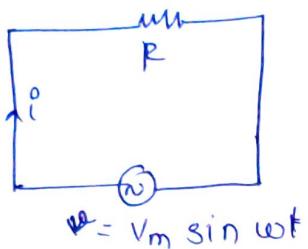


\Rightarrow AC fundamentals.

$$\text{Form factor} = \frac{\text{E rms}}{\text{Avg.}}$$

$$= 1.11$$

\rightarrow AC through pure resistor.



$$V = i^o R$$

$$\Rightarrow \frac{V}{R} = i^o \quad \Rightarrow \quad i^o = \frac{V_m \sin \omega t}{R} \Rightarrow i^o = I_m \sin \omega t$$

Note: Voltage & current in ~~same~~ phase.

\rightarrow Instantaneous Power

$$\begin{aligned} P_{\text{inst}} &= V^o i^o \\ &= V_m I_m \sin^2 \omega t \end{aligned}$$

$$P_{\text{inst}} = V_m I_m \left[\frac{1 + \cos 2\omega t}{2} \right]$$

\rightarrow Average Power over cycle

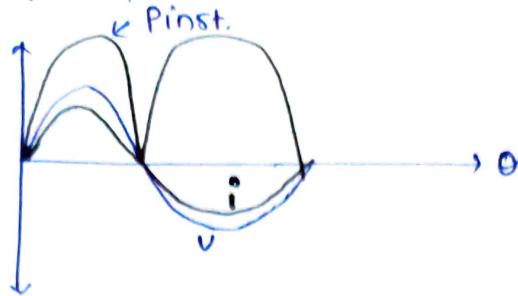
$$\Rightarrow P_{\text{av}} = \frac{\int_0^{2\pi} P_{\text{inst}} d\theta}{2\pi} = \frac{V_m I_m}{2\pi \times 2} \left[\theta - \frac{\sin 2\omega t}{2} \right]_0^{2\pi}$$

$$\Rightarrow P_{\text{av}} = \frac{V_m I_m}{4\pi} \times 2\pi$$

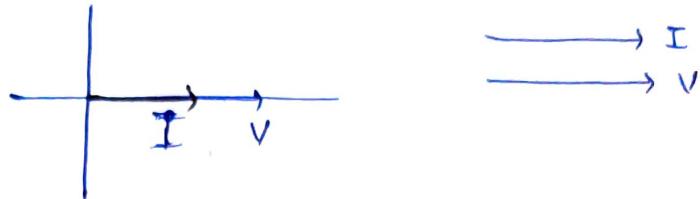
$$\Rightarrow P_{\text{av}} = \frac{V_m I_m}{2} = VI$$

\leftarrow RMS values.

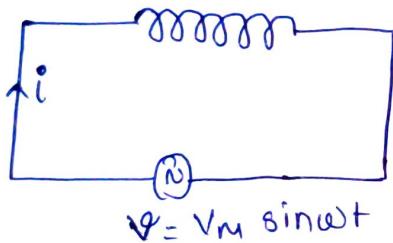
=) in wave form.



=) Phasor diagram.



=) AC through pure inductor.



$$\text{Now, } v = \omega L \frac{di}{dt}$$

$$\Rightarrow di = \frac{v}{\omega L} dt \Rightarrow i = \frac{V_m}{\omega L} \int \sin \omega t dt$$

$$\Rightarrow i = \frac{V_m}{\omega L} (-\cos \omega t)$$

$$\Rightarrow i = \underline{V_m} - I_m \sin(\pi/2 - \omega t)$$

$$\Rightarrow i = I_m \sin(\omega t - \pi/2)$$

$$XL = \omega L \leftarrow \text{inductive reactance.}$$

→ Current lags the voltage by $\pi/2$.

→ instantaneous power

$$\begin{aligned} P_{\text{inst.}} &= V_i \\ &= -V_m I_m \sin \omega t, \cos \omega t \end{aligned}$$

$$P_{\text{inst.}} = \frac{-V_m I_m}{2} \sin 2\omega t$$

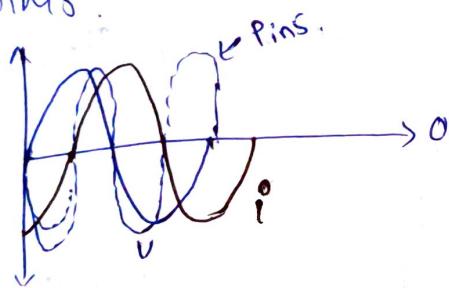
→ Average value of power:

$$P_{\text{avg}} = \frac{1}{2\pi} \int_0^{2\pi} P_{\text{inst.}} d\theta$$

$$= \frac{-V_m I_m}{2\pi \times 2} \int_0^{\pi} \sin 2\omega t d\theta = 0$$

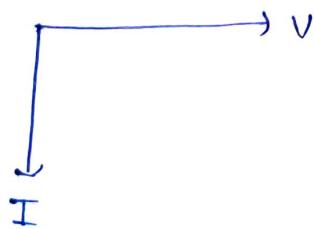
⇒ P_{avg} consumed by purely inductive circuit is always zero.

→ Wave forms.

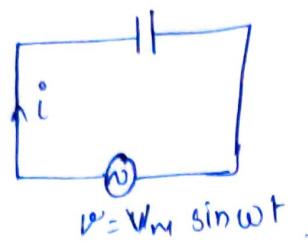


→ Frequency of fluctuation of power is twice the frequency of voltage or current.

→ phasor diagram



6
 \Rightarrow At through pure capacitor.



$$\text{Now, } q = CV$$

$$\Rightarrow q = C V_m \sin \omega t$$

$$\Rightarrow \frac{dq}{dt} = i = C V_m \frac{d \sin \omega t}{dt} = \omega C V_m \cos \omega t$$

$$\Rightarrow i = \frac{V_m}{(1/\omega C)} \cos(\omega t + \pi/2)$$

$$\Rightarrow i = \frac{V_m}{X_C} \sin(\omega t + \pi/2) = I_m \sin(\omega t + \pi/2)$$

$$[\textcircled{1} \times C = \frac{1}{\omega C}]$$

↑
capacitive
reactance.

\Rightarrow Current leads the voltage by $\pi/2$.

\rightarrow instantaneous power

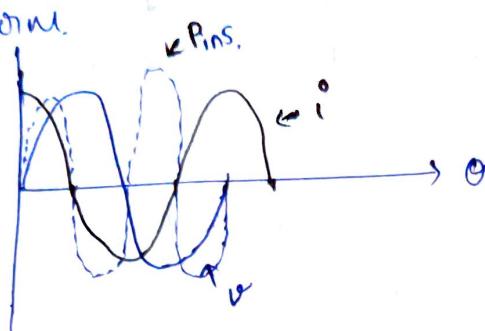
$$\begin{aligned} P_{\text{inst.}} &= V \cdot i \\ &= V_m I_m \sin \omega t \cdot \sin(\omega t + \pi/2) \\ &= \frac{V_m I_m}{2} \sin 2\omega t \end{aligned}$$

\rightarrow Average power over a cycle

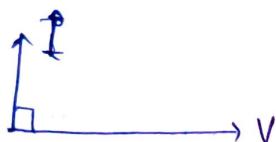
$$P_{\text{avg.}} = \frac{1}{2\pi} \int_0^{2\pi} P_{\text{inst.}} d\theta = 0.$$

- The average power consumed by pure capacitor is always zero.
- Frequency of fluctuation of power is twice the freq. of voltage or current.

→ Wave form.



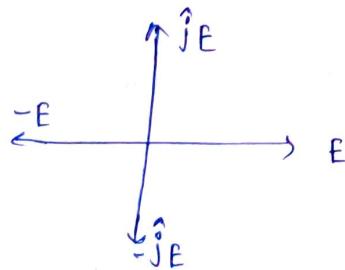
→ Phasor diagram.



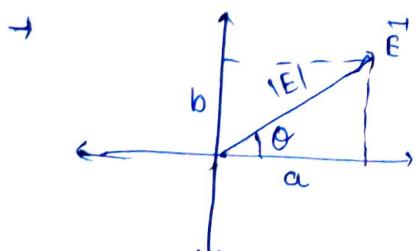
⇒ complex representation.

→ operator $\langle j \rangle$

→ if any quantity is multiplied by $\langle j \rangle$ → rotation by $\pi/2$ in anti-clockwise way.



Representation of vector.



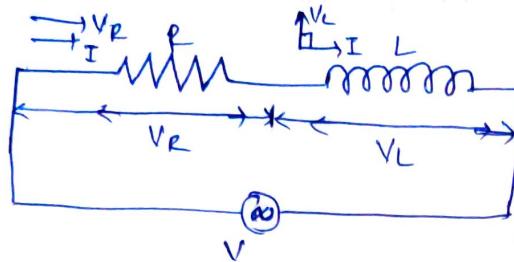
Rectangular form

Polar form

$$\begin{aligned}
 \vec{E} &= a + j b \\
 &= |E| \cos \theta + j |E| \sin \theta \\
 &= |E| (\cos \theta + j \sin \theta) \\
 &= |E| e^{j\theta} = |E| \angle \theta
 \end{aligned}$$

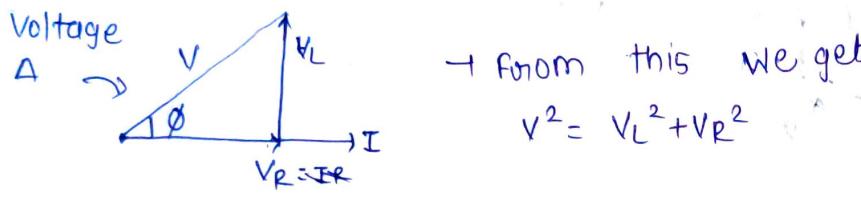
\Rightarrow some circuits. (series circuits).

① Series R-L circuit.



V = rms value of applied voltage.

I = rms value of current
 $V_R = IR$, $V_L = IX_L$



$$\Rightarrow \sqrt{V^2} = \sqrt{I^2 X_L^2 + I^2 R^2}$$

$$\Rightarrow V^2 = I^2 (X_L^2 + R^2)$$

$$\Rightarrow V^2 = I^2 Z^2$$

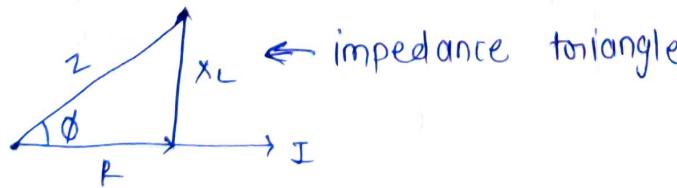
$$\Rightarrow Z = \sqrt{R^2 + X_L^2}$$

$$\text{& } \tan \phi = \frac{V_L}{V_R}$$

($\therefore Z$ = impedance of the circuit.)

(when ϕ is phase diff. b/w V & I).

From above we can say current lags the voltage by angle ϕ .



& therefore ϕ can be $\tan^{-1} \frac{X_L}{R}$

$$\therefore \tan \phi = \frac{X_L}{R}$$

Now, if $V = V_m \sin \omega t$

$$i = I_m \sin(\omega t - \phi)$$

where $I_m = \frac{V_m}{Z}$

→ Instantaneous power

$$P_{in} = V_i = V_m I_m \sin \omega t \sin(\omega t - \phi)$$

$$\boxed{P_{in} = \frac{V_m I_m}{2} [\cos \phi - \cos(2\omega t - \phi)]}$$

→ Average value of power over cycle:

$$\begin{aligned} P_{av} &= \frac{1}{2\pi} \int_0^{2\pi} P_{in} d\theta \\ &= \frac{1}{2\pi} \left[\int_0^{2\pi} \frac{V_m I_m}{2} \cos \phi - \int_0^{2\pi} \cos \frac{V_m I_m}{2} \cos(2\omega t - \phi) d\theta \right] \\ &= \frac{V_m I_m}{(2\pi)(2)} \left[\cos \phi [0]_0^{2\pi} + [\sin(2\omega t - \phi)]_0^{2\pi} \right] \\ &= \frac{V_m I_m}{2(2\pi)} (\cos \phi (2\pi - 0) + 0) \end{aligned}$$

$$\Rightarrow \boxed{P_{av} = \frac{V_m I_m}{2} \cos \phi}$$

$$\Rightarrow \boxed{P_{av} = VI \cos \phi}$$

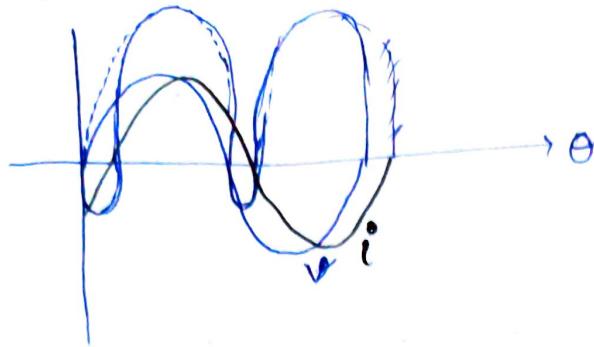
$\begin{cases} V = \text{rms value of } V_m \\ I = \text{rms value of } I_m \end{cases}$

Also, $\boxed{P_{av} = \Phi I Z \frac{R}{Z}}$,
 $\Rightarrow \boxed{P_{av} = I^2 R}$

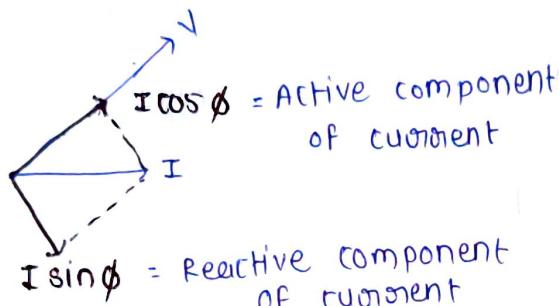
$\cos \phi$ is known as power factor of circuit & also we know that

$$\cos \phi = \frac{R}{Z} = \frac{VR}{V^2}$$

→ Wave form



→ Phasor & Resolution of current in components



$$\text{P}_{\text{active}} = VI \cos \phi = I^2 R \text{ (watt) (Active power)}$$

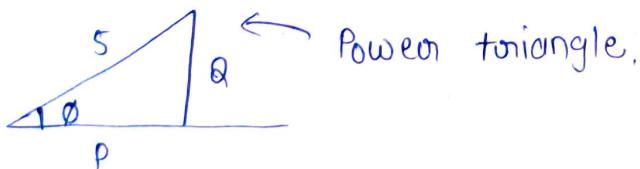
$$\text{Q} = \text{P}_{\text{reactive}} = VI \sin \phi = I^2 X_L \text{ (VAR) (Reactive power)}$$

↑ reactive volt amperes.

$$P = VI$$

$$S = P = I^2 Z \text{ (VA) (Apparent power)}$$

↑ volt amperes.



→ Example:

$$\textcircled{1} \quad R = 200 \Omega \quad \textcircled{2} \quad L = 638 \text{ mH}$$

$$e = 200 \sin \omega t$$

Find expression of current, power factor & power

$$\text{we have } \omega = 100 \pi$$

$$\textcircled{3} \quad X_L = \omega L = 200 \cdot 433$$

$$\text{Q. } Z = \sqrt{R^2 + X_L^2} = \sqrt{(200)^2 + (200 \cdot 4\pi)^2} = \sqrt{(200)^2 + (200 \cdot 4\pi)^2} = 283.15 \Omega$$

$$\theta = \tan^{-1} \frac{X_L}{R} = \tan^{-1} \frac{200 \cdot 4\pi}{200} = 45.06^\circ = 45^\circ$$

$$\text{Q. } I = \frac{V}{Z} = \frac{200}{\sqrt{2} \cdot 283.15} = 0.5 \text{ A}$$

$$i = I_m \sin(\omega t - \phi) = 0.5 \sin(100\pi t - \pi/4)$$

$$\cos \phi = \cos 45^\circ = 0.707 \text{ (lag)}$$

$$\text{Power} = 50 \text{ Watt}$$

\Rightarrow in complex form

$$I = V/Z = \frac{200}{\sqrt{2}} \angle 0$$

$$\text{But } Z = R + jX_L = 283.15$$

All other done same.

$$\text{Q. } v(t) = 141.4 \sin(314t + 10^\circ)$$

$$i(t) = 14.14 \sin(314t + 20^\circ)$$

Find PF, power and elemen values.

$$\cos \phi = \text{PF} = \frac{\cos}{(10 - (-20))} = \cos 30 = 0.866 \text{ (lag)}$$

$$P = V I \cos \phi = 865.7 \text{ watts}$$

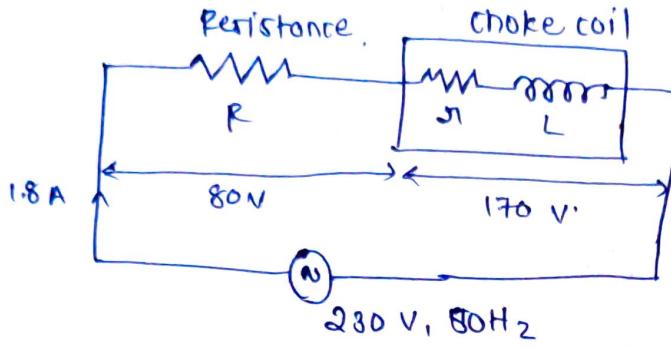
$$Z = \frac{V_m}{I_m} = 10$$

$$\text{Also } R = Z \cos \phi = 10 \times 0.866 = 8.66 \Omega$$

$$X_L = Z \sin \phi = 10 \times 0.5 = 5 \Omega$$

$$\Rightarrow S = 314 \times L$$

③ A pure resistance is connected in series with a choke coil across 230 volt, 50Hz supply. The current passing through circuit is 1.8A & voltage across the resistance & choke coil are 80V & 170V respectively. Calculate the resistance & inductance of the choke coil, & also determine power consumed by circuit.



$$\therefore R = \frac{V_R}{I} = \frac{80}{1.8} = 44.44 \Omega$$

→ Impedance of choke coil,

$$\therefore Z = \frac{V_{\text{choke}}}{I} = \frac{170}{1.8} = 94.44 \Omega = \sqrt{\omega^2 + X_L^2}$$

$$\therefore (94.44)^2 = \omega^2 + X_L^2 \quad - \textcircled{1}$$

→ Total impedance of circuit,

$$Z = \frac{V}{I} = \frac{230}{1.8} = 127.77 \Omega$$

$$Z = \sqrt{(R+\omega)^2 + X_L^2}$$

$$Z_1 = R + j\omega, \quad Z_2 = \omega + jX_L \quad \therefore Z = (R+\omega) + jX_L$$

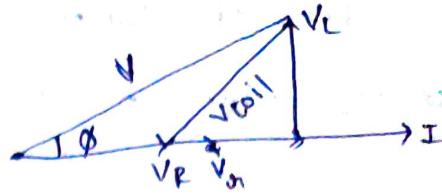
$$\therefore Z = (127.77)^2 = (44.44)^2 + 88.88\omega + \omega^2 + X_L^2$$

$$\therefore (127.77)^2 = (44.44)^2 + 88.88\omega + (94.44)^2$$

$$\omega = 61.1 \Omega, \quad X_L = 72, \quad L = 229 \text{ mH.}$$

overall power factor

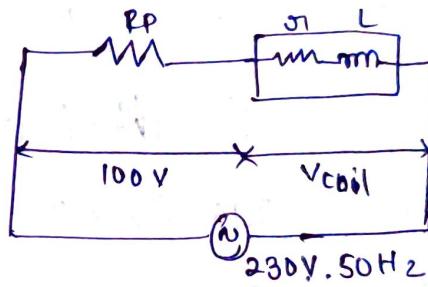
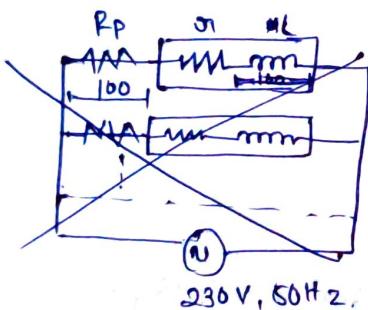
$$\cos \phi = \frac{R+o\Omega}{Z} = 0.826 \text{ (16g)}$$



$$\text{Now, } P = VI \cos \phi$$

$$\therefore P = 280 \times 1.8 \times 0.826 \\ = 342 \text{ Watt.}$$

4. It is desired to run a bank of ~~10 watt-long hundred~~
~~100 watt, 100 volt.~~
 lamps in parallel from a
 230 V, 50 Hz supply by inserting a choke coil in series
 with the bank of lamps. If choke coil has a power
 factor of 0.2) find resistance & inductance of choke coil.



X we have $\cos \phi = 0.2$ wrong answer

$$V = 230, V_L = 100 \quad \therefore V_R = \sqrt{(230)^2 - (100)^2} = 207.12.$$

$$\text{P} = VI \cos \phi$$

$$\therefore 100 = 230 \times I \times 0.2 \quad \therefore I = 50/23 \text{ A.}$$

$$\therefore o\Omega = \frac{V_R}{I} = 95.27 \Omega$$

$$X_L = 46$$

X

$$\text{Now } 46 = 2\pi \times 50 \times L$$

$$\therefore L =$$

$R_p = \text{Req. of } 8 \text{ bulbs}$

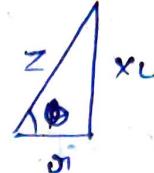
$$P = V^2 \cos \theta$$

$$P = \frac{V_p^2}{R}$$

$$\Rightarrow R = \frac{(100)^2}{100}$$

$\Rightarrow R = 100 \Omega$ → Resistance of single lamp.

$$\therefore R_p = 10 \Omega$$



$$\therefore \cos \theta = 0.2 = \frac{R}{Z_{\text{coil}}}$$

$$Z_{\text{coil}}$$

$$\therefore R^2 = 0.04 (R^2 + X_L^2) \quad (I = 10 \text{ A})$$

$$Z_{\text{circuit}} = \frac{V}{I} = \frac{230}{10} = 23$$

$$(23)^2 = (R + jZ)^2 + (X_L)^2$$

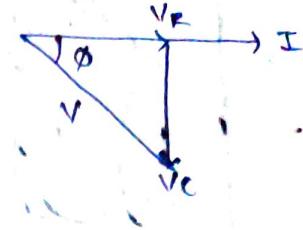
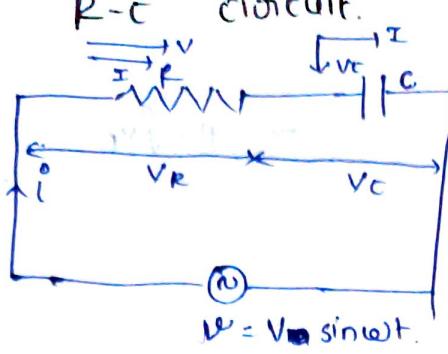
$$\therefore 8.96 jZ^2 = X_L^2$$

$$\therefore 529 = 100 + 20jZ + j^2 4.96 jZ^2 \cdot jZ^2 + \frac{3.96 jZ^2}{0.04}$$

$$\therefore jZ = 3.76 \Omega$$

$$X_L = 18.38 \Omega$$

\Rightarrow Series R-C circuit.



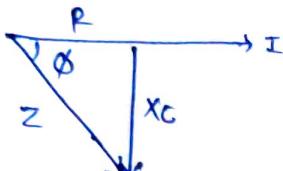
$$V^2 = V_R^2 + V_C^2$$

$$\Rightarrow V^2 = I^2(R^2 + X_C^2)$$

$$\Rightarrow V^2 = I^2 Z^2$$

$$\therefore Z = \sqrt{R^2 + X_C^2}$$

Z impedance of circuit.



$$\Rightarrow \boxed{\phi = \tan^{-1} \frac{X_C}{R}}$$

\rightarrow Current leads the applied voltage by angle ϕ .

\rightarrow if $V(t) = V_m \sin \omega t$

$$\text{then } i(t) = I_m \sin(\omega t + \phi)$$

$$\text{where } I_m = \frac{V_m}{Z}$$

\rightarrow P instantaneous = $v(t)i(t)$

$$= \frac{V_m I_m}{2} 2 \sin \omega t \sin(\omega t + \phi)$$

$$\boxed{P_{in} = VI [\cos \phi - \cos(2\omega t + \phi)]}$$

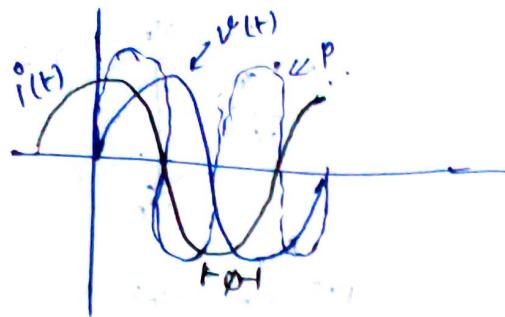
\rightarrow Average value of power over a cycle.

$$P_{av} = \frac{1}{2\pi} \int_0^{2\pi} P_{in} d\theta \Rightarrow P_{av} = \frac{1}{2\pi} VI \int_0^{2\pi} [\cos \phi - \cos(2\omega t + \phi)] d\theta$$

$$\Rightarrow P_{av} = \frac{VI}{2\pi} \times 2\pi \times \cos \phi$$

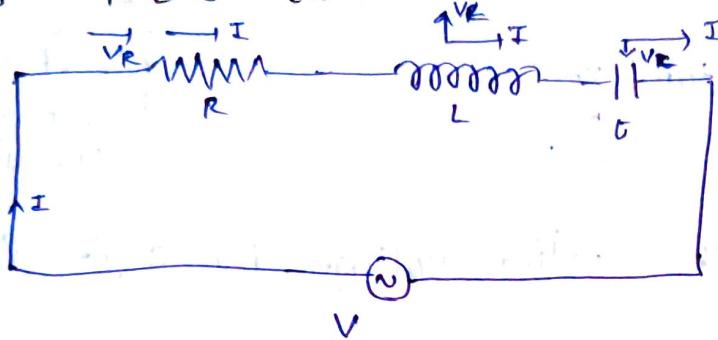
$$\Rightarrow \boxed{P_{av} = VI \cos \phi}$$

→ Waveform



$$Z = R - jX_C$$

* Series R-L-C circuit.



Three possibilities

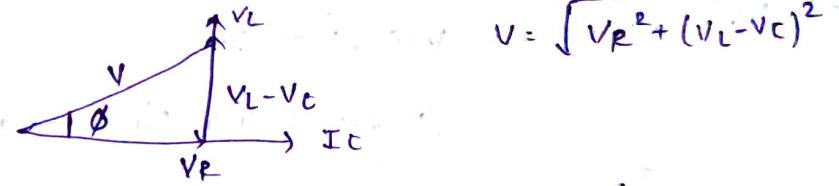
$$\textcircled{1} \quad X_L > X_C$$

$$\textcircled{2} \quad X_L \approx X_C$$

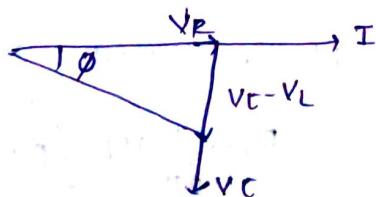
$$\textcircled{3} \quad X_L < X_C$$

Now, four

$$\textcircled{1} \quad X_L > X_C$$



$$\textcircled{2} \quad X_L < X_C$$



Now, from both ① & ②.

$$V_p^2 = V^2 + (V_L - V_C)^2$$

$$\Rightarrow V^2 = I^2 [R^2 + (X_L - X_C)^2]$$

$$\Rightarrow V = IZ$$

where, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

\Rightarrow phase angle

$$\phi = \tan^{-1} \frac{X_L - X_C}{R}$$

Also, $Z = R + j(X_L + X_C)$

$$\Rightarrow V = V_m \sin(\omega t)$$

$$\Rightarrow I = I_m \sin(\omega t \pm \phi)$$

$$\Rightarrow I_m = \frac{V_m}{Z}$$

NOW, +ve sign if $X_L > X_C$
 -ve sign if $X_C > X_L$

Now instantaneous power.

$$P_{\text{inst}} = V \cdot I = \frac{V_m I_m}{2} \cos \phi \sin(\omega t \pm \phi)$$

$$\Rightarrow P_{\text{inst}} = \frac{V_m I_m}{2} [\cos \phi - \cos(\omega t \pm \phi)]$$

Now, Average power over a cycle.

$$P_{\text{avg.}} = \int_0^{2\pi} \frac{P_{\text{inst}} d\theta}{2\pi}$$

$$\Rightarrow P_{\text{avg.}} = \frac{V_m I_m}{2} \cos \phi$$

$$\Rightarrow P_{\text{avg.}} = VI \cos \phi$$

Now in vector form,

$$I = \frac{V \angle \phi}{Z}$$

$$\Rightarrow I = \frac{V_r \angle \phi}{|Z| \angle I\phi}$$

$$\therefore I = \frac{V_r}{|Z|} \angle +\phi$$

③ $X_L = X_C$

→ It becomes purely resistive.

→ Resonance condition.

NOW $X_L = X_C$

$$\Rightarrow \omega L = \frac{1}{\omega C}$$

$$\Rightarrow \omega^2 = \frac{1}{LC}$$

$$\Rightarrow \boxed{\omega = \frac{1}{\sqrt{LC}} \text{ rad/s}}$$

$$\Rightarrow \boxed{f_{\text{res}} = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}}$$

① → voltage & current are in phase

② → P.F. $\cos \phi = 1$ or $\phi = 0$.

③ → The impedance is minimum.

④ → Current is maximum.

⑤ → Voltage across L or C is more than the supply voltage. (Voltage magnification).

→ This voltage magnification is equal to quality factor of the circuit.

$$\text{Q-factor} = \frac{V_L}{V} = \frac{X_L}{R} = \frac{\omega_m L}{R} = \frac{1}{R\sqrt{LC}}$$

Now, $V_L = QV$ & Q is always greater than 1.

A150.

$$Q\text{-factor} = 2\pi f$$

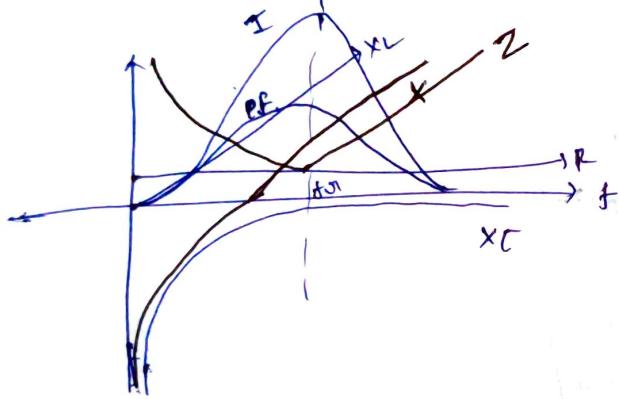
Max. stored energy
Energy dissipate per cycle:

$$\text{Power loss} = \omega t \times \frac{\frac{1}{2} I_m^2 L}{\left(\frac{I_m}{\sqrt{2}}\right)^2 \cdot R(t)}$$

$$\omega t = \frac{2\pi f L \times t}{R} = \frac{\omega_0 L}{R} = \frac{1}{R} \int \frac{L}{C}$$

⇒ Variation of parameters with frequency.

→ Graphical representation of Resonance.



① Resistance independent of F.

② $X_L = 2\pi f L$
∴ $X_L \propto f$

③ $X_C = \frac{1}{2\pi f C}$
∴ $X_C \propto \frac{1}{f}$

Rectangular hyperbola.

④ Net Reactance $x = X_L - X_C$.

⑤ Impedance $Z = \sqrt{R^2 + x^2}$

⑥ Current $I = \frac{V}{Z}$

⑦ Power

= Numericals

- ① A resistor & capacitor are connected in series across 150 V AC supply. The current taken by circuit is 6 amp. When frequency of supply is 50 Hz, current reduced to 5 A. When frequency of supply is 40 Hz. Determine value of R & C.

$$Z_1 = \frac{150}{6} = 25 \quad \Rightarrow \quad 62.5 = R^2 + \frac{1}{(4\pi f)^2 C^2} \quad - ①$$

$$Z_2 = \frac{150}{5} = 30 \quad \Rightarrow \quad 900 = R^2 + \frac{1}{(80\pi)^2 C^2} \quad - ②$$

By ① & ② we get.

$$900 = 625 - \frac{1}{(100\pi)^2 C^2} + \frac{1}{(80\pi)^2 C^2}$$

$$\Rightarrow 275 = \frac{1}{\pi^2 C^2} \left[\frac{1}{806400} - \frac{1}{10000} \right]$$

$$\Rightarrow C^2 = \frac{1}{275\pi^2} \left[\left(\frac{1}{80}\right)^2 - \left(\frac{1}{100}\right)^2 \right]$$

$$\Rightarrow C^2 = \frac{1}{\pi^2 275} \left[\left(\frac{1}{80} + \frac{1}{100}\right) \left(\frac{1}{80} - \frac{1}{100}\right) \right]$$

$$\Rightarrow C^2 = \frac{1}{\cancel{\pi^2} \cancel{275} 8} \quad \Rightarrow C = 1.43 \times 10^{-4} F$$

$$\boxed{C = 143 \mu F}$$

By putting value in ①, we get

$$\boxed{R = 11.64 \Omega}$$

(Q) Voltage $V = 100 \sin(314t)$ is applied to a circuit consisting of 25Ω resistor & $80\mu F$ capacitor in series. Determine

- An expression for current flowing at any instant.
- Power consumed.
- Potential difference across capacitor at the instant when current is $\frac{I_m}{2}$ of it

Solution: $\omega = 314$

$$\therefore Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

$$\therefore Z = 47 \Omega$$

Now, $\phi = \tan^{-1} \frac{X_C}{R} = \phi = \tan^{-1} \frac{39.8}{25} = 57.8^\circ$

$$\therefore i = \frac{V_m}{Z} \sin \left(\frac{\omega t}{2} + 57.8^\circ \right)$$

$$\therefore i = 2.127 \sin(314t + 57.8^\circ)$$

$$\therefore i = 2.127 \sin(314t + 1)$$

Now $P = I^2 R = \frac{(2.127)^2}{2} \times 25 = 56.5$.

Now, $V_C = i X_C$

$$\therefore V_C = 2.127(99.8) \sin(314t + 1 - \pi/2)$$

$$\therefore V_C = 84.65 \sin(314t + 1 - \pi/2)$$

Now at $I = \frac{I_m}{2}$

$$\therefore \frac{1}{2} = \sin(314t + 1)$$

$$\therefore 314t + 1 = \pi/6$$

$$\therefore V_C = 73.5 \text{ V}$$

③ A circuit takes current of 3A. and power factor of 0.6 lagging when connected to 175V, 50Hz supply. another circuit takes current of 5A. at PF 0.707 leading when connected to same supply. if the two circuits are now connected in series across 230V, 50Hz supply calculate, I, P, & PF.

CASE 1

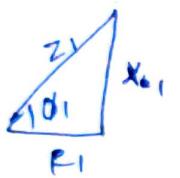
$$V_1 = 175 \text{ V}$$

$$\omega = 50 \text{ Hz}$$

$$I_1 = 3 \text{ A.}$$

$$\cos \phi_1 = 0.6 \text{ (lag)}$$

$$Z_1 = \frac{V_1}{I_1} = \frac{175}{3} = 58.33 \Omega$$



$$\therefore R_1 = 35 \Omega$$

$$X_1 = 46.66 \Omega$$

$$\therefore Z_1 = 35 + j46.66 \Omega$$

CASE 2

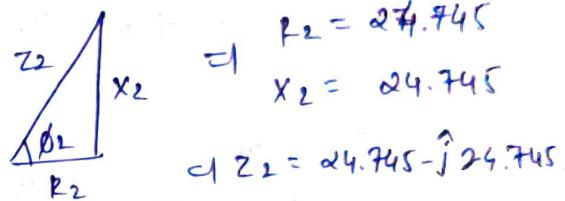
$$V_2 = 175 \text{ V}$$

$$f_2 = 50 \text{ Hz}$$

$$I_2 = 5 \text{ A}$$

$$\cos \phi_2 = 0.707 \text{ (lead)}$$

$$Z_2 = \frac{V_2}{I_2} = \frac{175}{5} = 35 \Omega$$



$$R_2 = 24.745$$

$$X_2 = 24.745$$

$$\therefore Z_2 = 24.745 - j24.745$$

CASE 3.

$$V = 230 \text{ V.}$$

$$\text{Now, } Z = Z_1 + Z_2$$

$$\therefore Z = 59.745 + j21.915$$

$$\therefore \angle Z = 63.6^\circ \angle 20.14^\circ$$

$$\text{PF } \cos \phi = 0.988 \text{ (lag)}$$

$$I = \frac{230}{63.64} = 3.62 \angle -20.14^\circ$$

$$P = (3.62)^2 \times 59.745$$

$$\therefore P = 783 \text{ Watt.}$$

④ A resistance of $24\ \Omega$, capacitance of $150\ \mu F$ & inductance of $0.16\ H$ are connected in series with each other to supply of $240\ V$, $50\ Hz$. calculate current in circuit, potential diff. across each, frequency to which the supply would be connected so that current would be at unity power factor & find current at this frequency.

we have

$$R = 24\ \Omega \quad C = 150\ \mu F \quad L = 0.16\ H$$

$$V = 240\ V \quad f = 50\ Hz.$$

$$X_L = 50.26\ \Omega$$

$$\therefore X_C = 21.22\ \Omega$$

$$\Rightarrow \therefore Z = \sqrt{576 + (29.04)^2} = 37.67\ \Omega \angle 50.43^\circ$$

$$\text{Now, } I = \frac{V}{Z} = \frac{240}{37.67 \angle 50.43^\circ} = 6.37 \angle -50.43^\circ$$

$$\text{Now, } V_R = 152.88\ V \quad V_L = 320.15\ V \quad V_C = 135.17\ V$$

$$f_{oi} = \frac{1}{2\pi\sqrt{LC}} = 4.87 \times 10^{-3}\ Hz \quad 32.5\ Hz.$$

$$\Rightarrow I_{oi} = \frac{240}{24} = 10\ A. \quad \text{Now, } Q = \frac{V_L}{V} = 2.09$$

⑤ A resistance, a capacitor and a variable inductance are connected in series across $200\ V$, $60\ Hz$ supply. the maximum current which can be obtain by varying the L is $314\ mA$. & voltage across capacitor is $300\ V$, calculate the elements well.

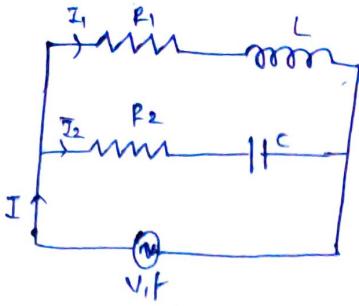
$$R = ? \quad C = ? \quad L = ? \\ V_C = 300 \quad A_{oi} = 314 \times 10^{-3}\ A. \quad f = 60\ Hz$$

$$R = \frac{V}{A_{oi}} = \frac{200}{314 \times 10^{-3}} = 636.94\ \Omega$$

$$X_C = \frac{V_C}{I} = \frac{800}{0.364} = 2200 \Omega \quad \Rightarrow \quad C = 3.33 \mu F$$

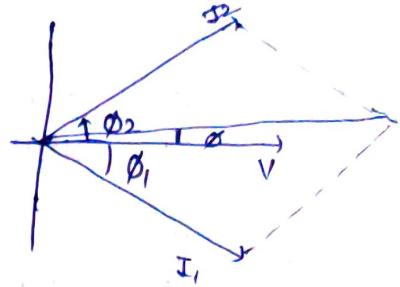
$$X_L = X_C \quad \Rightarrow \quad L = 3.04 H$$

\Rightarrow Parallel circuits.



$$Z_1 = R_1 + jX_L = |Z_1| \angle \phi_1 = \sqrt{R_1^2 + X_L^2} \angle \tan^{-1} \frac{X_L}{R_1}$$

$$Z_2 = R_2 + jX_C = |Z_2| \angle \phi_2 = \sqrt{R_2^2 + X_C^2} \angle \tan^{-1} \frac{X_C}{R_2}$$



$$\text{Now, } I_1 = \frac{V \angle 0}{Z_1 \angle \phi_1} = |I_1| \angle -\phi_1$$

$$I_2 = \frac{V \angle 0}{Z_2 \angle \phi_2} = |I_2| \angle \phi_2$$

$$I = I_1 + I_2$$

$$\Rightarrow I = a_1 + j b_1 + a_2 + j b_2$$

$$\Rightarrow I = (a_1 + a_2) + (b_2 - b_1) j$$

$$I = \frac{V}{Z_1} + \frac{V}{Z_2} \Rightarrow \frac{V}{Z} = V \left(\frac{1}{Z_1} + \frac{1}{Z_2} \right) \Rightarrow \frac{1}{Z} = \frac{Z_2 + Z_1}{Z_1 Z_2}$$

$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$\frac{V}{Z} = V \cdot Y \rightarrow \text{Admittance}$$

$$Y = \frac{1}{Z} \text{ mho.}$$

$$Y = G + jB$$

susceptance.

G \downarrow conductance B_L \downarrow inductive
 B_C \downarrow capacitive

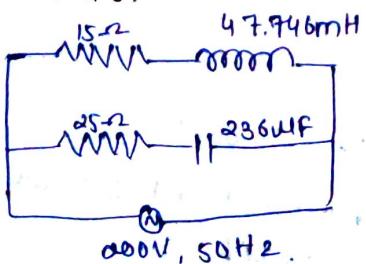
$$\text{Let } Y_1 = \frac{1}{Z_1} = \frac{1}{R_1 + jX_L} = \frac{R_1 - jX_L}{R_1^2 + X_L^2}$$

$$Y_1 = g_L - j b_L$$

$$\text{Similarly, } Y_2 = \frac{R_2 + jX_C}{R_2^2 + X_C^2} = g_C + j b_C$$

\Rightarrow Numericals.

1)



find I, P.F & power

$$X_L = \omega \pi f L = 15 \Omega$$

$$X_C = \frac{1}{\omega \pi f C} = 13.68 \Omega = 13.5 \Omega$$

$$Z_1 = 15 + j15 = 21.21 \angle 45^\circ$$

$$Z_2 = 25 + j13.5 = 28.40 \angle -28.33^\circ$$

$$\text{Now, } I_1 = \frac{V}{Z_1} = \frac{200 \angle 0^\circ}{21.21 \angle 45^\circ} = 9.42 \angle -45^\circ = 6.67 - j6.67$$

$$I_2 = \frac{V}{Z_2} = \frac{200 \angle 0^\circ}{28.4} = 7.04 \angle 28.33^\circ = 6.19 + j3.34$$

$$\text{Now, } I = I_1 + I_2 = 12.86 - j3.33$$

$$\therefore I = 13.28 \angle 14.5^\circ$$

$$\text{Now, PF is } \cos 14.5^\circ = 0.96 \text{ (lag)}$$

$$\begin{aligned} \text{Power} &= VI \cos \phi \\ &= 200 \times 13.28 \times 0.96 \\ &= 2549.96 \text{ W} \end{aligned}$$

- 2) It is desired to raise the power factor of circuit to unity by connecting a capacitor in parallel to circuit. Determine the value of capacitance.

$$\begin{aligned} I &= I_1 + I_2 + I_C \\ \therefore I &= 12.86 - j3.33 + \frac{200}{-jX_C} \end{aligned}$$

$$P.F = 1$$

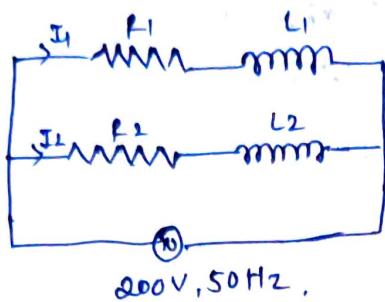
\Rightarrow imaginary part of $I = 0$

$$\therefore -3.33 + \frac{200}{X_C} = 0 \quad \therefore X_C = 60.06 \Omega$$

$$C = 52.9 \mu F \approx 53 \mu F$$

$$\therefore \frac{200^2}{3.33} = 2\pi \times 50 \times C \quad \therefore C = 0.19 \mu F$$

③ A choke coil takes a current of 10 A. at R, dissipates 1410 watt when connected to a 200 V, 50 Hz supply. When another coil is connected in parallel with it, the total current taken from supply is 20 A. at a p.f of 0.866. Determine the current and overall p.f. when the coils are connected in series across same supply.



$$\text{Now, } I_1 = 10 \text{ A. } \cos \phi = P_1 = 1410 \text{ W}$$

$$I = 20 \text{ A. } \cos \phi = 0.866$$

$$\text{Now, } P_1 = VI \cos \phi_1$$

$$\therefore \cos \phi_1 = \frac{1410}{10 \times 200} = 0.705 \quad \text{--- (1)}$$

$$Z = R + jX_L$$

$$\therefore P_1 = \frac{V^2}{100 \times R_1} \Rightarrow R_1 = 14.1 \Omega \quad \text{--- (2)}$$

$$\text{Now, } \cos \phi_1 = \frac{R_1}{Z} \Rightarrow Z_1 = \frac{R_1}{\cos \phi_1} \Rightarrow Z_1 = 20 \Omega \quad \text{--- (3)}$$

$$\therefore Z_1 = \frac{14.10}{10.00} + j \frac{14.18}{10.00} \Rightarrow X_{L1} = \frac{14.18}{10.00} \Omega$$

$$\therefore L_1 = \frac{14.18}{100 \pi} = 4.5 \text{ mH} \quad \text{--- (4)}$$

$$I = 10 \angle -45.17^\circ = 7.05 - j7.09$$

$$I = 20 \angle -30^\circ = 17.32 - j10$$

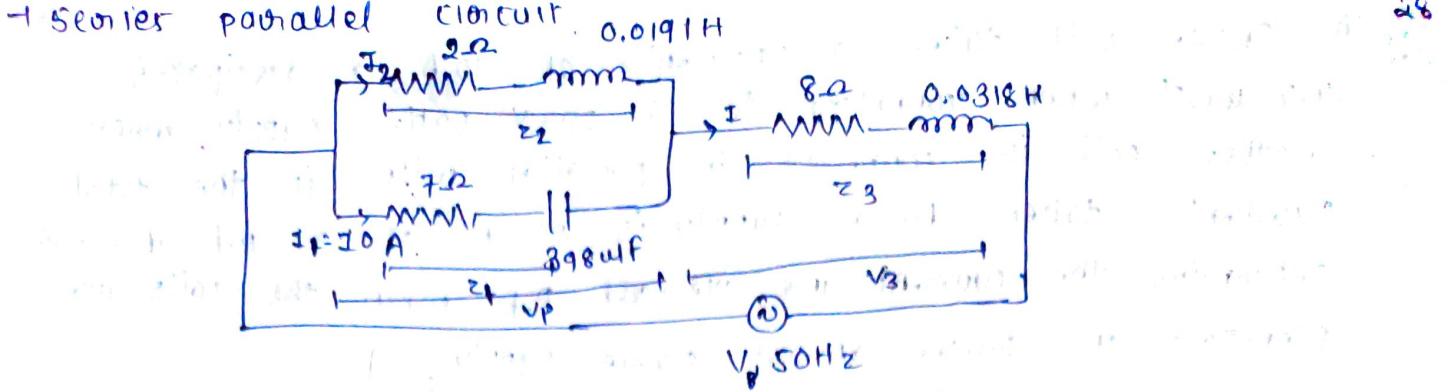
$$I - I_1 = I_2 = 10.27 - j2.91 = 10.67 \angle -15^\circ$$

$$Z_2 = \frac{200}{10.67 \angle -15^\circ} = 18.94 \angle +15^\circ = 18.03 + j5.10 \quad \text{--- (5)}$$

$$Z = 32.13 + j19.28 = 37.39 \angle 30.96^\circ \quad \text{--- (6)}$$

$$I = \frac{V}{Z \cos \phi} = \frac{5.98}{5.33} \angle -15.7^\circ$$

$$\text{P.F} = 0.86 \text{ (lag)}$$



$$V = ? \quad z_1 = 7 - j 8 = 10.63 \angle -48.81^\circ$$

$$z_2 = 2 + j 15.99 = 6.32 \angle 71.56^\circ$$

$$z_3 = 8 + j 9.98 = 12.8 \angle 51.34^\circ$$

$$\text{Now, } Z = \frac{z_1 z_2}{z_1 + z_2} + z_3$$

let I_1 is reference. $\therefore I_1 = 10 \angle 0^\circ \text{ A}$.

$$\text{Now, } V_p = I_1 z_1 = 10 \angle 0^\circ \cdot 10.63 \angle -48.81^\circ$$

$$\boxed{V_p = 106.3 \angle -48.81^\circ} \quad \text{--- (1)}$$

$$\text{Now, } I_2 = \frac{106.3 \angle -48.81^\circ}{6.32 \angle 71.56^\circ} = 16.81 \angle -120.37^\circ = -8.81 \angle 14.5^\circ$$

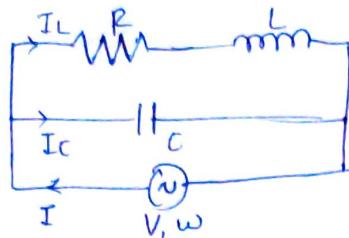
$$\begin{aligned} \text{Now, } I &= I_1 + I_2 \\ &= 10 \angle 0^\circ - 8.81 \angle 14.5^\circ = 12.8 \angle -38.09^\circ \end{aligned}$$

$$\therefore V_3 = \frac{23.5 \angle -38.09^\circ}{12.8 \angle -38.09^\circ} = 300 \angle 0^\circ$$

$$\cancel{V_3} = V_3 = 14.58 \angle -38.09^\circ \cdot 12.8 \angle 51.34^\circ \\ = 186.624 \angle 13.25^\circ$$

Resonance in parallel circuit.

① R-L parallel with C.



This circuit is under resonance when inductive component of total current is zero.

$$\Rightarrow I_{\text{current}} = [I_C - I_L \sin \phi_L]$$

$$\Rightarrow I_C = I_L \sin \phi_L$$

$$\Rightarrow \frac{V}{X_C} = \frac{V}{X_L} \cdot \frac{X_L}{Z_L}$$

$$\Rightarrow Z_L^2 = X_L \cdot X_C$$

$$\Rightarrow Z_L^2 = \frac{L}{C}$$

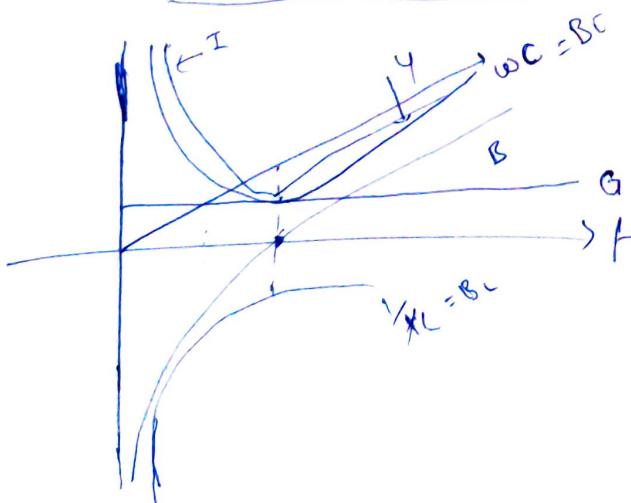
$$\Rightarrow Z_L = \sqrt{\frac{L}{C}}$$

$$\text{OR. } \frac{L}{C} = R^2 + L^2$$

$$(\omega \pi f_0 L)^2 = R^2 + \frac{L}{C} - R^2$$

$$\Rightarrow \omega \pi f_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$\therefore f_0 = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$



you will get
current magnification.

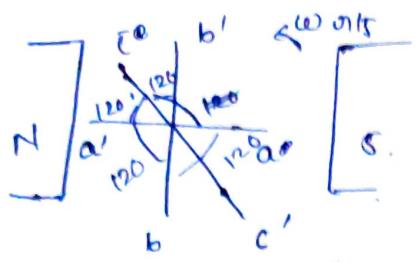
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* Poly phase circuits *

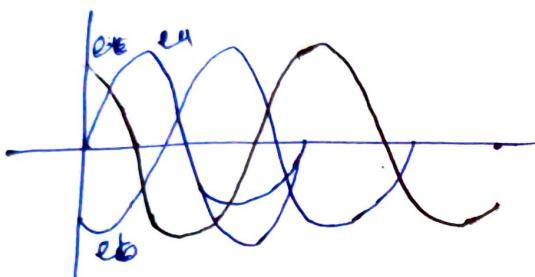


$$e_a = E_m \sin \omega t$$

$$e_b = E_m \sin (\omega t - 120^\circ)$$

$$e_c = E_m \sin (\omega t - 240^\circ)$$

- Three identical coils rotating in anti-clockwise direction, in a uniform magnetic field, 120° from each other.



- Resultant instantaneous emf.

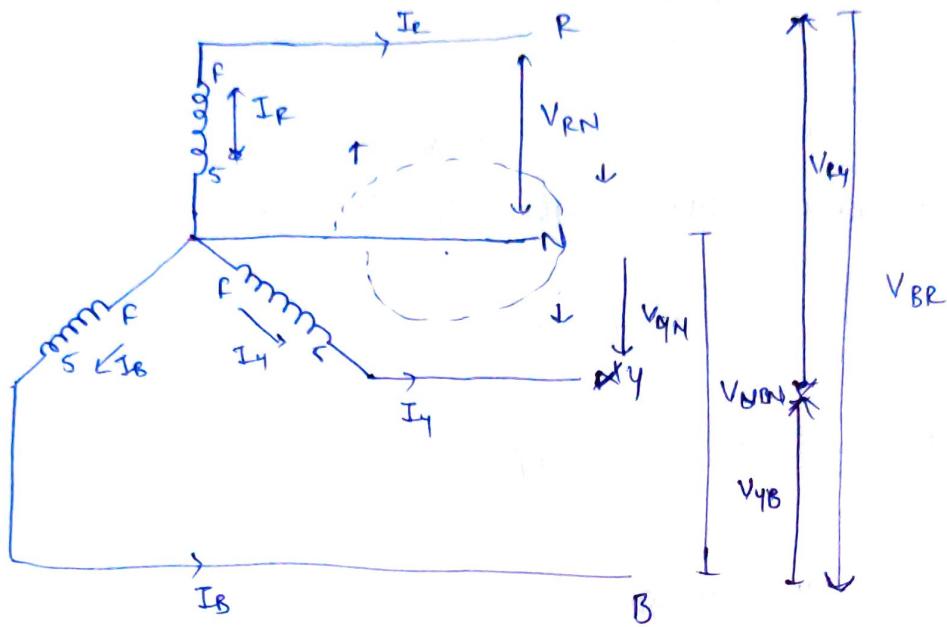
$$= e_a + e_b + e_c$$

$$= E_m [\sin \omega t + \sin (\omega t - 120^\circ) + \sin (\omega t - 240^\circ)]$$

$$= E_m [\sin \omega t + \alpha \sin (\omega t - 180^\circ) \cos 60^\circ]$$

$$= 0$$

\Rightarrow Star connection :



\rightarrow sometimes it's called Y -connection.
 R = Red Y = Yellow B = Blue N = Black

- $\rightarrow V_{RN} = V_{YN} = V_{BN}$ are called phase voltage = V_{ph} .
- $\rightarrow V_{RY} = V_{BR} = V_{YB} = V_L$ are called line voltages.
- $\rightarrow I_R = I_Y = I_B = I_L$ are called line currents & current passing through winding is called phase current.

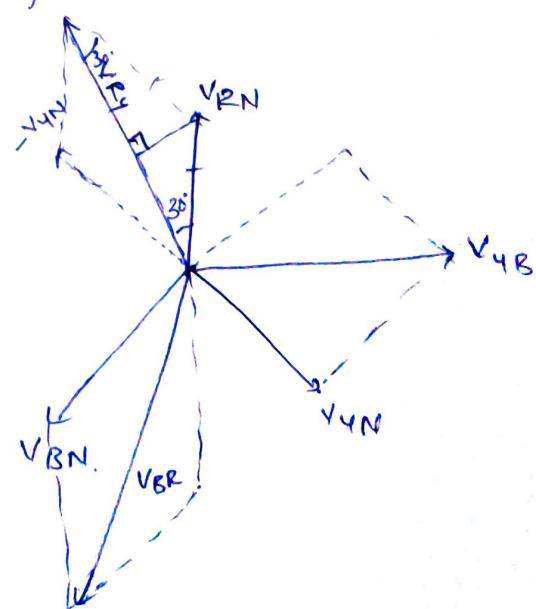
Now, from fig. $I_L = I_{ph}$

$$\text{Also, } V_{RN} - V_{RY} - V_{YN} = 0$$

$$\therefore V_{RN} = V_{RN} - V_{YN}$$

$$\text{Similarly } V_{YB} = V_{YN} - V_{BN}$$

$$V_{BR} = V_{BN} - V_{RN}$$



NOW, from fig.

$$\begin{aligned} V_{Ry} &= \sqrt{3} V_{RN} \cos 30^\circ \\ &= \sqrt{3} V_{RN} \end{aligned}$$

∴ $V_L = \sqrt{3} V_{ph}$

→ Total power $P = \frac{3}{2} V_{ph} I_{ph} \cos \phi$.
active.
 ϕ is phase diff. b/w V_{ph} & I_{ph} .

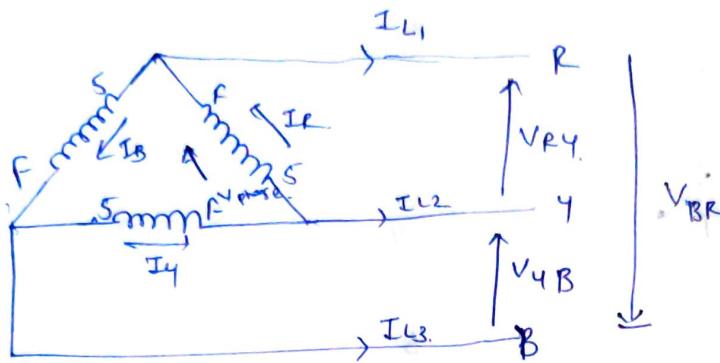
∴ $P = \frac{3}{2} \frac{V_L}{\sqrt{3}} I_L \cos \phi$

∴ $P = \sqrt{3} V_L I_L \cos \phi$ watts

→ Power reactive = $3 V_{ph} I_{ph} \sin \phi$
 $= \sqrt{3} V_L I_L \sin \phi$ VAR

→ Power apparent = $3 V_{ph} I_{ph}$
 $= \sqrt{3} V_L I_L$ VA

\Rightarrow Delta connection: Δ connection.



\rightarrow Now from fig.

$$V_L = V_{ph}$$

$$\begin{aligned} I_{L1} &= I_R - I_B \\ I_{L2} &= I_R - I_Y \\ I_{L3} &= I_B - I_R \end{aligned}$$

$$\& I_L = \sqrt{3} I_{ph}$$

Now, Power

$$\begin{aligned} P_{active} &= 3 V_{ph} I_{ph} \cos \phi \\ &= \sqrt{3} V_L I_L \cos \phi \end{aligned}$$

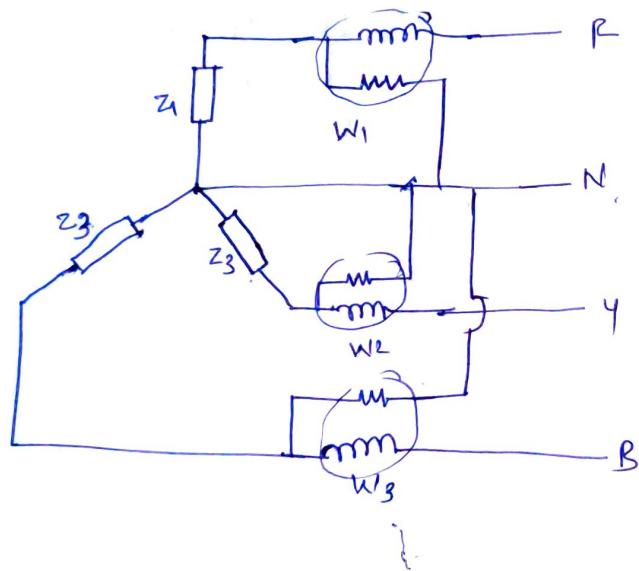
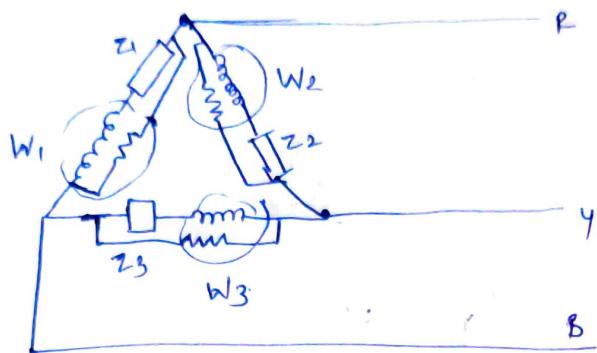
$$\begin{aligned} P_{reactive} &= 3 V_{ph} I_{ph} \sin \phi \\ &= \sqrt{3} V_L I_L \sin \phi \end{aligned}$$

$$\begin{aligned} P_{apparent} &= 3 V_{ph} I_{ph} \\ &= \sqrt{3} V_L I_L \end{aligned}$$

=) Measurement of Power

→ Three no. of ways

i) Simplest one is 3 wattmeter method.

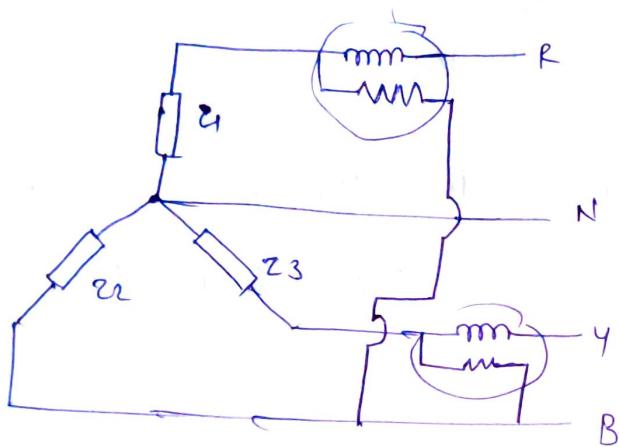
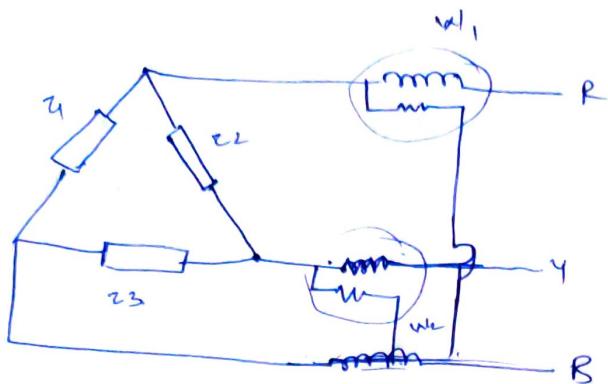


→ if $Z_1 = Z_2 = Z_3$ then it is balanced.

Now, total Power

$$P_{\text{Active}} = W_1 + W_2 + W_3$$

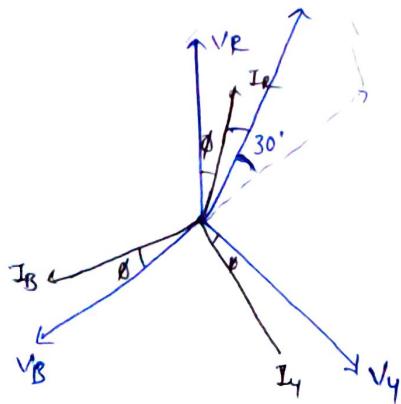
ii) 2 wattmeter method.



→ connect the current coil of wattmeter in any two lines and pressure coil across the remaining line.

Now, current in $W_1 = I_R$ & voltage in $W_1 = V_{RB}$.

& Now, assume that the phase current is lagging by angle ϕ w.r.t to respective phase voltage.

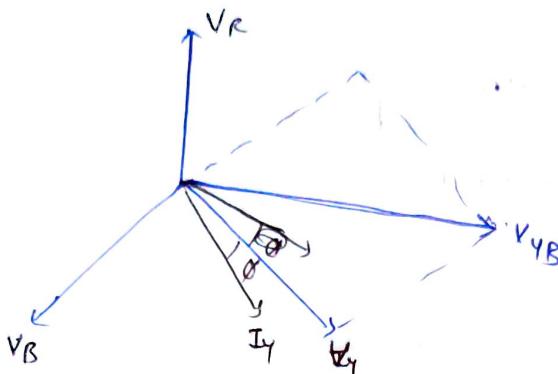


From fig. Angle b/w V_{RB} & I_R is $(30 - \phi)$

$$\therefore W_1 = V_{RB} I_R \cos(30 - \phi)$$

Now, current in $W_2 = I_y$ & phase voltage = V_{YB}

$$= V_y - V_B$$



From fig. Angle b/w V_{YB} & I_y is $(30 + \phi)$

$$\therefore W_2 \text{ reading} = V_{YB} I_y \cos(30 + \phi)$$

$$\text{Now, } P_{\text{total}} = W_1 + W_2$$

$$= V_{RB} I_R \cos(30 - \phi) + V_{YB} I_y \cos(30 + \phi)$$

$$= V_L I_L [\cos(30 - \phi) + \cos(30 + \phi)]$$

$$= V_L I_L [\cos 30 \cdot \cos \phi + \sin 30 / \sin \phi + \cos 30 \cdot \cos \phi - \sin 30 / \sin \phi]$$

$$= \alpha V_L I_L \frac{\sqrt{3}}{2} \cos \phi$$

$P = \sqrt{3} V_L I_L \cos \phi$

→ Power.

Similarly :

$$\text{Power} = I_{L1} - I_{L2}$$

$$= 2 V_L I_L \sin 30^\circ \sin \phi$$

$$= V_L I_L \sin \phi$$

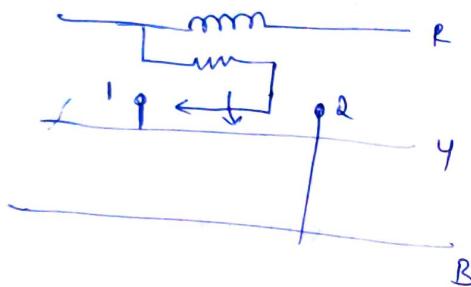
∴ $\boxed{\text{total reactive power} = \sqrt{3} (W_1 - W_2)}$

① IMP. OF REACTIVE power

②. INAC voltage
is multiple of "

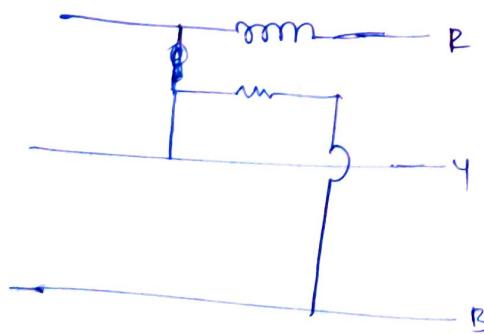
iii) One wattmeter method.

From ACTIVE power.



→ Explanation same as (ii)

= Reactive power by one wattmeter method.



Now, current in coil = I_R voltage = V_{4B}

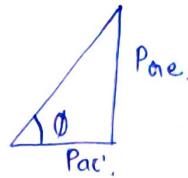
8 Angle b/w V_{YB} & I_R = $(90 - \phi)$

$$\text{∴ } W = V_{YB} I_R \cos(90 - \phi) \\ = V_L I_L \sin \phi$$

$$\text{∴ } \boxed{P_{\text{true}} = \sqrt{3} W}$$

⇒ Power factor by two wattmeter.

$$\tan \phi = \frac{P_{\text{true}}}{P_{\text{ac}}} = \frac{\sqrt{3} (W_1 - W_2)}{(W_1 + W_2)}$$



⇒ Variation of wattmeter readings.

$$W_1 = \sqrt{3} V_L I_L \cos(30 - \phi), \quad W_2 = V_L I_L \cos(30 + \phi)$$

① $\phi = 0$. (Resistive load).

$$W_1 = W_2.$$

② $0 < \phi < 60$
Both wattmeter will read positive values. They are diff.

③ $60 \leq \phi < 90$
then $W_2 = 0$, then $P_{\text{total}} = W_1$

④ $90 < \phi < 180$

$$W_1 = +ve \quad \& \quad W_2 = -ve$$

so to read the values interchange the terminals
of current coil or pressure coil.

Usually pressure coil,

consider the reading with negative sign.

⑤ $\phi = 90^\circ$ (Pure inductive or pure capacitive circuit).
Both wattmeter will read equal & oppo. reading.

=) Practice Application:

Numericals :

① 3 identical coils having resistance of $20\ \Omega$ & inductance of $63.661\ \text{mH}$. are connected

i) in star & ii) in delta.

across $440\ \text{V}, 50\ \text{Hz}$, 3- ϕ supply. calculate for each case

i) line current & phase current

ii) V_{ph} & V_L iii) P.F iv) Reading on each of two

wattmeter connected to measure P and total P .

$$\text{Here, } R_{ph} = 20\ \Omega \quad L_{ph} = 63.661 \times 10^{-3}\ \text{H}$$

$$\therefore X_{ph} = 20\ \Omega \approx 19.9996$$

$$\begin{aligned} \therefore Z_{ph} &= 20 + j20 \\ &= 28.28 \angle 45^\circ \end{aligned}$$

$$\text{Now, } V_L = 440\ \text{V}$$

i) Star connection,

$$V_L = 440 \Rightarrow V_{ph} = \frac{440}{\sqrt{3}} = 254.03\ \text{V}$$

$$\text{Now, } I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{254.03}{28.28 \angle 45^\circ} = 8.983 \angle -45^\circ\ \text{A.}$$

$$\therefore I_L = I_{ph} = 8.983\ \text{A}$$

$$\text{Now, } \phi = 45^\circ \Rightarrow \cos \phi = \text{P.F} = 0.707 \text{ (lag)}$$

$$\text{Now, } W_1 = V_L I_L \cos(30 - \phi)$$

$$= 440 \times 8.983 \times \cos(-15^\circ)$$

$$W_1 = 3817.84\ \text{watt.}$$

$$W_2 = V_L I_L \cos(30 + \phi)$$

$$= 1022.987\ \text{watt.}$$

$$\therefore \text{total } P = 4840.827\ \text{watts.}$$

ii) Delta connection

$$V_L = V_{ph} = 440$$

$$\therefore I_{ph} = \frac{440}{\sqrt{3} \times 28.28 \angle 45^\circ} = 15.55 \angle -45^\circ$$

$$I_L = \frac{I_{ph} \sqrt{3}}{\sqrt{8}} = 26.93 \text{ A}$$

$$\text{Now, } W_1 = V_L I_L \cos(30^\circ - \phi) \\ = 11445.44$$

$$W_2 = V_L I_L \cos(30^\circ + \phi) \\ = 2998.47$$

$$\therefore \text{Total } P = 14443.91 \text{ Watt.}$$

$$\phi = 45^\circ \text{ (lag)} \quad \therefore \cos \phi = \text{P.F} = 0.707 \text{ (lag)}$$

② The total real power absorbed by a 3- ϕ balanced load is 60 KW and reactive power 1000 KVAR determine the power factor of load & readings of 2 wattmeter connected to measure power.

It's given that $P_{active} = 60000 \text{ W}$
 $P_{reactive} = 1000000 \text{ VAR}$

we know that

$$\tan \phi = \frac{P_{reactive}}{P_{active}} = \frac{10^6}{6 \times 10^4} = \frac{100}{6} \Rightarrow \phi = \tan^{-1} \frac{50}{3}$$

$$\therefore \cos \phi = 0.0598$$

$$\text{Now, } W_1 + W_2 = 60000 \quad \& \quad \sqrt{3}(W_1 - W_2) = 10^6$$

$$\therefore W_1 = 318.675 \text{ KW}$$

$$W_1 - W_2 = 577350.2642$$

$$\therefore W_2 = -258.675 \text{ KW.}$$

- ③ A balanced delta connected load is supplied from 3- ϕ , 400V, 50Hz supply. The current in each phase is 16A. & it lags applied voltage by angle of 36.86° calculate.
- i) V_{ph}
 - ii) I_L
 - iii) Load parameter if it is two element circuit
 - iv) Total power
 - v) Reading of two wattmeters.

here, it is delta connection so we know that

i) $V_{ph} = V_L$

$\Rightarrow \boxed{V_{ph} = 400V}$

ii) Also, $I_L = \sqrt{3} I_{ph}$

$\therefore \boxed{I_L = 27.71A}$

iii) load parameter are resistance & inductor if current lags the voltage.

$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = 25$$

$R = 20.00 \Omega$ $X_L = 14.996 \approx 15 \Omega$ $L_{ph} = 47.75 \text{ mH.}$
--

iv) Now, $P = \sqrt{3} V_L I_L \cos \phi$
 $= 15360.43 \text{ W}$

$\therefore \boxed{P = 15.36 \text{ kW}}$

v) Now, ~~$W_1 + W_2 = 15360.4 \text{ W}$~~

Now, $W_1 = V_L I_L \cos(30 - \phi)$
 $= 11004.65 \text{ W}$

$W_2 = V_L I_L \cos(30 + \phi)$
 $= 8868.35 \text{ W.}$

(4) Power input of star connected load is measured by 2 wattmeter method. Readings of wattmeter are.
 $W_1 = 100 \text{ kW}$, $W_2 = 50 \text{ kW}$.

If the circuit is connected to a supply of 1100 V, 50 Hz. determine load parameters for two element circuit.

As it is star connected.

$$I_L = I_{ph} \quad \text{&} \quad \underline{\underline{V_L}} = \underline{\underline{V_{ph}}} / \sqrt{3}$$

$$V_L = 1100 \text{ V.} \quad \therefore V_{ph} = 635.08 \text{ V.}$$

$$\text{Now, } P_{\text{active}} = W_1 + W_2 = 150 \text{ kW}$$

$$P_{\text{reactive}} = \sqrt{3}(W_1 - W_2) = 50\sqrt{3} \text{ kW} = 86.6 \text{ kW.}$$

$$\text{Now, } \text{P.F.} = \cos \left[\tan^{-1} \frac{P_{\text{reactive}}}{P_{\text{active}}} \right] = 0.866 \text{ (lag)}$$

$$\text{Now, } 150000 = \sqrt{3} V_L I_L \times 0.866$$

$$\therefore I_L = 90.9 \text{ A.}$$

$$\therefore Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{635.08}{90.9} = 6.9865$$

$$= 6.05 + j3.49$$

$$\therefore R_{ph} = 6.05 \Omega$$

$$X_{Lph} = 3.49 \Omega.$$

$$\therefore L = 11.1 \text{ mH}$$

Note:

In capacitive circuit $W_1 < W_2$

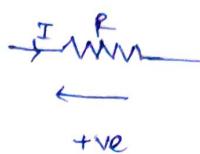
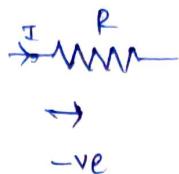
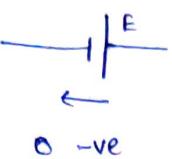
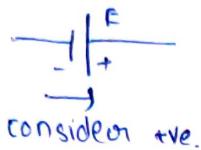
In inductive circuit $W_1 > W_2$.

Networks

Topic - 1 . Mesh Current Analysis.

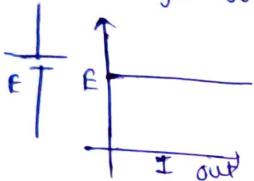
KVL → Algebraic sum of voltage around closed path is zero.

KCL → Algebraic sum of current at a junction is zero.



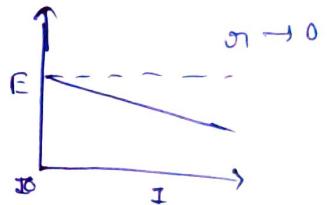
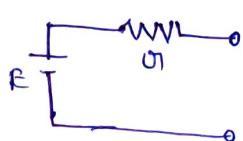
→ ① Ideal source.

① ideal voltage source.



$$E = V_L$$

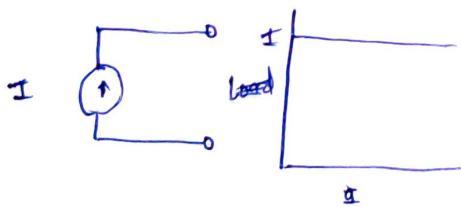
Practical source.



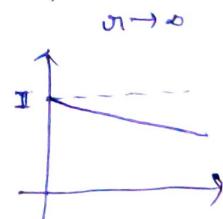
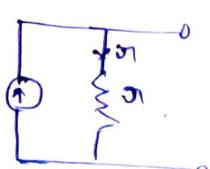
→ r is called internal resistance of the source.

→ For voltage source, it should be small as possible.

② ideal current source



Practical source.



H PDC link.

- No. of eqn required = No. of meshes (loops)
- Assume the mesh current. } same
- Assume the direction of traversal.
- While writing the mesh eqn, while applying KVL to n^{th} mesh assume that $I_m >$ any other current.
- While assuming mesh currents, make sure that atleast one current is passing through every element of the network.

→ Method

Make only 1 current through source.

$$I_1 = -2 \text{ A.}$$

$$100 - 5(I_1 + I_2) - 10(I_1 + I_2 - I_3) - 30(I_2 - I_3) - 10(I_2) = 0.$$

$$50 - 30(I_3 - I_2) - 10(I_3 - I_2 - I_1) - 20I_3 = 0.$$

$$100 - 5(I_2 - 2) - 10(I_2 - 2 - I_3) - 30(I_2 - I_3) - 10I_2 = 0$$

$$\cancel{100} - 55I_2 + 40I_3 = 0$$

$$\Rightarrow 55I_2 - 40I_3 = 130 \quad \Rightarrow 11I_2 - 8I_3 = 26 \quad [\times 9] \quad \textcircled{1}$$

$$44I_2 - 32I_3 = 104$$

$$50 - 30(I_3 - I_2) - 10(I_3 - I_2 + 2) - 20I_3 = 0$$

$$30 - 60I_3 + 40I_2 = 0$$

$$60I_3 - 40I_2 = 30$$

$$\Rightarrow 60I_3 - 40I_2 = 30 \quad \Rightarrow 12I_3 - 8I_2 = 6.$$

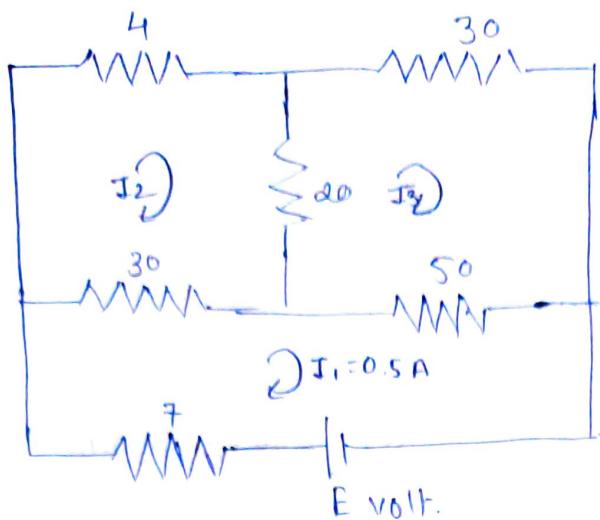
$$34I_3 = 137$$

$$6I_3 - 4I_2 = 3 \quad [\parallel] \quad \textcircled{2}$$

$$I_3 = 137/34 = 4.029 \text{ A.}$$

(4).

Q. 1



Find value of E.
50 input current
in 7 ohm is 0.5 A.

$$\begin{aligned} E - 7I_1 - 30(I_1 - I_2) - 50(I_1 - I_3) &= 0 \\ -4I_2 - 20(I_2 - I_3) - 30(I_2 - I_1) &= 0 \\ -30I_3 - 50(I_3 - I_1) - 20(I_3 - I_2) &= 0 \end{aligned}$$

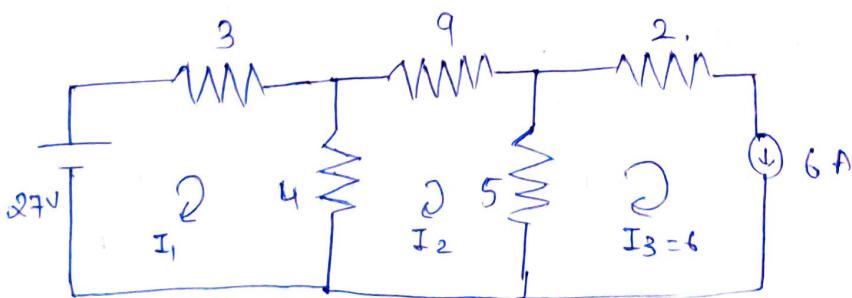
$$\begin{bmatrix} 7 & -30 & -50 \\ -30 & +54 & -20 \\ -50 & -20 & 100 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E \\ 0 \\ 0 \end{bmatrix}$$

By Cramers Rule.

$$I_1 = \frac{\begin{vmatrix} E & -30 & -50 \\ 0 & 54 & -20 \\ 0 & -20 & 100 \end{vmatrix}}{\Delta} = 0.5 = \frac{E(5400 - 400)}{\Delta} = \frac{5000E}{\Delta} = 0.5$$

$$\therefore E = 44.1 \text{ V}$$

Q. 2).



Find power dissipation in 4 ohm.

$$\therefore I_3 = 6 \text{ A}$$

$$27 - 3I_1 - 4(I_1 - I_2) = 0$$

$$-9I_2 - 5(I_2 - I_1) - 4(I_2 - I_1) = 0$$

$$27 - 7I_1 + 4I_2 = 0$$

$$30 + 5I_1 - 18I_2 = 0$$

$$+7I_1 - 4I_2 = 27] \times 5$$

$$-5I_1 + 18I_2 = 30] \times 7$$

$$\Rightarrow +35I_1 + 20I_2 = 27 \times 135$$

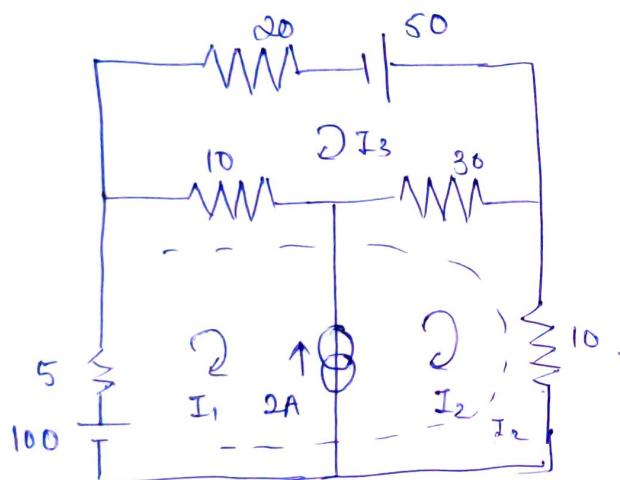
$$-35I_1 + 126I_2 = 210$$

$$\Rightarrow +106I_2 = 345$$

$$I_2 = 345 / 106$$

$$\therefore P = I^2 R = 180.7W$$

8)



$$I_2 - I_1 = 2$$

$$50 - (I_3 - I_2) 30 - 10(I_3 - I_1) - 20I_3 = 0$$

$$100 - 5I_1 - 10(I_1 - I_3) + 2 = 0$$

$$-10I_2 - 30(I_2 - I_3) = 0$$

$$50 - 30(I_3 - I_1 - 2) - 10(I_3 - I_1) - 20I_3 = 0$$

$$50 - 60I_3 + 40I_1 + 60 = 0$$

$$60I_3 - 40I_1 = 110 \quad \text{--- } ①$$

$$30I_3 - 40(2 + I_1) = 0$$

$$30I_3 - 40I_1 = +80 \quad \text{--- } ②$$

$$30I_3 = 30$$

$$I_3 = 1$$

$$I_1 = -5/4$$

Find out current passing through 30 Ω .

Super mesh technique.

consider 2 loops to avoid current source.

$$\therefore I_2 - I_1 = 2 \quad \text{Eqn.}$$

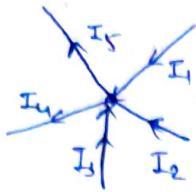
$$100 - 5I_1 - 10(I_1 - I_3) - 30(I_2 - I_3)$$

$$-10I_2 = 0$$

$$100 - 15I_1 - 40I_3 - 40I_2 = 0$$

$$100 - 15I_1 - 40I_3 - 40I_1 = 80$$

KCL \circ In any electrical circuit, the algebraic sum of current meeting at a point is always zero.



$$I_1 + I_2 + I_3 - I_4 - I_5 = 0.$$

$$\text{P.e } i_1 + i_2 + i_3 = i_{\text{out}}$$

\Rightarrow Nodal voltage analysis

i) Junction \circ It is a point in a network where three or more branches are there meeting.

-i) identify no. of junctions.

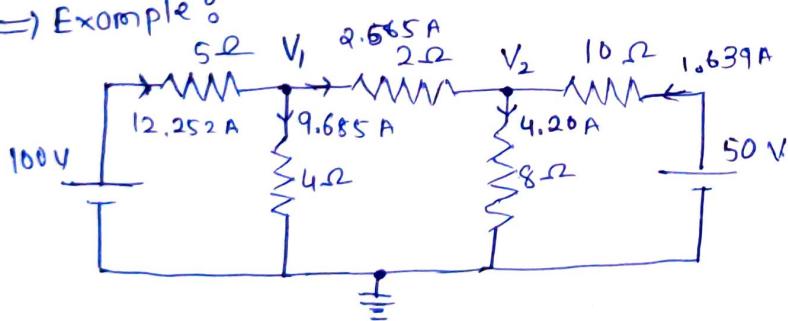
-ii) consider one of the junction as reference, (so we can say its potential is zero. & assume the load voltage for remaining junctions).

-iv) Current flows from higher to lower potential.

-vi) While applying KCL to write node eqn., assume that all branch currents are leaving the node.

$\star \checkmark$ no. of eqn = no. of junctions - 1.

\Rightarrow Example \circ



Now, At Node - 1.

$$\frac{V_1 - 100}{5} + \frac{V_1 - 0}{4} + \frac{V_1 - V_2}{2} = 0 \Rightarrow 4V_1 - 400 + 5V_1 + 10V_1 - 10V_2 = 0 \\ \Rightarrow 19V_1 - 10V_2 = 400$$

$$\frac{V_2 - 50}{10} + \frac{V_2 - 0}{8} + \frac{V_2 - V_1}{2} = 0 \Rightarrow 4V_2 - 200 + 5V_2 + 20V_2 - 20V_1 = 0$$

$$29V_2 - 20V_1 = 200.$$

$$\therefore V_2 = 33.61$$

$$V_1 = 88.74$$

$$Y = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix}$$

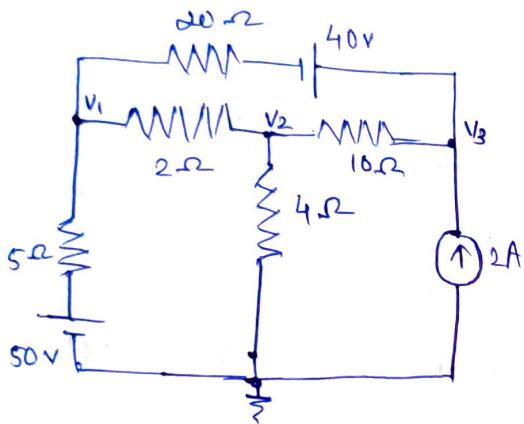
$$[Y, V = I]$$

Y_{11} = sum of all branch admittance connected to Junction 1. (Always +ve)

Y_{12} = sum of all branch admittances connected b/w Junction 0 or node 1 & node 2. (Always -ve).

$$I = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

I_1 = sum all source current connect to Junction 1
 $-$ = current toward junction, consider it with +ve sign.



Find power supplied by 50V source.

$$\frac{V_1 - 50}{5} + \frac{V_1 - V_2}{2} + \frac{V_1 + 40 - V_3}{20} = 0$$

$$\left(\frac{1}{5} + \frac{1}{2} + \frac{1}{20}\right)V_1 - \frac{1}{2}V_2 - \frac{1}{20}V_3 = 10 - 2 = 8$$

$$\frac{3V_1}{4} - \frac{1}{2}V_2 - \frac{1}{20}V_3 = 8$$

$$\frac{V_2 - V_1}{2} + \frac{V_2 - V_3}{10} + \frac{V_2}{4} = 0 \Rightarrow \left(\frac{1}{2} + \frac{1}{10} + \frac{1}{4}\right)V_2 - \frac{V_1}{2} - \frac{V_3}{10} = 0$$

$$= \frac{17}{20}V_2 - \frac{V_1}{2} - \frac{V_3}{10} = 0$$

$$\frac{V_3 - V_2}{10} + \frac{V_3 - 40 - V_1}{20} - 2 = 0 \Rightarrow \left(\frac{1}{10} + \frac{1}{20}\right)V_3 - \frac{V_2}{10} - \frac{V_1}{20} = 4$$

$$\Rightarrow \frac{3}{20}V_3 - \frac{V_2}{10} - \frac{V_1}{20} = 4$$

find only V_1 & than $\frac{50-V_1}{5} = I_1$,
 & power is $V_1 I_1$,

NOW,

$$\begin{bmatrix} 0.75 & -0.5 & -0.05 \\ -0.5 & 0.85 & -0.1 \\ -0.05 & -0.1 & 0.15 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \\ 4 \end{bmatrix}$$

NOW, $V_1 = \frac{\begin{bmatrix} 8 & -0.5 & 0.05 \\ 0 & 0.85 & -0.1 \\ 4 & -0.1 & 0.15 \end{bmatrix}}{\Delta} = \frac{0.94 + 0.03}{0.08 + 0.2 + -0.17} = \frac{0.97}{0.11} = 8.81$

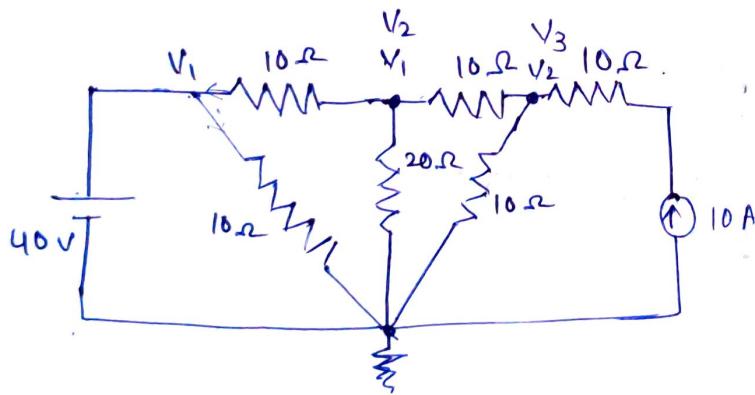
$$V_1 = 8.81 \text{ V}$$

$$\Rightarrow I_1 = \frac{50 - 8.81}{5} = 8.236 \text{ A}$$

$$P = 50 \times 8.236$$

$P = 411.81 \text{ W}$. is power supplied by 50 V supply.

3)



find out power supplied by 40 V supply source.

$$\text{Now, } \frac{V_1 - 40}{10} + \frac{V_1}{20} + \frac{V_1 - V_2}{10} = 0 \Rightarrow \frac{1}{10} \left(\frac{1}{10} + \frac{1}{10} + \frac{1}{20} \right) V_1 - \frac{V_2}{10} = 4 \quad (1)$$

$$\frac{V_2 - V_1}{10} + \frac{V_2}{10} + \frac{V_2 - 10}{10} = 0 \Rightarrow \left(\frac{1}{10} + \frac{1}{10} + \frac{1}{10} \right) V_2 - \frac{V_1}{10} = 10. \quad (2)$$

$$\begin{bmatrix} 0.25 & -0.1 \\ -0.1 & 0.2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 10 \end{bmatrix}$$

$$V_1 = \frac{\begin{bmatrix} 4 & -0.1 \\ 10 & 0.2 \end{bmatrix}}{0.065} \times 0.1 = \frac{2.4}{0.065} = 37.84 \text{ V} \quad \frac{1.8}{0.04} = 45$$

$$V_2 = \frac{\begin{bmatrix} 0.25 & 4 \\ -0.1 & 10 \end{bmatrix}}{0.065} = \frac{2.9}{0.065} = 44.61 \text{ V.} \quad \frac{2.9}{0.04} = 72.5$$

$$I_1 = \frac{V_1 - 40}{10} = 0.616 \text{ A}$$

$$I_1 = \frac{5}{10} = 0.5$$

$$\therefore I_{40} = 4 + 0.616 = 4.616$$

$$I = 4 - 0.5 = 3.5$$

$$\therefore P = 40 \times 4.616 \text{ Watt}$$

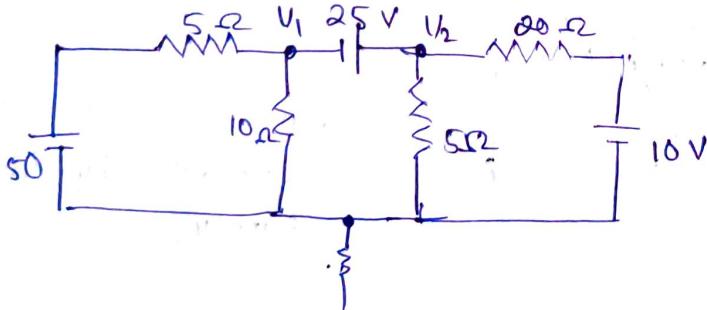
~~white wrongy solution~~

$$\boxed{P = 184.64 \text{ Watt}} \quad \boxed{P = 140 \text{ Watt}}$$

$$V_1 = 40 \text{ V.}$$

$$I_1 = 0.5$$

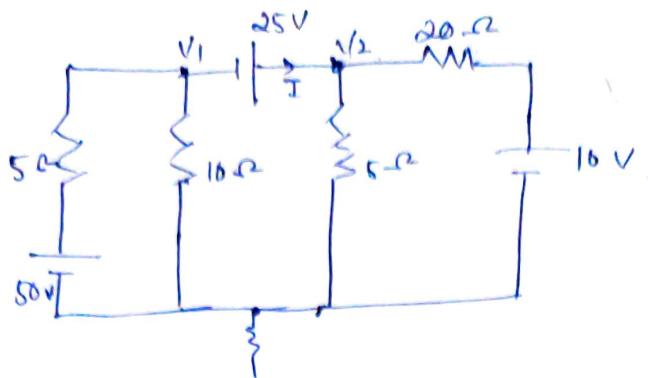
4)



$$\frac{V_1 - 50}{5} + \frac{V_1}{10} + V_1 + 25 - V_2 =$$

$$\frac{V_2 - 10}{20} + \frac{V_2}{5}$$

4)



Assume I in 25V branch.

$$\frac{V_1 - 50}{5} + \frac{V_1}{10} - I = 0 \quad \text{--- (1)}$$

$$\frac{V_2 - 10}{20} + \frac{V_2}{5} - I = 0. \quad \text{--- (2)}$$

$$V_2 - V_1 = 25 \quad \text{--- (3)}$$

$$\Rightarrow 0.2V_1 - 10 + 0.1V_1 - I = 0 \Rightarrow 0.3V_1 - I = 10 \quad \text{--- (3)}$$

$$0.05V_2 + 0.2V_2 - I - 0.5 = 0 \Rightarrow 0.25V_2 - I = 0.5 \quad \text{--- (4)}$$

Now $\rightarrow (3) - (4)$, we get.

~~$$0.3V_1 - 0.25V_2 = 9.5 \quad \text{--- (6)}$$~~

Now, $(6) \times 4 + (5)$, we get.

~~$$\begin{aligned} V_2 - V_1 &= 25 \\ + 1.2V_1 - V_2 &= 38.0 \end{aligned} \Rightarrow 0.2V_1 = 38$$~~

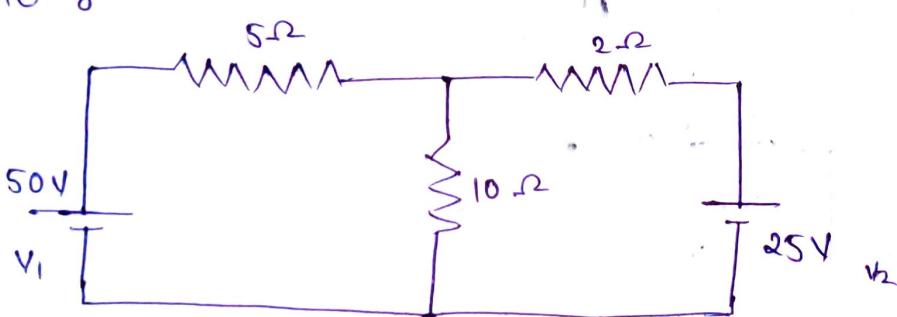
$$\Rightarrow V_1 = 38 \times 5$$

=1 Superposition theorem,

- In a linear, bilateral network containing more than one energy sources, the resultant current in any branch of the network is the algebraic sum of the currents that would be produced, by each energy source taken separately, with all the other energy sources being replaced by their respective internal resistances.
- If internal resistances are not given, consider sources as ideal sources & replace voltage source by short circuit & current source by open circuit.

Example :

1)



Find out current passing through $10\ \Omega$.

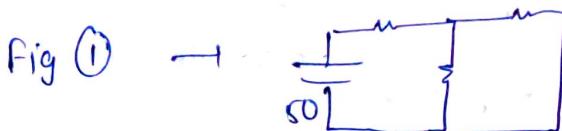
$$I_1 = 7.5 \text{ A} \quad I_2 = 4.6875 \text{ A}$$

$$I_{10} = 1.25 \quad I_2 = 1.5625$$

$$\text{Ans} \quad \frac{V_1 - 50}{5} + \frac{V_1}{10} + \frac{V_1 - 25}{2} = 0. \quad \Rightarrow 0.8V_1 = 22.5$$

$$V_1 = 28.125$$

$$I = 2.8125$$



→ to get I_1 through $10\ \Omega$.



→ to get I_2 through $10\ \Omega$.

$$I_1 + I_2 = I \text{ through } 10\ \Omega$$

5)

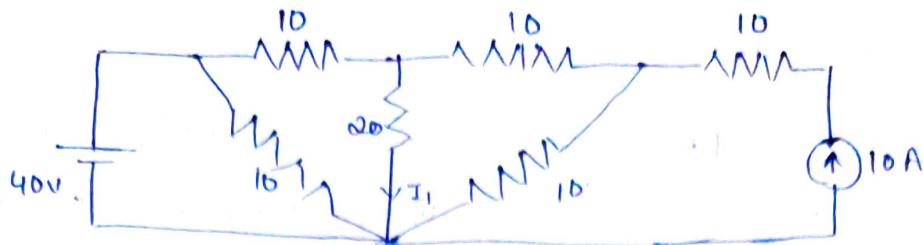
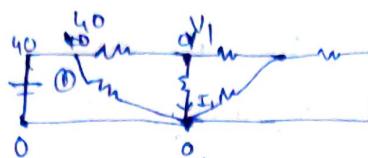


Fig. →



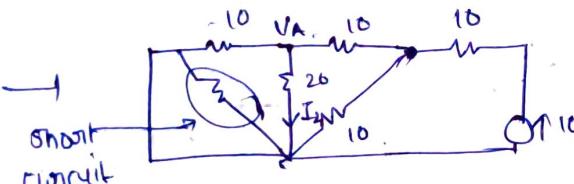
$$\frac{V}{2\Omega} + \frac{V-40}{10} + \text{At Junc (1)} \quad \frac{V}{2\Omega} + \frac{V-40}{10} + \frac{V}{2\Omega} = 0.$$

$$\frac{40-V_1}{10} + \frac{V_1}{4\Omega}$$

$$\Rightarrow \frac{4V}{20} - 4 = 0 \quad V = 20 \text{ V}$$

$$I_1 = 1 \text{ A}$$

Fig →



$$\frac{2VA}{20} + \frac{VA}{20} + \frac{2VA - 2VB}{20} = 0 \Rightarrow \frac{10VA}{20} - \frac{4VB}{20} = 0$$

$$\frac{VB}{10} + \frac{VB - VA}{10} + \cancel{\frac{VB}{10}} = 10 \Rightarrow \frac{3VB}{10} - \frac{VA}{10} = 100.$$

$$20VA - 12VB = 0$$

$$12VB - 4VA = 100$$

$$\frac{2VB}{10} = VA$$

$$2VB = 100$$

$$VA = 3.8 \cancel{46} 25$$

$$2VB - VA = 100$$

$$\frac{5VA}{20} - \frac{2VB}{20} = 0 \Rightarrow 5VA - 2VB = 0.$$

$$2VB - VA = 500$$

$$4VA$$

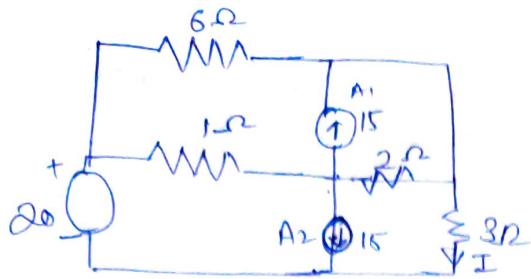
$$VA = 25$$

$$VB = 500$$

$$\frac{1000}{5}$$

$$5VA = 1500 - 500$$

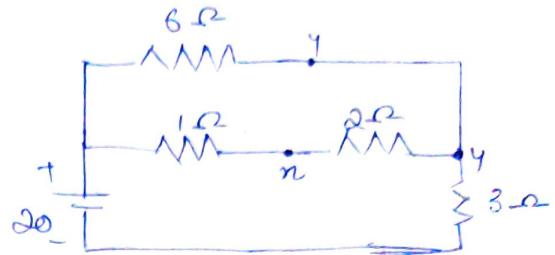
6)



Find current through 3Ω .

Now, for $20V$ source.

$$\text{Now, } I_1 = \frac{20}{5} = 4A.$$



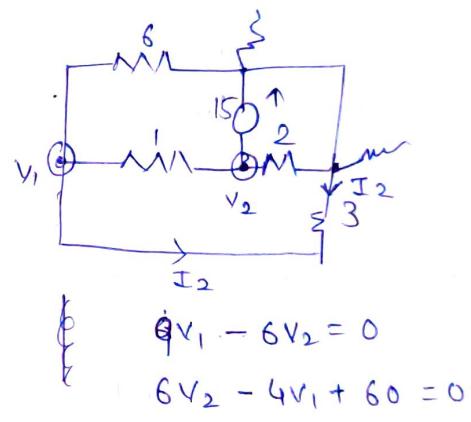
Now, for $A_1 (15V)$ source.

$$\frac{V_1}{6} + \frac{V_1 - V_2}{1} + \frac{V_1}{3} = 0$$

$$\frac{V_1}{2} + V_1 - V_2 = 0 \\ \Rightarrow 3V_1 - 2V_2 = 0.$$

$$\frac{V_2 - V_1}{1} + 15 + \frac{V_2}{2} = 0 \\ \Rightarrow 3V_2 - 2V_1 + 30 = 0.$$

$$\therefore I_2 = \frac{0 - (-12) - V_1}{3} = 4A$$



$$5V_1 = -60 \\ \Rightarrow V_1 = -12.$$

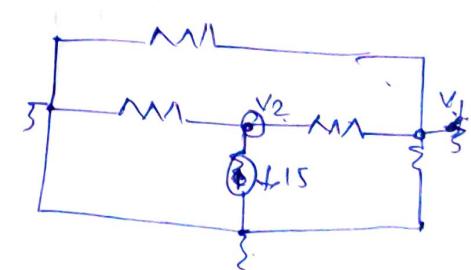
Now, for $A_2 (15V)$ source.

$$\frac{V_1}{6} + \frac{V_1}{3} + \frac{V_2 - V_1}{2} = 0 \\ \Rightarrow 3V_1 - 3V_2 = 0 \\ \Rightarrow 3V_1 = 3V_2$$

$$\frac{V_2}{1} + \frac{V_2 - V_1}{2} + 15 = 0 \\ \Rightarrow 3V_2 - 2V_1 + 30 = 0 \\ \Rightarrow V_1 = -6 \\ \Rightarrow V_2 = -6$$

$$I_3 = \frac{V_2}{2} = -3A$$

$$\therefore I = 4 + 4 - 2 = 6A$$



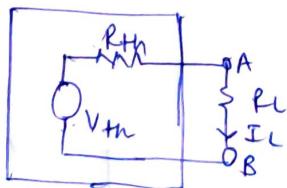
$$I_3 = \frac{-6}{3} = -2.$$

\Rightarrow Thevenin's theorem.

- Any two terminals linear, bilateral, active networks can be replaced by single voltage source V_{th} in series with single resistance R_{th} .



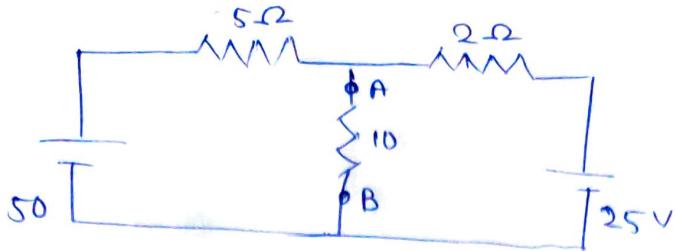
- The thevenin's equivalent voltage V_{th} is the open circuit voltage measured between terminals A & B.
- The thevenin's equivalent resistance R_{th} is the resistance of the network as seen from output terminals A & B with all energy sources being replaced by their internal resistances.
- IF internal resistances are not given then consider the sources as ideal sources and replace voltage source by short-circuit & current source by open circuit.



$$I_L = \frac{V_{th}}{R_L + R_{th}}$$

Numericals :

1) Nu



find current through 10.

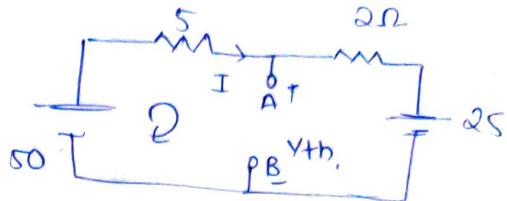
Now, for V_{th} .

$$50 - 5 \times \frac{25}{7} - V_{th} = 0.$$

$$V_{th} = 50 - \frac{25}{7} = 0$$

$$V_{th} = \frac{225}{7} = 32.145$$

$$R_{th} = 10/7$$



$$I = \frac{50 - 25}{7} = \frac{25}{7} = 3.571 \text{ A.}$$

$$\therefore I_L = \frac{V_{th}}{R_L + R_{th}} = \frac{32.145}{1.428 + 10} = 2.81 \text{ A.}$$