Quantum Mechanics

Rutherford → Continuous spectrum → Line spectrum Shown

Bohr explained it by introducing orbits.

Black Body emits and absorbs same amount of energy (radiation)

Black body → cavity ~~modulator~~ radiator.

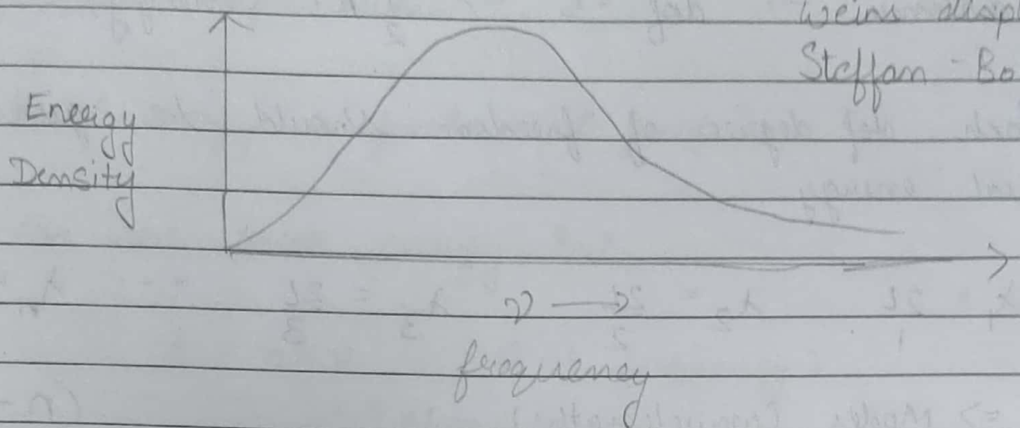
There is no difference between spectrum by black body. (Each maintained ~~by~~ at certain temp.) put in temp bath.

On this, plank stated that he can only explain this by considering radiation in chunks.

old physics → Amplitude and intensity for energy (radiation)

photoelectric effect → frequency for energy

Quantum Mechanics.



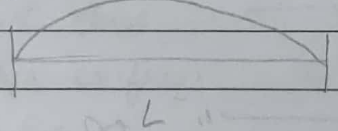
Weins displacement
Steffan - Boltzman

Black body only depends on temperature.

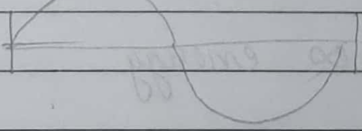
B.b.$_t$ < Sur. → absorbing.
B.b.$_t$ > Sur. → emitting

Cavity of length 'L'
Modes → No. of wave lengths
Acc. to classical physics, break L and can fit any no. of modes (wave length)

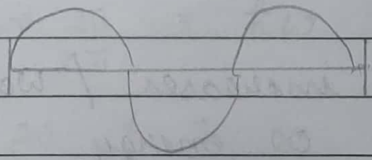
Standing wave → Node at end (0 Amplitude)



$\lambda_1 = \frac{2L}{1}$



$\lambda_2 = L = \frac{2L}{2}$



$\lambda_3 = \frac{2L}{3}$

$\lambda_n = \frac{2L}{n}$ $n \in [1, \infty)$ ~~only integer~~

- Equipartition of energy
  The total energy would be each degree of freedom has energy contributed $\frac{1}{2}kT$
  Monoatomic → 3 d.o.f. ⇒ $\frac{3}{2}kT$ (P.E. = 0)

diatomic → dof. = 5 ⇒ $\frac{5}{2}kT$ (Energy)

Each ~~dof~~ degree of freedom should be given equal energy.

$\lambda_1 = \frac{2L}{1}$ $\lambda_2 = \frac{2L}{2}$ $\lambda_3 = \frac{2L}{3}$ - - - $\lambda_n = \frac{2L}{n}$

($n \to \infty$)

$\frac{n}{2}$ ⇒ Modes (wavelengths)

n ⇒ Degree of freedom.

Acc to equipartition of energy,

λ_1 should have energy $\frac{1}{2}kT$

λ_2 " ——— " ——— " $2(\frac{1}{2}kT)$

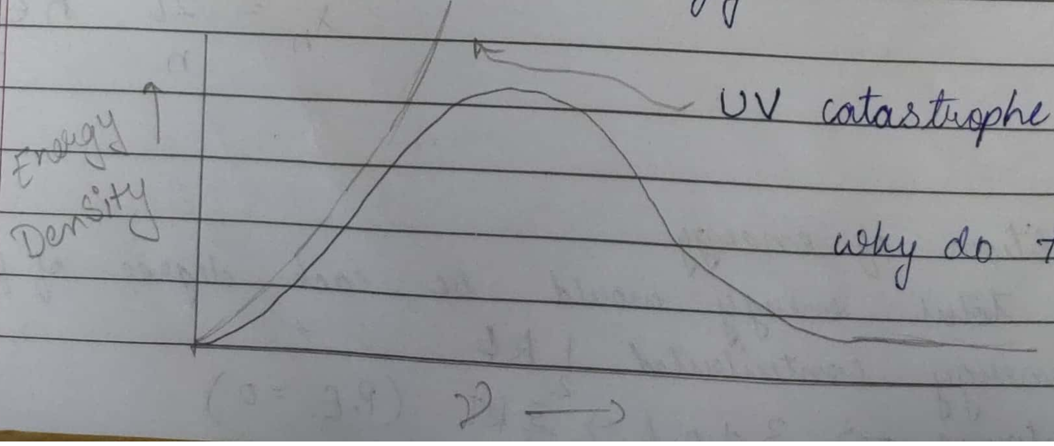
λ_3 " ——— " ——— " $3(\frac{1}{2}kT)$

⋮

λ_∞ " ——— " ——— " ∞

∴ ~~A~~ s/s has ∞ energy.

∴ As frequency increases / wavelength decreases, ~~has~~ can have ∞ energy.



why do the graph falls.

o Planck's explaination

Total energy is fixed. (E fixed)

We can break energy. But,

$E = nh\nu$ ($\nu \rightarrow$ frequency)
↳ Always remain.

$n \Rightarrow$ No. of mode.

Breaking energy.

Let there be $10h\nu$ energy.

So,

$E = 1(10h\nu) = 2(5h\nu) = ---- = 5(2h\nu) = 10(1 \cdot h\nu)$

Can't break energy. ← Now, no. of mode would fall. ← And thus at higher ~~en~~ freq. energy drops ← Solving UV catastrophe

Formula

$E = h\nu = h\nu \frac{2\pi}{2\pi} = \hbar\omega$ $\hbar = \frac{h}{2\pi}$ $\omega = 2\pi f = 2\pi\nu$

$p = \frac{h}{\lambda} = \frac{h}{\lambda}\frac{2\pi}{2\pi} = \frac{\hbar}{k}$ $k = \frac{\lambda}{2\pi}$

$\nu = \frac{E}{h}$ $\lambda = \frac{h}{p}$ $p = \gamma \cdot m \cdot v$

$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

Q A certain 660 Hz tuning fork be considered as a harmonic oscillator of vibrational energy of 0.04 J. Compare energy quanta of this tuning fork with that of an atomic oscillator, absorbing and emitting radiation of frequency 5.00×10^{14} Hz

$$E_1 = h\nu = 660 \times 6.62 \times 10^{-34}$$
$$= 4.37316 \times 10^{-31} \text{ J}$$

$$E_2 = h\nu = 6.626 \times 10^{-34} \times 5 \times 10^{14}$$
$$= 33.13 \times 10^{-20}$$
$$= 3.313 \times 10^{-19}$$

Q Find the de-Broglie wavelength of

i) 46 gm golf ball moving at $30 \, ms^{-1}$

ii) An electron with $v = 10^7$ m/s.

i) $\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34}}{46 \times 10^{-3} \times 30} = 4.8 \times 10^{-34}$ m

ii) $\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34}}{9.1 \times 10^{-31} \times 10^{7}} = 7.28 \times 10^{-11}$ m

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Q Neutrons produced in a reactor are used for chain reaction after they are thermalised, i.e., their KE. are reduced to that of the energy of the air molecule at room temperature. Taking room temp as 300K, estimate the de-Broglie wavelength of such neutron ($m_n = 1.67 \times 10^{-27}$ kg)

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Speed of ~~wave~~ particle, $v_p = \lambda \cdot \nu$ (classical mechanics)

$E = h\nu = \gamma mc^2$

$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

$v_p = \frac{\cancel{h}}{\cancel{\gamma} m v} \cdot \frac{\cancel{\gamma} mc^2}{\cancel{h}}$ (used de-Broglie eq^n)

$\lambda = \frac{h}{p} = \frac{h}{\gamma m v}$

$v_p = \frac{c^2}{v} > c$

$\hookrightarrow$ Not possible. $\Rightarrow$ Some problem with classical mechanics formula.

Waves / Beats.

$y_1 = A\cos(kx - \omega t)$

$y_2 = A\cos\{(k+\Delta k)x - (\omega + \Delta\omega)t\}$

$y_1 + y_2 = A\left[\cos(kx - \omega t) + \cos\{(k+\Delta k)x - (\omega+\Delta\omega)t\}\right]$

$= A\left[2\cos\left(\frac{2kx + \Delta kx - 2\omega t - \Delta\omega t}{2}\right)\right.$

$\left.\cos\left(\frac{-\Delta kx - \Delta\omega t}{2}\right)\right]$

$= 2A\cos\left(kx - \omega t + \cancel{\frac{\Delta kx - \Delta\omega t}{2}}\right)\cos\left(\frac{\Delta kx + \Delta\omega t}{2}\right)$ Neglect

$= 2A\cos(kx - \omega t)\cos\left(\frac{\Delta kx + \Delta\omega t}{2}\right)$

$v_p = \frac{\omega}{k} = \frac{c^2}{v}$

$v_g = \frac{\Delta\omega}{\Delta k} = \frac{d\omega}{dk}$

$\uparrow$
wave group velocity

$\omega = 2\pi\nu = 2\pi\frac{\gamma mc^2}{h} = \frac{2\pi mc^2}{h\sqrt{1 - \frac{v^2}{c^2}}}$

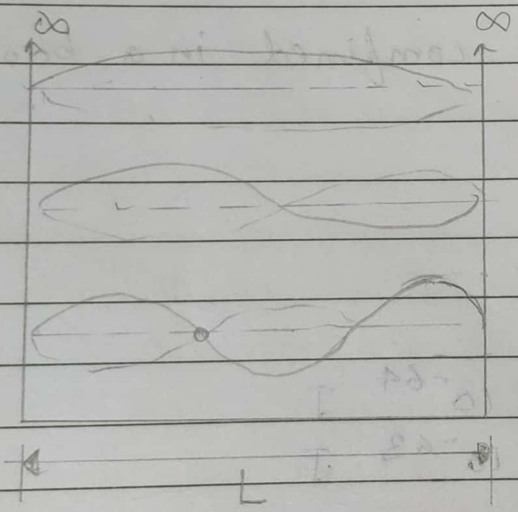
$$k = \frac{2\pi}{\lambda} = \frac{2\pi p}{h} = \frac{2\pi \gamma m v}{h}$$

$$= \frac{2\pi m v}{h\sqrt{1-\frac{v^2}{c^2}}}$$

$$v_g = \frac{d\omega}{dk} = \frac{d\omega/dv}{dk/dv} = v$$

Particle in a box.

Particle is confined in a box (with ∞ length of wall).



Particle cannot attain energy to climb over the wall.

Waves should be standing, i.e., Amplitude at wall should be 0.

$$\therefore \lambda_n = \frac{2L}{n}$$

Larger the length larger the discretisation

Non-Realistic case :-

$$KE = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$p = \frac{h}{\lambda}$$

$$\therefore E_n = \frac{h^2/\lambda^2}{2m}$$

$$\lambda = \frac{2L}{n}$$

$$\therefore E_n = \frac{h^2 n^2}{2 \cdot 2^2 L^2 m}$$

$$E_n = \frac{h^2 n^2}{8L^2 m} \qquad n = 1, 2, 3, \dots$$

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Q An electron confide in a box of 0.10 nm length, find its permitad energies.

$$E_n = \frac{h^2 n^2}{8L^2 m}$$

$E_1 = 6.025 \times 10^{-18} = 6.02 \times 10^{-18}$ J

$E_2 = 2.41 \times 10^{-17} = 2.41 \times 10^{-17}$ J

$E_3 = 5.422 \times 10^{-17} = 5.422 \times 10^{-17}$ J

$E_4 = 9.64 \times 10^{-17} = 9.64 \times 10^{-17}$ J

$E_5 = 1.5061 \times 10^{-16} = 1.50 \times 10^{-16}$ J

$\vdots$

$E_n = 6.02 \times 10^{-18}\, n^2$ J.

Q A 10gm marble is confined in a box of 10cm length

$$E_n = \frac{h^2 n^2}{8L^2 m}$$

$E_1 = 5.48 \times 10^{-64}$ J

$E_2 = 2.20 \times 10^{-63}$ J

$\vdots$

$E_n = 5.48 \times 10^{-64}\, n^2$ J

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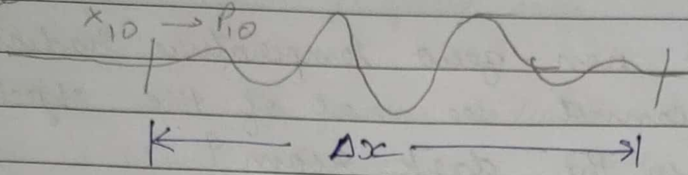
$x_1 \to p_1$ $\Delta p = P(\lambda_{10}) - P(\lambda_1)$

combination of 10 wave

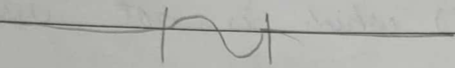
$x_{10} \to p_{10}$



Δx

Amplitude shows the probability of finding the e^- there.

combination of 20 wave.



Δx

$$\Delta p = p(\lambda_{20}) - p(\lambda_1)$$

You can only know

Assignment - 1

Questions - Answers

Q1 All bodies at non-zero temperature radiate Then why we cannot see most of the objects when they are placed in the dark room?

At room temperature, bodies emit radiation in the infrared radiation (IR) which is not visible to the naked eye

Q2 What is a black body? How would the radiations characteristics of an ordinary object differ from its matching black body?

A blackbody is an idealized physical body that absorbs all incident electromagnetic radiation, regardless of frequency or angle of incedence.

The radiation characteristics of an ordinary body unlike a black body depend on the amount of radiation absorbed and spectrum released will differ. For example, a blackbody acts as an ideal emmitter whereas an ordinary body doesn't, i.e., it emits as much ~~as~~ or more thermal radiative energy as any other body at the same temperature.

Q3 Define spectral radiancy. What is its relation with total reading?

Spectral radiancy is just the rate at which energy is radiated per unit surface area for a small wavelength ~~interval~~ interval. Radiance in the radiant flux emitted, reflected, transmitted or recieved by a given Surface per unit projected area.

Q4. Calculate ratio of powers radiated by the two Slab A & B having surface temperature 5700°c & 8100°c resp.

$P \propto T^4$ ...... (from stefans law)

$$\frac{P_1}{P_2} = \left(\frac{T_1}{T_2}\right)^4 = \left(\frac{5700}{8100}\right)^4 = 0.245$$

∴ The ratio of power radiated by two ~~star~~ Slabs is 0.245.

Q5. write down mathematical formula of
(i) weins displacement law.
(ii) Stefen - Boltzmann's law
(iii) Plank's law
(iv) Rayleighs jeans law.

(i) The spectral radiance of black body radiation per unit wavelength peaks at the wavelength λ_{max} given by

$$\lambda_{max} = \frac{b}{T}$$

T = Temperature
b = wein's displacement const.
$b = 2.898 \times 10^{-3}$ m K.

(ii) The total energy radiated per unit surface area of a black body accross all wavelengths per unit time is directly proportional to the fourth power of Temperature (T)

$$E = \sigma T^4$$

where, $\sigma = 5.67 \times 10^{-8}\ W m^{-2} K^{-4}$ is called stefan's boltzmann's const.

(iii) Planck's Law

$$u(\nu)\,d\nu = \frac{8\pi h}{c^3} \frac{\nu^3 d\nu}{(e^{h\nu/kT} - 1)}$$

(iv) Rayleigh Jeans Law

$$u(\nu)\,d\nu = \frac{8\pi kT}{c^3} \nu^2 d\nu$$

Q6. Discuss two most prominent results of photoelectric effect which couldn't be understood classically.

The two prominent results from photoelectric effect which could not be explained by classical mechanics are :-

① There is no time interval b/w arrival of photons and emission of photoelectrons.

② Light is present as packets of energy (or as photons). The kinetic energy of emitted photoelectrons depends on the frequency of incident light.

Q7. why cannot the compton effect be detected with visible light?

Compton effect is observed when a light of suitable incident frequency knocks off an electron of an atom. Visible light doesn't have enough incident energy to knock off an electron. The maximum wavelength change is compton wavelength is 4.852 pm (i.e. twice compton wavelength of e^-). Hence, the shift in wavelength for visible light is less than 0.01% of initial wavelength making the effect hardly detectable.

Q8. Explain the physical significance of compton wavelength

$$\lambda' - \lambda = \lambda_c (1 - \cos\phi)$$

where, $\lambda_c = \frac{h}{mc} \rightarrow$ Compton wavelength

The compton wavelength gives the scale of the wavelength change of incident photon.

Q9. Explain why not all photoelectrons have same KE even though work function is constant.

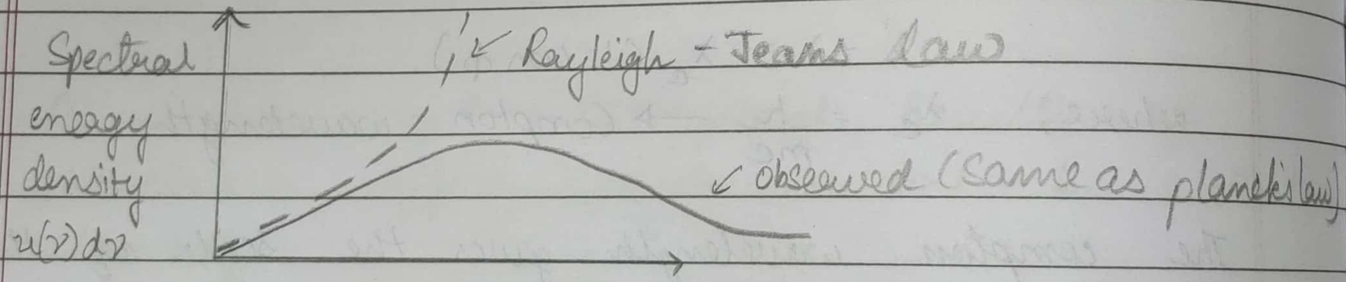
Initial energies and interelectronic collisions contribute to Kinectic energies of the electron before emission which causes change in their KE despite const. work function.

Q10 Discuss atleast one result except blackbody radiation, photoelectric effect, compton effect that could not be explained classically.

The discrepency known as UV catastrophe couldn't be explaind by classical physics Rayleigh - Jeans Formula

$$u(\nu)d\nu = \frac{8\pi kT}{c^3}\nu^3 d\nu \quad \text{(classical physics)}$$

As frequency increases towards the UV end of the spectrum, the formula predicts that energy density should increase as ν^2 in the limit of infinitely high frequencies, $u(\nu)d\nu$ should also go to infinity.



Planck's radiation formula:

$$u(\nu)d\nu = \frac{8\pi h}{c^3}\frac{\nu^3 d\nu}{(e^{h\nu/kT}-1)}$$

Assignment-2

Q1 In a thermonuclear explosion the temperature in the fireball is momentarily 10^7 K. Find the wavelength at which the radiation emitted is a maximum.

Using wein's displacement law,

$$\lambda_m = \frac{2.898 \times 10^{-3}}{10^7}$$

$$= 2.898 \times 10^{-10} \text{ m}$$

$$= 2.898 \text{ Å}$$

Q2. At what wavelength does the human body emit its maximum temperature radiation?

Assumption - human body temp. = $37°C$ = 310 K

Using wein's displacement law,

$$\lambda_m = \frac{2.898 \times 10^{-3}}{310}$$

$$= 9.348 \times 10^{-6}$$

$$= 9.348 \ \mu m$$

Q3. A sphere of radius r is maintained at a surface temp. T by an internal heat source. The sphere is surrounded by a thin, concentric shell of radius 2r. Both object emit and absorb as blackbodies what is the temperature of the shell?

Q4. Cosmic background radiation peaks at a wavelength of about 1 mm. what is the temperature of the universe?

Using weins displacement law

$$T = \frac{b}{\lambda_m}$$

$$= \frac{2.898 \times 10^{-3}}{10^{-3}}$$

$$= 2.898\ K$$

$$= -270.102^{\circ}C$$

Q5 what is the wavelength of a photon whose energy is equal to the rest mass energy of an electron?

Rest mass energy of $e^- = m_e c^2$

$$= (9.31 \times 10^{-31})(3 \times 10^{8})^2$$

$$m_o c^2 = \frac{hc}{\lambda}$$

$$9.31 \times 10^{-31} \times 9 \times 10^{16} = \frac{6.625 \times 10^{-34} \times 3 \times 10^{8}}{\lambda}$$

$$\lambda = 0.237 \times 10^{-3} \times 10^{-8}$$

$$= 0.237\ pm$$

Q6. The energy required to remove an e^- from sodium is 2.3 eV. Does sodium show a photoelectric effect for yellow light, with $\lambda = 5890 \mathring{A}$? What is the cutoff wavelength for photoelectric emission from sodium?

$$E = \frac{12375}{\lambda} eV = \frac{12375}{5840} = 2.101 eV$$

This energy isn't sufficient to remove e^-.

$$\text{cutoff wavelength } (\lambda) = \frac{12375}{2.3} = 5380.4 m$$

Q7 If we assume that stellar surfaces behave like blackbodies we can get a good estimate of their temperature by measuring λ_{max}. For the sun $\lambda_{max} = 5100 \mathring{A}$, whereas for the North star $\lambda_{max} = 3500 \mathring{A}$. Find the surface temperature of these stars. Using Stefan's law, and the temperature just obtained, determine the power radiated from $1 cm^2$ of stellar surface.

λ_{max} for Sun $= 5100 \mathring{A}$

$$T_{Sun} = \frac{b}{\lambda_{max}} = \frac{2.898 \times 10^{-3}}{5100 \times 10^{-10}} = 5680.39 K.$$

λ_{max} for north star $= 3500 \mathring{A}$

$$T_{NS} = \frac{2.898 \times 10^{-3}}{3500} = 8277.14 K$$

$$P = \sigma A T^4 = 26613.639 W$$

$$P_{Sun} = \sigma A T_{Sun}^4 = 5903.311 W$$

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Q8. If we look into a cavity whose walls are kept at a const. temp. no details of the interior are visible Explain.

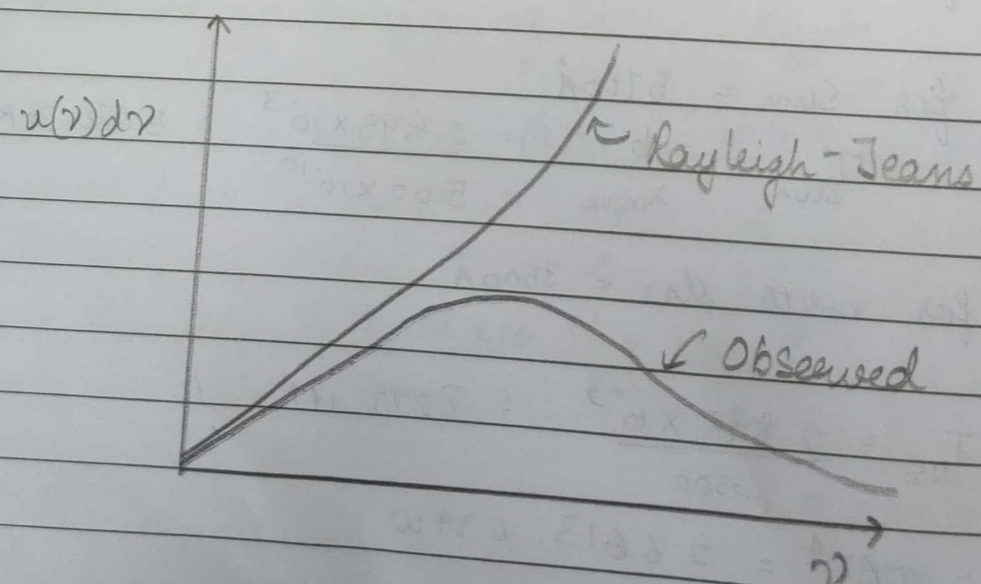
At room temp and at other const. temp., radiations emitted are necessarily not in visible range. Therefore, the inside isn't visible.

Q9 What is the origin of the UV catastrophe?

Acc. to Reighleigh Jeans formula,

$$u(\nu)\, d\nu \propto \nu^2$$

As freq. increases towards UV end of spectrum formula predicts that energy density should increase directly proportional to ν^2. In the limit of infinitely high frequencies, energy density should go to infinity. But that is not the case. Therefore, this discrepancy was known as UV catastrophe



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Q10 Why is it that even for incident radiation that is monochromatic, photoelectrons are emitted with a spread of velocities?.

Acc. to Compton effect interaction of photon with each other, photoelectrons are emitted with a spread of velocities.

Q11 Photoelectric emission confirms

(a) Dual nature of radiation
(b) The intensity of incident light
(c) wave nature of radiation
(d) Electromagnetic nature of radiation

Q12 The cut-off potential is a function of

(a) The frequency of incident light

Wave function

A microscopic particle acts if certain aspects of its behaviour are governed by an associated de-Broglie wave or wave function.

In dealing with very simple cases like motion of free particle and particle in a box, we have applied the aspects of matter wave successfuly.

However, it does not tell us how the wave propagates. Though this postulates have been successfuly have been predicting wavelength of these particles.
But only in cases where wavelength is const.

We must have quantitative relation b/w property of particle and associated wave funcⁿ which describe the wave.

Rules

(i) ψ must be continuous and single valued

(ii) $\frac{\partial \psi}{\partial x}, \frac{\partial \psi}{\partial y}, \frac{\partial \psi}{\partial z}$ must be continuous and single valued.

(iii) ψ must be Normalizable, i.e., $\psi \to 0$ $x \to \pm\infty$

$$\int_{-\infty}^{\infty} |\psi|^2 dx = 1 \longrightarrow \text{Does not come}$$

(Assume ψ)

↳ Total probability

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To make the integral $= 1$, we normalize it

Example $\int_{-\infty}^{\infty} |\psi|^2 dx = 100 \longrightarrow \frac{1}{100}\int_{-\infty}^{\infty} |\psi|^2 dx = 1$

$\psi = A + iB$

$|\psi|^2 = \psi^* \psi$

$= (A - iB)(A + iB)$

$= A^2 + B^2 \geq 0$

$\int |\psi|^2 dV \propto P$

$V \rightarrow$ Volume

$P \rightarrow$ Probability

4