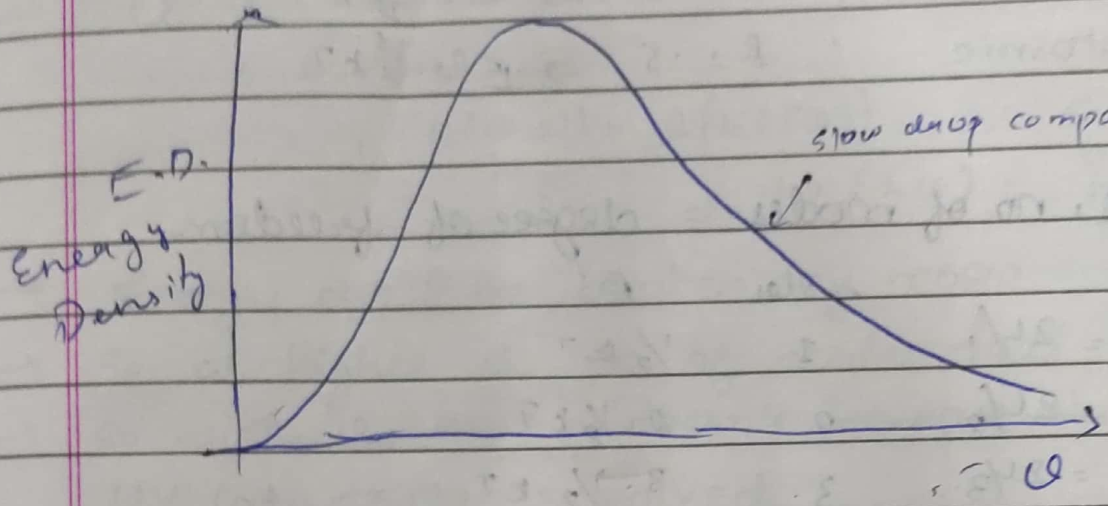
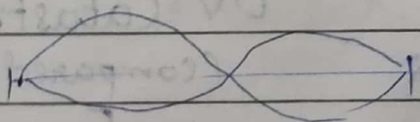
Quantum Physics

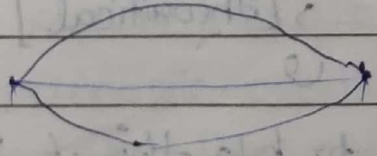


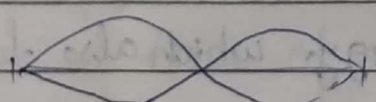
[observed]

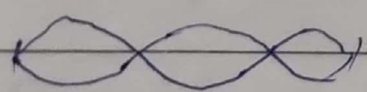
→ Black body cavity length = L $\longleftarrow L \longrightarrow$

→ Standing waves are formed in cavity. No. of ~~full~~ standing waves formed in L length is called no. of modes.

Ex :-    modes = 2   ($\lambda_1 = L$)

→  $\lambda_1 = \frac{2L}{1}$ $n = 1$ $E = \frac{1}{2}kT$

 $\lambda_2 = \frac{2L}{2}$ $n = 2$ $E = 2 \cdot \frac{1}{2} kT$ $\longrightarrow$ $\lambda_n = \frac{2L}{n}$ $n \to n$

 $\lambda_3 = \frac{2L}{3}$ $n = 3$ $E = 3 \cdot \frac{1}{2} kT$

n half waves
$n \in \{1, 2, \ldots\}$

∞ $E = \infty$

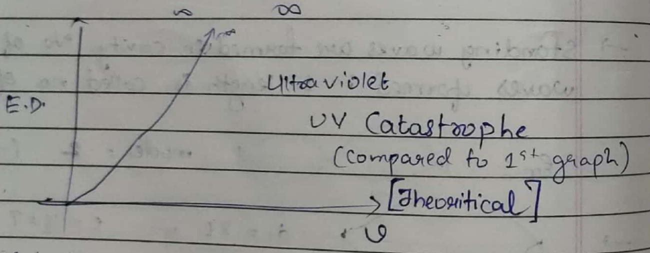
→ Equipartition of Energy / Boltzmann's law :-
The kinetic energy of each degree of freedom of a gas molecule is $\frac{1}{2} kT$.

Monoatomic : $f = 3 \Rightarrow E = \frac{3}{2}kT$

Diatomic : $f = 5 \Rightarrow E = \frac{5}{2}kT$

→ Here, no. of modes = degree of freedom

	modes	E
$\lambda_1 = 2L/1$	1	$\frac{1}{2}kT$
$\lambda_2 = 2L/2$	2	$2\cdot\frac{1}{2}kT$
$\lambda_3 = 2L/3$	3	$3\cdot\frac{1}{2}kT$
⋮	⋮	⋮
	∞	∞



→ Adopting classical physics to ~~b/o~~ this it lead to this catastrophe of ∞ energy which also fails to explain observed graph

⇒ Planck's Explanation:-

→ Total energy is fixed.

→ It be broke into any ~~no~~ no. of parts but it will follow $E = nh\nu$

$n \rightarrow$ no. of modes

→ Let $E = 10h\nu$; $[n \neq 10]$.

$E = nh\nu$

$= 1(h(10\nu)) = 2(h(5\nu)) = \dots = 5\cdot(h(2\nu))$

$= 10\cdot(h\nu) = $????

→ Energy can't be broken any more

→ So at higher ν, no. of modes fall

→ So with ν, energy doesn't increase always and UV catastrophe is solved.

☆ $E = h\nu\cdot\frac{2\pi}{2\pi} = \hbar\omega$ $\quad \hbar = \frac{h}{2\pi}$ $\quad \omega = 2\pi\nu$

$p = \frac{h}{\lambda}\cdot\frac{2\pi}{2\pi} = \frac{\hbar}{k}$ $\quad k = \frac{\lambda}{2\pi}$

Q: A certain 660 Hz tuning fork be considered as a harmonic oscillator of vibrational energy 0.04 J. Compare the energy quanta of this tuning fork with that of an atomic oscillator absorbing & emitting radiation of frequency 5.00×10^{17} Hz.

⇒ $E_f = h\nu = 6.626\times10^{-34}\times660 = 4.37\times10^{-31}$ J

$E_a = h\nu = 6.626\times10^{-34}\times5\times10^{17} = 3.31\times10^{-16}$ J

Particle Wave

$E = h\nu$

$p = \frac{h}{\lambda}$

Wave Particle

$\nu = \frac{E}{h}$

$\lambda = \frac{h}{p}$

De-Broglie (Read as De-Broy)

$p = \gamma \cdot m v$

$= \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \cdot mv$

γ comes into play only at high speeds (0.8c 0.9c etc)

For macroscopic world $\gamma = 1$

Q:- Find De-Broglie wavelengths of

1) 46 g golf ball moving at a speed of 30 m/s

2) an e^- with velocity of 10^7 m/s

$\Rightarrow \gamma = \frac{1}{\sqrt{1-\frac{10^{14}}{9\times10^{16}}}} = \sqrt{\frac{900}{899}}$

1) $\lambda_1 = \frac{h}{46\times30} \times 1000 = 4.8 \times 10^{-34}$ m

2) $\lambda_e = \frac{h}{\gamma m_e v} = \frac{6.626\times10^{-34}}{1.00055\times9.1\times10^{-31}\times10^7} = 7.27\times10^{-11}$ m

Q:-) Neutrons produced in a reactor are used for chain reaction after they are thermalised i.e. their kinetic energies are reduced to that of the energy of the air molecules at room temperature. Taking room temp. as 300K estimate the De-Broglie's wavelengths of such neutron.

$m_n = 1.67 \times 10^{-27}$ kg.

$\Rightarrow$ K.E. $= \frac{3}{2} k_B T =$

Speed of wave (v_p) [Phase velocity]

$v_p = f \cdot v$

$E = h\nu = \gamma mc^2$

$$\gamma = \left(\sqrt{1 - \frac{v^2}{c^2}}\right)^{-1} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\lambda = \frac{h}{p} = \frac{h}{\gamma m v}$$

$$v_p = \frac{\cancel{h}}{\cancel{\gamma m} v} \cdot \frac{\cancel{\gamma m} c^2}{\cancel{h}} = \frac{c^2}{v}$$

$$v_p = \frac{c^2}{v} > c$$

Beats:-

$y_1 = A\cos(kx - \omega t)$

$y_2 = A\cos((k+\Delta k)x - (\omega + \Delta\omega)t)$

$k + \Delta k \approx k$

$\omega + \Delta\omega \approx k$

$y = y_1 + y_2$

$$= A \cdot 2\cos\left(\frac{kx - \omega t + kx - \omega t}{2}\right)\cos\left(\frac{\cancel{kx} - \cancel{\omega t} - \cancel{kx} + \cancel{\omega t} - \Delta kx + \Delta\omega t}{2}\right)$$

$$= 2A\cos(kx - \omega t)\cos\left(\frac{\Delta\omega t - \Delta k x}{2}\right)$$

$v_p \rightarrow$ wave velocity $= \frac{\omega}{k} = \frac{c^2}{v}$

$v_g \rightarrow$ Group velocity $= \frac{\Delta\omega}{\Delta k} = \frac{d\omega}{dt}$
(Amplitude velocity)

$\omega = 2\pi\nu$ $\quad$ $\nu \rightarrow$ frequency

$$= 2\pi \cdot \frac{\gamma mc^2}{h}$$

$$= \frac{2\pi mc^2}{h\sqrt{1 - \frac{v^2}{c^2}}}$$

$k = \frac{2\pi}{\lambda}$

$k = \frac{2\pi p}{h} = \frac{2\pi m v}{h}$

$$K = \frac{2\pi}{\lambda}$$

$$= \frac{2\pi p}{h} = \frac{2\pi m v}{h\sqrt{1 - \frac{v^2}{c^2}}}$$

$$v_g = \frac{d\omega}{dt} = \frac{d\omega}{dv} \times \frac{1}{dk/dv}$$

$$\frac{d\omega}{dv} = \frac{2\pi mc^2}{h}\left(\frac{-1}{2}\right)\left(1 - \frac{v^2}{c^2}\right)^{-3/2}\left(\frac{-2v}{c^2}\right)$$

$$\frac{dk}{dv} = \frac{2\pi m v}{h}\left(\frac{-1}{2}\right)\left(1 - \frac{v^2}{c^2}\right)^{-3/2}\left(\frac{-2v}{c^2}\right)$$

$$v_g = \frac{\frac{2\pi mc^2}{h}\left(\frac{-1}{2}\right)\left(1 - \frac{v^2}{c^2}\right)^{-3/2}\left(\frac{-2v}{c^2}\right)}{\frac{2\pi m v}{h}\left(\frac{-1}{2}\right)\left(1 - \frac{v^2}{c^2}\right)^{-3/2}\left(\frac{-2v}{c^2}\right)}$$

$$\frac{dk}{dv} = \frac{2\pi m}{h}\frac{d}{dv}\left[v\left(1-\frac{v^2}{c^2}\right)^{-1/2}\right]$$

$$= \frac{2\pi m}{h}\left[v\left(\frac{-1}{2}\right)\left(1-\frac{v^2}{c^2}\right)^{-3/2}\left(\frac{-2v}{c^2}\right) + \left(1-\frac{v^2}{c^2}\right)^{-1/2}\right]$$

$$= \frac{2\pi m}{h}\left[\left(1-\frac{v^2}{c^2}\right)^{-1/2}\left[\frac{v^2}{c^2}\left(1-\frac{v^2}{c^2}\right)^{-1} + 1\right]\right]$$

$$= \frac{2\pi m}{h} \times \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\left[\frac{\cancel{v^2/c^2} + 1 - \cancel{v^2/c^2}}{1-v^2/c^2}\right]$$

$$= \frac{2\pi m}{h\sqrt{1-v^2/c^2}}\left[\frac{1}{1-v^2/c^2}\right]$$

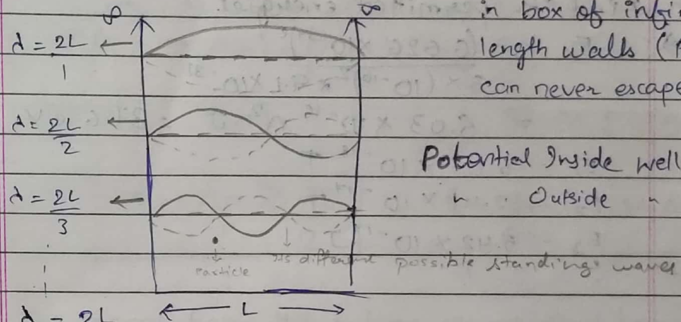
$$= \frac{2\pi m}{h(1-v^2/c^2)^{3/2}}$$

$$V_g = \frac{d\omega}{dv} \times \frac{1}{dk/dv} = v$$

$$= \frac{\frac{2\pi m c^2}{h}\left(\frac{-1}{2}\right)\left(1-\frac{v^2}{c^2}\right)^{-3/2}\left(\frac{-2v}{c^2}\right)}{\frac{2\pi m}{h}\left(1-\frac{v^2}{c^2}\right)^{-3/2}}$$

$$V_g = V$$

※ Particle in a Box



A particle is confined (non relativistic) in box of infinite length walls (Particle can never escape)

Potential Inside well = 0
" " Outside " = ∞

$$\lambda_n = \frac{2L}{n}$$

Non realistic case (x neglected)

$$KE = \frac{1}{2}mv^2 = \frac{p^2}{2m} = E_n$$

$$p_n = \frac{h}{\lambda_n} = \frac{nh}{2L} \rightarrow \boxed{E_n = \frac{n^2h^2}{8L^2m}} \quad n = 1, 2, 3, \ldots$$

→ $\lambda = \frac{2L}{n} \Rightarrow n \neq 0$ bcoz $\lambda = \infty$ so the particle can be anywhere but we said it is in the well.

1) Trapped particle cannot have arbitary Energy.

2) Energy of trapped particle cannot be 0.

3) We can't see discretization due to small value of h.

Q:) An electron confined in a box 0.10 nm length. Find its permitted energies

$\Rightarrow$ $E_n = \frac{(6.626 \times 10^{-34})^2}{8 \times (10^{-10})^2 \times 9.1 \times 10^{-31}}$

$= 6.03 \times 10^{-18} n^2 \, J = 37.6 \, eV$

$E_1 = 6.03 \times 10^{-18} \, J$

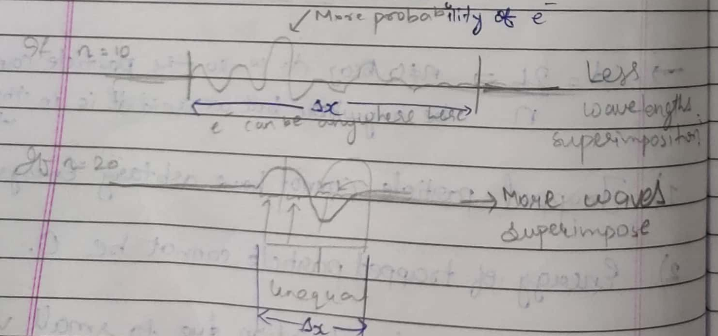
$E_2 = 2.41 \times 10^{-17} \, J$

$E_3 = 5.42 \times 10^{-17} \, J$

Q:) A particle of mass m = 10g is confined in a box of 10 cm length

$\rightarrow$ $E_n = \frac{n^2 h^2}{8L^2 m} = n^2 \frac{(6.626 \times 10^{-34})^2}{8 \times (0.1)^2 (10^{-2})}$

$= 5.48 \times 10^{-64} n^2 \, J$



Let n = 10 has wavelengths $\lambda_1, \lambda_2, \ldots \lambda_{10}$

$\Delta p = p(\lambda_{10}) - p(\lambda_1)$

If $\Delta x \uparrow$ $\Delta p \downarrow$

If n = 20 $\lambda_1, \lambda_2, \ldots, \lambda_{20}$

$\Delta p = p(\lambda_{20}) - p(\lambda_1)$

If $\Delta x \downarrow$ $\Delta p \uparrow$

$\rightarrow$ $\boxed{\Delta x \cdot \Delta p \geq \frac{\hbar}{2}}$ $\hbar = \frac{h}{2\pi}$ $\Delta E \cdot \Delta t \geq \frac{\hbar}{2}$

"We cannot know the future as we don't know present"

Q:- A measurement confirms the position of a proton to an accuracy of $\pm 1.00 \times 10^{-11}$ m. Find the uncertainity in proton's position after 1.00s.

$\Rightarrow$ i) Remains constant ii) Increases iii) Decreases ($v << c$) mass is constant

$\Delta x_0 = \pm 1.00 \times 10^{-11}$ m at t = 0s

$\Delta x = ?$ at t = 1.00s

$\Delta p \cdot \Delta x_0 \geq \frac{\hbar}{2}$

$\Delta v \geq \frac{\hbar}{2m\Delta x_0}$

$\Delta x_1 = t \cdot \Delta v \geq \frac{\hbar t}{2m\Delta x_0}$

≥ 3152.45 m

Black body radiation
Wein's displacement → Derivation from Ch-9
Stefan - Boltzmann

photoelectric, Pair production, Compton

Ch 2 2.2 2.3 2.4 2.7

Ch 3 3.1 - 3.7 & 3.9

uncertainity → Application

Extra
1.8, 1.9
For confusion of relativity & non-relativity

→ We have learned that a microscopic particle acts as if certain aspects of its behaviour are governed by an associated De Broglie wave or wavefunction.

→ In dealing with very simple case like motion of a particle & particle in a box we have applied this aspects of matter waves successfully.

→ However, It doesn't tell us how the wave propagates.

→ Though these postulates have been successfully predicting the wavelengths of this particles but only in cases where this wavelengths are constant.

→ We must have a quantitative reln b/w the properties of a particle & the properties of the associated ~~wa~~ wavefunction that is describing the wave.

1. → ψ must be continuous & single valued

2. → $\frac{\partial \psi}{\partial x}$ must be continuous & single valued (for y, z too)

3. → ψ must be Normalizable i.e. $\psi \to 0$ $x \to \pm \infty$ (y, z)

ψ should be complex

$\psi = A + iB$ A, B → real fns

$|\psi|^2 = \psi \cdot \psi^* = (A+iB)(A-iB)$

$= A^2 + B^2 \geq 0$

$\int_{-\infty}^{\infty} |\psi|^2 dx = 1$

$\int |\psi|^2 dV \propto P$ → Probability