Answers to TUT-6

let

1.) $f(x,y,z) = \frac{x^2}{4} + y^2 + \frac{z^2}{9} - 3$ $P(-2,1,-3)$

$\frac{\partial f}{\partial x} = \frac{2x}{4}$ $\frac{\partial f}{\partial y} = 2y$ $\frac{\partial f}{\partial z} = \frac{2z}{9}$

$f_x|_p = (-1)$ $f_y|_p = 2$ $f_z|_p = -2/3$

Eq^n of Tangent plane

$$(x-x_0)\left(\frac{\partial f}{\partial x}\right)_p + (y-y_0)\left(\frac{\partial f}{\partial y}\right)_p + (z-z_0)\left(\frac{\partial f}{\partial z}\right)_p = 0$$

$$(x+2)(-1) + (y-1)\,2 + (z+3)\,-2/3 = 0$$

$$-3(x+2) + 6(y-1) + -2(z+3) = 0$$

$$-3x + 6y - 2z - 6 - 6 - 6 = 0$$

Ans:(1) $3x - 6y + 2z + 18 = 0$ $[3x - 6y + 2z = -18]$

Eq^n of ~~Tangent~~ Normal line

$$\frac{x-x_0}{\left(\frac{\partial f}{\partial x}\right)_p} = \frac{y-y_0}{\left(\frac{\partial f}{\partial y}\right)_p} = \frac{z-z_0}{\left(\frac{\partial f}{\partial y}\right)_p}$$

$$\frac{(x+2)}{-1} = \frac{(y-1)}{2} = \frac{(z+3)}{(-2/3)}$$

Ans: $\frac{(x+2)}{-1} = \frac{(y-1)}{2} = \frac{(3z+9)}{-2}$

2.) $f(x,y,z) = x^2+y^2+z^2-3$ $P(1,1,1)$

$\frac{\partial f}{\partial x} = 2x$ $\frac{\partial f}{\partial y} = 2y$ $\frac{\partial f}{\partial z} = 2z$

$f_x|_p = 2$ $f_y|_p = 2$ $f_z|_p = 2$

Eq^n of Tangent Plane,

$$(x-x_0)\left(\frac{\partial f}{\partial x}\right)\Big|_p + (y-y_0)\left(\frac{\partial f}{\partial y}\right)\Big|_p + (z-z_0)\left(\frac{\partial f}{\partial z}\right)\Big|_p = 0$$

$$(x-1)(2) + (y-1)\,2 + (z-1)\,2 = 0$$

$$[x+y+z=3] \quad -(1)$$

Eq^n of Normal Line

$$\frac{(x-x_0)}{(\partial f/\partial x)|_p} = \frac{(y-y_0)}{(\partial f/\partial y)|_p} = \frac{(z-z_0)}{(\partial f/\partial z)|_p}$$

Ans: $\left[\frac{(x-1)}{2} = \frac{(y-1)}{2} = \frac{(z-1)}{2}\right] -(2)$

Ans: $\frac{(x-1)}{1} = \frac{(y-1)}{1} = \frac{(z-1)}{1}$

3.) Plane $4x-6y-z+14=0$ is tangent to $f(x,y,z) = x^2+3y^2+2z$

Let point be (x_0, y_0, z_0)

$\therefore$ Plane $4x-6y-z+14=0$ is tangent plane

$$(x-x_0)\frac{\partial f}{\partial x}\Big|_p + (y-y_0)\left(\frac{\partial f}{\partial y}\right)\Big|_p + (z-z_0)\left(\frac{\partial f}{\partial z}\right)\Big|_p = 0$$

$\frac{\partial f}{\partial x} = 2x$

$$(x-x_0)\,2x_0 + (y-y_0)\,6y_0 + (z-z_0)\,2 = 0$$

$\frac{\partial f}{\partial y} = 6y$

$$2x_0\,x + 6y_0\,(y) + 2z - 2z_0 - 2x_0^2 - 6y_0^2 = 0$$

$\frac{\partial f}{\partial z} = 2$

$$-x_0\,x - 3y_0\,(y) - z + z_0 + x_0^2 + 3y_0^2 = 0$$

{Comparing coefficients} $4x - 6y - z + 14 = 0$

$-x_0 = 4 \qquad [x_0 = -4]$

$-3y_0 = -6 \qquad [y_0 = 2]$

$x_0^2 + 3y_0^2 + z_0 = 14$

$16 + 12 + z_0 = 14 \qquad z_0 = 14 - 28$

$= -14$

Ans: The point is $(-4, 2, -14)$

4.> $R = \frac{E}{I}$ $\qquad I = 20 \pm 0.1$ A

$E = 120 \pm 0.05$ V

$R = 6$

max error $= \delta R = (?)$ $\qquad$ % error $= \frac{\delta R}{R} \times 100\%$

$\log R = \log E - \log I$

$\frac{\delta R}{R} = \frac{\delta E}{E} \pm \frac{\delta I}{I}$

$\delta R = R\left(\frac{0.05}{120} \pm \frac{0.1}{20}\right)$

$\delta R = 6\left(4.16 \times 10^{-4} \pm 5 \times 10^{-3}\right)$

$\delta R = 6 \times \cancel{54.16}\ (45.84) \times 10^{-4}$

$= \cancel{324.96 \times 10^{-4}}$

$= \cancel{0.032496}$

$\delta R = 275.04 \times 10^{-4}$

Ans $\boxed{\delta R = 0.0275}$

% Error $= \frac{\delta R}{R} \times 100 = 0.4583$

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5.) $P = \frac{E^2}{R}$ $E = 200 - 5$ V $\delta E = -5$ V

$R = 8 - 0.20\ \Omega$ $\delta R = -0.20\ \Omega$

$\log P = 2\log(E) - \log(R)$

$P = \frac{E^2}{R}$

$\frac{\delta P}{P} = 2\frac{\delta E}{E} \quad \frac{\delta R}{E}$

$= \frac{(200)^2}{(8)}$

$\frac{\delta P}{P} = 2\frac{(-5)}{(200)} - \frac{(-0.20)}{(8)}$

$= 5000$ W

$= \frac{-10}{200} + \frac{0.20}{8}$

$= (5000)\left(\frac{-1}{20} + \frac{0.20}{8}\right)$

$= 5000\ (-0.025)$

$[\delta P = -125]$

The power decreases by 125 W.

6.) $T = 2\pi\sqrt{L/g}$ $\frac{\delta L}{L} \times 100\% = 1\%$ $\frac{\delta g}{g} \times 100\% = 2.5\%$

$\log T = \log(2\pi) + \frac{1}{2}\log(L) - \frac{1}{2}\log(g)$

$\frac{\delta T}{T} = (0) + \frac{\delta L}{(2)L} - \frac{\delta g}{2g}$

$\frac{\delta T}{T} \times 100\% = \frac{1}{2}\left(\frac{\delta L}{L} \times 100\%\right) - \frac{1}{2}\left(\frac{\delta g}{g} \times 100\right)$

$= \frac{1}{2}(1) - \frac{1}{2}(2.5)$

$\frac{\delta T}{T} \times 100 = \frac{-1.5}{2}$

$\frac{\delta T}{T} \times 100 = -0.75$

The maximum error in Time period

Ans: is 0.75%.

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7.) $I = \frac{PLAN}{33000} = \frac{PL(\pi d^2)N}{33000}$

$r\%$ error in P, L, N & 'd', find error in I

$\log I = \log P + \log(L) + 2\log(d) + \log N + \log(\pi) - \log(33000)$

$$\frac{\delta I}{I} = \frac{\delta P}{P} + \frac{\delta L}{L} + 2\frac{\delta d}{d} + \frac{\delta N}{N}$$

$$\frac{\delta I}{I} \times 100\% = \left(\frac{\delta P}{P} \times 100\%\right) + \left(\frac{\delta L}{L} \times 100\%\right) + 2\left(\frac{\delta d}{d} \times 100\%\right) + \left(\frac{\delta N}{N} \times 100\%\right)$$

$$= r\% + r\% + 2r\% + r\%$$

Ans: Max^m Error in I $= 5r\%$

8.) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ Area of Ellipse $= \pi ab$

$\frac{\delta a}{a} \times 100\% = 1\%$ $\frac{\delta b}{b} \times 100\% = 1\%$

$\log(A) = \log(\pi) + \log(a) + \log(b)$

$$\frac{\delta A}{A} = 0 + \frac{\delta a}{a} + \frac{\delta b}{b}$$

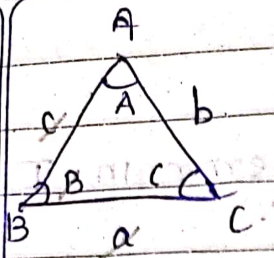
$$\frac{\delta A}{A} \times 100\% = \frac{\delta a}{a} \times 100 + \frac{\delta b}{b} \times 100$$

$$= 1\% + 1\%$$

$$\frac{\delta A}{A} \times 100\% = 2\%$$

Ans: Maximum possible error in Area $= 2\%$

9.)



S.t.b $\angle A, \angle C$, side b

$$A = \frac{1}{2}(b)(c)\sin(A)$$

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{(c)}$$

$$c = \frac{\sin(C)\,(a)}{\sin(A)} \quad -(1)$$

$$\sin(\pi-(A+C)) = \frac{b}{a}\sin(A) \qquad \sin(B)$$

$$\text{Area} = \frac{1}{2}(b)\left(\frac{\sin(C)}{\cancel{\sin(A)}}(a)\right) \cancel{\sin}(A) \qquad a = \frac{b\sin(A)}{\sin(\pi-(A+C))}$$

$\longrightarrow$ using sine rule

$$= \frac{1}{2}\, b\, \sin(C)\, \frac{b\sin(A)}{\sin(\pi-(A+C))}$$

$$\boxed{\cancel{(A)}P = \frac{b^2}{2}\,\frac{\sin(A)\sin(C)}{\sin(A+C)}} \quad \longleftarrow (1)$$

let Area of Triangle be P $A = \dfrac{b^2\sin(C)}{2}\,\dfrac{\sin(A)}{\sin(A+C)}$

(A = Angle A) Differentiate w.r.t to 'A'

$$\frac{\delta P}{\delta A} = \frac{b^2\sin(C)}{2}\,\frac{(\sin(A+C)\cos(A) - \sin(A)\cos(A+C))}{(\sin(A+C))^2}$$

$$= \frac{b^2\sin(C)}{2}\,\frac{(\sin(A+C-A))}{(\sin(A+C))^2}$$

$$\frac{\delta P}{\cancel{\delta}} = \left(\frac{b^2\sin(C)}{2}\,\frac{\sin(C)}{(\sin(A+C))^2}\right)\delta A \quad -(2)$$

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$$\frac{\delta P}{P} = \frac{\frac{b^2}{2}(\sin(C))^2 (\delta A)}{\frac{b^2}{2} \frac{(\sin(A+C))^2}{\frac{\sin(A)\ \sin(C)}{\sin(A+C)}}}$$

Ans: $\frac{\delta P}{P} = \frac{\sin(A)\ \sin(C)\ \delta A}{\sin(A+C)}$, Hence Found.

10.) $[(2.92)^3 + (5.78)^3]^{1/5}$

$= x + \delta x$

let $U = [x^3 + y^3]^{1/5}$ $\quad x = 3 - 0.08$

$U^5 = x^3 + y^3$ $\quad y = 6 - 0.22$

$5U^4\ \delta U = 3x^2\ \delta x + 3y^2 . \delta y$ $\quad = y + \delta y$

$$(\delta U) = \frac{3x^2\ \delta x + 3y^2\ \delta y}{5U^4}$$

$$= \frac{3}{5} \frac{((3)^2(-0.08) + (6)^2(-0.22))}{(3)^4}$$

$$= \frac{1}{15}(-0.08 + -0.88) = \frac{-0.96}{15} = -0.064$$

$U = [(3)^3 + (6)^3]^{1/5} = ((3)^3(1+8))^{1/5} = (3)$

Ans: $U + \delta U = 3 - 0.064$

$= 2.936$
