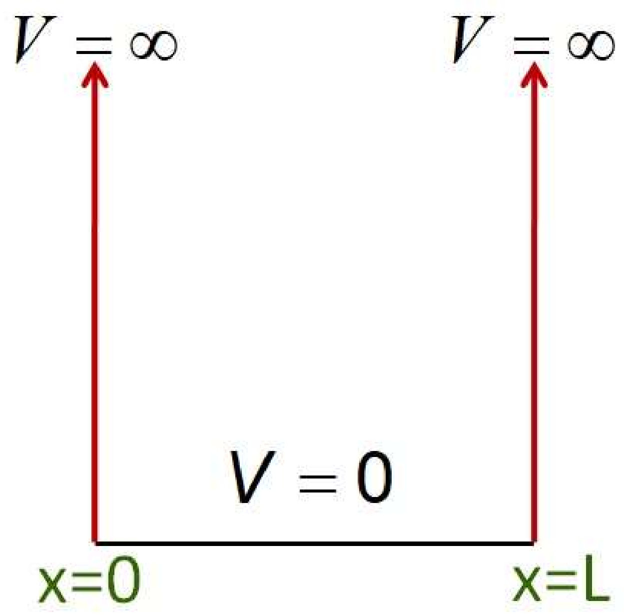
Wave function: Superposition, Measurement and Interpretation

Particle in a Finite Square Box

Particle in a Box

$$V(x) = 0 \quad for \quad 0 \le x \le L$$
$$= \infty \quad for \quad x < 0 \text{ or } x > L$$



$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L} x\right) \quad for \quad 0 \le x \le L$$
$$= 0 \qquad elsewhere$$

$$E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2} \qquad n \ge 1$$

Time dependent wave function $\Psi_n(x,t) = \psi_n(x) e^{-iE_n t/\hbar}$

Superposition of wave functions

Take linear combination of all wave functions ψ_n

$$\Psi(x,t) = c_1\psi_1(x)e^{-iE_1/\hbar} + c_2\psi_2(x)e^{-iE_2/\hbar} + \ldots\ldots$$

$$\therefore \Psi(x,t) = \sum_n c_n\psi_n(x)e^{-iE_n/\hbar}$$

C_n is a coefficient for ψ_n

$$P(x,t) = \Psi^*(x,t)\Psi(x,t) = \sum_l c_l^*\psi_l^*(x)e^{iE_l/\hbar}\sum_n c_n\psi_n(x)e^{-iE_n/\hbar}$$

$$= \sum_{l,n} c_l^* c_n \psi_l^*(x)\psi_n(x)e^{-i(E_n-E_l)t/\hbar}$$

Probability is a function of time.

Superposition is not a stationary state.

Consider the superposition $$\Psi(x,t)=\sum_n c_n\psi_n(x)e^{-iE_n/\hbar}$$

$$\Psi^*(x,t)\Psi(x,t)=\sum_l c_l^*\psi_l^*(x)e^{iE_l/\hbar}\sum_n c_n\psi_n(x)e^{-iE_n/\hbar}$$

Normalization demands $$\int_{-\infty}^{\infty}\Psi^*(x,t)\Psi(x,t)dx=1$$

$$\int_{-\infty}^{\infty}\Psi^*(x,t)\Psi(x,t)dx=\sum_{l,n}c_l^*c_ne^{-i(E_n-E_l)t/\hbar}\int_{-\infty}^{\infty}\psi_l^*(x,t)\psi_n(x,t)dx$$

We know that $$\int_{-\infty}^{\infty}\psi_l^*(x,t)\psi_n(x,t)dx=\delta_{\ln}$$

$$\int_{-\infty}^{\infty}\Psi^*(x,t)\Psi(x,t)dx=\sum_{l,n}c_l^*c_ne^{-i(E_n-E_l)t/\hbar}\delta_{\ln}=\sum_n\left|c_n\right|^2$$

$$\therefore\sum_n\left|c_n\right|^2=1$$

$$\Psi(x,t)=\sum_n c_n\psi_n(x)e^{-iE_n/\hbar} \qquad \sum_n |c_n|^2=1$$

$|c_n|^2$ is the probability of finding the particle with energy E_n

Energy corresponding to superposition $\Psi(x,t)$

$$\langle E\rangle=\int_{-\infty}^{\infty}\Psi^*(x,t)H\Psi(x,t)dx=\sum_{l,n}c_l^*c_ne^{-(E_n-E_l)}\int_{-\infty}^{\infty}\psi_l^*(x)H\psi_n(x)dx$$

$$\int_{-\infty}^{\infty}\psi_l^*(x)H\psi_n(x)dx=E_n\int_{-\infty}^{\infty}\psi_l^*(x)\psi_n(x)dx=E_n\delta_{\ln}$$

$$\therefore\langle E\rangle=\sum_{l,n}c_l^*c_ne^{-(E_n-E_l)t/\hbar}E_n\delta_{\ln} \qquad \therefore\langle E\rangle=\sum_n|c_n|^2E_n$$

Examples: $\psi_1(x)=\sqrt{\frac{2}{L}}\sin\left(\frac{\pi}{L}x\right)$ $\psi_4(x)=\sqrt{\frac{2}{L}}\sin\left(\frac{4\pi}{L}x\right)$

Construct superposition $\Psi(x,t)=c_1\psi_1(x)e^{-iE_1t/\hbar}+c_4\psi_4(x)e^{-iE_4t/\hbar}$

Normalization of $\Psi(x,t)$ **demands** $|c_1|^2+|c_4|^2=1$

$$\Psi(x,t)=\psi_1(x)e^{-iE_1t/\hbar}+\psi_4(x)e^{-iE_4t/\hbar}$$ ➡ ***Not normalized***

$$\Psi(x,t)=\frac{1}{\sqrt{2}}\psi_1(x)e^{-iE_1t/\hbar}+\frac{1}{\sqrt{2}}\psi_4(x)e^{-iE_4t/\hbar}$$ ➡ ***Normalized,***

Equal contents of each function

$$\Psi(x,t)=\frac{1}{\sqrt{5}}\psi_1(x)e^{-iE_1t/\hbar}+\frac{2}{\sqrt{5}}\psi_4(x)e^{-iE_4t/\hbar}$$ ➡ ***Normalized,***

Unequal contents of each function

Now calculate average energy

$$\langle E \rangle = \sum_n |c_n|^2 E_n$$

$$\Psi(x,t) = \frac{1}{\sqrt{2}} \psi_1(x) e^{-iE_1 t/\hbar} + \frac{1}{\sqrt{2}} \psi_4(x) e^{-iE_4 t/\hbar}$$

$$\Rightarrow \langle E \rangle = \frac{1}{2} E_1 + \frac{1}{2} E_4$$

$$\Psi(x,t) = \frac{1}{\sqrt{5}} \psi_1(x) e^{-iE_1 t/\hbar} + \frac{2}{\sqrt{5}} \psi_4(x) e^{-iE_4 t/\hbar}$$

$$\Rightarrow \langle E \rangle = \frac{1}{5} E_1 + \frac{4}{5} E_4$$

Wave function, Measurement and Interpretation

> *"I think I can safely say that nobody understands quantum mechanics"* —— **Richard Feynman**

- **The state of a system is represented by wave function ψ**

Wave function is the ***only thing necessary*** *to describe a system; the wave function can be used to obtain* ***any property*** *of the system.*

Suppose we measure the position of a particle.

ψ contains all information. So the ***act of measurement*** *has* ***reduced the wave function*** *to a* ***point*** *and the position is made known!*

- If a particle is in a state ψ and an ideal measurement of variable A will yield one of its eigenvalues α with probability $P(\alpha)$. The state of the system will change from ψ to ξ ($A\,\xi = \alpha\,\xi$) and

$$P(\alpha) = \left| \int_{-\infty}^{\infty} \xi^{*} \psi \, d\Omega \right|^{2}$$

(Copenhagen Interpretation)

$\boxed{\text{Measurement Device}} \Rightarrow \psi \Rightarrow \zeta$ + Measurement result

Collapse of wave function

Consider a quantum system consisting of n states of energy E_n and wave function ψ_n

$$H\psi_n = E_n \psi_n$$

Wave function of the system must contain all information

$$\Psi = \sum_n c_n \psi_n e^{-iE_n/\hbar}$$

C_n is a coefficient for ψ_n

Now we make measurement of energy

Energy Measurement Device

$$\Psi(x,t) = \sum_n c_n \psi_n(x) e^{-iE_n/\hbar}$$

$$\psi_m + E_m$$

Wave function collapse

$$P(E_m) = |c_m|^2$$

***Example:* Consider a superposition of two wave function of a particle in a box**

$$\Psi(x,t) = \frac{1}{\sqrt{5}}\psi_1(x)e^{-iE_1t/\hbar} + \frac{2}{\sqrt{5}}\psi_4(x)e^{-iE_4t/\hbar}$$

If no measurement is done on the system,

$\Psi(x,t)$ ***will continue to evolve in time.***

Now if a measurement is done at time 't' and the system is found to have energy E_4.

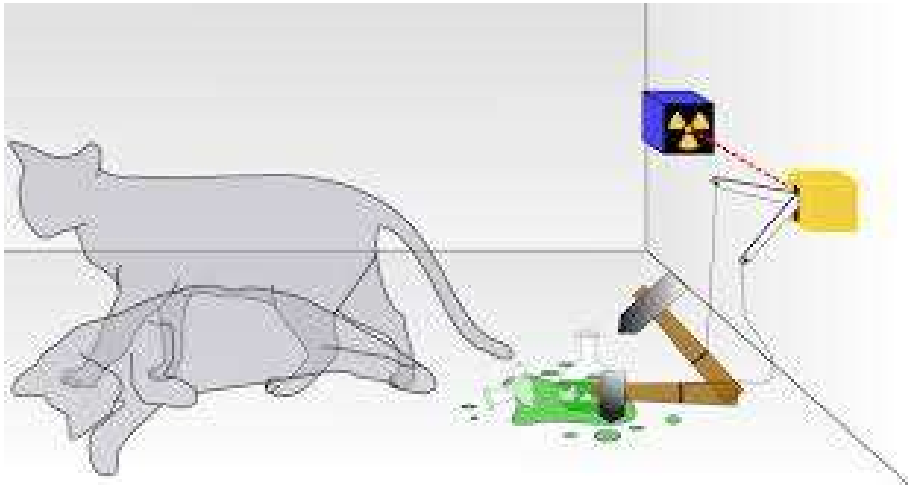
This implies that the wave function has taken the form

$$\Psi(x,t) = \frac{2}{\sqrt{5}}\psi_4(x)e^{-iE_4t/\hbar}$$

and for t'>t, it continues to evolve as $\Psi(x,t') = \frac{2}{\sqrt{5}}\psi_4(x)e^{-iE_4t'/\hbar}$

"Wave function collapse"

Schrodinger Cat Paradox

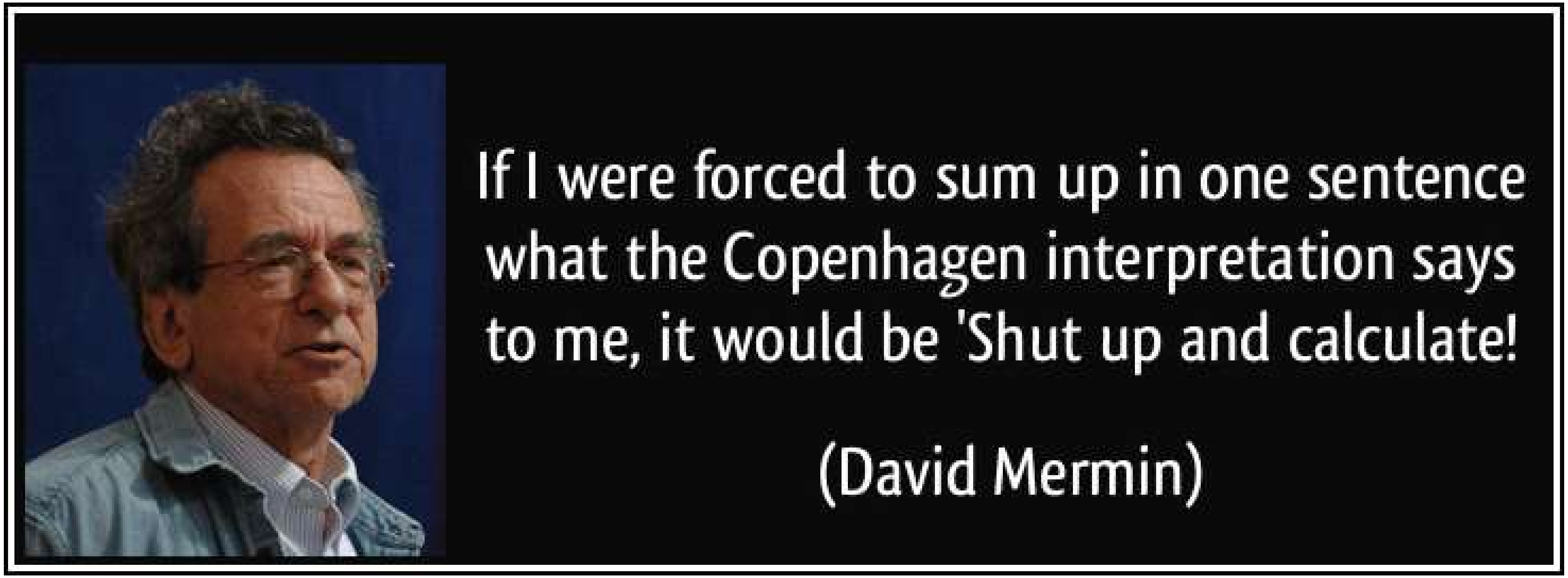


$$\psi_{cat} = \alpha\psi_{alive} + \beta\psi_{dead}$$

- Put a cat in a box containing a sealed bottle of cyanide.
- A hammer can break the bottle of cyanide, if it gets a trigger from a counter.
- The α-counter generates the trigger if it detects an α particle.
- Radioactive atoms of half life $T_{1/2}$ is kept close to the α-counter

Before making measurement, the system will evolve as per ψ_{cat}

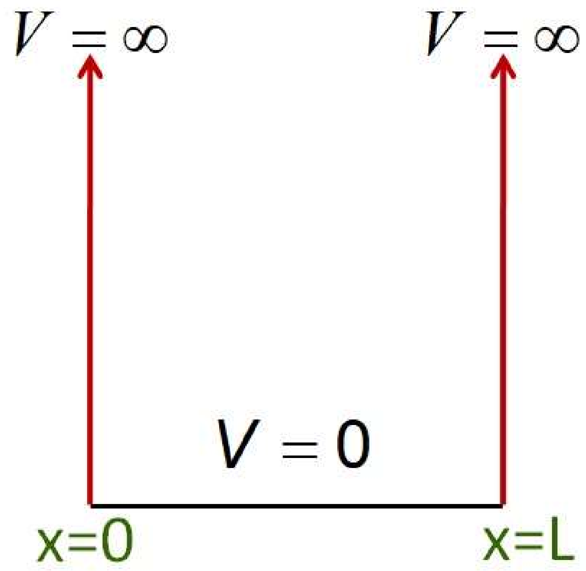
$$\boxed{\text{Act of Measurement}} \Rightarrow \psi_{cat} = \begin{matrix} \alpha\psi_{alive} \\ +\ \beta\psi_{dead} \end{matrix} \begin{matrix} \nearrow \ \psi_{alive} \\ \text{Or} \\ \searrow \ \psi_{dead} \end{matrix}$$



Shut up and calculate!

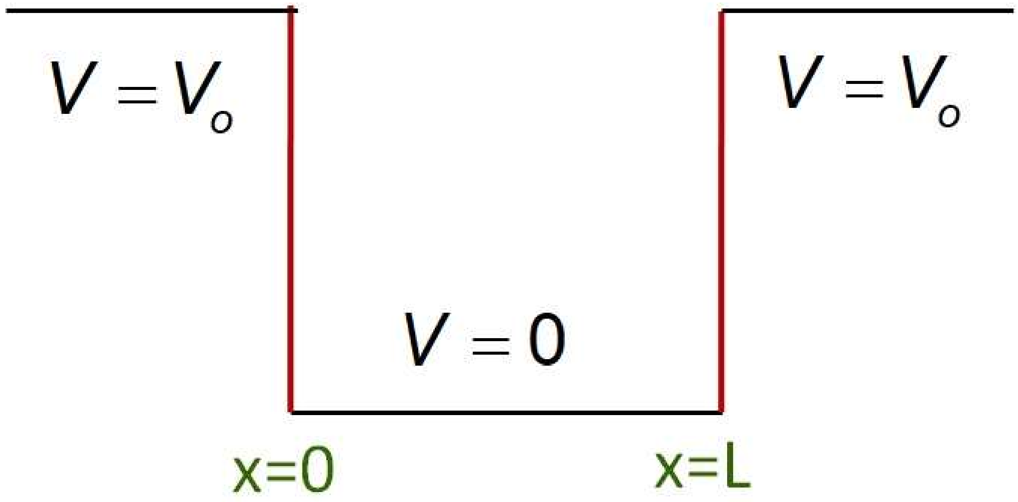
Particle in a finite box

Infinite box

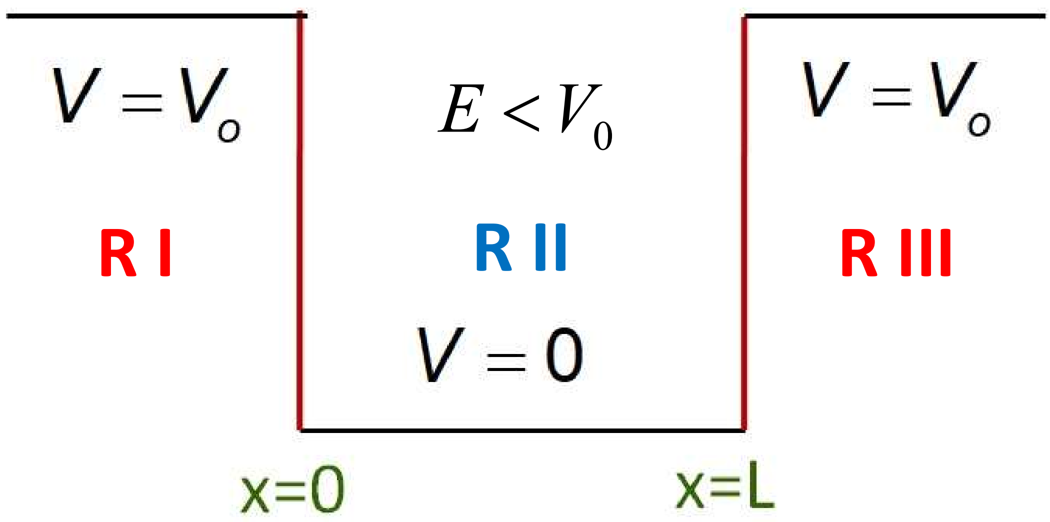


$$V(x) = 0 \quad for \quad 0 \le x \le L$$
$$= \infty \quad for \quad x < 0 \text{ or } x > L$$

Finite box



$$V(x) = 0 \quad for \quad 0 \le x \le L$$
$$= V_0 \quad for \quad x < 0 \text{ or } x > L$$



$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2}+V(x)\psi(x)$$
$$= E\psi(x)$$

R I: $$\frac{\partial^2\psi(x)}{\partial x^2} = -\frac{2m(E-V_0)}{\hbar^2}\psi(x) = \alpha^2\psi(x)$$

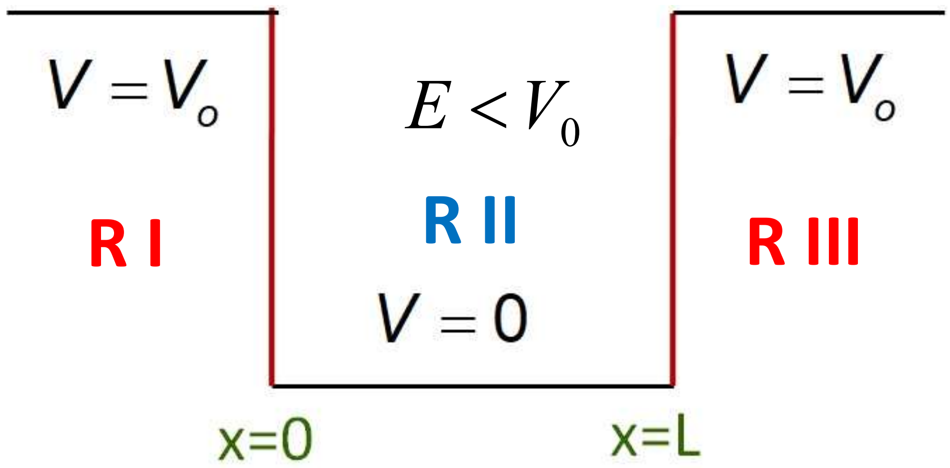
R II: $$\frac{\partial^2\psi(x)}{\partial x^2} = -\frac{2mE}{\hbar^2}\psi(x) = -k^2\psi(x)$$

R III: $$\frac{\partial^2\psi(x)}{\partial x^2} = -\frac{2m(E-V_0)}{\hbar^2}\psi(x) = \alpha^2\psi(x)$$

$$\alpha^2 = \frac{2m(V_0-E)}{\hbar^2}$$

$$V_0 > E$$

$$k^2 = \frac{2mE}{\hbar^2}$$



$$\alpha^2 = \frac{2m(V_0 - E)}{\hbar^2}$$

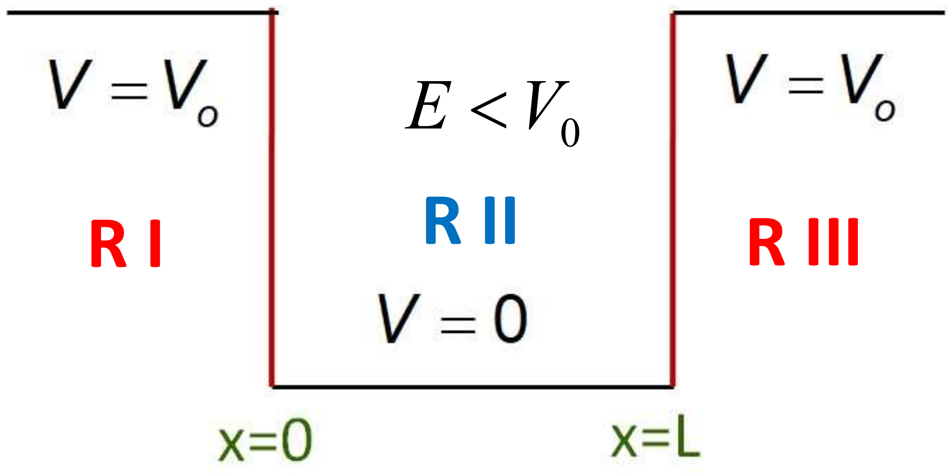
$$k^2 = \frac{2mE}{\hbar^2}$$

R I: $$\frac{\partial^2 \psi}{\partial x^2} = \alpha^2 \psi \Rightarrow \psi_1(x) = Ae^{\alpha x} + Be^{-\alpha x}$$

R II $$\frac{\partial^2 \psi}{\partial x^2} = -k^2 \psi \Rightarrow \psi_2(x) = C\sin(kx) + D\cos(kx)$$

R III $$\frac{\partial^2 \psi}{\partial x^2} = \alpha^2 \psi \Rightarrow \psi_3(x) = Ge^{\alpha x} + Fe^{-\alpha x}$$

Unknowns (7): A, B, C, D, F, G and E(energy)



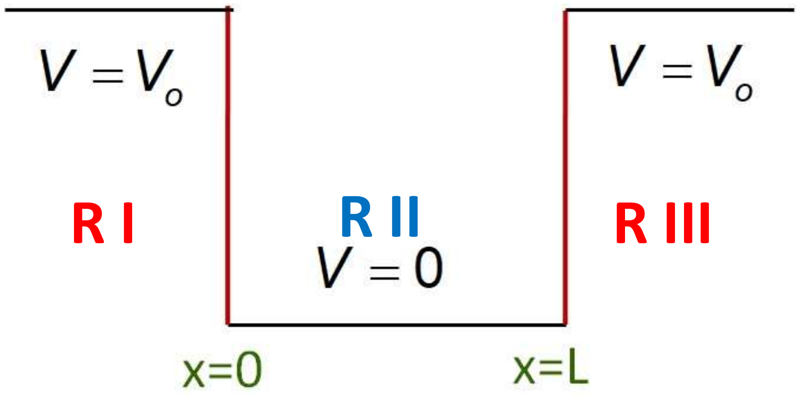
R I: $\psi_1(x) = Ae^{\alpha x} + Be^{-\alpha x}$ $\Rightarrow$ $\psi_1(x) = Ae^{\alpha x}$

$$x \to -\infty,\ e^{-\alpha x} \to \infty \Rightarrow B = 0$$

R II $\psi_2(x) = C\sin(kx) + D\cos(kx)$

R III $\psi_3(x) = Ge^{\alpha x} + Fe^{-\alpha x}$ $\Rightarrow$ $\psi_3(x) = Fe^{-\alpha x}$

$$x \to \infty,\ e^{\alpha x} \to \infty \Rightarrow G = 0$$



R I: $\psi_1(x) = Ae^{\alpha x}$

R II: $\psi_2(x) = C\sin(kx) + D\cos(kx)$

R III: $\psi_3(x) = Fe^{-\alpha x}$

Continuity at x=0

$$\psi_1(0) = \psi_2(0)$$

$$\boxed{\therefore A = D}$$

Continuity of derivative at x=0

$$\left.\frac{\partial \psi_1}{\partial x}\right|_{x=0} = \left.\frac{\partial \psi_2}{\partial x}\right|_{x=0}$$

$$\left.\alpha A e^{\alpha x}\right|_{x=0} = \left[kC\cos(kx) - kD\sin(kx)\right]_{x=0}$$

$$\boxed{\therefore \alpha A = kC}$$

Continuity at x=L

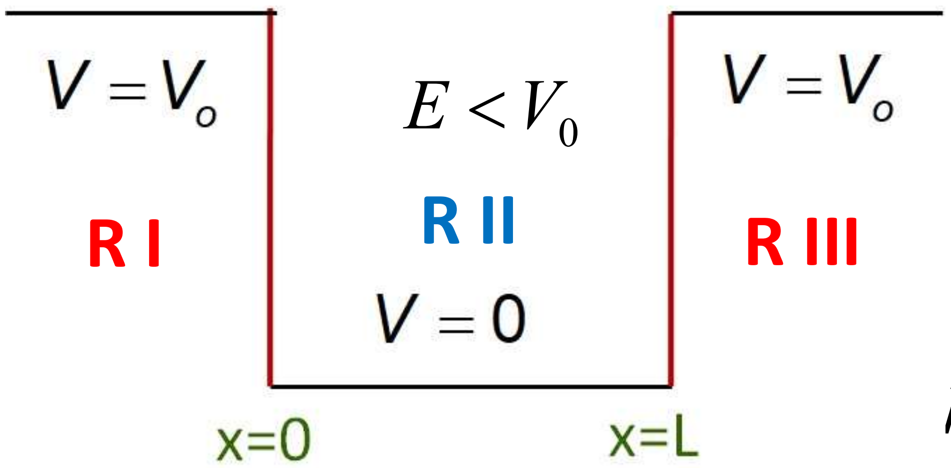
$$\psi_2(L) = \psi_3(L)$$

$$\boxed{C\sin(kL) + D\cos(kL) = Fe^{-\alpha L}}$$

Continuity of derivative at x=L

$$\left.\frac{\partial \psi_2}{\partial x}\right|_{x=0} = \left.\frac{\partial \psi_3}{\partial x}\right|_{x=0}$$

$$\boxed{kC\cos(kL) - kD\sin(kL) = -\alpha Fe^{-\alpha L}}$$



$$A = D$$

$$\alpha A = kC$$

$$C\sin(kL) + D\cos(kL) = Fe^{-\alpha L}$$

$$kC\cos(kL) - kD\sin(kL) = -\alpha Fe^{-\alpha L}$$

R I: $\psi_1(x) = Ae^{\alpha x}$ $\qquad A = D$

R II: $\psi_2(x) = C\sin(kx) + D\cos(kx)$ $\qquad C = (\alpha / k)A$

R III: $\psi_3(x) = Fe^{-\alpha x}$ $\qquad F = Ae^{\alpha L}\left[(\alpha / k)\sin(kL) + \cos(kL)\right]$

A to be determined by normalization

R I: $\psi_1(x) = Ae^{\alpha x}$ $A = D$

R II: $\psi_2(x) = C\sin(kx) + D\cos(kx)$ $C = (\alpha / k)A$

R III: $\psi_3(x) = Fe^{-\alpha x}$ $F = Ae^{\alpha L}[(\alpha / k)\sin(kL) + \cos(kL)]$

Normalization

$$\int_{-\infty}^{0} \psi_1^*(x)\psi_1(x)dx + \int_{0}^{L} \psi_2^*(x)\psi_2(x)dx + \int_{L}^{\infty} \psi_3^*(x)\psi_3(x)dx = 1$$

$$|A|^2 \int_{-\infty}^{0} e^{2\alpha x}dx + |A|^2 \int_{0}^{L} [(\alpha / k)\sin kx + \cos kx]^2 dx$$

$$+|A|^2 e^{2\alpha L}[(\alpha / k)\sin kL + \cos kL]^2 \int_{L}^{\infty} e^{-2\alpha x}dx = 1$$

So all constants are now known!

$$A = D$$

$$C = (\alpha / k)A$$

$$C\sin(kL) + D\cos(kL) = Fe^{-\alpha L}$$

$$kC\cos(kL) - kD\sin(kL) = -\alpha Fe^{-\alpha L}$$

Expressing all coefficient in terms of A

$$(\alpha / k)A\sin(kL) + A\cos(kL) = Fe^{-\alpha L} \qquad (i)$$

$$\alpha A\cos(kL) - kA\sin(kL) = -\alpha Fe^{-\alpha L} \qquad (ii)$$

$$\frac{(ii)}{(i)} \Rightarrow \frac{(\alpha / k)\cos(kL) - \sin(kL)}{(\alpha / k)\sin(kL) + \cos(kL)} = -\frac{\alpha}{k}$$

$$\frac{(\alpha/k)\cos(kL)-\sin(kL)}{(\alpha/k)\sin(kL)+\cos(kL)}=-\frac{\alpha}{k}$$

$$\frac{(\alpha/k)-\tan(kL)}{(\alpha/k)\tan(kL)+1}=-\frac{\alpha}{k}$$

$$(\alpha/k)-\tan(kL)=-(\alpha/k)^2\tan(kL)-(\alpha/k)$$

$$\left[1-(\alpha/k)^2\right]\tan(kL)=2(\alpha/k)$$

$$\tan(kL)=\frac{2\alpha k}{(k^2-\alpha^2)}$$

$$\alpha=\sqrt{2m(V_0-E)/\hbar^2}$$

$$k=\sqrt{2mE/\hbar^2}$$

Solve numerically or graphically to obtain energy E

➡ ***Quantized energy levels***

Graphical solution

$$k = \sqrt{2mE/\hbar^2}$$

$$\tan(kL) = \frac{2\alpha k}{(k^2 - \alpha^2)}$$

$$\alpha = \sqrt{2m(V_0 - E)/\hbar^2}$$

$$\tan(kL) = \frac{2\tan(kL/2)}{1 - \tan^2(kL/2)} = \frac{2(\alpha/k)}{1 - (\alpha/k)^2}$$

$$\tan(\theta) = \frac{2\tan(\theta/2)}{1 - \tan^2(\theta/2)}$$

$$\therefore \tan(kL/2) = \frac{\alpha}{k}$$

Define $k_0 = \sqrt{2mV_0/\hbar^2}$

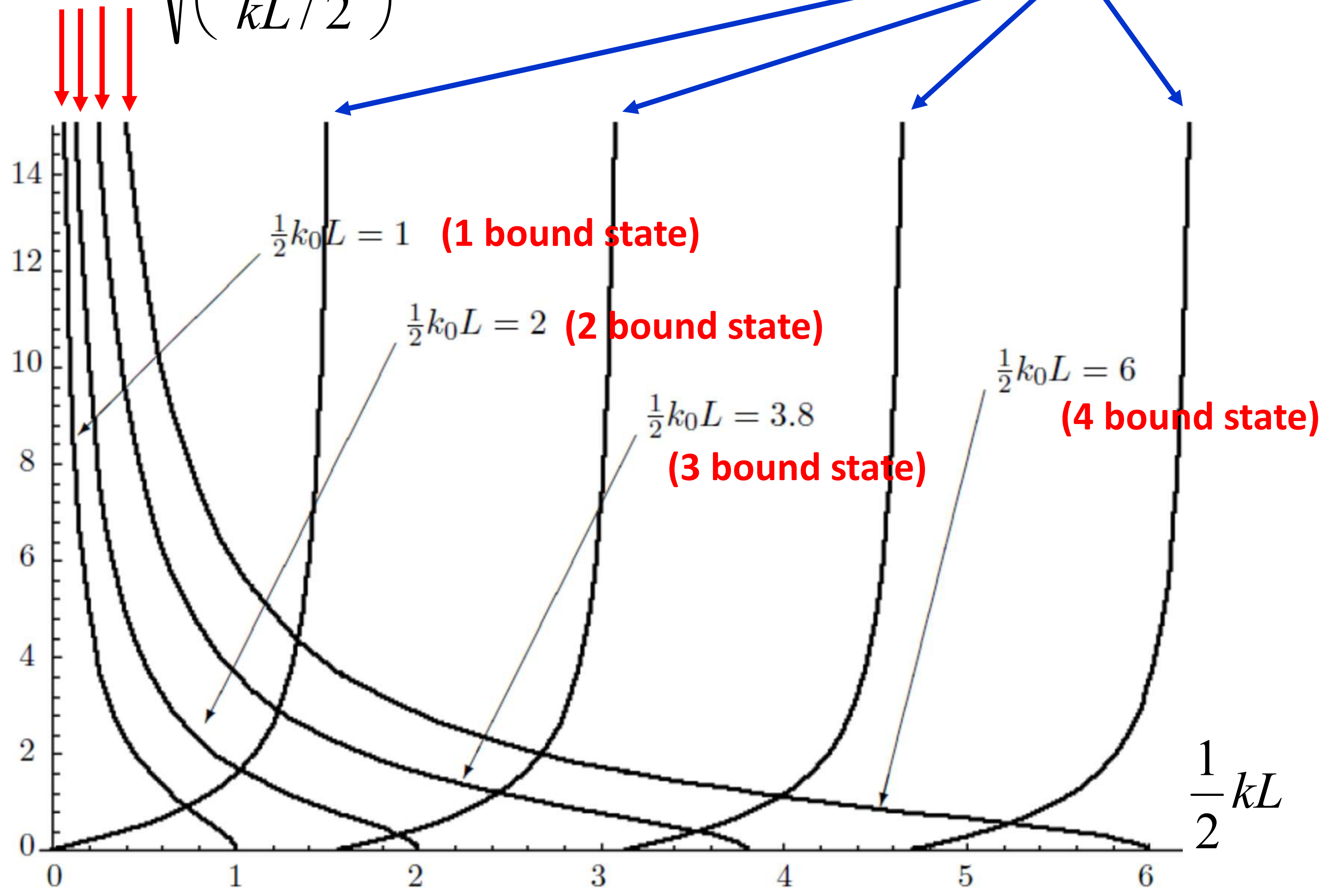
$$\therefore \frac{\alpha}{k} = \sqrt{\frac{V_0 - E}{E}} = \sqrt{\frac{V_0}{E} - 1} = \sqrt{\left(\frac{k_0}{k}\right)^2 - 1} = \sqrt{\left(\frac{k_0 L/2}{kL/2}\right)^2 - 1}$$

Plot $\tan(kL/2)$ **and** $f(kL) = \sqrt{\left(\frac{k_0 L/2}{kL/2}\right)^2 - 1}$ **vs** $kL/2$

Intersection points give the quantized energy levels

$$f(kL) = \sqrt{\left(\frac{k_0 L/2}{kL/2}\right)^2 - 1}$$

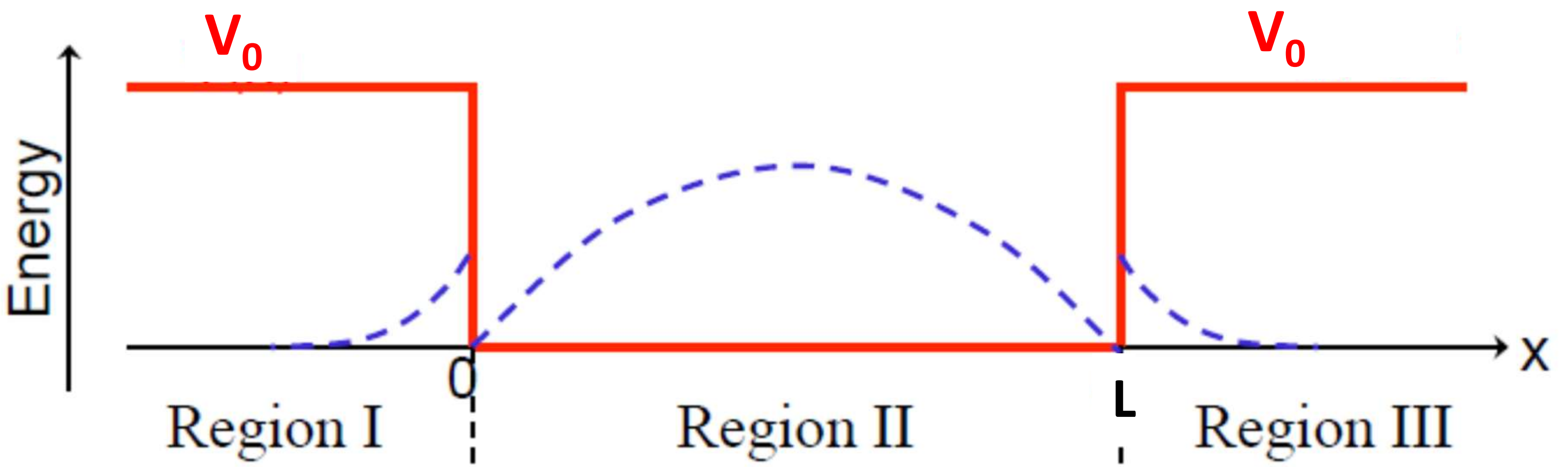
$\tan(kL/2)$



$$k_0 = \sqrt{2mV_0/\hbar^2}$$

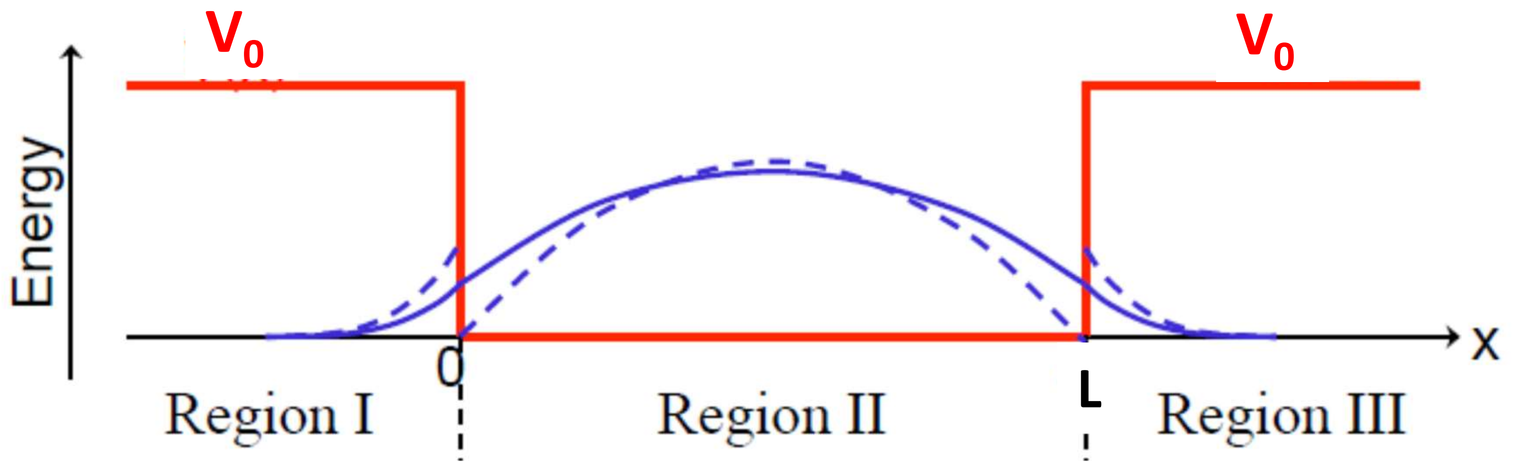
As V_o increases, it admits more and more bound states

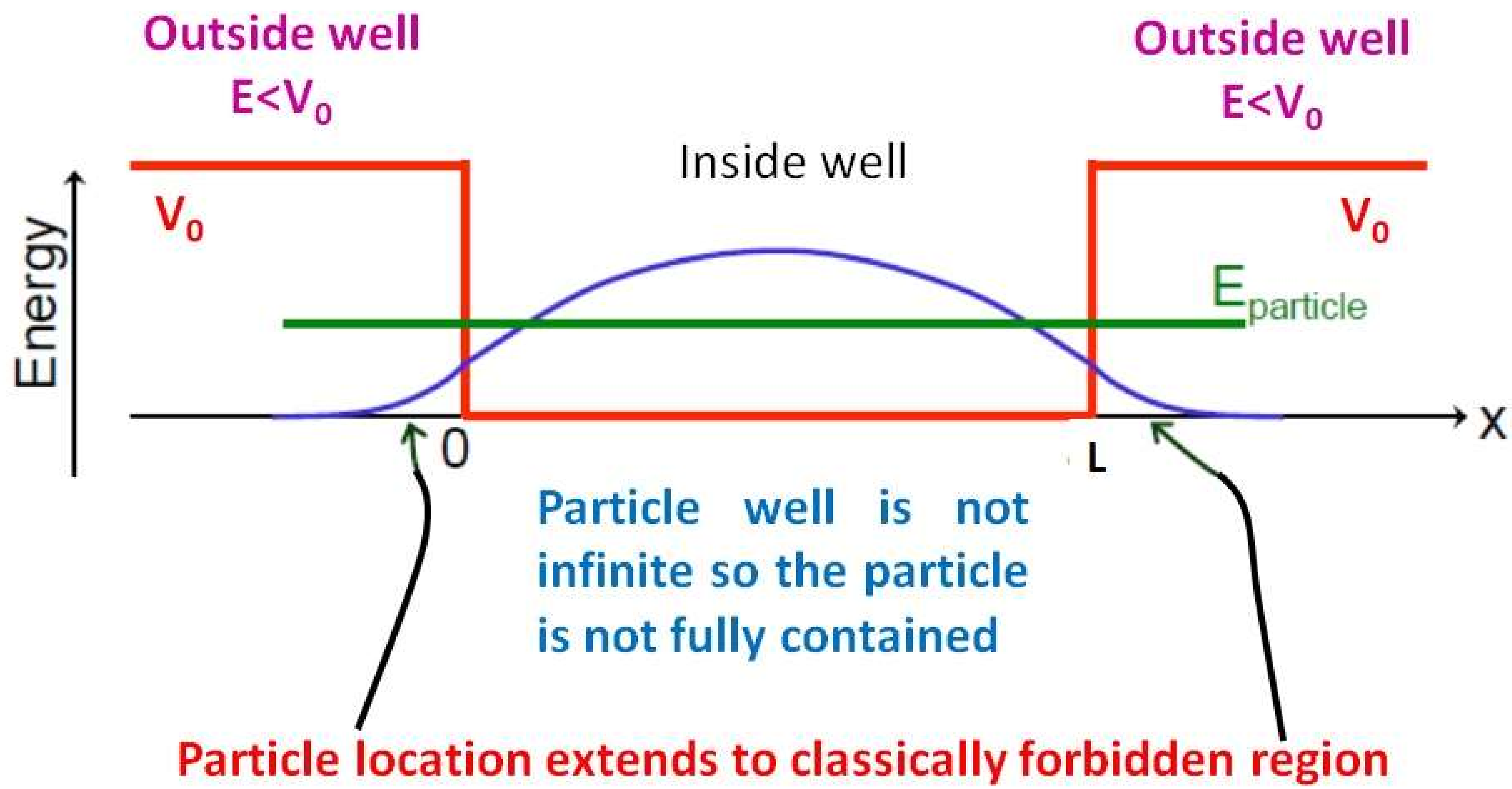
Wave function: Pictorial representation



$$\psi_1(x) = Ae^{\alpha x} \qquad \psi_2(x) = C\sin(kx) + D\cos(kx) \qquad \psi_3(x) = Fe^{-\alpha x}$$

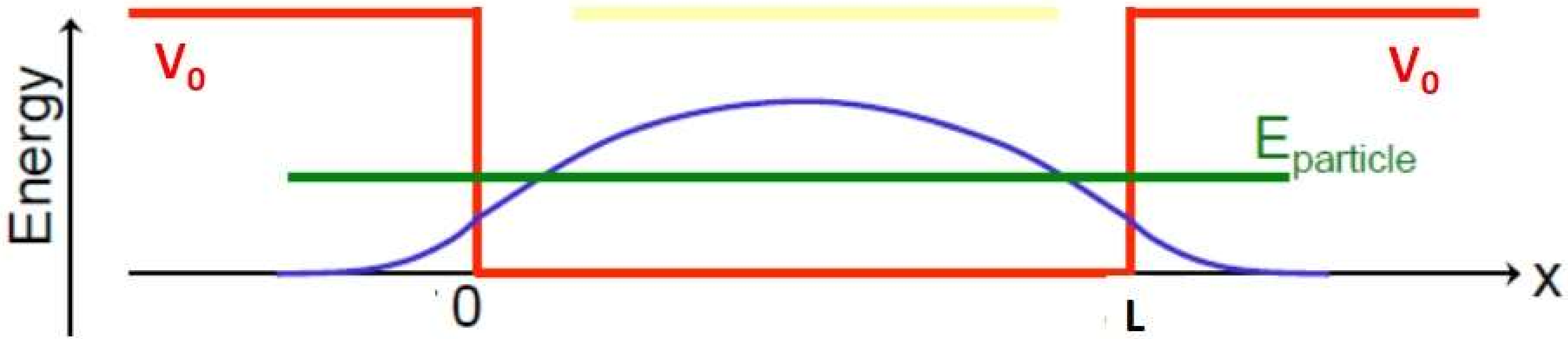
By matching boundary conditions, we achieved





Classically forbidden region: *Particle has total energy less than the potential energy*

Approximate energy expression:



$$\psi_1(x) = Ae^{\alpha x}$$

$$\psi_1(\delta) = \psi_1(0)/e$$

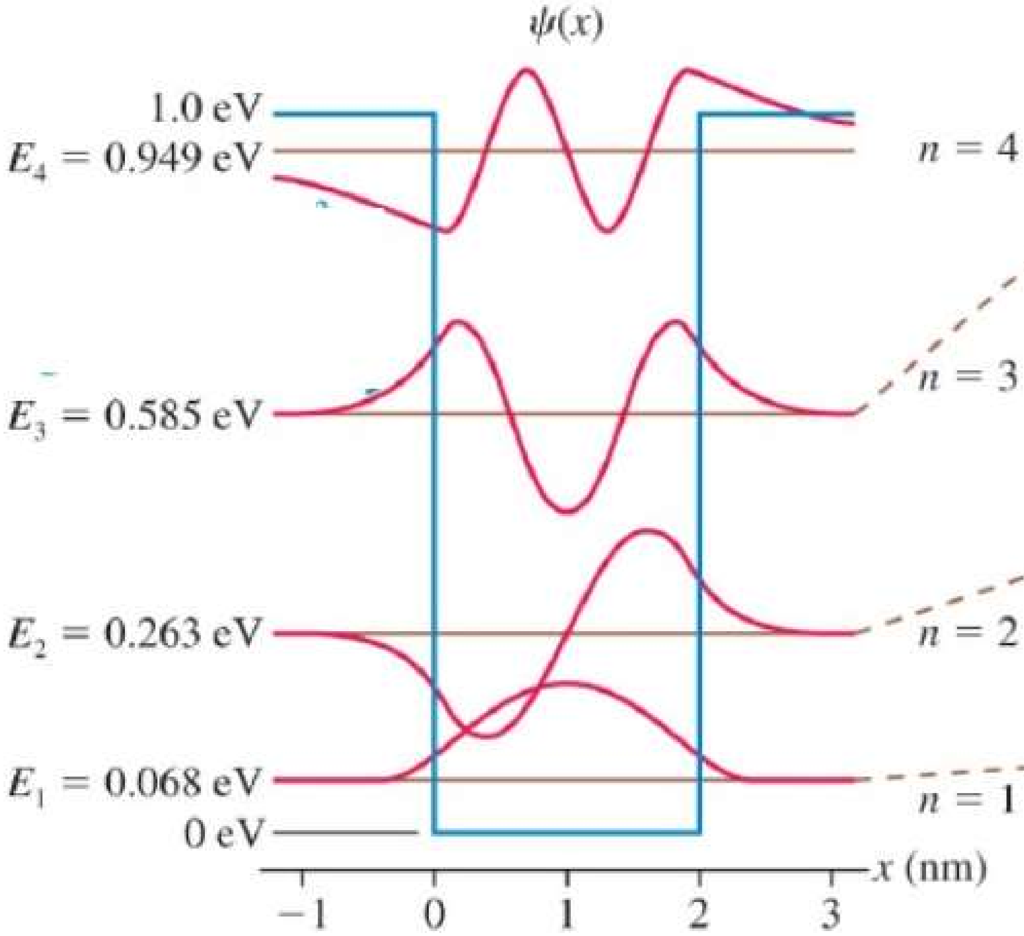
Define Penetration depth= $\delta = \dfrac{1}{\alpha} = \dfrac{\hbar}{\sqrt{2m(V_0 - E)}}$

Effective dimension of the potential well = $L + 2\delta$

Approximate energies $E_n \approx \dfrac{n^2\pi^2\hbar^2}{2m(L+2\delta)^2}$

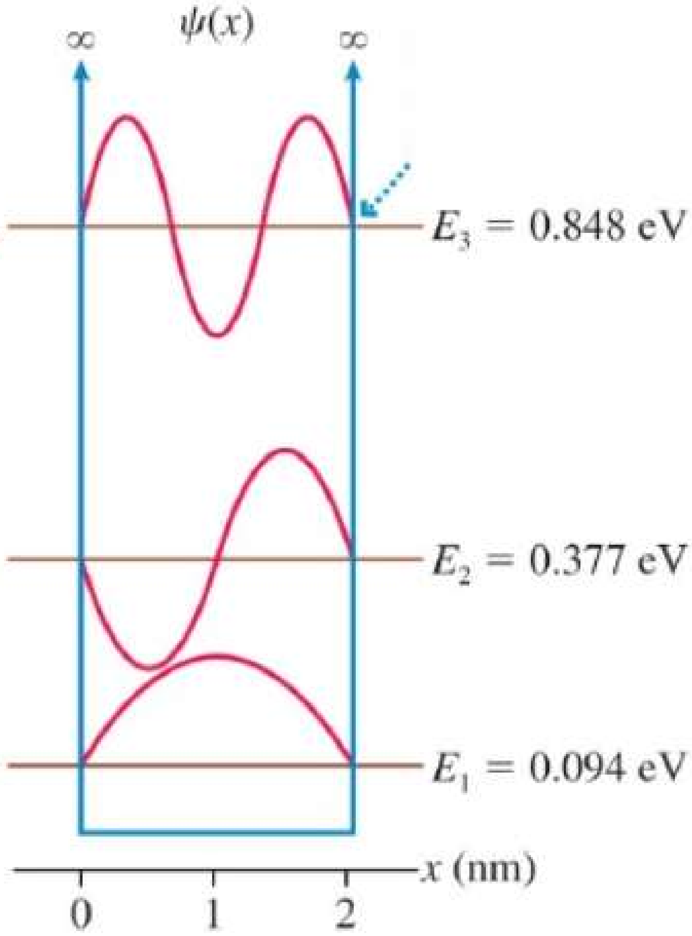
Comparison of finite and infinite potential wells

Electron in a finite well with L=2 nm and V_0=1 eV



Wave functions extend into classically forbidden region

Electron in an infinite well with L=2 nm and V_0= ∞



Wave functions are zero at the wall

Energy quantization is a result of combined efforts:

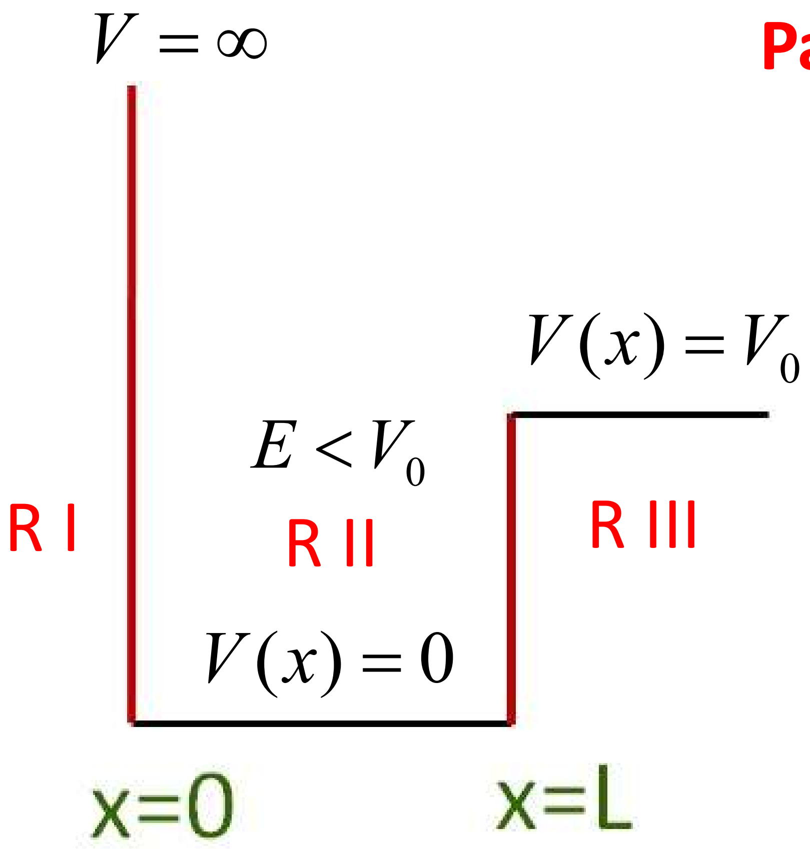
$\psi(x)$ is a solution to the independent Schrodinger Equation **and that**

the solution satisfies the boundary conditions.

Both these are rooted in the nature of the physical problem.

How many bound states exist in a given potential shall depend on the depth V_0 .

Particle in semi infinite potential



R I: $\psi(x)=0$

R II: $\psi(x)=A\sin kx$

R III: $\psi(x)=Ce^{-\alpha x}$

$$\alpha=\sqrt{2m(V_0-E)}/\hbar$$

$$k=\sqrt{2mE}/\hbar$$

Continuity at x=L $\Rightarrow$ $A\sin kL=Ce^{-\alpha L}$

Continuity of derivative at x=L $\Rightarrow$ $Ak\cos kL=-\alpha Ce^{-\alpha L}$

$$\boxed{\therefore \cot(kL)=-\alpha/k}$$

$$\cot(kL) = -\alpha / k$$

$$\alpha = \sqrt{2m(V_0 - E)} / \hbar$$

$$k = \sqrt{2mE} / \hbar$$

$$k_0 = \sqrt{2mV_0} / \hbar$$

$$\cot(kL) = -\sqrt{\frac{V_0 - E}{E}} = -\sqrt{\frac{V_0}{E} - 1}$$

$$\cot(kL) = -\sqrt{\left(\frac{k_0}{k}\right)^2 - 1} = -\sqrt{\left(\frac{k_0 L}{kL}\right)^2 - 1}$$

Plot **(1)** $\cot(kL)$ vs kL

(2) $\sqrt{(k_0 L / kL)^2 - 1}$ vs kL **for given** $k_0 L$

Intersection points give the quantized energy levels

Normalization of wave function

R I: $\psi(x) = 0$

R II: $\psi(x) = A\sin kx$

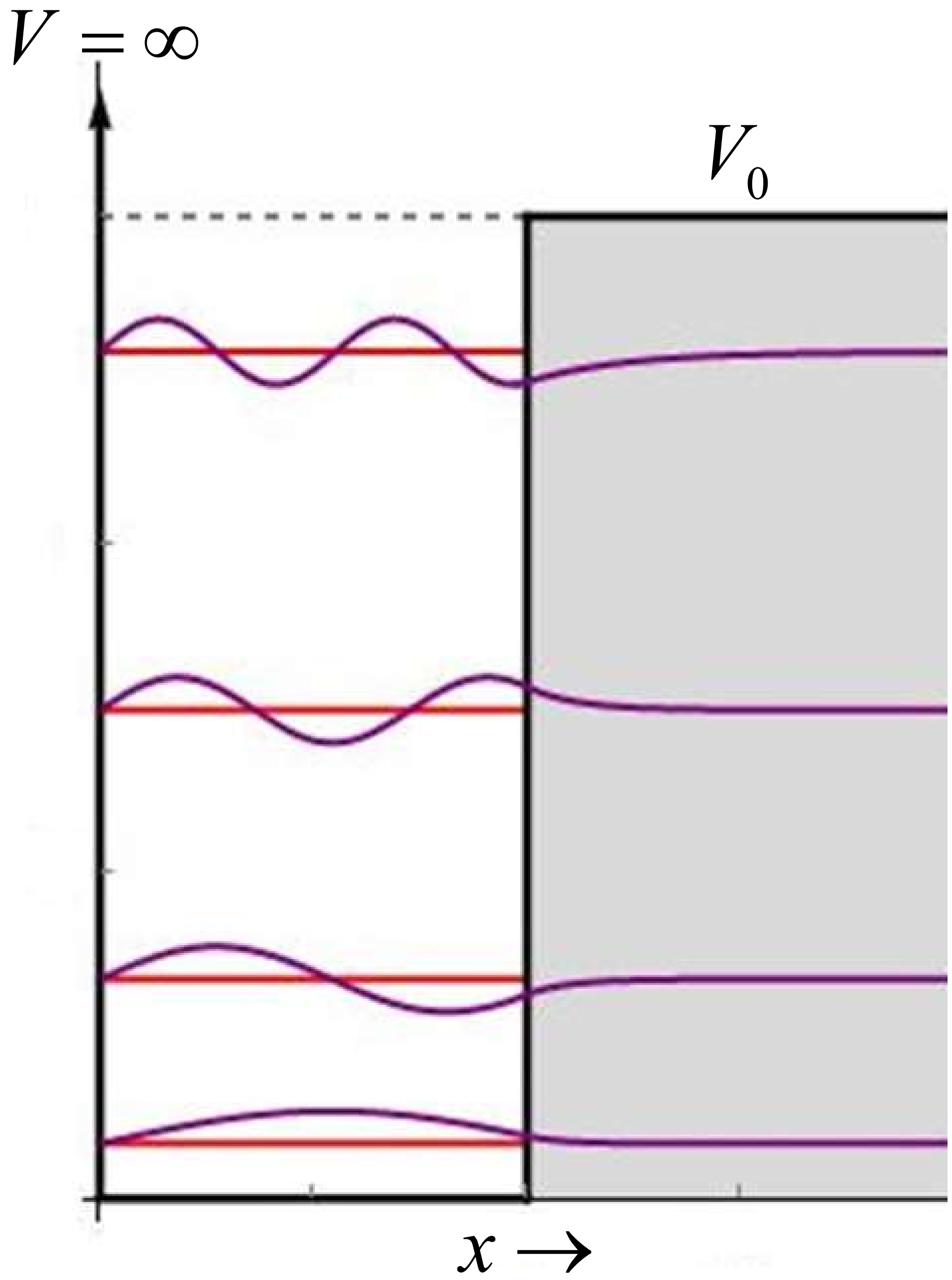
R III: $\psi(x) = Ce^{-\alpha x}$

$$A\sin kL = Ce^{-\alpha L}$$

$$C = Ae^{\alpha L}\sin kL$$

$$|A|^2\int_0^L \sin^2(kx)dx + |A|^2 e^{2\alpha}\sin^2(kL)\int_L^\infty e^{-2\alpha x}dx = 1$$

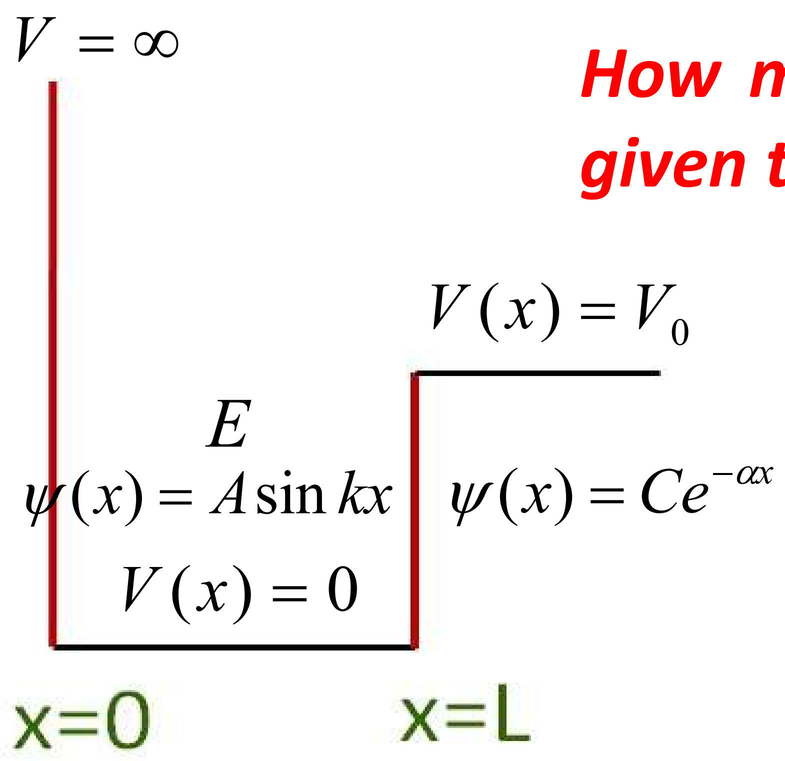
$$\therefore A = \left(\frac{L}{2} - \frac{1}{4k}\sin(2kL) + \frac{1}{\alpha}\sin^2(kL)\right)^{-1/2}$$



Wave functions

Wave functions extend in the classically forbidden region on one side

How many bound levels can we have given the value of V_0?



$E < V_0$ Particle is confined within the box therefore bound energy levels

$E > V_0$ Particle is free to escape

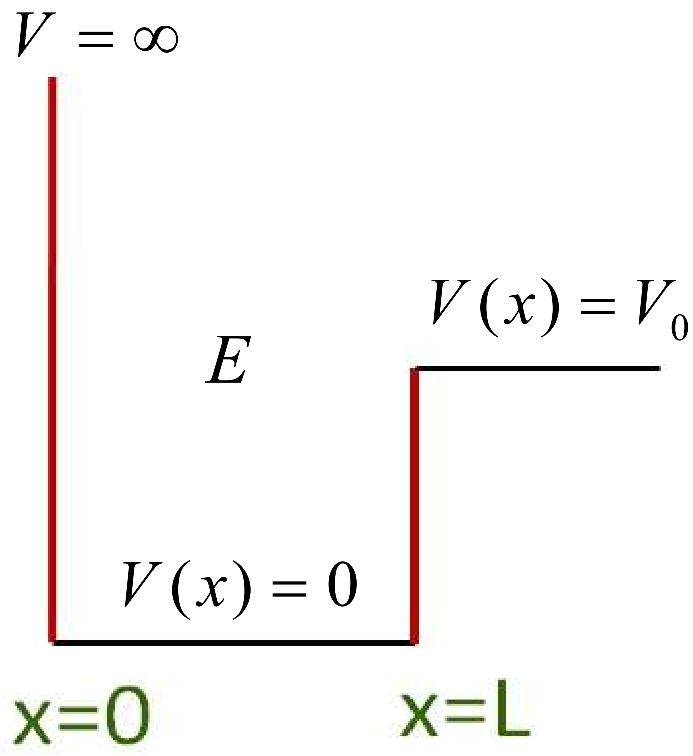
Let $E = V_0$ $\Rightarrow$

$$\alpha = \sqrt{2m(V_0 - E)}/\hbar = 0$$

$$k = \sqrt{2mE}/\hbar = k_0 = \sqrt{2mV_0}/\hbar$$

Continuity of derivative at x=L $\Rightarrow$ $Ak\cos kL = -\alpha Ce^{-\alpha L} = 0$

Since, $A \neq 0$ and $k \neq 0$, $\Rightarrow$ $\cos(kL) = 0$



Thus for $E = V_0$

$$k = \sqrt{2mE}/\hbar = k_0 = \sqrt{2mV_0}/\hbar$$

$$\cos(kL) = 0$$

$$kL = (2n'-1)\frac{\pi}{2} \qquad n' = 1,2,3,\ldots$$

n' denote allowed energy levels when $E=V_0$

$$\therefore k_0 L = \frac{\sqrt{2mV_0}}{\hbar} = (2n'-1)\frac{\pi}{2}$$

$$\therefore V_0 = (2n'-1)^2 \frac{\pi^2\hbar^2}{8mL^2} \qquad n' = 1,2,3,\ldots$$

$$V_0 = (2n'-1)^2 \frac{\pi^2\hbar^2}{8mL^2} \qquad n' = 1,2,3,\ldots$$

To support **1 level** $V_0 > \frac{\pi^2\hbar^2}{8mL^2}$

2 level $V_0 > \frac{9\pi^2\hbar^2}{8mL^2}$

To support **Only 1 level** $\frac{\pi^2\hbar^2}{8mL^2} < V_0 < \frac{9\pi^2\hbar^2}{8mL^2}$

Only 2 level $\frac{\pi^2\hbar^2}{8mL^2} < V_0 < \frac{25\pi^2\hbar^2}{8mL^2}$

$$V_0 = (2n'-1)^2 \frac{\pi^2\hbar^2}{8mL^2} \qquad n' = 1,2,3,\ldots$$

$$(2n_{\max} - 1)^2 = \frac{V_0}{\left(\frac{\pi^2\hbar^2}{8mL^2}\right)} \Rightarrow n_{\max} = \frac{1}{2}\sqrt{\frac{V_0}{\left(\frac{\pi^2\hbar^2}{8mL^2}\right)}} + \frac{1}{2}$$

n_{max}= maximum number of bound levels

$$\therefore n_{\max} = \sqrt{\frac{V_0}{\left(\frac{\pi^2\hbar^2}{2mL^2}\right)}} + \frac{1}{2}$$

$$\therefore n_{\max} = \sqrt{\frac{V_0}{E_1}} + \frac{1}{2}$$

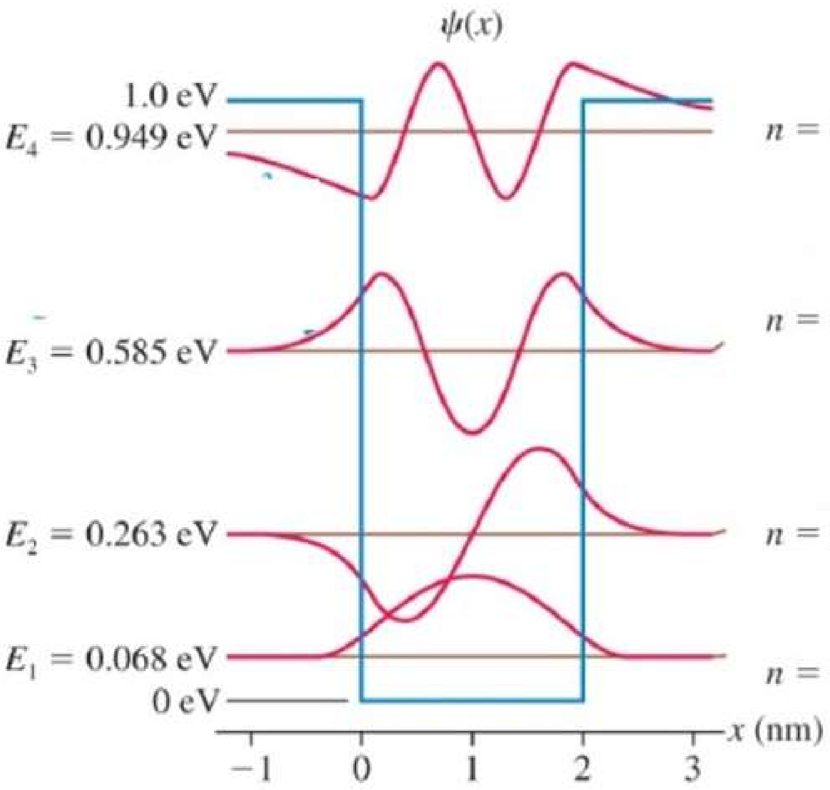
E_1 is the ground state energy of particle in an infinite box

For a particle in infinite potential

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

$$E_1 = \frac{\pi^2\hbar^2}{2mL^2}$$

(See slide 28)



Number of bound states

$$n_{\max} = \sqrt{\frac{V_0}{\left(\frac{\pi^2\hbar^2}{2mL^2}\right)}} + \frac{1}{2}$$

Caution: This expression is for semi-infinite box. We are trying to apply it to finite box. This will give a ball park figure!

$$\left(\frac{\pi^2\hbar^2}{2mL^2}\right) = 0.094 \text{ eV}$$

$$n_{\max} = \sqrt{\frac{1\,\text{eV}}{0.094\,\text{eV}}} + \frac{1}{2}$$

$$n_{\max} = 3.76 \sim 4$$