De Broglie Wave and Wave Particle Duality

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Light: Dual Nature

Wave Nature

Interference

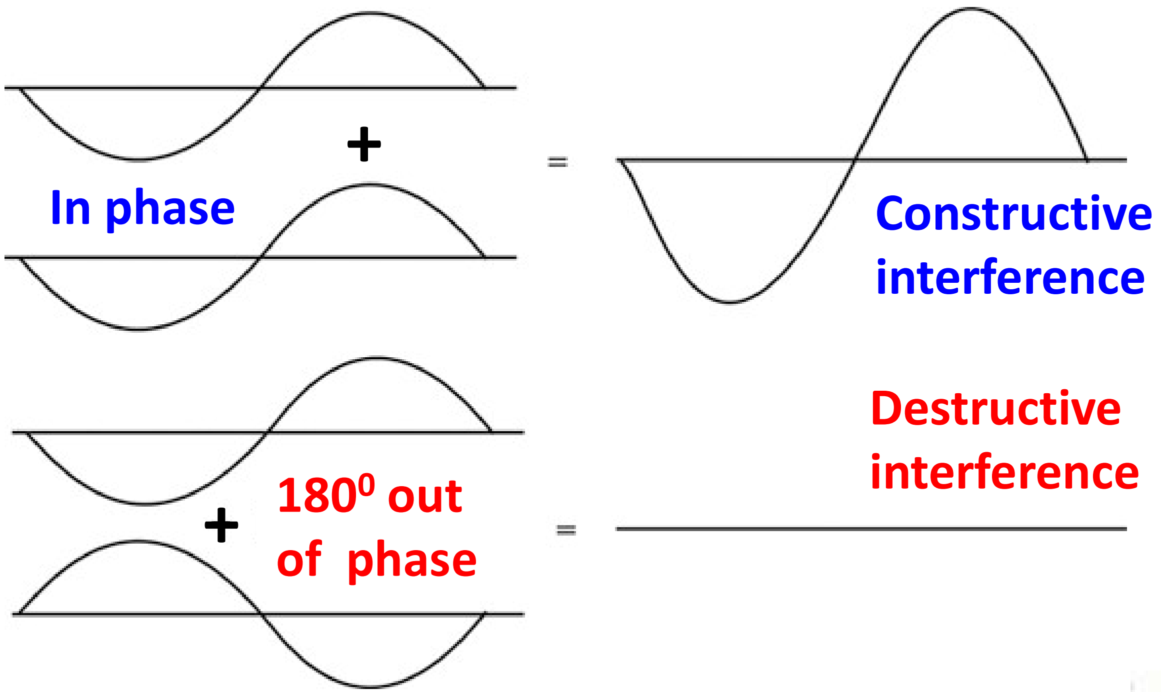
Diffraction

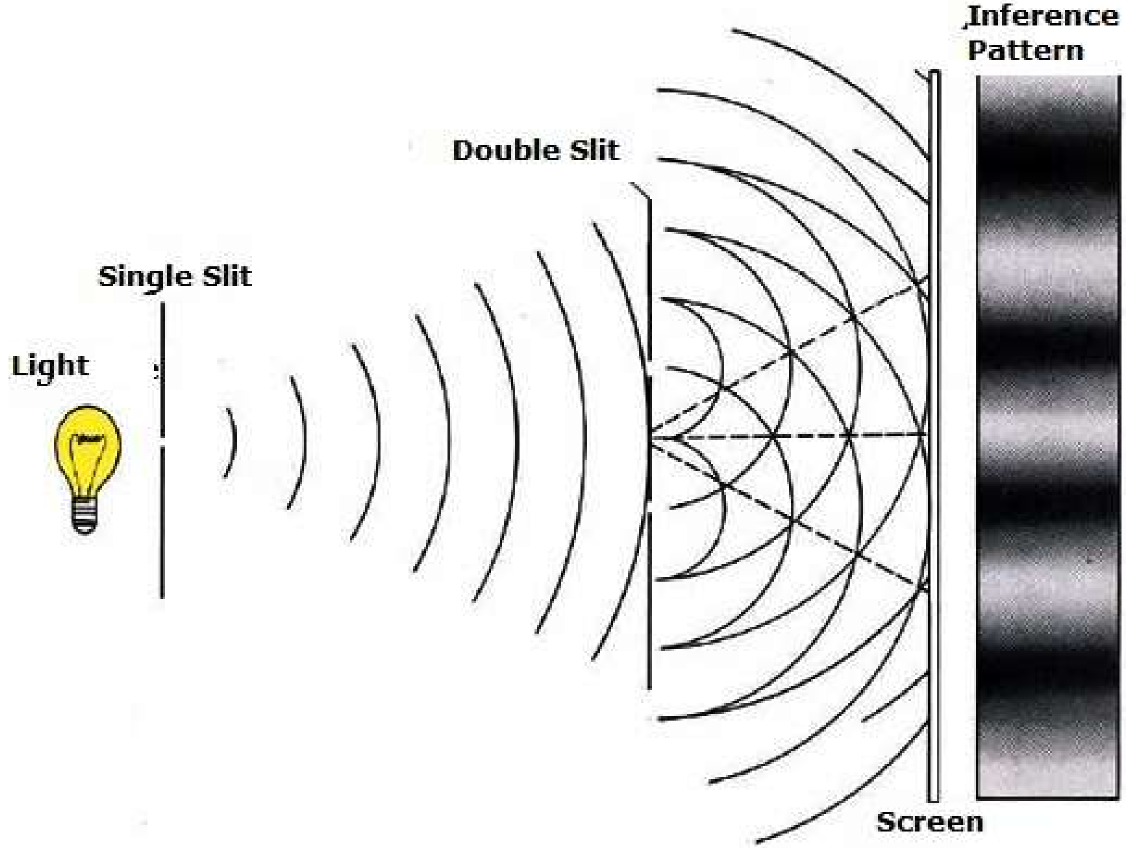
Particle Nature

Photoelectric effect

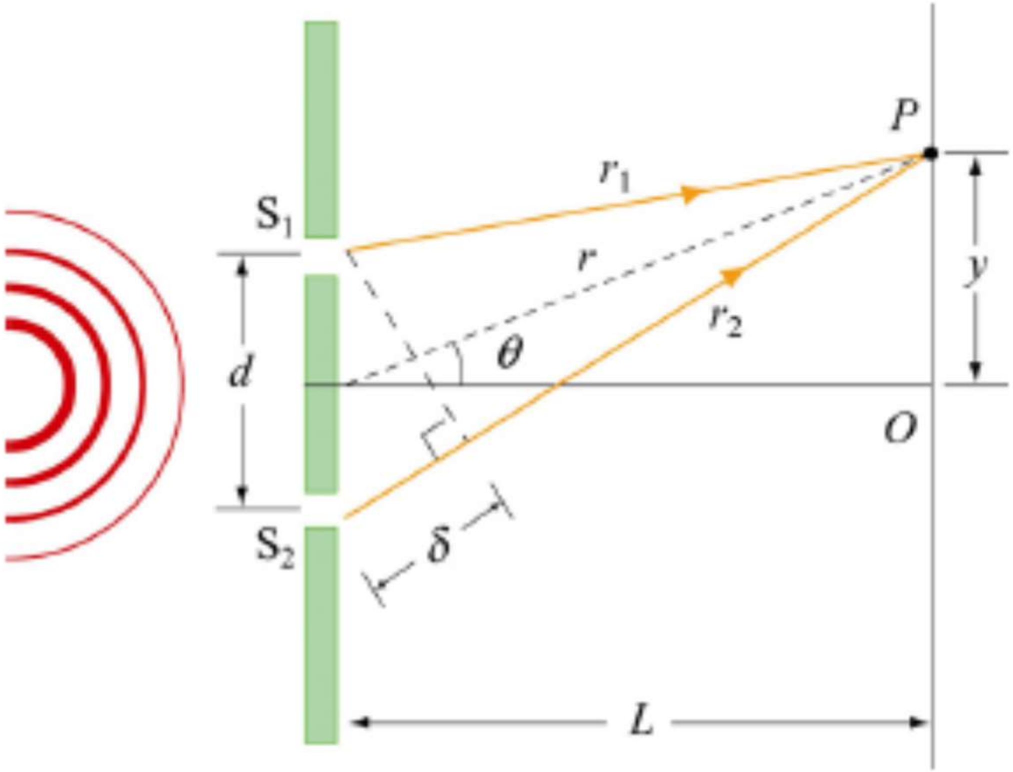
Compton effect

Interference of waves





Young's double slit experiment



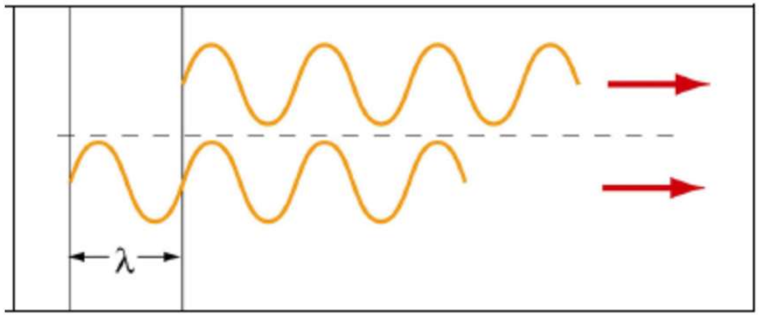
Young's double slit experiment

Path difference between two rays:

$$\delta = r_2 - r_1 = d \sin\theta$$

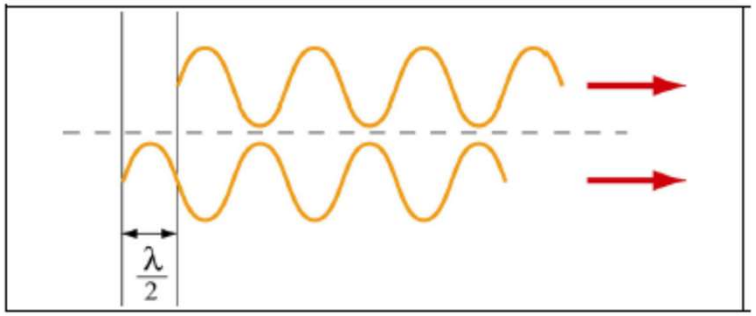
Constructive interference

$$\delta = d \sin\theta = m\lambda$$

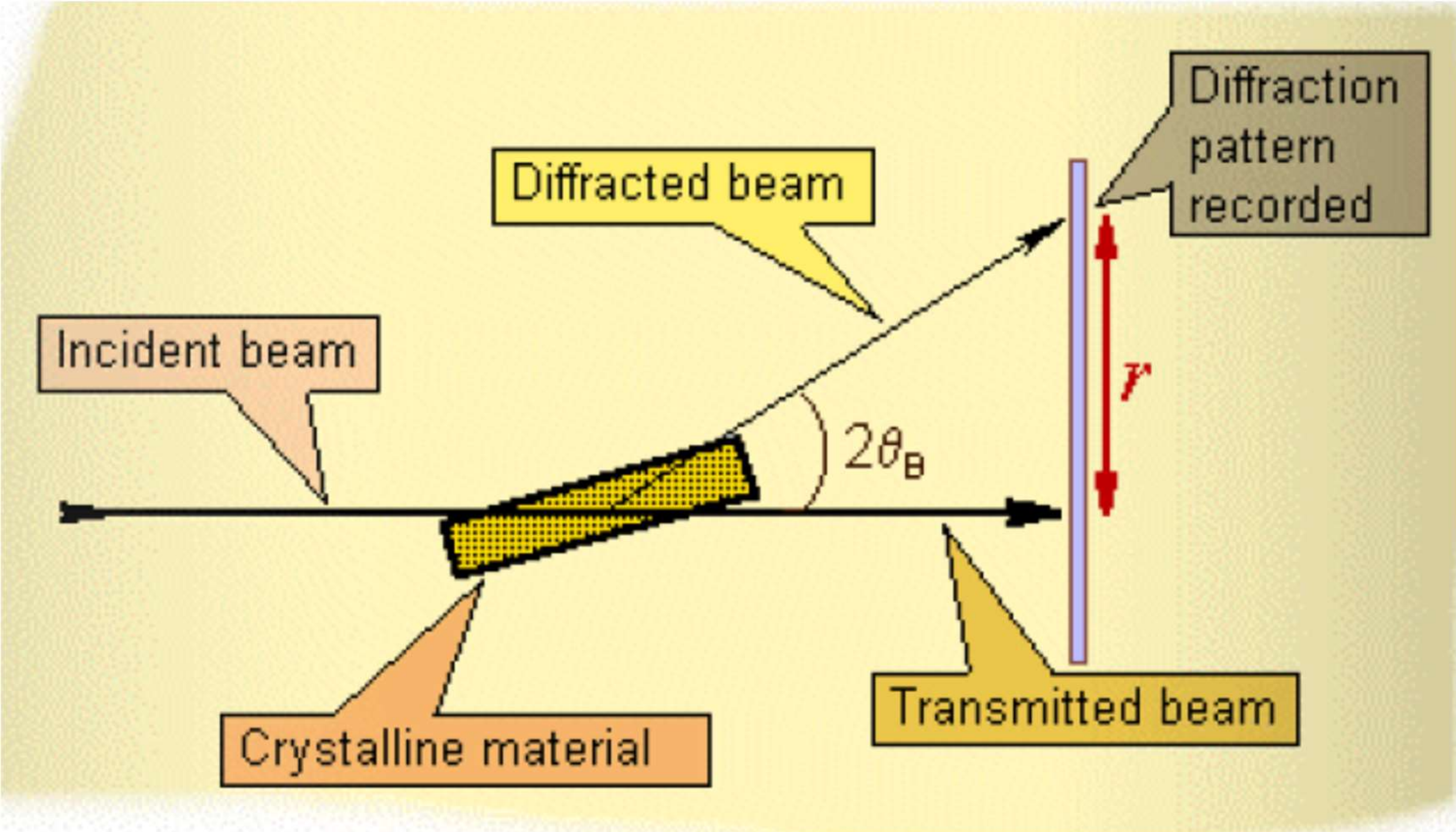


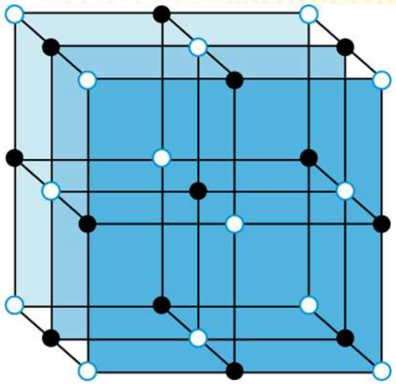
Destructive interference

$$\delta = d \sin\theta = (m + 1/2)\lambda$$



X-ray Diffraction





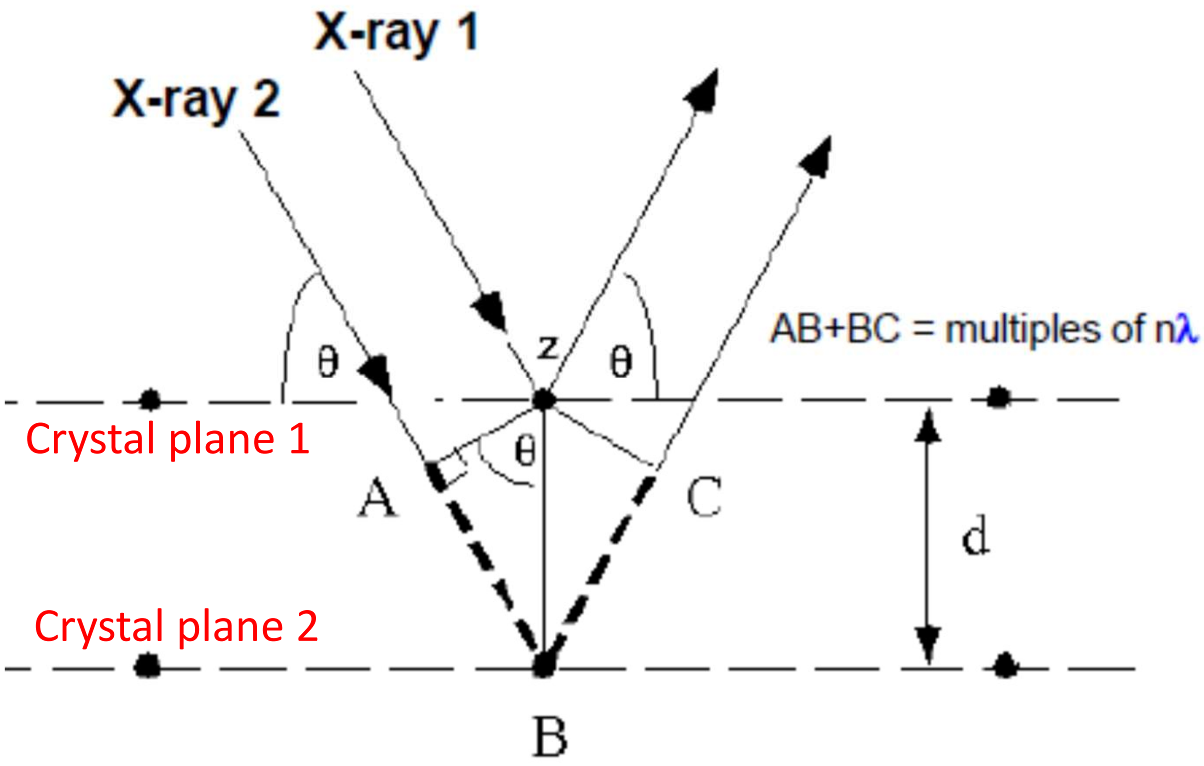
Crystal plane 1

Crystal plane 2

d

Spacing between planes

X-ray Diffraction (1912)



Atoms in successive planes will scatter constructively at an angle θ, if the path difference is integral multiple of wavelength.

Path difference = AB+BC = nλ

AB = BC = d sinθ

Bragg Equation

$$n\lambda = 2d\sin\theta$$

Wave particle Duality: Light

Does light consist of waves or particles?

Phenomenon	Can be explained as waves	Can be explained as particles
Reflection	$\surd$	$\surd$
Refraction	$\surd$	$\surd$?
Interference	$\surd$	$\times$
Diffraction	$\surd$	$\times$
Polarization	$\surd$	$\times$
Photoelectric effect	$\times$	$\surd$
Compton scattering	$\times$	$\surd$

- **Photon can behave like particle or a wave depending upon experiment.**

- **Isn't it logical to assume that the particles may also have some wave property?**

Enters de Broglie (1923)

- *If wave behave like particles, then why not particles behave like waves?*

- ***Everything (matter and radiation) has both wave and particle properties; which property you see depends on the experiment you perform***

de Broglie Wavelength

For a photon, momentum $p = h\nu / c = h / \lambda$

So for a particle of momentum *p*, the wavelength is

$$\lambda_{dB} = h / p = h / m\mathrm{v} = h / \gamma m_0 \mathrm{v}$$

λ_{dB} = de Broglie wavelength

Example 1: *A ball of m=150 g, v=40m/s*

$$\lambda_{dB} = \frac{6 \cdot 626 \times 10^{-34}\, Js}{(0 \cdot 15 kg)(40 m / s)}$$

$$= 1 \cdot 1 \times 10^{-34}\, m$$

Atomic dimension:
10^{-10} m

Nuclear dimension:
10^{-14} m

Example 2: *An electron accelerated through 100V, $v=5.9\times 10^6$ m/s*

$$\lambda_{dB} = \frac{6.626\times10^{-34}\,Js}{(9.11\times10^{-31}\,kg)(5.9\times10^{6}\,m/s)} = 1.2\times10^{-10}\,m$$

λ_{dB} for macroscopic objects are negligibly small

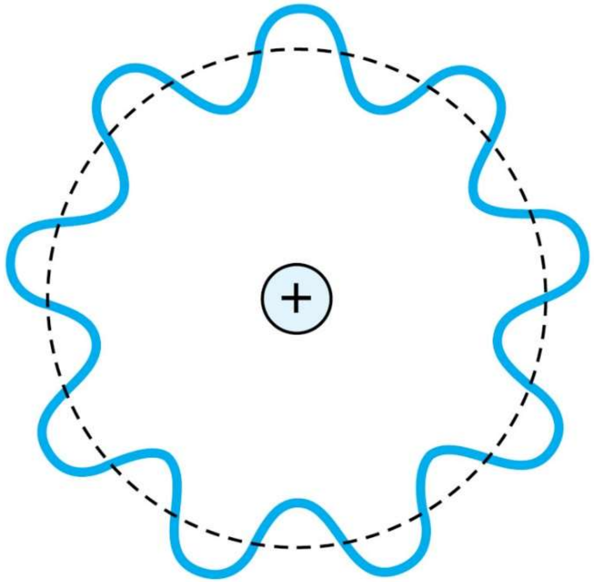
λ_{dB} are important for microscopic objects like electron

Objects that are large in the absolute sense have the property that the wavelength λ_{dB} associated with them are completely negligible compared to their size. Therefore, large particles only manifest particle nature.

de Broglie hypothesis and Bohr's Quantization condition

One of Bohr's assumptions concerning his hydrogen atom model was that in a stationary state

$$L = m_e \mathrm{v r} = n\hbar$$



The de Broglie wavelength of electron is

$$\lambda_{dB} = h / m_e \mathrm{v}$$

The orbits are stable if the perimeter contains integer multiple of de Broglie wavelengths

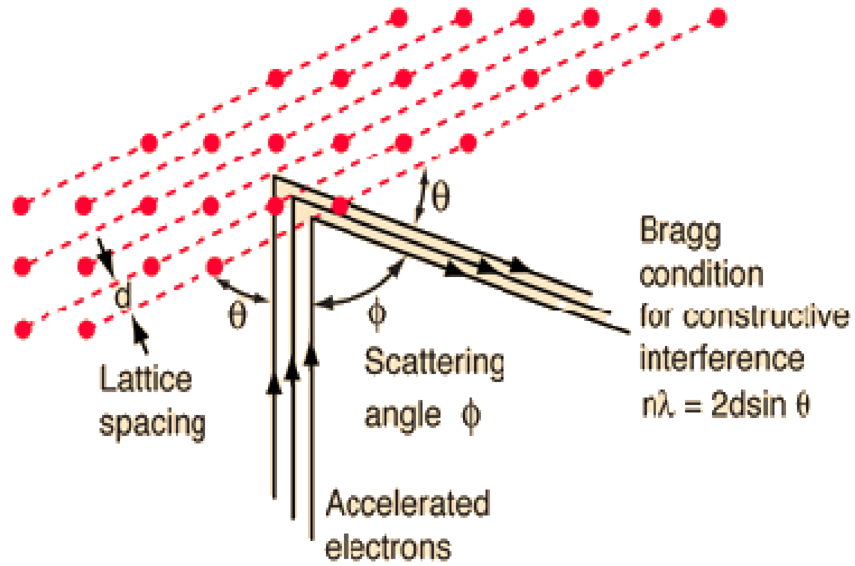
$$2\pi r = n\lambda_{dB} = nh / m_e \mathrm{v} \quad \Rightarrow \quad m_e \mathrm{v} r = nh / 2\pi = n\hbar$$

Testing of de Broglie hypothesis

Wave properties of electron

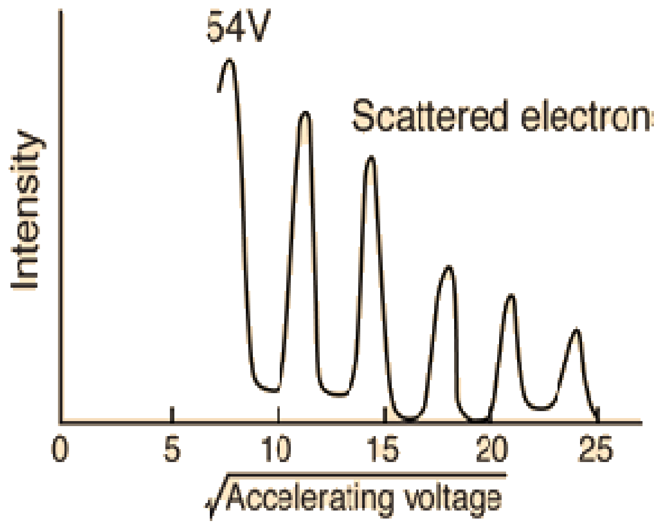
Davisson-Germer Experiment (1927)

Electrons were diffracted from a Nickel crystal similar to x-rays The diffraction peaks satisfy the Bragg condition



$$\frac{1}{\lambda}=\frac{n}{2d\sin\theta}=\frac{p}{h}=\frac{\sqrt{2mE}}{h}=\frac{\sqrt{2meV}}{h}$$

Electron wavelength · *Bragg law* · *deBroglie relationship* · *Acceleration through voltage V*



de Broglie wavelength of electron: $\lambda_{dB} = \dfrac{h}{\sqrt{2meV}}$

λ_{dB} = 1.67 A^o = 1.67 $\times$ 10^{-10} m for 54 eV electron

Can be varied by varying accelerating voltage eV

Bragg Equation: $n\lambda = 2d\sin\theta$

λ Should be of the same magnitude as d

Microscope

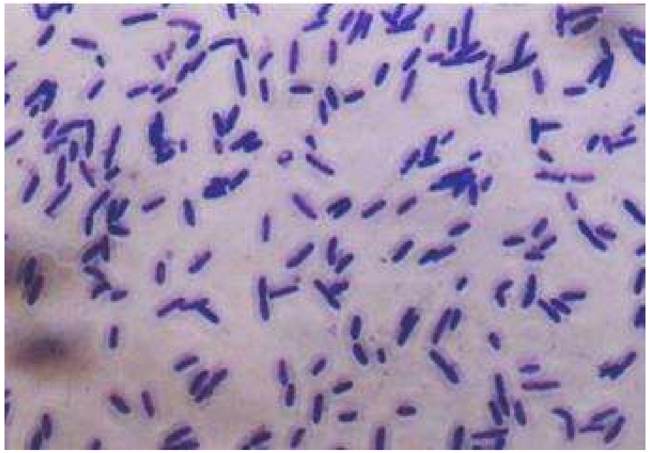
Resolution ~ λ shorter λ, better is resolution

Electron Microscope

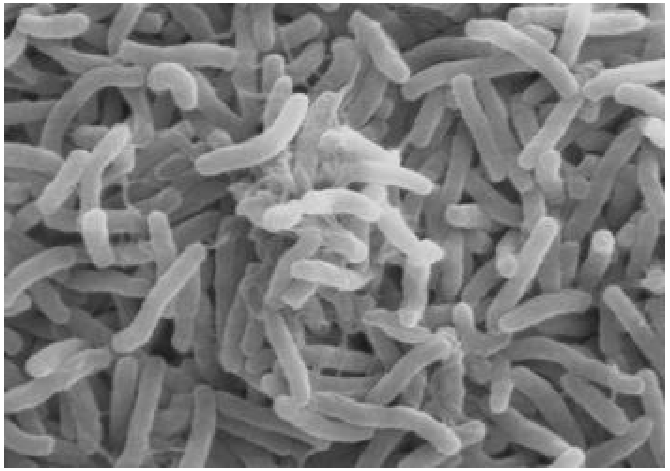
Wavelength versus Size

With a visible light microscope, we are limited to being able to resolve objects which are at least about $0.5*10^{-6}$ m = 0.5 µm = 500 nm in size.

This is because visible light, with a wavelength of ~500 nm cannot resolve objects whose size is smaller than it's wavelength.



Bacteria, as viewed using visible light



Bacteria, as viewed using electrons!

Electron Microscope

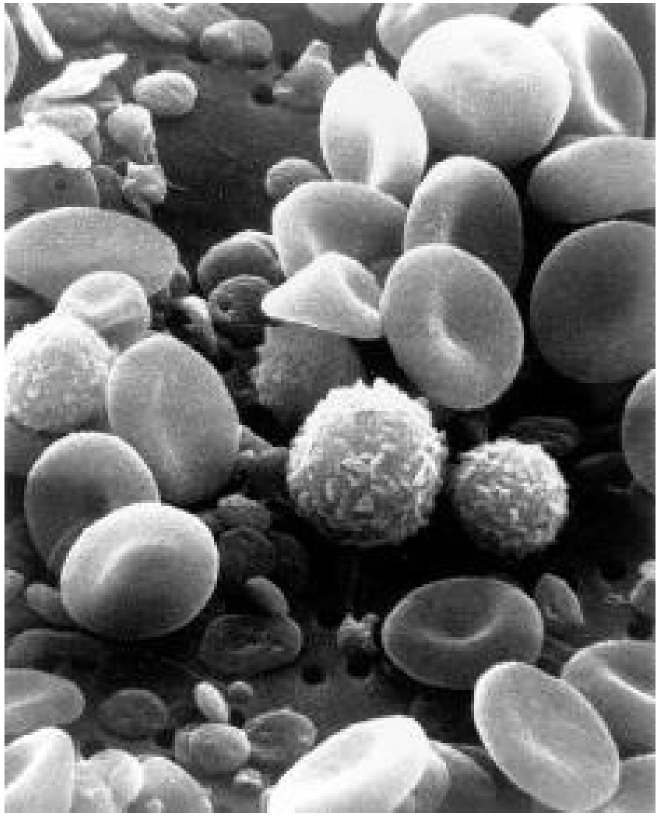
The electron microscope is a device which uses the wave behavior of electrons to make images which are otherwise too small for visible light!

Application of electron wave:
Electron microscope (1931), Nobel Prize 1986 (Ernst Ruska)

The best optical microscope using UV light have magnification of ~ 2000 and **resolution of ~ 100 nm**, however, electron microscope using 100 kV electrons has magnification of as much as 10^7 and **resolution of 0.2 nm.**

Scanning Electron Microscope (SEM)

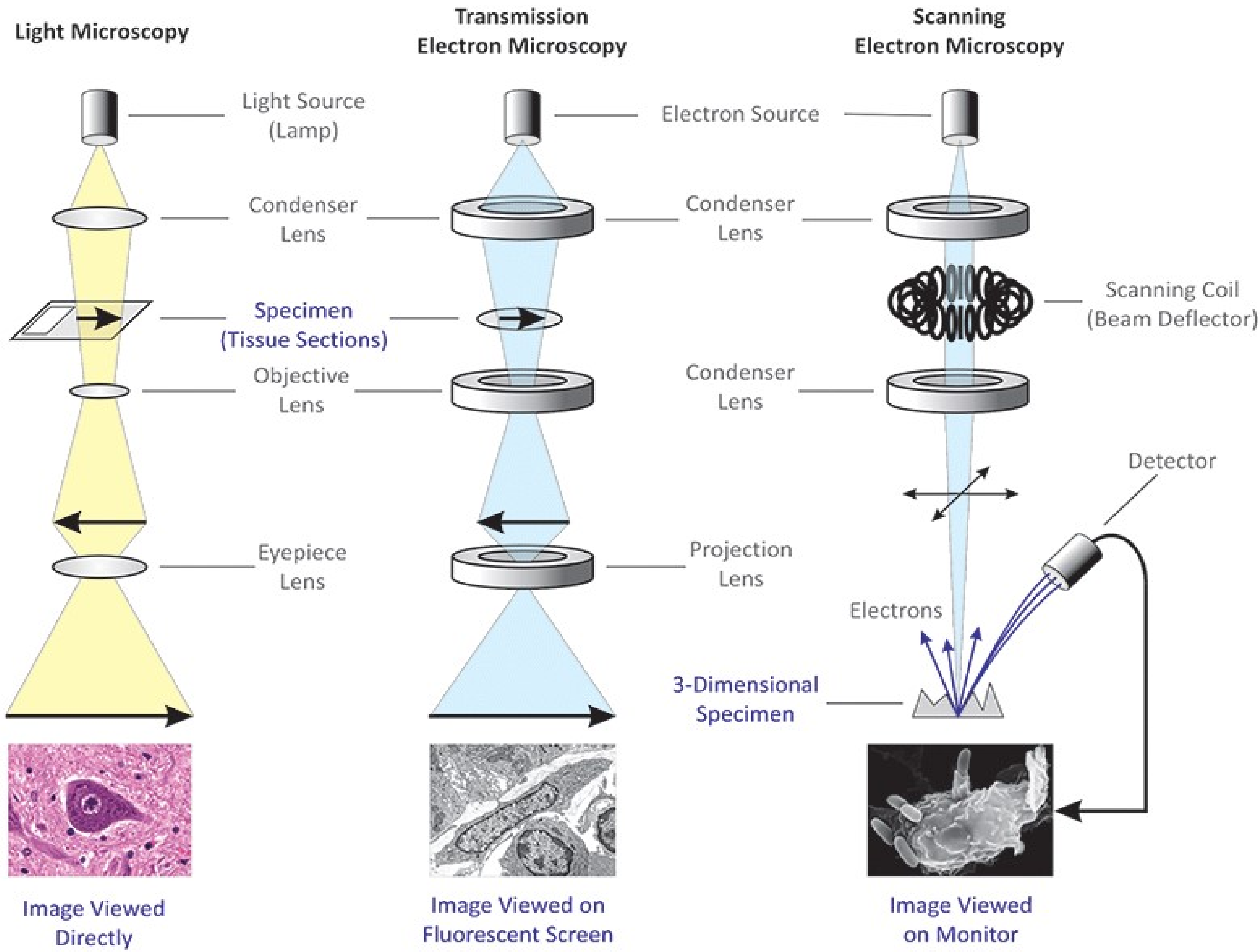
3-d images by SEM. An electron beam (20 KeV) is sharply focused on a specimen and scanned. The low energy scattered secondary electrons are detected.



SEM can resolve features as small as 5 nm. This is about 100 times better than can be done with visible light microscopes!

IMPORTANT POINT:

High energy particles can be used to reveal the structure of matter !



Light Microscopy
Transmission Electron Microscopy
Scanning Electron Microscopy
Light Source (Lamp)
Electron Source
Condenser Lens
Condenser Lens
Scanning Coil (Beam Deflector)
Specimen (Tissue Sections)
Objective Lens
Condenser Lens
Detector
Eyepiece Lens
Projection Lens
Electrons
3-Dimensional Specimen
Image Viewed Directly
Image Viewed on Fluorescent Screen
Image Viewed on Monitor

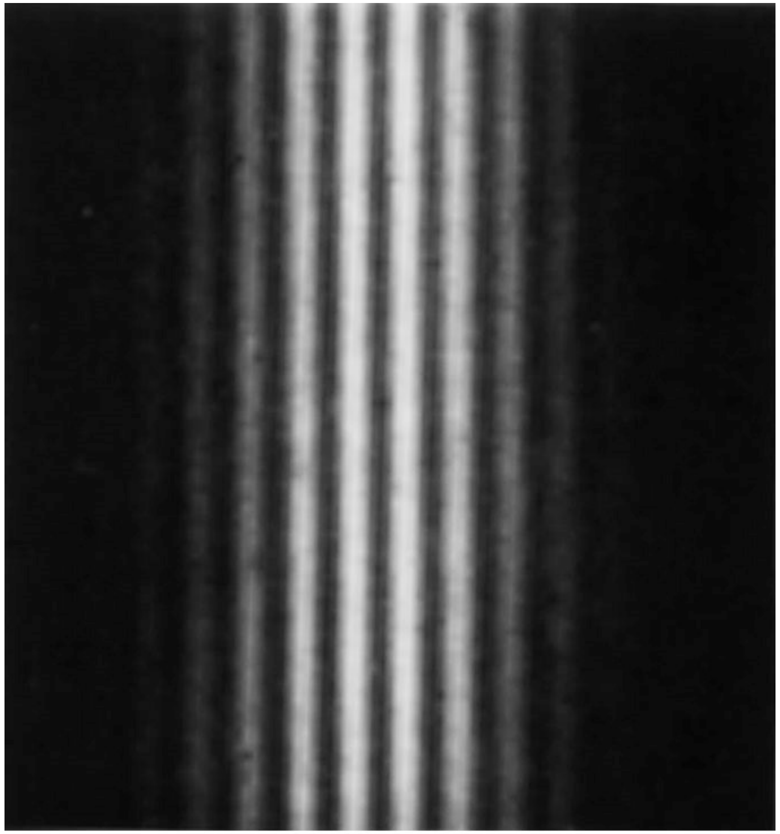
Interference with Electrons

Electron Double Slit Experiment

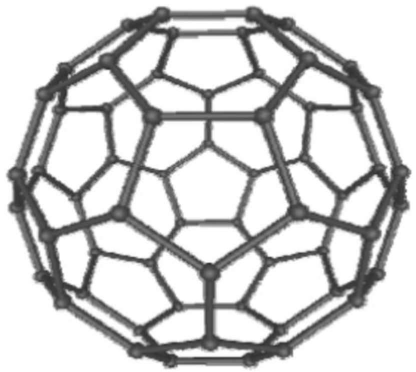
C. Jönsson (Tübingen, Germany, 1961):

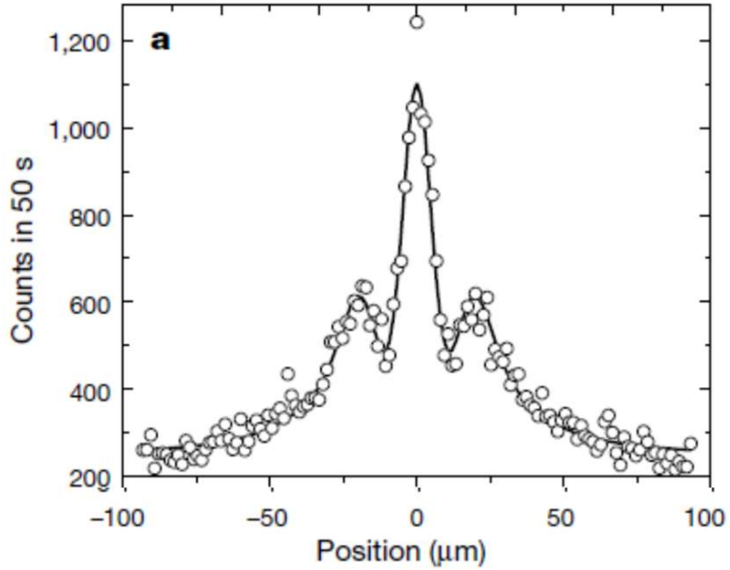
Double-slit interference effects for electrons by constructing very narrow slits and using relatively large distances between the slits and the observation screen.

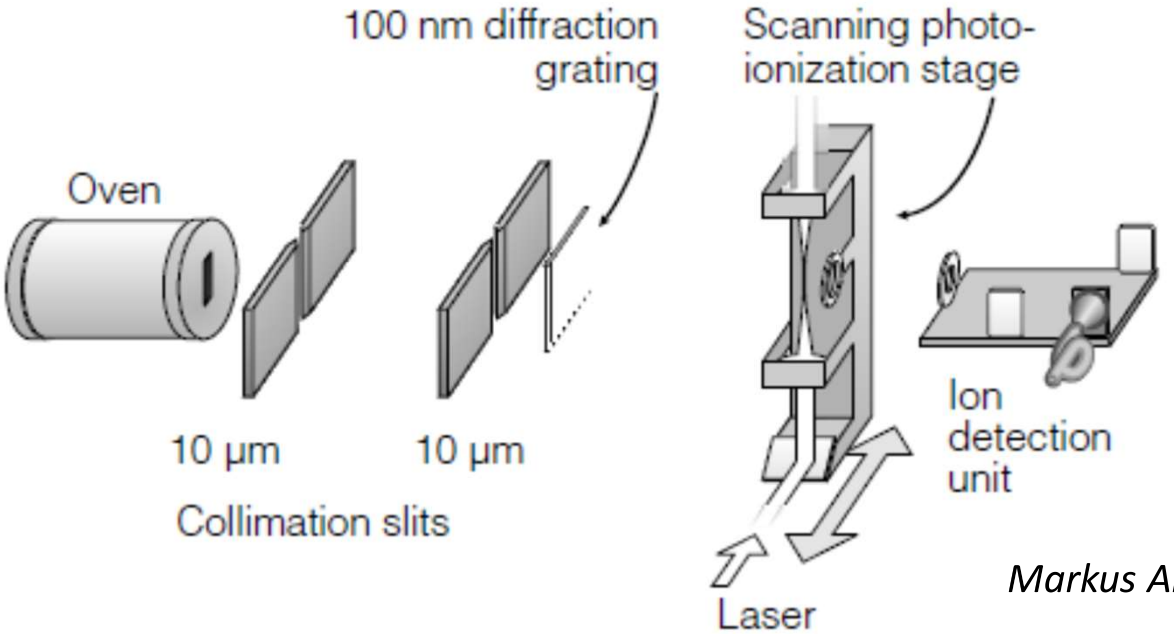
This experiment demonstrated that precisely the same behavior occurs for both light (waves) and electrons (particles).



Wave Property of C_{60} molecule



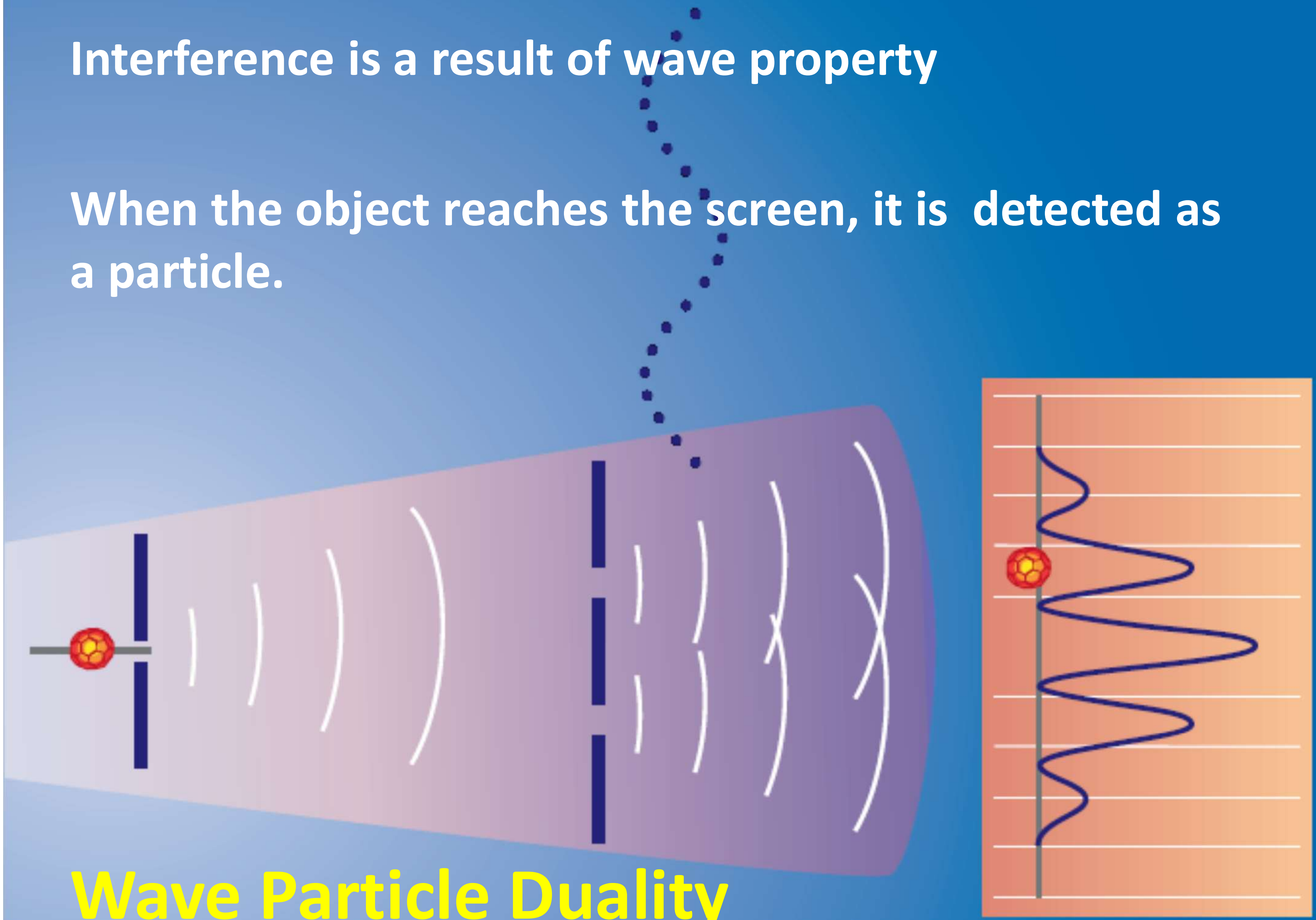




Markus Arndt et al, Nature 401, 680 (1999)

Interference is a result of wave property

When the object reaches the screen, it is detected as a particle.



Wave Particle Duality

Life is strong and fragile. It's a paradox... It's both things, like quantum physics: It's a particle and a wave at the same time. It all exists all together.

Joan Jett