6 BEAMS AND CABLES

6·1 Analysis of Beams :

In the previous chapter, it has been noticed that the truss members were subjected to axial forces only. Now we will analyse the *member which is subjected to a transverse load*. For a simple beam having symmetrical section and supports, and loading also symmetrical about the same plane then the beam system becomes *coplanar.*

The beam shown in Fig. 6·1 is subjected to various loads. Left hand support is of *hinge* while right is a *roller. Loads perpendicular to the axis of the beam will give rise to shear and bending in the beam. Nonperpendicular load will produce axial force besides shear and bending type of force.*

Fig 6·1

At present, our discussion will be limited to analysis of *determinate beams* only, i.e. *beams having only three unknowns.* It is also possible to determine more than three unknown reactions in case of beams having *additional internal hinges. Internal hinge will give us one more condition that is the moment at this point will be zero moreover the reaction at internal hinge will not be acting means the reaction is zero.* Various determinate beams are shown in Fig 6·2.

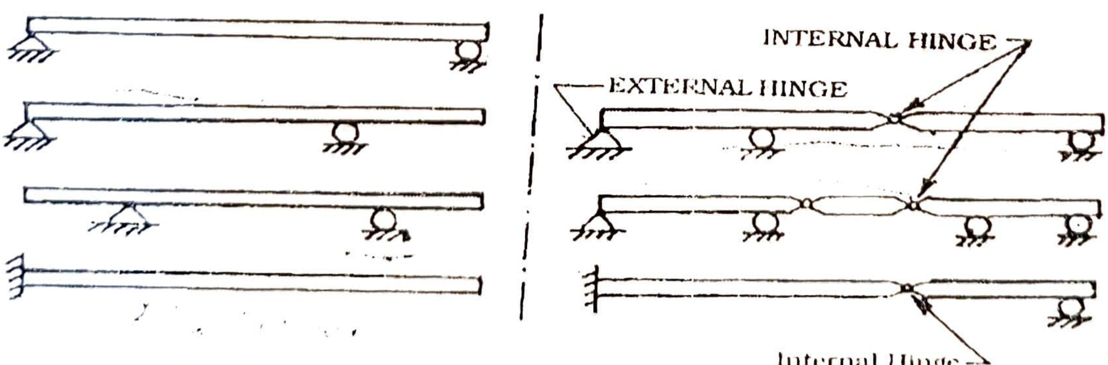


Fig 6·2 Statically determinate beams.

A beam having **three unknown** reactions is called **determinate** **m**, because these three unknowns can be determined by using ce equilibrium equations, viz. $\Sigma F_x = 0$, $\Sigma F_y = 0$ and $\Sigma M_{any\ point} = 0$.

Roller support *has one unknown reaction perpendicular to surface on which roller moves,* **hinged support** *has two knowns perpendicular to each other or one unknown reaction th unknown angle of reaction, while* **fixed support** *offers three known reaction two reaction components perpendicular to each her and a moment. At internal hinge the sum of the moments will zero with zero reaction.*

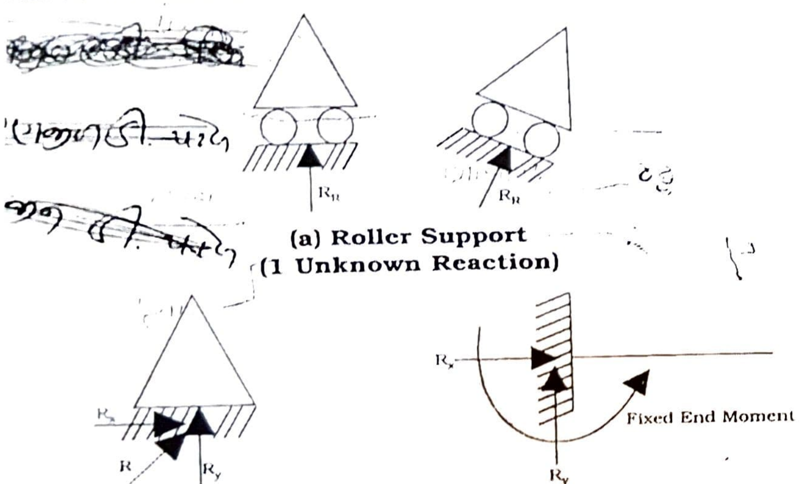


Fig 6·3 Support Conditions and Unknowns

6·2 Shear Force and Bending Moment in a Beam :

For the beam shown in Fig 6·1, **support reactions** can be determined by using the free body diagram of the entire beam and applying the equations of equilibrium. The **internal forces** at any point x can be obtained by cutting the beam at this section and considering the free body diagram of the either part (i.e. left or right part). These unknown forces will have equal magnitude but opposite sense in each part as shown in Fig 6·3.

The magnitude of each unknown can be obtained by considering the equilibrium of either part.

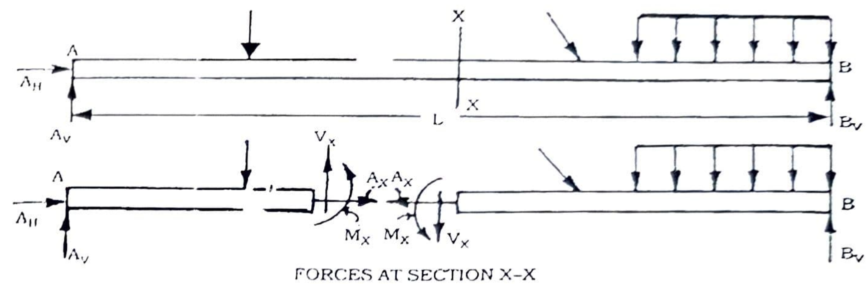


Fig 6·4

Sign convention selected here is shown in Fig 6·5, which is similar to the sign conventions used in most text books on engineeing mechanics. Shear force acting downward and moment acting counter clockwise on left hand part is designated as 'positive.' *In other words, the* ***shear forces*** *acting on right and left sides which* ***cause clockwise rotation*** *and* ***moments*** *which cause* **"sagging"** *(bottom of beam in tension & top in compression) are called* **positive.** On right hand part, positive will be opposite to it.

Shear Forces causing clockwise rotation are positive

Moments causing sagging of beam are positive

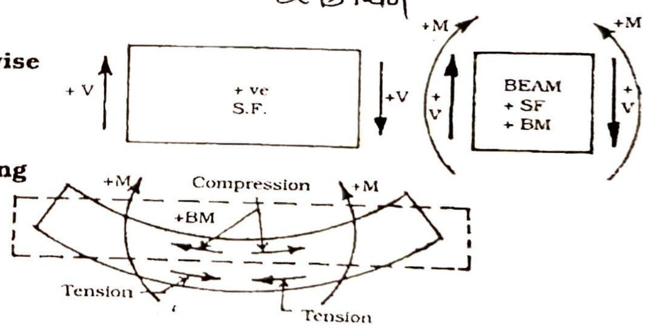


Fig 6·5

The same can be visualized otherway by considering a small beam portion. Here **positive shear** tends to rotate the block in a **clockwise direction** and **positive moment** creates **tension in bottom fibres** and **compression in top fibres.**

6·3 Shear Force and Bending Moment Diagrams :

By following the procedure narrated in the previous section, we can find the shear force and bending moment at any point on the beam. But for design purpose, we are interested to know the variation of shear and moment along the axis of the beam. These variations will be discontinuous due to presence of point loads, moments and change in uniform loading. *The graphical variation of shear and moment are known as shear force diagram and bending moment diagrams*, respectively. For drawing these, one need to know the variation of shear and moment as a function of distance from either support at various sections.

6·4 Relations between Distributed Load, Shear Force and Bending Moment :

For the beam subjected to various loadings, the above mentioned procedure of drawing shear force and bending moment diagram becomes tedious and time consuming. In such a case, one may use the ***relations available between load, shear and moment.***

Consider an infinitesimal element Δx of the beam shown in Fig 6·6. Continuous variation of shear force and bending moment is assumed along the element. Now the free body diagram of the element will be considered for obtaining various relations.

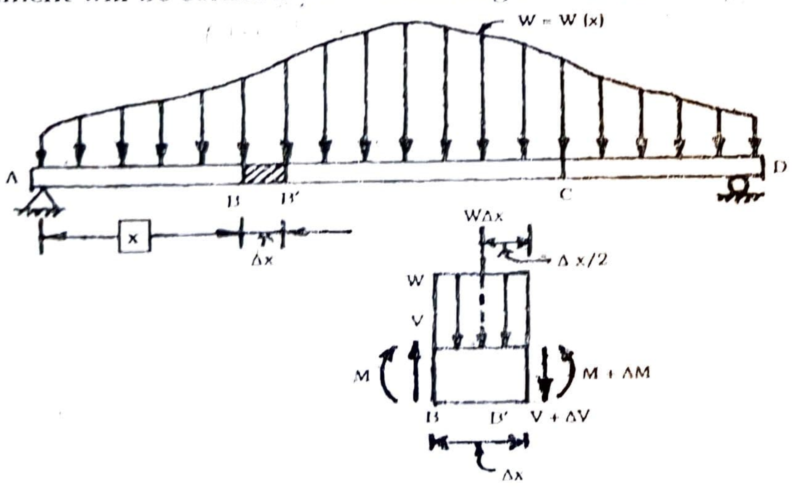


Fig 6·6

(i) Relations between load and shear :

Considering the sum of the vertical components of the forces acting on the free body as zero, we have

$V - (V + \Delta V) - w\,\Delta x = 0$

Dividing by Δx and taking the limit $\Delta x \to 0$, the above equatio becomes,

$$\boxed{\frac{dV}{dx} = -w}$$

i.e. the slope of shear force diagram is equal to the negativ distributed load intensity. — ढाल

Integrating the above equation between two points B and C along the beam,

$$V_B - V_C = \Delta V_{BC} = -\int_C^B w\,dx$$ क्षेत्रफल से फर्क

$$\boxed{V_B - V_C = -\text{(area under load curve between C and B)}}$$

i.e. change in shear force is equal to negative area unde loading curve.

(2) Relations between shear and bending moment :

Considering the sum of the moments about B on the free body as zero, we have

$$(M + \Delta M) - M - V\,\Delta x + w\,\Delta x \times \frac{\Delta x}{2} = 0$$

$$\Delta M = V\,\Delta x - 1/2\, w\,(\Delta x)^2$$

Dividing by Δx and taking the limit as $\Delta x \to 0$, the above equation becomes,

$$\boxed{\frac{dM}{dx} = V}$$

i.e. slope of moment diagram is equal to the shear.

Integrating the equation between two points B and C along the beam,

$$M_B - M_C = \Delta M_{BC} = \int_C^B V\,dx$$

$$\boxed{M_B - M_C = \text{area under shear diagram between C and B}}$$

i.e. change in moment is equal to the area under shear diagram.

From the alove relations we can say that if the loading curve w = w (x) is a polynomial of degree n, then V = V (x) will be a curve of degree n + 1 and M = M (x) will be a curve of degree n + 2. At points, where point loads or moment is acting, the discontinuities occure in the shear and moment diagrams. Hence applications of these relations is restricted for such load cases.

13

5 Cables :

In many engineering applications, we often encounter relatively
xible cables or chains that are used to support loads. *Cables*
n main load-carrying component in case of suspension bridges
d trolley wires, where in the weight of cable itself may be neglected.
hile in transmission line and guy wires for high towers, the cable
eight may be important and need to be included in analysis.

To facilitate computations, we will make the assumption that
e cable is perfectly flexible and inextensible. Due to which the cable
ill be subjected to only tensile force and will have constant length
fore and after the loading is applied.

) Cable subjected to Concentrated Loads :

If *flexible cable* (that its resistance to bending is small and may
: neglected) of *negligible weight* is supporting several concentrated
ads, then cable takes the form of several straight line segments
here each segment is subjected to a constant tensile force.

Consider the cable shown in Fig 6·7 subjected to three
oncentrated loads.

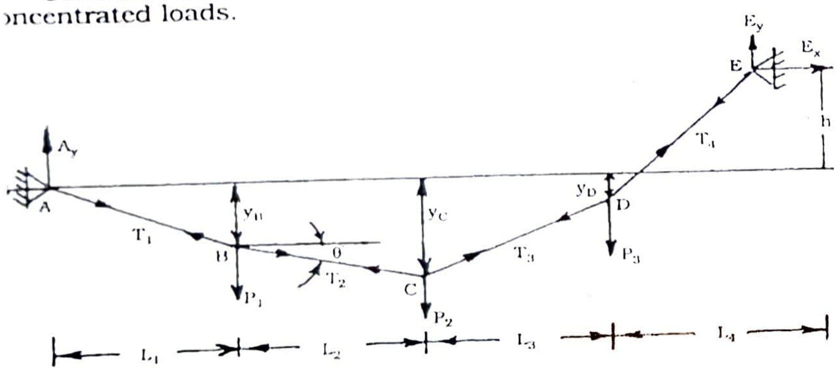


Fig 6·7

Here the distances - h, L_1, L_2, L_3, L_4 and the loads P_1, P_2 and P_3
re known. For the problem, we need to determine the tension in
he four segments, four components of reactions at A and E and the
hree sags Y_B, Y_C and Y_D. Thus we have got total eleven unknowns.
We can write two equations of force equilibrium at each of the points
A, B, C, D and E which gives total ten equations. Thus we need
some more information to solve such type of problems, which may
e the total cable length or sag at one of the points.

Another approach of solving such problems is to obtain support
reactions. Here unknown will be four components of reactions at A
and E. Using three equations of equilibrium by considering the rigid
body equilibrium of entire cable, we can find out three unknowns
so here also one more unknown can be obtained from the geometry

of the cable. Once the reaction components are obtained we can start the analysis joint by joint from either support.

Considering point B, for example, we draw the freebody diagram of the portion of cable AB. Writing $\cdot F_x = 0$ and $\cdot F_y = 0$, we obtain the components of the force T_2 representing tension in the portion of cable to the right of B. We observe that $T_2 \cos \theta = A_x$, the *horizontal component of the tension is the same at any point of the cable.*

Tension T is maximum when cos θ is minimum i.e. the portion of cable which has the largest angle of inclination θ. Clearly, this portion of cable must be adjacent to one of the two supports of cable.

(2) Cable subjected to Distributed Load : वितरित

Consider a weightless cable AB carries a *uniformly distributed load (w) along the horizontal.* In suspension bridges, such type of case exists as the *weight of cables is very small compared to the weight of the roadway.*

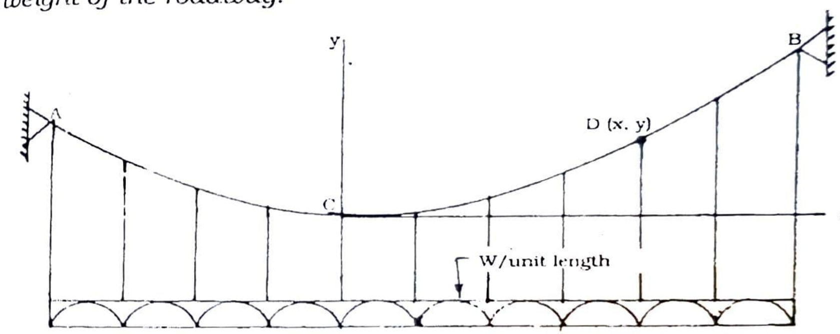


Fig 6·8

Consider origin at the lowest point C of the cable. The free-body diagram of CD portion is also shown.

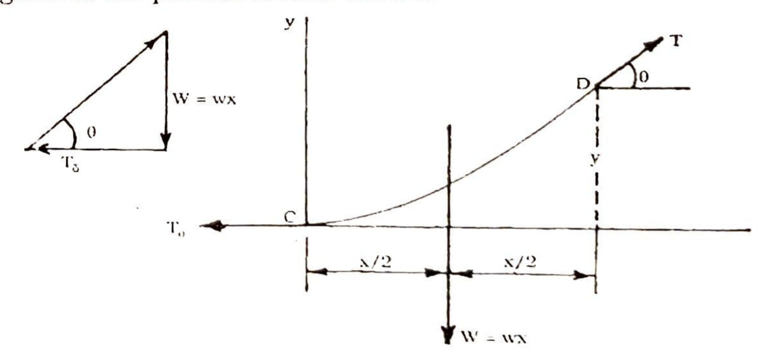


Fig 6·9

Here, $\boxed{T\cos\theta = T_0}$ and, $\boxed{T\sin\theta = W = wx}$

From the force triange,

$$\boxed{T = \sqrt{T_0^2 + w^2x^2}} \quad \text{and,} \quad \boxed{\tan\theta = \frac{wx}{T_0}}$$

It should be noted here that the *horizontal component of the tension T is the same at any point. The tension T is minimum at the lowest point (= T_o) and maximum at one of the two supports.*

Taking moment about D,

$$\Sigma M_D = 0, \; +↺, \quad w\,x\,\frac{x}{2} - T_0\,y = 0$$

$$\therefore \quad \boxed{y = \frac{wx^2}{2\,T_0}}$$

This is the equation of a parabola with the vertical axis. The vertex of parabola is at the origin of coordinates.

When the supports A and B of the cable have the same elevation, and span (L), sag (h) and load w per unit horizontal length are given, the minimum tension T_0 may be determined by substituting **x = L/2** and **y = h** in the above equation of parabola.

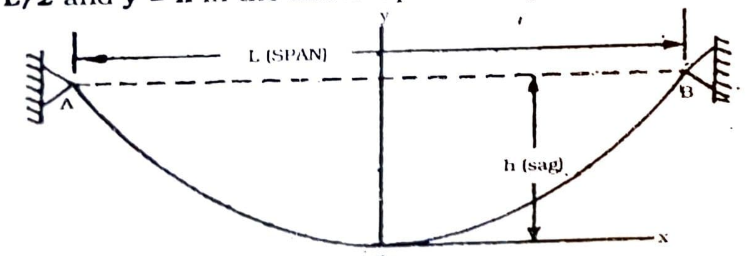
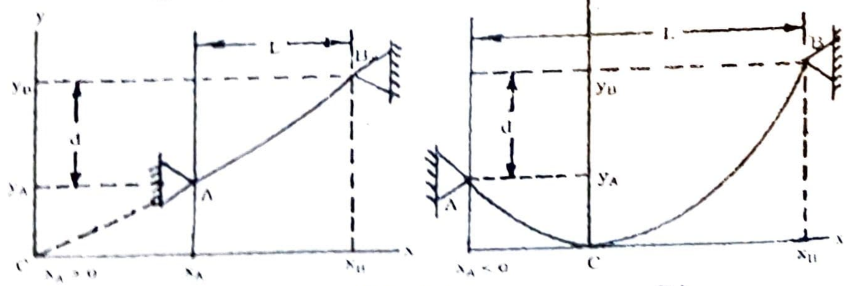


Fig 6·10

When the support have different elevations, the position of the lowest point of the cable is not known and coordinates x_A, y_A and x_B, y_B of the supports must be determined.

Here, $\mathbf{x_B - x_A = L}$ and $\mathbf{y_B - y_A = d}$



(a) Fig 6·11 (b)

The length of the cable from its lowest point to its support B may be obtained from the equation

$$S_B = \int_0^{x_B} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

Differentiating $y = \dfrac{wx^2}{2T_0}$, we obtain $\dfrac{dy}{dx} = \dfrac{wx}{T_0}$

and substituting this into above equation and using the binomial theorem to expand the radical in an infinite series, we have

$$S_B = \int_0^{x_B} \left(1 + \frac{w^2x^2}{2T_0^2} - \frac{w^4x^4}{8\,T_0^4} + \text{----}\right)dx$$

$$= x_B\left(1 + \frac{w^2x_B^2}{6\,T_0^2} - \frac{w^4x_B^4}{40\,T_0^4} + \text{-------}\right)$$

since, $w\,x^2_B/2\,T_o = y_B$.

$$S_B = x_B\left[1 + \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2 - \frac{2}{5}\left(\frac{y_B}{x_B}\right)^4 + \ldots\ldots\right]$$

In most cases, the series converges for values of the ratio y_B/x_B less than 0·5 which is much smaller, only first two terms of the series need to be computed.

*Hence, Length of cable **CB** (S_B) May be obtained as*

$$\boxed{S_B = x_B\left[1 + \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2\right]}$$

(3) Cables subjected to Self Weight :

Let us consider a cable AB carrying a *uniformly distributed load (w) along the cable itself.* Cables hanging under their own weight are loaded in this way. ***The deflected shape of the cable under its own weight is called catenary.***

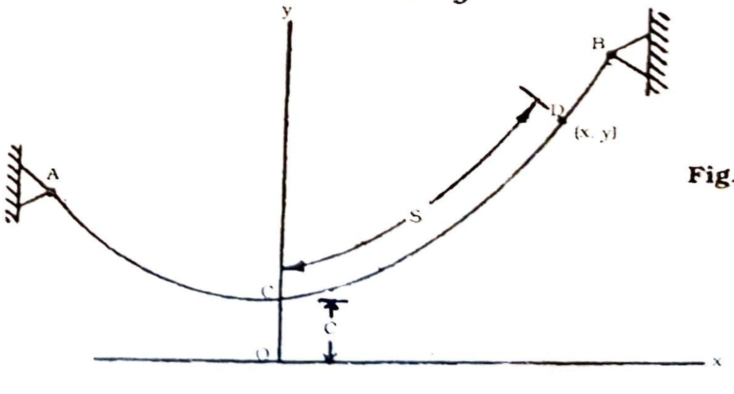


Fig. 6·12 (a)

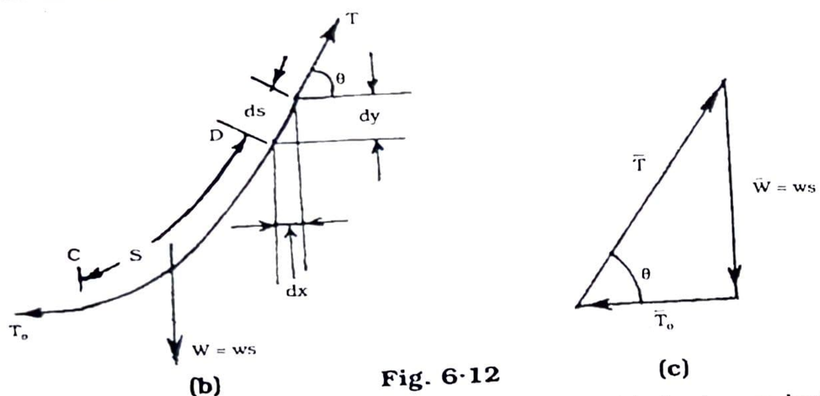


Fig. 6·12

The free - body diagram of CD portion of cable is shown, but to determine horizontal distance between D and resultant (W) we shall consider small element also.

Now, resultant, $\boxed{W = ws}$

and, $T = \sqrt{T_0^2 + w^2}$

$= \sqrt{T_0^2 + w^2 s^2}$

Let us introduce the constant $\boxed{c = T_0/w}$ for simplifying the computations, and **select origin O at a distance c below C.**

Here, $\boxed{T_0 = wc,}$ and $\boxed{T = w\sqrt{c^2 + s^2}}$

Now, $\cos\theta = \dfrac{T_0}{T}$ and $dx = ds \cos\theta$

Hence, $dx = ds\cos\theta = \dfrac{T_0}{T}\,ds = \dfrac{w\,c\,ds}{w\sqrt{c^2 + s^2}} = \dfrac{ds}{\sqrt{1 + s^2/c^2}}$

Integrating from C (0, c) to D (x, y), we get

$$x = \int_0^s \frac{ds}{\sqrt{1 + s^2/c^2}} = c\left[\sinh^{-1}\frac{s}{c}\right]_0^s = c\sinh^{-1}\frac{s}{c}$$

We may write

$$\boxed{s = c\sinh\frac{x}{c}}$$

Now,

$dy = dx\tan\theta$, similarly, $\tan\theta = \dfrac{W}{T_0}$

$\therefore\ dy = dx\tan\theta = \dfrac{W}{T_0}\,dx = \dfrac{s}{c}\,dx = \sinh\dfrac{x}{c}\,dx$

Integrating from C (0, c) to D (x, y), we get

$$y - c = \int_0^x \sinh\frac{x}{c}\,dx = c\left[\cosh\frac{x}{c}\right]_0^x = c\left(\cosh\frac{x}{c} - 1\right)$$

(Here, sinh 0 = 0, cosh 0 = 1)

$\therefore\ y - c = c\cosh\dfrac{x}{c} - c$

$\therefore\ \boxed{y = c\cosh\dfrac{x}{c}}$

This is the equation of catenary with vertical axis.

From above equations of s and y (squaring both sides of both equations, subtracting and using $\cosh^2 z - \sinh^2 z = 1$), we get a relation

$$\boxed{y^2 - s^2 = c^2}$$

Thus above equations of T_0, W and T will be

$\boxed{T_0 = wc}$ $\boxed{W = ws}$ and $\boxed{T = wy}$

Important Notes : (1) If the weight of the cable is negligible, and the ratio of the sag to the span is small, assume the cable to be parabolic. (2) When the weight of the cable is considerable, catenary shape of the cable should be taken.

(3) Numerical values of **sinh x** and **cosh x** are computed on most calculatators either **directly** or **from definitions,**

$$\boxed{\sinh x = \frac{1}{2}(e^x - e^{-x}),\ \cosh x = \frac{1}{2}(e^x + e^{-x})}$$

IMPORTANT EQUATIONS

1. **Relations between load and shear :**

$\dfrac{dV}{dx} = -w$ and $\Delta V_{BC} = -\displaystyle\int_C^B w\,dx$

$V_B - V_C =$ *–(area under load curve between C and B).*

2. **Relations between shear and bending moment :**

$\dfrac{dM}{dx} = V$ and $\Delta M_{BC} = \displaystyle\int_C^B V\,dx$

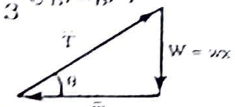
$M_B - M_C =$ *area under shear diagram between C and B.*

Cables with concentrated loads : Using entire cable, determine ...tions at supports, then tensions in segments are to be ...ermined by considering free-body of individual segments.

Parabolic cable : *UDL along horizontal*

...os $\theta = T_0$, $W = wx$, $y = \dfrac{wx^2}{2T_0}$, $s_B = x_B \left[1 + \dfrac{2}{3}(y_B/x_B)^2\right]$

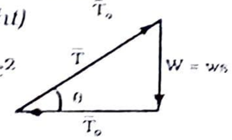
...in $\theta = wx$, $T = \sqrt{T_0^2 + w^2x^2}$, $\tan\theta = \dfrac{wx}{T_0}$



Catenary : *UDL along cable itself (self weight)*

$W = ws$, $c = \dfrac{T_0}{w}$, $s = c \sinh \dfrac{x}{c}$, $y^2 - s^2 = c^2$

$T_0 = wc$, $T = wy$ $y = c \cosh \dfrac{x}{c}$



If the weight of the cable is negligible, and the ratio of the ...to the span is small, assume the cable to be parabolic. When ...weight of the cable is considerable, catenary shape of the ...le should be taken.

SOLVED EXAMPLES

Determine the shear force and bending moment at the beam ...tions passing through points B and D of the beam shown below.

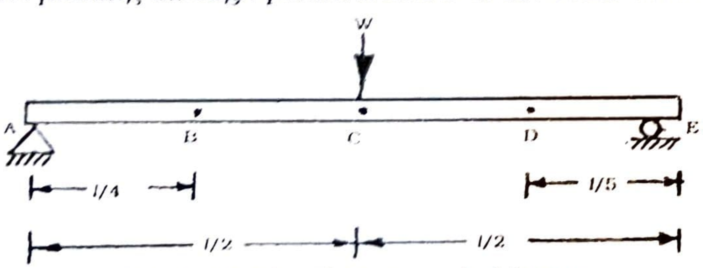


Fig 6·13 Simply supported Beam

There being symmetric in beam and loading both, the supports ... offer half the reactions. The free body diagram of the entire ...m is drawn here.

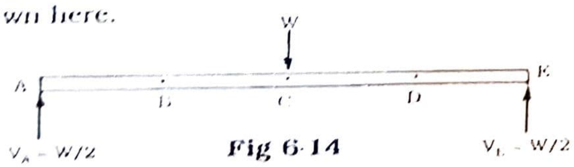


Fig 6·14

For obtaining the magnitude of shear force and bending moment ... the point B, the left segment of AB is drawn below.

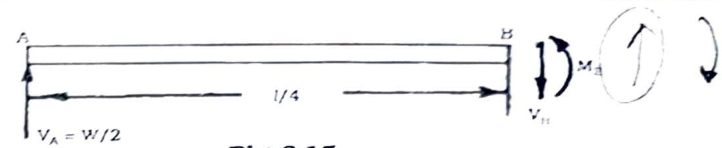


Fig 6·15

Observe the positive sign for shear force and bending moment at the section, which is consistent with the sign conventions adopted.

Using two equilibrium equations on section AB,

$\Sigma F_y = 0$, ↑+ve, $V_A - V_B = 0$

$\boxed{V_B = W/2}$

$\Sigma M_B = 0$, +ve, $V_A \times l/4 - M_B = 0$

$\boxed{M_B = \dfrac{Wl}{8}}$

Thus both shear force and bending moment are positive at section B and sign shown on the section is correct. The same result can be obtained by considering the right hand part of section B i.e. segment BE.

Now, for obtaining shear force and bending moment at point D, right hand part of point D is considered i.e. segment DE. Free body diagram of DE is shown below.

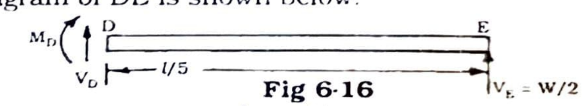


Fig 6·16

Using two equations of equilibrium, we have

$\Sigma F_y = 0$, ↑+ve, $V_D + V_E = 0$

$\boxed{V_D = -W/2}$

$\Sigma M_D = 0$, ↺+ve, $+V_E \times l/5 + M_D = 0$

$\boxed{M_D = \dfrac{Wl}{10}}$

Here shear force is negative, hence sign for it will be opposite to the assumed one i.e. it will be downward on the right hand section.

2. Obtain the values of shear force and bending moment at the beam sections passing through points B and C for the cantilever beam subjected to uniformly distributed load shown below.

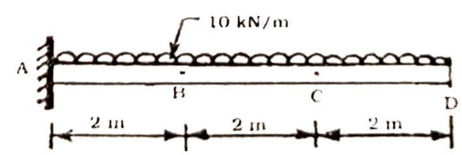


Fig 6·17
Cantilever beam.

For cantilever type of beam there is no need to find support reactions. The magnitude of shear force and bending moment at the point can be obtained by considering the free-body diagram of the free portion. In this case we need to consider the right hand part of the section.

For obtaining the shear force and bending moment at point B, right hand part i.e. portion BD will be considered. Free body diagram of part BD is shown here.

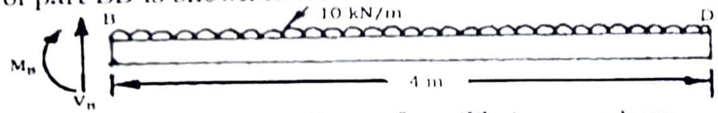


Fig 6·18

Using the two equations of equilibrium, we have

$$\Sigma F_y = 0, \uparrow +ve, \quad V_B - 10 \times 4 = 0$$

$$\boxed{V_B = 40 \text{ kN}}$$

$$\Sigma M_B = 0, \circlearrowleft +ve, \quad M_B + 10 \times 4 \times 4/2 = 0$$

$$\boxed{M_B = -80 \text{ kN.m}}$$

Shear force and bending moment at point C can be obtained by considering right hand part of section C. Free body diagram of part CD is shown below.

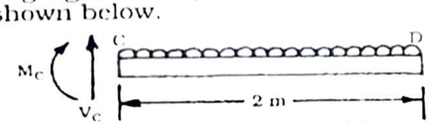


Fig 6·19

Considering two equations of equilibrium, we have

$$\Sigma F_y = 0, \uparrow +ve, \quad V_c - 10 \times 2 = 0$$

$$\boxed{V_c = 20 \text{ kN}}$$

$$\Sigma M_c = 0, \circlearrowleft +ve, \quad M_c + 10 \times 2 \times 2/2 = 0$$

$$\boxed{M_c = -20 \text{ kN.m}}$$

3. Obtain the magnitude of shear force and bending moment at the beam sections passing through points B and D for the overhang beam shown here.

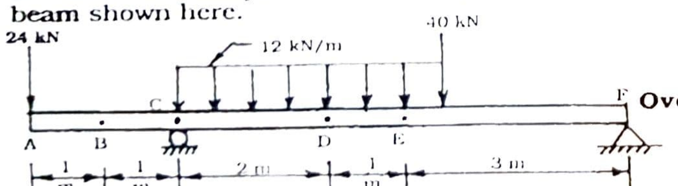


Fig 6·20 Overhang beam

Here we need to obtain support reactions, which can be obtained by considering the free body diagram of the entire beam shown here.

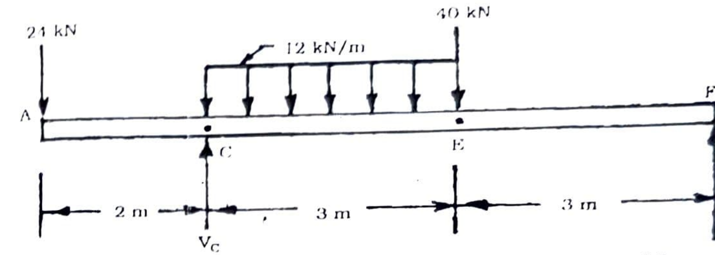


Fig 6·21

The two unknowns V_C and V_F can be obtained by considering only two equations of equilibrium.

$$\Sigma M_F = 0, \circlearrowleft +ve, \quad -24 \times 8 + V_C \times 6 - 12 \times 3 \times (3 + \frac{3}{2}) - 40 \times 3 = 0$$

$$\boxed{V_C = 79 \text{ kN}}$$

$$\Sigma F_y = 0, \uparrow +ve, \quad -24 + V_C - 12 \times 3 - 40 + V_F = 0$$

$$\boxed{V_F = 21 \text{ kN}}$$

The value of V_F can be obtained otherwise by considering one more moment equation as

$$\Sigma M_C = 0, \circlearrowleft +ve, \quad -24 \times 2 + 12 \times 3 \times \frac{3}{2} + 40 \times 3 - V_F \times 6 = 0$$

$$\boxed{V_F = 21 \text{ kN}}$$

Now, considering the left hand part of section B we can obtain the magnitude of shear force and bending moment at point B. Free-body diagram for the part is shown here.

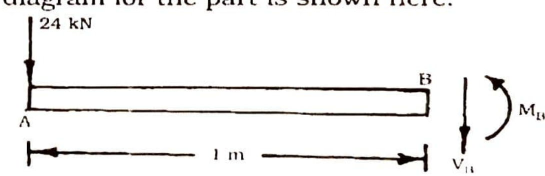


Fig 6·22

From two equations of equilibrium, we have

$$\Sigma F_y = 0, \uparrow +ve, \quad -24 - V_B = 0$$

$$\boxed{V_B = -24 \text{ kN}}$$

$$\Sigma M_B = 0, \circlearrowleft +ve, \quad -24 \times 1 - M_B = 0$$

$$\boxed{M_B = -24 \text{ kN.m}}$$

Note the negative sign for shear force in this cantilever portion. In previous problem it was positive for the reason that cantilever was towards right. Bending moment has maintained the same negative sign.

The magnitude of shear force and bending moment at point D n be obtained by considering either part. Here right hand part of ction D is considered. Free body diagram of the portion DF is own below.

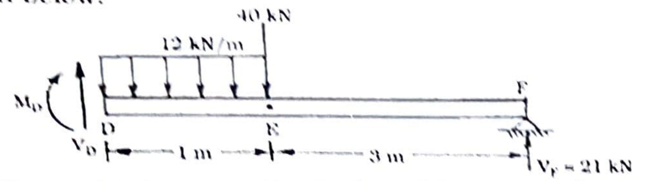


Fig 6.23

Now using two equations of equilibrium, we have.

$\Sigma F_y = 0$, ↑+ve, $V_D - 12 \times 1 - 40 + V_F = 0$

$\boxed{V_D = 31 \text{ kN}}$

$\Sigma M_D = 0$, ↻+ve, $M_D + 12 \times 1 \times \frac{1}{2} + 40 \times 1 - V_F \times 4 = 0$

$\boxed{M_D = 38 \text{ kN.m}}$

Note that here positive sign for shear force and bending moment nich is maintained as at point B of example 1.

Determine the shear force and moment at the beam sections ssing through points B and D of the beam shown here.

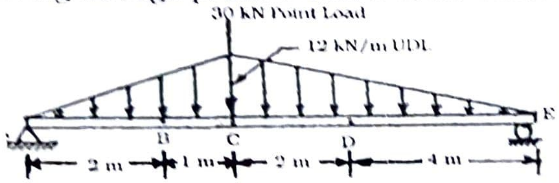


Fig 6.24 Simply supported beam

Here point load of 30 kN is acting at C in addition to triangular ading acting on the complete beam.

The free body diagram of the beam is shown below.

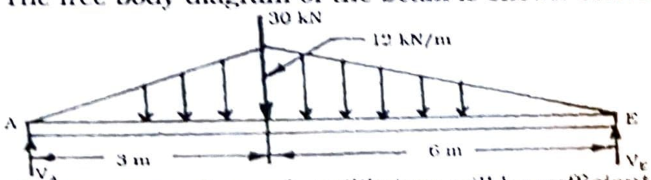


Fig 6.25

Only two equations of equilibrium will be sufficient to give two nknown reactions.

$\Sigma M_A = 0$, ↻+ve, $(1/2 \times 3 \times 12) \times (2/3 \times 3) + 30 \times 3 + (1/2 \times 6 \times 12)(3 + 1/3 \times 6) - V_E \times 9 = 0$

$\boxed{V_E = 34 \text{ kN}}$

$\Sigma F_y = 0$, ↑+ve, $V_A - 1/2 \times 3 \times 12 - 30 - 1/2 \times 6 \times 12 + V_E = 0$

$\boxed{V_A = 50 \text{ kN}}$

Unknown V_A can be obtained otherwise by using moment equation at joint E.

$\Sigma M_E = 0$, ↻+ve, $V_A \times 9 - (1/2 \times 3 \times 12)(6 + 1/3 \times 3) - 30 \times 6 - (1/2 \times 6 \times 12)(2/3 \times 6) = 0$

$\boxed{V_A = 50 \text{ kN}}$

The free-body diagram of the left segment of AB is shown here.

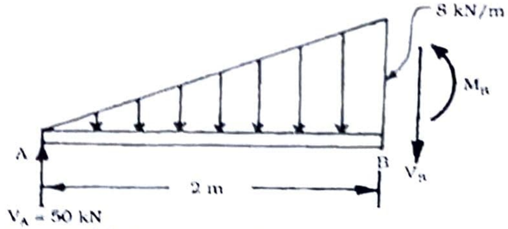


Fig 6.26

Using equilibrium equations,

$\Sigma F_y = 0$, ↑+ve, $V_A - (1/2 \times 2 \times 8) - V_B = 0$

$\boxed{V_B = 42 \text{ kN}}$

$\Sigma M_B = 0$, ↻+ve, $V_A \times 2 - (1/2 \times 2 \times 8)(1/3 \times 2) - M_B = 0$

$\boxed{M_B = 94.67 \text{ kN.m}}$

The same results could have been obtained by considering right segment BE.

Now, for obtaining shear and moment at point D, right segment DE is considered, which is shown below.

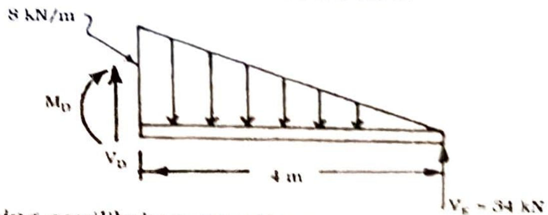


Fig 6.27

Using equilibrium equations,

$\Sigma F_y = 0$, ↑+ve, $V_D - 1/2 \times 4 \times 8 + V_E = 0$

$\boxed{V_D = -18 \text{ kN}}$

$\Sigma M_D = 0$, ↻+ve, $M_D + (1/2 \times 4 \times 8)(1/3 \times 4) - V_E \times 4 = 0$

$\boxed{M_D = 114.67 \text{ kN.m}}$

5. Determine the axial force, shear force and bending moment at the section passing through B of the cantilever beam shown here.

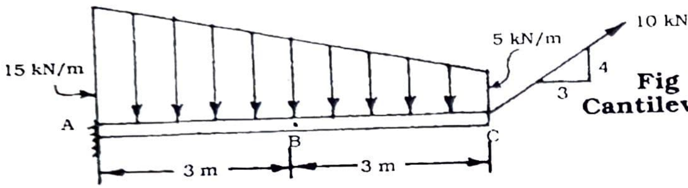


Fig 6·28
Cantilever beam

Here the required values at B can be obtained by two ways. In case, if we want to consider the left part of section B i.e. AB, then we need to find reactions at A considering free-body diagram of the entire beam and using three equations of equilibrium. Otherwise, we can consider the right part of the section, i.e. BC and find the required values straightway. Here second approach is selected.

Free-body of the right part of section B is shown below.

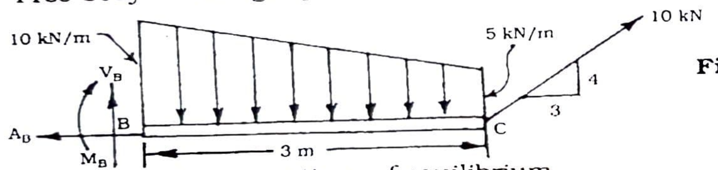


Fig 6·29

Now using three equations of equilibrium

$\Sigma F_x = 0, \ \overrightarrow{+ve}, \quad - A_B + (0{\cdot}6)\ 10 = 0$

$$\boxed{A_B = 6 \text{ kN}}$$

$\Sigma F_y = 0, \ \uparrow +ve, \quad V_B - (5 \times 3) - 1/2\ (5 \times 3) + (0{\cdot}8) \times 10 = 0$

$$\boxed{V_B = 14{\cdot}5 \text{ kN}}$$

$\Sigma M_B = 0, \circlearrowright +ve, \quad M_B + (5 \times 3)\ (1/2 \times 3) + (1/2 \times 5 \times 3)\ (1/3 \times 3) - (0{\cdot}8) \times 10 \times 3 = 0$

$$\boxed{M_B = -\ 6 \text{ kN.m}}$$

6. Draw the shear force and bending moment diagram for the simply supported beam subjected to point load as shown here.

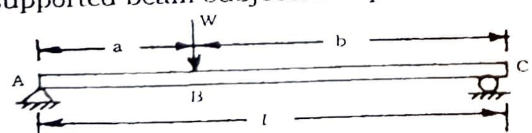


Fig 6·30

The support reactions can be obtained by considering the equilibrium of entire beam as shown below.

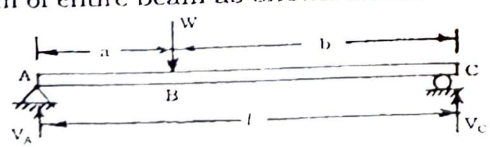


Fig 6·31

$\Sigma M_A = 0, \circlearrowright +ve, \quad W\,a - V_c\, l = 0$

$$\boxed{V_C = \frac{W\,a}{l}}$$

$\Sigma F_y = 0, \uparrow +ve, \quad V_A - W + V_C = 0$

$$\boxed{V_A = \frac{W\,b}{l}}$$

General equation for shear force and bending moment in the region AB can be obtained by cutting beam at distance x from A within AB and considering equilibrium of either part. Here free-body diagram of the left hand part is shown.

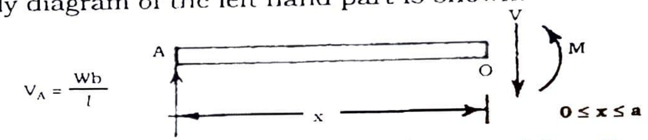


Fig 6·32

Direction selected for shear force and bending moment is positive as per the sign convention adopted.

$\Sigma F_y = 0, \uparrow +ve, \quad V_A - V = 0$

$$\boxed{V = \frac{W\,b}{l}} \qquad \text{———— (1)}$$

$\Sigma M_O = 0, \circlearrowright +ve, \ V_A\,x - M = 0$

$$\boxed{M = \frac{W\,b\,x}{l}} \qquad \text{———— (2)}$$

In the same way free-body diagram of the left hand part within BC is drawn below.

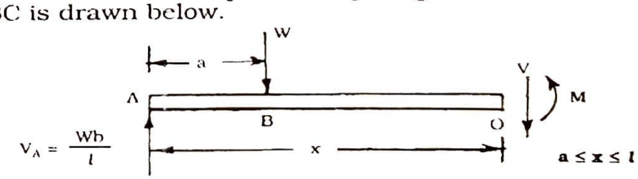


Fig 6·33

Using two equations of equilibrium, we have

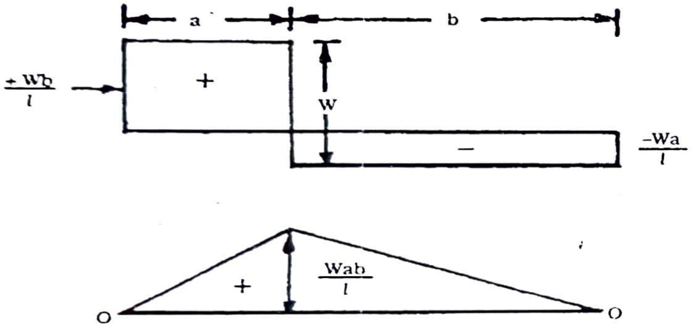
$\Sigma F_y = 0, \uparrow +ve, \quad V_A - W - V = 0$

$$\boxed{V = -\frac{W\,a}{l}} \qquad \text{———— (3)}$$

$\Sigma M_O = 0 \circlearrowright +ve, \quad V_A\,x - W\,(x - a) - M = 0$

$$\boxed{M = \frac{Wa\,(l-x)}{l}} \qquad \text{———— (4)}$$

From the equations for shear force i.e. equations (1) and (3), we can see that shear force is maintaining constant value in the particular portion. Within portion AB shear force has positive value and in portion BC it is negative i.e. at point B there is sudden drop of the magnitude W. While from equations for bending moment i.e. equations (2) and (4), we can see that bending moment variation is linear. In portion AB it increases linearly from the value zero to positive value of Wab/l. In portion BC it decreases linearly from the positive value of Wab/l to value zero. Note that at supports, value of bending moment is zero and under the load it has got unique value Wab/l obtained from either of two equations. Finally the shear force and bending moment diagrams can be drawn as shown below.



Shear force diagram

Bending-moment diagram

Fig 6·34

7. Draw the shear and bending moment diagram for the beam shown here.

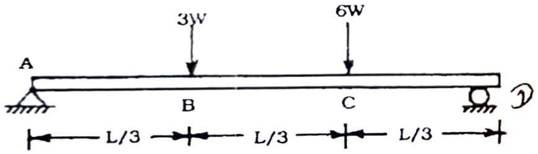


Fig 6·35

Considering the free-body of the entire beam the reactions can be obtained.

$$\Sigma M_A = 0, \circlearrowright+ve, \quad 3W \times \frac{L}{3} + 6W \times \frac{2L}{3} - V_D \times L = 0$$

$$\boxed{V_D = 5W}$$

$$\Sigma F_y = 0, \uparrow+ve, \quad V_A + V_D - 3W - 6W = 0$$

$$\boxed{V_A = 4W}$$

Support reactions are shown below.

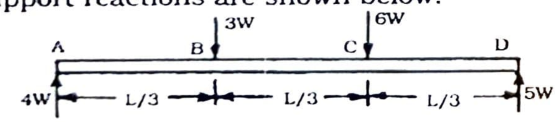


Fig 6·36

The beam is cut at an imaginary section at a distance x from A within AB and free-body diagram of the left part is shown below. The unknowns, shear and moment on the section are shown in positive direction as per the convention.

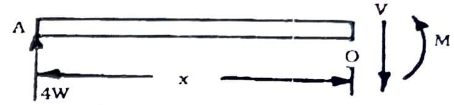


Fig 6·3

$$\Sigma F_y = 0, \uparrow ve, \quad 4W - V = 0$$

$$V = 4W \qquad \text{—— (1)}$$

$$\Sigma M_O = 0, \circlearrowright +ve, \; 4W\,x - M = 0$$

$$M = 4W\,x \qquad \text{—— (2)}$$

Now a free-body diagram of section at a distanc x from A within BC is shown here.

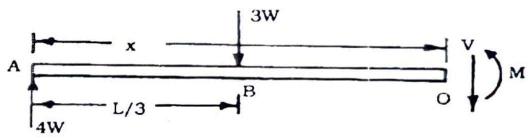


Fig 6·3

$$\Sigma F_y = 0, \uparrow+ve, \quad 4W - 3W - V = 0$$

$$V = W \qquad \text{—— (3)}$$

$$\Sigma M_o = 0, \circlearrowright +ve, \; 4W\,x - 3W\,(x - L/3) - M = 0$$

$$M = W\,x + WL \qquad \text{—— (4)}$$

Now a free-body diagram of section at a distance x from A within CD is shown below.

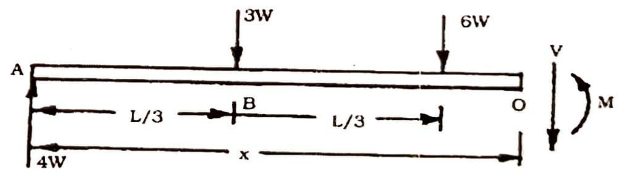


Fig. 6·3

$$\Sigma F_y = 0, \uparrow+ve, \quad 4W - 3W - 6W - V = 0$$

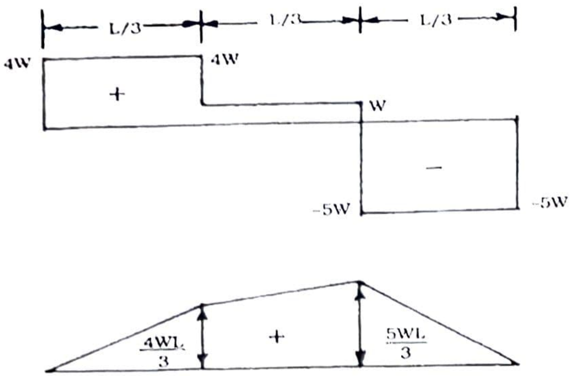
$$V = -5W \qquad \text{—— (5)}$$

$$\Sigma M_O = 0, \circlearrowright +ve, \; 4W\,x - 3W\,(x - \frac{L}{3}) - 6\,W\,(x - \frac{2L}{3}) - M = 0$$

$$M = -5Wx + 5WL \qquad \text{—— (6)}$$

Shear and moment diagrams can be plotted using the above equations (1) to (6) in the respective regions as shown here.

14



Shear force diagram

Bending moment diagram

Fig. 6·40

8. Draw shear force and bending moment diagram for the simply supported beam of span l, subjected to uniformly distributed load as shown below.

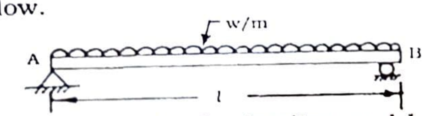


Fig 6·41

There being symmetry for loading and beam, reactions at A and B will be half of the total load. The free body diagram of entire beam will be as shown here.

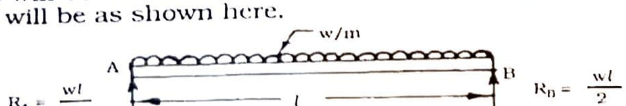


Fig 6·42

General equation for shear force and bending moment at any section at distance x from support A can be obtained by considering the free body of either portion. Here it is shown for the left hand part of the beam.

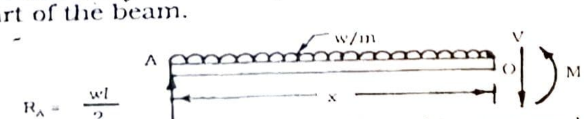


Fig 6·43

From the two equations of equilibrium, we have

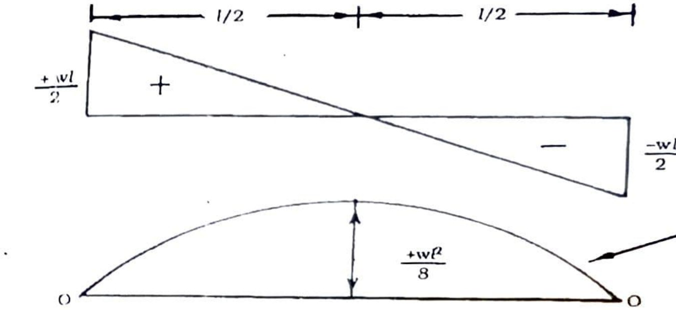
$\Sigma F_y = 0, \uparrow +ve, \quad R_A - wx - V = 0$

$$\boxed{V = w\,(l/2 - x)}$$

$\Sigma M_o = 0, \circlearrowright +ve, \quad R_A\,x - wx\,\dfrac{x}{2} - M = 0$

$$\boxed{M = \frac{wx\,(l - x)}{2}}$$

Here variation of shear force is linear. It has positive value of magnitude $wl/2$ at support A and it reduces to negative value of same magnitude at support B. It crosses the side at centre, say at point C. While bending moment has got zero value at both the supports and variation is parabolic with maximum value of $+\,wl^2/8$ at centre. The complete diagrams will be as shown below.



Shear force diagram

Bending moment diagram

Fig 6·44

9. Draw the shear force and bending moment diagram for the simply supported beam subjected to uniformly distributed load on right-hand half of the span as shown below.

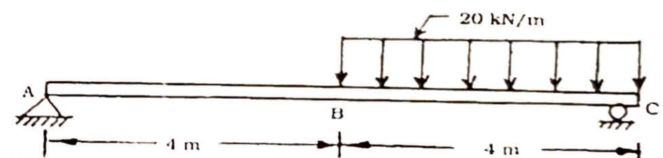


Fig 6·45

Reactions for beam can be obtained from two equations of equilibrium as

$\Sigma M_A = 0, \circlearrowright +ve, \quad 20 \times 4 \times (4 + 4/2) - R_C \times 8 = 0$

$$\boxed{R_C = 60 \text{ kN}}$$

$\Sigma F_y = 0, \uparrow +ve, \quad R_A - 4 \times 20 + R_c = 0$

$$\boxed{R_A = 20 \text{ kN}}$$

This can be obtained otherwise by using one more equation of equilibrium for moment as

$\Sigma M_c = 0, \circlearrowright +ve, \quad R_A \times 8 - 20 \times 4 \times \dfrac{4}{2} = 0$

$$\boxed{R_A = 20 \text{ kN}}$$

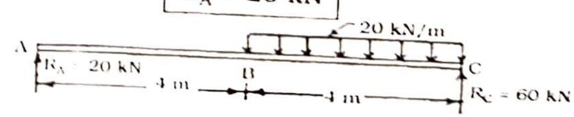


Fig 6·46

Now general equation for variation of shear force and bending moment can be obtained by considering the sections in portion AB and BC. Free body diagram of left hand part of beam at distance x from A within AB will be as shown here.

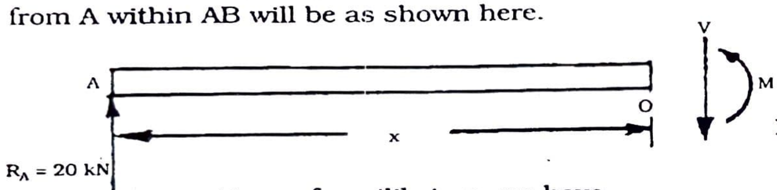


Fig 6·47

From equations of equilibrium, we have

$\Sigma F_y = 0, \uparrow$+ve, $R_A - V = 0$

$$\boxed{V = 20 \text{ kN}} \qquad \text{——— (1)}$$

$\Sigma M_O = 0, \circlearrowright$+ve, $R_A\, x - M = 0$

$$\boxed{M = 20\, x \text{ kN. m}} \qquad \text{——— (2)}$$

Free body diagram of left hand part of beam at distance x from A within BC will be as shown below.

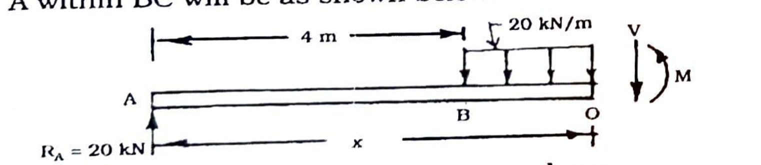


Fig 6·48

From equations of equilibrium, we have

$\Sigma F_y = 0, \uparrow$+ve, $R_A - 20\,(x-4) - V = 0$

$$\boxed{V = 100 - 20\,x} \qquad \text{——— (3)}$$

$\Sigma M_O = 0, \circlearrowright$+ve, $R_A\, x - 20\,(x-4)\dfrac{(x-4)}{2} - M = 0$

$$\boxed{M = 20x - 10\,(x-4)^2} \qquad \text{——— (4)}$$

From the above equations (1) and (2), we can note that when shear force is constant the variation of bending moment is linear. While from equations (3) and (4) we can note that when variation of shear force is linear, bending moment variation is parabolic.

From equation (3), we can see that shear force has got positive value of magnitude 20 kN at B and decreases constantly. At support C the shear force has negative value of magnitude 60 kN. Value will be zero at point x = 5 m.

Bending moment at point B is + 80 kN.m and becomes zero at support C, in between the variation is parabolic as observed from equation (4). For obtaining maximum, differentiate equation (4) with respect to x and equate it to zero.

$$\frac{dM}{dx} = 20\,x - 100 = 0$$

$$\therefore\ x = 5\text{m}$$

Value will be $20 \times 5 - 10\,(5-4)^2$

$$M_{max} = 90 \text{ kN. m}$$

Thus we can observe that bending moment diagram has attained the maximum value at the same section where shear force is zero. The diagrams will be as shown below.

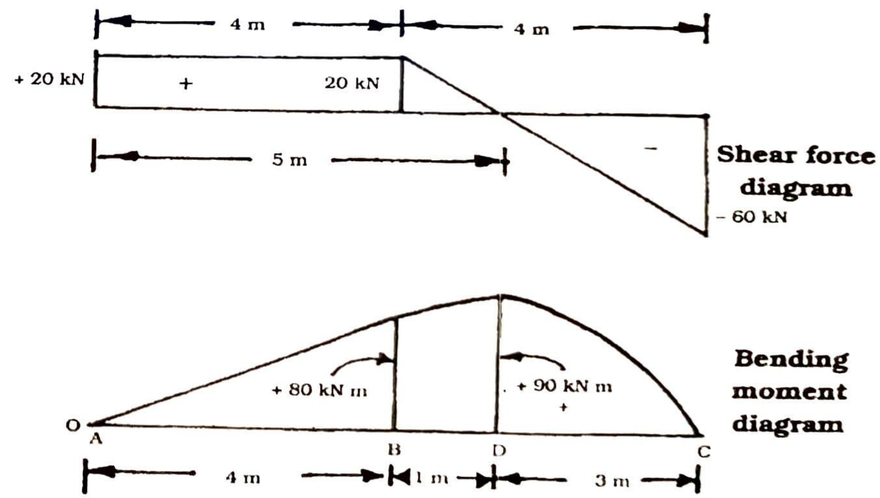


Fig 6·49

Here the shape of the bending moment diagram should be noted. When shear force line is horizontal, the bending moment line is inclined and when shear force line is inclined, the bending moment line is curve.

Now the curvature of bending moment diagram is required to be noted. It is convex in BC portion.

To check the curvature, use the equation $\frac{dM}{dx} = V$. *Slope of moment diagram is equal to shear force at that point.*

Slope of B. M. diagram at C is maximum, ∴ SF should be maximum at C. Slope of B.M. diagram at D is Zero, ∴ SF should be zero at D.

10. Prepare shear force and bending moment diagram for the beam shown below.

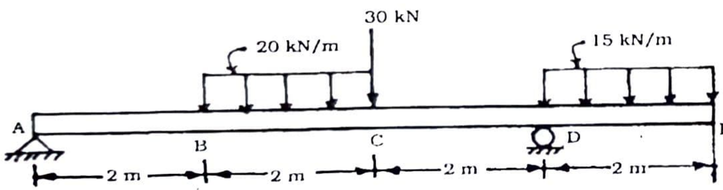


Fig. 6.50

Reactions at supports can be obtained using equations of equilibrium as.

$$\Sigma \mathbf{M_A} = \mathbf{0}, \circlearrowright +\text{ve}, \quad 20 \times 2 \times \left(2 + \frac{2}{2}\right) + 30 \times 4 + 15 \times 2 \times \left(6 + \frac{2}{2}\right) - R_D \times 6 = 0$$

$$\boxed{R_D = 75 \text{ kN}}$$

$$\Sigma \mathbf{F_y} = \mathbf{0}, \uparrow +\text{ve}, \quad R_A - 20 \times 2 - 30 + R_D - 15 \times 2 = 0$$

$$\boxed{R_A = 25 \text{ kN}}$$

General equations for shear force and bending moment can be obtained for various portions by considering the equilibrium of either part of the section. Here we need to consider four parts, namely AB, BC, CD and DE.

Free body diagram of the left part of beam at distance x from A within AB will be as shown here.

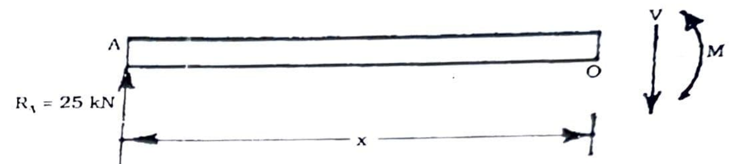


Fig. 6.51

From equations of equilibrium, we have

$$\Sigma \mathbf{F_y} = \mathbf{0}, \uparrow +\text{ve}, \quad R_A - V = 0$$

$$\boxed{V = 25 \text{ kN}} \qquad \text{———— (1)}$$

$$\Sigma \mathbf{M_O} = \mathbf{0}, \circlearrowright +\text{ve}, \quad R_A \cdot x - M = 0$$

$$\boxed{M = 25 \cdot x \text{ kN m}} \qquad \text{———— (2)}$$

Similarly, free body diagram of the left part of beam at distance x from A within BC will be as shown below.

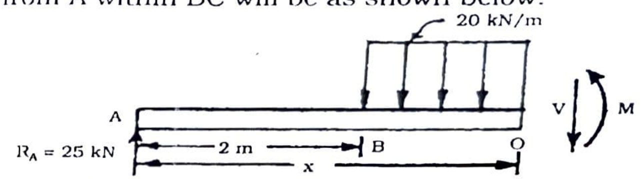


Fig. 6.52

From equations of equilibrium, we have

$$\Sigma \mathbf{F_y} = \mathbf{0}, \uparrow +\text{ve}, \quad R_A - 20\,(x - 2) - V = 0$$

$$\boxed{V = 65 - 20\,x \text{ kN}} \qquad \text{———— (3)}$$

$$\Sigma \mathbf{M_O} = \mathbf{0}, \circlearrowright +\text{ve}, \quad R_A \cdot x - 20\,(x - 2)\,\frac{(x - 2)}{2} - M = 0$$

$$\boxed{M = -10x^2 + 65x - 40 \text{ kN m}} \qquad \text{—— (4)}$$

Free-body diagram of the left part of beam at distance x from A within CD will be as shown below.

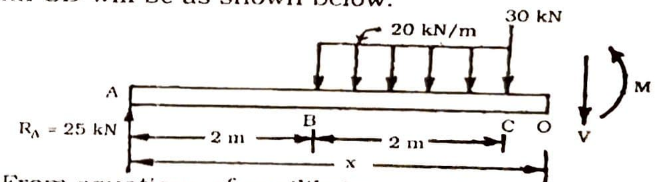


Fig 6.53

From equations of equilibrium, we have

$$\Sigma \mathbf{F_y} = \mathbf{0}, \uparrow +\text{ve}, \quad R_A - 20 \times 2 - 30 - V = 0$$

$$\boxed{V = 45 \text{ kN}} \qquad \text{———— (5)}$$

$$\Sigma \mathbf{M_O} = \mathbf{0}, \circlearrowright +\text{ve}, \quad R_A \cdot x - 20 \times 2 \times \left(x - 4 + \frac{2}{2}\right) - 30\,(x - 4) - M = 0$$

$$\boxed{M = -45x + 240 \text{ kN·m}} \qquad \text{———— (6)}$$

Free body diagram of the left part of beam at distance x from A within DE will be as shown here.

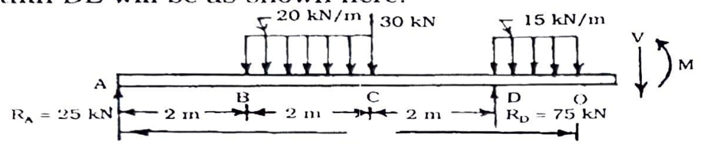


Fig 6·54

From equations of equilibrium we have

$\Sigma F_y = 0$, ↑+ve, $R_A - 20 \times 2 - 30 + R_D - 15\,(x - 6) - V = 0$

$$\boxed{V = 120 - 15\,x \text{ kN}} \quad \text{——— (7)}$$

$\Sigma M_O = 0$, ↻ +ve, $R_A \cdot x - 20 \times 2\,(x - 6 + 2 + \frac{2}{2}) - 30\,(x - 6 + 2) + R_D\,(x - 6) - 15\,(x - 6)\,(x - 6)/2 - M = 0$

$$\boxed{M = -\,7{\cdot}5x^2 + 120x - 480 \text{ kN m}} \quad \text{—— (8)}$$

Using equations (1) to (8), it will be possible to draw shear force and bending moment diagrams. At the junction points, the numerical values for shear force and bending moment can be varified from the equations available for adjacent portions. We will be also interested to know the location where shear force crosses the sign and hence value of maximum bending moment at such points. From the study of equations, we can see that from equations (3) and (7), we can have locations where shear force has got zero value.

From equation (3)

$$V = 65 - 20\,x = 0$$

$$x = 3{\cdot}25 \text{ m}$$

and value of maximum bending moment from equation (4) will be

$$M = -\,10 \times 3{\cdot}25^2 + 65 \times 3{\cdot}25 - 40$$
$$= 65{\cdot}625 \text{ kN·m}$$

From equation (7)

$$V = 120 - 15\,x = 0$$
$$x = 6 \text{ m}$$

and value of maximum bending moment from equation (8) will be

$$M = -\,7{\cdot}5 \times 6^2 + 120 \times 6 - 480$$
$$= -\,30 \text{ kN·m}$$

Here we can see that **shear force is changing sign at two locations,** hence **two maximum value of bending moments** are obtained which are **having opposite signs.**

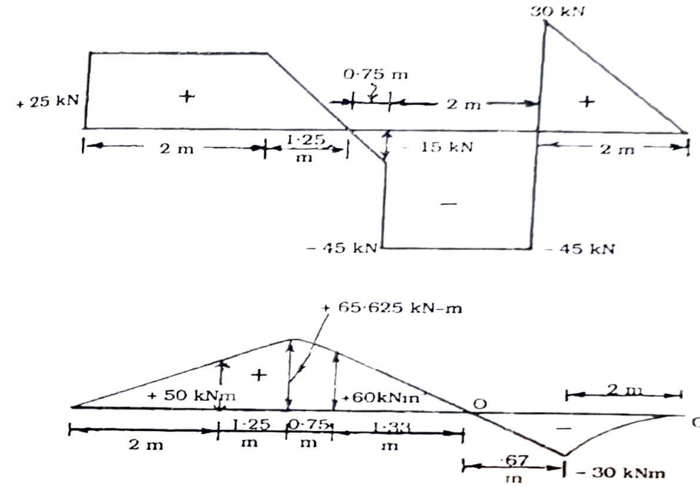


Fig 6·55

Here, from bending moment diagram, we can observe that at x = 5·33 m bending moment changes the sign. This point is known as ***point of centraflexure at which the nature of curvature of deformed beam changes.***

11. Draw shear force and bending moment diagram for the both side overhang beam shown here.

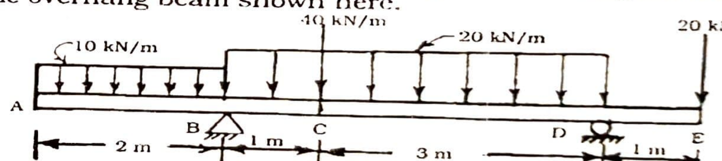


Fig 6·56

Here support reactions can be obtained by considering equilibrium of entire beam as

$\Sigma M_B = 0$, +ve,↻ $-\,10 \times 2 \times \frac{2}{2} + 40 \times 1 + 20 \times 4 \times \frac{4}{2} - R_D \times 4 + 20 \times 5 = 0$

$$\boxed{R_D = 70 \text{ kN}}$$

$\Sigma F_y = 0$, ↑+ve, $-\,10 \times 2 + R_B - 40 - 20 \times 4 + R_D - 20 = 0$

$$\boxed{R_B = 90 \text{ kN}}$$

Now general expression for shear force and bending moment can be obtained by considering equilibrium of either part of beam various portions. We need to consider four parts as AB, BC, CD and DE.

Free body diagram of the left part of beam at distance x from A within portion AB will be shown below.

AB

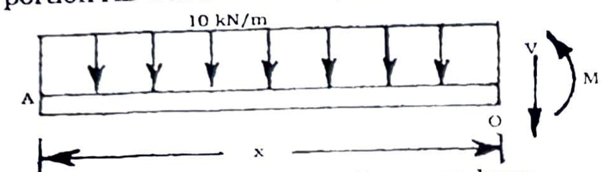


Fig. 6·57

Considering equilibrium equations, we have

$\Sigma F_y = 0$, ↑+ve, $\quad -10 \cdot x - V = 0$

$$\boxed{V = -10x \text{ kN}} \qquad \text{——— (1)}$$

$\Sigma M_O = 0$, ↻+ve, $\quad -10 \cdot x \dfrac{x}{2} - M = 0$

$$\boxed{M = -5x^2 \text{ kN·m}} \qquad \text{——— (2)}$$

Similary, we will consider equilibrium of free body diagram of the left part of beam in various portions as below.

BC

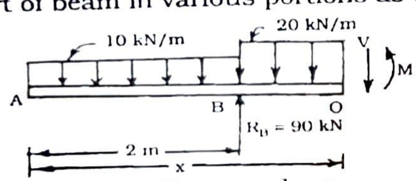


Fig. 6·58

From equilibrium equations, we have

$\Sigma F_y = 0$, ↑+ve, $\quad -10 \times 2 + R_B - 20\,(x - 2) - V = 0$

$$\boxed{V = (110 - 20\,x) \text{ kN}} \qquad \text{——— (3)}$$

$\Sigma M_O = 0$, ↻+ve, $\quad -10 \times 2 \times \left(x - 2 + \dfrac{2}{2}\right) + R_B\,(x-2)$

$$-20\,(x-2)\,\frac{(x-2)}{2} - M = 0$$

$$\boxed{M = (-10x^2 + 110\,x - 200) \text{ kN·m}} \qquad \text{——— (4)}$$

CD

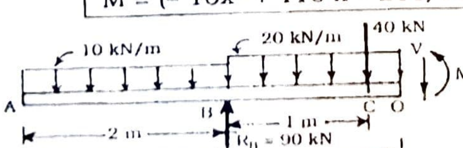


Fig. 6·59

From equilibrium equations, we have

$\Sigma F_y = 0$, ↑+ve, $\quad -10 \times 2 + R_B - 40 - 20\,(x - 2) - V = 0$

$$\boxed{V = (70 - 20x) \text{ kN}} \qquad \text{——— (5)}$$

$\Sigma M_O = 0$, ↻+ve, $\quad -10 \times 2 \times \left(x - 2 + \dfrac{2}{2}\right) + R_B\,(x-2) - 40 \times (x - 2 - 1)$

$$-20\,(x-2)\,\frac{(x-2)}{2} - M = 0$$

$$\boxed{M = (-10x^2 + 70\,x - 80) \text{ kN·m}} \qquad \text{——— (6)}$$

DE

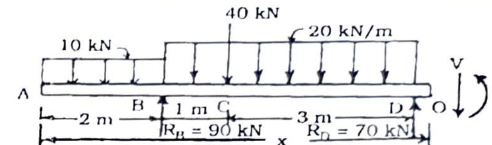


Fig. 6·60

From equilibrium equations, we have

$\Sigma F_y = 0$, ↑+ve, $\quad -10 \times 2 + R_B - 40 - 20 \times 4 + R_D - V = 0$

$$\boxed{V = 20 \text{ kN}} \qquad \text{——— (7)}$$

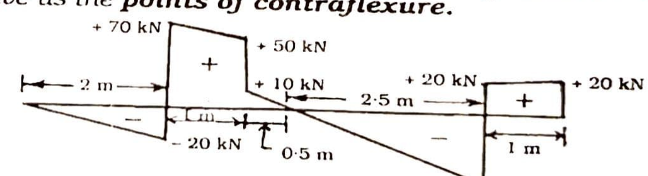
$\Sigma M_O = 0$, ↻+ve, $\quad -10 \times 2 \times \left(x - 6 + 4 + \dfrac{2}{2}\right) + R_B \times (x - 6 + 4)$

$$-40 \times (x - 6 + 3)$$

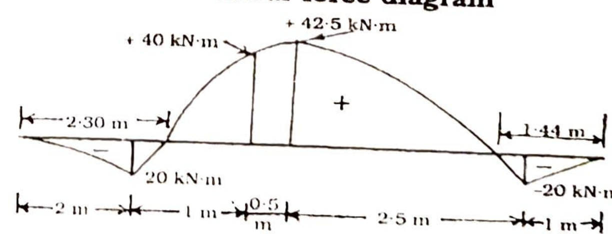
$$-20 \times 4\left(x - 6 + \frac{4}{2}\right) + R_D\,(x - 6) - M = 0$$

$$\boxed{M = (20\,x - 140) \text{ kN·m}} \qquad \text{——— (8)}$$

Now using equations (1) to (8), we can draw shear force and bending moment diagram for the beam. Before that, we will be interested to know the locations where shear force becomes zero and at the same locations value of maximum bending moment which is to be obtained. *The locations where bending moment changes sign, will give us the* ***points of contraflexure.***



Shear force diagram



Bending moment diagram

Fig. 6·61

From the shear force diagram, we can see that it crosses base at distance 2m, 3·5 and 6m from free end A. At the same locations bending moment diagram has peak values as – 20, kN·m, + 42·5 kN·m, – 20 kN·m respectively. The point of contraflexures are at distance of 2·30 m and 5·56 m from A.

2. Draw shear force and bending moment diagram for the beam shown here.

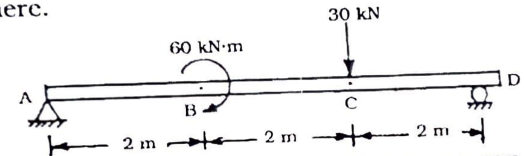


Fig 6·62

Considering equilibrium equations, we can obtain support actions as

$\Sigma M_A = 0$, ↺+ve, $60 + 30 \times 4 - R_D \times 6 = 0$

$\boxed{R_D = 30 \text{ kN}}$

$\Sigma F_y = 0$, ↑+ve, $R_A - 30 + R_D = 0$

$\boxed{R_A = 0}$

Now we can consider the equlibrium of one part in various portions to obtain general expressions for shear force and bending moment.

B

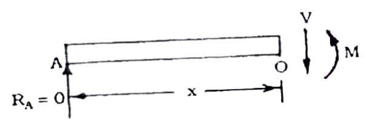


Fig. 6·63

$\Sigma F_y = 0$, ↑+ve, $R_A - V = 0$

$\boxed{V = 0}$ ——— (1)

$\Sigma M_O = 0$, ↺+ve, $R_A \cdot x - M = 0$

$\boxed{M = 0}$ ——— (2)

BC

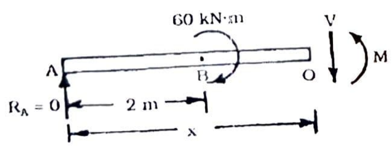


Fig. 6·64

$\Sigma F_y = 0$, ↑+ve, $R_A - V = 0$

$\boxed{V = 0}$ ——— (3)

$\Sigma M_O = 0$, ↺+ve, $60 - M = 0$

$\boxed{M = 60 \text{ kN·m}}$ ——— (4)

CD

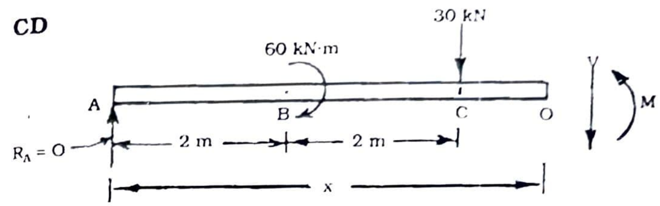


Fig. 6·65

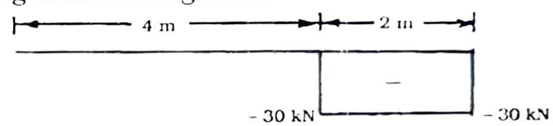
$\Sigma F_y = 0$, ↑+ve, $R_A - 30 - V = 0$

$\boxed{V = -30 \text{ kN}}$ ——— (5)

$\Sigma M_o = 0$, ↺+ve, $60 - 30(x - 4) - M = 0$

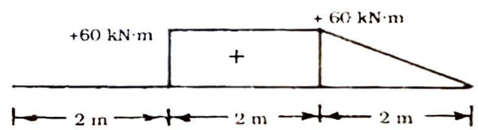
$\boxed{M = (-30x + 180) \text{ kN·m}}$ ——— (6)

Now using the equations (1) to (6), we can prepare shear force and bending moment diagrams.



Shear force diagram

Fig. 6·66



Bending moment diagram

13. Draw shear force and bending moment diagrams for the cantilever beam shown below.

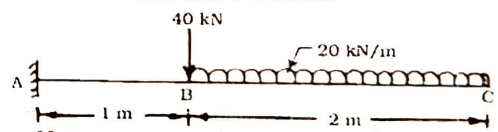


Fig 6·67
Cantilever beam

Here we can obtain general expressions for shear force and bending moment by considering the right hand part of the section. For the portions BC and AB the **distance x can be measured from the free end C. In doing so we have not to find support reactions.**

BC

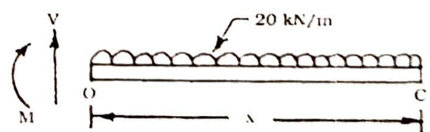


Fig. 6·68

$\Sigma F_y = 0$, ↑+ve, $V - 20 \times x = 0$

$\boxed{V = (20x) \text{ kN}}$ ——— (1)

$\Sigma M_o = 0, \circlearrowright +ve, \quad M + 20 \times x \times \frac{x}{2} = 0$

$\boxed{M = (-10x^2)\ kN{\cdot}m}$ ——— **(2)**

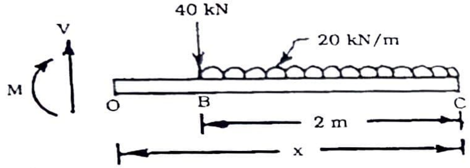


Fig 6·69

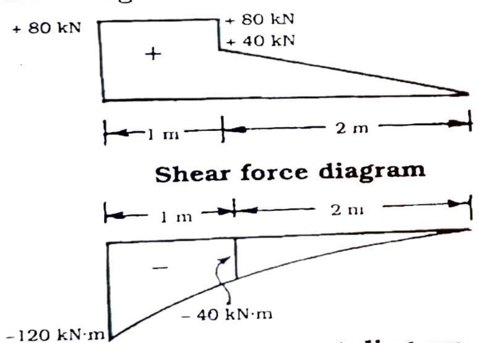
$\Sigma F_y = 0, \uparrow +ve, \quad V - 40 - 20 \times 2 = 0$

$\boxed{V = 80\ kN}$ ——— (3)

$\Sigma M_o = 0, \circlearrowright +ve, \quad M + 40\ (x - 2) + 20 \times 2\ (x - 2 + \frac{2}{2}) = 0$

$\boxed{M = (-80x + 120)\ kN{\cdot}m}$ ——— **(4)**

Using equations (1) to (4), we can prepare the shear force and :nding moment diagrams as



Shear force diagram

Bending moment diagram

Fig. 6·70

14. Draw shear force and bending moment diagram for the :antilever beam shown here.

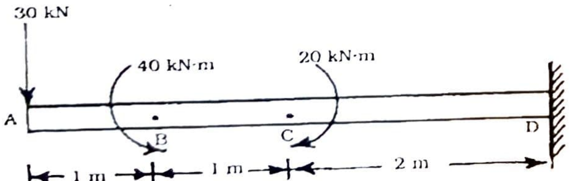


Fig. 6·71 Cantilever beam

Similar to previous example we can consider three portions AB, BC and CD.

AB

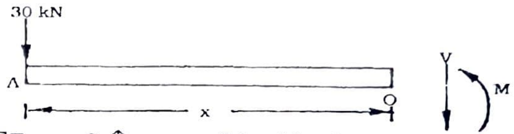


Fig 6·72

$\Sigma F_y = 0, \uparrow +ve, \quad -30 - V = 0$

$\boxed{V = -30\ kN}$ ——— (1)

$\Sigma M_o = 0, \circlearrowright +ve, \quad -30\ x - M = 0$

$\boxed{M = (-30\ x)\ kN{\cdot}m}$ ——— (2)

BC

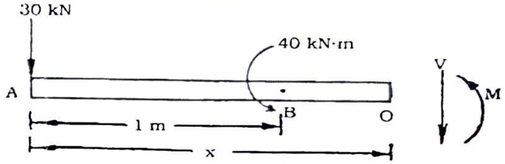


Fig. 6·73

$\Sigma F_y = 0, \uparrow +ve, \quad -30 - V = 0$

$\boxed{V = -30\ kN}$ ——— **(3)**

$\Sigma M_o = 0, \circlearrowright +ve, \quad -30\ x - 40 - M = 0$

$\boxed{M = (-30x - 40)\ kN{\cdot}m}$ ——— **(4)**

CD

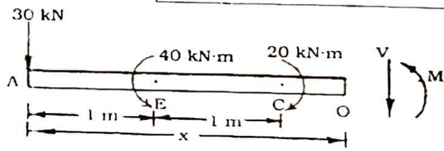


Fig 6·74

$\Sigma F_y = 0, \uparrow +ve, \quad -30 - V = 0$

$\boxed{V = -30\ kN}$ ——— **(5)**

$\Sigma M_o = 0, \circlearrowright +ve, \quad -30\ x - 40 + 20 - M = 0$

$\boxed{M = (-30x - 40 + 20)\ kN{\cdot}m}$ ——— **(6)**

Using equations (1) to (6), we can draw the shear force and bending moment diagrams for the beam.

Shear force diagram

Bending moment diagram

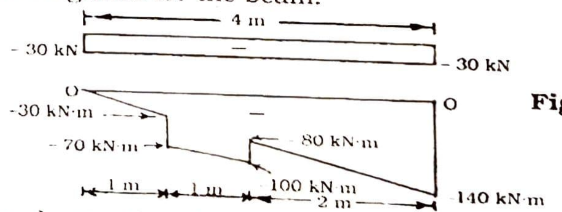


Fig. 6·75

Here we can observe that the shear force diagram has negative values allthroughout, while bending moment is negative as was the case in previous problem. Slope of bending moment diagram is same for the beam except the discontinuity of bending moment due to action of applied moments.

15. Determine the tension in each segment of the cable shown below. Also obtain the elevation of points B and D.

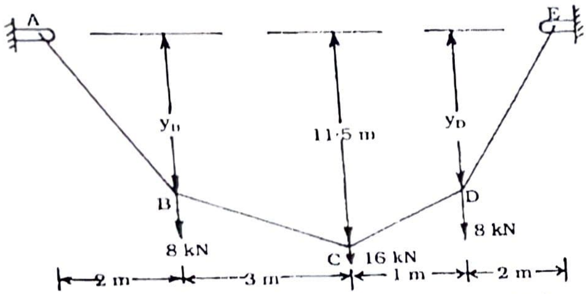


Fig. 6·76

Analysis can be performed by two procedures. In the first procedure the four support reactions, four cable tensions and two elevations i.e. the total ten unknowns are obtained considering the joint equilibrium at each points (five joints – A to E) which will give ten equations. While in another procedure, the direct approach is used which is adopted here.

If support reactions at A and E i.e. V_A, H_A, V_E and H_E are not to be determined and only four cable tensions plus two unknown distances (y_B and y_D) - total six unknowns are needed then by considering the equilibrium equations at B, C and D joints (Six equations) one can find these unknowns easily.

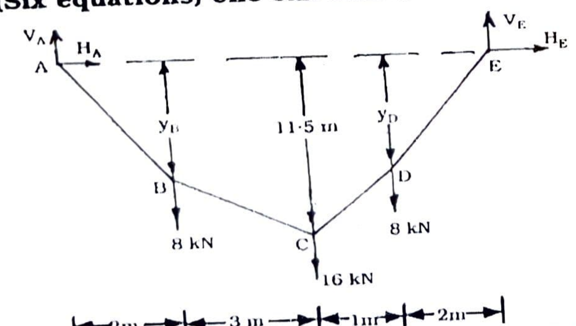


Fig. 6·77

Considering the free body of the entire cable,

$\Sigma M_A = 0$, ↻ +ve, $8 \times 2 + 16 \times 5 + 8 \times 6 - V_E \times 8 = 0$

$\boxed{V_E = 18 \text{ kN}}$

$\Sigma F_y = 0$, ↑+ve, $V_A - 8 - 16 - 8 + V_E = 0$

$\boxed{V_A = 14 \text{ kN}}$

$\Sigma F_x = 0$, $\overrightarrow{+ve}$, $H_A + H_E = 0$ ——— (1)

Now the fourth equation can be obtained by using the positi of joint C. We can use the equilibrium of either side of joint C. Bo will be able to give the remaining two horizontal reaction Considering the free body of the *left side of joint* C.

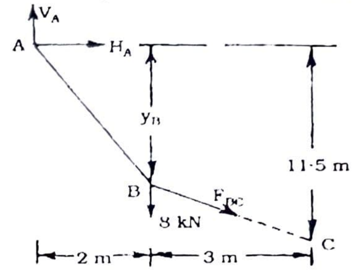


Fig 6·7

$\Sigma M_C = 0$, ↻ +ve, $V_A \times 5 + H_A \times 11{\cdot}5 - 8 \times 3 = 0$

$\boxed{H_A = -4 \text{kN}}$

Using equation (1), we have

$\boxed{H_E = 4 \text{ kN}}$

This can be obtained otherwise by considering the *equilibriu* of *right hand side of joint* C.

$\Sigma M_C = 0$, ↻ +ve, $-V_E \times 3 + H_E \times 11{\cdot}5 + 8 \times 1 = 0$

$\boxed{H_E = 4 \text{ kN}}$

Once the support reactions are obtained, we can start analyzir joint after joint. Use the force equilibrium at each joint.

Joint A

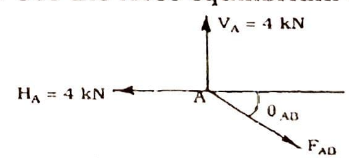


Fig. 6·7

$\Sigma F_x = 0$, $\overrightarrow{+ve}$, $-H_A + F_{AB} \cos \theta_{AB} = 0$

$\Sigma F_y = 0$, ↑+ve, $V_A - F_{AB} \sin \theta_{AB} = 0$

$\boxed{\theta_{AB} = 74{\cdot}05°}$

$\boxed{F_{AB} = 14{\cdot}56 \text{ kN}}$

Knowing the angle we can obtain the value of y_B

$y_B = 2 \times \tan \theta_{AB}$

$\boxed{y_B = 7 \text{ m}}$

Joint B

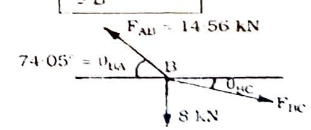


Fig. 6·8

15

$\Sigma F_x = 0, \overrightarrow{+ve}, \quad -F_{BA} \cos \theta_{BA} + F_{BC} \cos \theta_{BC} = 0$

$\Sigma F_y = 0, \uparrow +ve, \quad F_{BA} \sin \theta_{BA} - F_{BC} \sin \theta_{BC} - 8 = 0$

$\boxed{\theta_{BC} = 56{\cdot}31^\circ}$

$\boxed{F_{BC} = 7{\cdot}21 \text{ kN}}$

Joint C

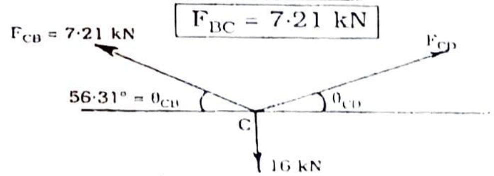


Fig 6·81

$\Sigma F_x = 0, \overrightarrow{+ve}, \quad -F_{CB} \cos \theta_{CB} + F_{CD} \cos \theta_{CD} = 0$

$\Sigma F_y = 0, \uparrow +ve, \quad F_{CB} \sin \theta_{CB} + F_{CD} \sin \theta_{CD} - 16 = 0$

$\boxed{\theta_{CD} = 68{\cdot}20^\circ}$

$\boxed{F_{CD} = 10{\cdot}77 \text{ kN}}$

Joint D

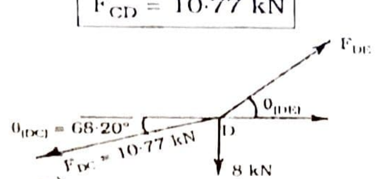


Fig. 6·82

$\Sigma F_x = 0, \overrightarrow{+ve}, \quad -F_{DC} \cos \theta_{DC} + F_{DE} \cos \theta_{DE} = 0$

$\Sigma F_y = 0, \uparrow +ve, \quad -F_{DC} \sin \theta_{DC} + F_{DE} \sin \theta_{DE} - 8 = 0$

$\boxed{\theta_{DE} = 77{\cdot}47^\circ}$

$\boxed{F_{DE} = 18{\cdot}44 \text{ kN}}$

Knowing the angle we can obtain value of y_D.

$y_D = 2 \times \tan \theta_{DE}$

$\boxed{y_D = 9\text{m}}$

Elevation can be obtained otherwise by using moment equilibrium at joint D for right hand part

$\Sigma M_D = 0, \circlearrowright +ve, \quad -V_E \times 2 + H_E \times y_D = 0$

$\boxed{y_D = 9\text{m}}$

Now joint E can provide two numerical checks.

Joint E

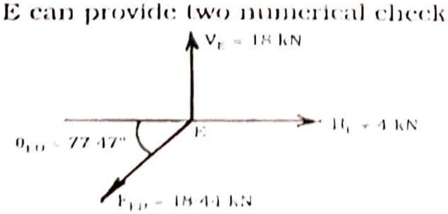


Fig 6·83

$\Sigma F_x = 0, \overrightarrow{+ve}, \quad -F_{ED} \cos \theta_{ED} + H_E$

$= -18{\cdot}44 \cos 77{\cdot}47 + 4 = 0$ **(check)**

$\Sigma F_y = 0, \uparrow +ve, \quad V_E - F_{ED} \sin \theta_{ED}$

$= 18 - 18{\cdot}44 \sin 77{\cdot}47 = 0$ **(check).**

Now we can prepare the diagram indicating results there-in.

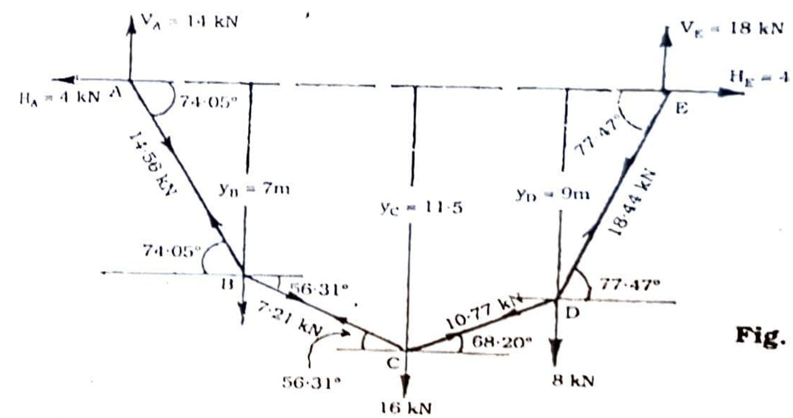


Fig. 6·84

From the results shown we can observe that the *member having least slope has got minimum force i.e. member BC, while the member having greatest slope has got maximum force i.e.* member DE. **This is due to the horizontal component of force being constant.**

16. A light cable of suspension bridge supports 9 N/m load between the two columns having span length 50 m. If the sag of the cable is 0·6 m, determine the maximum force developed in the cable, slope of cable at supports and the required length of the cable. *(As the cable is weightless, the* ***parabolic shape*** *of the cable is to be considered).*

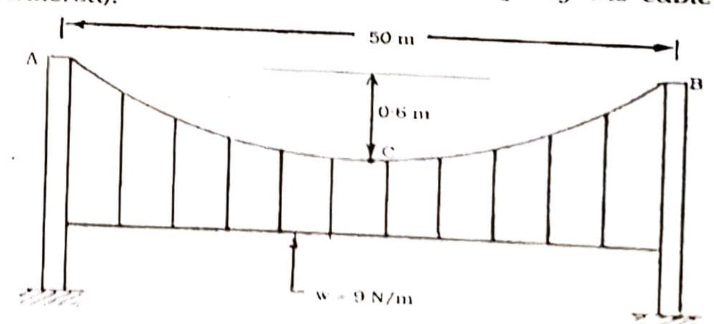


Fig. 6·85

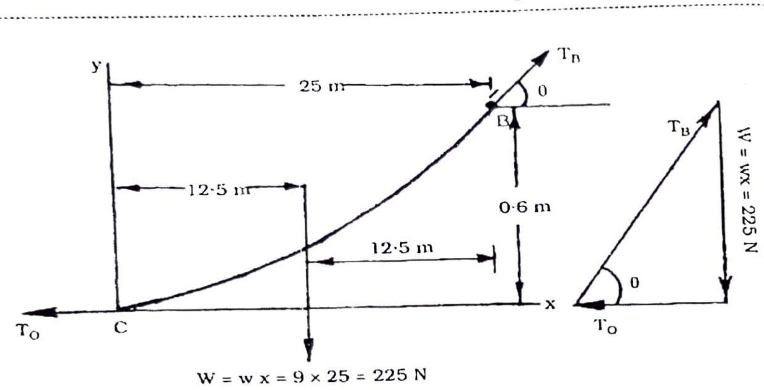


Fig. 6·86

The freebody diagram of cable **CB** is shown. The total load of CB will act halfway between C and B.

Total load $W = w\,x = 9 \times 25 = 225$ N

Taking moment about B,

$\Sigma M_B = 0$, +ve, $T_0 \times 0{\cdot}6 - w\,x \times 12{\cdot}5 = 0$

$0{\cdot}6\, T_0 - 225 \times 12{\cdot}5 = 0$

$\therefore\ T_0 = 4687{\cdot}5$ N

From the force triangle,

$T_B = \sqrt{T_0^2 + w^2x^2} = \sqrt{4687{\cdot}5^2 + 225^2} = 4692{\cdot}9$ N

$\boxed{T_B = T_{max} = 4692{\cdot}9 \text{ N}}$

From the force triangle,

$$\tan\theta = \frac{W}{T_0} = \frac{225}{4687{\cdot}5} = 0{\cdot}048$$

$\therefore\ \boxed{\theta = \text{slope of cable at support} = 2{\cdot}75^\circ}$

Now, length of the cable (CB),

$$s_B = x_B\left[1 + \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2\right]$$

$$= 25\left[1 + \frac{2}{3}\left(\frac{0{\cdot}6}{25}\right)^2\right]$$

$= 25{\cdot}0096$ m

The total length of the cable required (AB)

$= 2 \times 25{\cdot}0096$ m

$\boxed{\text{Length} = 50{\cdot}0192 \text{ m}}$

17. Determine the minimum and maximum tension in the uniform cable, the deflection curve and length of the cable if the weight of the cable is 5 N/m and having span 20 m, sag 6 m.

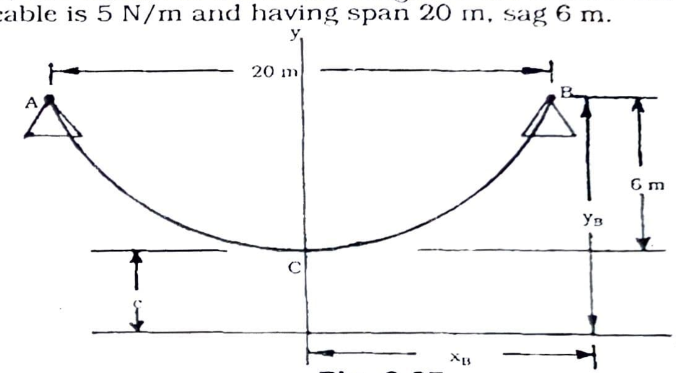


Fig. 6·87

Equation of cable : *The origin of coordinates is placed at a distance c below the lowest point of the cable.*

The equation of the cable is given by

$$y = c\cosh\frac{x}{c}$$

The coordinates of point B are

$x_B = 10$ m $\qquad y_B = 6 + c$

Substituting the coordinates into the equation of cable, we get

$$6 + c = c\cosh\frac{10}{c}$$

$$\therefore\ \frac{6}{c} + 1 = \cosh\frac{10}{c}$$

The value of c is determined by assuming successive trial values, as shown below, i.e. putting the different value of c and checking the equality of left and right portions of the equation.

[Here, $\cosh x = 1/2\,(e^x + e^{-x})$]

c	$\frac{6}{c}$	$\frac{10}{c}$	$\frac{6}{c} + 1$	$\cosh\frac{10}{c}$
18	0·333	0·555	1·333	1·158
25	0·24	0·4	1·24	1·081
10	0·6	1·0	1·6	1·54
9	0·667	1·11	1·667	1·682
8	0·75	1·25	1·75	1·83
9·5	0·63	1·05	1·63	1·60
9·2	0·65	1·087	1·65	1·65

Taking $c = 9{\cdot}2$,

We have, $y_B = 9{\cdot}2 + 6 = 15{\cdot}2$ m.

Maximum & minimum Tension :

$T_{min} = T_C = w\,c = (5\ \text{N/m})\,(9{\cdot}2\ \text{m})$ $\boxed{T_{min} = 46\ \text{N}}$

$T_{max} = T_B = w\,y_B = (5\ \text{N/m})\,(15{\cdot}2\ \text{m})$ $\boxed{T_{max} = 76\ \text{N}}$

ength of Cable :

Half length of cable can be found from equation, $y_B^2 - s_{CB}^2 = c^2$

$$\therefore\ s_{CB}^2 = y_B^2 - c^2 = (15{\cdot}2)^2 - (9{\cdot}2)^2$$

$$\therefore\ s_{BC} = 12{\cdot}1\ \text{m}$$

Therefore, $\boxed{\text{Total length of cable AB} = 24{\cdot}2\ \text{m}}$

The equation of deflected shape of cable is

$$y = c \cosh \frac{x}{c}$$

$$\boxed{y = 9{\cdot}2 \cosh \frac{x}{9{\cdot}2}}$$

THEORY RELATED QUESTIONS

1. Give examples of determinate beams. Also give examples of ndeterminate beams having degree of indeterminancy one, two and hree.

2. Indicate the sign conventions for shear force and bending noment adopted by you. Explain it with the help of shear force and bending moment diagram for the beams shown below.

(a)

(b)

(c)

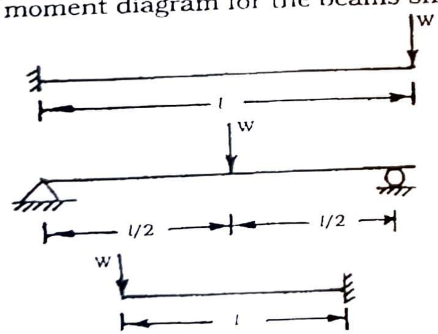


Fig. 6·88

3. Develop the relations between distributed load, shear force and bending moment. Explain it with the help of example.

4. Describe the steps for analysing cable or rope subjected to concentrated loads.

EXERCISES

6·1 to 6·8. Find the shear force and bending moment at the sections passing through the points shown therein for the beams shown below.

6·1 **Points B & D**

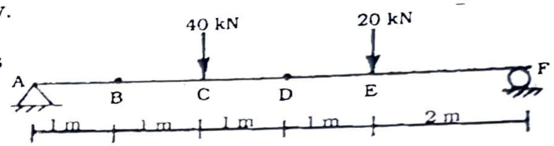


Fig. 6·89

6·2 **Points C & E**

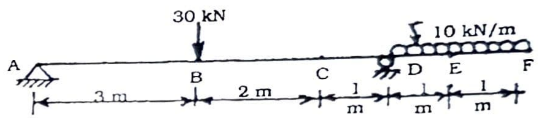


Fig. 6·90

6·3 **Points B & D**

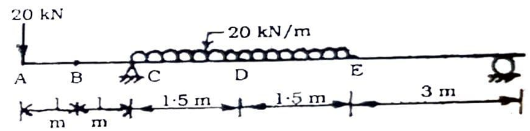


Fig. 6·91

6·4 **Points B, D & G**

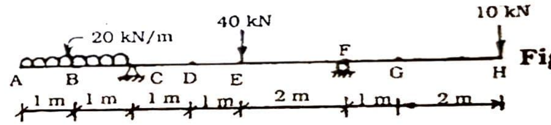


Fig. 6·92

6·5 **Points B & D**

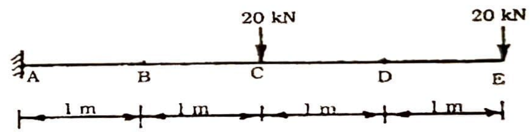


Fig. 6·93

6·6 **Points B & D**

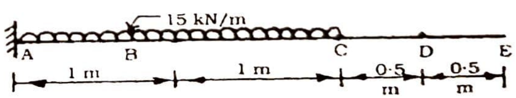


Fig. 6·94

6·7 **Points B & E**

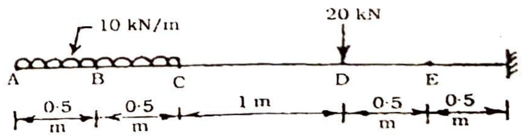


Fig. 6·95

6·8 **Points B, D & F**

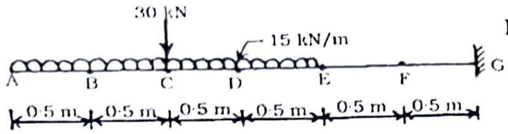


Fig. 6·96

6·9 To 6·22 Draw shear force and bending moment diagrams for the beams shown below.

6·9

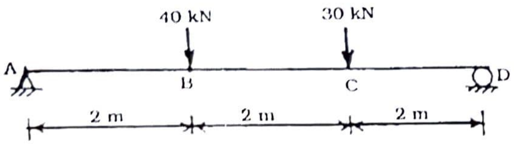


Fig. 6·97

6·10

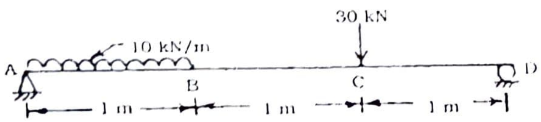


Fig. 6·98

6·11

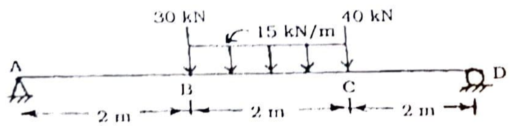


Fig. 6·99

6·12

Fig. 6·100

6·13

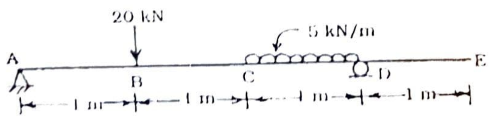


Fig. 6·101

6·14

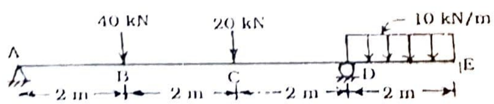


Fig. 6·102

6·15

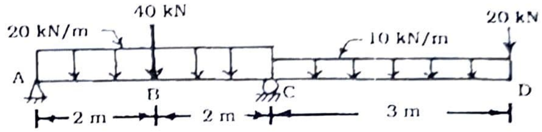


Fig. 6·103

6·16

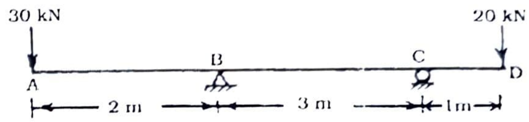


Fig. 6·104

6·17

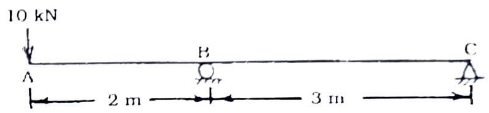


Fig. 6·105

6·18

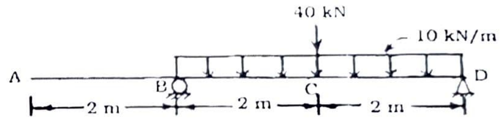


Fig. 6·106

6·19

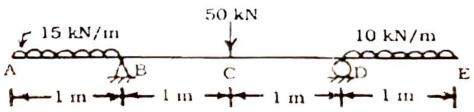


Fig. 6·107

6·20

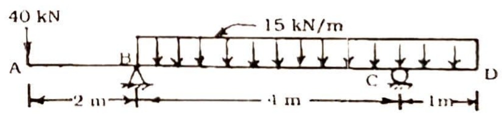


Fig. 6·108

6·21

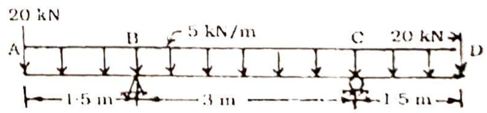


Fig. 6·109

6·22

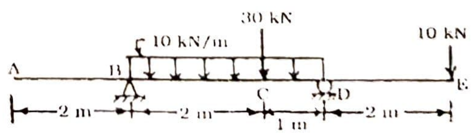


Fig. 6·110

6·23 To 6·27 Determine the tensions in the cable due to loads shown below (Reactions at supports are not needed). Consider only two equilibrium equations at intermediate cable joints.

6·23

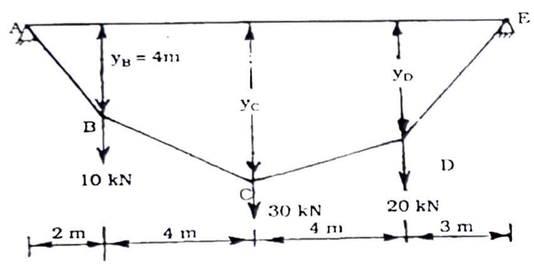


Fig. 6·111

6·24

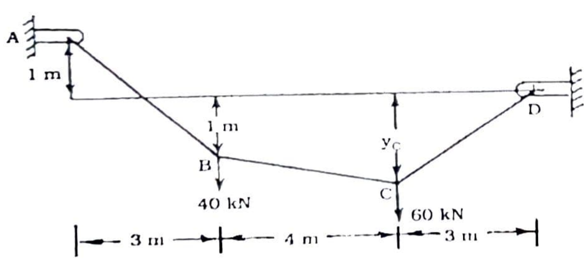


Fig. 6·112

6·25

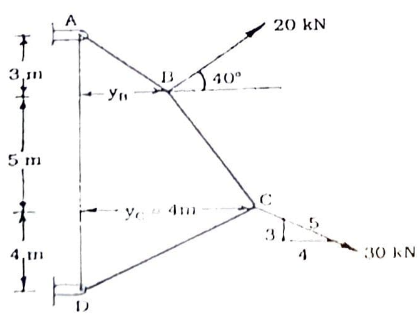


Fig. 6·113

6·26

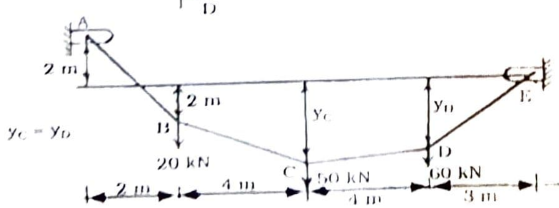


Fig. 6·114

6·27

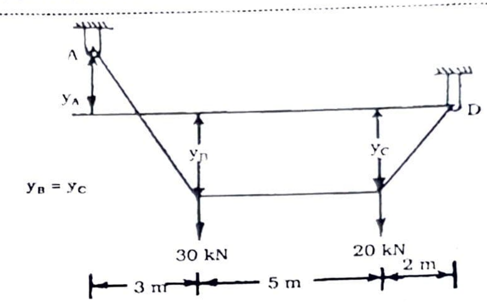


Fig. 6·115
T_{CD} = 60 kN

6·28 An electric wire spanning for 25 m between two poles has a mass per unit length of 400 g/m. Determine the smallest value of sag assuming parabolic-shape and if maximum tension in the wire is 300 N.

6·29 Determine the maximum tension in the cable and its sag if cable length is 22 m. The telephone wire has a mass of 450 g/m. Poles are at a distance of 20 m.

6·30 Determine the required sag s so that the resultant horizontal force extented by the cable at B is zero.

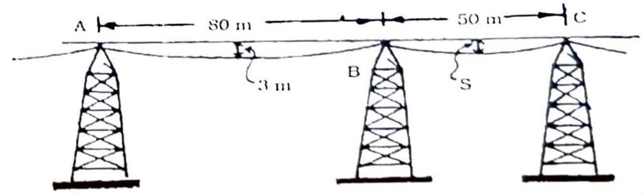


Fig 6·116

6·31 A cable is attached at B and passes over a small pulley at A. A counterweight of 400 N is attached to a cable after it passes over at A. knowing the span AB = 15 m and sag = 5m, determine **(a)** the length of cable from A to B **(b)** the mass per unit length of the cable. Neglect mass of cable at counterweight.

SOLUTIONS OF EXERCISES

6·1	V_B = + 33·33 kN,	M_B = + 33·33 kN·m
	V_D = − 6·67 kN,	M_D = + 59·99 kN·m.
6·2	V_C = − 18·33 kN,	M_C = − 1·65 kN·m
	V_E = + 10·0 kN,	M_E = − 5·0 kN·m
6·3	V_B = − 20·0 kN,	M_B = − 20·0 kN·m
	V_D = + 21·67 kN,	M_D = + 15·01 kN·m
6·4	V_B = − 20·0 kN,	M_B = − 10·0 kN·m
	V_D = + 22·5 kN,	M_D = − 17·5 kN·m
	V_G = + 10·0 kN,	M_G = − 20·0 kN·m
6·5	V_B = 40·0 kN,	M_B = − 80·0 kN·m
	V_D = + 20·0 kN,	M_D = − 20·0 kN·m

6·6 $V_B = +15{\cdot}0$ kN, $M_B = -7{\cdot}5$ kN·m

$V_D = 0{\cdot}0$ kN, $M_D = 0{\cdot}0$ kN·m

6·7 $V_B = -5{\cdot}0$ kN, $M_B = -1{\cdot}25$ kN·m

$V_E = -30{\cdot}0$ kN, $M_E = -30{\cdot}0$ kN·m

6·8 $V_B = -7{\cdot}5$ kN, $M_B = -1{\cdot}875$ kN·m

$V_D = -52{\cdot}5$ kN, $M_D = -31{\cdot}875$ kN·m

$V_F = -60{\cdot}0$ kN, $M_F = -90{\cdot}0$ kN·m

6·9 NOTE : While joining two points by curve in B.M. diagram, see the magnitudes of S.F. at these two points. Slope of B.M. diagram is equal to shear force. Maximum slope of B. M. at a point means maximum S.F. at that point.

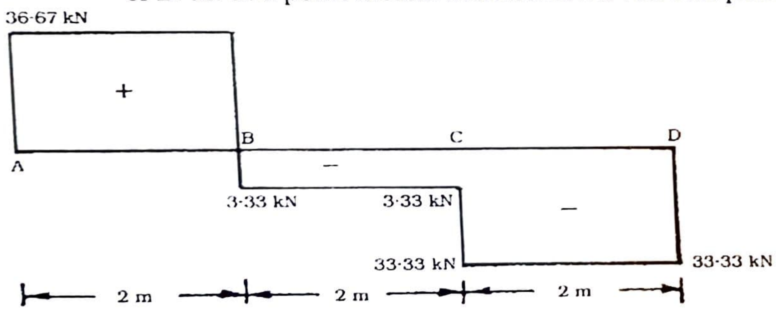


Fig. 6·117 (a) Shear Force Diagram

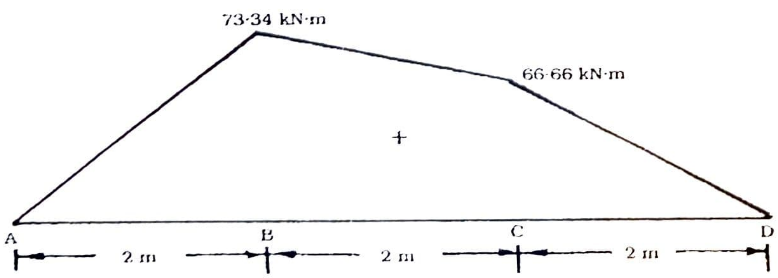


Fig 6·117 (b) Bending Moment Diagram

6·10

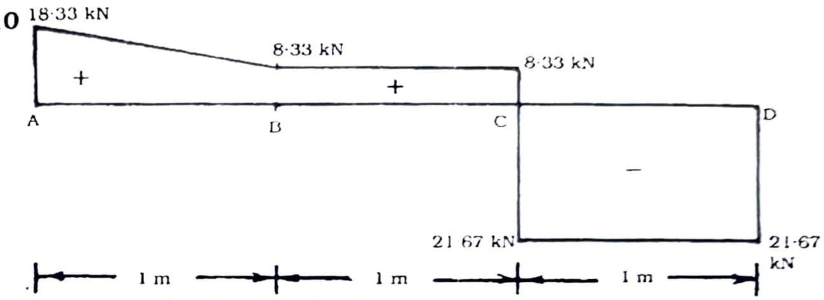


Fig. 6·118 (a) Shear Force Diagram

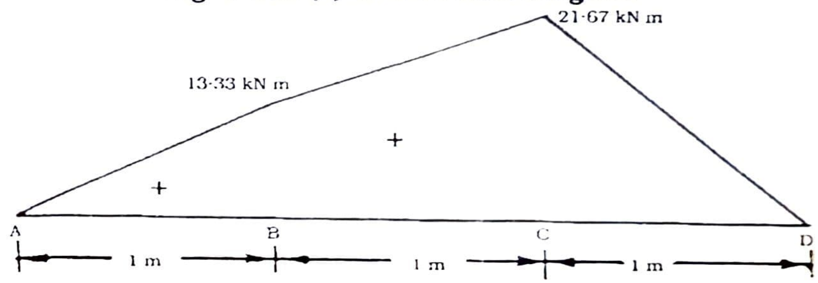


Fig. 6·118 (b) Bending Moment Diagram

6·11

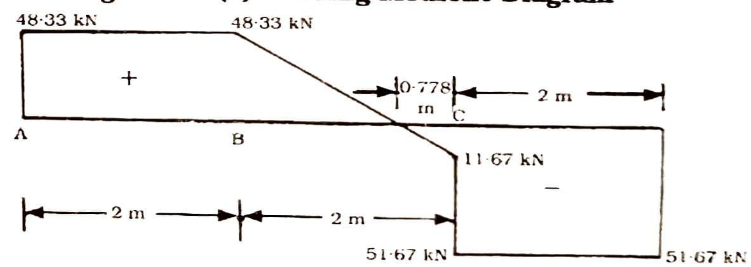


Fig. 6·119 (a) Shear Force Diagram

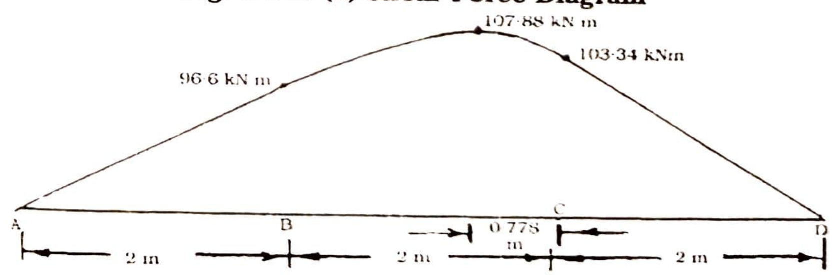


Fig. 6·119 (b) Bending Moment diagram

6·12

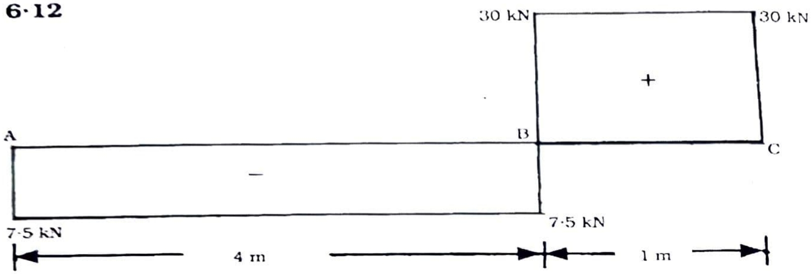


Fig. 6·120 (a) Shear Force Diagram

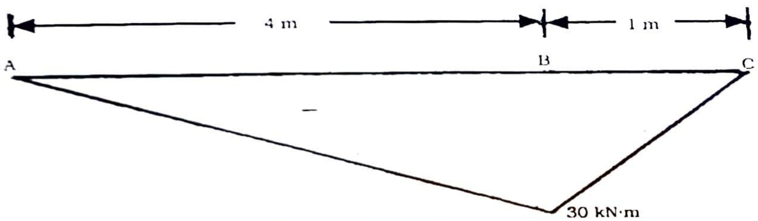


Fig. 6·120 (b) Bending Moment Diagram

6·13

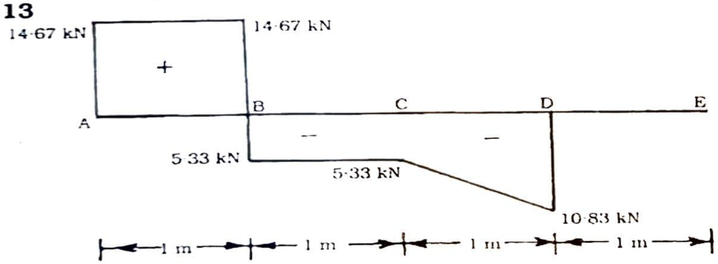


Fig. 6·121 (a) Shear Force Diagram

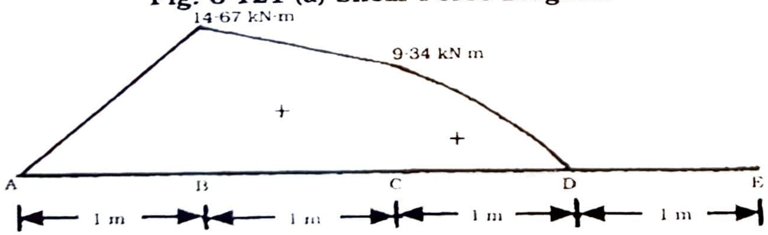


Fig 6·121 (b) Bending Moment Diagram

6·14

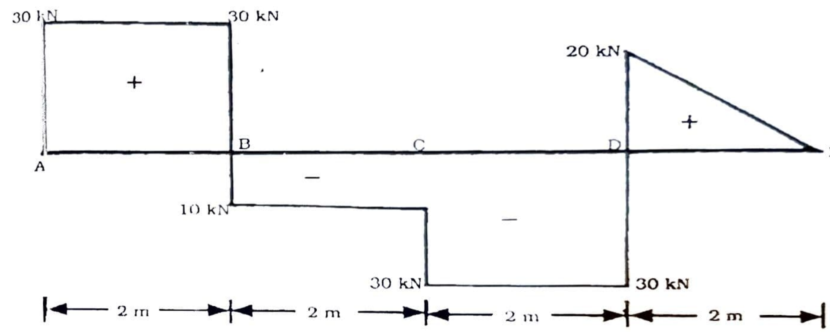


Fig. 6·122 (a) Shear Force Diagram

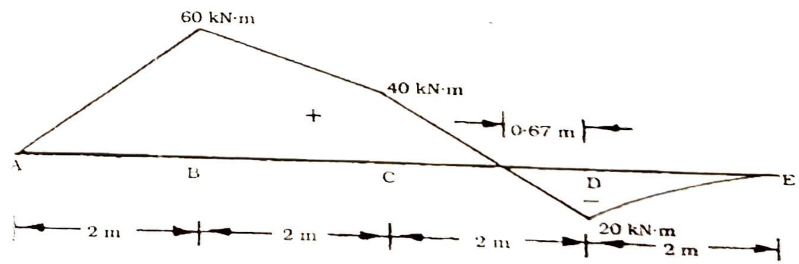
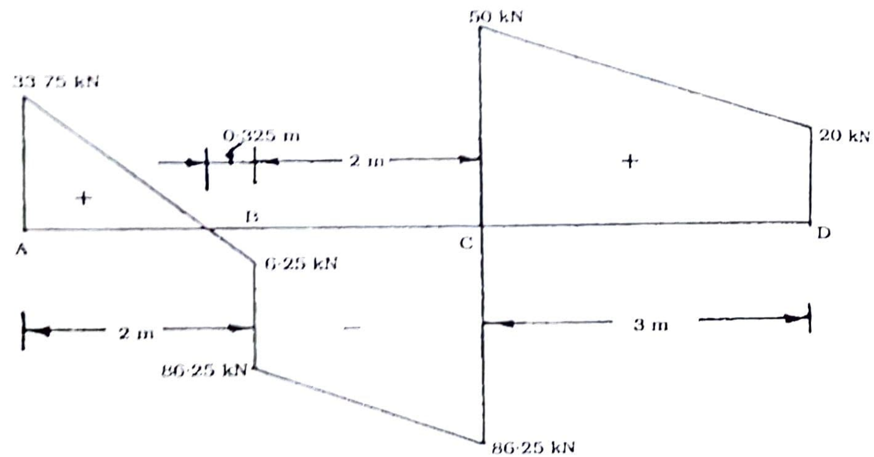


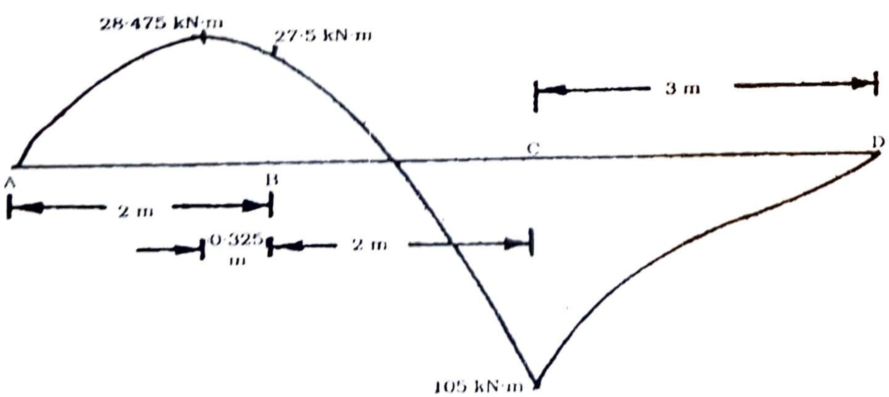
Fig. 6·122 (b) Bending Moment Diagram

6·15



Here,
AB = BC = 2m
CD = 3m

Fig. 6·123 (a) Shear Force Diagram



Note the shapes of the curves.

Fig. 6·123 (b) Bending Moment Diagram

6·16

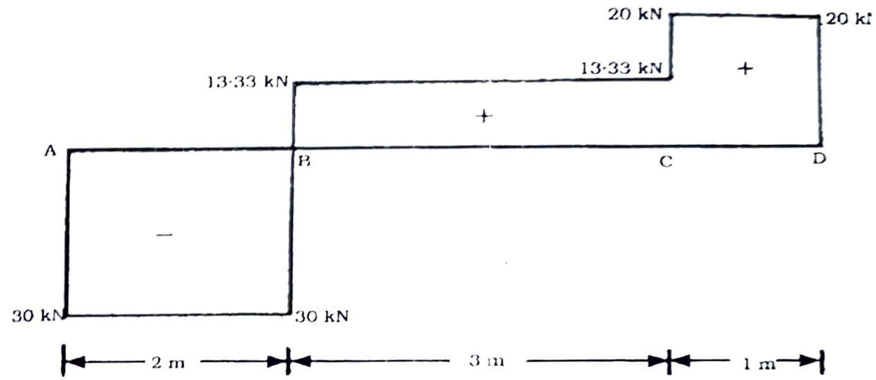


Fig. 6·124 (a) Shear Force Diagram

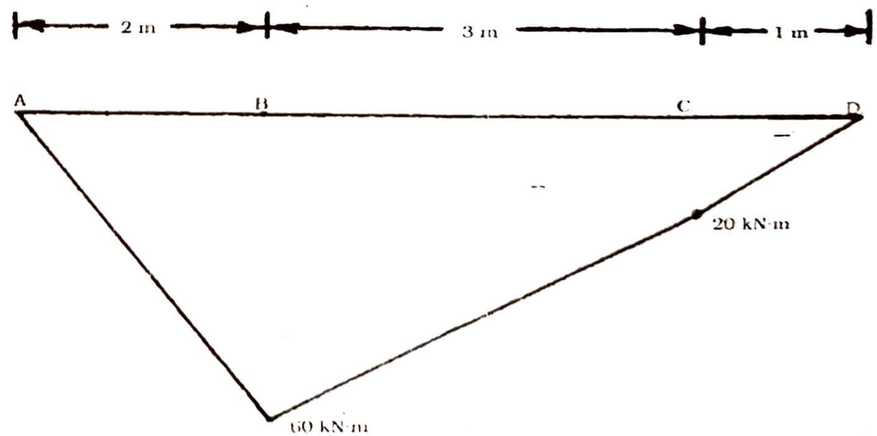


Fig. 6·124 (b) Bending Moment Diagram

·17

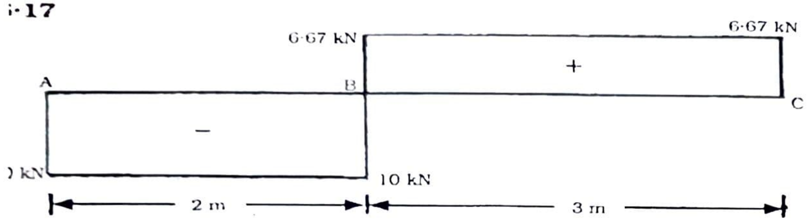


Fig. 6·125 (a) Shear Force Diagram

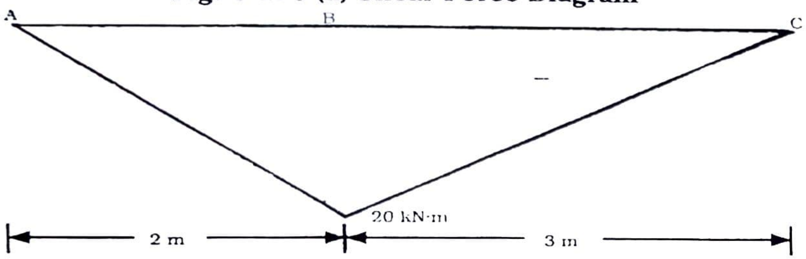


Fig. 6·125 (b) Bending Moment Diagram

·18

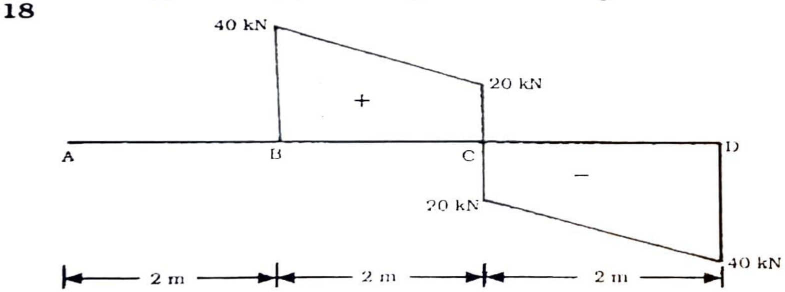


Fig. 6·126 (a) Shear Force Diagram

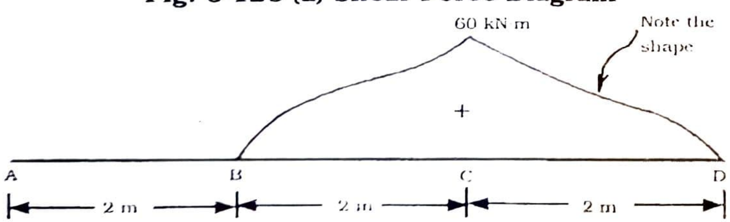


Fig. 6·126 (b) Bending Moment Diagram

6·19

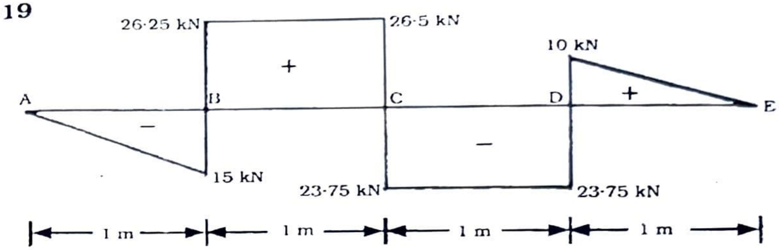


Fig. 6·127 (a) Shear Force Diagram

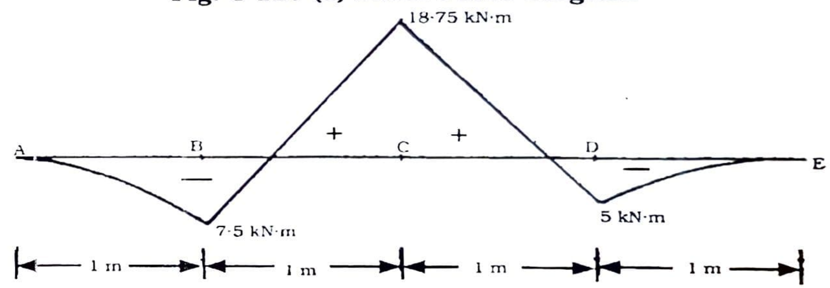


Fig. 6·127 (b) Bending Moment Diagram

6·20

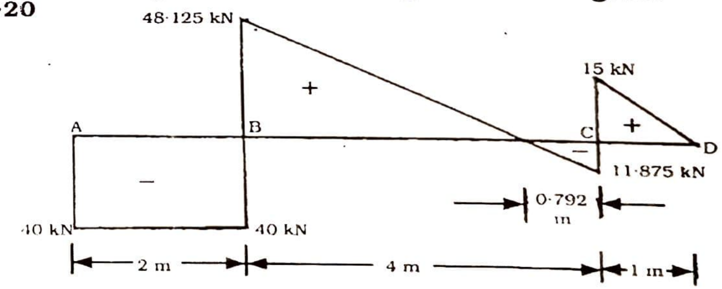


Fig. 6·128 (a) Shear Force Diagram

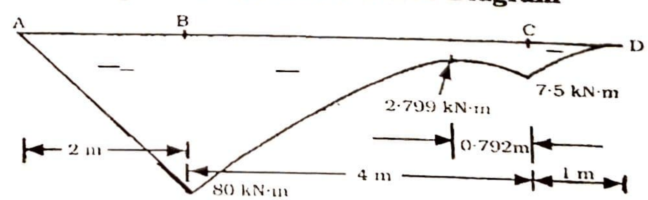


Fig. 6·128 (b) Bending Moment Diagram

6·21

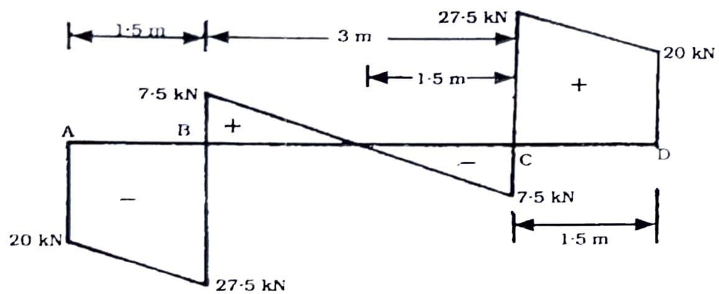


Fig. 6·129 (a) Shear Force Diagram

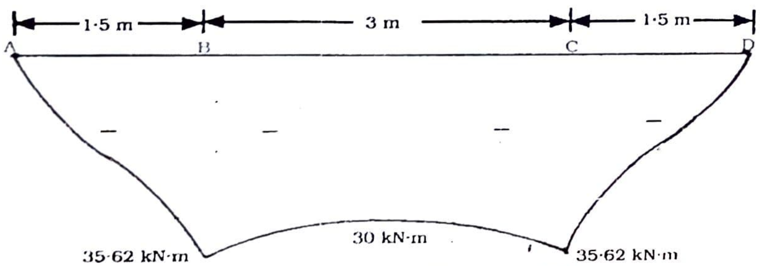


Fig 6·129 (b) Bending Moment Diagram

6·22

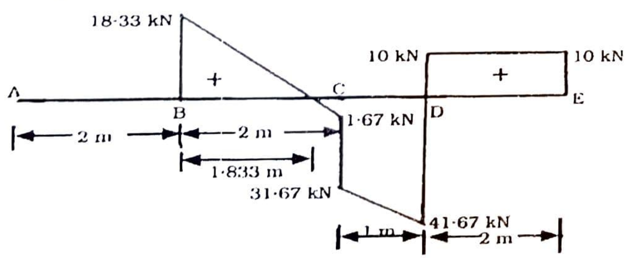


Fig. 6·130 (a) Shear Force Diagram

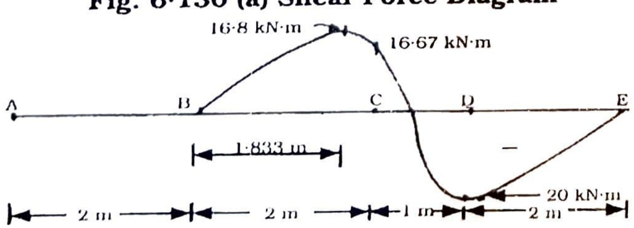


Fig. 6·130 (b) Bending Moment Diagram

6·23 Note : As unknowns are more in problems of cables, method of section can be used for getting more equations.

Take $\Sigma M_A = 0$, $R_{EV} = 30{\cdot}77$ kN, $R_{AV} = 29{\cdot}23$ kN

Considering equilibrium of joint A, B, C and D.

$T_{AB} =$ 32·66 kN, $T_{BC} = 24{\cdot}15$ kN, $T_{DE} = 34{\cdot}05$ kN,

$Y_C =$ 9·27m, $Y_D = 6{\cdot}32$ m, $T_{CD} = 43{\cdot}08$ kN

6·24 $\mathbf{M_A}$: $R_{Dy} \times 10 + R_{Dx} \times 1 = 40 \times 3 + 60 \times 7$

$\mathbf{M_B}$: (Right side only) $R_{Dy} \times 7 = R_{Dx} \times 1 + 60 \times 4$

Hence, $R_{Dy} =$ 45·88 kN, $R_{Dx} = 81{\cdot}18$ kN $(\rightarrow)$

$R_{Ay} =$ 54·12 kN, $R_{Ax} = 81{\cdot}18$kN $(\leftarrow)$

$T_{BA} =$ 97·57 kN, $T_{BC} = 82{\cdot}40$ kN, $T_{CD} = 93{\cdot}36$ kN, $Y_C =$ 1·7 m

6·25 $\mathbf{\Sigma M_A = 0}$: $-20 \sin 40° \, y_B - 20 \cos 40° \times 3 - 30 \times \frac{4}{5} \times 8$

$+ 30 \times \quad \times 4 + V_{DH} \times 12 = 0$

$\mathbf{\Sigma M_B}$ **(Bottom)** = **0 :** $-30 \times \frac{4}{5} \times 5 + 30 \times \frac{3}{5} \times (4 - y_B)$

$+ V_{DH} \times 9 - V_{DV} \times y_B = 0$

$\mathbf{\Sigma M_C}$ **(Bottom)** = **0 :** $4\,V_{DH} - 4\,V_{DV} = 0 \quad \therefore V_{DV} = V_{DH}$

From above equations, $y_B = 3{\cdot}01$ m, $V_{DV} = V_{DH} = 17{\cdot}06$ kN.

Considering equilibrium of joint B and C, $T_{BC} = 35{\cdot}74$ kN

$T_{CD} =$ 24·12 kN, $T_{AB} = 31{\cdot}43$ kN

6·26 Note : Take y_C - y_D, $R_{AV} = 62{\cdot}5$ kN, $R_{EV} = 67{\cdot}5$ kN

$R_{AH} =$ 31·25 kN $(\leftarrow)$, $R_{EH} = 31{\cdot}25$ kN $(\rightarrow)$, $T_{DE} = 74{\cdot}38$kN,

$T_{CD} =$ 32·11 kN, $T_{CB} = 52{\cdot}74$ kN, $T_{AB} = 69{\cdot}84$ kN, $y_C = 7{\cdot}44$ m

$y_D = 6{\cdot}48$ m.

6·27 $\mathbf{M_C}$: (Right side) : $+ R_{DH} \times y_B - R_{DV} \times 2 = 0 \qquad R_{DV} =$ 20 kN$\uparrow$

$\mathbf{M_B}$: (Right side) : $R_{DH} \times y_B + 20 \times 5 - R_{DV} \times 7 = 0$

$R_{AV} =$ 30 kN$\uparrow$

Considering equilibrium of joints A, B, C and D.

$y_B =$ $y_C = 0{\cdot}70$ m, $T_{AB} = 64$ kN, $T_{BC} = 56{\cdot}60$ kN, $T_{CD} = 60$ kN

$R_{AH} =$ 56·60 kN $(\leftarrow)$, $R_{DH} = 56{\cdot}60$ kN $(\rightarrow)$, $y_A = 0{\cdot}883$ m

6·28 Maximum tension will be at poles and minimum tension (horizontal) will be at center of the span.

$$T = \sqrt{T_o^2 + w^2x^2} \qquad \text{Now, } y = \frac{wx^2}{2T_o}$$

$$300 = \sqrt{T_o^2 + (0.4 \times 9.81)^2 (25/2)^2} \qquad h = \frac{(0.4 \times 9.81)\left(\frac{25}{2}\right)^2}{2 \times 295.96}$$

$$\therefore \quad T_o = 295.96 \text{ N} \qquad \text{Sag} = 1.04 \text{ m}$$

6·29

$$s_B = x_B \left[1 + \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2\right] \qquad y = \frac{wx^2}{2T_o}$$

$$\frac{22}{2} = 10\left[1 + \frac{2}{3}\left(\frac{sag}{10}\right)^2\right] \qquad 3.87 = \frac{0.45 \times 9.81 \times 10^2}{2T_o}$$

$$\therefore \quad \text{Sag} = 3.87 \text{ m} \qquad T_o = 57.03 \text{ N}$$

$$T_{max} = \sqrt{T_o^2 + w^2x^2}$$
$$= \sqrt{57.03^2 + (0.45 \times 9.81)^2 (10)^2}$$
$$= 72.12 \text{ N.}$$

6·30 AB Portion :

$$y = \frac{wx^2}{2T_o}, \quad 3 = \frac{w(40)^2}{2T_o} \qquad \therefore T_o = 266.67 \text{ w}$$

$$T = \sqrt{T_o^2 + w^2x^2}$$
$$= \sqrt{(266.67 \text{ w})^2 + w^2 (40)^2}$$
$$= 269.65 \text{ w}$$

$$\tan\theta = \frac{wx}{T_o} = \frac{w \times 40}{266.67 \text{ w}}$$

$$\therefore \quad \theta = 8.53° \qquad T\cos\theta = 266.67 \text{ w}$$

BC Porition :

$$y = \frac{wx^2}{2T_o}, \quad s = \frac{w(25)^2}{2T_o}$$

$$T_o = 312.5 \text{ w/s}$$

$$T = \sqrt{T_o^2 + w^2x^2}$$

$$= \frac{w}{s}\sqrt{(312.5)^2 + (25)^2 s^2}$$

$$\cos\theta = \frac{T_o}{\sqrt{T_o^2 + w^2x^2}}$$

$$= \frac{312.5 \text{ w}}{s} \times \frac{1}{\frac{w}{s}\sqrt{(312.5)^2 + (25)^2 s^2}}$$

Now, horizotal resultant is zero

$$\therefore \quad T_{AB}\cos\theta_{AB} = T_{BC}\cos\theta_{BC}$$

$$\text{but, } T\cos\theta = 266.67 \text{ w} = \frac{312.5 \text{ w}}{s}$$

$$\therefore \quad s = 1.17 \text{ m}$$

6·31

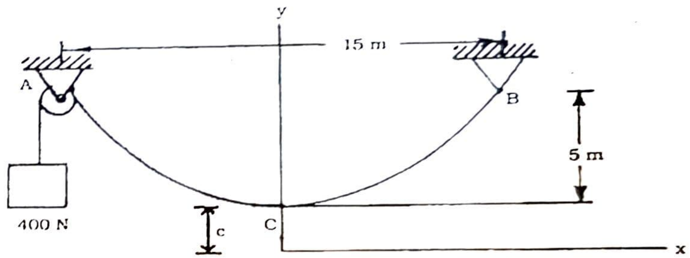


Fig. 6·131

The origin of coordinates is placed at a distance c below the lowest point of the cable. The equation of cable is

$$y = c\cosh\frac{x}{c}$$

The coordinates of B are

$$x_B = 7.5 \text{ m}, \quad y_B = 5 + c$$

Substituting in above equation

$$5 + c = c\cosh\frac{7.5}{c}$$

$$\therefore \quad \frac{5}{c} + 1 = \cosh\frac{7.5}{c}$$

The value of c is determined by taking trial value of it.

c	$\frac{5}{c}$	$\frac{7.5}{c}$	$\frac{5}{c}+1$	$\cosh\frac{7.5}{c}$
10	0·5	0·75	1·5	1·295
20	0·25	0·375	1·25	1·07
7	0·71	1·071	1·71	1·63
6·4	0·781	1·172	1·781	1·768
6·2	0·806	1·21	1·806	1·825
6·3	0·794	1·19	1·794	1·796

Here, $\cosh x = \frac{1}{2}(e^{x} + e^{-x})$

Taking c = 6·3 m, we have

$$y_B = 6{\cdot}3 + 5 = 11{\cdot}3 \text{ m}$$

Mass per unit length of cable :

Here, $T_{max} = 400$ N

$$T_B = 400 = w\,y_B = w\,(11{\cdot}3 \text{ m})$$

$$\therefore w = 35{\cdot}39 \text{ N/m}$$

$$\therefore \text{Mass per unit length of cable} = \frac{w}{g} = \frac{35{\cdot}39}{9{\cdot}81}$$

$$m = 3{\cdot}61 \text{ kg/m}$$

Length of Cable :

Half length CB is to be determined by equation

$$y_B^2 - s_{CB}^2 = c^2$$

$$\therefore s_{BC}^2 = y_B^2 - c^2 = (11{\cdot}3)^2 - (6{\cdot}3)^2$$

$$\therefore s_{BC} = 9{\cdot}38 \text{ m}$$

$$\therefore \text{Length of cable AB} = 2 \times 9{\cdot}38 \doteq 18{\cdot}76 \text{ m}$$

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