10 KINETICS OF PARTICLES

In the previous chapter, we have studied a *kinematics* in which only the *geometry of the motion* is considered *without* considering the *cause of motion* i.e. *without* considering the *force* applied for creating motion.

Now, in the *kinetics* we have to establish the *relations between forces acting on the body, mass of the body* and *motion of the body*. Thus the *cause of motion*, i.e. the *force* required or applied for creating a motion is *to be considered* here.

10·1. Newton's Second Law of Motion :

It states that "If the resultant force acting on a particle is not zero, the particle will have an acceleration proportional to the magnitude of the resultant and in the direction of this resultant force.

Fig 10·1

Under the action of the force, the particle will be observed to move in a straight line and in the direction of the force. If the particle is applied repeatedly with forces F_1, F_2, F_3 etc. of *different magnitude and/or direction*, then we find each time that it moves with accelerations $\bar{a}_1$, $\bar{a}_2$, $\bar{a}_3$ with magnitudes a_1, a_2, a_3 *proportional to the* magnitudes of the forces F_1, F_2, F_3.

Thus,

$$\frac{F_1}{a_1} = \frac{F_2}{a_2} = \frac{F_3}{a_3} = \text{constant.}$$

The constant value is a characteristic of the particle and is called *mass of the particle* (m). We can say

$$\boxed{\bar{F} = m\,\bar{a}}$$

when a particle is subjected simultaneously to *several forces*, it can be stated that

$$\boxed{\Sigma\,\bar{F} = m\,\bar{a}}$$

where $\Sigma\,\bar{F}$ = *sum*, or *resultant*, of all forces acting on the particle.

10·2. Linear Momentum of a Particle and Rate of Change of Linear Momentum :

We know that, *acceleration is a rate of change of velocity*.

$$\bar{a} = \frac{d\,\bar{v}}{dt}$$

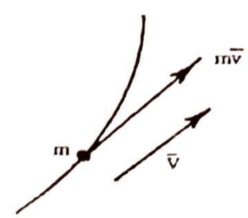


Fig. 10·2

and hence

$\Sigma\,\bar{F} = m\,\bar{a}$ becomes

$$\Sigma\,\bar{F} = m\,\frac{d\,\bar{v}}{dt}$$

$$\therefore \quad \Sigma\,\bar{F} = \frac{d}{dt}(m\,\bar{v})$$

but $\boxed{m\,\bar{v} = \bar{L}}$ $\quad \bar{L}$ = Linear momentum

$\therefore$ $\boxed{\Sigma\,\bar{F} = \dot{\bar{L}}}$ $\quad \dot{\bar{L}}$ = Rate of change of linear momentum.

Here, the $m\,\bar{v}$ is a vector called *linear momentum* or simply *momentum*. It has the *same direction as the velocity* of the particle and magnitude, L = mv

where v = *speed* of the particle.

Thus the above last equation can be stated as "*the resultant of the forces acting on the particle is equal to the rate of change of the linear momentum of the particle.*"

When $\dot{\bar{L}} = \dfrac{d\,(m\,\bar{v})}{dt} = 0$, then $\Sigma\,\bar{F} = 0 \;\therefore\; \boxed{m_1 v_1 = m_2 v_2}$

Thus, "*if the resultant force acting on a particle is zero, the linear momentum of the particle remains constant, both in magnitude and* *direction.* "This is the ***principle of conservation of linear momentum for a particle*** **(alternative statement of Newton's First Law).**

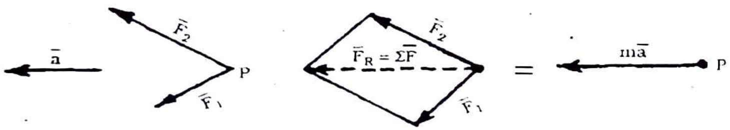
10·3. Equations of Motion :

When a particle of mass m is acted upon by several forces, the resultant force is determined by a vector summation of all the forces. i.e.

$$\boxed{\bar{F}_R = \Sigma\, \bar{F}}$$

and for this more general case, the equation of motion may be written as,

$$\boxed{\Sigma\, \bar{F} = m\, \bar{a}}$$



Free-body diagram (a) **Kinetic diagram (b)**

Fig. 10·3

It will be found more convenient **to replace the above equation of motion by *equivalent equations involving scalar quantities.***

(1) Rectangular Components :

We know that

$$\Sigma\, \bar{F} = m\, \bar{a}$$

$$\therefore \quad \Sigma F_x\, \bar{i} + \Sigma F_y\, \bar{j} + \Sigma F_z\, \bar{k} = m\,(a_x\, \bar{i} + a_y\, \bar{j} + a_z\, \bar{k})$$

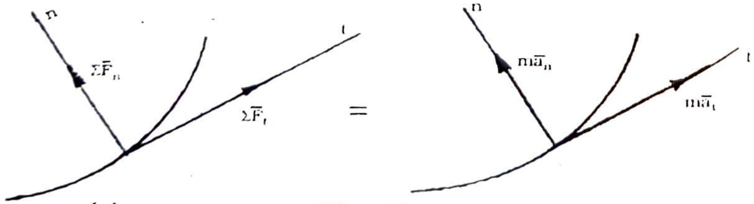
From which,

$$\boxed{\begin{aligned} \Sigma F_x &= m\, a_x = m\, \ddot{x} \\ \Sigma F_y &= m\, a_y = m\, \ddot{y} \\ \Sigma F_z &= m\, a_z = m\, \ddot{z} \end{aligned}}$$

Scalar eqns. of rectangular components.

(2) Tangential and Normal Components :

When particle *moves over a curved path* which is known, the equation of motion for the particle may be written in the *normal and tangential directions.*



(a) **Fig. 10·4** **(b)**

Thus,

$$\boxed{\begin{aligned} \Sigma F_t &= m a_t \\ \Sigma F_n &= m a_n \end{aligned}}$$

but, we know that

$$a_t = \frac{dv}{dt} \quad \text{and} \quad a_n = \frac{v^2}{\rho}$$

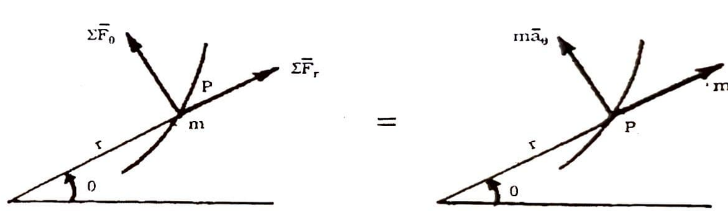
hence,

$$\therefore \quad \boxed{\begin{aligned} \Sigma F_t &= m\, \frac{dv}{dt} \\ \Sigma F_n &= m\, \frac{v^2}{\rho} \end{aligned}}$$

Scalar eqns. of tangential and normal components

(3) Radial and Transverse Components :

The *polar coordinates of* particle P, moving in a plane under the action of several forces, are r and θ.



(a) **(b)**

Fig. 10·5

The radial and transverse components of forces and acceleration of the particle can be equated as

$$\boxed{\begin{aligned} \Sigma F_r &= m a_r \\ \Sigma F_\theta &= m a_\theta \end{aligned}}$$

but, we know that

$$a_r = \ddot{r} - r\,\dot{\theta}^2 \quad \text{and}$$
$$a_\theta = r\,\ddot{\theta} + 2\,\dot{r}\,\dot{\theta}$$

hence,

$$\boxed{\begin{aligned} \Sigma F_r &= m\,(\ddot{r} - r\,\dot{\theta}^2) \\ \Sigma F_\theta &= m\,(r\,\ddot{\theta} + 2\,\dot{r}\,\dot{\theta}) \end{aligned}}$$

Scalar eqns. of radial and tangential components

10·4. Dynamic Equilibrium :

The equation of motion can also be rewritten in the form,

$$\boxed{\Sigma\, \bar{F} - m\, \bar{a} = 0}$$

The vector $- m\, \bar{a}$ is referred to as ***inertia vector*** or ***inertia force*** having **magnitude ma** and of ***direction opposite to** that of* **the *acceleration.***

Thus if we **add inertia vector** to the system of forces then the state of **"equilibrium"** created which is called the **dynamic equilibrium.** *The particle may thus be considered to be in equilibrium under the given forces and the inertia vector.*

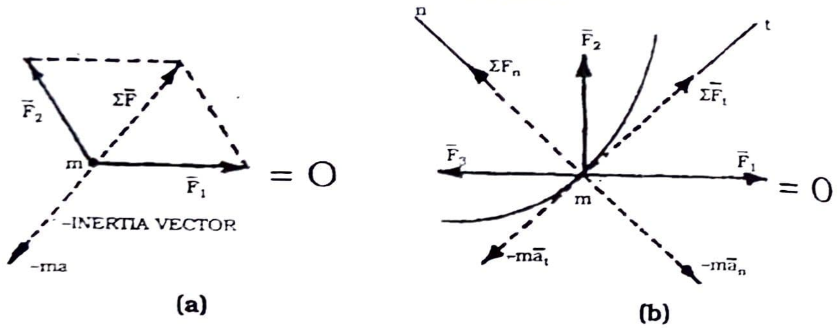


Fig. 10·6

We may draw in tip-to-tail fashion all the coplanar forces, including inertia vector, to form a *closed - vector polygon.*

Using *Rectangular components*, for **dynamic equilibrium,**

the conditions are $\boxed{\Sigma F_x = 0 \quad \Sigma F_y = 0}$ **including inertia vector**

Using *Tangential & Normal components*, for *dynamic equilibrium*,

the conditions are $\boxed{\Sigma F_t = 0 \quad \Sigma F_n = 0}$ **including inertia vector**

$-m\,\bar{a}_t =$ ***tengential component of inertia vector*** *provides resistance hence* ***offers to a change in speed*** *of the particle.*

$-m\,\bar{a}_n =$ ***normal component of inertia vector,*** *(centrifugal force) due to which particle* ***tries to leave its curved path.***

Static Equilibrium : *(Newton's First law)*

$F_R = \Sigma\,\bar{F} = 0,\ a = 0,$ *particle at rest* or *move with constant velocity.*

Dynamic Equilibrium : (Newton's Second law)

$\Sigma\,\bar{F} - m\,\bar{a} = 0$ Here, $\Sigma\,\bar{F} \neq 0$ but *with inertia vector, particle remains in equilibrium.*

$-m\,\bar{a}$ *depends upon characteristic of particle.*

The dynamic equilibrium equation (Newton's second law) expresses that ***if we add the vector –ma to the forces acting on the particle, wa obtain a system of vectors*** equivalent to zero [Fig. 10·6 (a)].

The vector –ma, of magnitude ma and of **direction opposite** *to that of the acceleration, is called an* **inertia vector.**

10·5 Angular Momentum of a Particle. Rate of Change of Angular Momentum :

The moment about Oof the linear momentum $m\,\bar{v}$ *is called* ***the moment of momentum or the angular momentum*** *of the particle about Oat that instant and is denoted by* $\bar{H}_o$.

Thus, $\boxed{\bar{H}_o = \bar{r} \times m\,\bar{v}}$

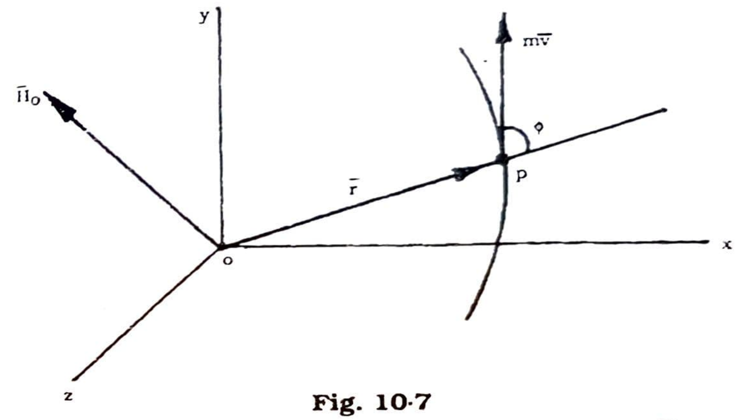


Fig. 10·7

$\bar{H}_o$ is a vector perpendicular to the plane containing $\bar{r}$ and $m\,\bar{v}$.

The magnitude of $\bar{H}_o$ is

$$\boxed{H_o = r\,mv\sin\phi}$$

where ϕ = angle between $\bar{r}$ and $m\bar{v}$. Again the determinant form of H_o will be

$$\bar{H}_o = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ x & y & z \\ mv_x & mv_y & mv_z \end{vmatrix}$$

Where, x, y, z = coordinates of particle P

mv_x, mv_y, mv_z = components of linear momentum mv.

Here again,

$$\bar{H}_o = H_x\,\bar{i} + H_y\,\bar{j} + H_z\,\bar{k}$$

∴ The components of $\bar{H}_o$ will be,

$$H_x = m\,(yv_z - zv_y)$$
$$H_y = m\,(zv_x - xv_z)$$
$$H_z = m\,(xv_y - yv_x)$$

Using polar coordinate system, resolving **linear momentum** in to radial and transverse components as shown below, we may write

$$H_o = r\,mv \sin\phi = r\,mv_0$$

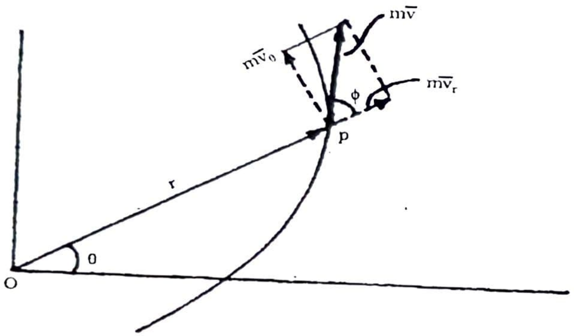


Fig. 10·8

but $\boxed{v_0 = r\dot{\theta}}$

$\therefore$ $\boxed{H_o = m\,r^2\dot{\theta}}$

Rigid Body is made up of number of particles, and let ω is the angular velocity of the plane body at the instant considered. The angular momentum $\mathbf{H_G}$ of a plane body about its mass center G may be computed as

$$H_G = \omega \sum_{i=1}^{n} r_i^2 \Delta m_i$$

Recalling that the sum $\Sigma r_i^2 \Delta m_i$ represents the moment of inertia ($\bar{I}$) of the plane body about a centroidal axis perpendicular to the slab,

$$\boxed{H_G = \bar{I}\omega}$$

The derivative of $\bar{H}_o$ *with respect to t is*

$$\frac{d\bar{H}_o}{dt} = \dot{H}_o = \frac{d}{dt}(\bar{r} \times m\bar{v}) = \dot{\bar{r}} \times m\bar{v} + \bar{r} \times m\dot{\bar{v}}$$

but $\dot{\bar{r}} = \bar{v}$ and $\dot{\bar{v}} = \bar{a}$

$$\dot{H}_o = \bar{v} \times m\bar{v} + \bar{r} \times m\bar{a}$$

but $\bar{v}$ and $m\bar{v}$ vectors are collinear

$$\dot{H}_o = 0 + \bar{r} \times \Sigma\bar{F}$$

$$= 0 + \Sigma \bar{M}_o$$

$$\therefore \boxed{\Sigma M_o = \dot{H}_o}$$

Which states that, the ***sum of the moments about O of the forces acting on the particle is equal to the rate of change of moment of the momentum (angular momentum) of the particle about O.***

For plane rigid body

$$H_G = \bar{I}\omega = \bar{I}\alpha$$

Thus the rate of change of the angular momentum of the plane rigid body is represented by a vector of the same direction as α (that is, perpendicular to the pane body) and of magnitude $\bar{I}\alpha$.

10·6. Plane Motion of a Rigid Body : D' Alembert's Principle :

Consider a rigid plane (slab) of mass m moving under the action of several external forces $\bar{F}_1$, $\bar{F}_2$, $\bar{F}_3$ etc., contained in the plane of the slab (Fig. 10·9).

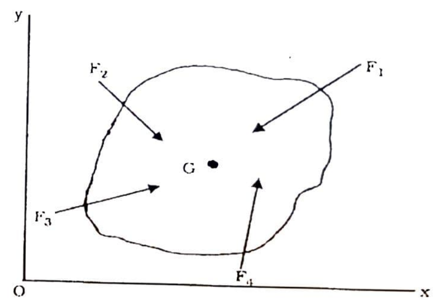


Fig. 10·9

Writing the **fundamental equations of motion** in scalar form, we have

$$\Sigma F_x = m\bar{a}_x$$

$$\Sigma F_y = m\bar{a}_y$$

$$\Sigma M_G = \bar{I}\alpha$$

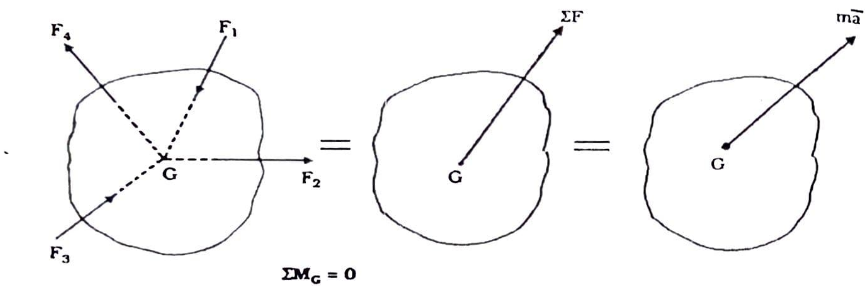
Where, α = Angular acceleration $= \dfrac{d^2\theta}{dt^2}$

$\bar{I}$ = moment of Inertia about a *centroidal axis perpendicular to the plane.*

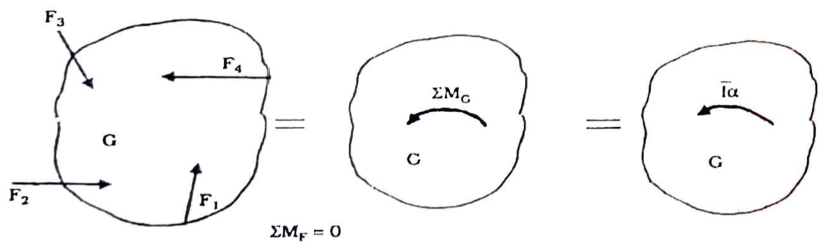
Thus, *the motion of the plane slab is completely defined by the force resultant and moment resultant about G of the external forces acting on it.*

D' Alembert's Principle after the french mathematician Jean le Rond d'Alembert (1717 – 1783) states that, "*the external forces acting on a* ***rigid body*** *are equivalent to the effective forces of the various* ***particles*** *forming the body.*"

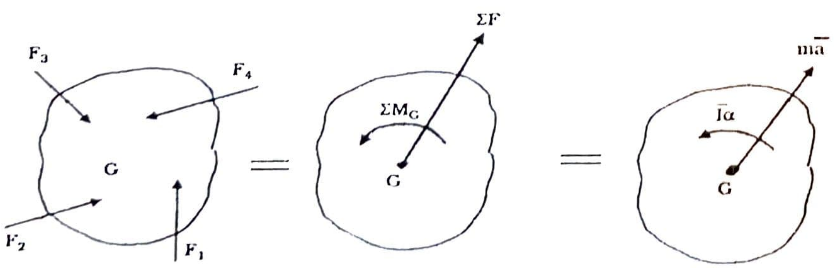
The effective forces shown in fig. 10·9 can be replaced by a vector $m\bar{a}$ attached at the mass center G of the plane slab and a couple of moment $\bar{I}\alpha$.



Only Translation, Concurrent Forces
Angular Acceleration α = O, No Rotation
Fig. 10·10



Only Centroidal Rotation, Nonconcurrent Forces
Linear Acceleration = $\bar{a}$ = 0, No Translation
Fig. 10·11



Both Translation & Rotation, Nonconcurrent Forces
Linear Acceleration ($\bar{a}$) & Angular Acceleration (α) ≠ 0
Fig. 10·12

In a particular case of a body in *translation* (Fig. 10·10), the angular acceleration of the body is identically equal to zero and its effective forces reduce to the vector $m\bar{a}$ attached at G. Thus, *the resultant of the external forces acting on a rigid body in translation passes through the mass center of the body (G) and is equal to $m\bar{a}$.*

When a plane slab rotates about a fixed axis perpendicular to the reference plane and passing through its mass center G, we say that the body is in *centroidal rotation.* Since the acceleration $\bar{a}$ is equal to zero, the effective forces of the body reduce to the couple $\bar{I}\alpha$ (Fig. 10·11). Thus, *the external forces acting on a body in centroidal rotation are equivalent to a couple of moment $\bar{I}\alpha$.*

The most general plane motion of a rigid body symmetrical with respect to the reference plane may be replaced by the *sum of translation and centroidal rotation* (Fig. 10·12).

According to Newton's second Law : **Under Motion**	**Under Equilibrium Condition**
$\left.\begin{aligned}\Sigma F &= m\bar{a}\\ \Sigma M &= \bar{I}\alpha\end{aligned}\right\}$ **(A)**	$\left.\begin{aligned}\Sigma F - m\bar{a} &= 0\\ \Sigma M - \bar{I}\alpha &= 0.\end{aligned}\right\}$ **(B)**
	Where, $-m\bar{a}$ is called **Inertia Force OR D' Alemberts Force** and $-\bar{I}\alpha$ is called **Inertia Torque OR D' Alembert's Torque.**

Above **(A)** and **(B)** equations will give the same final result. *They differ only in the manner of application of the concept.*

10·7 Motion under a Central Force. Conservation of Angular Momentum.

Central Force : *It is the only **force** $\bar{F}$, **directed toward or away from a fixed point O,** acting on the particle P and **due to which the particle is moving.***

The point O is referred as the center of force.

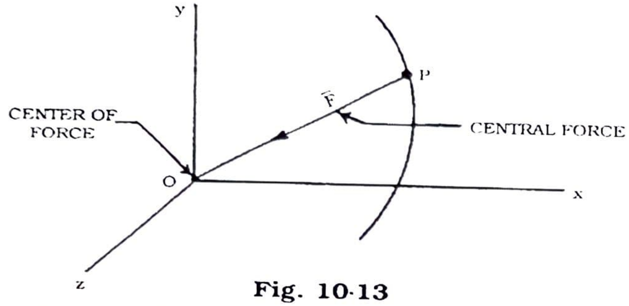


Fig. 10·13

Here the particle is moving due to central force $\bar{F}$ passing through O, hence moment of this force about O is zero as $\bar{r}$ and $\bar{F}$ are collinear.

Then, $\Sigma \bar{M}_o = 0$ at any given instant as $\bar{F}$ passes through O.

$\therefore$ $\dot{\bar{H}}_o = 0$ .... for all values of t.

$\therefore$ $\boxed{\bar{H}_o = \bar{r} \times m\,\bar{v} = \text{constant}}$

We can conclude that *the* ***angular momentum of a particle moving under a central force is constant, both in magnitude and direction.***

And **$\bar{r}$ must be perpendicular to $\bar{H}_o$. Thus, a particle under a central force moves in a fixed plane perpendicular to $\bar{H}_o$.**

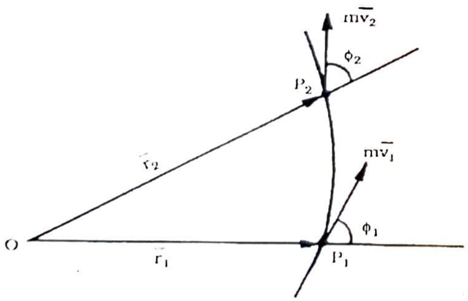


Fig. 10·14

Since, H_o = constant, $mr^2\dot{\theta} = H_o$ = constant

and $r\,mv\sin\phi$ = constant

$\therefore$ $r_1 mv_1 \sin\phi_1 = r_2 mv_2 \sin\phi_2$

$\boxed{(\bar{H}_o)_1 = (\bar{H}_o)_2}$

The above equation is known as the ***conservation of angular momentum. This occurs when the particle is subjected to only a central force.***

10·8 Work of a Force :

In mechanics, *a* ***force $\bar{F}$ does work only when it undergoes a displacement in the direction of the force.*** A particle is moved from A to A′ due to force $\bar{F}$ acting at A. *The vector AA′ is called* ***displacement $d\bar{r}$*** *and its* ***magnitude is ds.***

The *work of force* $\bar{F}$ *is a scalar quantity* as it is a *dot product* of $\bar{F}$ and $d\bar{r}$.

$\boxed{dU = \bar{F} \cdot d\bar{r}}$

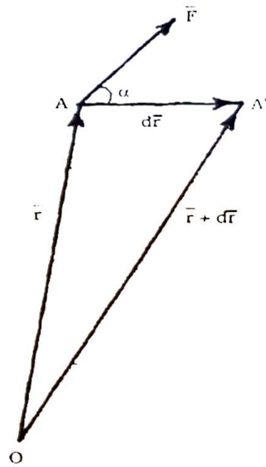


Fig. 10·15

$dU = F\,ds\cos\alpha$

where, F = magnitude of force $\bar{F}$

ds = magnitude of displacement $d\bar{r}$

α = angle between $\bar{F}$ and $d\bar{r}$

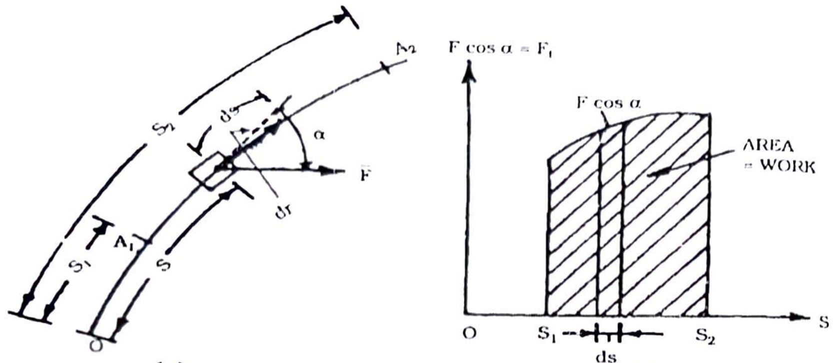
Thus, **work = force × displacement along force.**

In *SI system*, unit of work is

joule (J) = (N) × (m).

(1) Work of a Variable Force :

The work of $\bar{F}$ during a *finite displacement* of the particle from A_1 to A_2 is obtained *by integrating* the equation $dU = \bar{F} \cdot d\bar{r}$ along the path described by the particle.



(a) (b)

Fig. 10.16

From above figure, we may write

$$\boxed{U_{1-2} = \int_{A_1}^{A_2} \bar{F} \cdot d\bar{r} = \int_{S_1}^{S_2} (F\cos\alpha)\, ds = \int_{S_1}^{S_2} F_t\, ds}$$

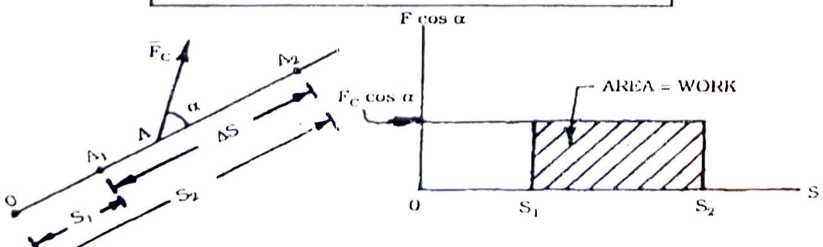
Here $F\cos\alpha = F_t$ = *tangential component of the force F*, means component of force along the displacement.

Work can also be determined by measuring the area closed between $F_t - S$ curve and S axis as shown in above graph.

(2) Work of a Constant Force Moving along a Straight Line :

When a particle moving in a straight line is acted upon by *a force F of constant magnitude and constant direction*, the work done is equal to

$$\boxed{U_{1-2} = (F_c\cos\alpha)\, \Delta S = F_c\cos\alpha\, (S_2 - S_1)}$$



(a) (b)

Fig. 10.17

The above equation is explained in the figure (a) as the work done is a product of the force in the direction of displacement and

the displacement. In figure (b) it is explained by measuring area of $F_c \cos\alpha - S$ graph and S axis.

(3) Work of a Weight (Work of a Force of Gravity) :

The work of the weight $\bar{W}$ of a body exerted on that body is obtained by the *components of $\bar{W}$ in to equation of work,*

$$U_{1-2} = \int_{A_1}^{A_2} F_x dx + F_y dy + F_z dz).$$

With the y axis chosen upward, we have

$F_x = 0,\ F_y = -W,\ F_z = 0.$

Hence

$$dU = -W dy$$

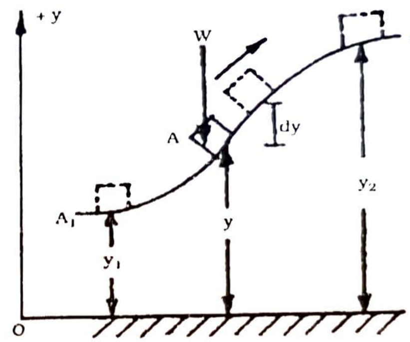
$$U_{1-2} = -\int_{y_1}^{y_2} W\, dy$$

$$= W y_1 - W y_2$$

OR

$$U_{1-2} = -W (y_2 - y_1)$$

$$\boxed{U_{1-2} = -W\Delta y}$$



(+ Δy)Particle moving up.

(–W) Force downward

(–U) Work done

Fig. 10.18

When body moves : *(i) upward* : W *negative*, Δy *positive*,

U_{1-2} negative

***(ii) down ward* :** W *negative*, Δy *negative*,

U_{1-2} positive.

Work done is positive if the direction of displacement and the direction of applied force are same.

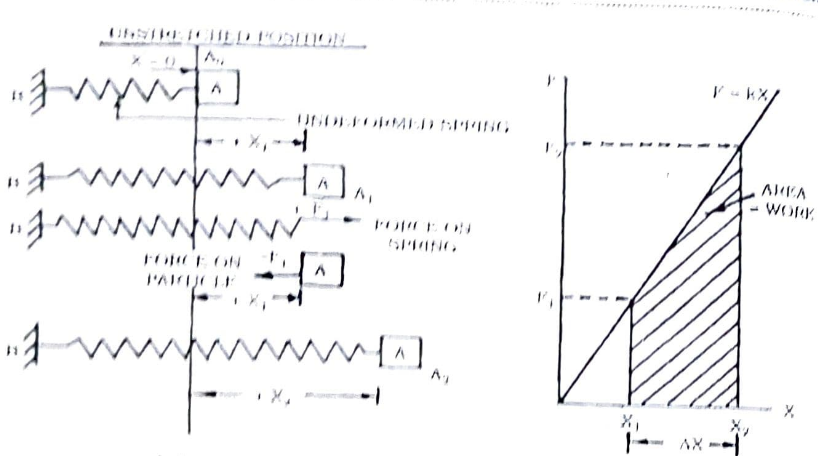
(4) Work of the Force Exerted by a Spring :

The magnitude of the force $\bar{F}$ exerted by the elastic spring on body is proportional to the deflection x of the spring measured from its unstretched position.

$$F \propto x$$

$$F = k\, x$$

where *k is the* ***spring constant*** (or **stiffness**) expressed in N/m. or kN/m.



(a) Fig. 10.19 (b)

In above figure, a particle A attached to spring is shown. From unstretched position of spring (x = 0), the spring is stretched to position A_1 (here $x = + x_1$) and then to position A_2 ($x = + x_2$). **Here the force acting on spring is + F where as the force acting on particle is − F (same magnitude, opposite direction). The displacement is positive.**

The work of the force F exerted by *the spring on the body* (− F) during a finite displacement from A_1 **($x = x_1$)** to A_2 **($x = x_2$)** is obtained by

$$dU = (-F)(dx) = -kx\,dx$$

$$U_{1-2} = -\int_{x_1}^{x_2} kx\,dx = -\left(\frac{1}{2}k\,x_2^2 - \frac{1}{2}k\,x_1^2\right)$$

$$\therefore \quad U_{1-2} = -\left(\frac{1}{2}k\,x_2^2 - \frac{1}{2}k\,x_1^2\right)$$

$$\text{OR} \quad U_{1-2} = -\left(\frac{1}{2}k\,\Delta x^2\right)$$

Note : If

	Direction of spring force acting on the particle	*Direction of displacement of the particle*		*Work Done*
(1)	+ F → A	+ Δx →	**Same**	$+U_{1-2}$ (Positive)
(2)	← − F A	+ Δx →	**Opposite**	$-U_{1-2}$ (Negative)

10.9. Kinetic Energy of a Particle. Principle of work and Energy.

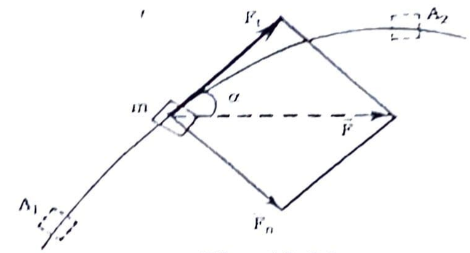


Fig. 10.20

A particle m is acted upon by a force $\overline{F}$ and moving along a path either *rectilinear* or *curved.*

Newton's second law in terms of tangential components of the force and of the *acceleration*

$$F_t = m\,a_t$$

$$= m\frac{dv}{dt} \quad \text{where,} \quad v = \text{speed of the particle}$$

$$= m\frac{dv}{ds}\frac{ds}{dt}$$

$$F_t = mv\frac{dv}{ds} \quad \text{because} \quad v = \frac{ds}{dt}$$

$$\therefore \quad F_t\,ds = mv\,dv$$

In above figure,
at A_1, $s = s_1$ and $v = v_1$
at A_2, $s = s_2$ and $v = v_2$

Integrating from A_1 to A_2

$$\int_{s_1}^{s_2} F_t\,ds = m\int_{v_1}^{v_2} v\,dv$$

$$\therefore \quad U_{1-2} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$\therefore \quad \boxed{U_{1-2} = T_2 - T_1}$$ .... **Principle of work and Energy.**

Here, $\boxed{T = \frac{1}{2}mv^2}$ T = *kinetic energy of a particle*

(scalar quantity)

Unit of T = N·m = **joule.**

Principle of work and Energy : *The work of the force F is equal to the change in kinetic energy of the particle.*

The above equation can be written as

$$\boxed{T_1 + U_{1-2} = T_2}$$

Kinetic Energy at A_1	+	Work done due to F between A_1 and A_2	=	Kinetic Energy at A_2

10·10. Potential Energy (V) :

(1) Energy : *It may be defined as the* ***capacity for doing work.***

(2) Kinetic Energy (T) : *It is the* ***energy comes from the motion*** *of the particle.*

(3) Potential Energy (V) : *It is the* ***energy comes from the position*** *of the particle, measured from a fixed datum or reference plane.*

(i) Gravitational Potential Energy (V_g) :

(Potential energy of weight)

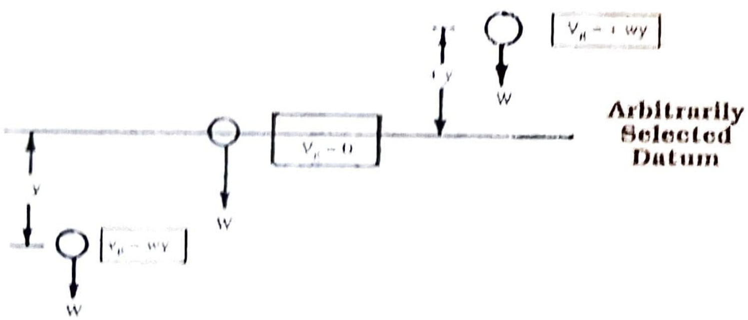


Fig. 10·21

(a) Positive V_g : y positive, *W has the capacity of doing work when the particle is moved back down to the datum.*

(b) Negative V_g : y–negative, *W does negative work when the particle is moved back up to the datum.*

(c) $V_g = 0$: y = 0 at datum.

In general, **y is positive upward.**

$$\boxed{V_g = W \cdot y}$$

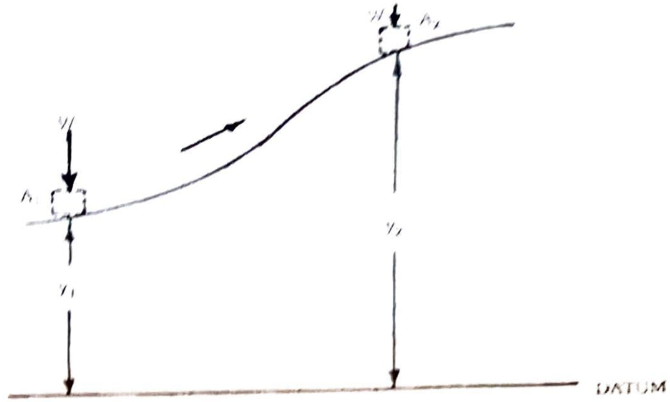


Fig. 10·22

In above figure, Work done = Change in potential energy

$$U_{1-2} = W\,y_1 - W\,y_2$$

$$U_{1-2} = (V_g)_1 - (V_g)_2$$

Here, $(V_g)_2 > (V_g)_1$ → **Potential energy increases during displacement** → **Work U_{1-2} negative**

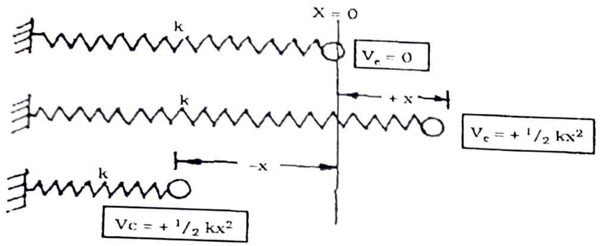
Here displacement (Δy) is against the force W hence work done is negative.

(ii) Elastic Potential Engrgy (V_e) :
(Potential energy due to spring)

When an *elastic spring is elongated or compressed* a distance x *from its unstretched position,* the *elastic potential energy* V_e due to the spring's configuration can be expressed as

$$\boxed{V_e = + \frac{1}{2}\,kx^2}$$

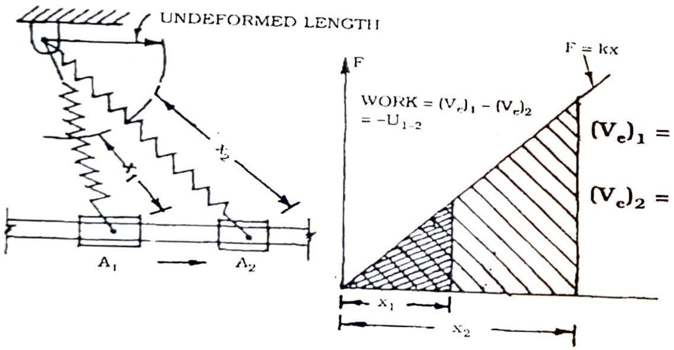
Here, V_e is ***always positive since, in the deformed position, the force of the spring has the capacity for always doing positive work on the particle*** *when the spring is returned to its unstretched position as shown below.*



V_e always positive.

Fig. 10·23

From the *above figure it can be noticed* that if the *spring is released from the deformed position, the force acting on the particle and displacement both are in the same direction, hence the potential energy of the spring to do the work during deformed position will always be positive.*



$$(V_e)_1 = \frac{1}{2} kx_1^2$$

$$(V_e)_2 = \frac{1}{2} kx_2^2$$

(a) Fig. 10·24 (b)

In above figure also, V_e is always positive but before deformed position the work of the force (F) exerted by the spring on the body is negative. ***The work of the elastic force depends only upon the initial and final deflections of the spring.***

10·11. Conservation of Energy :

Conservative Forces : *A force* $\bar{F}$ *acting on a particle A is said to be* ***conservative if its work U_{1-2} is independent of the path*** *followed by the particle A as it moves from A_1 to A_2.*

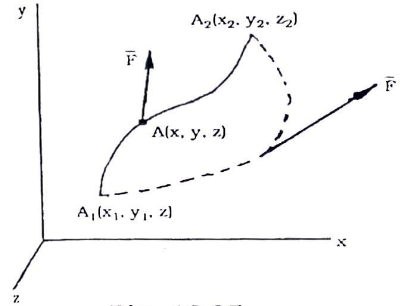


Fig. 10·25

We may write

$$\boxed{U_{1\to2} = V_{(x1,\ y1,\ z2)} - V_{(x2,\ y2,\ z2)}}$$

$\therefore \quad U_{1-2} = V_1 - V_2$

Thus, U_{1-2} depends upon potential energy of A_1 and A_2, not the path A_1 to A_2.

If A_2 is chosen to coincide with A_1, that is, if the particle is moving round and finally coming to A_1 again (closed path).

then $\quad V_1 = V_2 \quad$ and $\quad U_{1\to2} = 0$

The weight of the particle and force exerted by the spring are the conservative forces, of which the work may be expressed as a change in potential energy.

We know the **principle of work and energy** that

$$U_{1-2} = T_2 - T_1$$

but $\quad U_{1-2} = V_1 - V_2$

Hence, $\quad \boxed{T_1 + V_1 = T_2 + V_2}$

This is the ***equation of conservation of energy,*** *which indicates that when a* ***particle moves under the action of conservative forces, the sum of kinetic energy and of the potential energy of the particle remains constant.***

T + V = Total Mechanical Energy (E)

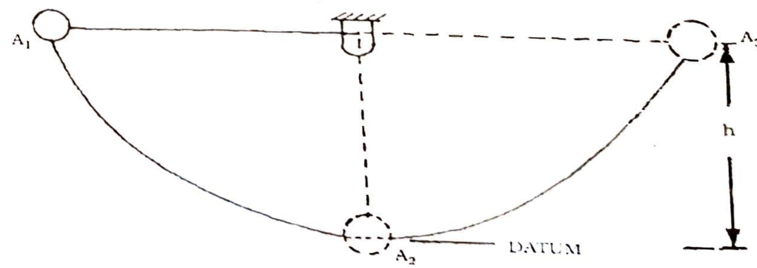


Fig 10·26

Consider the *example of pendulum* as shown in figure.

At A_1, Velocity $v_1 = 0 \quad \therefore T_1 = \frac{1}{2} m v_1^2 = 0$

$$\text{and } V_1 = Wh$$

$$\therefore T_1 + V_1 = Wh \qquad \text{—— (i)}$$

At A_2, Velocity $v_2 = \sqrt{2gh} \quad \therefore T_2 = \frac{1}{2} m v_2^2 = \frac{1}{2}\frac{W}{g}(2gh) = Wh$

$$\text{and } V_2 = 0 \quad \therefore T_2 + V_2 = Wh \qquad \text{—— (ii)}$$

At A_3, Velocity $v_3 = 0 \quad \therefore T_3 = 0$

$$\text{and } V_3 = Wh \quad \therefore T_3 + V_3 = Wh \qquad \text{—— (iii)}$$

Hence **total mechanical energy remains same at A_1, A_2 and even at A_3.**

Potential energy at A_1 is converted in to kinetic energy at A_2 which is again converted in to potential energy at A_3.

Kinetic energy will have the same value at any two points located at the same level.

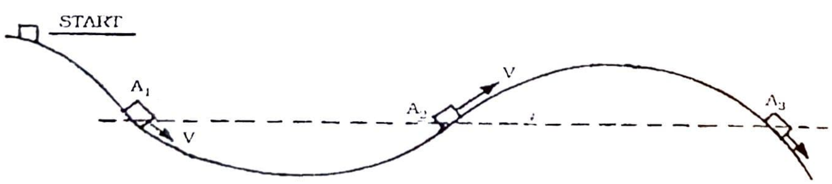


Fig. 10·27

As long as the only forces acting on the particle are its weight and normal reaction of the path (no friction), the particle will have the same speed at A_1, A_2 and A_3 located at the same level.

***Weight of a particle** and the **force exerted by a spring** are **conservative forces** whereas **friction forces** are **nonconservative forces**. In other words, the work of a friction force can not be expressed as a change in potential energy. The **work of friction force is always negative** since the friction forces have a direction opposite to that of the displacement of the body on which they act.*

10·12. Principle of Impulse and Momentum :

A third basic method for the solution of problems dealing with the ***impulsive motion of particles***, is based on the *principle of impulse and momentum*.

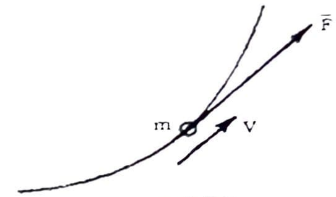


Fig. 10·28

A particle of mass m is acted upon by a force $\bar{F}$. *Newton's second law* may be expressed in the form

$$\bar{F} = \frac{d}{dt}(m\,\bar{v}), \qquad \text{where, } m\,\bar{v} = \textit{linear momentum.}$$

$$\therefore \quad \bar{F}\,dt = d\,(m\,\bar{v})$$

Integrating $\int_{t_1}^{t_2} \bar{F}\,dt = m\,\bar{v}_2 - m\,\bar{v}_1$

$$\therefore \quad \boxed{m\,\bar{v}_1 + \int_{t_1}^{t_2} \bar{F}\,dt = m\,\bar{v}_2} \quad \text{...... \textbf{Principle of Impulse and Momentum.}}$$

where, $\int_{t_1}^{t_2} \bar{F}\,dt$ is known as **impulse**

(it is a vector) **of the force $\bar{F}$ during the time interval.**

$$\boxed{Imp_{1-2} = \int_{t_1}^{t_2} \bar{F}\,dt}$$

again,

$$\boxed{Imp_{1-2} = \bar{i}\int_{t_1}^{t_2} F_x\,dt + \bar{j}\int_{t_1}^{t_2} F_y\,dt + \bar{k}\int_{t_1}^{t_2} F_z\,dt}$$

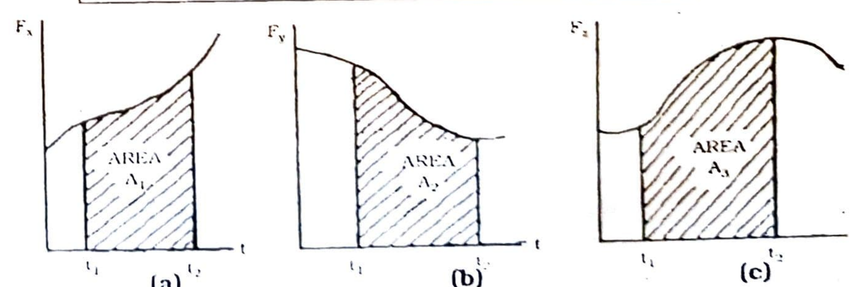


Fig. 10·29

$Imp_{1-2} = (\text{Area } A_1)\ \bar{i} + (\text{Area } A_2)\ \bar{j} + (\text{Area } A_3)\ \bar{k}$

where, Area A_1 = *x component of impulse* $(Imp_{1-2})_x$ of force $\bar{F}$

Area A_2 = *y component of impulse* $(Imp_{1-2})_y$ of force $\bar{F}$

Area A_3 = *z component of impulse* $(Imp_{1-2})_z$ of force $\bar{F}$

Unit of impulse is N.s

but N.s = (kg. m/s²) · s = kg. m/s

which is the unit of linear momentum. Thus *dimensionally linear momentum and impulse are same.*

The **principle of linear impulse and momentum** states that ***initial momentum of the particle at t_1 plus the impulse of the force $\bar{F}$ during the time interval t_1 to t_2 is equivalent to the final momentum of the particle at t_2.***

Thus **final momentum** may be obtained by *adding vectorially* its *initial momentum* and the *impulse.*

Initial momentum + **Impulse** = **Final momentum.**

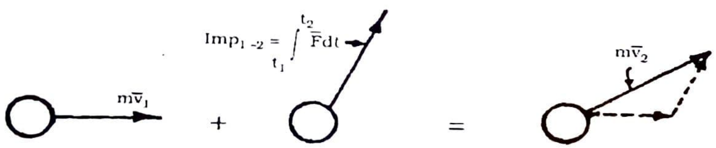


Fig. 10·30

$\boxed{m\ \bar{v}_1 + Imp_{1-2} = m\ \bar{v}_2}$ **for single particle**

$\Sigma m_i\ (\bar{v}_i)_1 + \Sigma(Imp_i)_{1-2} = \Sigma m_i\ (\bar{v}_i)_2$ **for a system of particles**

A very large force acting in a very short time interval on a particle and produce a definite change in momentum, is called ***impulsive force.*** and the *resulting motion* an ***impulsive motion.***

The average force $\bar{F}$ exerted by the bat on the base ball is very large as time interval Δt is very short and hence resulting impulse $\bar{F}\ \Delta t$ is large enough to change the sense of motion of ball.

Weight of the body and the force exerted by a spring are non-impulsive forces which may be neglected since the corresponding impulse $\bar{F}\ \Delta t$ is very small.

IMPORTANT EQUATIONS

1. **Newton's Second Law of Motion :**

$$\Sigma\ \bar{F} = m\ \bar{a}$$

2. **Linear Momentum ($\bar{L}$) :** $\bar{L} = m\ \bar{v}$

3. (i) **Rate of change of Linear Momentum ($\dot{\bar{L}}$) :**

$$\dot{\bar{L}} = \Sigma\ \bar{F}$$

(ii) If $\Sigma\ \bar{F} = 0$, $\dot{\bar{L}} = 0\ \therefore\ m_1\ v_1 = m_2\ v_2$ – **Conservation of Momentum**

4. **Equations of Motion :** $\Sigma\ \bar{F} = m\ \bar{a}$

(i) **Rectangular Components :**
$\Sigma F_x = m\ a_x$
$\Sigma F_y = m\ a_y$
$\Sigma F_z = m\ a_z$

(ii) **Tangential and Normal components :**

$$\Sigma F_t = ma_t = m\frac{dv}{dt}$$

$$\Sigma F_n = ma_n = m\frac{v^2}{\rho}$$

(iii) **Radial and Transverse components**

$$\Sigma F_r = ma_r = m(\ddot{r} - r\ \dot{\theta}^2)$$

$$\Sigma F_\theta = ma_\theta = m\ (r\ \ddot{\theta} + 2\ \dot{r}\ \dot{\theta})$$

5. **Dynamic Equlibrium :**

$$\Sigma\ \bar{F} - m\ \bar{a} = 0$$

6. **Angular Momentum ($\bar{H}_o$) :**

(i) $\bar{H}_o = \bar{r} \times m\ \bar{v}$

(ii) $H_o = r\ mv\sin\phi$

(iii) $\bar{H}_o = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ x & y & z \\ mv_x & mv_y & mv_z \end{vmatrix}$

(iv) $H_x = m\ (yv_z - zv_y)$
$H_y = m\ (zv_x - xv_z)$
$H_z = m\ (xv_y - yv_x)$

(v) $H_o = mr^2\dot{\theta}$

7. **Rate of change of Angular Momentum ($\dot{\bar{H}}_o$) :**

$$\dot{\bar{H}}_o = \Sigma\ \bar{M}_o$$

8. **Motion under Central Force :**

Moment of central force $\Sigma\ \bar{M}_o = 0$

$\therefore\ \dot{\bar{H}}_o = 0$

$\therefore\ \bar{H}_o = \bar{r} \times m\ \bar{v} = \text{constant.}$

9. Conservation of Angular Momentum of a Particle subjected to Central Force.

$$(\bar{H}_o)_1 = (\bar{H}_o)_2$$

10. Work of a Force (U) : $dU = \bar{F} \cdot d\bar{r}$

$$dU = F\, ds \cos\alpha$$

(i) Work of a variable force :

$$U_{1-2} = \int_{S_1}^{S_2} F_t\, ds$$

(ii) Work of a constant force moving along a straight line :

$$U_{1-2} = F_c \cos\alpha\,(S_2 - S_1)$$

(iii) Work of a weight :

$U_{1-2} = -W\Delta y$ .... **(W and Δy in opposite direction)**

$U_{1-2} = W\Delta y$ .... **(W and Δy in same direction)**

(iv) Work of a spring force :

$U_{1-2} = -(\frac{1}{2} k\, \Delta x^2)$ .... **spring force and Δx in opposite direction.**

11. Kinetic Energy (T) : $T = \frac{1}{2} mv^2$

12. Principle of work and energy :

$$T_1 + U_{1-2} = T_2$$

13. Potential energy (V) :

(i) Potential energy of weight (V_g) :

$$V_g = Wy$$

(ii) Potential energy due to spring (V_e) :

$$V_e = \frac{1}{2} k\, x^2$$

14. Conservation of Energy :

$$U_{1-2} = V_1 - V_2$$

$$T_1 + V_1 = T_2 + V_2$$

15. Impulse :

$$Imp_{1-2} = \int_{t_1}^{t_2} \bar{F}\, dt$$

16. Principle of Impulse and Momentum :

$m\bar{v}_1 + \int_{t_1}^{t_2} \bar{F}\, dt = m\bar{v}_2$ .... **For single particle**

Initial momentum + Impulse$_{1-2}$ = Final momentum.

$\sum m_i (v_i)_1 + \sum (Imp_i)_{1-2} = \sum m_i (v_i)_2$ .... **For a system of particles**

SOLVED EXAMPLES

1. A block of mass 100 kg rests on an inclined surface. Find the magnitude of the force P required to give the block an acceleration of 2 m/s² in the upward direction along the plane. μ_k between block and surface is 0·2.

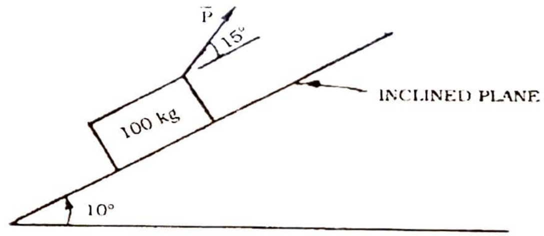
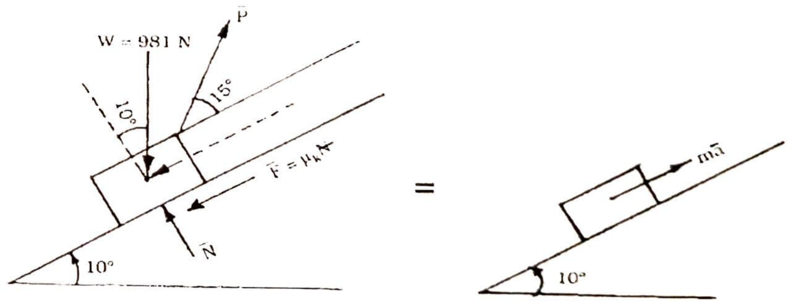


Fig. 10·31



(a) (b)

Fig. 10·32

The *free-body diagram* is shown in above figure. The forces acting on the block are equivalent to vector ma.

Here, $W = mg = (100\text{ kg})(9{\cdot}81\text{ m/s}^2) = 981$ N.

and $F = \mu_k N = 0{\cdot}2$ N

$m\,a = (100\text{ kg})(2\text{ m/s}^2) = 200$ N.

We may resolve the forces *along the inclined plane and perpendicular to inclined plane.*

(i) Along the inclined plane :

$$-W \sin 10^\circ - F + P \cos 15^\circ = ma$$

$$-981 \sin 10^\circ - 0{\cdot}2\ N + P \cos 15^\circ = 100\ (2)$$

$$-0{\cdot}2\ N + 0{\cdot}97\ P = 370{\cdot}35 \qquad \text{—— (1)}$$

(ii) Perpendicular to inclined plane :

$$N - W \cos 10^\circ + P \sin 15^\circ = 0$$

$$\therefore \quad N = 966{\cdot}1 - 0{\cdot}2P \qquad \text{—— (2)}$$

Putting value of N from eq. (2) in to eq. (1)

$$-0{\cdot}2\,(966{\cdot}1 - 0{\cdot}26\ P) + 0{\cdot}97\ P = 370{\cdot}35$$

$$\therefore \quad -193{\cdot}22 + 0{\cdot}052\ P + 0{\cdot}97\ P = 370{\cdot}35$$

$$\therefore \quad 1{\cdot}022\ P = 563{\cdot}57$$

$$\boxed{P = 551{\cdot}44\ N}$$

2. The two blocks shown are originally at rest. Neglecting the masses of the pulleys and the effect of friction in the pulleys and between the block and the inclines, determine **(a)** the acceleration of each block, **(b)** the tension in the cable.

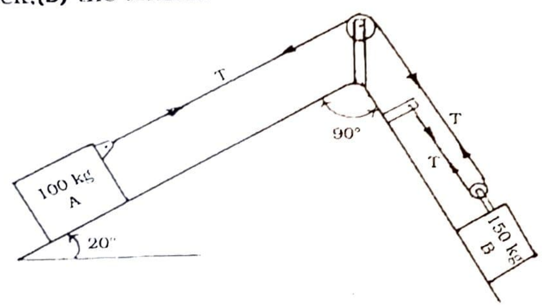
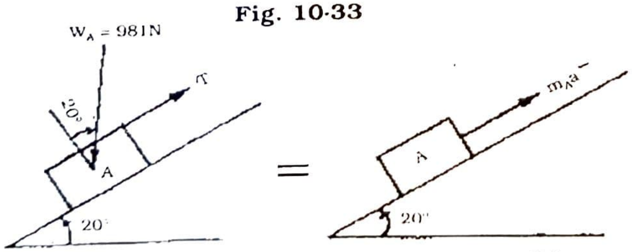
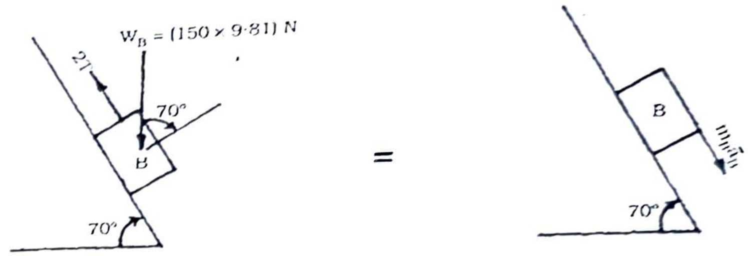


Fig. 10·33



(a) (b)

Fig. 10·34



(a) (b)

Fig. 10·35

(1) **Block A :**

$$T - W_A \sin 20^\circ = m_A\, a_A$$

$$T - 981 \sin 20^\circ = 100\, a_A$$

$$T = 100\, a_A + 335{\cdot}52 \qquad \text{—— (1)}$$

(2) **Block B :**

$$-2T + W_B \sin 70^\circ = m_B\, a_B$$

$$\therefore \quad -2T + 1382{\cdot}76 = 150\, a_B$$

$$\therefore \quad T = 691{\cdot}38 - 75\, a_B \qquad \text{—— (2)}$$

(3) **Relation of a_A and a_B :**

If block A moves through x_A up, block B moves down through

$$x_B = \frac{1}{2}\, x_A$$

Differentiating twice with respect to t, we have

$$a_B = \frac{1}{2}\, a_A \qquad \text{—— (3)}$$

Now, putting the value of eq. (3) in eq. (2)

$$T = 691{\cdot}38 - 75\,(1/2\ a_A)$$

$$\therefore \quad T = 691{\cdot}35 - 37{\cdot}5\, a_A \qquad \text{—— (4)}$$

Equating eq. (1) and eq (4)

$$100\, a_A + 335{\cdot}52 = 691{\cdot}35 - 37{\cdot}5\, a_A$$

$$\therefore \quad 137{\cdot}5\, a_A = 355{\cdot}83$$

$$\therefore \quad \boxed{a_A = 2{\cdot}59\ m/s^2}$$

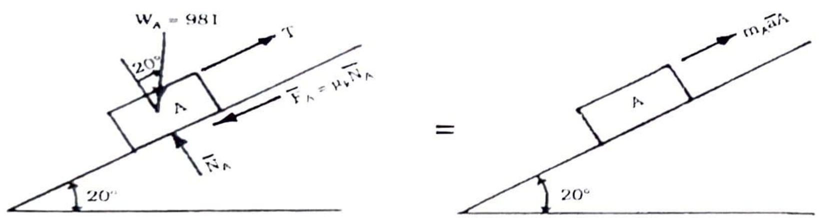
but $\quad a_B = \dfrac{1}{2}\, a_A$

$$\therefore \quad \boxed{a_B = 1{\cdot}295\ m/s^2}$$

and $\quad T = 100\, a_A + 335{\cdot}52$

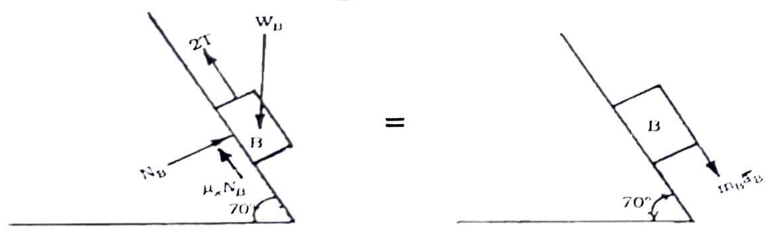
$\therefore \quad \boxed{T = 594{\cdot}52 \text{ N}}$

3. Solve example 2, assuming that the coefficients of friction between the blocks and inclines are μ_s = **0·25** and μ_k = **0·20**.



(a) (b)

Fig. 10·36



(a) (b)

Fig. 10·37

Here, **if blocks are in motion, we have to consider μ_k not μ_s.**

(1) Block A :

Along plane :

$T - W_A \sin 20° - F_A = m_A A_A$

$T = 335{\cdot}52 + 100\, a_A + 0{\cdot}2\, N_A$ ——— (1)

Perpendicular to plane :

$N_A - W_A \cos 20° = 0$

$\therefore \quad N_A = 921{\cdot}84$ N ——— (2)

Hence eq. (1) becomes

$T = 335{\cdot}52 + 100\, a_A + 184{\cdot}37$ ——— (3)

(2) Block B :

Along plane :

$-2T + W_B \sin 70° - \mu_k N_B = m_B a_B$

$T = 691{\cdot}38 - 75 a_B - 0{\cdot}1\, N_B$ ——— (4)

Perpendicular to plane :

$N_B - W_B \cos 70° = 0$

$N_B = 503{\cdot}28$ N ——— (5)

Hence eq. (4) becomes

$T = 691{\cdot}38 - 75\, a_B - 50{\cdot}33$ ——— (6)

(3) Relation of a_A and a_B :

Here also, $x_B = 1/2\, x_A$ ——— (7)

$\therefore \quad a_B = 1/2\, a_A$

Now putting the value of eq. (7) in to (6)

$T = 691{\cdot}38 - 75\,(1/2\, a_A) - 50{\cdot}33$

$\therefore \quad T = 691{\cdot}38 - 37{\cdot}5\, a_A - 50{\cdot}33$ ——— (8)

Equating eq. (3) and eq. (8)

$335{\cdot}52 + 100\, a_A + 184{\cdot}37 = 691{\cdot}38 - 37{\cdot}5\, a_A - 50{\cdot}33$

$\therefore \quad 137{\cdot}5\, a_A = 121{\cdot}16$

$\therefore \quad \boxed{a_A = 0{\cdot}88 \text{ m/s}^2}$

but $a_B = 1/2\, a_A$

$\therefore \quad \boxed{a_B = 0{\cdot}44 \text{ m/s}^2}$

and $T = 335{\cdot}52 + 100\, a_A + 184{\cdot}37$

$\therefore \quad \boxed{T = 607{\cdot}89 \text{ N}}$

4. Knowing that the system shown below starts from rest, find the velocity at t = 1·2 s of **(a)** collar A. **(b)** collar B. *Neglect the masses of the pulleys and the effect of friction.*

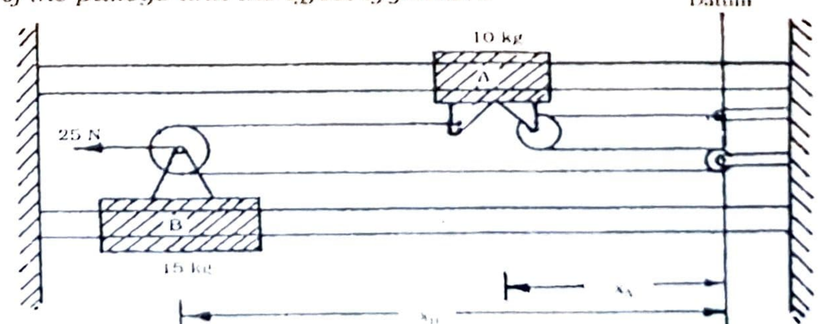
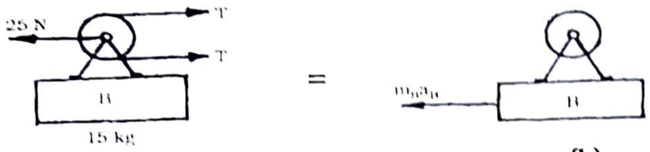
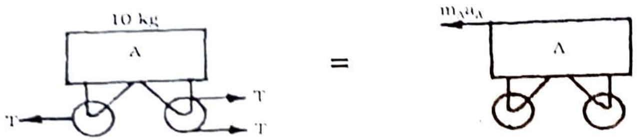


Fig. 10·38



(a) (b)

Fig. 10·39



(a) (b)

Fig. 10·40

Initially *we assume acceleration of A & B collars same.*

Block B : (*Equation of motion*)

$$\overset{+}{\leftarrow}\Sigma F_x = ma_B, \quad 25 - 2T = m_B a_B$$

$$25 - 2T = 15 a_B$$

$$T = 12{\cdot}5 - 7{\cdot}5 a_B \qquad \text{—— (1)}$$

Block A : (*Equation of motion*)

$$\overset{+}{\leftarrow}\Sigma F_x = ma_A, \quad -2T + T = m_A a_A$$

$$-2T + T = 10 a_A$$

$$T = -10 a_A \qquad \text{—— (2)}$$

Relation between a_B and a_A :

Let x_B = distance between B and right end as shown in figure.

x_A = distance between A and right end as shown.

Length of cable = $x_B + (x_B - x_A) + 2x_A = 2x_B + x_A$

Before and after motion, the length of the cable remains constant.

$$2x_B + x_A = \text{constant}$$

Differentiating with respect to t

once, $\quad 2v_B + v_A = 0$

and twice, $\quad 2a_B + a_A = 0$

$$\therefore \quad a_B = -\frac{1}{2} a_A \qquad \text{—— (3)}$$

Here, $2\Delta x_B + \Delta x_A = 0$

$\therefore \ \Delta x_A = -2\Delta x_B$

In other way, if we move collar B by say 1 cm, then A will be moved in opposite direction by 2 cm and hence if acceleration of B is say a_B then acceleration of A is $a_A = -2 a_B$.

Now, $\quad T = 12{\cdot}5 - 7{\cdot}5 a_B$

$$= 12{\cdot}5 - 7{\cdot}5 \left(-\frac{1}{2} a_A\right)$$

$$= 12{\cdot}5 + 3{\cdot}75 a_A \qquad \text{—— (4)}$$

Equating eq.ⁿ (4) and eq. (2)

$$12{\cdot}5 + 3{\cdot}75 a_A = -10 a_A$$

$$\therefore \quad a_A = -0{\cdot}909 \text{ m/s}^2$$

and $a_B = 0{\cdot}455 \text{ m/s}^2$

Means B moves toward left whereas A moves toward right.

Velocity at t = 1·2 s

(a) **of collar A :** $v_A = u_A + a_A t$

$v_A = 0 - 0{\cdot}909 \times 1{\cdot}2$

$$\boxed{v_A = -1{\cdot}09 \text{ m/s (towards right)}}$$

(b) **of collar B :** $v_B = u_B + a_B t$

$v_B = 0 + 0{\cdot}455 \times 1{\cdot}2$

$$\boxed{v_B = 0{\cdot}546 \text{ m/s (towards left).}}$$

5. A 6-kg block B rests as shown on the upper surface of a 10 kg wedge A. Neglecting friction, determine **(a)** the acceleration of A, **(b)** the acceleration of B relative to A, immediately after the system is released from rest.

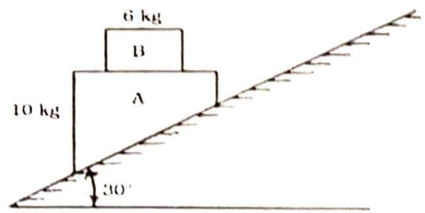
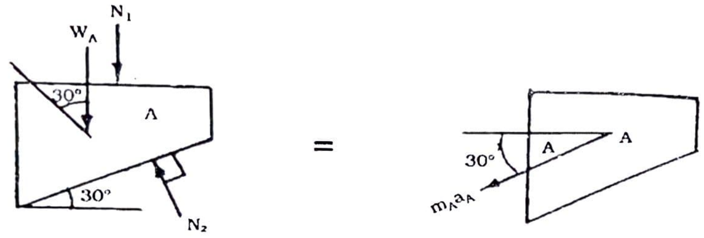


Fig. 10·41

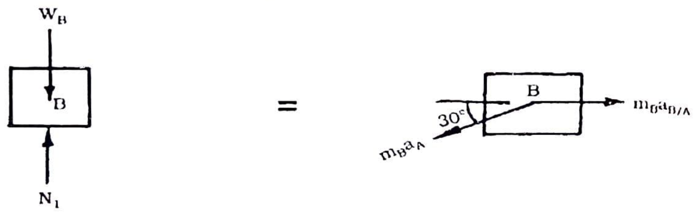
(a) (b)

Fig. 10·42



(a) (b)

Fig. 10·43



(a) (b)

Fig. 10·44

Let *us first examine the accelerations of wedge and of the block.* **(Kinematics – without regard to forces).**

Wedge A : As wedge is resting on inclined surface, its acceleration a_A is also inclined. We assume that it is directed downward.

Block B : The acceleration $\bar{a}_B$ of block B may be expressed as the sum of the acceleration of A and of the acceleration of B relative to A.

$$\bar{a}_B = \bar{a}_A + \bar{a}_{B/A}$$

Now, the free-body diagrams are drawn for wedge A and of block B and *Newton's second law* is applied. ***(Kinetics - with regard to forces).***

Wedge A : *Along the inclined plane :*

$$W_A \sin 30° + N_1 \sin 30° = m_A a_A$$

$$(10)(9{\cdot}81) \sin 30° + N_1 \sin 30° = 10\, a_A.$$

$$N_1 = 20a_A - 98{\cdot}1 \quad \text{——— (1)}$$

Block B : $\Sigma F_x = m_B a_x$, $\overrightarrow{+ve}$, $0 = m_B a_{B/A} - m_B a_A \cos 30°$

$$\therefore \quad a_A = 1{\cdot}155\, a_{B/A} \quad \text{——— (2)}$$

$\Sigma F_y = m_B a_y$, $\uparrow$+ve, $N_1 - W_B = -m_B a_A \sin 30°$

$$N_1 = (6)(9{\cdot}81) - (6)\, a_A \sin 30°$$

$$N_1 = 58{\cdot}86 - 3\, a_A \quad \text{——— (3)}$$

Equating eq. (1) and eq. (3)

$$20\, a_A - 98{\cdot}1 = 58{\cdot}86 - 3a_A$$

$$\therefore \quad \boxed{a_A = 6{\cdot}82 \text{ m/s}^2 \; \swarrow 30°}$$

Putting this value in eq. (2)

$$\boxed{a_{B/A} = 5{\cdot}90 \text{ m/s}^2 \rightarrow}$$

6. A 30 kg box rests on a 20 kg trolley. Coefficient of friction between box and the trolley is $\mu = 0{\cdot}3$.

(I) If the box is not to slip with respect to the trolley, determine (a) the max. allowable magnitude of P, (b) the corresponding acceleration of the trolley.

(II) If the box is allowed to slip with respect to the trolley in which $\mu_k = 0{\cdot}2$ between box and trolley and force P = 150 N is applied horizontally, find the acceleration of **(i)** trolley **(ii)** the box and **(iii)** the box with respect to trolley.

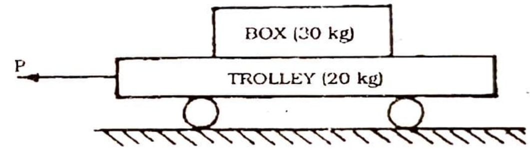


Fig. 10·45

(I) If the box is not to slip (Box is stationary) :

Trolley :

(a) **Fig. 10·46** (b)

$\Sigma F_x = ma_x$, $\quad P = 50\, a \quad \therefore \quad a = \dfrac{P}{50}$

Box : Box will be acted upon by frictional force F which is to be balanced by ma.

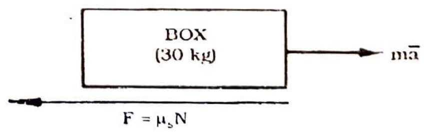


Fig. 10·47

$$\text{but } F = 0{\cdot}3 \times 30 \times 9{\cdot}81 = 30 \times \frac{P}{50}$$

$\boxed{P = 147{\cdot}15 \text{ N}}$ and $\boxed{a = 2{\cdot}943 \text{ m/s}^2}$

(II) Box is allowed to slip :

Here $P = 150$ N

$\mu_k = 0.2$

For Box :

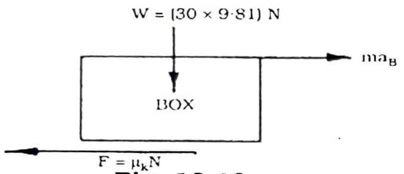


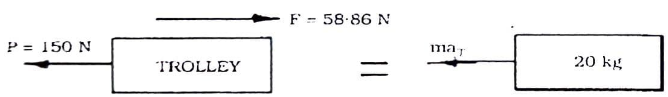
Fig. 10·48

$F = 0.2 \times 30 \times 9.81 = ma_B$

$\therefore \quad 58.86 = 30 \times a_B$

$\therefore \quad \boxed{a_B = 1.96 \text{ m/s}^2}$

For Trolley :



(a) **Fig. 10·49** (b)

$\Sigma F = ma_T$

$150 - 58.86 = 20 \times a_T$

$\therefore \quad \boxed{a_T = 4.56 \text{ m/s}^2}$

The acceleration of box with respect to trolley

$a_{B/T} = 4.56 - 1.96$

$\boxed{a_{B/T} = 2.6 \text{ m/s}^2}$

7. The 100 kg block A as shown is released from rest. If the mass of the pulleys and the cord is neglected, determine the speed of the 20 kg block B in 2 seconds.

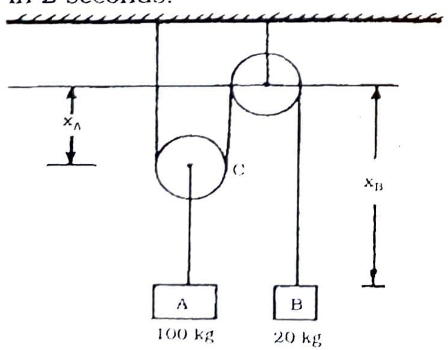


Fig. 10·50

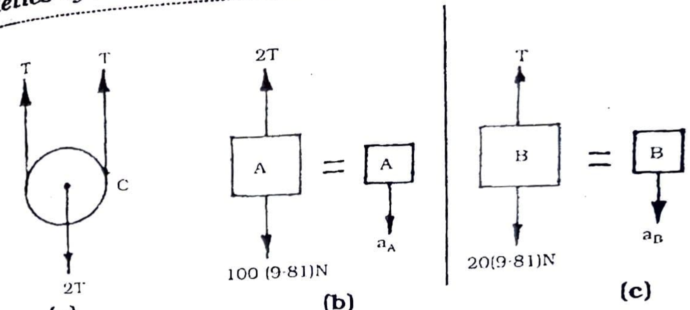


Fig. 10·51 Free-body diagrams

Initially we assume *all accelerations positive* (↓). Free - body diagrams are shown for pulley, block A and block B.

Equations of Motion :

Block A :

$+\downarrow\Sigma F_y = ma_y$, $100\,(9.81) - 2T = 100\, a_A$ ——— (1)

Block B :

$+\downarrow\Sigma F_y = ma_y$, $20\,(9.81) - T = 20\, a_B$ ——— (2)

Relation between a_A and a_B :

From the figure & as total length of cord remains same

$2\, x_A + x_B = \text{constant.}$

Differentiating twice with respect to time

$2a_A + a_B = 0$ ——— (3)

$\therefore \quad 2a_A = -a_B$

Putting this value of a_A in terms of a_B in eq. (1) ——— (4)

$100\,(9.81) - 2T = -50\, a_B$

From eq. (2) and eq. (4) and then after from eq. (3)

$T = 327$ N

$a_A = 3.27 \text{ m/s}^2$ (↓) Hence, a_A is *downward* and a_B *upward.*

$a_B = -6.54 \text{ m/s}^2$ (↑)

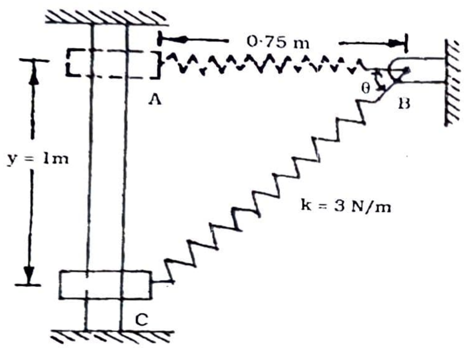
Now, the velocity of block B in 2 seconds is

$v_B = v_0 + a_B t$ for block B.

$= 0 + (-6.54)\,(2)$

$\boxed{v_B = 13.1 \text{ m/s}\uparrow}$

8. A smooth 4-kg collar C as shown is attached to a spring having a stiffness k = 3 N/m and an unstretched length of 0·75 m. If the collar is released from rest at A, determine its acceleration and the normal force of the rod on the collar at the instant y = 1 m.



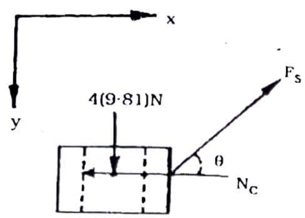


Fig 10·52 **Fig 10·53**

Free-body diagram of collar C is shown above. Equations of motion are written as under.

Equations of Motion :

$\overrightarrow{+}\Sigma F_x = ma_x,\quad -N_c + F_s \cos\theta = 0$ ——— **(1)**

$+\downarrow \Sigma F_y = ma_y,\quad 4\,(9{\cdot}81) - F_s \sin\theta = 4a$ ——— **(2)**

But $F_s = k\,x$

where, $x = CB - AB$

$= \sqrt{y^2 + (0{\cdot}75)^2} - 0{\cdot}75$

$= \sqrt{1^2 + (0{\cdot}75)^2} - 0{\cdot}75$

$\therefore\quad x = 0{\cdot}5$ m

$\therefore\quad F_s = k\,x$

$= 3 \times 0{\cdot}5$

$= 1{\cdot}50$ N ——— **(3)**

and, $\sin\theta = \dfrac{y}{\sqrt{y^2 + (0{\cdot}75)^2}} = \dfrac{1}{\sqrt{1^2 + (0{\cdot}75)^2}}$

$\therefore\quad \theta = 53{\cdot}1°$ ——— **(4)**

Substituting the value of F_s and θ in eq. (1) and eq. (2)

$-N_C + 1{\cdot}5 \cos 53{\cdot}1° = 0 \quad\therefore\quad \boxed{N_c = 0{\cdot}9\text{ N}}$

and $4\,(9{\cdot}81) - 1{\cdot}5 \sin 53{\cdot}1° = 4a \quad\therefore\quad \boxed{a = 9{\cdot}51\text{ m/s}^2\downarrow}$

9. The system shown in figure is released from rest. Determine **(i)** the tension in the cable **(ii)** the velocity of B after 4 seconds **(iii)** the velocity of A after it has moved 1·5 m.

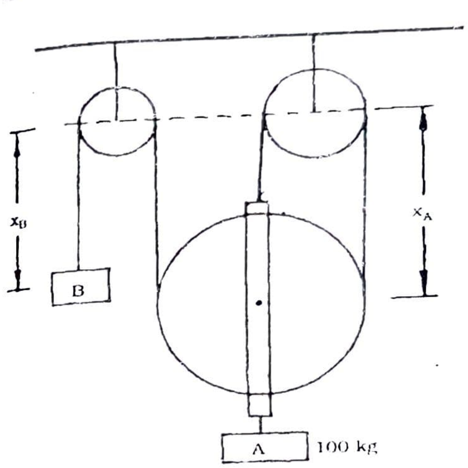
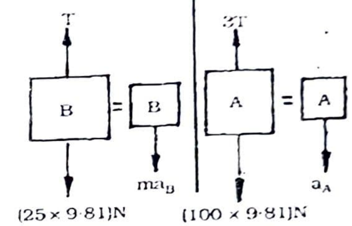


Fig. 10·54



(a) **(b)**

Fig. 10·55

Intially we assume accelerations of A and B downward. The free-body diagrams are also shown for block B and block A.

Equations of Motion :

Block B :

$\downarrow+\Sigma F_y = ma_y,\quad W_B - T = ma_B$

$25\,(9{\cdot}81) - T = 25\,a_B$ ——— **(1)**

Block A :

$\downarrow+\Sigma F_y = ma_y,\quad W_A - 3T = -\,m\,a_A$

$100\,(9{\cdot}81) - 3T = 100\,a_A$ ——— **(2)**

Relations of a_A and a_B :

As length of the cable remains constant

$x_B + 3\,x_A = \text{constant}.$

Differentiating twice with respect to time

$a_B + 3\,a_A = 0$ ——— **(3)**

Eq. (1) becomes

$25\,(9{\cdot}81) - T = 25\,(-\,3a_A)$ ——— **(4)**

From eq. (2) and eq. (4)

$a_A = 0{\cdot}754\text{ m/s}^2$

$a_B = -\,2{\cdot}262\text{ m/s}^2$

$\boxed{T = 301{\cdot}86\text{ N}}$

Now, velocity of B after 4 seconds is equal to

$$v_B = u_B + a_B t$$
$$= 0 + (-2.262) \times 4$$
$$= -9.048 \text{ m/s}.$$

and velocity of A after it has moved 1·5 m is

$$v_A^2 = u_A^2 + 2\, a_A s$$
$$= 0 + 2\,(0.754) \times 1.5$$

$$\therefore \quad \boxed{v_A = 1.504 \text{ m/s}}$$

10. Two masses of 200 kg and 80 kg are connected by an inextensible string. If a force of 1200 N is applied to the pulley as shown in figure, determine the acceleration of 80 kg mass and tension in the string. *Neglect the masses of the pulleys.*

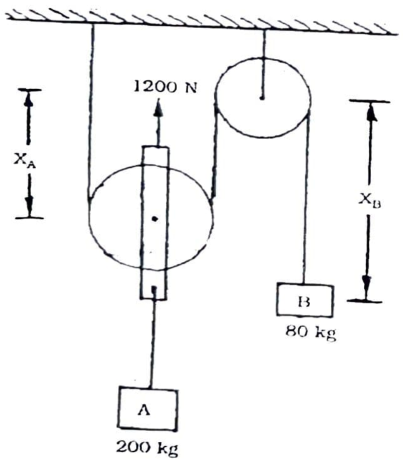
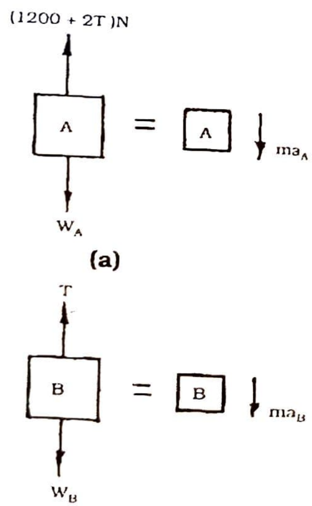


Fig. 10.56



(b)
Fig. 10.57

Initially we assume the accelerations of A and B downward.

Equations of Motion :

Block A :

$\downarrow + \Sigma F_A = ma_A$, $200\,(9.81) - 1200 - 2T = 200\, a_A$ —— **(1)**

Block B :

$\downarrow + \Sigma F_B = ma_B$, $80\,(9.81) - T = 80\, a_B$ —— **(2)**

Relation of a_A and a_B :

From figure,

$$2x_A + x_B = \text{constant}.$$
$$\therefore \quad 2v_A + v_B = 0$$
$$\text{and} \quad 2a_A + a_B = 0$$
$$\therefore \quad a_A = -a_B/2 \qquad \text{—— (3)}$$

Putting this value in eq. (1)

$$200\,(9.81) - 1200 - 2T = 200\,(-a_B/2) \qquad \text{—— (4)}$$

From eq. (2) and eq. (4)

$$\boxed{a_B = 3.11 \text{ m/s}^2}$$

and $\boxed{T = 536 \text{ N}}$

11. The system of three masses shown in figure is released from rest. Find the acceleration of mass m_2 and velocity of mass m_1 at time t = 5 seconds. *Neglect mass and friction of pulleys and strings.*

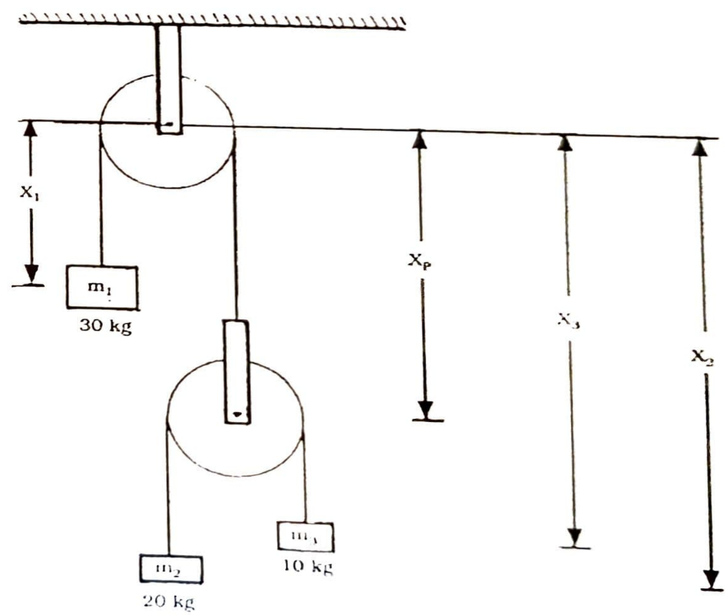
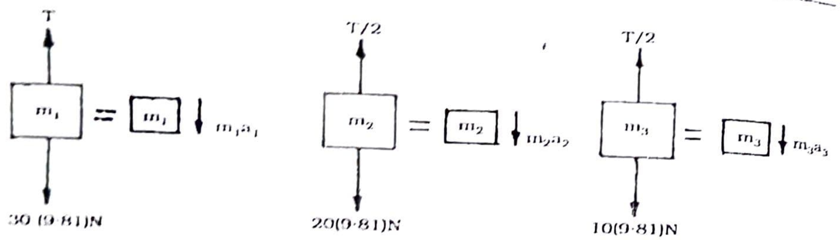


Fig. 10.58



(a) (b) (c)

Fig. 10·59

Assuming initially all accelerations downwards.

Equations of Motion :

Block m_1 :

$$30\ (9{\cdot}81) - T = 30\ a_1 \qquad \text{—— (1)}$$

Block m_2 :

$$20\ (9{\cdot}81) - \frac{T}{2} = 20\ a_2 \qquad \text{—— (2)}$$

Block m_3 :

$$10\ (9{\cdot}81) - \frac{T}{2} = 10\ a_3 \qquad \text{—— (3)}$$

Relationship of a_1, a_2 and a_3 :

From figure, length of two strings remain constant.

$$x_1 + x_p = \text{constant}$$

and $$(x_2 - x_p) + (x_3 - x_p) = \text{constant.}$$

therefore, from first equation of string

$$x_p = -x_1 + \text{constant}$$

Putting this in second equation we get

$$2x_1 + x_2 + x_3 = \text{constant.}$$

Hence $$2v_1 + v_2 + v_3 = 0$$

and $$2a_1 + a_2 + a_3 = 0 \qquad \text{—— (4)}$$

From eq. (1) to eq (4)

$$T = 277\ \text{N.}$$

$$a_1 = 0{\cdot}576\ \text{m/s}^2$$

$$\boxed{a_2 = 2{\cdot}885\ \text{m/s}^2}$$

and $$a_3 = -4{\cdot}04\ \text{m/s}^2.$$

Velocity of mass m_1 at time t = 5 s is equal to

$$v_1 = u_1 + a_1 t$$

$$= 0 + 0{\cdot}576 \times 5$$

$$\boxed{v_1 = 2{\cdot}88\ \text{m/s}}$$

12. A 2 kg ball revolves in a horizontal circle as shown. Knowing that L = 0·9 m and that the maximum allowable tension in the cord is 50N, determine **(a)** the maximum allowable speed, **(b)** the corresponding value of the angle θ.

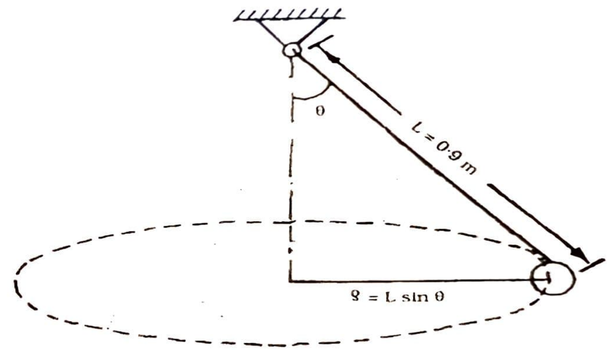
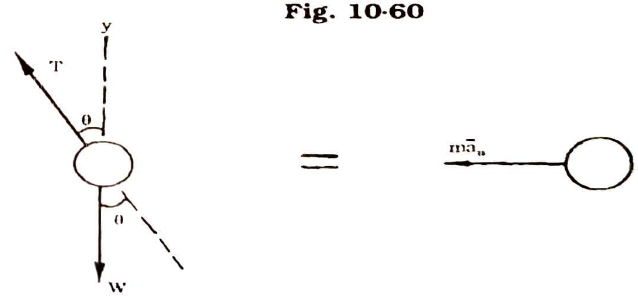


Fig. 10·60



(a) (b)

Fig. 10·61

Here the ball revolves in a *horizontal circular path* of radius $\rho = L \sin\theta$. The normal component $\bar{a}_n$ of the acceleration is directed toward the center of the path, its magnitude is $a_n = v^2/\varrho$ where v is the speed of the ball.

Applying *Newton's second law*,

$+\uparrow\Sigma F_y = 0:$ $\quad T\cos\theta - W = 0,$

$\therefore \quad 50\cos\theta - 2\,(9.81) = 0$

$\therefore \quad \boxed{\theta = 66.9^\circ}$

Equation of Motion :

$\leftarrow + \Sigma F_n = ma_n,$ $\quad T\sin\theta = m\,a_n$

$50\sin 66.9^\circ = 2\,a_n$

$\therefore \quad a_n = 23.0 \text{ m/s}^2$

but $a_n = \dfrac{v^2}{\rho}$ $\quad$ where $\quad \rho = L\sin\theta = 0.9\sin 66.9^\circ$

$\therefore \quad \rho = 0.83 \text{ m.}$

$\therefore \quad v^2 = a_n\,\rho = 23.0 \times 0.83 = 19.09$

$\therefore \quad \boxed{v = 4.37 \text{ m/s}}$

13. A small sphere of mass 5 kg is held as shown by two wires AB and CD. If wire AB is cut, determine the tension in the other wire CD **(a)** before AB is cut **(b)** immediately after AB has been cut.

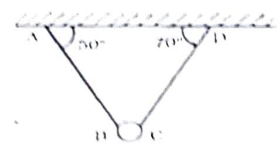
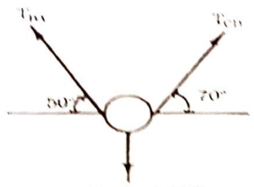
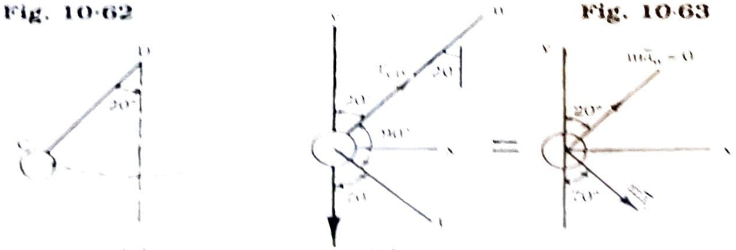


Fig. 10.62



Free-body diagram of sphere before AB is cut.
Fig. 10.63



Free-body diagram of sphere immediately after AB has been cut
Fig. 10.64

(a) Before AB is cut :

Free-body diagram of sphere before AB is cut is shown above.

$\Sigma F_x = 0, \quad \overrightarrow{+ve}, \quad -T_{BA}\cos 50^\circ + T_{CD}\cos 70^\circ = 0$

$-0.64\,T_{BA} + 0.34\,T_{CD} = 0$

$\therefore \quad T_{BA} = 0.53\,T_{CD} \qquad \text{—— (1)}$

$\Sigma F_y = 0, \quad \uparrow +ve, \quad T_{BA}\sin 50^\circ + T_{CD}\sin 70^\circ - W = 0$

$0.766\,T_{BA} + 0.94\,T_{CD} - 49.05 = 0$

$T_{BA} = 64.03 - 1.227\,T_{CD} \qquad \text{—— (2)}$

Equating eq. (1) and eq. (2)

$0.53\,T_{CD} = 64.03 - 1.227\,T_{CD}$

$\therefore \quad \boxed{T_{CD} = 36.44 \text{ N}}$

(b) Immediately after AB is cut :

$\Sigma F_t = ma_t, \quad W\cos 70^\circ = ma_t$

$\therefore \quad a_t = 3.555 \text{ m/s}^2 \qquad \text{—— (3)}$

$\Sigma F_n = ma_n, \quad T_{CD} - W\sin 70^\circ = ma_n$

$\therefore \quad a_n = 0.2\,T_{CD} - 9.22 \qquad \text{—— (4)}$

$\Sigma F_y = ma_y, \quad T_{CD}\cos 20^\circ - W = ma_n\cos 20^\circ - ma_t\cos 70^\circ$

$\therefore \quad a_n = 0.2\,T_{CD} - 9.146 \qquad \text{—— (5)}$

From eq. (4) and eq. (5), it can be concluded that either ***a_n must be zero*** or ***T_{CD} must be zero***, but ***T_{CD} can not be zero*** hence **$a_n = 0$.**

Putting this value

$\boxed{T_{CD} = 46.1 \text{ N}}$

14. The motion of a 2 kg block B in a horizontal plane is defined by the relations $r = 3t^2 - t^3$ and $\theta = 2t^2$, where r is expressed in meters, t in seconds, and θ in radians. Determine the radial and transverse components of the force exerted on the block when **(a)** t = 0 **(b)** t = 1 s.

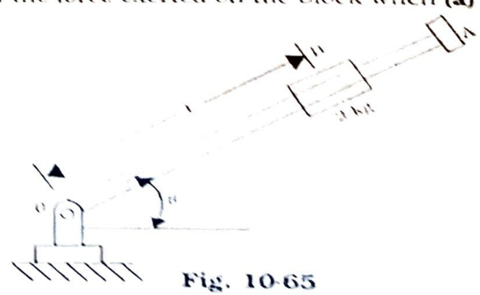
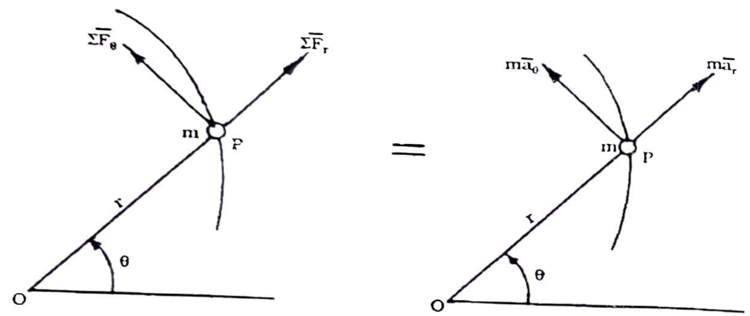


Fig. 10.65



(a) Fig. 10·66 (b)

Equations of Motion : *Using radial and transverse components*

$$\Sigma F_r = m a_r : \quad \Sigma F_r = m(\ddot{r} - r\dot{\theta}^2) \qquad \text{—— (1)}$$

$$\Sigma F_\theta = m a_\theta : \quad \Sigma F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta}) \qquad \text{—— (2)}$$

Now, $r = 3t^2 - t^3$ and $\theta = 2t^2$

$$\therefore \quad \dot{r} = \frac{dr}{dt} = 6t - 3t^2 \qquad \therefore \quad \dot{\theta} = \frac{d\theta}{dt} = 4t$$

and $\ddot{r} = \dfrac{d^2r}{dt^2} = \dfrac{d(\dot{r})}{dt} = 6 - 6t$ and $\ddot{\theta} = \dfrac{d^2\theta}{dt^2} = \dfrac{d(\dot{\theta})}{dt} = 4$

Putting these values in eq. (1) & eq. (2)

$$F_r = m(\ddot{r} - r\dot{\theta}^2) = 2\{(6 - 6t - (3t^2 - t^3)(4t)^2\}$$
$$= 2(6 - 6t - 48t^4 + 16t^5)$$
$$\therefore \quad F_r = 32t^5 - 96t^4 - 12t + 12 \qquad \text{—— (3)}$$

and

$$F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta}) = 2\{(3t^2 - t^3)(4) + 2(6t - 3t^2)(4t)\}$$
$$= 2(12t^2 - 4t^3 + 48t^2 - 24t^3)$$
$$\therefore \quad F_\theta = 120t^2 - 56t^3 \qquad \text{—— (4)}$$

$$\boxed{\text{(a) when } t = 0, \quad F_r = 12\text{ N}, \quad \text{and} \quad F_\theta = 0}$$

$$\boxed{\text{(b) when } t = 1\text{ s}, \quad F_r = -64\text{ N}, \quad \text{and} \quad F_\theta = 64\text{ N}}$$

15. A mass of 10 kg released from rest slides 300 mm along a 1 to 10 incline. It is stopped there by a spring whose constant is 2500 N/m. *Neglecting the effect of friction*, determine the maximum compression of the spring.

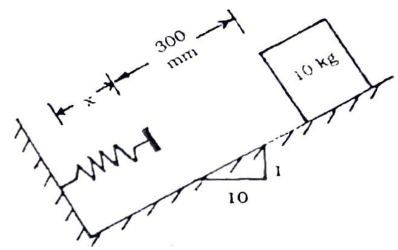


Fig. 10·67

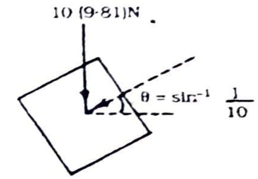


Fig. 10·68

Let x be the *maximum compression in the spring*

$$\tan\theta = \frac{1}{10} \qquad \therefore \theta = 5{\cdot}71° \text{ and } \sin\theta = 0{\cdot}1$$

Work done by the block $= W \sin\theta \times (x + 0{\cdot}3)$

$$U_B = 10 \times 9{\cdot}81 \times 0{\cdot}1\,(x + 0{\cdot}3)$$

Work done by the spring $= -\dfrac{1}{2}kx^2$

$$U_S = -\frac{1}{2} \times 2500\,(x^2)$$

But the *block comes to rest*, hence *total work done is zero.*

$$\therefore \quad U_B + U_S = 0$$

$$10 \times 9{\cdot}81 \times 0{\cdot}1\,(x + 0{\cdot}3) - \frac{1}{2} \times 2500\,(x^2) = 0$$

$$\boxed{x = 0{\cdot}052 \text{ m}}$$

16. The 20 kg block shown in figure rests on the smooth incline. If the spring is originally unstretched, determine the total work done by all the forces acting on the block when a horizontal force P = 200 N pushes the block up the plane by d = 2m.

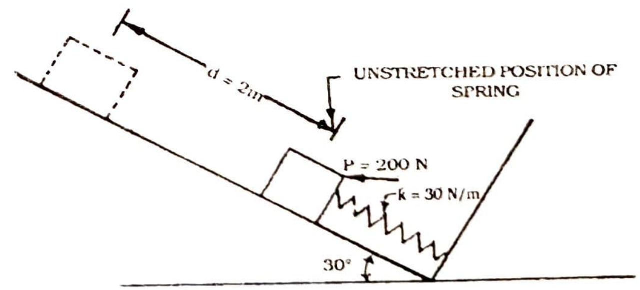
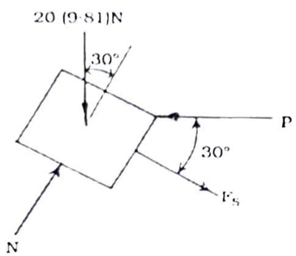


Fig. 10·69



Free-body diagram of block.

Fig. 10·70

Along the displacement, the *components of the forces* are to be found out for finding the work done.

Work done by different forces are calculated below :

(1) Horizontal force P : *This force is constant.*

Here the *displacement is in the direction of P cos 30° therefore work done is positive.*

$$U_P = (P \cos 30°)\, d$$
$$= (200 \cos 30°)2$$
$$= 346{\cdot}4 \text{ J.}$$

(2) Spring Force F_s : *Spring force is opposite to the displacement hence work done is negative.*

$$U_s = -\frac{1}{2}\, k\, x^2$$
$$= -\frac{1}{2}\,(30)\,(2)^2$$
$$= -60 \text{ J.}$$

(3) Weight W : *W sin 30° is opposite to the displacement hence workdone is negative.*

$$U_w = -(W \sin 30°)\, d$$
$$= -(20 \times 9{\cdot}81 \times 0{\cdot}5)2$$
$$= -196{\cdot}2 \text{ J}$$

(4) Normal Force N : *This force does* **no** *work since it is perpendicular to the displacement.*

$$U_N = 0.$$

Therefore, *Total work done* = *Work done by individual forces.*

$$U_{TOTAL} = U_P + U_S + U_W + U_N$$
$$= 346{\cdot}4 + (-60) + (-196{\cdot}2) + 0$$

$$\boxed{U_{TOTAL} = 90{\cdot}2 \text{ J}}$$

17. An 8-kg plunger is released from rest in the position shown and is stopped by two nested springs, the constant of the outer spring is k_1 = 3 kN/m and the constant of the inner spring is k_2 = 10 kN/m. If the maximum deflection of the outer spring is observed to be 150 mm, determine the height h from which the plunger was released.

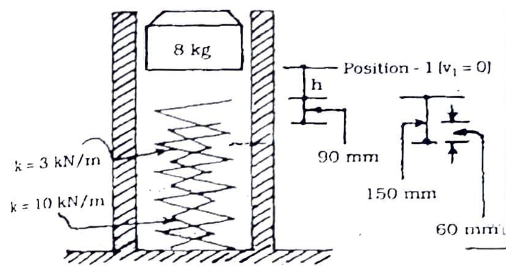


Fig. 10·71

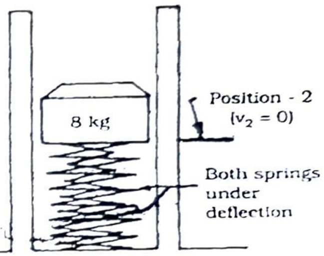


Fig. 10·72

Maximum deflection of outer spring = 150 mm

therefore, the *maximum deflection of inner spring* = (150 – 90) mm = 60 mm.

Applying the *principle of work and energy* between position (1) and position (2)

$$T_1 + U_{1-2} = T_2 \qquad \text{——— (1)}$$

$$\frac{1}{2}\, m v_1^2 + \{+W\,(h + 0{\cdot}150) - \frac{1}{2} \times k_1 \times x_1^2 - \frac{1}{2}\, k_2 \times x_2^2\} = \frac{1}{2}\, m v_2^2$$

but $v_1 = 0$ and $v_2 = 0$.

and $x_1 = 0{\cdot}150$m and $x_2 = 0{\cdot}060$ m.

∴ Equation (1) becomes,

$$0 + \{8 \times 9{\cdot}81\,(h + 0{\cdot}150) - \frac{1}{2}\,(0{\cdot}150)^2 \times 3000 - \frac{1}{2}\,(0{\cdot}060)^2 \times 10000\} = 0$$

∴ $78{\cdot}48\, h - 39{\cdot}978 = 0$

∴ $\boxed{h = 0{\cdot}509 \text{ m}}$

18. A system shown in figure released from rest. Determine the maximum velocity of the 10 kg mass. How high will the 10 kg mass rise above the floor ?

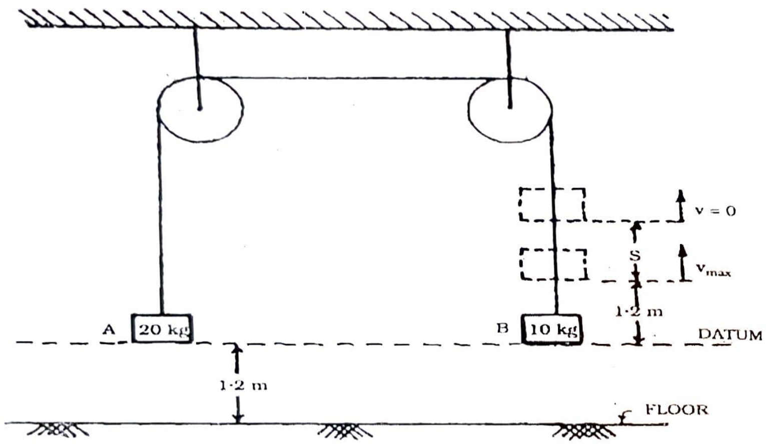


Fig 10.73

From the figure, it can be concluded that

velocity of A = *velocity of* B.

$v_A = v_B = v_{say}$ ——— **(1)**

Hence, if *A moves down by 1·2m, B moves up by 1·2 m.*

Now, using *work and energy principle,*

Initial Kinetic Energy + Work done = Final Kinetic Energy. of the system

$$0 + W_A(1\cdot2) - W_B(1\cdot2) = \frac{1}{2} m_A v^2 + \frac{1}{2} m_B v^2$$

$$\therefore \quad 0 + 20(9\cdot81)(1\cdot2) - 10(9\cdot81)(1\cdot2) = \frac{1}{2}(30)v^2$$

$$\frac{117\cdot72 \times 2}{30} = v^2$$

$$\therefore \quad \boxed{v_{max} = 2\cdot80 \text{ m/s}}$$

Mass 10 kg will further rise till its velocity becomes zero.

$$v^2 - u^2 = 2gs$$

$$0 - (2\cdot8)^2 = 2 \times 9\cdot81 \times s$$

$$\therefore \quad s = 0\cdot4 \text{ m}$$

Hence, mass 10 kg will rise from the floor by

a distance $1\cdot2 + 1\cdot2 + 0\cdot4 = \boxed{2\cdot8 \text{ m}}$

19. A 4 kg mass attached to a spring slides along a circular guide in horizontal plane without friction as shown in figure. What will be the velocity of mass as it passes through point C if it is released from rest at B ? The undeformed length of the spring is 200 mm and its stiffness is 200 N/m.

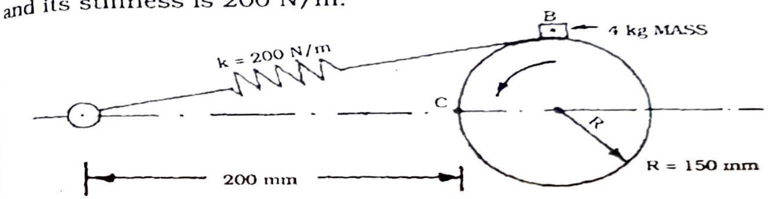


Fig. 10·74

Extension of the spring when the mass is at B.

$$= x = \sqrt{350^2 + 150^2} - 200$$

$$= 180\cdot79 \text{ mm}$$

$$= 0\cdot181 \text{ m.}$$

Using **Principle of work and energy :**

$$T_1 + U_{1-2} = T_2$$

$$0 + \frac{1}{2} kx^2 = \frac{1}{2} mv_c^2$$

$$\frac{1}{2}(200)(0\cdot181)^2 = \frac{1}{2}(4)v_c^2$$

$$\therefore \quad \boxed{v_c = 1\cdot28 \text{ m/s.}}$$

20. A particle of weight 500 N is moving with a horizontal velocity of 3·0 m/s at the top of the hill A. Determine the normal force exerted on the particle when it arrives at B, where the radius of curvature is ρ = 17 m. *Neglect friction.*

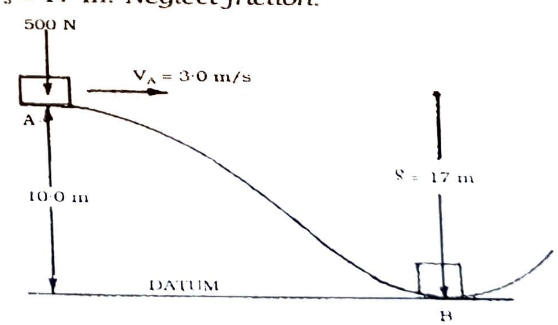


Fig. 10·75

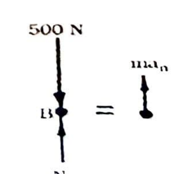


Fig. 10·76

Applying **Principle of conservation of energy** between points A and B,

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2} m_A v_A^2 + Wh = \frac{1}{2} m_B v_B^2 + 0$$

$$\frac{1}{2}\left(\frac{500}{9.81}\right)(3)^2 + (500)(10) = \frac{1}{2}\left(\frac{500}{9.81}\right) v_B^2 + 0$$

$$\therefore \quad v_B = 14.32 \text{ m/s}^2$$

Applying **Equation of Motion at B,**

$$+\downarrow \Sigma F_n = ma_n, \quad N_B - 500 = \left(\frac{500}{9.81}\right)a_n$$

$$\text{but } a_n = \frac{v^2}{\rho} = \frac{(14.32)^2}{17} = 12.06 \text{ m/s}^2$$

$$\therefore \quad N_B - 500 = \left(\frac{500}{9.81}\right) 12.06$$

$$\boxed{N_B = 1114.7 \text{ N}}$$

21. An airline employee tosses a suit case of mass 30 kg with a horizontal velocity of 10 m/s on to a baggage carrier of mass 70 kg which is resting on a horizontal surface. Knowing that the carrier can roll freely, determine the velocity of the carrier after the suitcase has slide to a relative stop on the carrier.

Using **Principle of conservation of linear momentum**

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v_3$$

$$30 \times 10 + 70 \times 0 = (30 + 70) v_c$$

$$\boxed{v_c = 3 \text{ m/s}}$$

22. The 200 kg crate as shown is originally at rest on the smooth horizontal surface. If a force of 200 N, acting at an angle of 45°, is applied to the crate for 10 s, determine the final velocity of the crate and the normal force which the surface exerts on the crate during the time interval.

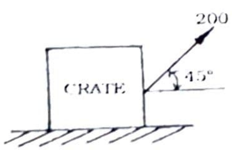
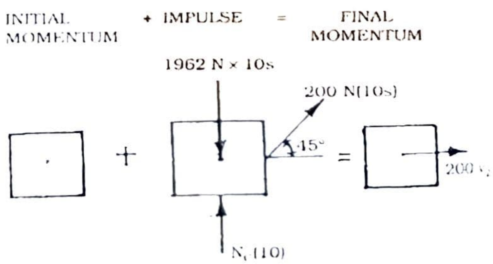


Fig. 10.77



(a) (b) (c)

Fig. 10.78

Here *force is constant*

$$\therefore \quad \textbf{Impulse} = F_c (t_2 - t_1)$$

$$\therefore \quad (Imp)_x = 200 \cos 45° (10)$$

$$\text{and} \quad (Imp)_y = 200 \sin 45° (10) + N_c (10) - 1962 (10)$$

Applying **Principle of impulse and momentum,**

$$(\overset{\rightarrow}{+}) \quad m (v_x)_1 + \int_{t_1}^{t_2} F_x \, dt = m (v_x)_2$$

$$O + 200 \cos 45° (10) = 200 v_2$$

$$\therefore \quad \boxed{v_2 = 7.07 \text{ m/s}}$$

$$(+\uparrow) \quad m (v_y)_1 + \int_{t_1}^{t_2} F_y \, dt = m (v_y)_2$$

Assuming that the *crate remains on the surface*

$$O + N_c (10) - 1962 (10) + 200 (\sin 45°)(10) = 0$$

$$\therefore \quad \boxed{N_c = 1820.6 \text{ N}}$$

23. Blocks A and B as shown have a mass of 3 kg and 5 kg respectively. If the system is released from rest, determine the velocity of block B in 6 seconds. *Neglect the mass of pulleys and cord.*

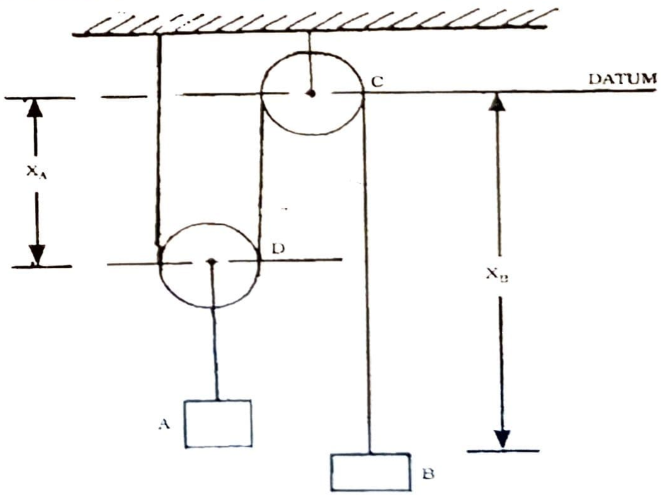


Fig. 10.79

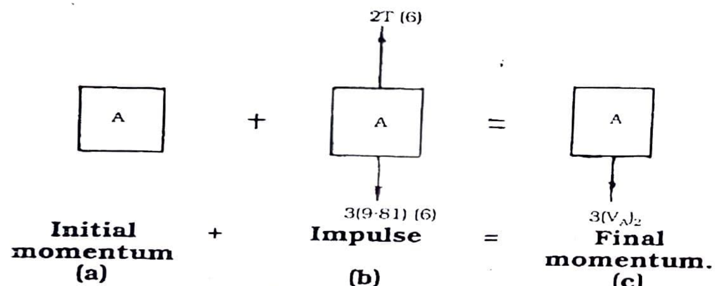


Fig. 10·80

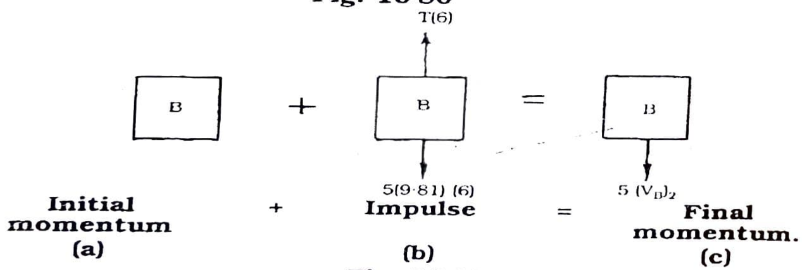


Fig. 10·81

Applying **Principle of Impulse and Momentum**

Block A :

$$(+\downarrow) \qquad m\,(v_A)_1 + \int_{t_1}^{t_2} F_y dt = m\,(v_A)_2$$

$$0 - 2T\,(6) + 3\,(9{\cdot}81)\,(6) = 3\,(v_A)_2 \qquad \text{—— (1)}$$

Block B :

$$(+\downarrow) \qquad m\,(v_B)_1 + \int_{t_1}^{t_2} F_y\, dt = m\,(v_B)_2$$

$$0 + 5\,(9{\cdot}81)\,(6) - T\,(6) = 5\,(v_B)_2 \qquad \text{—— (2)}$$

Relation between v_A and v_B :

Length of the cord remains same

$\therefore \quad 2\,x_A + x_B = \text{constant}$

differentiating w. r. to t

$2\,v_A + v_B = 0$

$\therefore \quad 2\,v_A = -\,v_B \qquad \text{—— (3)}$

From eq (1), eq (2) and eq (3)

$$\boxed{(v_B)_2 = 35{\cdot}8 \text{ m/s}\downarrow \quad T = 19{\cdot}2 \text{ N}}$$

24. A mass of 8 kg can slide freely on a smooth vertical rod as shown in figure. The mass is released from rest at a distance of 500 mm from the top of the spring. The spring constant is 60 N/mm.

Determine the relocity of A when the spring has compressed through 20 mm. The free length of the spring is 400 mm.

(Pune University)

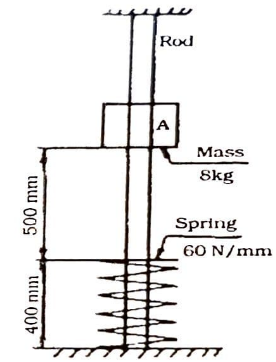


Fig. 10·82

Let initial rest position = Position 1 When $v_1 = 0$

and Final position = Position 2 when $v = v_2$ (Compression of spring = 20 mm)

Applying **Work – Energy Principle :**

(Kinetic Energy)$_1$ + (Work done by Gravitational Force) + (Work done by Spring Force) = (Kinetic Energy)$_2$

$$\frac{1}{2}\,mv_1^2 + mgh + \left(-\,\frac{1}{2}\,kx^2\right) = \frac{1}{2}\,mv_2^2$$

$$(0) + 8 \times 9{\cdot}8\,(0{\cdot}5 + 0{\cdot}020) + \left(-\,\frac{1}{2} \times \frac{60 \times 1000}{1000} \times \left(\frac{20}{1000}\right)^2\right) = \frac{1}{2} \times 8 \times v_2^2$$

$$\boxed{v_2 = 2{\cdot}684 \text{ m/s}}$$

25. A small toy-car of mass 2 kg is released from position A as shown in figure, and rolls freely along the smooth curved floor ABC in vertical plane. If the radii of curvature of the path at B and C are 1 m and 5 m respectively, find the reactions of the curved floor to the toy-car when it is at positions B and C. **(Pune University)**

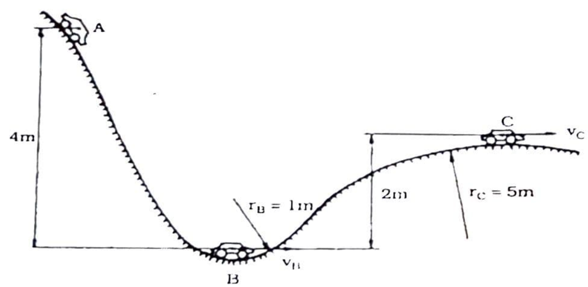


Fig. 10·83

Applying **Work – Energy Principle between Positions A and B :**

$$(\text{Kinetic Energy})_A + \left(\begin{array}{c}\text{Gravitational}\\ \text{Work done}\\ \text{between}\\ \text{A \& B}\end{array}\right) + \left(\begin{array}{c}\text{Friction}\\ \text{Work}\end{array}\right) = (\text{Kinetic Energy})_B$$

$$\frac{1}{2}\, m v_A^2 + mg(h_A - h_B) + \left(\begin{array}{c}\text{Smooth floor}\\ \text{Zero friction}\end{array}\right) = \frac{1}{2}\, m v_B^2$$

$$0 + 2 \times 9{\cdot}81\,(4) + 0 = \frac{1}{2} \times 2 \times v_B^2$$

$$\therefore \quad V_B = 8{\cdot}859 \text{ m/s}$$

Applying **Work – Energy Principle between Positions A and C :**

$$(\text{Kinetic Energy})_A + \left(\begin{array}{c}\text{Gravitational}\\ \text{Work done}\\ \text{between}\\ \text{A \& C}\end{array}\right) + \left(\begin{array}{c}\text{Friction}\\ \text{Work}\end{array}\right) = \left(\begin{array}{c}\text{Kinetic}\\ \text{Energy}\end{array}\right)_C$$

$$\frac{1}{2}\, m v_A^2 + mg\,(h_A - h_C) + (0) = \frac{1}{2}\, m v_C^2$$

$$0 + 2 \times 9{\cdot}81\,(4 - 2) + 0 = \frac{1}{2} \times 2 \times v_C^2$$

$$\therefore \quad v_c = 6{\cdot}264 \text{ m/s}$$

Now, **Forces acting on the car = Reaction by the floor on the car :**

Position B :

(1) Car's own weight = W = mg (↓)

(2) Centrifugal force $= ma_n = m\, \dfrac{v_B^2}{\rho_B}$ (↓)

Reaction by the floor at B $= \left[mg + m\, \dfrac{v_B^2}{\rho_B}\right]$ (↑)

$$= \left[2 \times 9{\cdot}81 + 2 \times \frac{8{\cdot}859^2}{1}\right]$$

$$\boxed{R_B = 176{\cdot}58 \text{ N } (\uparrow)}$$

Position C :

(1) Car's own weight = W = mg (↓)

(2) Centrifugal force $= ma_n = m\, \dfrac{v_C^2}{\rho_C}$ (↑)

Reaction by the floor at C $= \left[mg - m\, \dfrac{v_C^2}{\rho_C}\right]$ (↑)

$$= \left[2 \times 9{\cdot}81 - \frac{6{\cdot}264^2}{5}\right]$$

$$\boxed{R_C = 3{\cdot}925 \text{ N } (\uparrow)}$$

26. A train weighing 30 tonnes is travelling up an incline of 1 in 300 at a uniform speed of 36 km/hr. If the tractive resistance on the level track is 5g N/tonne, find the power developed.

(Gujarat University, Aug. 1987)

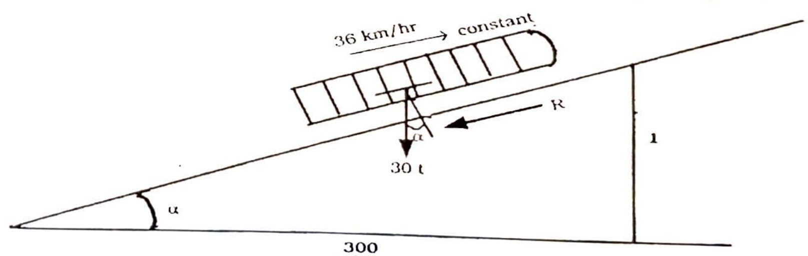


Fig. 10·84

Mass of the train = 30 × 1000 = 30,000 kg

Speed of the train $= \dfrac{30 \times 1000}{60 \times 60} = 10$ m/s

$\tan\alpha = 1/300 \therefore \alpha = 0{\cdot}19^\circ$ & $\sin\alpha = 0{\cdot}0033 = \tan\alpha$

Tractive Resistance = R = 5 g × 30 = 1470 N

Total resistance acting on the train along the path of the train

$= $ Component of weight + tractive resistance
$= (30{,}000 \times 9{\cdot}81) \sin\alpha + 1470$
$= 980{\cdot}99 + 1470$
$= 2450{\cdot}99 \text{ N} \approx 2451 \text{ N}$

As the speed of the train is constant, the tractive force acting on the train must be equal to the total resistance acting on the train.

Now, Power developed = Tractive force × Velocity

$= 2451 \times 10 = \boxed{24500 \text{ Watts}}$

27. A man weighing 600 N gets into an elevator, calculate force exerted by him on the floor of the elevator, **(a)** When it is moving down with uniform velocity of 2 m/s **(b)** Ascending with a acceleration of 3 m/s² **(c)** Descending with an acceleration of 4·9 m/s².

(Gujarat University, Dec. 1988)

Case (a) Velocity is uniform, acceleration = 0

Force exerted by the man on the floor of the elevator

= Man's own weight + Inertia force
= 600 + mass × acceleration
$= \boxed{600 \text{ N } (\downarrow)}$

Case (b) Acceleration = 3 m/s² (↑)
Inertia force will be down ward.

Force exerted by the man on the floor of the elevator

= Man's own weight (↓) + Inertia force (↓)

$$= 600 + \frac{600}{9{\cdot}81} \times 3$$

$= \boxed{783{\cdot}49 \text{ N } (\downarrow)}$

Case (c) Acceleration = 4·9 m/s² (↓)
Inertia force will be upward.

Force exerted by the man on the floor of the elevator

= Man's own weight (↓) + Inertia force (↑)

$$= 600 - \frac{600}{9.81} \times 4{\cdot}9$$

$= \boxed{300{\cdot}3 \text{ N } (\downarrow)}$

28. Three Blocks A, B and C as shown in figure have their masses 20 kg, 15 kg and 5 kg respectively. Determine the tension in each string and acceleration of each block.

(Saurashtra University, June 1991)

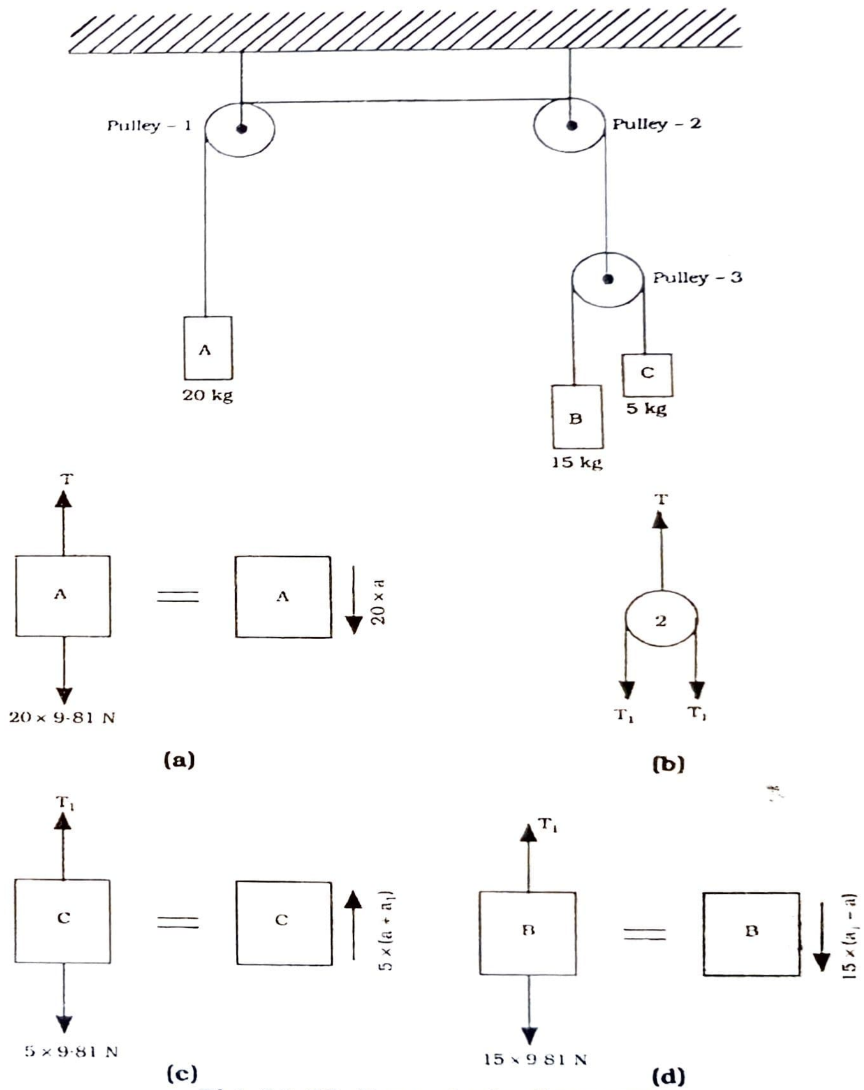


Fig. 10·85 Free - body diagrams

Let, acceleration of body A = a (downward) m/s^2

$\therefore$ acceleration of pulley – 3 = a (upward) m/s^2

Let, acceleration of body C (due to mass B) = a_1 (upward) m/s^2

(This acceleration is **about** pulley - 3)

$\therefore$ Net acceleration of body C = $a + a_1$ (upward) m/s^2

and, Net acceleration of body B = $a_1 - a$ (down ward) m/s^2

pplying **Equations of Motion :**

$\Sigma F_y = ma_y$, **Block A :** $20 \times 9{\cdot}81 - T = 20 \times a$ ... (1)

Block B : $(15 \times 9{\cdot}81) - T_1 = 15\,(a_1 - a)$ ... (2)

Block C : $T_1 - (5 \times 9{\cdot}81) = 5 \times (a + a_1)$ ... (3)

and, from equilibrium of **pulley - 3**

$T = 2\,T_1$ ... (4)

From eq. (1), $T = 196{\cdot}2 - 20\,a$ ... (5)

from eq. (4), $2T_1 = 196{\cdot}2 - 20\,a \;\therefore\; T_1 = 98{\cdot}1 - 10\,a$ ... (6)

Putting eq. (5) in eq. (2),

$15 \times 9{\cdot}81 - (98{\cdot}1 - 10\,a) = 15\,a_1 - 15\,a$

$\therefore\; 147{\cdot}15 - 98{\cdot}1 = 15\,a_1 - 15\,a - 10\,a$

$\therefore\; 15\,a_1 - 25\,a = 49{\cdot}05$ ... (7)

Putting eq. (6) in eq. (3)

$98{\cdot}1 - 10\,a - (5 \times 9{\cdot}81) = 5\,a + 5\,a_1$

$49{\cdot}05 = 15\,a + 5\,a_1$ ... (8)

From eq. (7) and (8),

$10\,a_1 - 40\,a = 0$

$\therefore\; a_1 = 4\,a$ ... (9)

$\therefore$ from eq. (8),

$$\boxed{\begin{aligned} a &= 1{\cdot}4\ m/s^2 \\ a_1 &= 5{\cdot}6\ m/s^2 \\ T &= 168{\cdot}2\ N \\ T_1 &= 84{\cdot}1\ N \end{aligned}}$$

THEORY RELATED QUESTIONS

1. **State :**
   - **(i)** Newton's Second Law of Motion.
   - **(ii)** Principle of Conservation of Linear Momentum.
   - **(iii)** Principle of Work and Energy.
   - **(iv)** Principle of Conservation of Energy.
   - **(v)** Principle of Impulse and Momentum.
2. **Explain :**
   - **(i)** Linear Momentum.
   - **(ii)** Dynamic Equilibrium.
   - **(iii)** Angular Momentum.
   - **(iv)** Central Force.
   - **(v)** Work of a Force.
   - **(vi)** Kinetic Energy.
   - **(vii)** Potential Energy.
   - **(viii)** Conservative Forces.
   - **(ix)** Impulse.
3. **Explain in brief :**
   - **(i)** Equation of Motion.
   - **(ii)** Equation of motion in
     - **(a)** rectangular components.
     - **(b)** tangential and normal components.
     - **(c)** radial and transverse components.
   - **(iii)** Relation between moment and rate of change of angular momentum.
   - **(iv)** Conservation of angular momentum.
   - **(v)** Work of
     - **(a)** variable force.
     - **(b)** constant force.
     - **(c)** weight.
     - **(d)** spring force.

EXERCISES

10·1 The 50-kg crate as shown rests on a horizontal plane for which the coefficient of kinetic friction is μ_k = 0·3. If the crate is ubjected to a 400-N towing force as shown, determine the velocity f the crate in 5 seconds from rest.

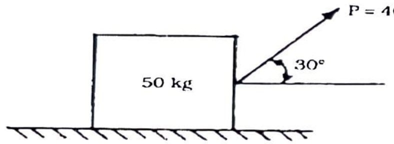


Fig. 10·86

.0·2 Block B has a mass m and is released from rest when it is on op of cart A, which has a mass of 3 × m. Determine the tension in ord CD needed to hold the cart from moving while B is sliding own A. *Neglect friction.*

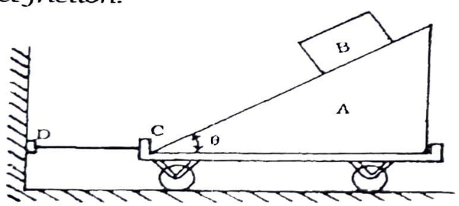


Fig. 10·87

0·3 The 1600 N cylinder at A is hoisted using the motor and the ulley system as shown. If the speed of point B on the cable is icreased at a constant rate from zero to v_B = 3 m/s in t = 5s, etermine the tension in the cable at B to cause the motion.

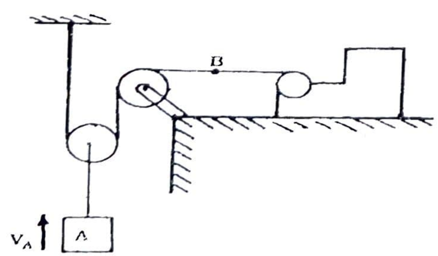


Fig. 10·88

10·4 The system shown is released from rest. *Assuming that the pulleys are frictionless and of negligible mass*, determine the acceleration of each block.

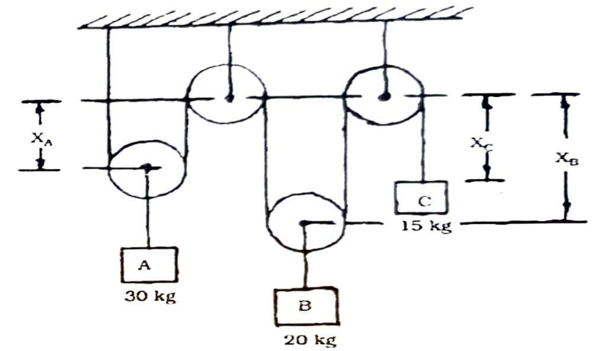


Fig. 10·89

10·5 A force of F = 60 N is applied to the cord as shown. Determine how high the 120 N block A rises in 2 s starting from rest. *Neglect the weight of pulleys and cord.*

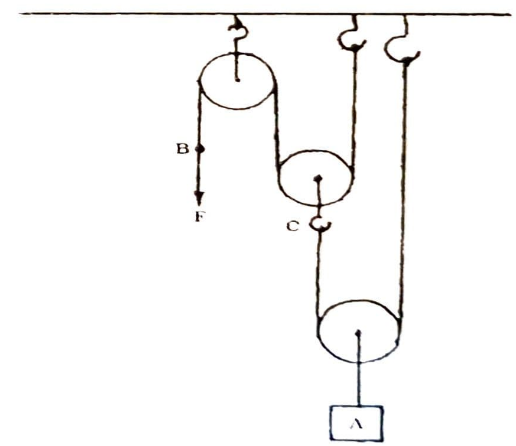


Fig. 10·90

10·6 Each of the three plates has a mass of 10 kg. If the coefficients of static and kinetic friction at each surface of contact are $\mu_s = 0{\cdot}3$ and $\mu_k = 0{\cdot}2$, respectively, determine the acceleration of each plate at the instant when the three horizontal forces are applied as shown.

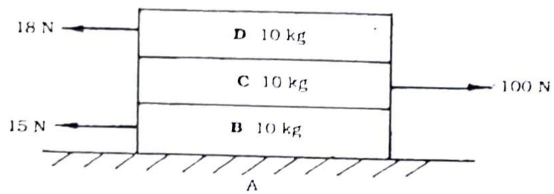


Fig. 10·91

10·7 Determine the banking angle θ of the circular track so that the wheels of the sports car shown in figure will not have to depend upon friction to prevent the car from sliding either up or down the curve. The car travels at a constant speed of 30 m/s. The radius of the track is 200 m.

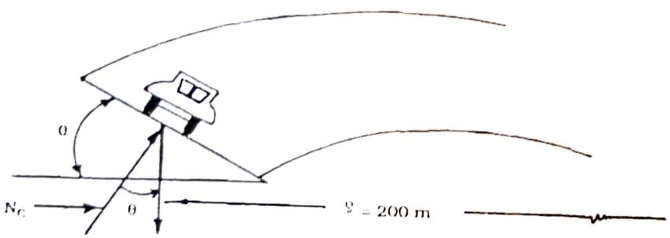


Fig. 10·92

10·8 The two blocks shown are originally at rest. Neglecting the masses of the pulleys and the effect of friction in the pulleys and between block A and the incline, determine **(a)** the acceleration of each block, **(b)** the tension in the cable.

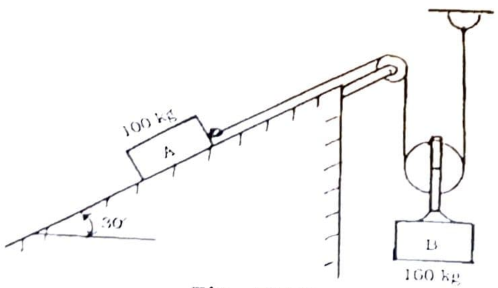


Fig. 10·93

10·9 A 1500 kg automobile is driven down a 5° incline at a speed of 90 km/h when the brakes are applied, causing a constant total braking force (applied by the road on the tires) of 8 kN. Determine the distance travelled by the automobile as it comes to stop.

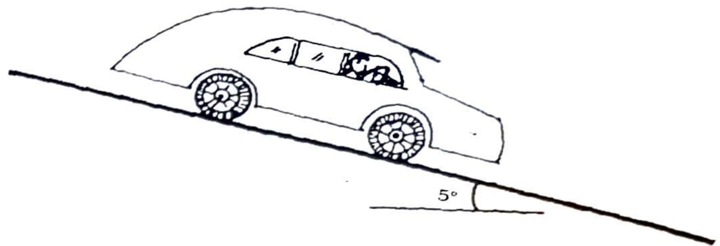


Fig. 10·94

10·10 A 8 kg block rests on the horizontal surface as shown. The spring which is not attached to the block, has a stiffness k = 500 N/m and is initially compressed 0·15 m from C to A. After the block is released from rest at A, determine its velocity when it passes point D. The coefficient of kinetic friction between the block and the plane is $\mu_k = 0{\cdot}2$.

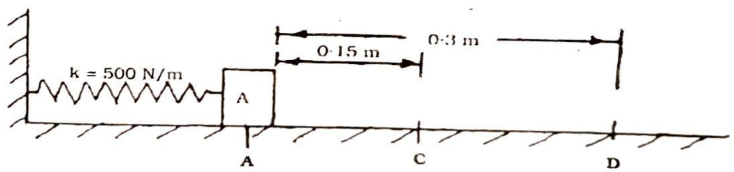


Fig. 10·95

10·11 A 1500-kg car starts from rest at point 1 and moves without friction down the track shown. **(a)** Determine the force exerted by the track on the car at point 2 where the radius of curvature of track is 5 m. **(b)** Determine the minimum safe value of the radius of curvature at point 3 (*i. e. when normal reaction becomes zero*).

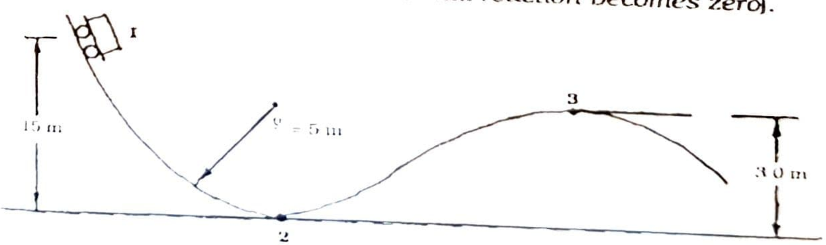


Fig. 10·96

10·12 The system shown is at rest when a constant 150-N force is applied to collar B. **(a)** If the force acts through the entire motion, determine the speed of collar B as it strikes the support at C. **(b)** After what distance d should the 150-N force be removed if the collar is to reach C with zero velocity ?

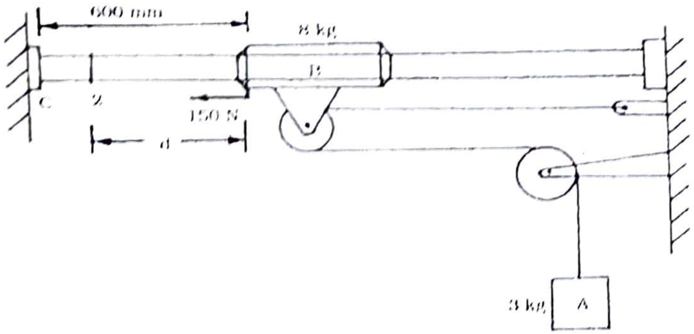


Fig. 10·97

10·13 A 4 kg block is attached to a cable and to a spring as shown. The constant of the spring is k = 1500 N/m and the tension in the cable is 12 N. If the cable is cut, determine the maximum displacement of the block.

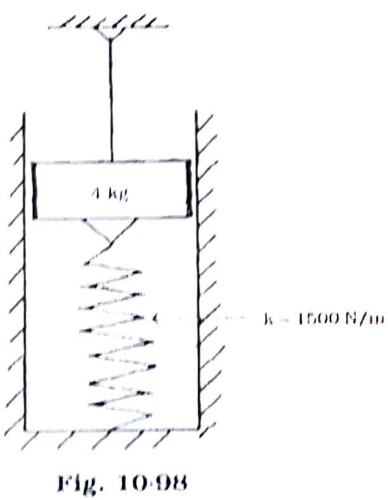


Fig. 10·98

10·14 A ram R shown has a mass of 150 kg and is released from rest 0·8 m from the top of a spring A that has a stiffness k_A = 13 kN/m. If a second spring B, having a stiffness k_B = 16 kN/m, is "nested" in A, determine the maximum deflection of A needed to stop the downward motion of the ram. The unstretched length of each spring is indicated in figure. *Neglect the mass of the springs.*

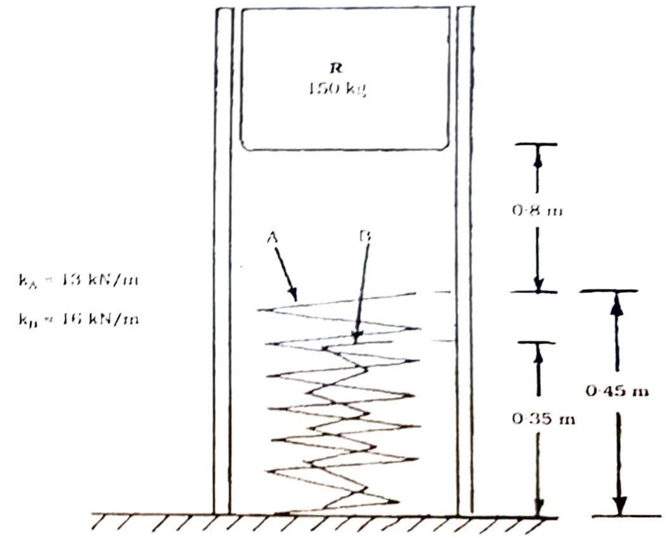


Fig. 10·99

10·15 A 2·5 kg block is at rest on a spring of constant 375 N/m. A 5 kg block is held above the 2·5 kg block so that it just touches it, and then is released. Determine the maximum force exerted on the blocks by the spring.

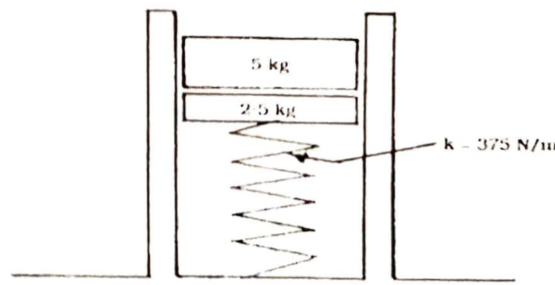


Fig. 10·100

10·16 A 12 kg package drops from a chute in to a 30 kg cart with a velocity of 3·2 m/s. Knowing that the cart is initially at rest and may roll freely, determine **(a)** the final velocity of cart, **(b)** the impulse exerted by the cart on the package.

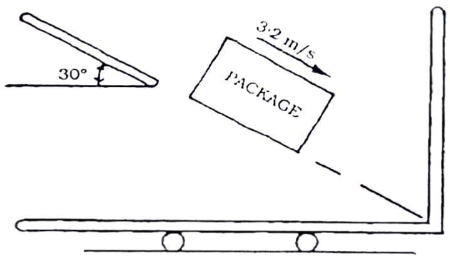


Fig. 10·101

10·17 Block A weighs 40 N and block B weighs 12 N. If B is moving downward with a velocity of $(v_B)_1 = 1$ m/s at $t = 0$; determine the velocity of A when $t = 1$ s. *Assume that the horizontal plane is smooth. Neglect the mass of the pulleys and cord.*

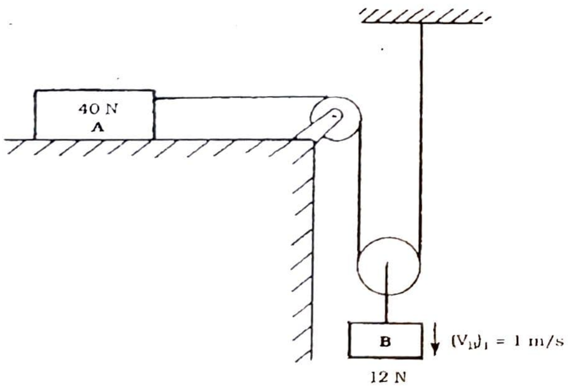


Fig. 10·102

10·18 The 15000 kg box car A is coasting freely at 1·5 m/s on the horizontal track when it encounters a tank car B having mass of 12000 kg and coasting at 0·75 m/s toward it as shown. If the cars meet and couple to gether, determine **(a)** the speed of both cars just after the coupling, and **(b)** the average force between them if the coupling takes place in 0·8 s.

Fig. 10·103

10·19 A 10 N ball is thrown in the direction shown with an initial speed of $v_A = 6$ m/s. Determine the time needed for it to reach its highest point B and the speed at which it is traveling at B. Use the *principle of impulse and momentum* for the solution.

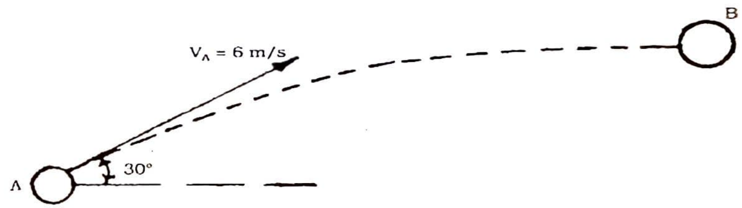


Fig. 10·104

10·20 An airline employee tosses a 15 kg suitcase with a horizontal velocity of 3 m/s on to a 40-kg baggage carrier. Knowing that the carrier is initilly at rest and can roll freely, determine **(a)** the velocity of the carrier after the suitcase has slid to a relative stop on the carrier, **(b)** the ratio of the final kinetic energy of the carrier and suitcase to the initial kinetic energy of the suitcase.

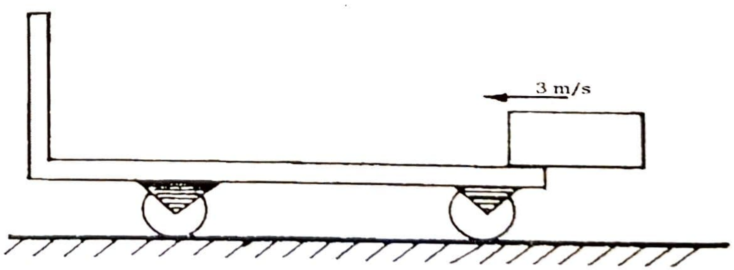


Fig. 10·105

SOLUTIONS OF EXERCISES

10·1 Resolving the forces in horizontal & vertical components & applying $\Sigma F = m\,a$

$400 \cos 30° - 0.3\, N_R = 50\, a$ .... **(1)**

$N_R - 50\,(9.81) + 400 \sin 30° = 0$ .... **(2)**

from which $N_R = 290.5$ N, $a = 5.19\ m/s^2$

$v_5 = u + at$

$v_5 = 0 + 5.19\,(5)$

$v_5 = 26\ m/s$

10·2 Resolving forces along inclined plane and perpendicular to inclined plane.

$\Sigma F = m\,a$

$mg \cos\theta = N_B$ ... **(1)**

$mg \sin\theta = m\,a$ ... **(2)**

and $T = N_B \sin\theta$

$\therefore\ T = mg \cos\theta \sin\theta$

$\therefore\ T = \dfrac{mg}{2} \sin 2\theta$

10·3 $v_B = u_B + a_B t$

$3 = 0 + a_B\,(5)$

$a_B = 0.6\ m/s^2$

$a_A = a_B/2$

$a_A = 0.3\ m/s^2$, $2T - 1600 = \dfrac{1600}{9.81} \times 0.3$

$2T - W_A = m_A\, a_A$

$T = 824.5$ N

10·4 Applying $\Sigma F = m\,a$

$2T - 30 \times 9.81 = 30\, a_A$ ... **(1)**

$2T - 20 \times 9.81 = 20\, a_B$ ... **(2)**

$T - 15 \times 9.81 = 15\, a_C$ ... **(3)**

and, From figure,

$2x_A + 2x_B + x_C = \text{constant}$

$\therefore\ 2a_A + 2a_B + a_C = 0$

$\therefore\ a_A = -1.63\ m/s^2,\ a_A = 1.63\ m/s^2\ (\downarrow)$

$a_B = 2.45\ m/s^2\ (\uparrow)$

$a_C = -1.63\ m/s^2,\ a_C = 1.63\ m/s^2\ (\downarrow)$

10·5 If F = 60 N, then tension in the first cord passing over pulley C is 60 N and is constant throughout. Hence tension in another cord passing over pulley A is 120 N.

$\Sigma F = m\,a$

On pulley C, (60 + 60) N upward force = 120 N downward force

Block A : $2 \times 120 - 120 = \dfrac{120}{9.81}\, a_A$

$\therefore\ a_A = 9.81\ m/s^2$

$S = ut + \dfrac{1}{2} at^2$

$h = 0 + \dfrac{1}{2}(9.81)2^2$

$= 19.62$ m

10·6 Applying $\Sigma F = m\,a$

$18 - 0.2 \times 10 \times 9.81 = 10 \times a_D$

$\therefore\ a_D = -0.162\ m/s^2\ (\rightarrow)$

$100 - 0.2 \times 10 \times 9.81 - 0.2 \times 20 \times 9.81 = 10 \times a_C$

$\therefore\ a_C = 4.11\ m/s^2\ (\rightarrow)$

$15 - 0.2 \times 20 \times 9.81 - 0.2 \times 30 \times 9.81 = 10 \times a_B$

$\therefore\ a_B = -96.6\ m/s^2 = 0$

$a_B = -96.6\ m/s^2$ **is not possible as friction forces required are too much hence a_B must be zero.**

10·7 Applying $\Sigma F = m\,a$

$N_C \cos\theta - mg = 0$ —— **(1)**

$N_C \sin\theta = m\dfrac{v^2}{g}$ —— **(2)**

Dividing (2) by (1)

$\tan\theta = \dfrac{v^2}{g} = \dfrac{30^2}{9.81\,(200)}$, $\theta = 24.64°$

10·8 Applying $\Sigma F = m\,a$

$T - 100 \times 9.81 \sin 30° = 100\, a_A$ ... **(1)**

$160 \times 9.81 - 2T = 160\, a_B$ ... **(2)**

and From figure $a_A = 2a_B$ ... **(3)**

$a_A = 2.1\ m/s^2$

$a_B = 1.05\ m/s^2$

$T = 700.7$ N

10·9 Applying **Principle of Work and Energy**

$T_1 + U_{1-1} = T_2$

$\dfrac{1}{2} mv_1^2 + (-8000\, d + (1500 \times 9.81 \sin 5°)\, d) = 0$

$\dfrac{1}{2} \times 1500 \times \left(\dfrac{90000}{3600}\right)^2 - 6717.5\, d = 0$

$d = 69.78$ m

10·10 Note : Here mass of Block is equal to 8 kg, otherwise the work done due to friction force will be more than the work due to spring force.

Applying **Principle of Work and Energy**.

$T_A + U_{A-D} = T_D$

$$\frac{1}{2} mv_A^2 + \{\frac{1}{2} k (0{\cdot}15)^2 - 0{\cdot}2 N_R (0{\cdot}3)\} = \frac{1}{2} mv_D^2$$

$$0 + \{\frac{1}{2}(500)(0{\cdot}15)^2 - 0{\cdot}2 (8 \times 9{\cdot}81)(0{\cdot}3)\} = \frac{1}{2} \times 8 \times v_D^2$$

$$v_D = 0{\cdot}48 \text{ m/s}$$

10·11 **(a)** Applying **Principle of Work and Energy** *between points 1 and 2.*

$$T_1 + U_{1-2} = T_2$$

$$0 + 1500 \times 9{\cdot}81 \times 15 = \frac{1}{2} \times 1500 \times v_2^2$$

$$v_2 = 17{\cdot}15 \text{ m/s}$$

Applying **Newton's Second Law.**

$$-W + N_2 = m a_n$$

$$-1500 \times 9{\cdot}81 + N_2 = 1500 \frac{v_2^2}{5}$$

$$N_2 = 102951{\cdot}75 \text{ N}$$

(b) Applying **Principle of Work and Energy** *between points 1 and 3.*

$$T_1 + U_{1-3} = T_3$$

$$0 + 1500 \times 9{\cdot}81 (12) = \frac{1}{2} \times 1500 \times v_3^2$$

$$v_3 = 15{\cdot}34 \text{ m/s}$$

Minimum safe value of radius of curvature occurs when $N_3 = 0$

$$W = m \frac{v_3^2}{\rho}$$

$$1500 \times 9{\cdot}81 = \frac{1500 (15{\cdot}34)^2}{\rho}$$

$$\rho = 24 \text{ m}$$

10·12 **(a)** Tension in string will be $3 \times 9{\cdot}81$ N

Applying **Principle of Work and Energy** *between B and C*

$$T_B + U_{B-C} = T_C$$

$$0 + (150 - 2 \times 3 \times 9{\cdot}81)\, 0{\cdot}6 = \frac{1}{2} \times 8 \times v_c^2 \quad \text{--- (i)}$$

$$v_c = 3{\cdot}7 \text{ m/s}$$

(b) Let at point 2 (at distance d from B) 150 N force is removed.

Applying **Principle of Work and Energy** *between point B and point 2* and then *point 2 and C.*

$$0 + (150 - 2 \times 3 \times 9{\cdot}81)\, d = \frac{1}{2} \times 8 \times v_2^2 \quad \text{--- (ii)}$$

$$\therefore \quad 22{\cdot}785\, d = v_2^2$$

and
$$\frac{1}{2} mv_2^2 - 2 \times 3 \times 9{\cdot}81 (0{\cdot}6 - d) = 0 \quad \text{--- (iii)}$$

$$4 \times 22{\cdot}785d - 35{\cdot}32 + 58{\cdot}86\, d = 0$$

$$\therefore \quad d = 0{\cdot}235 \text{ m}$$

10·13 Here at the block, the tension in the cable is acting upward and spring force is also acting upward whereas weight of the block is acting downward. Under these three forces the system is in equilibrium.

Before Cut :

$$12 - 4 \times 9{\cdot}81 + F_s = 0$$

$$F_s = 27{\cdot}24 \text{ N}$$

Intial Deflection of spring

$$x_o = \frac{27{\cdot}24}{1500} = 0{\cdot}0182 \text{ m}$$

After Cut :

$$T_1 + U_{1-2} = T_2$$

$$0 + 4 \times 9{\cdot}81 \times d - \frac{1}{2} \times 1500 \{(d + 0{\cdot}018)^2 - 0{\cdot}018^2\} = 0$$

$$\therefore \quad d = 0{\cdot}016 \text{ m}$$

$$= 16 \text{ mm}$$

10·14 If spring A is compressed by (x_A) m then spring B will be compressed by $(x_A - 0{\cdot}1)$m.

Applying ***principle of conservation of energy***

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 0 + \{\frac{1}{2} k_A x_A^2 + \frac{1}{2} k_B (x_A - 0{\cdot}1)^2 - Wh\}$$

$$\frac{1}{2}(13000)\, x_A^2 + \frac{1}{2}(16000)(x_A - 0{\cdot}1)^2 = 150 \times 9{\cdot}81 (0{\cdot}8 + x_A)$$

$$x_A = 0{\cdot}4 \text{ m}, \quad x_B = 0{\cdot}3 \text{ m}$$

10·15 Initial deflection due to 2·5 kg block is say x_0.

$2·5 \times 9·81 = F_s$ $T_1 + U_{1-2} = T_2$

$F_s = 24·53$ N $0 + 7·5 \times 9·81 \times d$

$x_0 = 24·53/375$ $-\frac{1}{2} \times 375\{(d + 0·065)^2 - 0·065^2\} = 0$

$= 0·065$ m $d = 0·262$ m

Max. force exerted by spring $= 375 \times (0·065 + 0·262)$

$= 122·6$ N.

10·16 Applying **Impulse – Momentum Principle :**

(a) Package and Cart :

$$m_p v_1 + \Sigma Imp_{1-2} = (m_p + m_c)v_2$$

$\overset{\rightarrow}{+}$ **x**, $m_p v_1 \cos 30° + 0 = (m_p + m_c)v_2$

$(12)(3·2)\cos 30° + 0 = (12 + 30)v_2$, $\therefore v_2 = 0·792$ m/s →

(b) **Package : $m_p v_1 + \Sigma Imp_{1-2} = m_p v_2$**

$\overset{\rightarrow}{+}$ **x**, $(12)(3·2)\cos 30° + F_x\Delta t = (10)(0·792)$

$F_x\Delta t = -25·34$ N·s

$+\uparrow$ **y**, $-m_p v_1 \sin 30° + F_y \Delta t = 0$

$-(12)(3·2)\sin 30° + F_y \Delta t = 0$

$F_y \Delta t = 19·2$ N·s

The **Impulse exerted on the package** = 31·8 N·s 37·15°

10·17 Applying **$\Sigma F = ma$** for block B and then A

$$12 - 2T = \frac{12}{9·81} a_B \quad ...(1)$$

$$T = \frac{40}{9·81} a_A \quad ...(2)$$

From figure, $2a_B = a_A$ ... **(3)**

$a_B = 0·68$ m/s² From figure, $v_A = 2v_B$

$v_B = u_B + a_B t$ $= 2 \times 1·68$

$v_B = 1 + 0·68(1)$ $v_A = 3·36$ m/s →

$= 1·68$ m/s.

10·18 (a) Applying **Principle of Conservation of Linear Momentum :**

$$m_A(v_A)_1 + m_B(v_B)_1 = (m_A + m_B)v_2$$

$(\overset{\rightarrow}{+})$ 15000 (1·5) 12000 (0·75) = 27000 v_2

$v_2 = 0·5$ m/s →

(b) Principle of Impulse and Momentum for car A.

$$m_A(v_A)_1 + \Sigma\int F\,dt = m_A v_2$$

$(\overset{\rightarrow}{+})$ $15000(1·5) - F_{avg}(0·8) = 15000(0·5)$ $\therefore F_{avg} = 18750$ N

10·19 Applying **Principle of Impulse and Momentum :**

$$m_A v_A + \Sigma Imp_{A-B} = m_A v_B$$

$+\uparrow$, $\frac{10}{9·81}(6 \sin 30°) + F_y \Delta t = \frac{10}{9·81}(0)$

$3·06 + \frac{10}{9·81}(-9·81)t = 0$

$t = 0·306$ s

Horizontal Velocity

$v_B = 6 \cos 30° = 5·2$ m/s

10·20 Applying **Principle of Impulse and Momentum.**

$$m_s v_1 + \Sigma Imp_{1-2} = (m_s + m_c)v_2$$

$\leftarrow +$, $15(3) + 0 = (15 + 40)v_2$

$v_2 = 0·82$ m/s

$T_2 = \frac{1}{2}(m_s + m_c)v_2^2 = 18·49$ J.

$T_1 = \frac{1}{2} m_s v_1^2 = 67·5$ J.

Energy Ratio $= \frac{T_2}{T_1} = 0·27$

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