Assignment

1. Form the following partial differential equation by eliminating arbitrary constants and arbitrary function:
   (i) $z=\left(x^2+a\right)\left(y^2+b\right)$
   (ii) $2z=\left(ax+y\right)^2+b$
   (iii) $z=F\left(x^2-y^2\right)$
   (iv) $z=x+y+f\left(xy\right)$

2. Find the directional derivative of the scalar function $f(x,y,z)=xyz$ in the direction of the outer normal to the surface $z=xy$ at the point $(3,1,3)$.
3. Find the work done when a force $\vec{F}=(x^2-y^2+x)\hat{i}-(2xy+y)\hat{j}$ moves the particle from origin to $(1,1)$ along the parabola $y^2=x$.
4. If $\vec{F}=2y\hat{i}-3\hat{j}+x^2\hat{k}$ and S is the surface of the parabolic cylinder $y^2=8x$ in the first octant bounded by the planes $y=4$ and $z=6,$ then evaluate $\iint_S \vec{F}\cdot\hat{n}\,dS$.
5. Evaluate $\iiint_V (2x+y)\,dV$, where V is the closed region bounded by the cylinder $z=4-x^2$ and the planes $x=0, y=0, y=2$ and $z=0$.
6. Verify Green's Theorem for $\int_C \left[(xy+y^2)dx+x^2dy\right]$, where c is the boundary by $y=x$ and $y=x^2$.
7. Use Divergence Theorem to evaluate $\iint_S \left[xz^2\,dydz+(x^2y-z^3)dzdx+(2xy+y^2z)dxdy\right]$, where S is the surface enclosing a region bounded by hemisphere $x^2+y^2+z^2=4$ above XY-plane.
8. Evaluate $\int_C \vec{F}\cdot\vec{dr}$ by Stoke's Theorem for $\vec{F}=yz\hat{i}-zx\hat{j}+xy\hat{k}$ and c is the curve of intersection of $x^2+y^2=1$ and $y=z^2$.
9. Find the rank of the following matrices:
   (i) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 10 \end{bmatrix}$ (ii) $\begin{bmatrix} 0 & 1 & 2 & -2 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{bmatrix}$
10. Solve the following equation by Gauss-elimination method:

$$2x_1+4x_2+x_3=3$$
$$3x_1+2x_2-2x_3=-2$$
$$x_1-x_2+x_3=3$$