TRUSSES

5 ANALYSIS OF PLANE TRUSSES

5·1. Introduction : *A truss is a structure composed of straight members connected at their end points.* The joint connections are usually, formed by bolting or welding. Truss which will be analyzed here is a plane truss and will be subjected to **loads** which act in its plane and **at joints** only. Mostly, the members of a truss are slender and unable to carry any lateral loads, therefore *loads on it must be acting at joints only.* Though, truss members are joined together by riveting **or welding** connections, *it is assumed that the members are pinned. Due to which the forces acting at each end of a member reduce to a single force, tensile or compressive.*

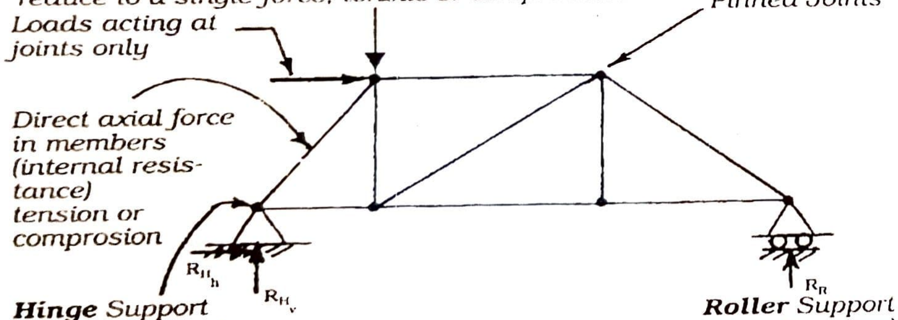
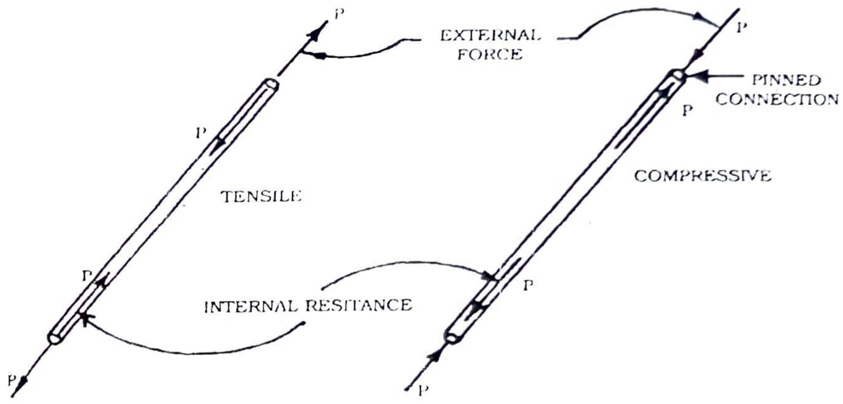


Fig. 5·1 TRUSS

Thus, because of the two important assumptions, that all loadings are applied at the joints and the members are joined together by smooth pins, *each truss member acts as a two force member. Here force in member will be directed along the axis of the member.*

TRUSS
- **Used to support large span** Roofs (industrial) Railway bridge
- **composed of straight members**
  - *External loads*
    - ***One Reaction at Roller Support***
    - ***Two Reactions at Hinge support***
    - ***Loads at joints only***
  - *Internal loads* — *Internal resistance in members* ***(tension or compression)***



Internal resistance
Fig 5·2 Member forces

From the figure it is clear that, first diagram shows forces on member which tend to elongate the member and the member is in tension and type of force acting on it will be tensile force. In second diagram, the forces tend to shorten the member and the member is in compression and type of force will be compressive force. Also note that action and reaction will be opposite, which are shown at pinned ends of members. Thus *on the member arrows directing inward indicate tension and arrows directing outward indicate compression.*

It is to be noted carefully that the tension and compression are indicated as internal resistance in the member, arrows directing towards centre indicate tension whereas arrows directing towards ends indicate compression.

tension **compression**

5·2. Stability and Determinancy of Trusses :

The reactions **(r)** *and forces in members* **(m)** *are unknowns in the truss* which are to be determined by considering *the equilibrium of the truss* as a whole as well as *each joint separately.* Considering the each joint in equilibrium, two equilibrium equations are available, viz. $\Sigma F_x = 0$ and $\Sigma F_y = 0$. Hence, if j number of joints are present, 2j equations are available.

The necessary condition for statically determinate truss (in which unknowns can be determined by equilibrium equations) is

$$\boxed{m = 2j - r}$$

where m = number of *members* = no.of forces in members (*unknowns*)

j = number of *joints*, hence 2j *equilibrium equations* are available.

r = number of *reactions* (One at roller, two at hinge) (*unknowns*)

The above equation can also be written as

$$\boxed{m + r = 2j}$$

where **m + r** is the total number of ***unknowns*** and **2j** is the number of available ***equations of statics*** while treating each joint as a free body.

Perfect truss is said to be a truss which satifies the necessary condition, m = 2j-3 and is also stable under the action of loadings, (means it will ***not*** *slide* or *collapsed*).

<u>Perfect Truss :</u>

Determinate + Stable

Determinate Truss : Unknowns = No. of Equilibrium Equations

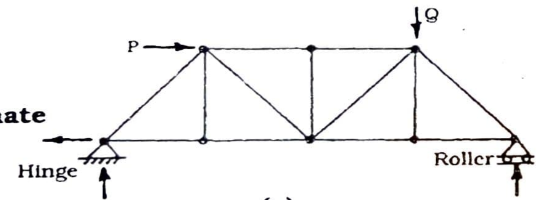
$m + r = 2j$

Stable Truss :

(i) Truss as a whole will remain in Static condition.

(ii) Each panel between the members will not be collapsed.

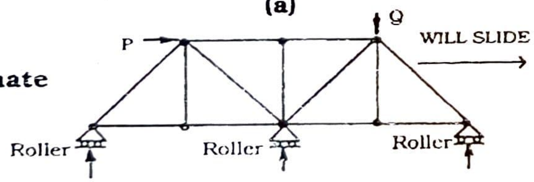
Stable Determinate



$m = 13$
$j = 8$
$r = 3$
$\mathbf{m + r = 2j}$

(a)

Unstable Determinate

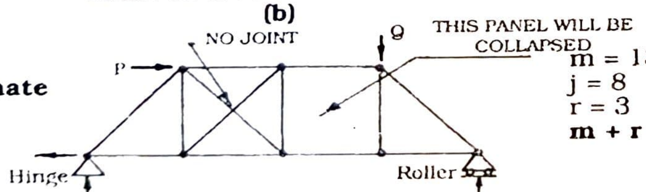


$m = 13$
$j = 8$
$r = 3$
$\mathbf{m + r = 2j}$

Truss as a whole is Unstable

(b)

Unstable Determinate

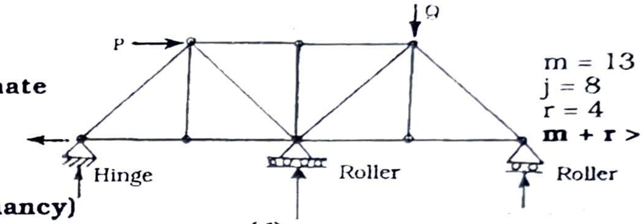


$m = 13$
$j = 8$
$r = 3$
$\mathbf{m + r = 2j}$

Panel is Unstable

(c)

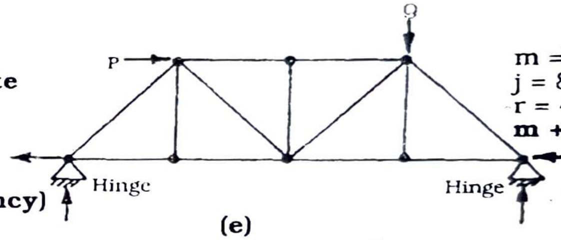
Stable Indeterminate (D.O.I. = 1) (One more reaction) (External Indeterminancy)



$m = 13$
$j = 8$
$r = 4$
$\mathbf{m + r > 2j}$

(d)

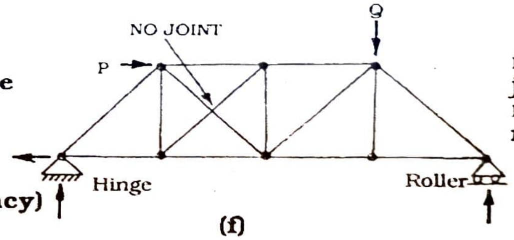
Stable Indeterminate (D.O.I. = 1) (One more reaction) (External Indeterminancy)



$m = 13$
$j = 8$
$r = 4$
$\mathbf{m + r > 2j}$

(e)

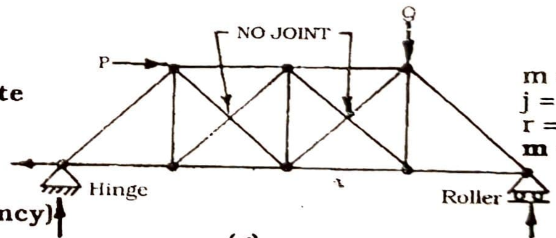
Stable Indeterminate (D.O.I. = 1) (One more member) (Internal Indeterminancy)



$m = 14$
$j = 8$
$r = 3$
$\mathbf{m + r > 2j}$

(f)

Stable Indeterminate (D.O.I. = 2) (Two more members) (Internal Indeterminancy)



$m = 15$
$j = 8$
$r = 3$
$\mathbf{m + r > 2j}$

(g)

Fig 5·3 Various Types of Trusses

The figure 5·3 shows various kinds of plane trusses. Figures 5·3 (a), 5·3 (b) and 5·3 (c) satisfy the necessary condition of statical determinancy out of whi ch truss of fig 5·3 (a) is stable and determinate. *The condition m+r = 2j is not always sufficient but also requires proper use of reactions and members for trusses to be* ***perfect.*** The truss shown in fig. 5·3 (b) will slide for general loading

Calculation → graphical method

Method of analysis

Joints   Sections

shown on it and that of fig 5·3 (c) will be collapsed in third panel under action of loads. Trusses of fig. 5·3 (d) and (e) have one more reaction which does not satisfy the equation m + r = 2j for determinancy and have got **degree of indeterminancy (D.O.I.)** of one though they are stable. Trusses of fig. 5·3 (f) and (g) have respectively one more and two more members then required, thus trusses are indeterminate to one degree and two degrees respectively. To be specific, fig. 5·3 (d) and (e) have ***external indeterminancy*** and fig. 5·3 (f) and (g) have ***internal indeterminancy.***

5·3. Methods of Analysis :

Commonly used methods for analyzing plane trusses (means determining forces in members) are method of joints and method of sections. In case of a non availability of calculating aid, one can use graphical method for analyzing truss.

In method of joints, at each joint the force system is coplanar and concurrent and summation of forces along two rectilinear axes will be zero which ensure translational or force equilibrium. Satisfying the equilibrium conditions at each joint, one can get the forces in the *members*. ***The Method of joint is most suited when forces in all the members are required to be obtained.***

In method of sections, we need to draw an imaginary line cutting truss in to two parts which passes through three or less then three members. Then for each part a free-body diagram is required to be drawn, where in, forces in the members cut by the section also need to be indicated. Application of equations of equilibrium can reveal the unknown member forces. ***The Method of section is best suited when we need to find forces in few particular members only. In such cases, the method of joints will turn out to be lengthy.***

Graphical method is used when we need to analyse the truss using drawing instruments only. The graphical method is simple and guick but introduces errors depending upon the accuracy of graphics.

5·4. Method of Joints :

In a plane truss, the lines of actions of all internal forces in members are known and hence the analysis reduces to find magnitude of the forces and their types i. e. tensile or compressive. For a simple truss, we can consider the *equilibrium of each joint*. *The unknowns are forces in members* **m** *plus three reactions, two at hinge support and one at roller.* ***These m+3 unknowns can be determined by two equilibrium equations at each joint which is equal to 2j.*** Thus, 2j=m+3, hence by drawing a free-body diagram of each joint and solving two equations of equilibrium ($\Sigma F_x=0$ and $\Sigma F_y=0$), one is capable to find forces in all the members and three reaction components also. In case of simply supported truss, the reactions can be obtained prior to the joint analysis by using three equilibrium equations of the whole rigid body (i.e. complete truss). Though, these are not independent equations but they help to locate joint where only two unknowns exist. For cantilever type of truss there is no need to write three equilibrium equations of rigid body, still they can help to check correctness of solutions at supports.

Steps to be followed :

Step 1. Draw a free - body diagram of an entire truss, showing there-in external forces acting on it, i.e. applied loads and reactions.

Step 2. Obtain magnitude and correct direction of reactions from equlibrium equations.

$$\boxed{\Sigma F_x = 0;\ \Sigma F_y = 0;\ \Sigma M_{\text{any point}} = 0}$$

Step 3. Redraw the truss showing there-in applied loads and reactions. ***Also assume tensile type of forces in all the members and show on them with arrow-heads directing inward.***

Step 4. Now locate a joint where only two unknown forces exist. Draw a free body diagram of the joint and write two equations of equilibrium.,

$$\boxed{\Sigma F_x = 0;\ \Sigma F_y = 0}$$

which will give unknown forces in two members.

Step 5. Analyse an adjacent joint where only two unknown member forces exist. Draw a free body diagram of the joint and find the unknown forces with the help of two equations of equlibrium, namely $\Sigma F_x = 0$ and $\Sigma F_y = 0$.

Step 6. Repeat the procedure shown in step 5 till all the member forces are obtained.

Step 7. Analysis of last but one joint gives one additional equation for numerical check of solution, while last joint gives two extra equations for numerical checks.

Step 8. ***Magnitude*** *of force being* ***positive*** *indicates that the assumption made about the type of force in member is correct i.e. force in such a member is of* ***tensile type. If magnitude is negative*** *then force type is* ***compressive.*** Now redraw the truss and show there-in the forces in each member with corrected arrowheads. *In case of* ***compressive force*** *show* ***arrow-heads directing outward*** *and indicate the magnitude of the force.*

5·5. Method of Sections :

As we have seen in previous section that the method of joints is a powerful method for the analysis of stable-determinate plane trusses. Unfortunately, the ***method of the joints*** *becomes* ***too lengthy*** *when we are interested only in finding the forces in few of the members of truss, particularly which is true for the truss having many members. In such a situation, the method of sections can be used with the great advantage.* The method requires **to cut a truss** in to **two parts** by an **imaginary line,** which passes through the particular members. Care must be taken to see that section cuts **maximum three members** only. Considering the equilibrium of an either part, it is possible to find the member forces lie on an imaginary cutting line.

Steps to be followed :

Step 1. As in the method of joints, here also we need to find the three reactions of the truss by considering the equilibrium of the whole truss and writing the three equations of equilibrium.

$$\Sigma F_x = 0;\ \Sigma F_y = 0;\ \Sigma M_{ANY\ POINT} = 0$$

In case of the cantilever type of truss, one may skip this step.

Step 2. Draw an ***imaginary line***, **cutting** the truss and passes through the **members** in which we **need to find the forces.** Take care that it cuts maximum three members only.

Step 3. Prepare a **free body diagram** of **either part** showing there in the loads, reactions and tensile forces for the members which are cut by an imaginary line. Note that the tensile force in member can be shown by arrowheads directing inward. In case of a cantilever truss if reactions are not obtained then a care should be taken to select the free portion first for analysis.

Step 4. Now three equations of equilibrium can be applied to find the forces in the members which are already cut. **Negative magnitude** of force indicates **compression.**

5·6. Graphical Method :

In the previous sections, *the analytical methods namely method of joints and method of sections* were discussed. In the case of a non availability of computing tool, one can depend on a graphical method, which is ***simple*** and *reliable*. **The method does not give exact results.**

Steps to be followed :

Step 1. Draw a truss to some convenient scale and mention the Bow's Notations to the various spaces. Firstly, name the spaces between the external loads and reactions, preferably in clockwise direction and then the internal spaces.

Step 2. Using space diagram, find the unknown reactions. In case of a cantilever truss, this step may be omitted.

Step 3. Draw the polygon, indicating there-in the loads and the reactions to some convenient scale.

Step. 4. Force polygon is drawn for the joint where only two unknown forces exist and then in sequence all polygons for all joints are be drawn to obtain Maxwell stress or force diagram. Here selection of joints is made in the same way as done in the method of joints.

Step 5. Then mark the arrows on the members of the space diagram in accordance with the closing of the force diagram, which will indicate the the kind of force acting on a particular member.

Step 6. Tabulate the results indicating magnitude and type of force for each member.

IMPORTANT EQUATIONS

(1) Two equilibrium equations for each **joint** :

$$\Sigma F_x = 0; \quad \text{and} \quad \Sigma F_y = 0$$

(2) Three equilibrium equations for rigid body (**whole truss** or **part**):

$$\Sigma F_x = 0;\ \Sigma F_y = 0; \ \text{and} \ \ \Sigma M_{ANY\ POINT} = 0$$

SOLVED EXAMPLES

1. Analyse the Scissors truss shown in Fig 5·4 by method of joints.

Step 1&2

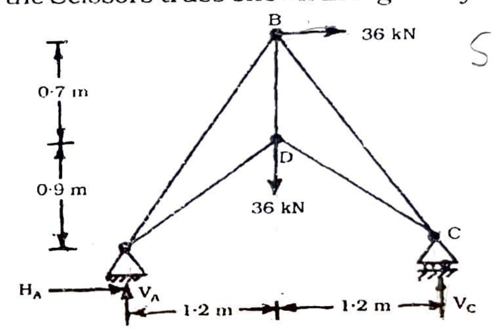


Fig. 5·4 Scissors Truss

The truss shown is of the variety of the simply supported type. In such a type, it is not possible to find a joint where only two unknowns exist. No doubt we have four joints which can give eight equations and are capable to find forces in five members and three reactions. But such a procedure needs a solution of eight simultaneous equations.

So for the simply supported type of trusses it is preferred *to find the external unknown reactions from the consideration of the free body of the whole truss and by using the three equilibrium equations.*

$\Sigma F_x = 0,\quad \overrightarrow{+ve},\quad H_A + 36 = 0$

$$\boxed{H_A = -36\ \text{kN}}$$

vertical

$\Sigma M_A = 0,\ \circlearrowright +ve,\quad 36 \times 1{\cdot}6 + 36 \times 1{\cdot}2 - V_c \times 2{\cdot}4 = 0$

$$\boxed{V_c = 42\ \text{kN}}$$

$\Sigma F_y = 0,\quad \uparrow +ve,\quad V_A + V_C - 36 = 0$

$$\boxed{V_A = -6\ \text{kN}}$$

Here note the reverse order of the equilibrium equations (Inplace of $\Sigma F_y = 0$ and $\Sigma M_A = 0$) selected. Instead of using $\Sigma F_y = 0$, even $\Sigma M_c = 0$ can also give the value of V_A in this case.

Step 3

Now redraw the truss showing correct directions of the reactions and assume tensile force in each member. **Note that arrowhead directing inward indicates the tensile force in the member.**

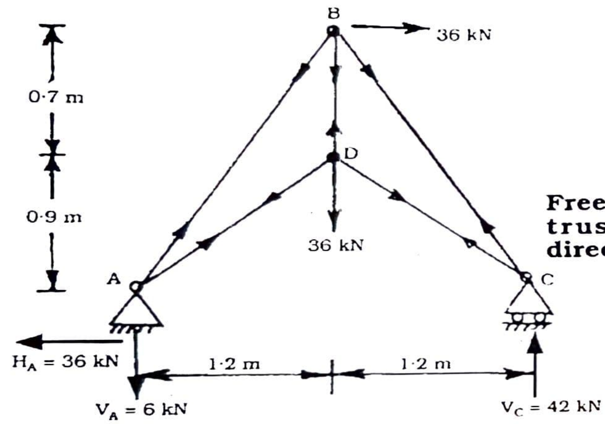


Fig 5.5.

Free - body diagram of truss with correct directions of reactions.

For the truss shown, we have two choices for starting an analysis of the joint, by either **joint A** or **joint C.** *Here, joint B is having three members hence three unknowns are to be determined with two equilibrium equations which is not possible. We can select either A or C, here joint C is selected.*

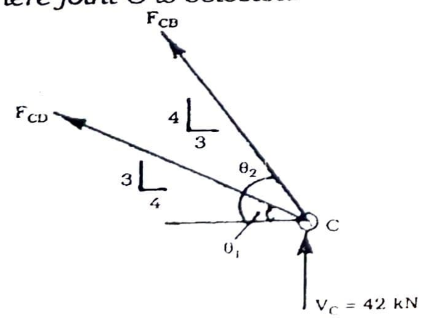


Fig 5·6

Free-body diagram of joint C.

Two equilibrium equations at joint C are

$$\Sigma F_x = 0, \quad \overrightarrow{+ve}, \quad -F_{CD} \cos\theta_1 - F_{CB}\cos\theta_2 = 0 \quad \ldots\ldots (1)$$

$$\Sigma F_y = 0, \quad \uparrow{+ve}, \quad F_{CD} \sin\theta_1 + F_{CB}\sin\theta_2 + 42 = 0 \quad \ldots\ldots (2)$$

Here $\theta_1 = 36{\cdot}87^\circ$ and $\theta_2 = 53{\cdot}13^\circ$

For which,
$$\cos\theta_1 = 0{\cdot}8$$
$$\cos\theta_2 = 0{\cdot}6$$
$$\sin\theta_1 = 0{\cdot}6$$
$$\sin\theta_2 = 0{\cdot}8$$

which can be obtained otherwise by observing the geometry at joint C.

$$\cos\theta_1 = \frac{1{\cdot}2}{1{\cdot}5} = 0{\cdot}8$$

$$\cos\theta_2 = \frac{1{\cdot}2}{2{\cdot}0} = 0{\cdot}6$$

$$\sin\theta_1 = \frac{0{\cdot}9}{1{\cdot}5} = 0{\cdot}6$$

$$\sin\theta_2 = \frac{1{\cdot}6}{2{\cdot}0} = 0{\cdot}8$$

In this problem we need the values of these two angles only.

From equation (1) and (2), we have

$$\boxed{F_{CD} = 90 \text{ kN} \quad \text{and} \quad F_{CB} = -120 \text{ kN}}$$

Here, F_{CB} is ***negative*** *which means that the assumption is wrong, the force is* ***compressive,*** *not tensile.*

Step4 *Now we should analyse the joint where only two unknown member forces exist.* We have three choices, i.e. all the remaining joints A, B and D. *The preferred and systematic way is to analyse the adjacent joint only i.e. we can select either joint B or joint D.* Here, we have selected joint B.

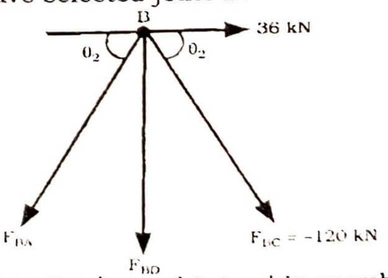


Fig 5·7

Free-body diagram of joint B.

Note that the force obtained in member BC is − 120 kN which indicates the compressive type of force in it, but **we will continue with the assumption of tensile type of force in the member** and **keeping the negative magnitude till the end of the problem.**

$$\Sigma F_x = 0, \quad \overrightarrow{+ve}, \quad -F_{BA}\cos\theta_2 + F_{BC}\cos\theta_2 + 36 = 0$$

$$\boxed{F_{BA} = -60 \text{ kN}}$$

$\Sigma F_y = 0$, $\uparrow$+ve, $\quad -F_{DA} \sin \theta_2 - F_{BD} - F_{BC} \sin \theta_2 = 0$

$$\boxed{F_{BD} = 144 \text{ kN}}$$

Now the unknown force remained is that of the member AD, which can be obtained by either analysing joint D or joint A. Here joint D is selected. For finding out force in the member AD, only one equation of equilibrium is sufficient, while the other can provide the numerical check in this case.

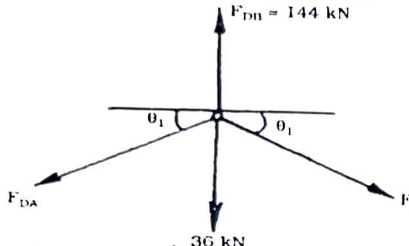


Fig. 5·8
Free-body diagram of joint D

$\Sigma F_x = 0$, $\overrightarrow{+ve}$, $\quad - F_{DA} \times \cos \theta_1 + F_{DC} \cos \theta_1 = 0$

$$\boxed{F_{DA} = 90 \text{ kN}}$$

$\Sigma F_y = 0$, $\uparrow$+ve. $\quad F_{DB} - F_{DA} \sin \theta_1 - F_{DC} \sin \theta_1 - 36$

$= 144 - 90 \times 0{\cdot}6 - 90 \times 0{\cdot}6 - 36 = 0$ **(check)**

So in turn, we have found out the forces in each member and yet an analysis of one joint A is possible, which can provide two additional numerical checks.

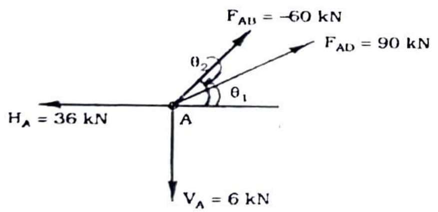


Fig. 5·9
Free-body diagram of joint A

$\Sigma F_x = 0$, $\overrightarrow{+ve}$, $\quad F_{AB} \cos \theta_2 + F_{AD} \cos \theta_1 - H_A$

$= -60 \times 0{\cdot}6 + 90 \times 0{\cdot}8 - 36$

$= 0$ **(check)**

$\Sigma F_y = 0$, $\uparrow$+ve, $\quad F_{AB} \sin \theta_2 + F_{AD} \sin \theta_1 - V_A$

$= -60 \times 0{\cdot}8 + 90 \times 0{\cdot}6 - 6$

$= 0$ **(check)**

After obtaining the force in each member and varifying the results numerically, we can redraw the truss showing therein the corrected force directions.

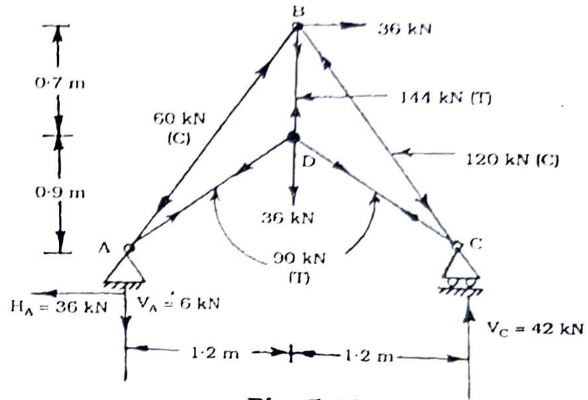


Fig. 5·10
Graphical presentation of results.

2. Analyse the truss shown in Fig 5·11.

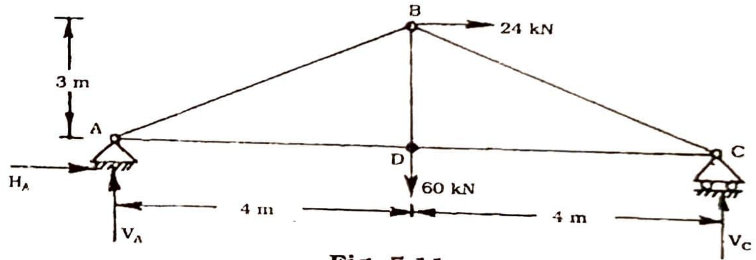


Fig. 5·11
Simply supported truss.

The numerical example selected is similar to the previous one. Following the steps systematically, we can obtain solution of the problem.

The three equations of equilibrium, *while* ***considering the free-body of the whole truss,*** give the three unknown reactions.

$\Sigma F_x = 0$, $\overrightarrow{+ve}$, $\quad 24 + H_A = 0$

$$\boxed{H_A = -24 \text{ kN}}$$

$\Sigma M_A = 0$, $\circlearrowright$+ve, $\quad 24 \times 3 + 60 \times 4 - V_c \times 8 = 0$

$$\boxed{V_c = 39 \text{ kN}}$$

$\Sigma F_y = 0$ $\quad \uparrow$+ve, $\quad V_A + V_c - 60 = 0$

$$\boxed{V_A = 21 \text{ kN}}$$

Now we can select a joint A or joint C for analysis, where only two unknown member forces exist. Here joint C is selected.

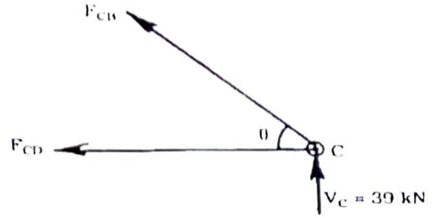


Fig. 5·12
Free-body diagram of joint C.

$\Sigma F_y = 0$, ↑+ve, $F_{CB} \sin\theta + V_c = 0$

$$\boxed{F_{CB} = -65 \text{ kN}}$$

$\Sigma F_x = 0$, $\overrightarrow{+ve}$, $-F_{CD} - F_{CB}\cos\theta = 0$

$$\boxed{F_{CD} = 52 \text{ kN}}$$

Now we have two choices of adjacent joints where only two unknown member forces exist, namely joint B and joint D. We have selected joint D.

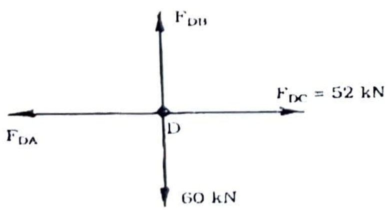


Fig 5·13
Free-body diagram of joint D

$\Sigma F_x = 0$, $\overrightarrow{+ve}$, $F_{DC} - F_{DA} = 0$

$$\boxed{F_{DA} = 52 \text{ kN}}$$

$\Sigma F_y = 0$, ↑+ve, $F_{DB} - 60 = 0$

$$\boxed{F_{DB} = 60 \text{ kN}}$$

Now only one unknown member force AB is remaining, which can be obtained by analysing joint B or joint A. One additional equilibrium equation will also provide numerical check.

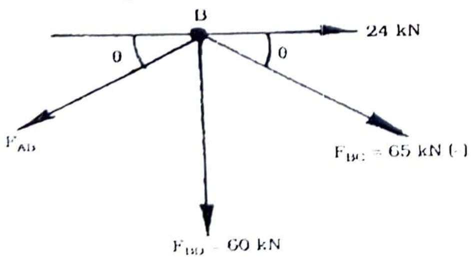


Fig. 5·14
Free-body diagram of joint B

$\Sigma F_x = 0$, $\overrightarrow{+ve}$, $-F_{BA}\cos\theta + 24 + F_{BC}\cos\theta = 0$

$$\boxed{F_{BA} = -35 \text{ kN}}$$

$\Sigma F_y = 0$, ↑+ve, $-F_{BA}\sin\theta - F_{BD} - F_{BC}\sin\theta$

$= -(-35) \times 0{\cdot}6 - 60 - (-65) \times 0{\cdot}6$

$= 0$ **(Check)**

Now, we have found out force in each and every member and yet one more joint A is available for analysis which can provide two numerical checks.

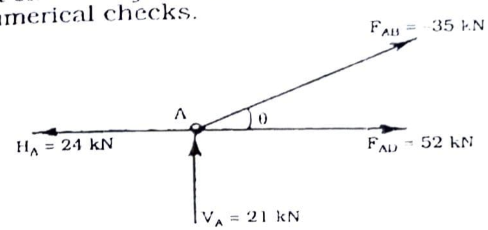


Fig. 5·15
Free-body diagram of joint A

$\Sigma F_x = 0$, $\overrightarrow{+ve}$, $F_{AD} + F_{AB}\cos\theta - H_A$

$= 52 + (-35) \times 0{\cdot}8 - 24$

$= 0$ **(check)**

$\Sigma F_y = 0$, ↑+ve, $F_{AB}\sin\theta + V_A$

$= (-35) \times 0{\cdot}6 + 21$

$= 0$ **(check)**

Now we can prepare the final diagram showing correct direction and magnitude of force in each member.

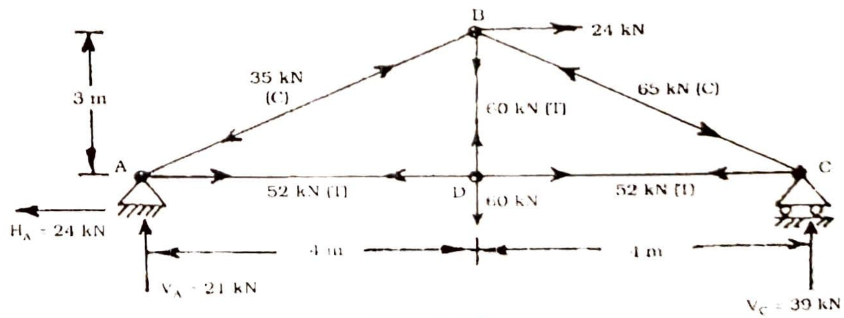


Fig. 5·16
Graphical representation of results.

3. Analyze the truss shown in Fig. 5·17 using method of joints.

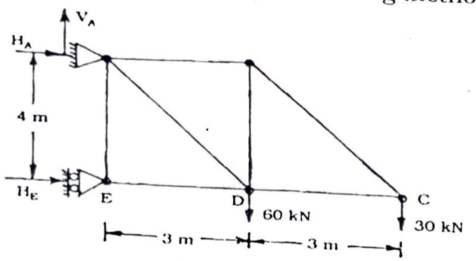


Fig 5·17

Cantilever truss

In cantilever type of truss, we can start the analyses from one joint to another and can solve the complete truss, as there exist a joint where only two unknowns are present (in this case joint C). Here, the **reactions can be determined by considering the equilibrium of joints only instead of considering the equilibrium of complete truss.** Thus using three equations of equilibrium we can find three reactions, which will provide numerical checks.

$\Sigma F_y = 0$, $\uparrow$+ve, $V_A - 60 - 30 = 0$

$\boxed{V_A = 90 \text{ kN}}$

$\Sigma M_A = 0$, $\circlearrowright$+ve, $30 \times 6 + 60 \times 3 - H_E \times 4 = 0$

$\boxed{H_E = 90 \text{ kN}}$

$\Sigma F_x = 0$, $\overrightarrow{+ve}$, $H_A + H_E = 0$

$\boxed{H_A = -90 \text{ kN}}$

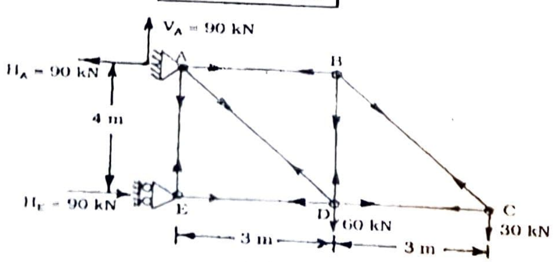


Fig. 5·18

After redrawing a truss showing correct directions of reactions and arrowheads for tensile force in each member, we can start analyses from free end where only two unknown forces exist.

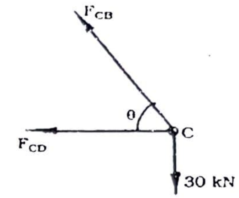


Fig. 5·19

Free-body diagram of joint C.

$\Sigma F_y = 0$, $\uparrow$+ve, $F_{CB} \sin \theta - 30 = 0$

$\boxed{F_{CB} = 37{\cdot}5 \text{ kN}}$

$\Sigma F_x = 0$, $\overrightarrow{+ve}$, $-F_{CD} - F_{CB} \cos \theta = 0$

$\boxed{F_{CD} = -22{\cdot}5 \text{ kN}}$

Now, we are forced to analyse joint B, where only two unknown forces exist and which is also adjacent to joint C.

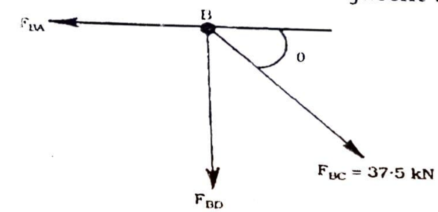


Fig. 5·20
Free-body diagram of joint B.

$\Sigma F_x = 0$, $\overrightarrow{+ve}$, $-F_{BA} + F_{BC} \cos \theta = 0$

$\boxed{F_{BA} = 22{\cdot}5 \text{ kN}}$

$\Sigma F_y = 0$, $\uparrow$+ve, $-F_{BD} - F_{BC} \sin \theta = 0$

$\boxed{F_{BD} = -30 \text{ kN}}$

Now adjacent joint where only two unknown member forces exist is joint D.

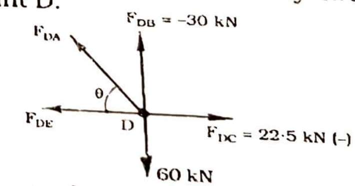


Fig 5·21
Free-body diagram of joint D

$\Sigma F_y = 0$, $\uparrow$+ve, $F_{DB} + F_{DA} \sin \theta - 60 = 0$

$\boxed{F_{DA} = 112{\cdot}5 \text{ kN}}$

$\Sigma F_x = 0$, $\overrightarrow{+ve}$, $F_{DC} - F_{DA} \cos \theta - F_{DE} = 0$

$\boxed{F_{DE} = -90 \text{ kN}}$

Now unknown member is AE, which can be obtained from either joint A or joint E, as both are adjacent to D.

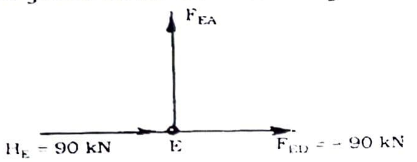


Fig 5·22
Free-body diagram of joint E

$\Sigma F_y = 0$, ↑+ve, $\boxed{F_{EA} = 0}$

$\Sigma F_x = 0$, $\overrightarrow{+ve}$, $F_{ED} + H_E$
$= -90 + 90$
$= 0$ **(check)**

Yet, we can analyze joint A, which will provide two additional checks.

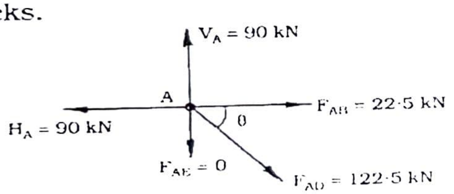


Fig 5·23
Free-body diagram of joint A

$\Sigma F_x = 0$, $\overrightarrow{+ve}$, $F_{AB} + F_{AD} \cos\theta - 90$
$= 22{\cdot}5 + 112{\cdot}5 \times 0{\cdot}6 - 90$
$= 0$ **(check)**

$\Sigma F_y = 0$, ↑+ve, $V_A - F_{AE} - F_{AD} \sin\theta$
$= 90 - 0 - 112{\cdot}5 \times 0{\cdot}8$
$= 0$ **(check)**

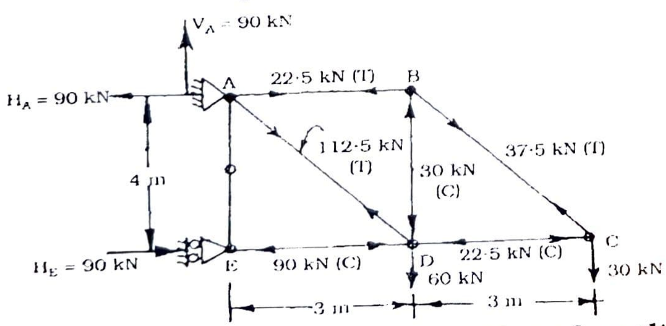


Fig. 5·24 Graphical representation of results

Here the member AE is having zero force. Though the member is to be provided in the truss for stability purpose. If it is not provided then the panel ADE will become weak and may be collapsed due to some inclined loading if applied.

The member AE having zero force is called Null member.

4. Analyze the truss shown in Fig 5·25

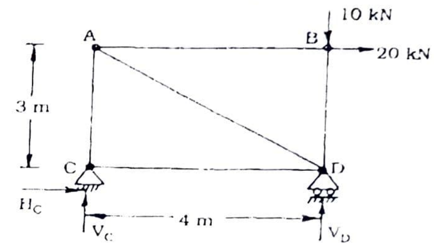


Fig 5·25

The truss shown is of the cantilever type which can be analyzed by taking joint after joint and starting from joint B, where only two unknown member forces exist. But use of three equations of equlibrium for finding out three unknown reactions will provide the numerical checks.

$\Sigma F_x = 0$, $\overrightarrow{+ve}$, $H_C + 20 = 0$
$\boxed{H_c = -20 \text{ kN}}$

$\Sigma M_c = 0$, ↻+ve, $10 \times 4 + 20 \times 3 - V_D \times 4 = 0$
$\boxed{V_D = 25 \text{ kN}}$

$\Sigma F_y = 0$, ↑+ve, $V_C + V_D - 10 = 0$
$\boxed{V_C = -15 \text{ kN}}$

Starting from free end B, where only two unknown member forces exist.

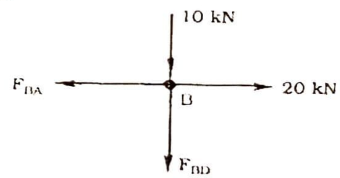


Fig 5·26
Free-body diagram of joint B

$\Sigma F_x = 0$, $\overrightarrow{+ve}$, $-F_{BA} + 20 = 0$
$\boxed{F_{BA} = 20 \text{ kN}}$

$\Sigma F_y = 0$, ↑+ve, $-10 - F_{BD} = 0$
$\boxed{F_{BD} = \ \ 10 \text{ kN}}$

Now the adjacent joints where only two unknown member forces exist are joint A and joint D. Here joint A is selected.

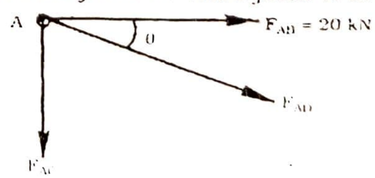


Fig 5·27
Free-body diagram of joint A.

$\Sigma F_x = 0, \quad \overrightarrow{+ve}, \quad F_{AB} + F_{AD} \cos\theta = 0$

$$\boxed{F_{AD} = -25 \text{ kN}}$$

$\Sigma F_y = 0, \quad \uparrow +ve, \quad -F_{AC} - F_{AD} \sin\theta = 0$

$$\boxed{F_{AC} = 15 \text{ kN}}$$

Now the unknown member is only CD, which can be obtained by analysing an adjacent joint C or joint D. In addition to this we will be having one more numerical check. Here joint C is selected.

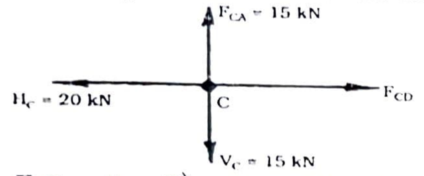


Fig 5·28 Free-body diagram of joint C

$\Sigma F_x = 0, \quad \overrightarrow{+ve}, \quad F_{CD} - H_c = 0$

$$\boxed{F_{CD} = 20 \text{ kN}}$$

$\Sigma F_y = 0, \quad \uparrow +ve, \quad F_{CA} - V_c$

$= 15 - 15$

$= 0$ **(check)**

Now, we have found out the force in each and every member and yet one more joint is available for analysis, namely joint D, which will provide two additional numerical checks.

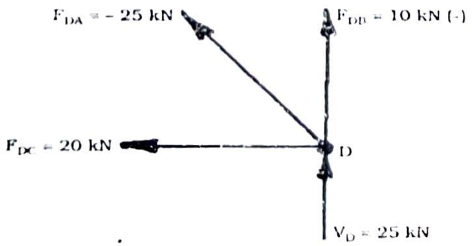


Fig 5·29 Free-body diagram of joint D.

$\Sigma F_x = 0, \quad \overrightarrow{+ve}, \quad -F_{DC} - F_{DA} \cos\theta$

$= -20 - (-25) \times 0{\cdot}8$

$= 0$ **(check)**

$\Sigma F_y = 0, \quad \uparrow +ve, \quad F_{DB} + F_{DA} \sin\theta + V_D$

$= (-10) + (-25) \times 0{\cdot}6 + 25$

$= 0$ **(check)**

Finally, we should redraw the truss showing correct arrow directions for member forces.

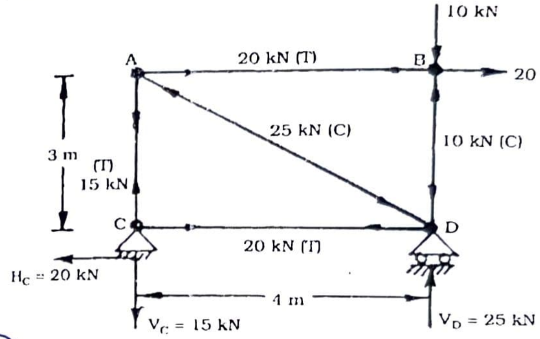


Fig 5·30 Graphical representation of results.

5. Analyze the simply supported truss shown in Fig 5·31.

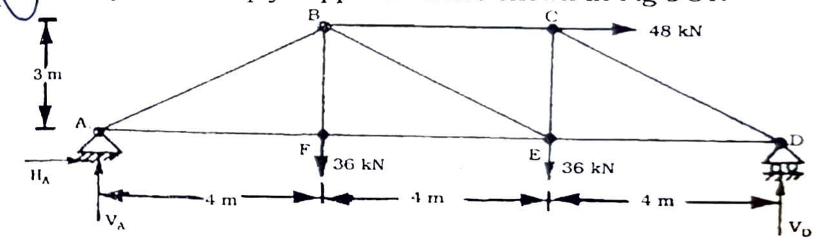


Fig 5·31

For the whole truss,

$\Sigma F_x = 0, \quad \overrightarrow{+ve}, \quad H_A + 48 = 0$

$$\boxed{H_A = -48 \text{ kN}}$$

$\Sigma M_A = 0, \quad \circlearrowright +ve, \quad 48 \times 3 + 36 \times 4 + 36 \times 8 - V_D \times 12 = 0$

$$\boxed{V_D = 48 \text{ kN}}$$

$\Sigma F_y = 0, \quad \uparrow +ve, \quad V_A + V_D - 36 - 36 = 0$

$$\boxed{V_A = 24 \text{ kN}}$$

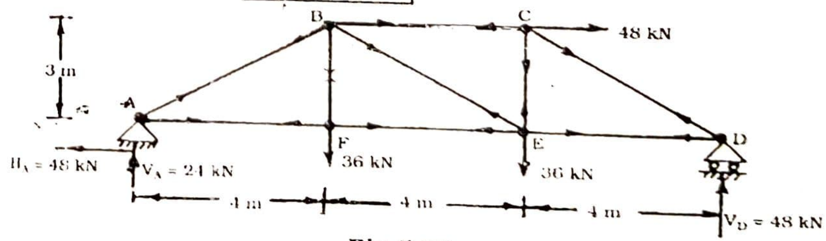


Fig 5·32

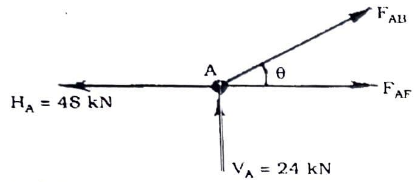


Fig 5·33 Free-body diagram of joint A

Joint A :

$\Sigma F_y = 0$, ↑+ve, $F_{AB} \sin\theta + V_A = 0$

$$\boxed{F_{AB} = -40 \text{ kN}}$$

$\Sigma F_x = 0$, $\overrightarrow{+ve}$, $F_{AF} + F_{AB}\cos\theta - H_A = 0$

$$\boxed{F_{AF} = 80 \text{ kN}}$$

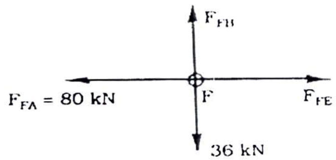


Fig 5·34 Free-body diagram of joint F

Joint F :

$\Sigma F_x = 0$, $\overrightarrow{+ve}$, $F_{FE} - F_{FA} = 0$

$$\boxed{F_{FE} = 80 \text{ kN}}$$

$\Sigma F_y = 0$, ↑+ve, $F_{FB} - 36 = 0$

$$\boxed{F_{FB} = 36 \text{ kN}}$$

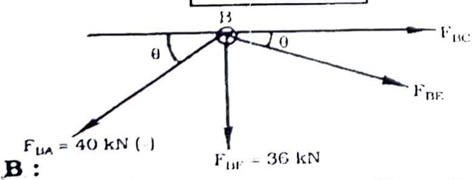


Fig 5·35 Free-body diagram of joint B

Joint B :

$\Sigma F_y = 0$, ↑+ve, $-F_{BE}\sin\theta - F_{BF} - F_{BA}\sin\theta = 0$

$$\boxed{F_{BE} = -20 \text{ kN}}$$

$\Sigma F_x = 0$, $\overrightarrow{+ve}$, $F_{BC} + F_{BE}\cos\theta - F_{BA}\cos\theta = 0$

$$\boxed{F_{BC} = -16 \text{ kN}}$$

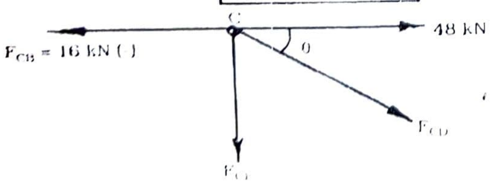


Fig 5·36 Free-body diagram of joint C.

Joint C :

$\Sigma F_x = 0$, $\overrightarrow{+ve}$, $48 + F_{CD}\cos\theta - F_{CB} = 0$

$$\boxed{F_{CD} = -80 \text{ kN}}$$

$\Sigma F_y = 0$, ↑+ve, $-F_{CD}\sin\theta - F_{CE} = 0$

$$\boxed{F_{CE} = 48 \text{ kN}}$$

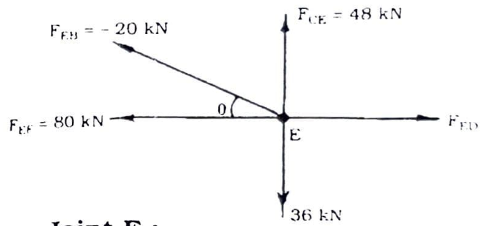


Fig 5·37 Free-body diagram of joint E

Joint E :

$\Sigma F_x = 0$, $\overrightarrow{+ve}$, $F_{ED} - F_{EB}\cos\theta - F_{EF} = 0$

$$\boxed{F_{ED} = 64 \text{ kN}}$$

$\Sigma F_y = 0$, ↑+ve, $F_{EC} + F_{EB}\sin\theta - 36$

$= 48 + (-20) \times 0{\cdot}6 - 36$

$= 0$ **(check)**

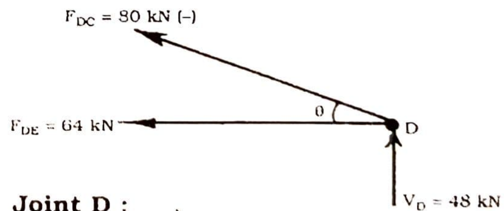


Fig 5·38 Free-body diagram of joint D

Joint D :

$\Sigma F_x = 0$, $\overrightarrow{+ve}$, $-F_{DE} - F_{DC}\cos\theta$

$= -64 - (-80) \times 0{\cdot}8$

$= 0$ **(check)**

$\Sigma F_y = 0$, ↑+ve, $F_{DC}\sin\theta + 48$

$= (-80) \times 0{\cdot}6 + 48$

$= 0$ **(check)**

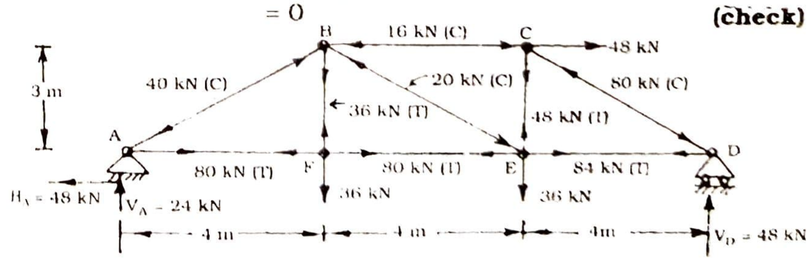


Fig. 5·39 Graphical representation of results.

3. Analyse the cantilever truss shown in Fig 5·40.

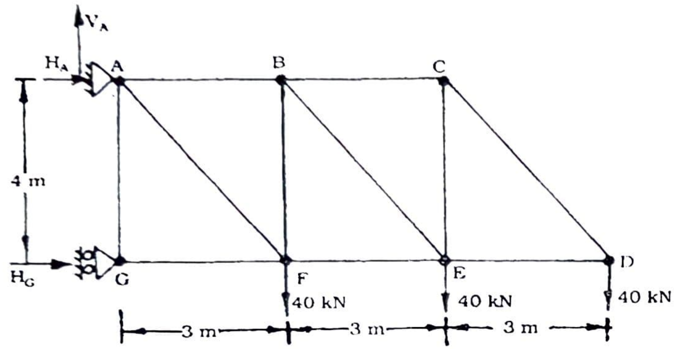


Fig 5·40

For a cantilever type of truss, it is possible to locate a joint having only two unknown member forces, which will be usually at free end and one can start analysis by taking joint after joint to find the solution. In case, if three equilibrium equations are used to find the three unknown reactions by considering the whole truss as a rigid body then analysis of truss by taking joint after joint can provide three numerical checks. Here also the same procudure is adopted.

For the whole truss.

$\Sigma F_y = 0$, $\uparrow$+ve, $V_A - 40 - 40 - 40 = 0$

$\boxed{V_A = 120 \text{ kN}}$

$\Sigma M_A = 0$, $\circlearrowright$+ve, $40 \times 3 + 40 \times 8 + 40 \times 12 - H_G \times 4 = 0$

$\boxed{H_G = 180 \text{ kN}}$

$\Sigma F_x = 0$, $\overrightarrow{+ve}$, $H_A + H_G = 0$

$\boxed{H_A = -180 \text{ kN}}$

After carefully observing the truss the reactions / loadings acting on it and the individual joints, one can conclude that member AG is a null member having zero force. This can be concluded from the observation of Joint G where there is no vertical component of force or reaction acting except force in AG.

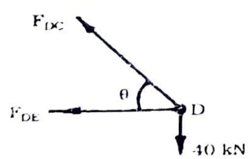


Fig 5·41
Free body diagram of joint D

Joint D :

$\Sigma F_y = 0$, $\uparrow$+ve, $F_{DC} \sin \theta - 40 = 0$

$\boxed{F_{DC} = 50 \text{ kN}}$

$\Sigma F_x = 0$, $\overrightarrow{+ve}$, $-F_{DE} - F_{DC} \cos \theta = 0$

$\boxed{F_{DE} = -30 \text{ kN}}$

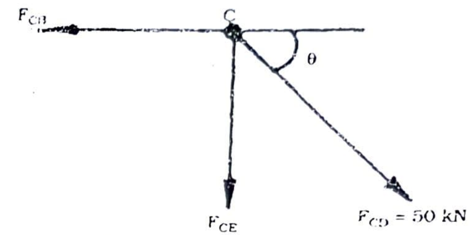


Fig 5·42
Free-body diagram of joint C

Joint C :

$\Sigma F_x = 0$, $\overrightarrow{+ve}$, $-F_{CB} + F_{CD} \cos \theta = 0$

$\boxed{F_{CB} = 30 \text{ kN}}$

$\Sigma F_y = 0$, $\uparrow$+ve, $-F_{CE} - F_{CD} \sin \theta = 0$

$\boxed{F_{CE} = -40 \text{ kN}}$

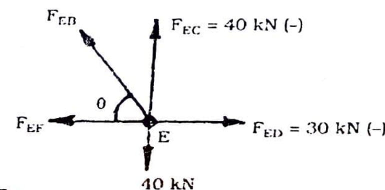


Fig 5·43
Free-body diagram of joint E

Joint E :

$\Sigma F_y = 0$, $\uparrow$+ve, $F_{EC} + F_{EB} \sin \theta - 40 = 0$

$\boxed{F_{EB} = 100 \text{ kN}}$

$\Sigma F_x = 0$, $\overrightarrow{+ve}$, $-F_{EF} - F_{EB} \cos \theta + F_{ED} = 0$

$\boxed{F_{EF} = -90 \text{ kN}}$

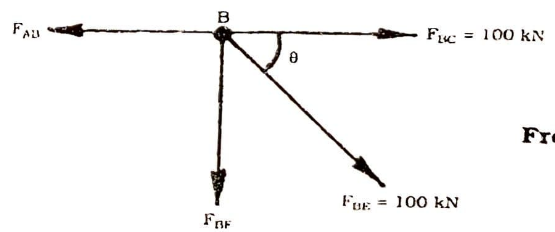


Fig 5·44
Free-body diagram of joint B.

Joint B :

$\Sigma F_x = 0$, $\overrightarrow{+ve}$, $-F_{BA} + F_{BC} + F_{BE} \cos \theta = 0$

$\boxed{F_{BA} = 90 \text{ kN}}$

$\Sigma F_y = 0$, $\uparrow$+ve, $-F_{BF} - F_{BE} \sin \theta = 0$

$\boxed{F_{BF} = -80 \text{ kN}}$

$\Sigma F_y = 0$, $\uparrow$+ve, $V_G + V_H = 0$

$\boxed{V_G = -320 \text{ kN}}$

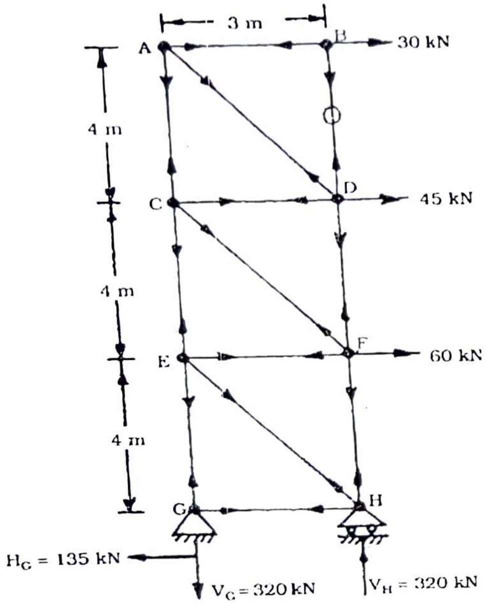


Fig 5.50

Here member BD is a null member (zero force member), by observing the joint B.

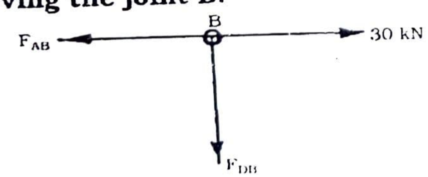


Fig 5.51
Free-body diagram of joint B.

Joint B :

$\Sigma F_x = 0$, $\overrightarrow{+ve}$, $-F_{BA} + 30 = 0$

$\boxed{F_{BA} = 30 \text{ kN}}$

$\Sigma F_y = 0$, $\uparrow$+ve, $-F_{BD} = 0$

$\boxed{F_{BD} = 0}$

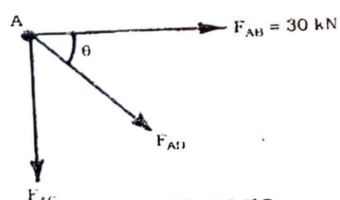


Fig 5.52
Free-body diagram of joint A.

Joint A :

$\Sigma F_x = 0$, $\overrightarrow{+ve}$, $F_{AB} + F_{AD} \cos\theta = 0$

$\boxed{F_{AD} = -50 \text{ kN}}$

$\Sigma F_y = 0$, $\uparrow$+ve, $-F_{AC} - F_{AD} \sin\theta = 0$

$\boxed{F_{AC} = 40 \text{ kN}}$

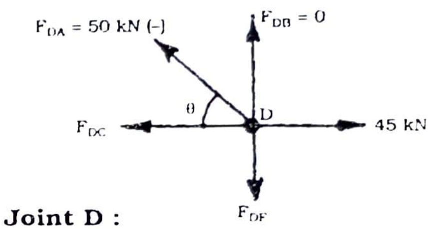


Fig 5.53
Free-body diagram of joint D.

Joint D :

$\Sigma F_x = 0$, $\overrightarrow{+ve}$, $-F_{DC} - F_{DA} \cos\theta + 45 = 0$

$\boxed{F_{DC} = 75 \text{ kN}}$

$\Sigma F_y = 0$, $\uparrow$+ve, $F_{DB} + F_{DA} \sin\theta - F_{DF} = 0$

$\boxed{F_{DF} = -40 \text{ kN}}$

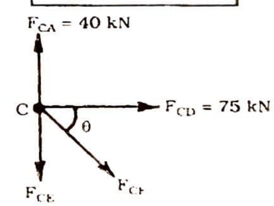


Fig 5.54
Free-body diagram of joint C

Joint C :

$\Sigma F_x = 0$, $\overrightarrow{+ve}$, $F_{CD} + F_{CF} \cos\theta = 0$

$\boxed{F_{CF} = -125 \text{ kN}}$

$\Sigma F_y = 0$, $\uparrow$+ve, $F_{CA} - F_{CE} - F_{CF} \sin\theta = 0$

$\boxed{F_{CE} = 140 \text{ kN}}$

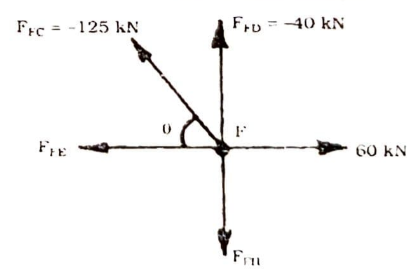


Fig 5.55
Free-body diagram of joint F

Joint F :

$\Sigma F_x = 0$, $\overrightarrow{+ve}$, $-F_{FE} - F_{FC}\cos\theta + 6 = 0$

$$\boxed{F_{FE} = 135 \text{ kN}}$$

$\Sigma F_y = 0$, $\uparrow$+ve, $F_{FD} + F_{FC}\sin\theta - F_{FH} = 0$

$$\boxed{F_{FH} = -140 \text{ kN}}$$

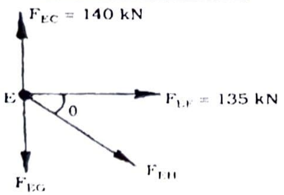


Fig 5.56 Free-body diagram of joint E

Joint E :

$\Sigma F_x = 0$, $\overrightarrow{+ve}$, $F_{EF} + F_{EH}\cos\theta = 0$

$$\boxed{E_{EH} = -225 \text{ kN}}$$

$\Sigma F_y = 0$, $\uparrow$+ve, $F_{EC} - F_{EH}\sin\theta - F_{EG} = 0$

$$\boxed{F_{EG} = 320 \text{ kN}}$$

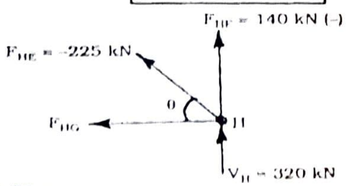


Fig 5.57 Free-body diagram of joint H

Joint H :

$\Sigma F_x = 0$, $\overrightarrow{+ve}$, $-F_{HE}\cos\theta - F_{HG} = 0$

$$\boxed{F_{HG} = 135 \text{ kN}}$$

$\Sigma F_y = 0$, $\uparrow$+ve, $F_{HF} + F_{HE}\sin\theta + V_H$

$= (-140) + (-225) \times 0.8 + 320$

$= 0$ **(check)**

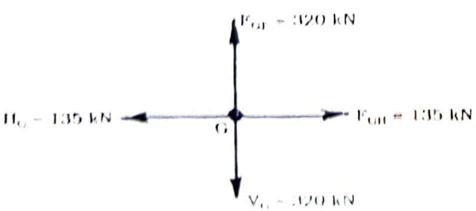


Fig 5.58 Free-body diagram of joint G.

Joint G :

$\Sigma F_x = 0$, $\overrightarrow{+ve}$, $-H_G + F_{GH}$

$= -135 + 135$

$= 0$ **(check)**

$\Sigma F_y = 0$, $\uparrow$+ve, $F_{GE} - V_G$

$= 320 - 320$

$= 0$ **(check)**

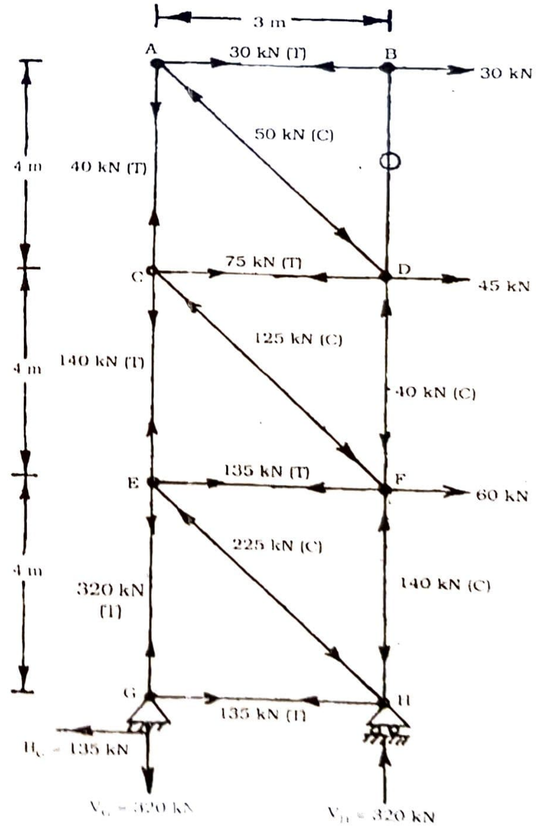


Fig 5.59 Graphical representation of results.

8. Find the forces in members **BC, BE and FE** of the truss shown in **example 5** by method of sections.

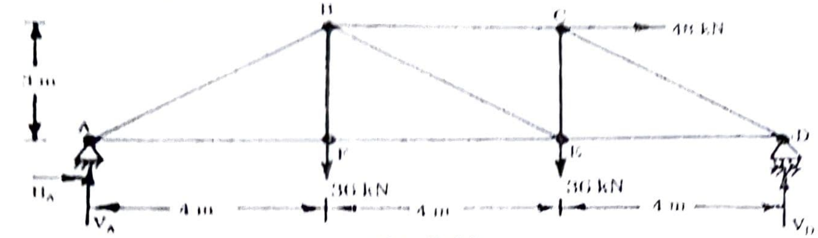


Fig. 5·60

For a simply supported truss it is a must to find first the reactions, which can be obtained as shown in the method of joints using three equations of equilibrium of the whole truss.

For the whole truss :

$\Sigma F_x = 0$, $\overrightarrow{+}$ve, $\quad H_A + 48 = 0$

$$\boxed{H_A = -48 \text{ kN}}$$

$\Sigma M_A = 0$, ↻+ve, $\quad 48 \times 3 + 36 \times 4 + 36 \times 8 - V_D \times 12 = 0$

$$\boxed{V_D = 48 \text{ kN}}$$

$\Sigma F_y = 0$, ↑+ve, $\quad V_A + V_D - 36 - 36 = 0$

$$\boxed{V_A = 24 \text{ kN}}$$

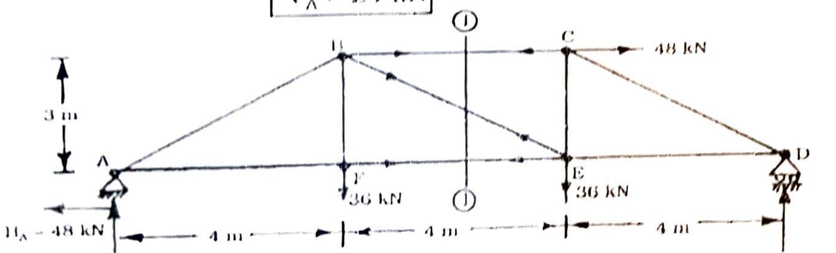


Fig 5·61

In the above diagram, the corrected directions of reactions are shown. An **imaginary line 1-1 cuts the truss in two parts** and also *passes through the members* ***BC, BE and FE*** *in which we need to find the forces.* Here also the tensile forces are assumed first and are shown with arrowheads directing inward on members. Now we are **free to select an either part of the truss** for analysis. We have selected the left part of the truss.

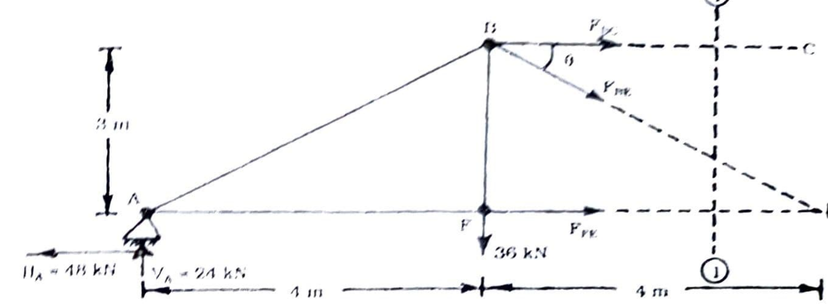


Fig. 5·62

Free body diagram of the left part of the section 1-1.

Equilibrium of left-part of section 1.1 :

$\Sigma F_x = 0$, $\overrightarrow{+}$ve, $\quad F_{BC} + F_{BE} \cos\theta + F_{FE} - 48 = 0$ .... **(1)**

$\Sigma F_y = 0$, ↑+ve, $\quad V_A - F_{BE} \sin\theta - 36 = 0$

$$\boxed{F_{BE} = -20 \text{ kN}}$$

$\Sigma M_B = 0$, ↻+ve, $\quad -F_{FE} \times 3 + V_A \times 4 + H_A \times 3 = 0$

$$\boxed{F_{FE} = 80 \text{ kN}}$$

Now from the equation (1), we have

$$\boxed{F_{BC} = -16 \text{ kN}}$$

Here the value of F_{BC} can be obtained otherwise also by using one more moment equation at a point which eliminates other two unknowns, i.e. say by writing moment equation about joint E.

$\Sigma M_E = 0$, ↻+ve, $\quad F_{BC} \times 3 + V_A \times 8 - 36 \times 4 = 0$

$$\boxed{F_{BC} = -16 \text{ kN}}$$

The member forces confirm the results determined in example 5.

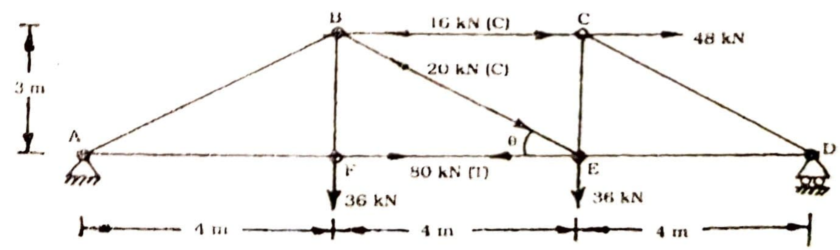


Fig. 5·63

By following the same procedure and **selecting even the right part of the truss,** one can obtain the same results.

11

Q. Find the forces in members **AB, BF and FE** of the cantilever truss shown in **example 6** by method of sections.

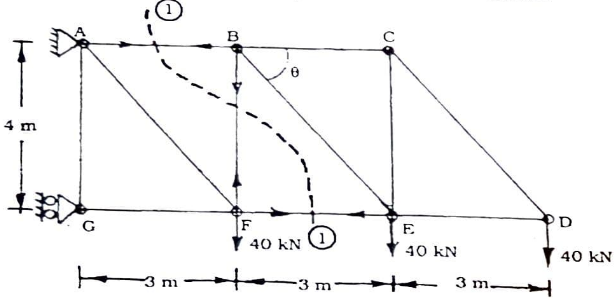


Fig 5.64

For a cantilever type of truss, there is no need to find even the port reactions. We can draw an imaginary cutting line and then ct a free end for the analysis.

The selected ***imaginary line 1-1*** *passes through the* ***members BF*** *and* ***FE*** *in which we need to find the forces.* Consider the -body diagram of the right part of the section line.

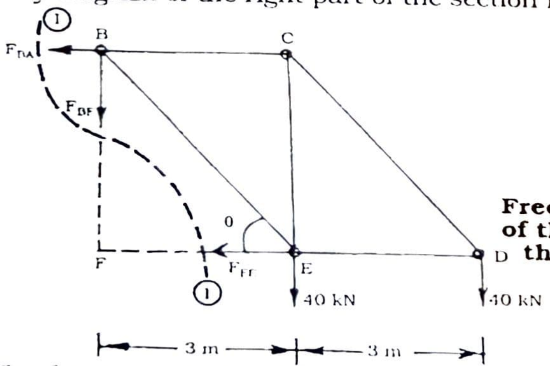


Fig 5.65 Free-body diagram of the right part of the section 1-1.

The force directions shown on the members indicate tensile :es. Now three equations of equilibrium can be applied to obtain three unknown forces.

Condition $\Sigma F_x = 0$ involve two unknowns, while $\Sigma F_y = 0$ is »able to give the force in member BF.

For right part of section 1–1 :

$\Sigma F_y = 0$, $\uparrow$+ve, $\quad -F_{BF} - 40 - 40 = 0$

$$\boxed{F_{BF} = -80 \text{ kN}}$$

Now $\Sigma M_B = 0$ can be used to obtain the force in member FE. *Through point B, both F_{BA} and F_{BF} are passing and hence will not induce any moment.*

$\Sigma M_B = 0$, ↻+ve, $\quad F_{FE} \times 4 + 40 \times 3 + 40 \times 6 = 0$

$$\boxed{F_{FE} = -90 \text{ kN}}$$

Now for obtaining the force in member BA, we can use $\Sigma F_x = 0$ or the moment equation. *For using $\Sigma M_{\text{ANY POINT}} = 0$, we need to locate a point which can eliminate the other two forces. Such point available is a point F, through which F_{BF} and F_{FE} are passing hence they will not create any moment.*

$\Sigma M_F = 0$, ↻+ve, $\quad -F_{BA} \times 4 + 40 \times 3 + 40 \times 6 = 0$

$$\boxed{F_{BA} = 90 \text{ kN}}$$

The same can be obtained even by using $\Sigma F_x = 0$

$\Sigma F_x = 0$, $\overset{\rightarrow}{+ve}$, $\quad -F_{BA} - F_{FE} = 0$

$$\boxed{F_{BA} = 90 \text{ kN}}$$

OR otherwise $\Sigma F_x = 0$ can be used for the numerical verification.

$\Sigma F_x = 0$, $\overset{\rightarrow}{+ve}$, $\quad -F_{BA} - F_{FE}$

$= -90 - (-90)$

$= 0$

(check)

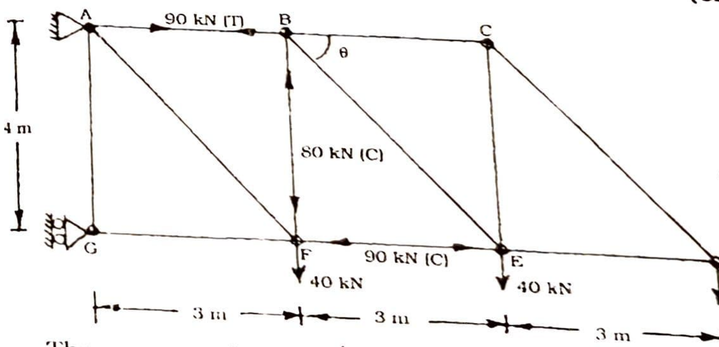


Fig 5.66

The same results were obtained also in example 6 by using the method of joints.

Now, for considering the left part of the section i.e. the supported part, we need to obtain support reactions first, before applying the equilibrium equations.

10. Find the forces in members **CE, CF, DF, EF and FH** of the vertical cantilever truss shown in **example 7** by method of sections.

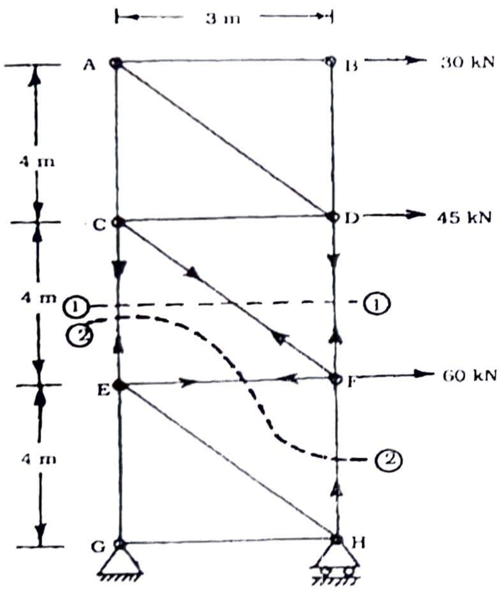


Fig 5·67
Vertical cantilever truss.

The truss being of cantilever type, there is no need to find the reactions at supports, provided the free upper part of the truss is selected for the analysis. Here the section 1-1 will give the forces in members CE, CF and DF whereas the section 2-2 will give the forces in remaining members EF and FH. Tensile forces are assumed initially in all the members.

Consider the upper free part of truss above the section 1-1.

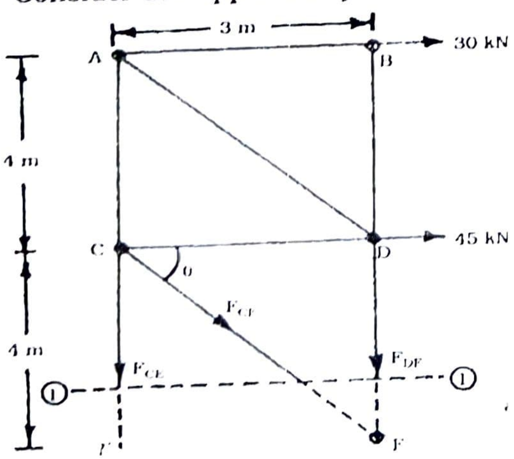


Fig. 5·68
Free-body diagram of upper part of section 1·1.

For upper part of section 1.1 :

$\Sigma F_x = 0$, $\overrightarrow{+ve}$, $F_{CF} \cos \theta + 30 + 45 = 0$

$$\boxed{F_{CF} = -125 \text{ kN}}$$

$\Sigma M_c = 0$, ↻+ve, $F_{DF} \times 3 + 30 \times 4 = 0$

$$\boxed{F_{DF} = -40 \text{ kN}}$$

$\Sigma M_F = 0$, ↻+ve, $30 \times 8 + 45 \times 4 - F_{CE} \times 3 = 0$

$$\boxed{F_{CE} = 140 \text{ kN}}$$

$\Sigma F_y = 0$ can be used for a numerical verification.

$\Sigma F_y = 0$, ↑+ve, $-F_{CE} - F_{CF} \sin \theta - F_{DF}$

$= -140 - (-125) \times 0.8 - (-40)$

$= 0$ **(check)**

Now, the upper free part of the section 2–2 can be considered for finding the force in members EF and FH.

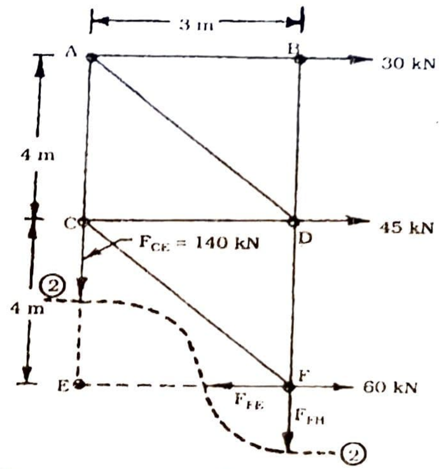


Fig 5·69
Free - body diagram of upper part of section 2·2.

For upper part of section 2–2 :

$\Sigma F_x = 0$, $\overrightarrow{+ve}$, $-F_{FE} + 30 + 45 + 60 = 0$

$$\boxed{F_{FE} = 135 \text{ kN}}$$

$\Sigma M_E = 0$, ↻+ve, $30 \times 8 + 45 \times 4 + F_{FH} \times 3 = 0$

$$\boxed{F_{FH} = -140 \text{ kN}}$$

$\Sigma F_y = 0$ can be used for the numerical verification.

$\Sigma F_y = 0$, ↑+ve, $-F_{CE} - F_{FH}$

$= -140 - (-140)$

$= 0$ **(check)**

1) Determine the forces in all members by method of joints and also check the forces in members **AB**, **FB** and **FE** by method of sections.

AB = 5m, $\cos\theta = \dfrac{8}{10}$

$\sin\theta = \dfrac{6}{10}$

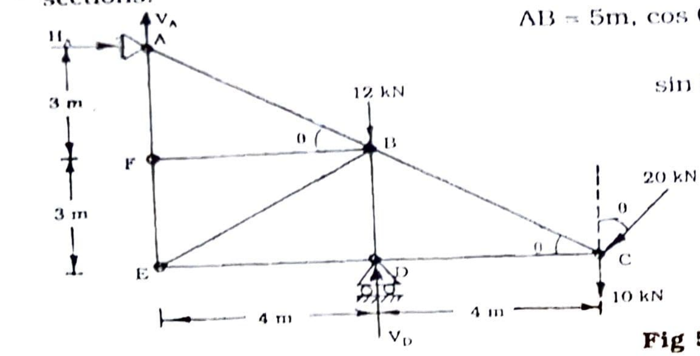


Fig 5·70

For the whole truss :

Reactions : $\Sigma F_x = 0$, $\overrightarrow{+ve}$, $H_A - 20\sin\theta = 0$

$\therefore H_A = 20\left(\dfrac{6}{10}\right)$

$\therefore \boxed{H_A = 12\text{ kN }(\rightarrow)}$

$\Sigma M_A = 0$, ↻ +ve

$12\times4 + 20\sin\theta\times6 + 20\cos\theta\times8 + 10\times8 - V_D\times4 = 0$

$\boxed{V_D = 82\text{ kN}\uparrow}$

$\Sigma F_y = 0$, ↑ +ve

$V_A - 12 - 20\cos\theta - 10 + V_D = 0$

$\boxed{V_A = -44\text{ kN}\downarrow}$

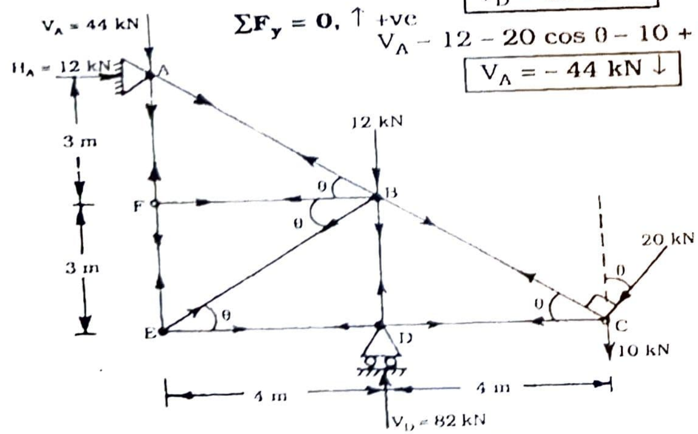


Fig 5·71
Free-body diagram of truss with reactions.

We may check at a glance all the joints to decide about the *zero force member* which will ultimately save our labour.

Joint F : At this joint, there is no external force is acting and the member FB is perpendicular to the members AF and FE. Hence the force in FB will be zero.

$\Sigma F_x = 0$, $\overrightarrow{+ve}$, $\boxed{F_{FB} = 0}$

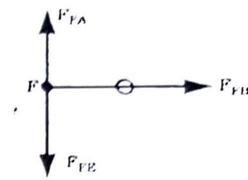


Fig 5·72
Free-body diagram of joint F.

$\Sigma F_y = 0$, ↑+ve, $F_{FA} - F_{FE} = 0$

$\boxed{F_{FA} = F_{F}}$

Joint A :

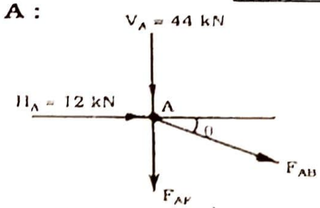


Fig 5·73
Free-body diagram of joint A

$\Sigma F_x = 0$, $\overrightarrow{+ve}$, $12 + F_{AB}\cos\theta = 0$

$\boxed{F_{AB} = -15\text{ kN}}$

$\Sigma F_y = 0$, ↑+ve, $-44 - F_{AF} - (-15\sin\theta) = 0$

$\therefore \boxed{F_{AF} = -35\text{ kN}}$

But, $F_{FA} = F_{FE}$

$\therefore \boxed{F_{FE} = -35\text{ kN}}$

Joint E :

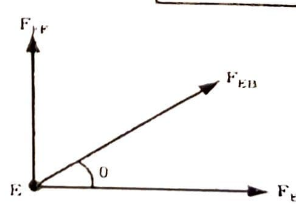


Fig 5·74
Free-body diagram of joint E.

$\Sigma F_y = 0$, ↑+ve, $F_{FE} + F_{EB}\sin\theta = 0$

$\boxed{F_{EB} = 58{\cdot}33\text{ kN}}$

$\Sigma F_x = 0$, $\overrightarrow{+ve}$, $F_{ED} + F_{EB}\cos\theta = 0$

$\boxed{F_{ED} = -46{\cdot}66\text{ kN}}$

Joint D :

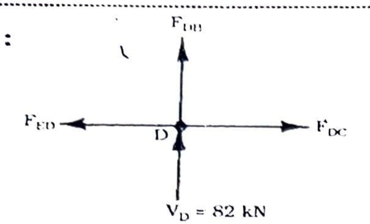


Fig 5·75 Free-body diagram of joint D.

$\Sigma F_x = 0$, $\overrightarrow{+ve}$, $-F_{ED} + F_{DC} = 0$

$$\boxed{F_{DC} = -46{\cdot}66 \text{ kN}}$$

$\Sigma F_y = 0$, $\uparrow$+ve, $F_{DB} + V_D = 0$

$$\boxed{F_{DB} = -82 \text{ kN}}$$

Joint B :

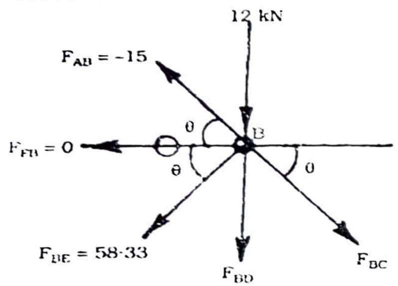


Fig 5·76 Free-body diagram of joint B.

$\Sigma F_x = 0$, $\overrightarrow{+ve}$, $-F_{BA} \cos\theta - F_{BE} \cos\theta + F_{BC} \cos\theta = 0$

$-(-15\times0{\cdot}8) - 58{\cdot}33\ (0{\cdot}8) + 0{\cdot}8\ F_{BC} = 0$

$$\boxed{F_{BC} = 43{\cdot}33 \text{ kN}}$$

$\Sigma F_y = 0$, $\uparrow$+ve, $-12 + F_{BA} \sin\theta - F_{BE} \sin\theta - F_{BD} - F_{BC} \sin\theta = 0$

$-12 + (-15\times0{\cdot}6) - 58{\cdot}33\ (0{\cdot}6) - (-82) - 0{\cdot}6\ F_{BC} = 0$

$$\boxed{F_{BC} = 43{\cdot}33 \text{ kN}} \quad \textbf{(check)}$$

Joint C :

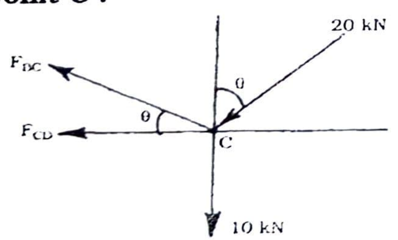


Fig 5·77 Free-body diagram of joint C

$\Sigma F_x = 0$, $\overrightarrow{+ve}$, $-F_{CD} - F_{BC} \cos\theta - 20 \sin\theta = 0$

$-(-46{\cdot}66) - 43{\cdot}33 \times 0{\cdot}8 - 20 \times 0{\cdot}6 = 0$

$0 = 0$ **(check)**

$\Sigma F_y = 0$, $\uparrow$+ve, $-10 - 20 \cos\theta + F_{BC} \sin\theta = 0$

$-10 - 20\ (0{\cdot}8) + 43{\cdot}33\ (0{\cdot}6) = 0$

$0 = 0$ **(check)**

Now, check the forces in members AB, FB and FE by ***method of sections.***

Consider the section 1–1 passing from all these three members.

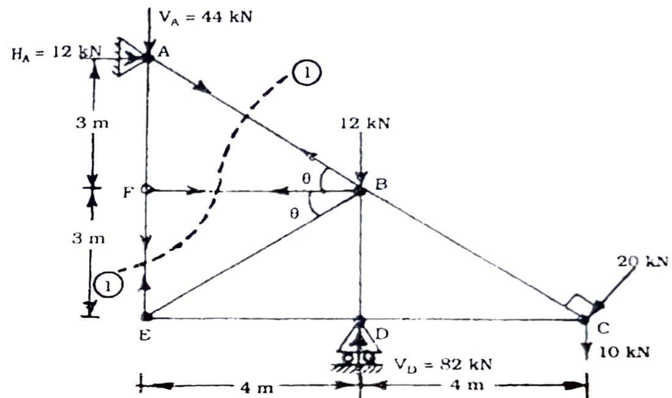


Fig 5·78

We may consider *left* OR *right* part of the section 1–1 under equilibrium.

Here, we consider the left part of the section 1–1 under equilibrium. In this case, F_{AB}, F_{FB} and F_{FE} will act as ***external forces*** as shown below.

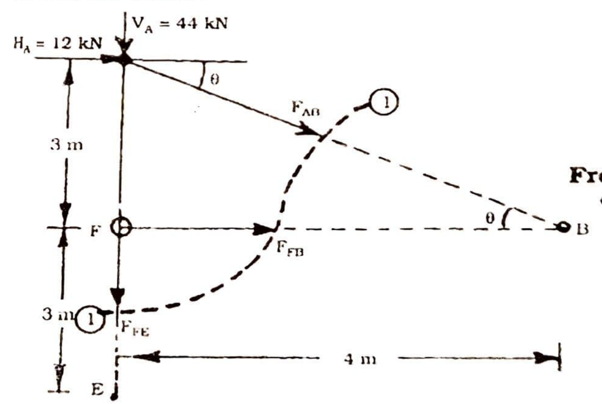


Fig 5·79 Free-body diagram of Left part of section 1–1.

Now, if we take moment about B, we can find out force in member FE directly as F_{AB} and F_{FB} are passing from B which do not create any moment.

For the left part of section 1.1 :

$\Sigma M_B = 0$, ↻ +ve,

$$-F_{FE} \times 4 - V_A \times 4 + H_A \times 3 = 0$$

$$-F_{FE} \times 4 - 44 \times 4 + 12 \times 3 = 0$$

$$\therefore \quad \boxed{F_{FE} = -35 \text{ kN}} \qquad \textbf{(check)}$$

and, moment about F will give the force in member AB as F_{FB} and F_{FE} are passing from F which do not create any mement.

$\Sigma M_F = 0$, ↻ +ve,

$$H_A \times 3 + F_{AB} \cos\theta \times 3 = 0$$

$$12 \times 3 + F_{AB}\,(0{\cdot}8) \times 3 = 0$$

$$\therefore \quad \boxed{F_{AB} = -15 \text{ kN}} \qquad \textbf{(check)}$$

Finally moment about A will give force in member FB as F_{AB} and F_{FE} are passing from A which do not create any moment.

$\Sigma M_A = 0$, ↻ +ve,

$$-F_{FB} \times 3 = 0$$

$$\therefore \quad \boxed{F_{FB} = 0} \qquad \textbf{(check)}$$

If we consider the right part of the section 1–1 under equilibrium, then we have to consider the reaction at D and the forces 10 kN, 12 KN, 20 kN, F_{AB}, F_{BF} and F_{FE} as external forces as shown below.

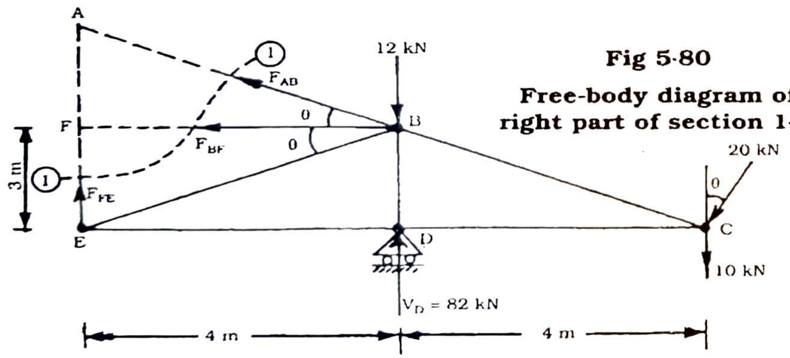


Fig 5·80

Free-body diagram of right part of section 1-1

In this case also, we can take the moments about point ***B, F and A.***

$\Sigma M_B = 0$, ↻ +ve.

$$F_{FE} \times 4 + 10 \times 4 + 20 \cos\theta \times 4 + 20 \sin\theta \times 3 = 0$$

$$\boxed{F_{FE} = -35 \text{ KN}} \qquad \textbf{(check)}$$

$\Sigma M_F = 0$, ↻ +ve.

$$-F_{AB} \sin\theta \times 4 + 12 \times 4 - 82 \times 4 + 10 \times 8$$

$$+ 20 \cos\theta \times 8 + 20 \sin\theta \times 3 = 0$$

$$\boxed{F_{AB} = -15 \text{ KN}} \qquad \textbf{(check)}$$

$$\boxed{F_{FB} = 0} \qquad \textbf{(check)}$$

The complete results are tabulated as under.

Reaction OR Member	Magnitude of Force	Nature of force Compression/ Tension.
V_A	44 kN	Downward (↓)
H_A	12 kN	Positive (→)
V_D	82 kN	Upward (↑)
AB	15 kN	Compression
BC	43·33 kN	Tension
CD	46·66 kN	Compression
DE	46·66 kN	Compression
EF	35 kN	Compression
FA	35 kN	Compression
FB	00 kN	**Zero Force Member**
BE	58·33 kN	Tension
BD	82 kN	Compression

For conveniency, it is better to show the results directly on the members as under.

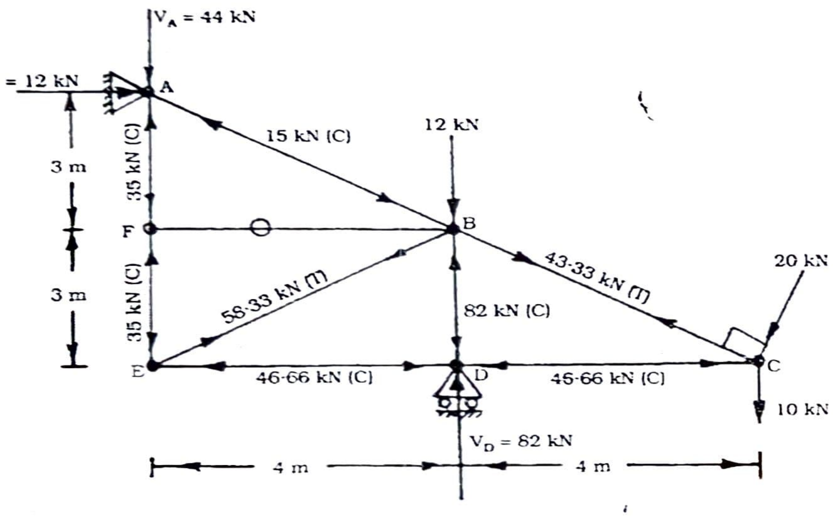


Fig 5·81

Graphical Presentation of Results.

2. Analyse the truss of example 1 graphically.

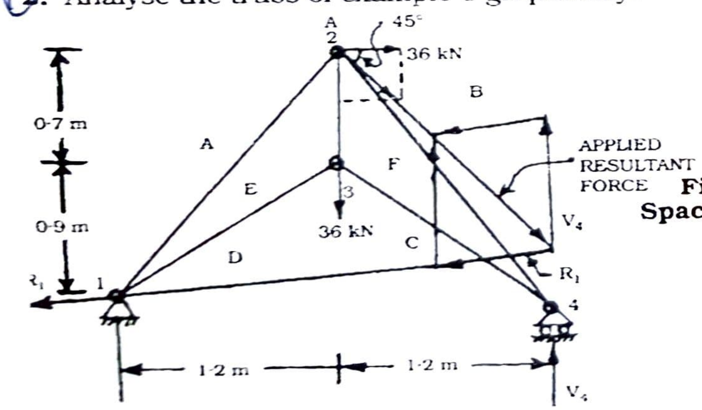


Fig. 5·82 Space diagram

Scale : 1 cm = 0·5 m

1 cm = 25 kN

Draw the truss to some convenient scale. The truss drawn according to its physical dimensions is called *space diagram*. Give *Bow's notations* to the spaces between loads and reactions inside and outside the truss. *Here, reaction can be obtained graphically after knowing the direction of the resultant load acting on the truss and the direction of reaction at right support. These two will intersect*

at a point which will help for deciding the direction of reaction at left support. Preparing the parallelogram of forces at junction point, we can find out the magnitude of the reactions. Now we can complete the polygon for the whole truss after considering it as a rigid body.

Horizontal 36 kN load at joint **2** is now called as **AB** force and is drawn according scale as **ab** force is *force diagram*. Similarly, **BC** is a reaction V_4 and is drawn as **bc**. The vertical downward load of 36 kN at joint **3** is drawn downward from **c** to **d**. Now parallel to **AE** member draw a line from **a** in force diagram. Similarly, parallel to **DE** draw a line from **d**. Thus these two lines will insect a point which will be called **e**. Thus comple the *force diagram*.

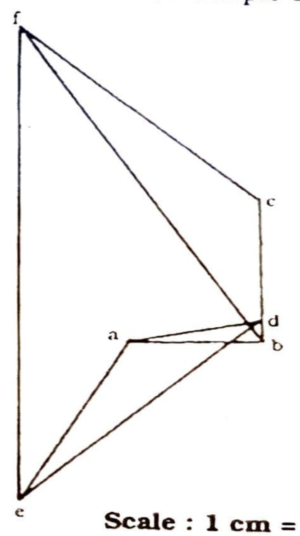


Fig 5·83 Maxwell force diagram

Scale : 1 cm = 20 kN

We can start drawing polygon for each joint in the sequence selected as in the case of method of joints. Thus we can prepare final diagram for the truss indicating there-in the forces acting in all members.

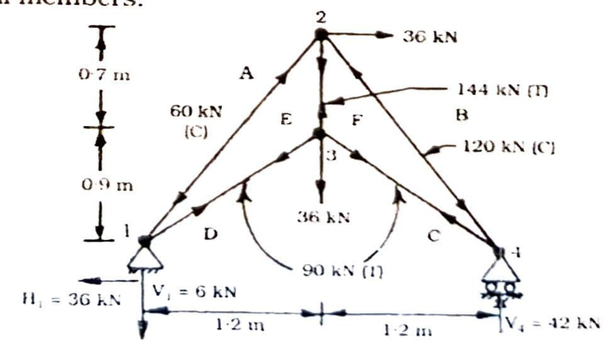


Fig 5·84 Results.

From Maxwell force diagram, the magnitude and type of force (tension or compression) for each member can be decided. Concentrate first on joint **1** and move along the **clockwise direction** about it. While moving from **d** to **a**, we can find H_1 and V_1 with their magnitude and direction. Then **a** to **e** gives direction to be placed on the member **AE** at joint **1.** In the same way, we can find the force direction to be placed on member **ED** while moving from joint **e** to **d** in Maxwell diagram. Length of each gives magnitude of the respective force. Same procedure can be adopted for joint **2**, joint **3** and joint **4**. The sign conventions adopted are same as per the previous sections.

THEORY RELATED QUESTIONS

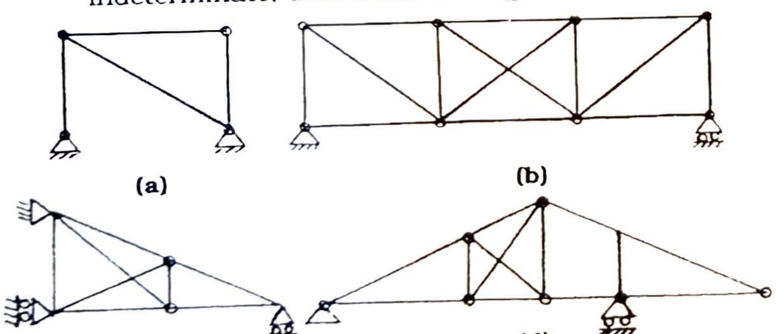
1. Write the assumptions to be made while analysing the plane truss.
2. Explain the stability and determinancy of the plane trusses with the help of examples.
3. Discuss the suitability of verious analytical methods of analyses of plane trusses.
4. Which are the steps to be followed while analysing the plane truss by method of joints ?
5. Explain the procedure to be adopted for method of sections.
6. Write briefly about the graphical method.

EXERCISES

5·1 Indicate whether the trusses shown in Fig 5·85 are

(i) Stable or unstable

(ii) Statically determinate or indeterminate. If they are indeterminate, then state the degree of indeterminancy.



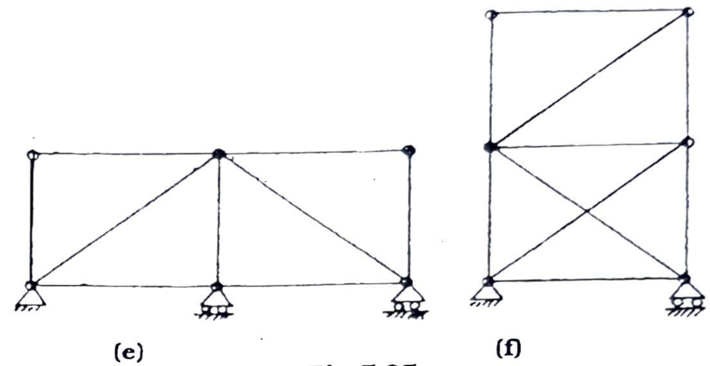


Fig 5·85

5·2 To 5·11 Analyse the trusses shown in Fig 5·86 to 5·95 by method of joints.

5·2

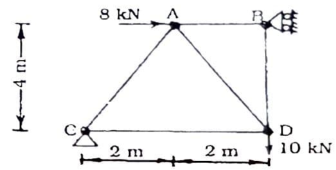


Fig 5·86

5·3

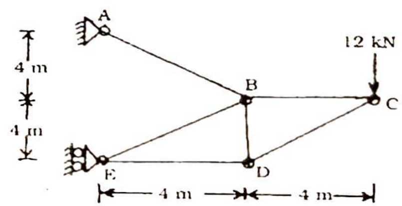


Fig 5·87

5·4

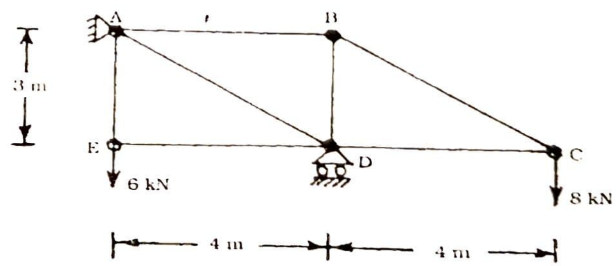


Fig 5·88

·5

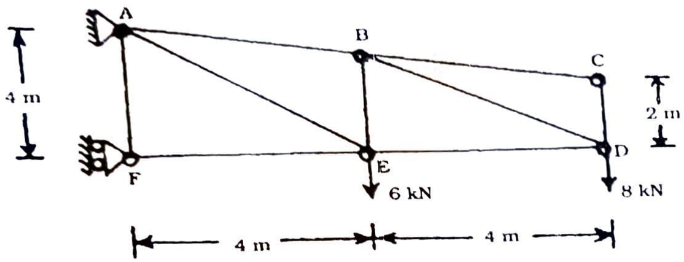


Fig 5·89

·6

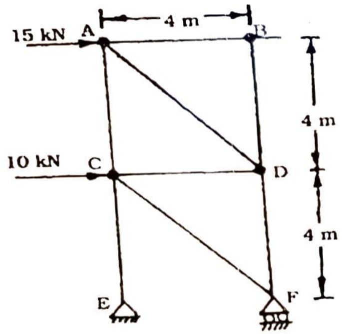


Fig 5·90

5·7

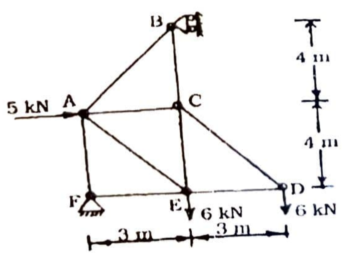


Fig 5·91

5·8

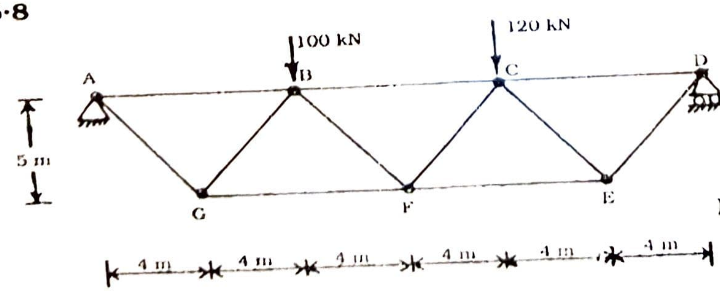


Fig 5·92

5·9

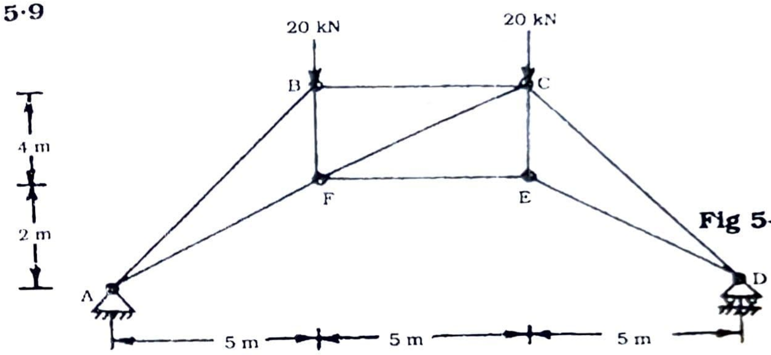


Fig 5·93

5·10

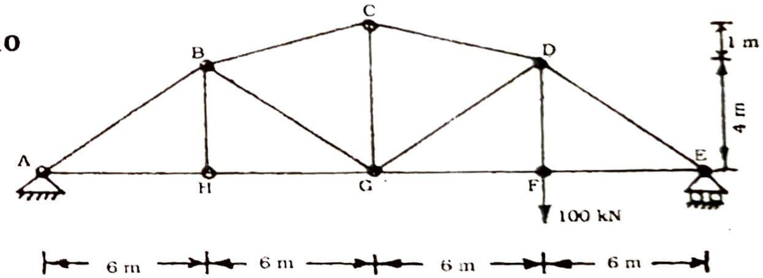


Fig 5·94

5·11

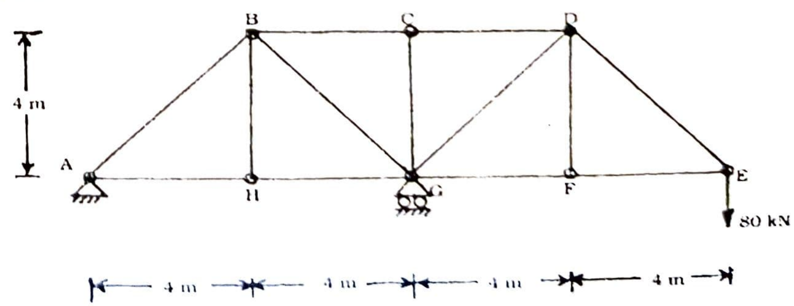


Fig 5·95

12

5·12 To 5·17 Determine the forces in few members only as indicated on the sides of the trusses shown in Fig 5·96 to 5·101.

5·12

Members AB, AD & CD

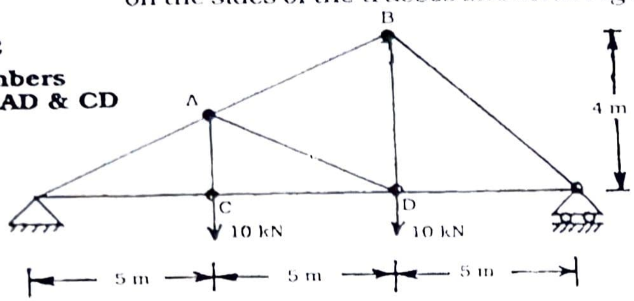


Fig 5·96

5·13

Members AB, AD & CD

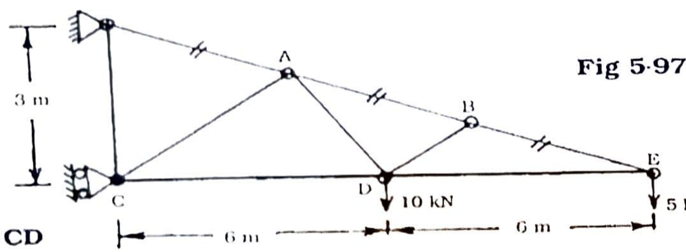


Fig 5·97

5·14

Members AB, AD & CD

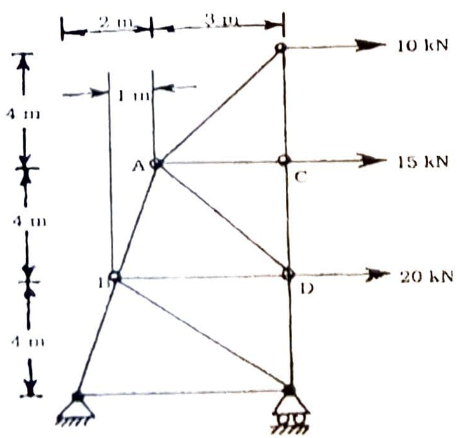


Fig 5·98

5·15

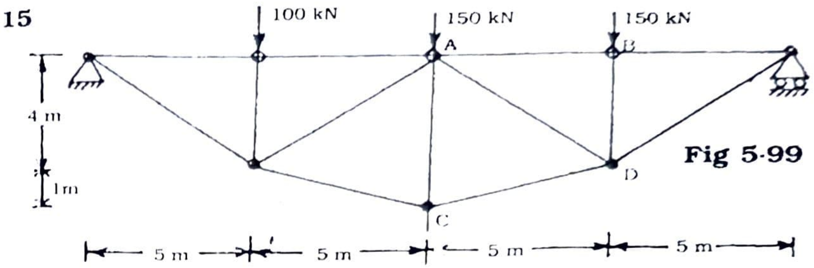


Fig 5·99

Members AB, AD & CD

5·16

Members AB, AD & CD

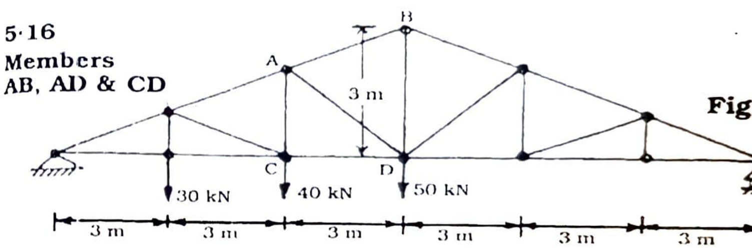


Fig 5·100

5·17

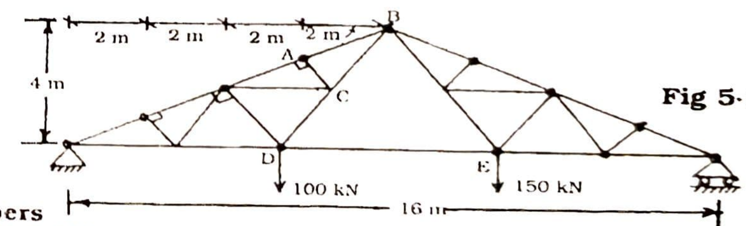


Fig 5·101

Members AB, BC & DE

5·18 To 5·21 Analyse the trusses shown in Fig 5·102 to 5·105 by graphical method. The Bow's notations are shown as A, B, C, D....... and joint number as 1,2,3......

5·18

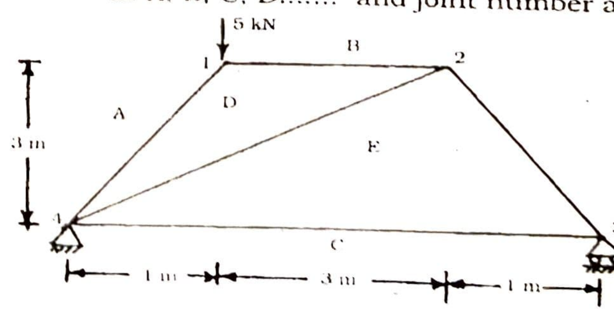


Fig 5·102

5·19

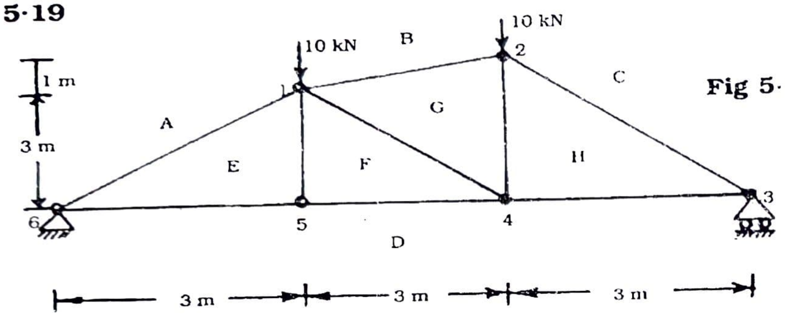


Fig 5·103

5·20

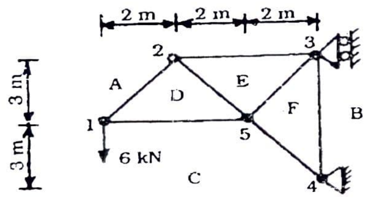


Fig 5·104

5·21

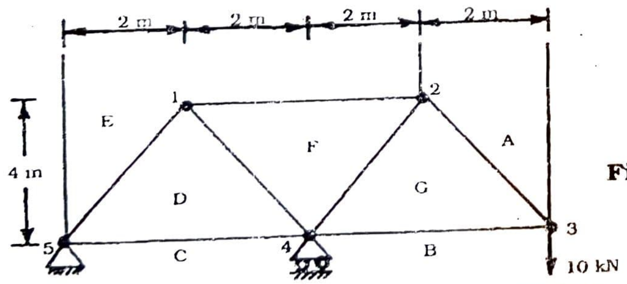


Fig 5·105

SOLUTIONS OF EXERCISES

5·1 (a) *Unstable* as one horizontal member is less for connecting supports.
determinate – m = 4, r = 4, j = 4, **m + r=2j.**

(b) *Stable, indeterminate* – m =14, r = 3, j = 8 **(D.O.I. = 1)**

(c) *Stable, indeterminate* – m = 8, r = 4, j = 5 **(D.O.I. = 2)**

(d) *Unstable* as one inclined member is less in third section (right side-near roller support).
determinate – m = 13, r = 3, j = 8, **m + r=2j.**

(e) *Stable, indeterminate* – m =9, r = 4, j = 6 (D.O.I. = 1)

(f) *Stable, indeterminate* – m =10, r = 3, j = 6 (D.O.I. = 1)

5·2 For the whole truss :

$\Sigma M_c =$ **0,** $8 \times 4 + 10 \times 4 - R_{B(H)} \times 4 = 0$, $R_{B(H)} = 18 kN (\leftarrow)$

$\Sigma F_y =$ **0,** $R_{C(V)} = 10$ kN ($\uparrow$)

$\Sigma F_x =$ **0,** $8 - 18 + R_{C(H)} = 0$, $R_{C(H)} = 10$ kN($\rightarrow$)

Now, start with joint B, then D, A and C in sequence.

Table

Member OR Reaction	Magnitude of Force	Nature of Force
AC	11·18 kN	Compression
AB	18 kN	Compression
AD	11·18 kN	Tension
BD	0·0	Zero Force
CD	5 kN	Compression
$R_{C(V)}$	10 kN	Upward
$R_{C(H)}$	10 kN	Towards right
$R_{B(H)}$	18 kN	Towards left

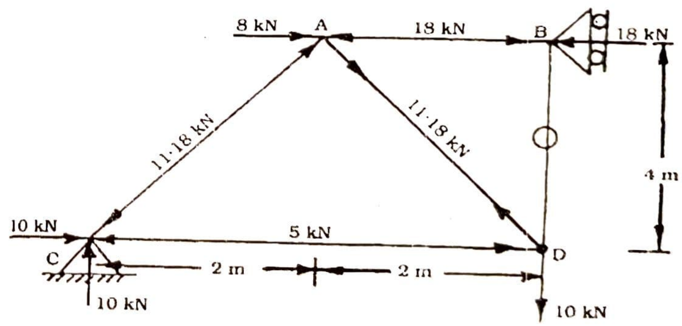


Fig. 5·106

5·3 For the whole truss :

$\Sigma M_A =$ **0,** $12 \times 8 - R_{E(H)} \times 8 = 0$, $R_{E(H)} = 12$ kN($\rightarrow$)

$\Sigma F_y =$ **0,** $R_{A(V)} = 12$ kN ($\uparrow$)

$\Sigma F_x =$ **0,** $R_{A(H)} = 0$, $R_{E(H)} = 12$ kN($\leftarrow$)

Now, start with joint C, then D, B, A and E sequenciely.

Here, joint A and E may be treated first.

At joint E, member ED is in same line with reaction at E and there is no other loading hence force in EB is zero.

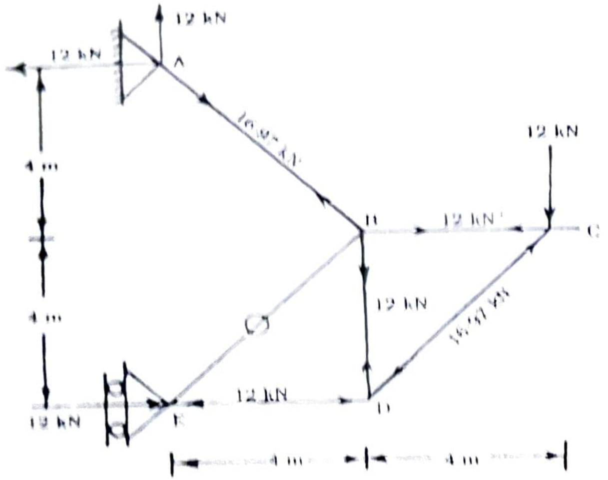


Fig. 5·107

5·4 For the whole truss :

$\Sigma M_A = 0$, $8 \times 8 - R_{D(V)} \times 4 = 0$, $R_{D(V)} = 16$ kN(↑)

$\Sigma F_y = 0$, $6 + 8 - 16 + R_{A(V)} = 0$, $R_{A(V)} = 2$ kN (↓)

$\Sigma F_x = 0$, There is no horizontal loading hence $R_{A(H)} = 0$.

Now, start with joint C or E, then B and A or D.

At joint E, vertical loading is in the same line with member AE hence force in ED will be zero.

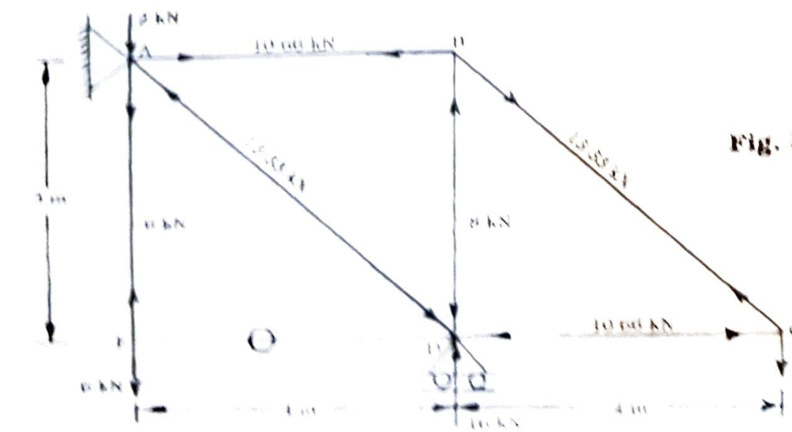


Fig. 5·108

5·5 For the whole truss :

At joint C, the horizontal component of force in BC will be zero as there is no other force in the same direction. Thus force in CD is also zero. At joint F, the reaction at roller support is in the same line with FE, hence force in AF is zero.

For the whole truss :

$\Sigma M_A = 0$, $6 \times 4 + 8 \times 8 - R_{F(V)} \times 4 = 0$, $R_{F(V)} = 22$ kN (→)

$\Sigma F_x = 0$, $R_{A(H)} = 22$ kN (←)

$\Sigma F_y = 0$, $6 + 8 - R_{A(V)} = 0$, $R_{A(V)} = 14$ kN (↑)

Now solve D joint, then B, E and A sequenciely

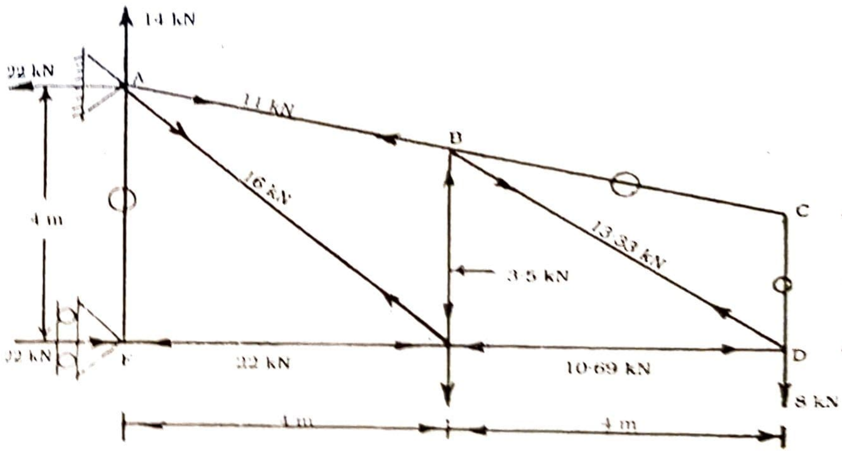


Fig. 5·109

5·6 The force in AB and BD will be zero as here is no external force at joint B.

For the whole truss :

$\Sigma M_E = 0$, $15 \times 8 + 10 \times 4 - R_{F(V)} \times 4 = 0$, $R_{F(V)} = 40$ kN (↑)

$\Sigma F_y = 0$, $R_{E(V)} = 40$ kN (↓)

$\Sigma F_x = 0$, $R_{E(H)} = 15 + 10 = 25$ kN (←)

Now start with joint A followed by D, C, F and E.

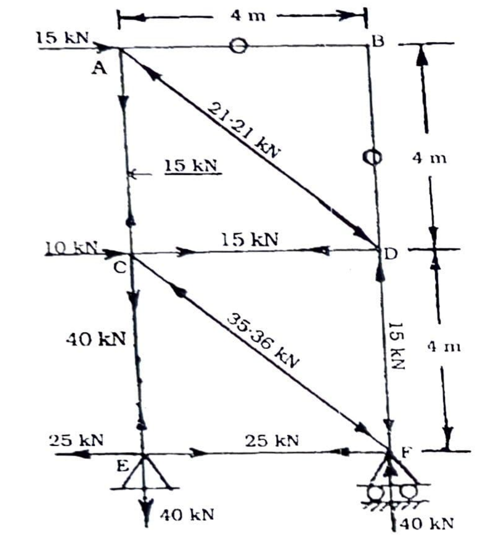


Fig. 5·110

5·7 For the whole truss :

$\Sigma M_F = 0$, $5 \times 4 + 6 \times 3 + 6 \times 6 - R_{B(H)} \times 8 = 0$, $R_{B(H)} = 9{\cdot}25$ kN (←)

$\Sigma F_x = 0$, $5 - 9{\cdot}25 + R_{F(H)} = 0$, $R_{F(H)} = 4{\cdot}24$ kN (→)

$\Sigma F_y = 0$, $6 + 6 - R_{F(V)} = 0$, $R_{F(V)} = 12$ kN (↑)

We may start from joint D, then B, C, E, A & F.

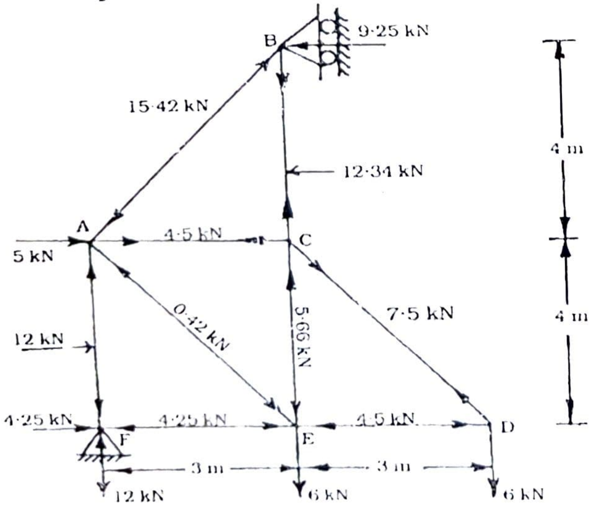


Fig. 5·111

5·8 In this truss, the external loadings are vertical only, hence reactions will be vertical.

For the whole truss :

$\Sigma M_A = 0$, $100 \times 8 + 120 \times 16 - R_D \times 24 = 0$, $R_D = 113{\cdot}33$ kN (↑)

$R_A = 100 + 120 - 113{\cdot}33 = 106{\cdot}67$ kN (↑)

First solve joint A and D, then G and E, then F & at the last B and C.

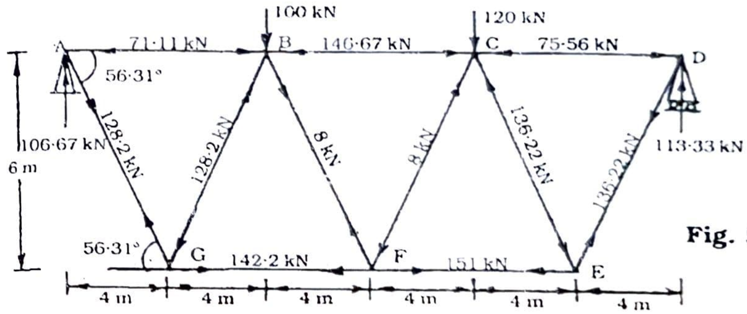


Fig. 5·112

5·9 The reactions will be vertical and equal as loadings are vertical and symmetrical.

$R_A = R_D = 20$ kN (↑)

Here we have to solve first A and D joints, then B, E, F and C.

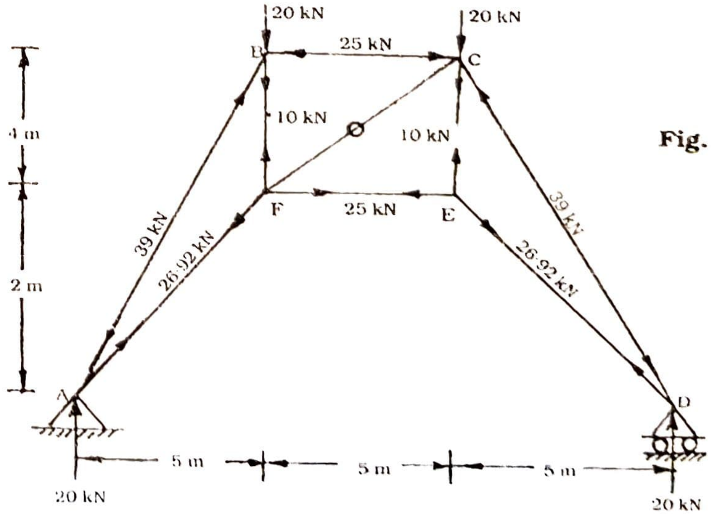


Fig. 5·113

5·10 Here, force in BH will be zero as there is no vertical loading at joint H. The reactions will be vertical.

For the whole trass :

$\Sigma M_A =$ **0**, $100 \times 18 = R_E \times 24 \therefore R_E = 75$ kN (↑)

$R_A = 100 - 75 = 25$ kN (↑)

Now, solve A and E joints, then H and. F, then B and D, then C, and at the last G.

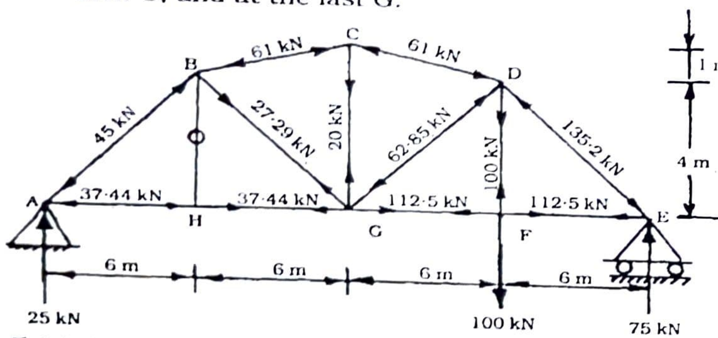


Fig. 5·114

5·11 Here force in BH, CG and DF will be zero as there is no loading and three members meeting at H, C and F joints.

For the whole trass :

$\Sigma M_A =$ 0, $80 \times 16 - R_G \times 8 = 0$, $R_G = 160$ kN (↑)

$\Sigma F_y =$ 0, $160 - 80 = R_{A(v)} = 80$ kN (↓)

Now, solve A and E joints, then H & F, then C, then B and D, then G.

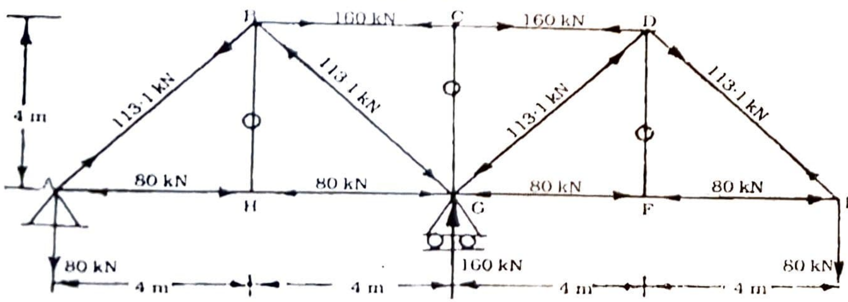


Fig. 5·115

5·12 Let section passing through members AB, AD & CD.
Considering *right* side of the section in equilibrium ($R_R = 10$ kN)

$\Sigma M_A =$ **0**, $F_{CD} \times 2 + 10 \times 5 - R_R \times 10 = 0$

$\therefore F_{CD} = 25$ kN (t)

$\Sigma M_B =$ **0**, $F_{AD} \times 5/5{\cdot}385 \times 4 + 25 \times 4 - 10 \times 5 = 0$

$\therefore F_{AD} = 13{\cdot}46$ kN (c)

$\Sigma M_D =$ **0**, $F_{AB} \times 5/5{\cdot}385 \times 4 + R_R \times 5 = 0$

$\therefore F_{AB} = 13{\cdot}46$ kN (c)

5·13 Let section passing throngh members AB, AD & CD.
Considering *right* side of the section under equilibrium.

$\Sigma M_A =$ **0**, $F_{CD} \times 2 + 10 \times$Ê2 $+ 5 \times 8 = 0$

$\therefore F_{CD} = 30$ kN (c)

$\Sigma M_D =$ **0**, $F_{AB} \cos 14° \times 1 + F_{AB} \sin 14° \times 2 - 5 \times 6 = 0$

$\therefore F_{AB} = 20{\cdot}6$ kN (t)

$\Sigma M_E =$ **0**, $F_{AD} \sin 45° \times 6 - 10 \times 6 = 0$

$\therefore F_{AD} = 14{\cdot}14$ kN (t)

5·14 Let section passing through members AB, AD & CD.
Considering *upper* part of the section in equilibrium.

$\Sigma M_D =$ **0**, $F_{AB} \times 4/4{\cdot}12 \times 3 - 10 \times 8 - 15 \times 4 = 0$, $F_{AB} = 48{\cdot}07$ kN (t)

$\Sigma M_A =$ **0**, $F_{CD} \times 3 + 10 \times 4 = 0 \quad \therefore F_{CD} = 13{\cdot}33$ kN (c)

$$\Sigma M_C = 0,\ 48{\cdot}07 \times \frac{4}{4{\cdot}12} \times 3 + F_{AD} \times \frac{4}{5} \times 3 - 10 \times 4 = 0,$$

$F_{AD} = 41{\cdot}67$ kN (c)

5·15 Let section passing through members AB, AD & CD.
Considering *right* side of the section in equilibrium.

$$\Sigma M_A = 0,\ F_{CD} \times \frac{5}{5{\cdot}1} \times 4 + F_{CD} \times \frac{1}{5{\cdot}1} \times 5 + 150 \times 5$$

$$- R_R \times 10 = 0,\ (R_R = 212{\cdot}5 \text{ kN})$$

$\therefore F_{CD} = 280{\cdot}6$ kN (t)

$\Sigma M_D =$ **0**, $F_{AB} \times 4 + R_R \times 5 = 0$, $F_{AB} = 265{\cdot}6$ kN (c)

$\Sigma M_B =$ **0**, $F_{CD} \times 5/5{\cdot}1 \times 4 + F_{AD} \times 5/6{\cdot}4 \times 4 - 212{\cdot}5 \times 5 = 0$

$F_{AD} = 12{\cdot}13$ kN (c)

5·16 Let section passing through members AB, AD & CD.
Considering *right* side of the section in equilibrium.

$\Sigma M_A =$ **0**, $F_{CD} \times 2 + 50 \times 3 - R_R \times 12 = 0$ ($R_R = 43{\cdot}33$ kN)

$F_{CD} = 185$ kN (t)

$\Sigma M_D = 0$, $F_{AB} \times \dfrac{3}{3 \cdot 16} \times 3 + 43 \cdot 33 \times 9 = 0$, $F_{AB} = 137$ kN (c)

$\Sigma M_{Hinge} = 0$, $F_{AD} \times 2/3 \cdot 6 \times 9 + R_R \times 18 - 50 \times 9 = 0$

$F_{AD} = 66$ kN (c)

.7 Load at D and E is 100 and 150 kN (downward) respectively.

Let section passing through members AB, AD & CD.
Considering right side of the section in equilibrium.

$\Sigma M_{Hinge} = 0$, $F_{BC} \times 0 \cdot 8 \times 8 - F_{BC} \times 0 \cdot 6 \times 4 + 150 \times 11 - R_R \times 16 = 0$

($R_R = 134 \cdot 38$ kN), $F_{BC} = 125$ kN (t)

$\Sigma M_B = 0$, $F_{DE} \times 4 + 150 \times 3 - R_R \times 8 = 0$, $F_{DE} = 156 \cdot 3$ kN (t)

$\Sigma M_D = 0$, $F_{AB} \times 2/2 \cdot 24 \times 4 - F_{AB} \times 1/2 \cdot 24 \times 3 - 150 \times 6 + R_R \times 11 = 0$

$F_{AB} = 259 \cdot 3$ kN (c)