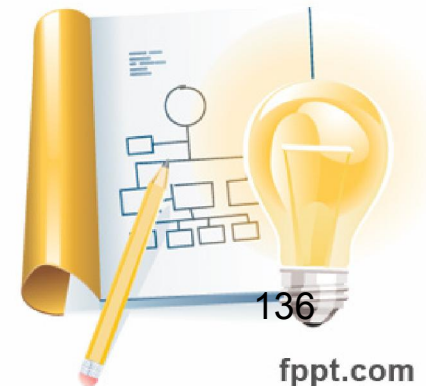


2's complement →



- (14) in 2's complement form

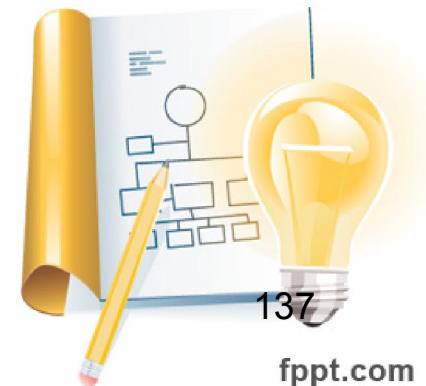
Binary Number		1	1	1	0
1's Complement		0	0	0	1
2's Complement		0	0	1	0
With Sign Bit	1	0	0	1	0





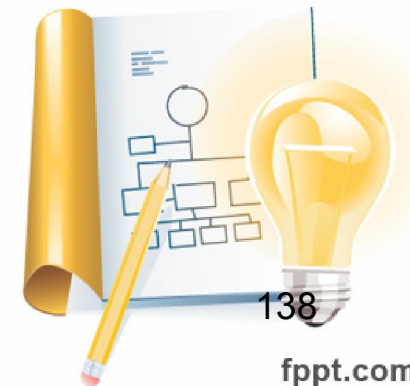
+(12) in 2's complement form

Binary Number		1	1	0	0
1's Complement		0	0	1	1
2's Complement		0	1	0	0
With Sign Bit	0	0	1	0	0



## Addition-Subtraction of signed number using 2's complement

1. Convert both numbers to equivalent binary form
2. Find the 2's complement of subtrahend
3. Add this 2's complement number to the minuend
4. If there is carry of 1, ignore it from the result to obtain the correct result.
5. If there is no carry, recompute the result
  - attach the negative sign to the obtained result



# Addition-Subtraction of signed number using 2's complement

1. Add  $(27)_{10}$  and  $(-11)_{10}$  using complementary representation for the negative value.

$$27 = 011011 \quad \text{and} \quad 11 = 001011$$

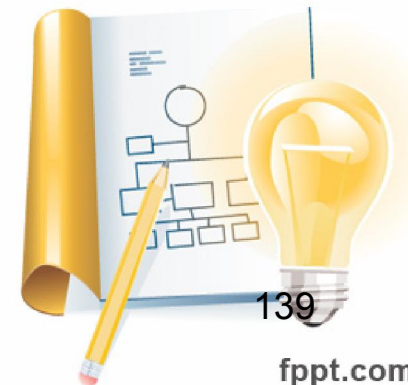
$$\begin{aligned} \text{2's complement of } (001011) &= \text{1's complement of } (001011) + 1 \\ &= 110100 + 1 \\ &= 110101 \end{aligned}$$

Add  $(011011)$  and  $(110101)$

$$\begin{array}{r} \phantom{+} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \\ \phantom{+} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \\ \phantom{+} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \\ + \phantom{0} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \\ \hline \textcircled{1} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \end{array}$$

Carry

Hence, result is  $(010000)_2$  or  $(16)_{10}$



# Addition-Subtraction of signed number using 2's complement

2. Subtract  $(25)_{10}$  from  $(42)_{10}$

$$25 = 011001 \quad \text{and} \quad 42 = 101010$$

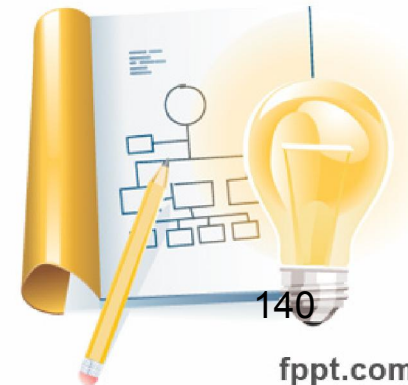
$$\begin{aligned} 2's \text{ complement of } (011001) &= 1's \text{ complement of } (011001) + 1 \\ &= 100110 + 1 \\ &= 100111 \end{aligned}$$

Add  $(101010)$  and  $(100111)$

$$\begin{array}{r} \phantom{+} \phantom{1} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \phantom{1} \\ \phantom{+} \phantom{1} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \phantom{1} \\ \phantom{+} \phantom{1} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \phantom{1} \\ + \phantom{1} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \phantom{1} \\ \hline \textcircled{1} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \phantom{1} \end{array}$$

Carry

Hence, Ignore carry and  
result is  $(010001)_2$  or  $(17)_{10}$



# Addition-Subtraction of signed number using 2's complement

3. Subtract  $(14)_{10}$  from  $(46)_{10}$

14 = 00001110      and   46 = 00101110

2's complement of (00001110) = 1's complement of (00001110) + 1

$$= 11110001 + 1$$

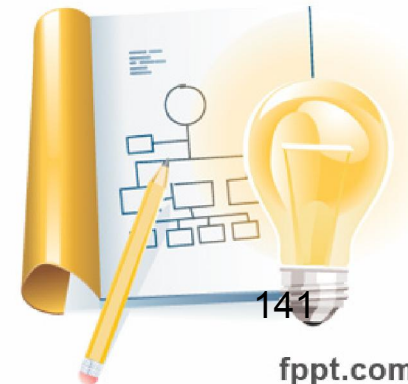
$$= 11110010$$

Add (00101110) and (11110010)

[illegible]

↓ Carry

Hence, Ignore carry and  
result is  $(00100000)_2$  or  $(32)_{10}$



# Addition-Subtraction of signed number using 2's complement

## 4. Subtract 84 from 68 ( $68 - 84$ )

binary of 84 = 1010100

binary of 68 = 1000100

1's complement of 84 = 0101011

2's complement of 84 =

$$\begin{array}{r} 0101011 \\ + \quad \quad 1 \\ \hline 0101100 \end{array}$$

Now, add  $1000100 + 0101100 = 1110000$

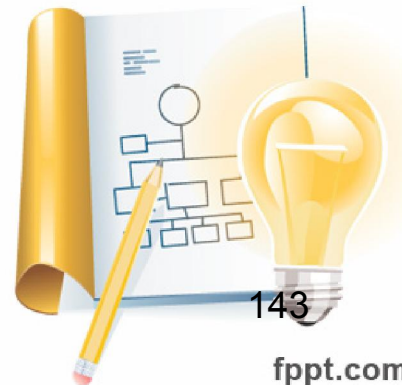
There is no carry, so we will take 2's complement of result

Answer : - 0010000



# Two's Complement Overflow Rules

- The rules for detecting overflow in a two's complement sum are simple:
  1. If the sum of two positive numbers yields a negative result, the sum has overflowed.
  2. If the sum of two negative numbers yields a positive result, the sum has underflowed.



# Overflow

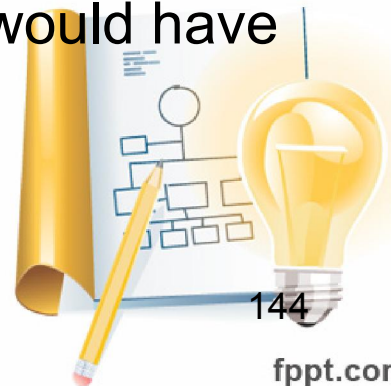
- **Example:** If we add two positive number  $7 + 6$  using 4-bit binary number, result should be  $+13$

$$\begin{array}{r} 0111 \text{ (7)} \\ + 0110 \text{ (6)} \\ \hline 1101 \end{array}$$

In signed notation, this is a result of  $-3$ , not  $+13$

(because in 4 bit binary system, 4<sup>th</sup> bit represent sign bit and only 3 bits represent magnitude of the number.

Therefore, an overflow has occurred, where result would have more bits than the original numbers.





# Underflow

- Example:** If we add two negative numbers -29 and -13 using 8-bit binary number, result should be -42

29 = 00011101

13 = 00001101

1's of 29 = 11100010

1's of 13 = 11110010

2's of 29 = 11100011

2's of 13 = 11110011

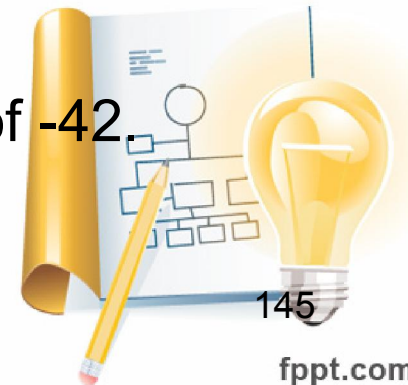
now, add 1 1 1 0 0 0 1 1

+ 1 1 1 1 0 0 1 1

1 1 1 0 1 0 1 1 0

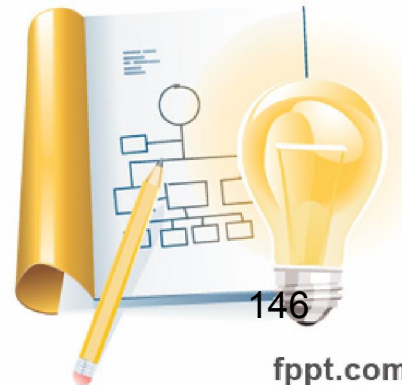
Result is 11010110

In signed notation, this is a result of +214 and not of -42.  
Therefore, an underflow has occurred.



# Cont...

- Overflow in two's complement occurs, not when a bit is carried out of the left column, but when there is a carry into the sign.
- A negative and positive added together cannot overflow, because the sum is between the addends.



- $-39 + 92 = 53$ :

$$\begin{array}{r}
 \boxed{1} \quad 1 \quad \quad 1 \quad 1 \\
 \quad 1 \quad 1 \quad 0 \quad 1 \quad 1 \quad 0 \quad 0 \quad 1 \\
 + \quad 0 \quad 1 \quad 0 \quad 1 \quad 1 \quad 1 \quad 0 \quad 0 \\
 \hline
 \quad 0 \quad 0 \quad 1 \quad 1 \quad 0 \quad 1 \quad 0 \quad 1
 \end{array}$$

Carryout without overflow. Sum is correct.

- $104 + 45 = 149$ :

$$\begin{array}{r}
 \quad 1 \quad 1 \quad \quad 1 \\
 \quad 0 \quad 1 \quad 1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \\
 + \quad 0 \quad 0 \quad 1 \quad 0 \quad 1 \quad 1 \quad 0 \quad 1 \\
 \hline
 \quad 1 \quad 0 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0 \quad 1
 \end{array}$$

Overflow, no carryout. Sum is not correct.

- $10 + -3 = 7$ :

$$\begin{array}{r}
 \boxed{1} \quad 1 \quad 1 \quad 1 \quad 1 \\
 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0 \\
 + \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 1 \\
 \hline
 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 1
 \end{array}$$

Carryout without overflow. Sum is correct.



# Examples

- $-75 + 59 = -16$ :

$$\begin{array}{cccccccc}
 & 1 & 1 & 1 & 1 & 1 & 1 & \\
 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\
 + & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\
 \hline
 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0
 \end{array}$$

No overflow nor carryout.

- $44 + 45 = 89$ :

$$\begin{array}{r}
 \phantom{+} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \\
 \phantom{+} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \\
 \phantom{+} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \\
 + \phantom{0} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \\
 \hline
 \phantom{0} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \phantom{0}
 \end{array}$$

No overflow nor carryout.

- $-1 + 1 = 0$ ;

$$\begin{array}{r}
 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \\
 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \\
 + \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \\
 \hline
 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0
 \end{array}$$

Carryout without overflow. Sum is correct.

- $127 + 1 = 128$ :

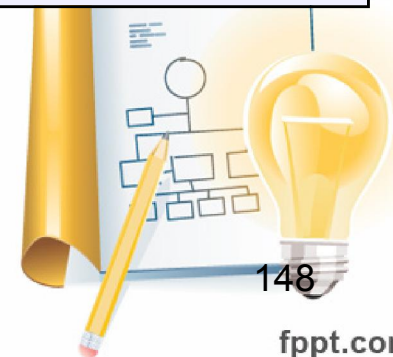
$$\begin{array}{r}
 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \\
 0 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \\
 + 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \\
 \hline
 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0
 \end{array}$$

Overflow, no carryout. Sum is not correct.

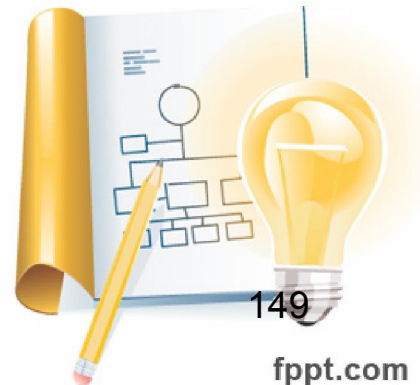
- $-103 + -69 = -172$ :

$$\begin{array}{r} 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \\ 1 \quad 0 \quad 0 \quad 1 \quad 1 \quad 0 \quad 0 \quad 1 \\ + 1 \quad 0 \quad 1 \quad 1 \quad 1 \quad 0 \quad 1 \quad 1 \\ \hline 0 \quad 1 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0 \quad 0 \end{array}$$

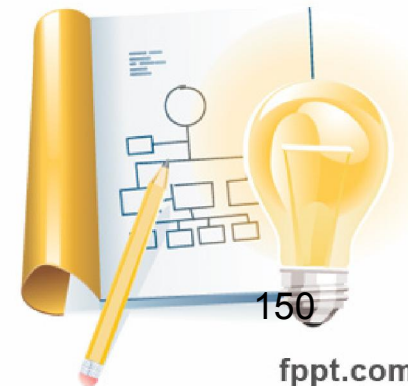
Overflow, with incidental carryout. Sum is not correct.



# Basic Logic Gates



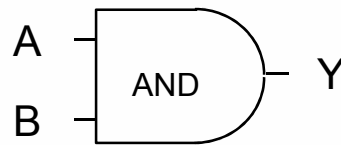
- Logic gate is an elementary building block of a digital circuit.
  - Which when combined with each other are able to perform complex logical and arithmetic operations.
- Two possible input
  - $0 = 0\text{v} = \text{False}$
  - $1 = +5\text{v} = \text{True}$



# AND Function

Text Description  $\Rightarrow$  Output Y is TRUE if inputs A AND B are TRUE, else it is FALSE.

Logic Symbol  $\Rightarrow$



Truth Table  $\Rightarrow$

INPUTS		OUTPUT
A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1
AND Gate Truth Table		

Boolean Expression  $\Rightarrow$

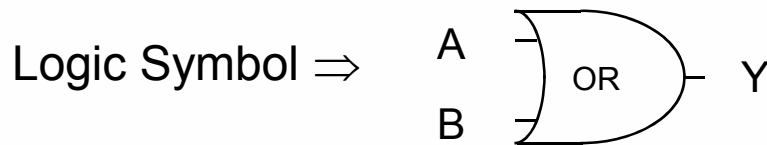
$$Y = A \times B = A \cdot B = AB$$

AND Symbol



# OR Function

Text Description  $\Rightarrow$  Output Y is TRUE if input A OR B is TRUE, else it is FALSE.



Truth Table  $\Rightarrow$

INPUTS		OUTPUT
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1
OR Gate Truth Table		

Boolean Expression  $\Rightarrow$   $Y = A + B$

$\swarrow$  OR Symbol

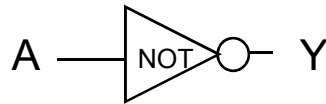




# NOT Function (inverter)

Text Description  $\Rightarrow$  Output Y is TRUE if input A is FALSE, else it is FALSE. Y is the inverse of A.

Logic Symbol  $\Rightarrow$



Truth Table  $\Rightarrow$

INPUT A	OUTPUT Y
0	1
1	0
NOT Gate Truth Table	

Boolean Expression  $\Rightarrow$

$$Y = \overline{A}$$

NOT Bar

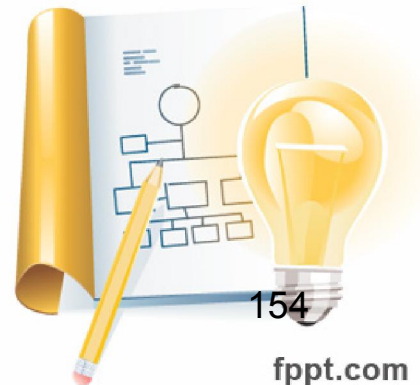
Alternative Notation

$$Y = A'$$

$$Y = !A$$

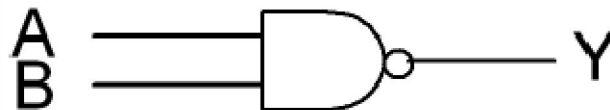
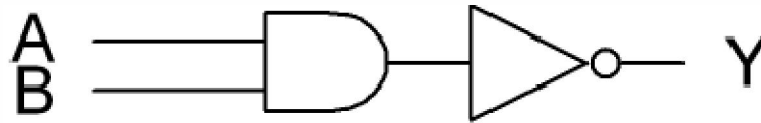


# Combination of Logic Gates



# NAND Function

- The term NAND is formed by the combination of NOT-AND
- implies an AND function with an inverted output.

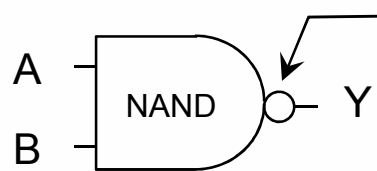


# NAND Function

Text Description  $\Rightarrow$

Output Y is FALSE if inputs A AND B are TRUE, else it is TRUE.

Logic Symbol  $\Rightarrow$



A bubble is an inverter

This is an AND Gate with an inverted output

Truth Table  $\Rightarrow$

INPUTS		OUTPUT
A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0
NAND Gate Truth Table		

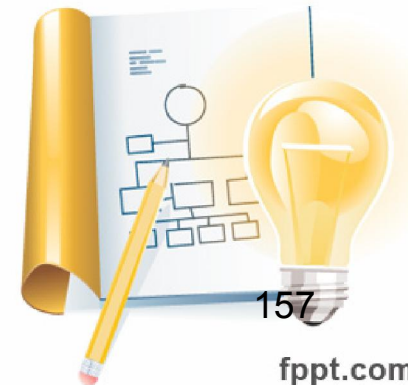
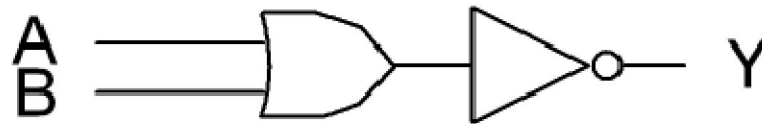
Boolean Expression  $\Rightarrow$

$$Y = \overline{A \times B} = \overline{AB}$$



# NOR Function

- The term NOR is formed by the combination of NOT-OR
- implies an OR function with an inverted output.

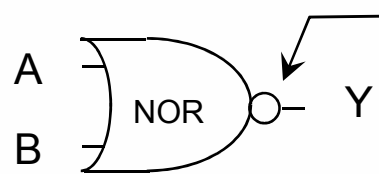


# NOR Function

Text Description  $\Rightarrow$

Output Y is FALSE if input A OR B is TRUE, else it is TRUE.

Logic Symbol  $\Rightarrow$



A bubble is an inverter.

This is an OR Gate with its output inverted.

Truth Table  $\Rightarrow$

INPUTS		OUTPUT
A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0
NOR Gate Truth Table		

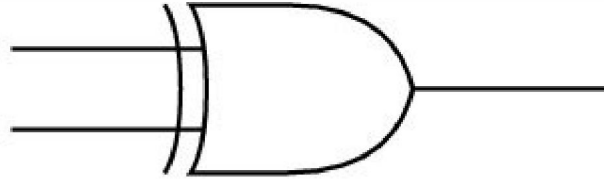
Boolean Expression  $\Rightarrow$

$$Y = \overline{A + B}$$

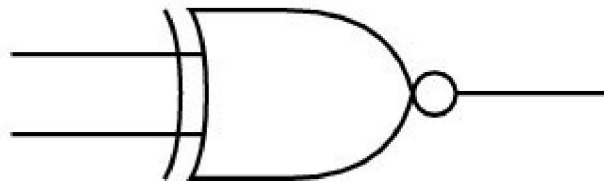


# XOR and XNOR

XOR

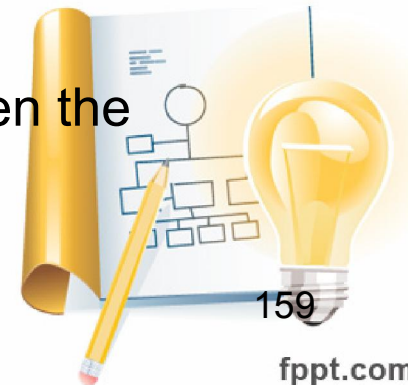


XNOR

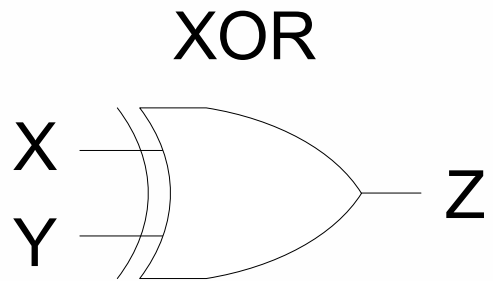


➤ XOR is an 'inequality' function → output is high(1) when the inputs are not equal to each other.

➤ XNOR is an 'equality' function → output is high(1) when the inputs are equal to each other.



# Exclusive-OR Gate



X	Y	Z
0	0	0
0	1	1
1	0	1
1	1	0

$$A \oplus B = A \cdot B' + A' \cdot B$$





# Exclusive-NOR Gate

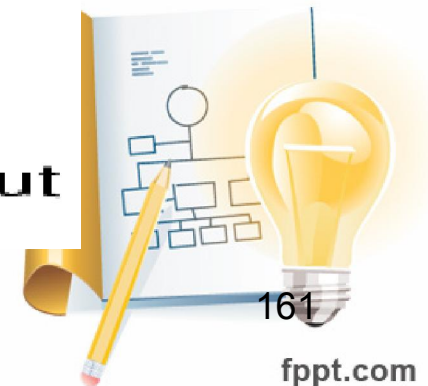
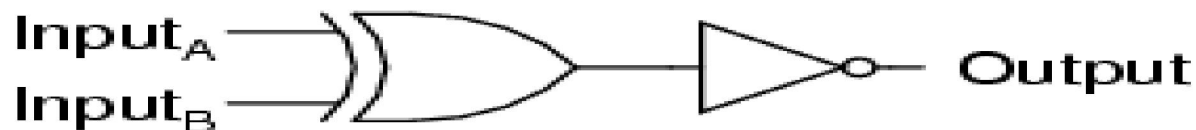
*Exclusive-NOR gate*



A	B	Output
0	0	1
0	1	0
1	0	0
1	1	1

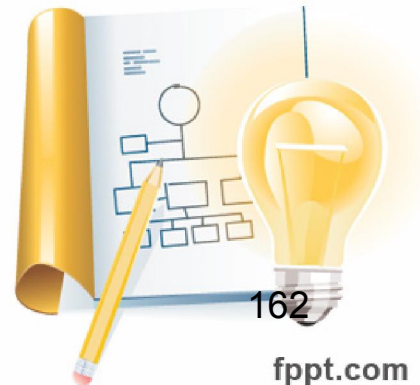
$$A \odot B = A \cdot B + A' \cdot B'$$

*Equivalent gate circuit*



# Boolean Algebra

- Boolean algebra is the mathematics of digital systems.
- It is extensively used in designing the circuitry that is used in computers.



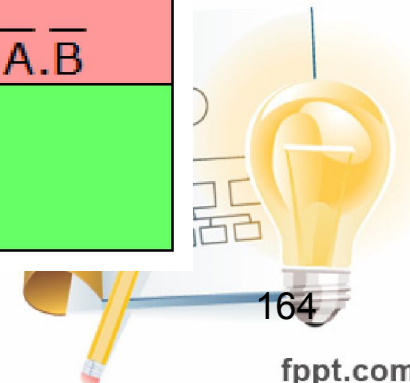
# Boolean Operations

- The **complement** is denoted by a bar. It is defined by
  - $\overline{0} = 1$  and  $\overline{1} = 0$ .
- The **Boolean sum**, denoted by + or by OR, has the following values:
  - $1 + 1 = 1$ ,  $1 + 0 = 1$ ,  $0 + 1 = 1$ ,  $0 + 0 = 0$
- The **Boolean product**, denoted by  $\cdot$  or by AND, has the following values:
  - $1 \cdot 1 = 1$ ,  $1 \cdot 0 = 0$ ,  $0 \cdot 1 = 0$ ,  $0 \cdot 0 = 0$



# Laws of Boolean Algebra

	AND Form	OR Form
Identify Law	$A.1 = A$	$A + 0 = A$
Zero and One Law	$A.0 = 0$	$A + 1 = 1$
Inverse Law	$A.\bar{A} = 0$	$A + \bar{A} = 1$
Idempotent Law	$A.A = A$	$A + A = A$
Commutative Law	$A.B = B.A$	$A + B = B + A$
Associative Law	$A.(B.C) = (A.B).C$	$A + (B + C) = (A + B) + C$
Distributive Law	$A + (B.C) = (A + B).(A + C)$	$A.(B + C) = (A.B) + (A.C)$
Absorption Law	$A(A + B) = A$	$A + AB = A$ $A + A'B = A + B$
DeMorgan's Law	$\overline{(A.B)} = \bar{A} + \bar{B}$	$\overline{(A + B)} = \bar{A}.\bar{B}$
Double Complement Law	$\overline{\bar{X}} = X$	

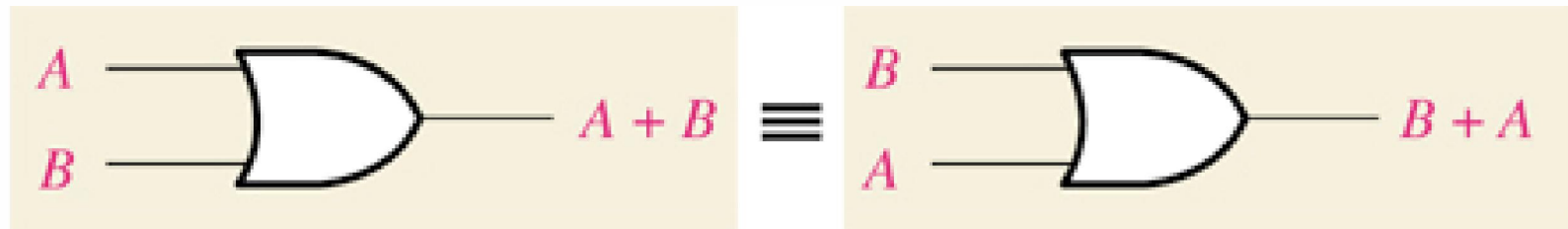


# Laws of Boolean Algebra

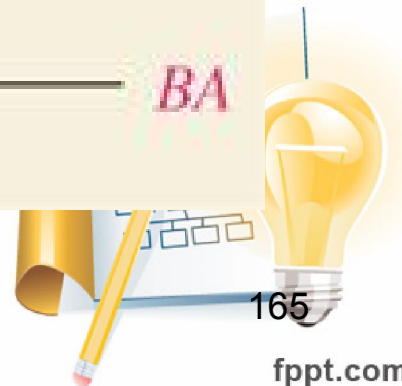
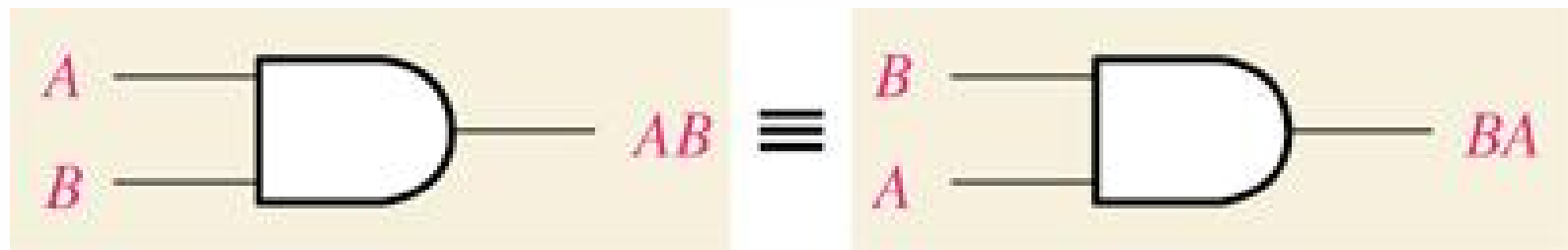
- Commutative Law

the order of literals does not matter

$$A + B = B + A$$



$$A B = B A$$

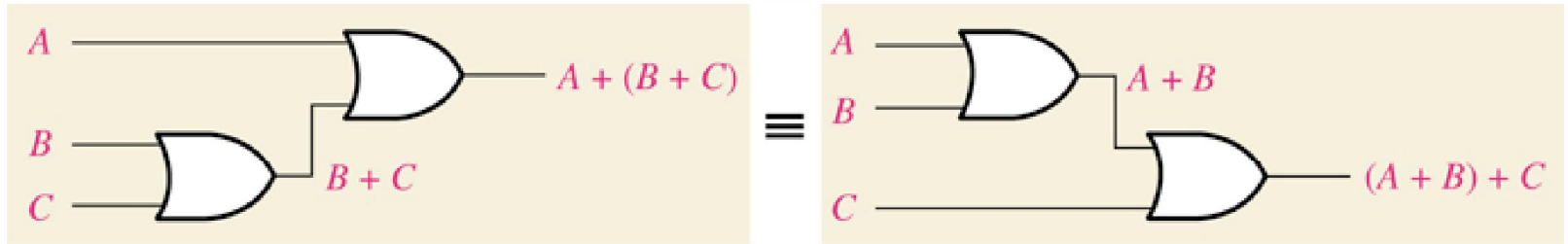


# Laws of Boolean Algebra

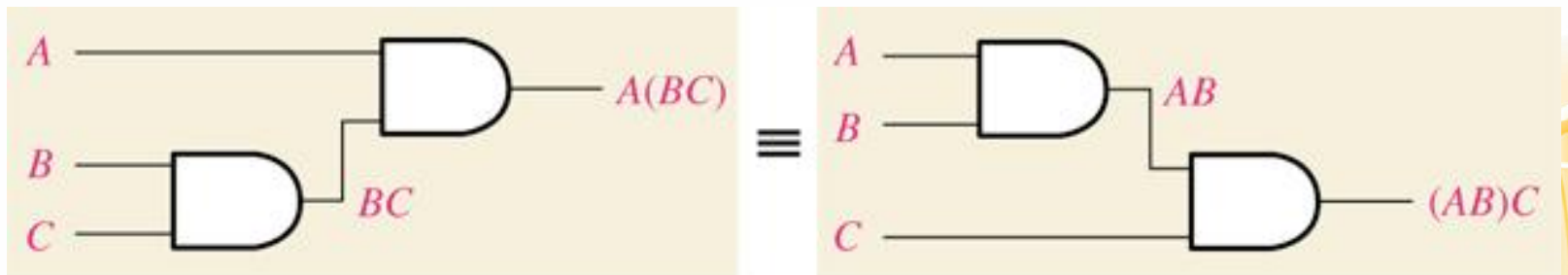
- Associative Law

the grouping of literals does not matter

$$A + (B + C) = (A + B) + C (=A+B+C)$$

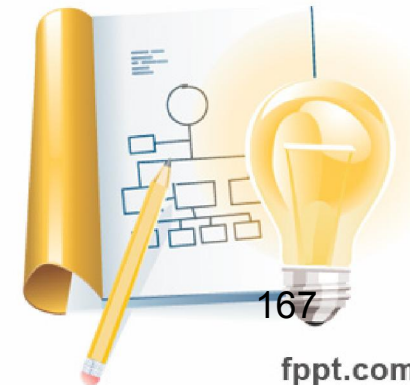


$$A(BC) = (AB)C (=ABC)$$



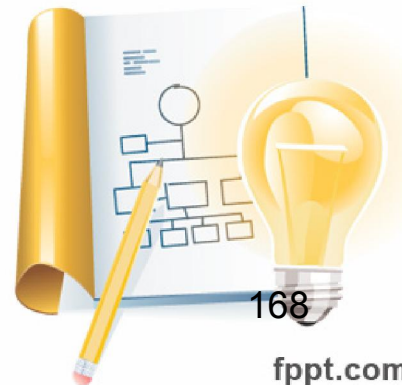
# Rules of Boolean Algebra

Rule Number	Boolean Expression
1	$A + 0 = A$
2	$A + 1 = 1$
3	$A \cdot 0 = 0$
4	$A \cdot 1 = A$
5	$A + A = A$
6	$A + A' = 1$
7	$A \cdot A = A$
8	$A \cdot A' = 0$
9	$A + AB = A$
10	$A + A'B = A + B$



## Why codes are used?

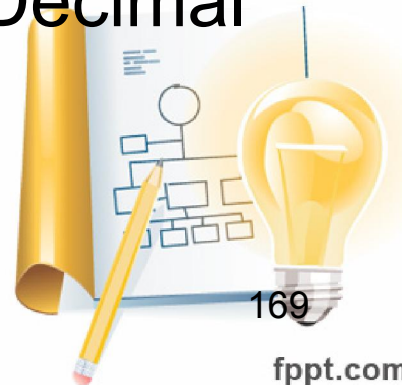
- Codes are used to represent the letters(A –Z, a-z) and special characters (such as +,-,\*,\$,&) in terms of 0's and 1's
- Every character can be represented by a combination of bits that is different from any other combination.





# Coding Systems

- BCD - Binary Coded Decimal
- ASCII - American Standard Code for Information Interchange
- EBCDIC - Extended Binary Coded Decimal Interchange Code



# Decimal and BCD

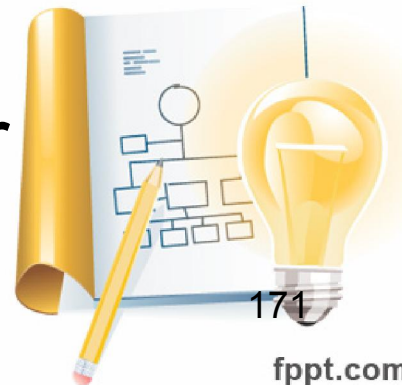
- The BCD is simply the 4 bit representation of the decimal digit.
- For multiple digit base 10 numbers, each symbol is represented by its BCD digit
- E.X.

5	3	1	9
0101	0011	0001	1001

Decimal Symbol	BCD Digit
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

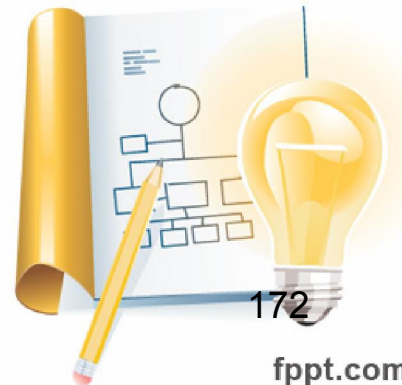


- BCD is fastest way to convert numbers from decimal to binary.
- However, it can represent only 16, ( $2^4$ ) symbols.
- The later version of BCD used a 6-bit code, which allows representing a max. of 64 that is  $2^6$  symbols.
- However, it is also not sufficient for modern computers.



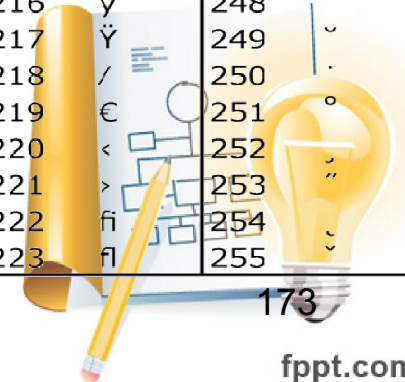
# ASCII

- It stands for American Standard Code for Information Interchange
- It is widely used in micro computers, data transmission.
- Code was originally designed as 7 bit code
- Later on, IBM developed a new version of ASCII called as ASCII-8.
  - They use of all 8 bits providing 256 symbols



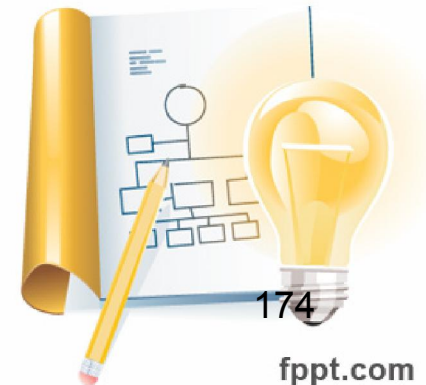
# ASCII

0	<NUL>	32	<SPC>	64	@	96	`	128	Ä	160	†	192	ì	224	‡
1	<SOH>	33	!	65	A	97	a	129	Å	161	°	193	í	225	·
2	<STX>	34	"	66	B	98	b	130	Ç	162	¢	194	î	226	,
3	<ETX>	35	#	67	C	99	c	131	É	163	£	195	ï	227	‘
4	<EOT>	36	\$	68	D	100	d	132	Ñ	164	§	196	ƒ	228	”
5	<ENQ>	37	%	69	E	101	e	133	Ö	165	•	197	≈	229	‰
6	<ACK>	38	&	70	F	102	f	134	Ü	166	¶	198	Δ	230	Â
7	<BEL>	39	'	71	G	103	g	135	á	167	β	199	«	231	Á
8	<BS>	40	(	72	H	104	h	136	à	168	®	200	»	232	Ê
9	<TAB>	41	)	73	I	105	i	137	â	169	©	201	...	233	È
10	<LF>	42	*	74	J	106	j	138	ä	170	™	202		234	Í
11	<VT>	43	+	75	K	107	k	139	ã	171	‘	203	À	235	Î
12	<FF>	44	,	76	L	108	l	140	å	172	”	204	Ã	236	Ï
13	<CR>	45	-	77	M	109	m	141	ç	173	≠	205	Ö	237	Ì
14	<SO>	46	.	78	N	110	n	142	é	174	Æ	206	Œ	238	Ó
15	<SI>	47	/	79	O	111	o	143	è	175	Ø	207	œ	239	Ô
16	<DLE>	48	0	80	P	112	p	144	ê	176	∞	208	-	240	•
17	<DC1>	49	1	81	Q	113	q	145	ë	177	±	209	—	241	Ò
18	<DC2>	50	2	82	R	114	r	146	í	178	≤	210	”	242	Ú
19	<DC3>	51	3	83	S	115	s	147	ì	179	≥	211	”	243	Û
20	<DC4>	52	4	84	T	116	t	148	î	180	¥	212	‘	244	Ü
21	<NAK>	53	5	85	U	117	u	149	ï	181	μ	213	’	245	ı
22	<SYN>	54	6	86	V	118	v	150	ñ	182	ð	214	÷	246	ˆ
23	<ETB>	55	7	87	W	119	w	151	ó	183	Σ	215	◊	247	˜
24	<CAN>	56	8	88	X	120	x	152	ò	184	Π	216	ÿ	248	—
25	<EM>	57	9	89	Y	121	y	153	ô	185	π	217	ÿ	249	˘
26	<SUB>	58	:	90	Z	122	z	154	ö	186	ƒ	218	/	250	˙
27	<ESC>	59	;	91	[	123	{	155	õ	187	ª	219	€	251	˚
28	<FS>	60	<	92	\	124		156	ú	188	º	220	<	252	¸
29	<GS>	61	=	93	]	125	}	157	ù	189	Ω	221	>	253	ˆ
30	<RS>	62	>	94	^	126	~	158	û	190	æ	222	fi	254	˜
31	<US>	63	?	95	_	127	<DEL>	159	ü	191	ø	223	fi	255	˘



- Determine the binary code of 'word' in the ASCII form.

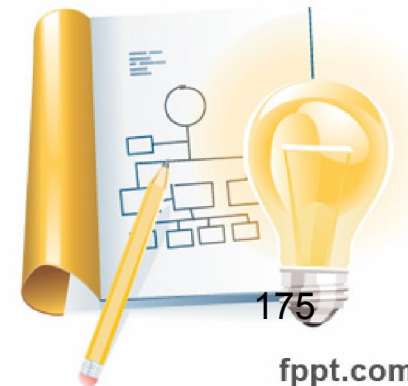
• w	o	r	d
119	111	114	100
01110111	01101111	01110010	01100110



# EBCDIC

- Extended Binary Coded Decimal Interchange Code uses 8 bits for each character.
  - Provides 256 different unique code

Character	Zone bits
A – I	1100
J – R	1101
S – Z	1110
0 - 9	1111
a – i	1000
j – r	1001
s – z	1010



## ALPHABETIC CHARACTERS

UPPERCASE				LOWERCASE			
PRINTS AS	EBCDIC			PRINTS AS	EBCDIC		
	IN BINARY	IN HEXA- DECIMAL			IN BINARY	IN HEXA- DECIMAL	
	<u>1111</u> 8421				<u>1111</u> 8421		
A	1100	0001	C 1	a	1000	0001	8 1
B	1100	0010	C 2	b	1000	0010	8 2
C	1100	0011	C 3	c	1000	0011	8 3
D	1100	0100	C 4	d	1000	0100	8 4
E	1100	0101	C 5	e	1000	0101	8 5
F	1100	0110	C 6	f	1000	0110	8 6
G	1100	0111	C 7	g	1000	0111	8 7
H	1100	1000	C 8	h	1000	1000	8 8
I	1100	1001	C 9	i	1000	1001	8 9
J	1101	0001	D 1	j	1001	0001	9 1
K	1101	0010	D 2	k	1001	0010	9 2
L	1101	0011	D 3	l	1001	0011	9 3
M	1101	0100	D 4	m	1001	0100	9 4
N	1101	0101	D 5	n	1001	0101	9 5
O	1101	0110	D 6	o	1001	0110	9 6
P	1101	0111	D 7	p	1001	0111	9 7
Q	1101	1000	D 8	q	1001	1000	9 8
R	1101	1001	D 9	r	1001	1001	9 9
S	1110	0010	E 2	s	1010	0010	A 2
T	1110	0011	E 3	t	1010	0011	A 3
U	1110	0100	E 4	u	1010	0100	A 4
V	1110	0101	E 5	v	1010	0101	A 5
W	1110	0110	E 6	w	1010	0110	A 6
X	1110	0111	E 7	x	1010	0111	A 7
Y	1110	1000	E 8	y	1010	1000	A 8
Z	1110	1001	E 9	z	1010	1001	A 9

## NUMERIC CHARACTERS

0	1111	0000	F 0	0	1111	0101	F 5
1	1111	0001	F 1	1	1111	0110	F 6
2	1111	0010	F 2	2	1111	0111	F 7
3	1111	0011	F 3	3	1111	1000	F 8
4	1111	0100	F 4	4	1111	1001	F 9



# EBCDIC vs. ASCII

Character	D	P	3
EBCDIC	1100 0100	1101 0111	1111 0011
ASCII	0100 0100	0101 0000	0011 0011

