L-3 BLACK BODY RADIATION, PLANK'S QUANTUM HYPOTHESIS SPECIFIC HEATS.

■ PLANK QUANTUM HYPOTHESIS:

→ Energy of oscillator quantized
→ Energy of particular mode of ν has value equal to $nh\nu$
→ All modes in thermal eqlb.
→ Eqlb established by Energy Xchange bet^n modes. happens through interaction with walls of cavity.

QUANTUM AVG.

$$\langle E\rangle = \frac{\sum_{n=0}^{\infty} nh\nu e^{-nh\nu/kT}}{\sum_{n=0}^{\infty} e^{-nh\nu/kT}} = \frac{h\nu}{e^{\frac{h\nu}{kT}}-1}$$

CLASSICAL AVG.

$$\langle E\rangle = \frac{\int_0^\infty E e^{-E/kT}\,dE}{\int_0^\infty e^{-E/kT}\,dE} = kT$$

$$\boxed{\langle E\rangle = \frac{h\nu}{e^{h\nu/kT}-1}}$$

Previously seen laws that were incomplete.

Wien Exponent.

$u(\nu,T) = A\nu^3 e^{-\beta\nu/T}$

for long λ low ν Not true

Rayleigh Jeans

$u(\nu,T) = \frac{8\pi\nu^2}{c^3}kT$

for (high ν / low λ) not true

$\int_0^\infty u(\nu,T)d\nu$ → diverges

UV catastrophe.

No. of mode in cavity / unit V / ~~...~~ in interval ν to $\nu+d\nu$: $\frac{8\pi\nu^2 d\nu}{c^3}$

$$\boxed{u(\nu,T) = \frac{8\pi\nu^2}{c^3}\langle E\rangle}$$

$$\therefore \boxed{u(\nu,T)\,d\nu = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{e^{h\nu/kT}-1}d\nu}$$

long λ Short ν

$\frac{8\pi\nu^2}{c^3}kT$ $\boxed{\text{Ray J.}}$

Short λ long ν

$A\nu^3 e^{-h\nu/kT}$ $\boxed{\text{Wien's}}$

conv. to λ:

$$u(\lambda,T)d\lambda = -\frac{8\pi hc}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1}d\lambda$$

now for λ_{max}

$$\frac{\partial(u(\lambda,T))}{\partial\lambda} = 0$$

$\lambda_{max}T =$ const.

Wien's Displacement

$$e_{Total} = \frac{c}{4}\int_0^\infty u(\nu)\,d\nu$$

$$= \frac{c}{4}\frac{8\pi h}{c^3}\left(\frac{kT}{h}\right)^4\int\frac{x^3dx}{e^x-1}$$

$$\Rightarrow \left(\frac{2\pi^5k^4}{15c^2h^2}\right)T^4$$

(**)

$$\boxed{\int_0^\infty \frac{x^3\,dx}{e^x-1} = \frac{\pi^4}{15}}$$

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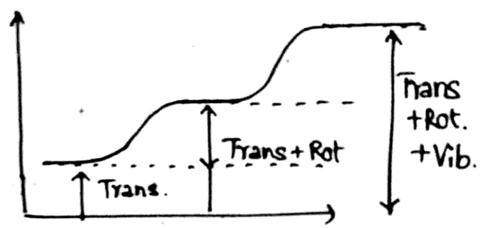
- **SPECIFIC HEATS OF GASES:**

$$C_V = \left(\frac{\partial \epsilon}{\partial T}\right)_V$$

ϵ is the energy of DoF.

C_V vs. T:



L-2 DoF, Equipartition of Energy
Black Body Radiation:

■ N Atom molecule:

• LINEAR: Trans. = 8
Rot. = 2
Vib. = 3N−5

• Non-LINEAR: Trans = 3
Rot. = 2
Vib. = 3N−6

T=3
R=3
Vib=3N-6
=3
Tot=9

Mono Atomic Trans = 3 Rot = 0 Vib = 0

Diatomic: Trans = 3 Rot = 2 Vib = 1.

■ EQUIPARTITION OF ENERGY:

$\frac{1}{2}kT$ for every DoF (Trans, Rot, KE, PE)

kT for each Vib DoF (Vib = KE + PE).

■ BOLTZMANN DISTRIBUTION:

$$f(E) = Ae^{\frac{-E}{kt}}$$

E is energy term.

$f(E)$ gives fraction of total having that energy.

$$\boxed{k = 1.38 \times 10^{-23}\ J\ K^{-1}}$$

$$\int_{-\infty}^{\infty} e^{-\alpha x^2}\,dx = \sqrt{\frac{\pi}{\alpha}}$$

$$\int_{-\infty}^{\infty} x^2 e^{-\alpha x^2}\,dx = \frac{\sqrt{\pi}}{2\alpha\sqrt{\alpha}}$$

①
■ KIRCHOFF'S THEOREM:

$$e(\nu) = J(\nu, T)\,A(\nu)$$

Black body $A(\nu) = 1$

$$e(\nu) = \frac{1}{A}\left(\frac{dQ}{dt}\right)d\nu$$

$J(\nu, T)$ univ. f^n for all black bodies.

■ STEFAN'S LAW:

for black body $\int_0^\infty e(\nu)\,d\nu = \sigma T^4$ $\sigma = 5.67 \times 10^{-8}\ W m^{-2} K^{-4}$

■ WIEN'S DISPLACEMENT LAW:

$\lambda_{max} T$ = const. = ~~2.98~~ 2.898×10^{-3} m.K

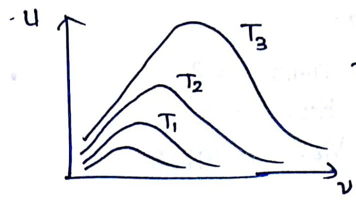
ENERGY DENSITY OF BLACK BODY:

$$J(\nu,T) = \frac{c}{4}\, u(\nu,T)$$

$$\uparrow \qquad\qquad\qquad \uparrow$$

$$\frac{1}{A}\left(\frac{dQ}{dT}\right)\frac{1}{d\nu} \qquad\qquad \left(\frac{dQ}{dV}\right)\frac{1}{d\nu}$$

$u(\nu,T)$ vs ν



$T_3 > T_2 > T_1$

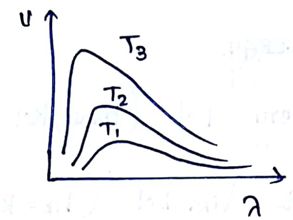
$\frac{T}{\nu_{max}} = c'$

WIEN'S EXPONENTIAL LAW:

$$u(\nu,T) = A\nu^3 e^{-\beta\nu/T}$$

Not agreeing for long λ. (short ν).

$u(\lambda,T)$ vs. λ



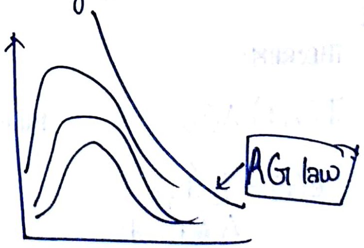
$\lambda_{max} T = c$

RAYLEIGH - JEAN'S LAW:

No. of Modes of Vibration per unit volume in frequency interval ν to $\nu + d\nu$ $= \frac{8\pi\nu^2}{c^3} d\nu$

⊛. $u(\nu,T) = \frac{8\pi\nu^2}{c^3} kT$ valid for $h\nu \lll kT$.

for High ν (diverges) ultraviolet catastrophe.



L-5

DE BROGLIE WAVES & WAVE PARTICLE DUALITY.

■ YDSE :

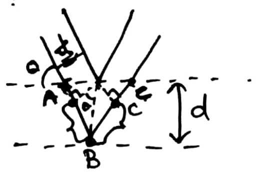
$\delta = P.D = r_2 - r_1 = d\sin\theta$

$y = D\tan\theta$

max: $d\sin\theta = n\lambda$

min: $d\sin\theta = \left(n+\frac{1}{2}\right)\lambda$

■ BRAGG'S EQ^n:



2 layer of crystal

$n\lambda = 2d\sin\theta = PD = AB + BC$

■ De Broglie's Hypothesis:

$p = \frac{h\nu}{c}$ $\therefore$ $p = \frac{h}{\lambda}$ $\lambda p = h$

$$\lambda_{dB} = \frac{h}{p} = \frac{h}{\gamma m_0 v} = \frac{h}{mv}.$$

Relation to Bohr : $L = m_e vr = n\hbar$

$\lambda_{dB} = \frac{h}{m_e v}$ $\therefore$ $2\pi r = \frac{nh}{m_e v} = n\lambda_{dB}$ ⊛ orbit stable if perimeter has integral multiple of λ

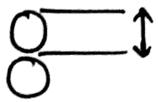
<u>Davisson - Germer Experiment</u>:

$$\frac{1}{\lambda} = \frac{n}{2d\sin\theta} = \frac{p}{h} = \frac{\sqrt{2mE}}{h} = \frac{\sqrt{2meV}}{h}$$

$$\lambda_{dB} = \frac{h}{\sqrt{2meV}} = \sqrt{\frac{150}{V}}\ \mathring{A}$$

$hc = \underline{12420\ eV\ \mathring{A}}$

Visibility of particle fringes affected if

 β = ~~dia~~ size of particle.