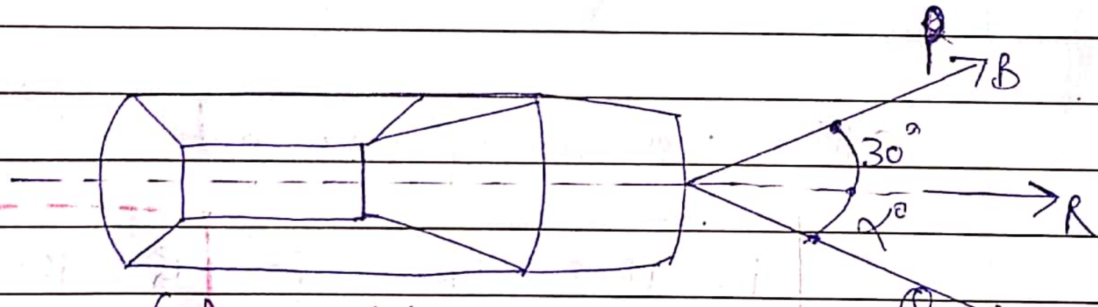
MID SEMESTER EXAMINATION – 2019

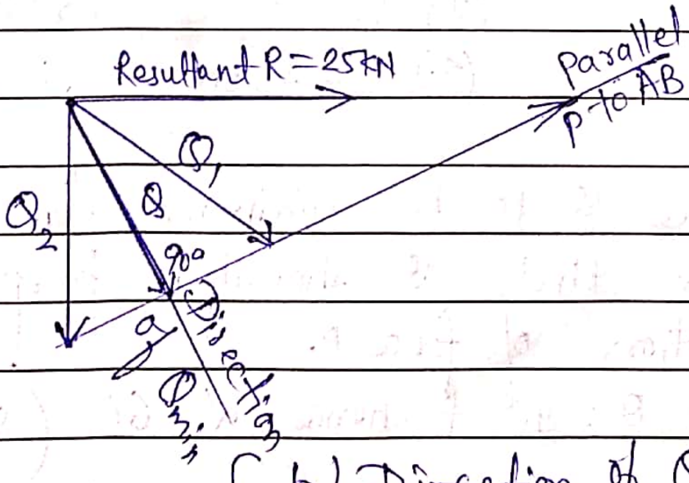
B.Tech - I - Semester 1

AM - 104 :- Engineering Mechanics

Soln → 1) a)



(a) Disabled car, $\alpha = 45°$



(b) Direction of Q minimum

Case (i) (1) $P = ?$ (2) $Q = ?$ (3) $\theta = (30^\circ + 45^\circ)$
(4) $\alpha_1 = 30^\circ$ (5) $R = 25 KN$, i.e. Resultant in direction of car

The resultant must be in the direction of the axis of automobile to move it in the direction of axis.

we have (1) $R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$
&
(2) $\tan\alpha_1 = \dfrac{Q\sin\theta}{P + Q\cos\theta}$

putting the values in above eqn, we get

$$25 = \sqrt{P^2 + Q^2 + 2PQ\cos 75} \quad \& \quad \tan 30^\circ = \frac{Q\sin 75}{P + Q\cos 75}$$

Solving above two eqn, we get

$\boxed{P = 18.322 \text{ KN}}$ & $\boxed{Q = 12.94 \text{ KN}}$

Case (ii) (1) $P = ?$ (2) $Q = ?$ (3) $\theta = (30 + \alpha)$
(4) $\alpha_1 = 30^\circ$ (5) $R = 25 KN$

For force Q to be minimum, from the fig. (b) it is seen that, Q should be perpendicular to the direction of force P.
i.e. angle $\theta = 90^\circ$ & hence $\alpha = 60^\circ$ (since $\theta = 30 + \alpha$)

Scanned by CamScanner

Now $R = \sqrt{P^2+Q^2+2PQ\cos\theta}$ & $\tan\alpha_1 = \dfrac{Q\sin\theta}{P+Q\cos\theta}$

$25 = \sqrt{P^2+Q^2+2PQ\cos 90}$ & $\tan 30 = \dfrac{Q\sin 90}{P+Q\cos 90}$

$25 = \sqrt{P^2+Q^2}$ & $\tan 30 = \dfrac{Q}{P}$

$Q = (P\tan 30)$

$625 = P^2+Q^2 = P^2+(P\tan 30)^2$

$\boxed{P = 21.65\,KN}$ & $\boxed{Q = 12.50\,KN}$

Sol^n - 1) (b)

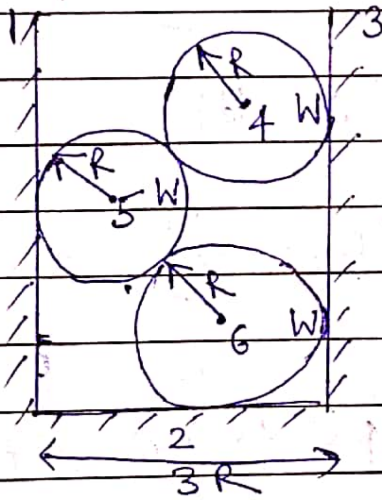


fig-(a) Given

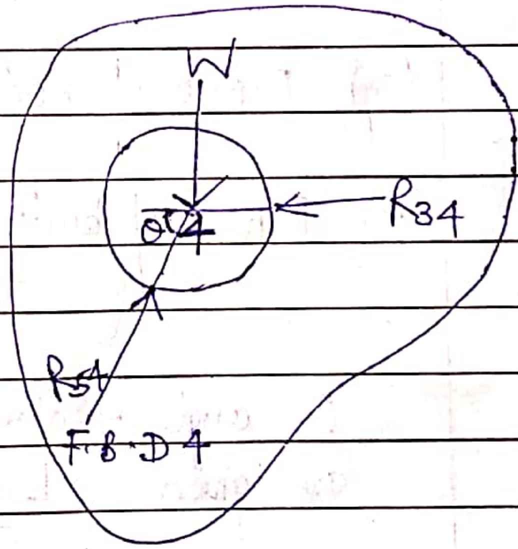


fig-(C) F.B.D-4

$\cos\theta = \dfrac{R}{(R+R)} \Rightarrow \theta = 60°$

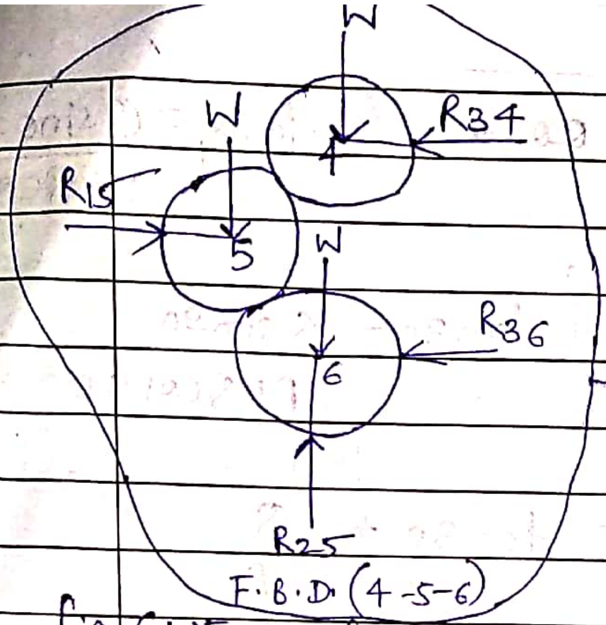


fig-(b) F.B.D. of 4-5-6

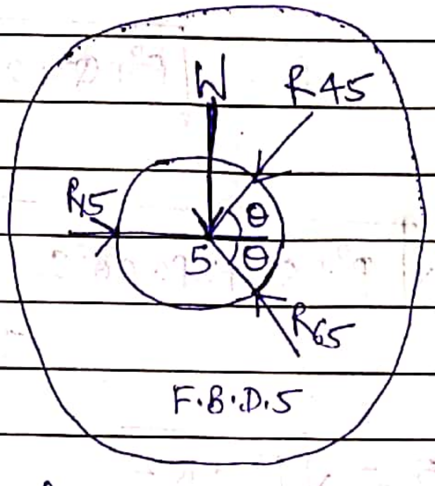


fig-(d) F.B.D.-5

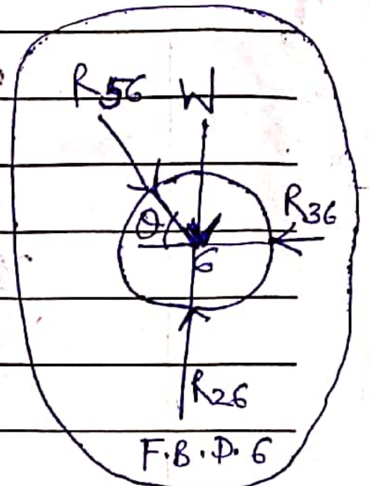


fig-(e) F.B.D. - 6

First prepare following free body diagrams:-

1) Combined 4-5-6 bodies F.B.D. [fig-(b)]

2) F.B.D. of body-4 [fig-(c)]

(3) F.B.D of body-5 [fig-(d)]

(4) F.B.D of body-6 [fig-(e)]

θ, angle of directions 45 & 56 with horizontal are given by

$$\cos\theta = \frac{[3R - R - R]}{[R+R]} = 0.5$$

$$\theta = 60^\circ$$

According to (1) $\Sigma H = 0$ & (2) $\Sigma V = 0$ for forces of fig-(b), we have

$(\rightarrow)$ $\boxed{R_{15} = R_{34} + R_{36} (\leftarrow)}$

$(\uparrow)$ $\boxed{R_{26} = [W + W + W] = 3W (\downarrow)}$

According to Lami's Theorem for forces of fig-(c), we have

$$\frac{R_{34}}{\sin(90+\theta)} = \frac{R_{54}}{\sin 90} = \frac{W}{\sin(180-\theta)}$$

$\therefore$ $\boxed{(1)\ R_{34} = \frac{W}{\sqrt{3}}}$ & $\boxed{(2)\ R_{54} = \frac{2W}{\sqrt{3}}}$

Scanned by CamScanner

According to Lami's Theorem, for forces of fig (e), we have

$$\frac{R_{26}-W}{\sin(180-\theta)} = \frac{R_{56}}{\sin 90} = \frac{R_{36}}{\sin(90+\theta)} = \frac{3W-W}{\sin(180-\theta)}$$

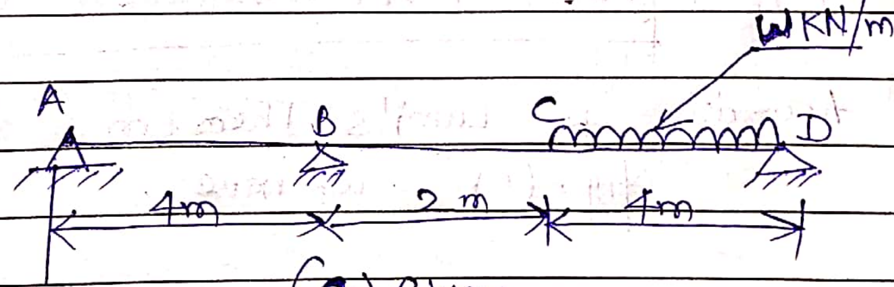
(since $R_{26}=3W$)

$\therefore$ (1) $\boxed{R_{56} = \frac{4W}{\sqrt{3}}}$ & $\boxed{(2)\ R_{36} = \frac{2W}{\sqrt{3}}}$

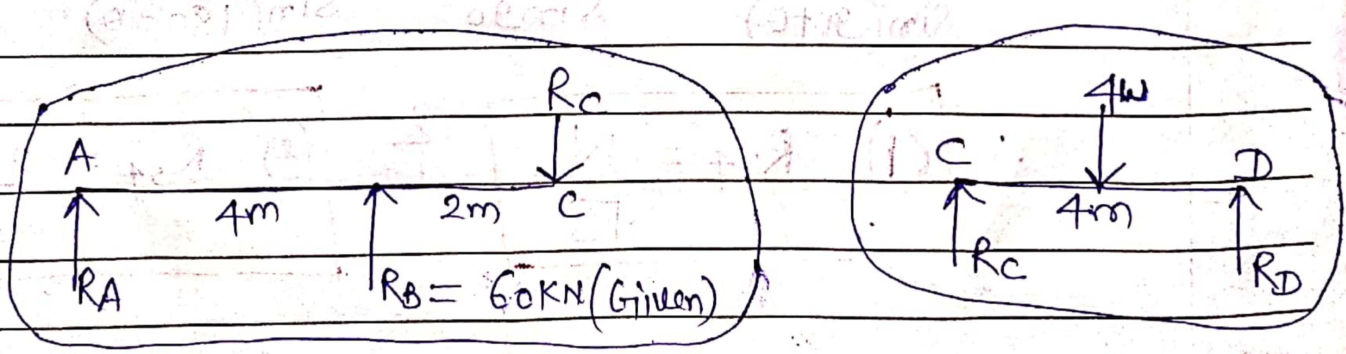
Now $\boxed{R_{15} = [R_{34}+R_{36}]}$

$$\boxed{R_{15} = \frac{W}{\sqrt{3}} + \frac{2W}{\sqrt{3}} = \sqrt{3}W}$$

Sol^n → 1 –(C)



(a) Given



(a) F.B.D of ABC & CD

First draw F.B.Ds of beam CD & beam ABC, as shown.

Now equating moments about A for forces, acting on beam ABC, we get

$$(\circlearrowright)\ R_C \times (4+2) = (60\times 4)\ (\circlearrowleft)$$

$R_C = 40\,KN\ (\downarrow)$ on ABC beam

$R_C = 40\,KN\ (\uparrow)$ on CD beam

Now $(\uparrow)\ R_A + 60 = 40\ (\downarrow)$

$R_A = -20\,KN\ (\uparrow)$

(change the direction, since negative)

$$\boxed{R_A = +20\,KN\ (\downarrow)}$$

Now equating moments about D for forces acting on CD, we get

$$(\circlearrowright)\ (40\times 4) = (4w)(2)\ (\circlearrowleft)$$

$\therefore$ $\boxed{w = 20\,KN/m \text{ U.D.L. on beam CD.}}$

Now according to $\Sigma V = 0$ for forces acting on beam CD, we get

Scanned by CamScanner

$(\uparrow)\ R_D + R_C = 4w\ (\downarrow)$

$R_D = (4 \times 20) - 40$

$\boxed{R_D = 40\ KN\ (\uparrow)}$

Soln:- (2) (a)

$OA^2 = (2)^2 + (1)^2 + (6)^2 = 41$

$\Rightarrow OA = \sqrt{41} = 6.403$

$OB^2 = (4)^2 + (-2)^2 + (5)^2 = 45.$

$\Rightarrow OB = \sqrt{45} = 6.708$

$OC^2 = (-3)^2 + (-2)^2 + (1)^2 = 14$

$\Rightarrow OC = \sqrt{14} = 3.742$

$OD^2 = (5)^2 + (1)^2 + (-2)^2 = 30$

$\Rightarrow OD = \sqrt{30} = 5.477$

unit vector along $OA = \vec{\lambda_1} = \frac{1}{6.403}(2\vec{i} + \vec{j} + 6\vec{k})$

similarly $\vec{\lambda_1} = 0.312\vec{i} + 0.156\vec{j} + 0.937\vec{k}$

$\vec{\lambda_2} = 0.596\vec{i} + 0.298\vec{j} + 0.745\vec{k}$

$\vec{\lambda_3} = -0.802\vec{i} + 0.535\vec{j} + 0.267\vec{k}$

$\vec{\lambda_4} = 0.913\vec{i} + 0.183\vec{j} + 0.365\vec{k}$

Scanned by CamScanner

$\vec{F_1} = 32\vec{\lambda_1} = 9.984\vec{i} + 4.992\vec{j} + 29.984\vec{k}$

$\vec{F_2} = 24\vec{\lambda_2} = 14.304\vec{i} - 7.152\vec{j} + 17.880\vec{k}$

$\vec{F_3} = 24\vec{\lambda_3} = -19.248\vec{i} - 12.84\vec{j} + 6.408\vec{k}$

$\vec{F_4} = 120\vec{\lambda_4} = 109.56\vec{i} + 21.96\vec{j} + 43.8\vec{k}$

Resultant $\vec{R} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4}$

$\vec{R} = 114.6\vec{i} + 6.96\vec{j} + 10.472\vec{k}$

$\boxed{|\vec{R}| = 115.288 \text{ KN}}$

Unit vector $\vec{\lambda} = 0.994\vec{i} + 0.060\vec{j} + 0.091\vec{k}$

$\boxed{\theta_x = 6.28^\circ \quad \theta_y = 86.54^\circ \quad \theta_z = 84.79^\circ}$

Scanned by CamScanner

Soln:- 2 → (b)

Force	From	To
10 KN	B(3,1,0)	A(0,1,2)
11.44 KN	A(0,1,2)	D(3,0,0)
7.85 KN	B(3,1,0)	C(3,0,2)

$$L_{BA} = \left[(-3)^2 + (0)^2 + (2)^2\right]^{1/2} = 3.606$$

$$L_{AD} = \left[(3)^2 + (-1)^2 + (-2)^2\right]^{1/2} = 3.742$$

$$L_{BC} = \left[(0)^2 + (-1)^2 + (2)^2\right]^{1/2} = 2.236$$

Unit vectors $\vec{\lambda}_{BA} = -0.832\vec{i} + 0.555\vec{k}$

$$\vec{\lambda}_{AD} = 0.802\vec{i} - 0.267\vec{j} - 0.534\vec{k}$$

$$\vec{\lambda}_{BC} = -0.447\vec{j} + 0.895\vec{k}$$

$$\vec{F}_{BA} = -8.32\vec{i} + 5.55\vec{k}$$

$$\vec{F}_{AD} = 9.175\vec{i} - 3.055\vec{j} - 6.109\vec{k}$$

$$\vec{F}_{BC} = -3.509\vec{j} + 7.026\vec{k}$$

$$\vec{R} = \Sigma\vec{F} = 0.855\vec{i} - 6.564\vec{j} + 6.467\vec{k}$$

Couple at C: $\vec{M}_C = (\vec{r}_1 \times \vec{F}_1) + (\vec{r}_2 \times \vec{F}_2) + (\vec{r}_3 \times \vec{F}_3)$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_B - x_C & y_B - y_C & z_B - z_C \\ -8.32 & 0 & 5.55 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_A - x_C & y_A - y_C & z_A - z_C \\ 9.175 & -3.055 & -6.109 \end{vmatrix}$$

$\vec{F_3}$ passes through C, hence moment contribution due to $F_3 = 0$,

$$\vec{M_C} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & -2 \\ 8.32 & 0 & 5.55 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3 & 1 & 0 \\ 9.175 & -3.055 & -6.109 \end{vmatrix}$$

$$\cancel{= \vec{i}(5.55) - \vec{j}(}$$

$$= -\vec{i}(5.55) - \vec{j}\left[-(-8.32)(-2)\right] + \vec{k}\left[-(-8.32)(1)\right]$$
$$+ \vec{i}\left[(+1)(-6.109)\right] - \vec{j}\left[(-3)(-6.109)\right] +$$
$$\vec{k}\left[(-3)(-3.055) - (9.175)(1)\right]$$

$$\vec{M_C} = -0.559\vec{i} - 1.687\vec{j} + 8.31\vec{k}$$

For ce couple system at C

$$\vec{R_{C_i}} = 0.855\vec{i} - 6.564\vec{j} + 6.467\vec{k}$$
$$= 9.254\left[0.092\vec{i} - 0.709\vec{j} + 0.7\vec{k}\right]$$

$$\boxed{\begin{aligned}\vec{M_C} &= -0.559\vec{i} - 1.687\vec{j} + 8.31\vec{k} \\ &= 8.498\left[-0.066\vec{i} - 0.199\vec{j} + 0.978\vec{k}\right]\end{aligned}}$$

Scanned by CamScanner

soln:- 2 → (OR) (b)

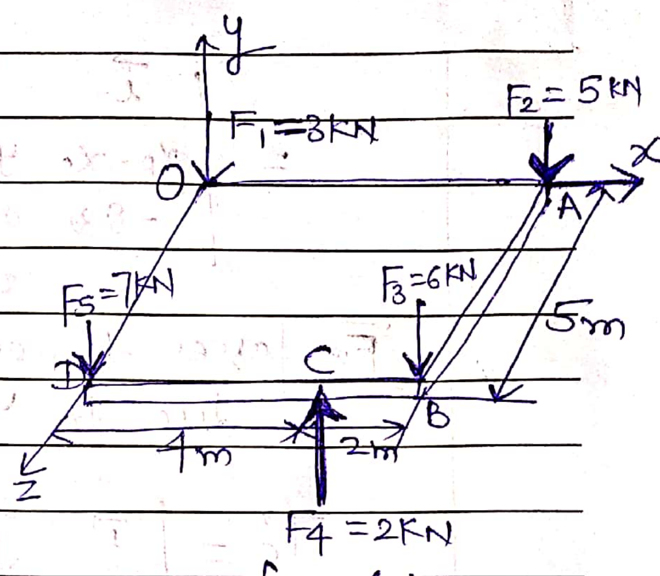


fig → (a)

Method I

(i) coordinates $O(0,0,0)$
$A(6,0,0)$, $B(6,0,5)$,
$C(4,0,5)$, $D(0,0,5)$

(ii) Force vector

$\vec{F_1} = -3\hat{j}$, $\vec{F_2} = -5\hat{j}$, $\vec{F_3} = -6\hat{j}$
$\vec{F_4} = 2\hat{j}$, $\vec{F_5} = -7\hat{j}$

(iii) Position vector

$\vec{r_1} = 0$, $\vec{r_2} = \vec{OA} = 6\hat{i}$

$\vec{r_3} = \vec{OB} = 6\hat{i} + 5z$

$\vec{r_4} = \vec{OC} = 4\hat{i} + 5z$

(iv) Moment vectors

$\vec{M_1} = \vec{r_1} \times \vec{F_1}$

$$\vec{M_1} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 0 & 0 & 0 \\ 0 & -3 & 0 \end{vmatrix}$$

$\boxed{\vec{M_1} = 0}$

$$\vec{M_2} = \vec{r_2} \times \vec{F_2}$$

$$\vec{M_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 0 & 0 \\ 0 & -5 & 0 \end{vmatrix}$$

$$\boxed{\vec{M_2} = -30k}$$

$$\vec{M_3} = \vec{r_3} \times \vec{F_3}$$

$$\vec{M_3} = \begin{vmatrix} i & \hat{j} & k \\ 6 & 0 & 5 \\ 0 & -6 & 0 \end{vmatrix}$$

$$\boxed{\vec{M_3} = 30\hat{i} - 36k}$$

$$\vec{M_4} = \vec{r_4} \times \vec{F_4}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 0 & 5 \\ 0 & 2 & 0 \end{vmatrix}$$

$$\boxed{\vec{M_4} = -10\hat{i} + 8k}$$

$$\vec{M_5} = \vec{r_5} \times \vec{F_5}$$

$$\vec{M_5} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 0 & 0 & 5 \\ 0 & -7 & 0 \end{vmatrix}$$

$$\boxed{\vec{M_5} = 35\hat{i}}$$

Scanned by CamScanner

(v) Resultant force vector

$$\vec{R} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4} + \vec{F_5}$$

$$\vec{R} = -3\hat{j} - 5\hat{j} - 6\hat{j} + 2\hat{j} - 7\hat{j}$$

$$\vec{R} = -19\hat{j} \ (KN)$$

~~Sol^n:- 3 (a) Prepare the table as below~~

~~Sr.No. shape Area (mm^2) $\vec{x}$ (mm) $\vec{y}$ (mm) $A\vec{x}$ (mm^3) $A\vec{y}$~~

Sol^n:- 3 (a) Prepare the table as below

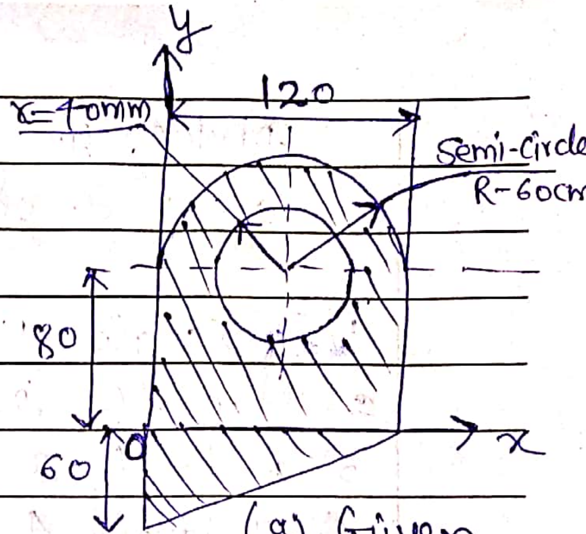
Sr. No.	Shape	Area (mm^2)	$\bar{x}$ (mm)	$\bar{y}$ (mm)	$A\bar{x}$ (mm^3)	$A\bar{y}$ (mm^3)
1)	Rectangle	$(120 \times 80) = 9600$	$\left(\frac{120}{2}\right) = 60$	$\left(\frac{80}{2}\right) = 40$	576000	384000
2)	Semi-Circle	$\left(\frac{\pi \cdot 60^2}{2}\right)$	+60	$\left(80 + \frac{4 \times 60}{3\pi}\right)$	339292.2	596390.8
(3)	Righ-angled Triangle	$\left(\frac{1}{2} \times 120 \times 60\right) = 3600$	40	$\left(\frac{-60}{3}\right) = -20$	144000	-72000
(4)	Circle (Hole)	$\pi \times 40^2 = -5026.55$	+60	+80	-301592.89	-402123.8
		$\Sigma A = 13828.32$			$\Sigma A\vec{x} = 757699.31$	$\Sigma A\vec{y} =$ [illegible]

Scanned by CamScanner

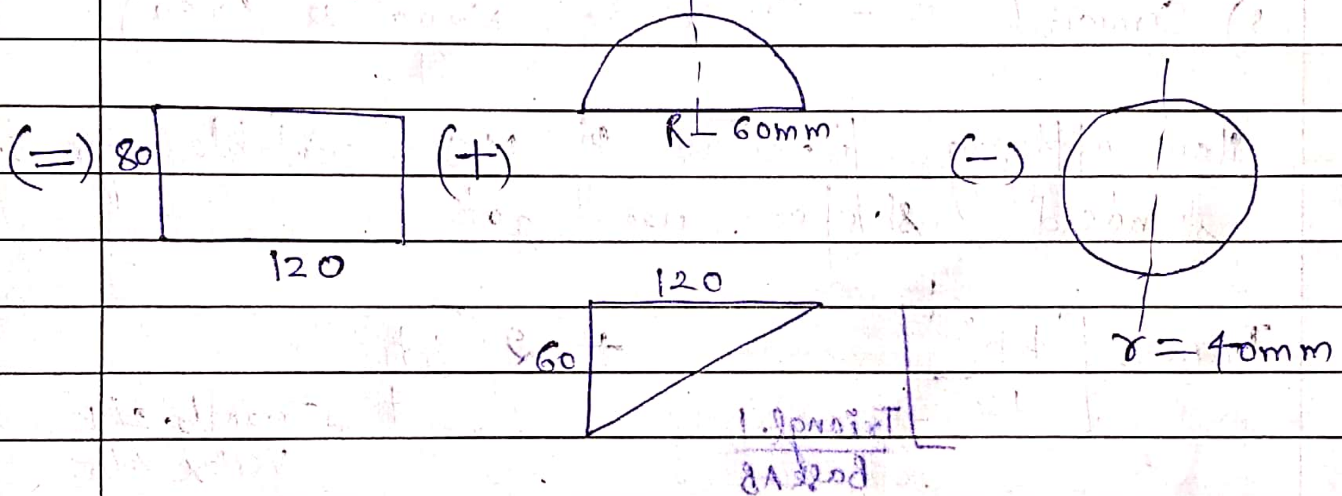
$\Sigma A = 13828.32$

$\Sigma A\bar{x} = 757699.31$

$\Sigma A\bar{y} = 506267$



(a) Given



(b) Division into standard shapes

Now, $\bar{x} = \dfrac{\Sigma A\bar{x}}{\Sigma A} = \dfrac{757699.31}{13828.32} = 54.7933\ mm$

$\bar{y} = \dfrac{\Sigma A\bar{y}}{\Sigma A} = \dfrac{506267}{13828.32} = 36.61\ mm$

Soln:- 3(b)

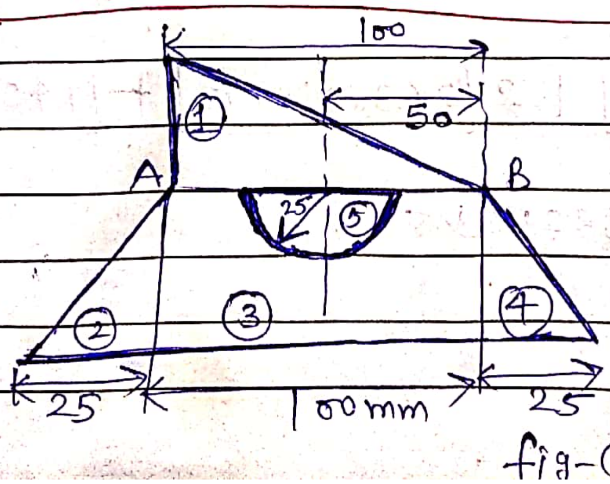


fig-(a)

Scanned by CamScanner

Divide the given composite area into 5-standard areas as under :-

1) Triangle 1 - (1) b = 100 (2) h = 60
2) Triangle 2 - (1) b = 25 (2) h = 75
3) Rectangle 3 - (1) b = 100 (2) d = 75
4) Triangle 4 - (1) b = 25 (2) h = 75
5) Semicircle 5 - (1) r = 25 (Negative Area)

Now applying formulae of M.I. suitable for 5 standard shapes, we get

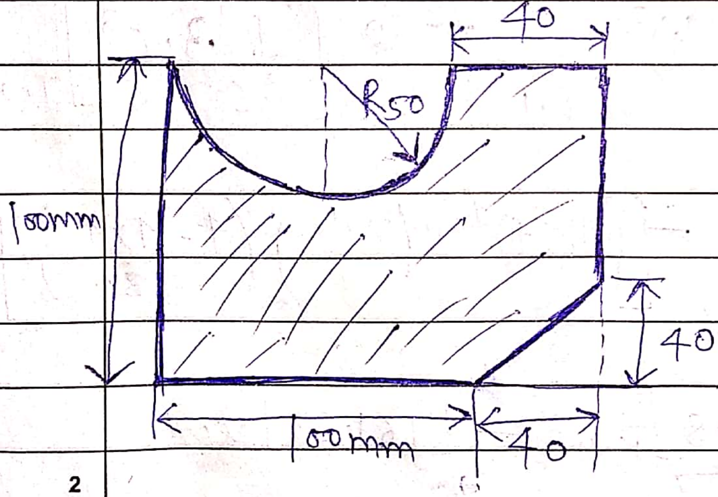
$$I_{AB} = \underbrace{\left[\frac{bh^3}{12}\right]}_{\text{Triangle.1 Base AB}} + 2\underbrace{\left(\frac{bh^3}{4}\right)}_{\text{Triangle.2/4 Vertex A/B}}$$

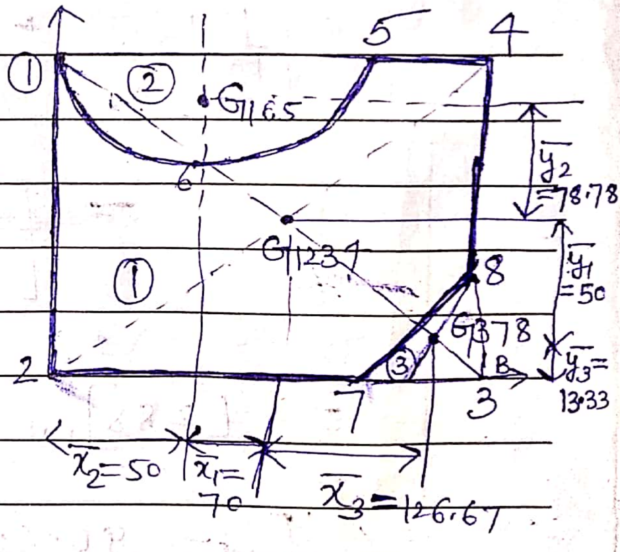
$$+ \underbrace{\left[\frac{bd^3}{3}\right]}_{\text{Rectangle.3 Base AB}} - \underbrace{\left[\frac{\pi r^4}{8}\right]}_{\text{semicircle.5 Diameter}}$$

$$= \left[\frac{100 \times 60^3}{12}\right]_1 + 2\left[\frac{25 \times 75^3}{4}\right]_{2/4} + \left[\frac{100 \times 75^3}{3}\right]_3 - \left[\frac{\pi}{8} \cdot 25^4\right]_5$$

$$= [1800000] + 2[2636718.75] + [14062500] - [153398]$$

$$= 20982539.5 \text{ mm}^4$$





Scanned by CamScanner

First divide the area into 3 standard areas and prepare the table as below.

3 standard Areas, their centroids & corresponding $\bar{x}$ & $\bar{y}$ are shown in figure

Sr. No.	Shape	Area (mm^2)	$\bar{x}$ (mm)	$\bar{y}$ (mm)	$A\bar{x}$ (mm^3)	$A\bar{y}$ (mm^3)
1)	Rectangle	$140 \times 100 = 14000$	$\frac{100+40}{2} = 70$	$\left(\frac{100}{2}\right) = 50$	980000	700000
2)	semicircle	$\frac{\pi \cdot 50^2}{2} = -3927$	$= 50$	$\left(100 - \frac{4 \times 50}{3\pi}\right) = 78.78$	-196350	-309369
3)	Triangle	$\frac{1}{2} \cdot 40 \cdot 40 = -800$	$\left(100+40-\frac{40}{3}\right) = 126.67$	$\frac{40}{3} = 13.33$	-101336	-10664
		$\Sigma A = 9273\ mm^2$			$\Sigma A\bar{x} = 682314$	$\Sigma A\bar{y} = 379967$

$\therefore$ (1) $\bar{x} = \frac{\Sigma A\bar{x}}{\Sigma A} = \frac{682314}{9273} = 73.58\ mm$

3 (2) $\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{379967}{9273} = 40.98\ mm$

Scanned by CamScanner

Now $I_{xxG} = \Sigma \left[I_{x_n x_n G_n} + A_n (\bar{y}_n - \bar{y})^2 \right]$

&

$I_{yyG} = \Sigma \left[I_{y_n y_n G_n} + A_n (\bar{x}_n - \bar{x})^2 \right]$

M.I. of total area about centroidal axis I_{xxG}

$$= \left[\frac{1}{12} \cdot 140 \cdot 100^3 + 14000(50 - 40.98)^2 \right]_1$$

$$- \left[0.11(50)^4 + 3927(78.78 - 40.98)^2 \right]_2$$

$$- \left[\frac{1}{36} \cdot 40 \cdot 40^3 + 800(13.33 - 40.98)^2 \right]_3$$

$$= (12,805,712)_1 - (6298554.68)_2 - (682729.11)_3$$

$$\boxed{I_{xxG} = 5824428.21 \ mm^4}$$

$$I_{yyG} = \left(\frac{1}{12} \cdot 100 \cdot 140^3 + 14000(73.58 - 70)^2 \right]_1$$

$$- \left[\frac{\pi}{8}(50)^4 + 3927(73.58 - 50)^2 \right]_2$$

$$- \left[\frac{1}{36} \cdot 40 \cdot 40^3 + 800(73.58 - 126.67)^2 \right]_3$$

$$= (23046096.27)_1 - (4637845.66)_2 - (2325949.59)_3$$

$$\boxed{I_{yyG} = 1608930 \ mm^4}$$

Scanned by CamScanner