8.1 Introduction

In most of the equilibrium problems that we have analysed up to this point, surfaces of contact have been assumed to be frictionless. The concept of a frictionless surface is, of course, an idealisation. All real surfaces have some roughness.

When a body moves or tends to move over another body, a force opposing the motion develops at the contact surfaces. Whenever a tendency exists for one contacting surfaces to slide along another surface, tangential force is generated between contacting surface. This force which opposes the movement or tendency of movement is called a ***frictional force*** *or simply* ***friction***.

When two bodies in contact are in motion then the frictional force is opposite to the relative motions.

Cause of Friction : Friction is due to *the resistance offered to motion by minutely projecting portions at the contact surfaces*. This microscopic projections gets interlocked. To a small extent, the material of two bodies in contact also produces resistance to motion due to intra molecular force of attraction, i.e., adhesive properties.

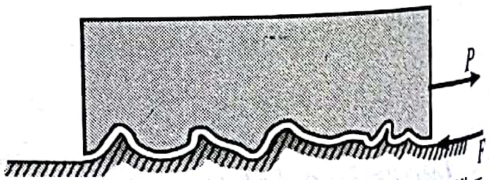


Fig. 8.1-i : Magnified Microscopic View of Rough Surface

8.2 Types of Friction

Dry Friction : Dry friction develops when *the unlubricated surface of two solids are in contact under a condition of sliding or a tendency to slide*. A frictional force tangent to the surfaces of contact is developed both during the interval leading up to impending slippage and while slippage takes place. The direction of the frictional force always opposes the relative motion or impending motion. This type of friction is also known as *Coulomb friction*.

Fluid Friction : Fluid friction is developed when *adjacent layers in a fluid (liquid or gas) are moving at different velocities*. This motion gives rise to frictional forces between fluid elements and these forces depend on the relative velocity between layers. Fluid frictions depends not only on the velocity gradients within the fluid but also on viscosity of fluid, which is a measure of its resistance to shearing action between fluid layers. Fluid friction is treated in the study of fluid mechanics and is beyond the scope of this text. So we are going to deal with dry friction only.

8.3 Mechanism of Friction

Consider a block of weight W resting on a horizontal surface as shown in Fig. 8.3-i. contacting surface possesses a certain amount of roughness. Let P be the horizontal fo applied which will vary continuously from zero to a value sufficient to just move the block then to maintain the motion. The free body diagram of the block shows active forces (i Applied force P and weight of block W) and reactive forces (i.e., normal reaction N a tangential frictional force F).

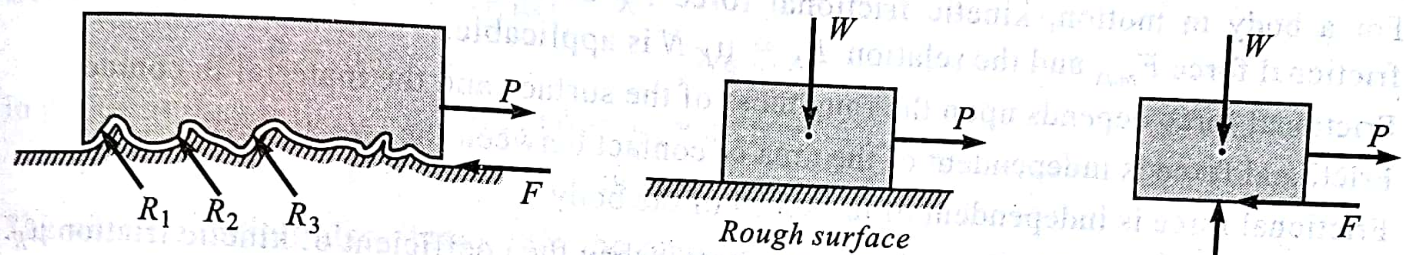


Fig. 8.3-i : Mechanism of Friction

Frictional force F has the remarkable property of adjusting itself in magnitude equal to th applied force P till the limiting equilibrium condition.

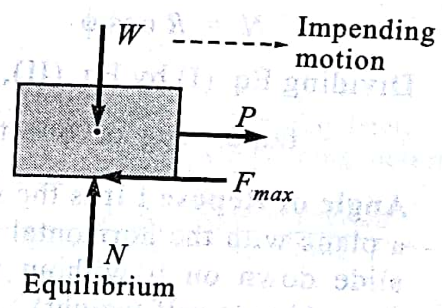
Limiting Equilibrium Condition : As applied force P increases, the frictional force F is equa in magnitude and opposite in direction. However, there is a limit beyond which the magnitude of the frictional force cannot be increased. If the applied force is more than this maximum frictional force, there will be movement of one body over the other. Once the body begins to move, there is decrease in frictional force F from maximum value observed under static condition. The frictional force between the two surface when the body is in motion is called *kinetic* or *dynamic friction* F_K.

Limiting Frictional Force (F_{max}) : It is the maximum frictional force developed at the surface when the block is at the verge of motion (impending motion).

Coefficient of Friction : By experimental evidence it is proved that limiting frictional force is directly proportional to normal reaction.

$$F_{max} \propto N$$

$$F_{max} = \mu_S N \Rightarrow \mu_S = \frac{F_{max}}{N} \quad \text{...(8.1)}$$

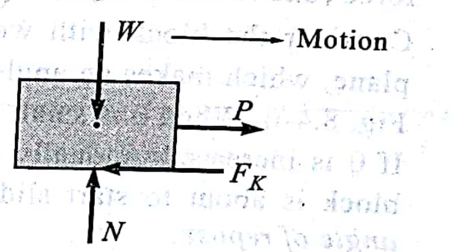


Coefficient of Static Friction : The ratio of limiting frictional force (F_{max}) and normal reaction (N) is a constant. This constant is called the *coefficient of static friction* (μ_S).

If a body is in motion, we have

$$F_K \propto N$$

$$F_K = \mu_K N \Rightarrow \mu_K = \frac{F_K}{N} \quad \text{...(8.2)}$$



Coefficient of Kinetic Friction : The ratio of kinetic frictional force (F_K) and normal reaction (N) is a constant. This constant is known as *coefficient of kinetic friction* (μ_K).

Kinetic friction is always less than limiting friction.

8.4 Laws of Friction

1. The frictional force is always tangential to the contact surface and acts in a direction opposite to that in which the body tends to move.
2. The magnitude of frictional force is self-adjusting to the applied force till the limiting frictional force is reached and at the limiting frictional force the body will have impending motion.
3. Limiting frictional force F_{max} is directly proportional to normal reactions (i.e., $F_{max} = \mu_S N$).
4. For a body in motion, kinetic frictional force F_K developed is less than that of limiting frictional force F_{max} and the relation $F_K = \mu_K N$ is applicable.
5. Frictional force depends upon the roughness of the surface and the material in contact.
6. Frictional force is independent of the area of contact between the two surfaces.
7. Frictional force is independent of the speed of the body.
8. Coefficient of static friction μ_S is always greater than the coefficient of kinetic friction μ_K.

Angle of Friction : It is the angle made by the resultant of the limiting frictional force F_{max} and the normal reaction N with the normal reactions.

Consider the block with weight W and applied force P.

When the block is at the verge of motion, limiting frictional force F_{max} will act in opposite direction of applied force and normal reaction N will act perpendicular to surface as shown in Fig. 8.4-i. We can replace the F_{max} and N by resultant reaction R which acts at an angle ϕ to the normal reaction.

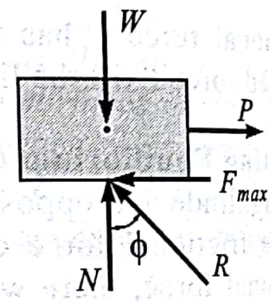


Fig. 8.4-i

This angle ϕ is called as the *angle of friction*.

From Fig. 8.4-ii, we have

$$F_{max} = R \sin \phi$$

$$\mu_S N = R \sin \phi \quad \text{... (I)} \quad (\because F_{max} = \mu_S N)$$

$$N = R \cos \phi \quad \text{... (II)}$$

Dividing Eq. (I) by Eq. (II), we get

$$\tan \phi = \mu_S \text{ or } \phi = \tan^{-1} \mu_S \quad \text{...(8.3)}$$

Angle of Repose : It is the minimum angle of inclination of a plane with the horizontal at which the body kept will just slide down on it without the application of any external force (due to self-weight).

Consider the block with weight W resting on an inclined plane, which makes an angle θ with horizontal as shown in Fig. 8.4-ii. When θ is small the block will rest on the plane. If θ is increased gradually a slope is reached at which the block is about to start sliding. This angle θ is called the *angle of repose*.

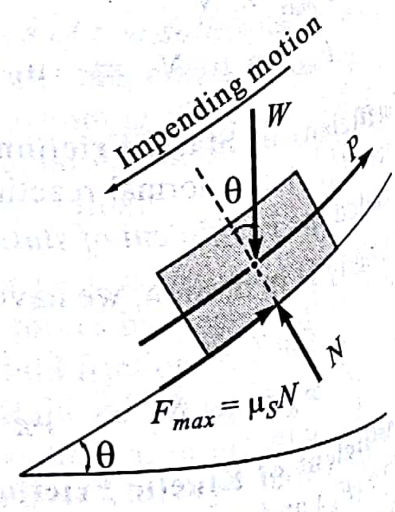


Fig. 8.4-ii

For limiting equilibrium condition, we have

$$\Sigma F_x = 0$$

$$\mu_S N - W \sin\theta = 0$$

$$W \sin\theta = \mu N \quad \text{... (I)}$$

$$\Sigma F_y = 0$$

$$N - W\cos\theta = 0$$

$$W\cos\theta = N \quad \text{... (II)}$$

Dividing Eq. (I) by Eq. (II), we get

$$\tan\theta = \mu_S \quad \text{...(8.3)}$$

In previous discussion, we had $\tan\phi = \mu_S$ which shows

Angle of friction ϕ = Angle of repose θ

The above relation also shows that the angle of repose is independent of the weight of the body and it depends on μ.

Cone of Friction : When the applied force P is just sufficient to produce the impending motion of given body, angle of friction ϕ is obtained which is the angle made by resultant of limiting friction force and normal reaction with normal reaction as shown in Fig. 8.4-iii. If the direction of applied force P is gradually changed through 360°, the resultant R generates a right circular cone with semi vertex angle equal to ϕ. This is called the *cone of friction*.

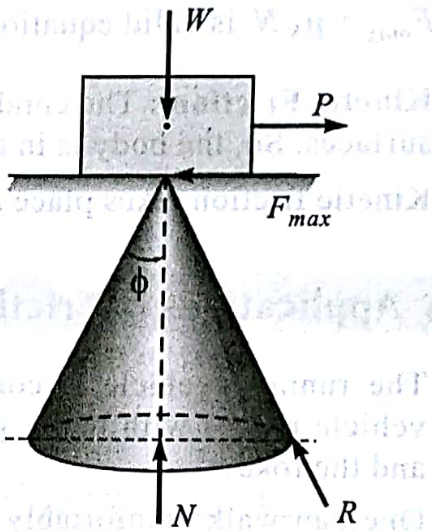


Fig. 8.4-iii

8.5 Types of Friction Problems

The above discussion can be represented by a graph with applied force P v/s frictional force F as shown in Fig. 8.5-i.

Referring to the graph we may now recognize three distinct types of problems. Here, we have static friction, limiting friction and kinetic friction.

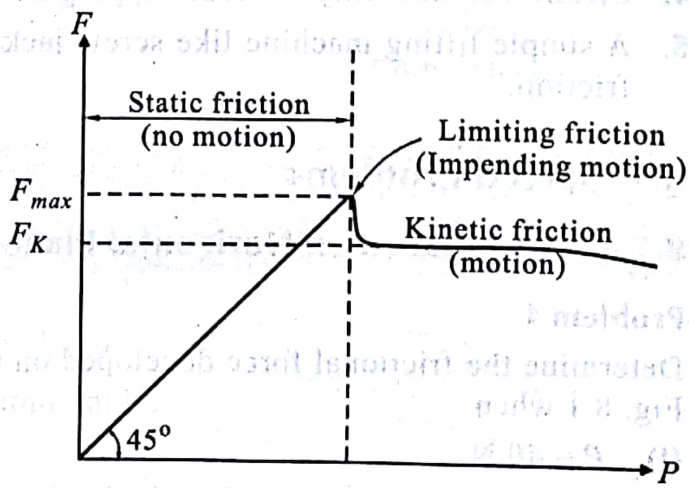


Fig. 8.5-i

1. **Static Friction :** If in the problem there is neither the condition of impending motion nor that of motion then to determine the actual force, we first assume static equilibrium and take F as a frictional force required to maintain the equilibrium condition.

Here, we have three possibilities

(i) $F < F_{max}$ $\Rightarrow$ Body is in static equilibrium condition which means body is purely at rest.

(ii) $F = F_{max}$ $\Rightarrow$ Body is in limiting equilibrium condition which means impending motion and hence $F = F_{max} = \mu_S N$ is valid equation.

(iii) $F > F_{max}$ $\Rightarrow$ Body is in motion which means $F = F_K = \mu_K N$ is valid equation (this condition is impossible, since the surfaces cannot support more force than the maximum frictional force. Therefore, the assumption of equilibrium is invalid, the motion occurs).

2. **Limiting Friction :** The condition of impending motion is known to exist. Here a body which is in equilibrium is on the verge of slipping which means the body is in limiting equilibrium condition.

   $F_{max} = \mu_S N$ is valid equation.

3. **Kinetic Friction :** The condition of relative motion is known to exist between the contacting surfaces. So, the body is in motion.

   Kinetic friction takes place $F_K = \mu_K N$ is valid equation.

8.6 Applications of Friction

1. The running vehicle is controlled by applying brake to its tire because of friction. The vehicle moves with better gripe and does not slips due to appropriate friction between tire and the road.
2. One can walk comfortably on the floor because of proper gripping between floor and the sole of the shoes. It is difficult to walk on oily or soapy floor.
3. Belt and pulley arrangement permits loading and unloading of load effectively because of suitable friction.
4. Lift moves smoothly without slipping due to proper rope and pulley friction combination.
5. A simple lifting machine like screw jack functions effectively based on principle of wedge friction.

8.7 Solved Problems

8.7.1 Body Placed on Horizontal Plane

Problem 1

Determine the frictional force developed on the block shown in Fig. 8.1 when

(i) $P = 40$ N,

(ii) $P = 80$ N. Coefficient of static friction between block and floor is $\mu_S = 0.3$ and $\mu_K = 0.25$ and

(iii) Also find the value of P when the block is about to move.

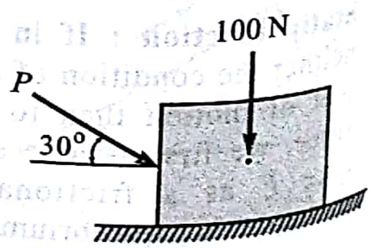


Fig. 8.1

Example 5.4. *What is the value of P in the system shown in Fig. 5.8(a) to cause the motion of 500N block to the right side ? Assume the pulley is smooth and the coefficient of friction between other contact surfaces is 0.20.*

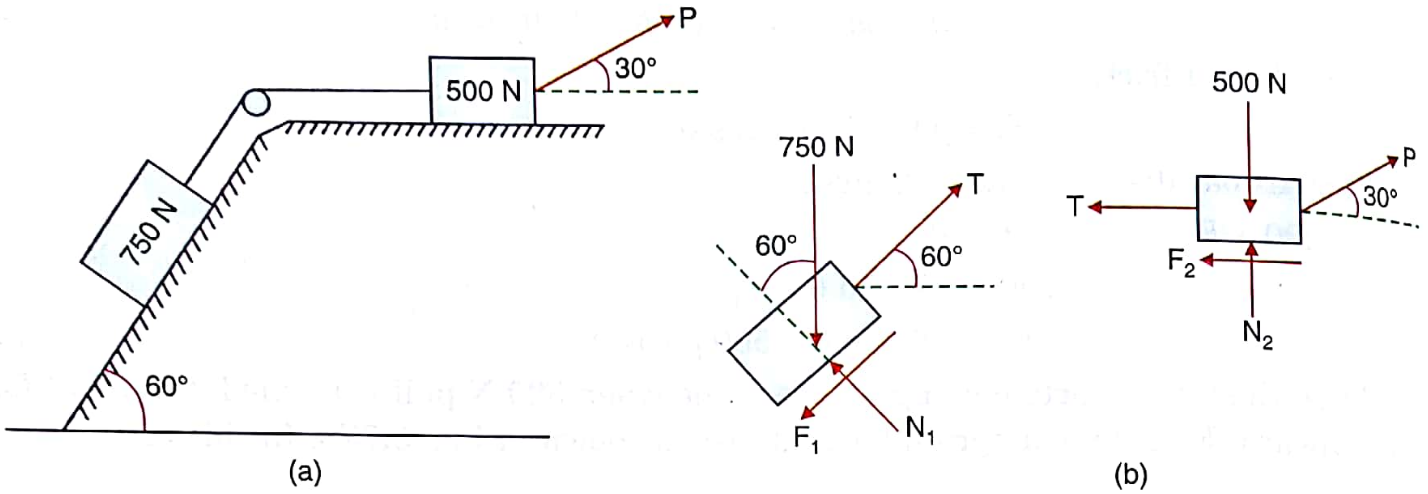


Fig. 5.8

Solution. Free body diagrams of the blocks are as shown in Fig. 5.8(*b*). Consider the equilibrium of 750 N block.

ΣForces normal to the plane = 0, gives

$$N_1 - 750 \cos 60° = 0 \quad \text{or} \quad N_1 = 375 \text{ N.}$$

Since the motion is impending

$$F_1 = \mu N_1 = 0.2 \times 375 = 75 \text{ N.}$$

ΣForces parallel to the plane = 0, gives

$$T - F_1 - 750 \sin 60° = 0$$

$$\therefore \quad T = F_1 + 750 \sin 60° = 75 + 750 \sin 60° = 724.52\text{N.}$$

Consider the equilibrium of 500 N block,

$$\Sigma V = 0, \text{ gives}$$

$$N_2 - 500 + P \sin 30° = 0$$

or $$N_2 + 0.5P = 500$$

i.e., $$N_2 = 500 - 0.5P$$

From law of friction,

$$F_2 = 0.2\, N_2 = 0.2(500 - 0.5P) = 100 - 0.1P$$

$$\Sigma H = 0, \text{ gives}$$

$$P \cos 30° - T - F_2 = 0$$

$$P \cos 30° - 724.52 - (100 - 0.1P) = 0$$

$$\therefore \quad P (\cos 30° + 0.1) = 724.52 + 100 = 824.52$$

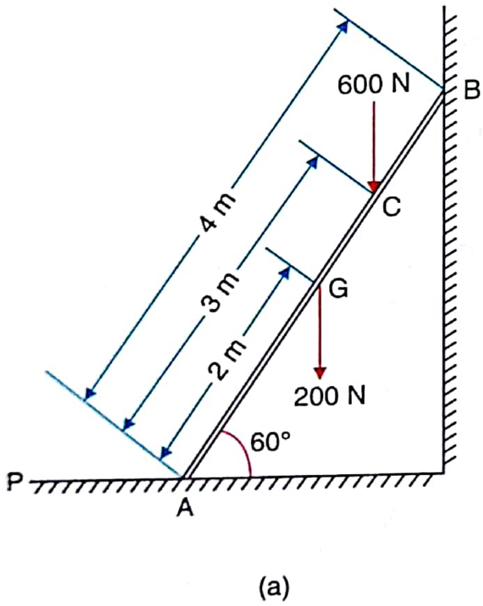
$$\therefore \quad \mathbf{P = 853.52 \text{ N}} \quad \textbf{Ans.}$$

Example 5.5. *Two blocks connected by a horizontal link AB are supported on two rough planes as shown in Fig. 5.9(a). The coefficient of friction between the block A and horizontal surface is 0.4. The limiting angle of friction between block B and inclined plane is 20°. What is the smallest weight W of the block A for which equilibrium of the system can exist, if the weight of block B is 5 kN?*

5.5 PROBLEMS INVOLVING NON-CONCURRENT FORCE SYSTEMS

There are many practical problems of non-concurrent force systems involving friction. In these cases, apart from law of friction, three equations of equilibrium are to be used. The method of solving such problems is illustrated below with typical problems.

Example 5.11. *A ladder of length 4 m, weighing 200 N is placed against a vertical wall as shown in Fig. 5.15(a). The coefficient friction between the wall and the ladder is 0.2 and that between the floor and the ladder is 0.3. In addition to self weight, the ladder has to support a man weighing 600 N at a distance of 3 m from A. Calculate the minimum horizontal force to be applied at A to prevent slipping.*



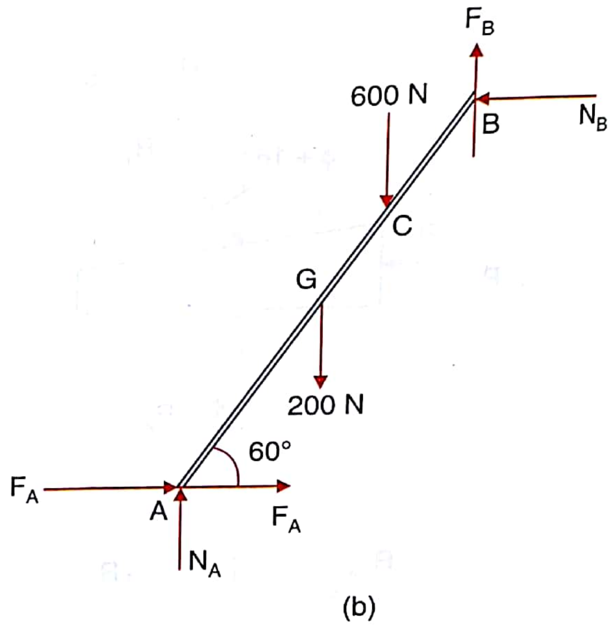


Fig. 5.15

Solution. The free body diagram of the ladder is as shown in Fig. 5.15(*b*).

$$\Sigma M_A = 0, \text{ gives}$$

$$N_B \times 4 \sin 60° + F_B \times 4 \cos 60° - 600 \times 3 \cos 60° - 200 \times 2 \cos 60° = 0$$

Dividing throughout by 4 and rearranging, we get,

$$0.866\, N_B + 0.5\, F_B = 275 \quad ...(1)$$

From the law of friction,

$$F_B = 0.2\, N_B \quad ...(2)$$

Substituting this in Eqn. (1), we get

$$N_B(0.866 + 0.5 \times 0.2) = 275$$

$\therefore$ $N_B = 284.68$ newton ...(3)

$\therefore$ $F_B = 0.2 \times 284.68 = 56.934$ newton ...(4)

$\Sigma V = 0$, gives

$$N_A - 200 - 600 + F_B = 0$$

$\therefore$ $N_A = 200 + 600 - 56.934 = 743.066$ newton

$\therefore$ $F_A = 0.3\ N_A = 0.3 \times 743.066 = 222.92$ newton.

$\Sigma H = 0$, gives

$$P + F_A - N_B = 0$$

$\therefore$ $P = N_B - F_A = 284.68 - 222.92$

i.e., **P = 61.76 newton. Ans.**

Example 5.12. *The ladder shown in Fig. 5.16(a) is 6m long and is supported by a horizontal floor and vertical wall. The coefficient of friction between the floor and the ladder is 0.25 and between wall and the ladder is 0.4. The self weight of the ladder is 200 N and may be considered as concentrated at G. The ladder also supports a vertical load of 900 N at C which is at a distance of 1 m from B. Determine the least value of α at which the ladder may be placed without slipping. Determine the reactions developed at that stage.*

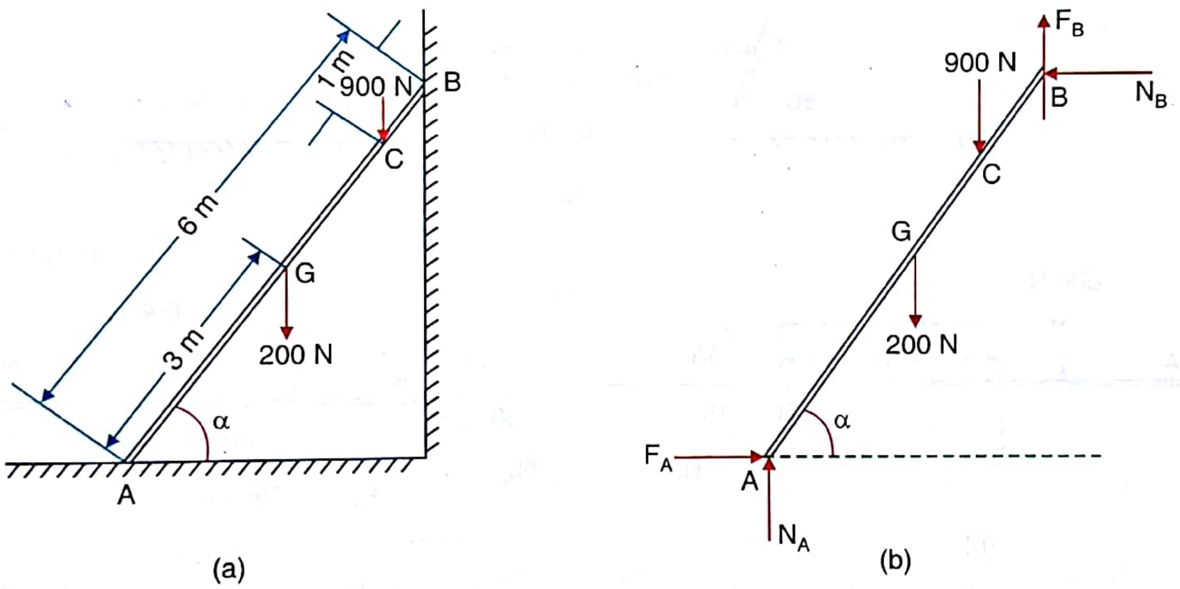


Fig. 5.16

Solution. Free body diagram of ladder for this case is as shown in Fig. 5.16(*b*). From the law of friction,

$F_A = 0.25\ N_A$ ...(1)

and $F_B = 0.4\ N_B$ ...(2)

$\Sigma V = 0$, gives

$$N_A - 200 - 900 + F_B = 0$$

i.e., $N_A + 0.4\ N_B = 1100$ ...(3)

$\Sigma H = 0$, gives

$$F_A - N_B = 0$$

i.e., $0.25\ N_A = N_B$ ...(4)

Wedge : A tapper shaped block with very less angle which are used for lifting or shifting or holding the heavy block by very less effort is called a *wedge*. The lifting or shifting of the distance is very small. While installing heavy machinery horizontal levelling is required with zero error. It is possible to adjust the small height by inserting wedge as a packing. Sometimes, combination of wedge is also used to push or shift heavy bodies by little distance. Simple lifting machine such as screw jack is based on principle of wedge which is used to raise or lower the heavy load by small effort.

Problem 21

A block of mass 150 kg is raised by a 10° wedge weighing 50 kg under it and by applying a horizontal force at it as shown in Fig. 8.21. Taking coefficient of friction between all surfaces of contact as 0.3, find what minimum force should be applied to raise the block.

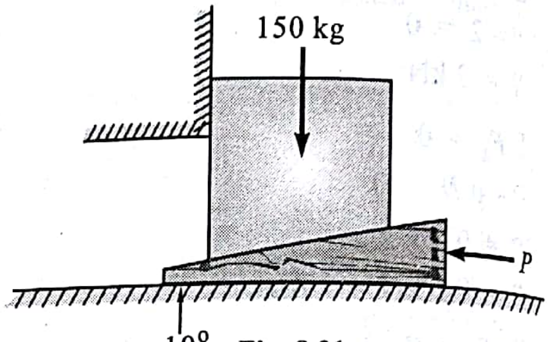
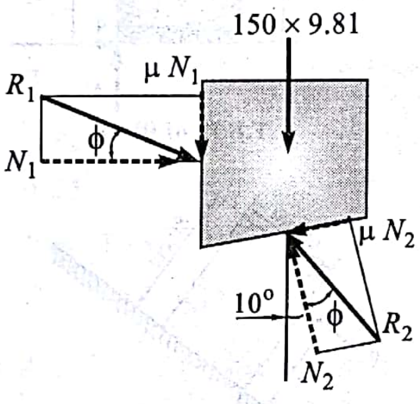


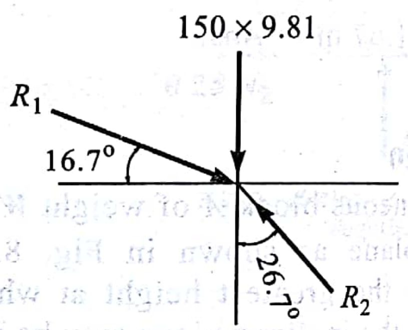
Fig. 8.21

Solution

(i) Consider the F.B.D. of 150 kg block



F.B.D. of Block



$\tan \phi = \mu = 0.3$

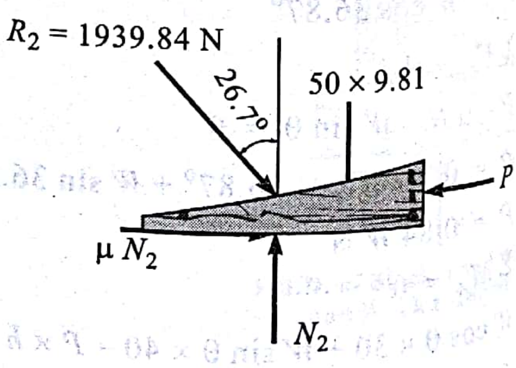
$\therefore \ \phi = 16.7°$

By Lami's theorem, we have

$$\frac{R_2}{\sin (90 - 16.7)°} = \frac{150 \times 9.81}{\sin (90 + 16.7 + 26.7)°}$$

$\therefore \ R_2 = 1939.84 \text{ N}$

(ii) Consider the F.B.D. of the wedge



F.B.D. of Wedge

$\Sigma F_y = 0$

$N_2 - (50 \times 9.81) - 1929.84 \cos 26.7° = 0$

$N_2 = 2223.5 \text{ N}$

$\Sigma F_x = 0$

$\mu N_2 + 1939.84 \sin 26.7° - P = 0$

$P = (0.3 \times 2223.5) + 1939.84 \sin 26.7°$

$P = 1538.66 \text{ N}$ ***Ans.***

Problem 24

Determine the force P required to move the block A of weight 5000 N up the inclined plane. Coefficient of friction between all contact surfaces is 0.25. Neglect the weight of the wedge and the wedge angle is 15 degrees.

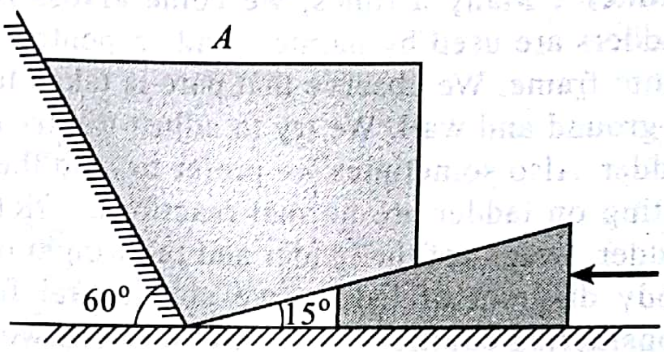


Fig. 8.24

Solution

(i) Consider the F.B.D. of Block A

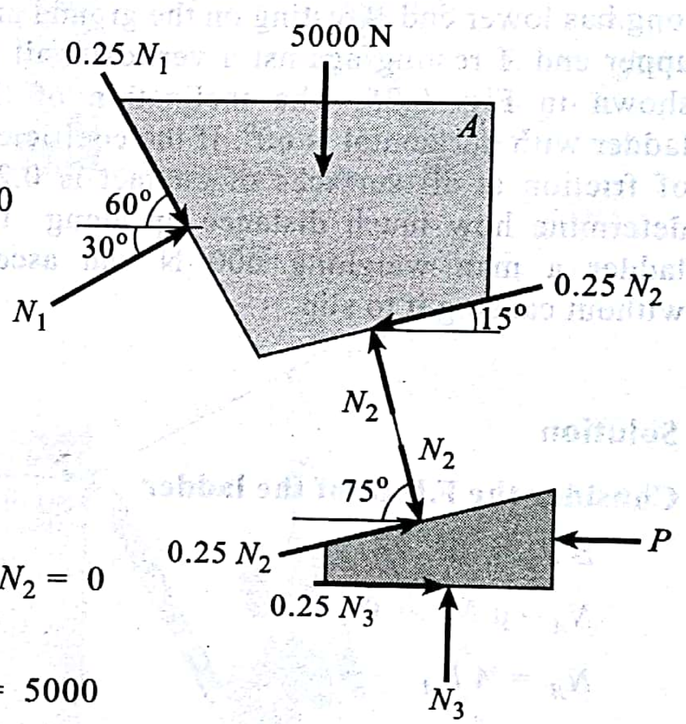
$\Sigma F_x = 0$

$$N_1 \cos 30° + 0.25N_1 \cos 60° - 0.25N_2 \cos 15° - N_2 \cos 75° = 0$$

$$0.860N_1 + 0.125N_1 - 0.241N_2 - 0.2588N_2 = 0$$

$$0.991N_1 = 0.499N_2$$

$$\therefore N_1 = 0.5165N_2$$



F.B.D. of Block A and Wedge

$\Sigma F_y = 0$

$$N_2 \sin 75° + N_1 \sin 30° - 5000 - 0.25N_1 \sin 60° - 0.25N_2 \sin 15° = 0$$

$$0.966N_2 + 0.5N_1 - 5000 - 0.2165N_1 - 0.0647N_2 = 0$$

Substituting the value of N_1

$$0.966N_2 + 0.2583N_2 - 0.1118N_2 - 0.0647N_2 = 5000$$

$$\therefore N_2 = 4772.3585 \text{ N}$$

(ii) Consider the F.B.D. of Wedge B

$\Sigma F_y = 0$

$$N_3 + 0.25N_2 \sin 15° - N_2 \sin 75° = 0$$

$$N_3 + 308.79 - 4609.744 = 0$$

$$N_3 = 4300.95 \text{ N}$$

$\Sigma F_y = 0$

$$0.25N_3 + 0.25N_2 \cos 15° + N_2 \cos 75° - P = 0$$

$$1075.23 + 1152.436 + 123.17 = P$$

$P = 3462.84$ N ***Ans.***

Ladder : Many a times, we come across the uses of ladder for attending the higher height. Ladders are used by painters and carpenters who want to peg a nail in the wall for mounting a photo frame. We observe that care is taken to place the ladder at appropriate angle with respect to ground and wall. We try to adjust the friction offered by the ground and wall in contact with ladder. Also sometimes we prefer to hold the ladder by a person for safety purposes. The forces acting on ladder are normal reactions, frictional forces between the ground, the wall and the ladder, weight of the ladder and the weight of the man climbing the ladder. Considering the free body diagram of ladder, we get general force system. The simplification of the system by considering equilibrium condition can be worked out by following equations :

$$\Sigma F_x = 0 ; \quad \Sigma F_y = 0 \text{ and } \Sigma M = 0$$

Problem 25

A uniform ladder weighing 100 N and 5 meters long has lower end B resting on the ground and upper end A resting against a vertical wall as shown in Fig. 8.25. The inclination of the ladder with horizontal is 60°. If the coefficient of friction at all surfaces of contact is 0.25, determine how much distance up along the ladder a man weighing 600 N can ascent without causing it to slip.

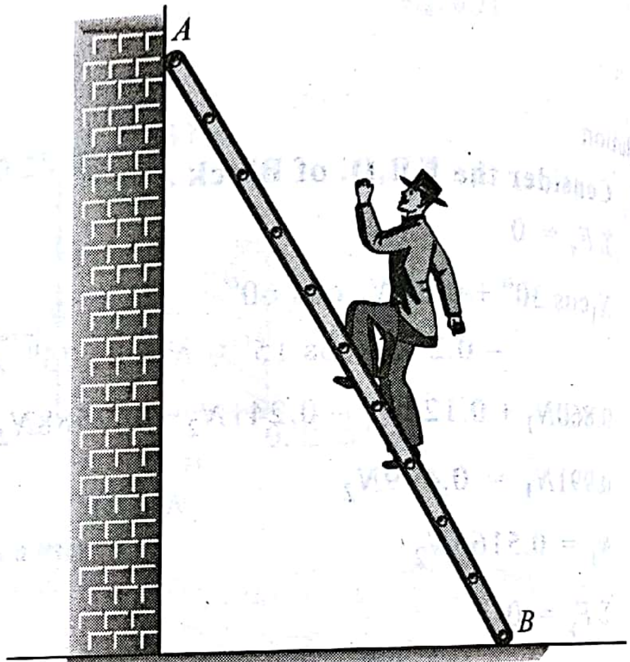


Fig. 8.25

Solution

Consider the F.B.D. of the ladder

$$\Sigma F_x = 0$$

$$N_A - \mu N_B = 0$$

$$N_B = 4 N_A$$

$$\Sigma F_y = 0$$

$$\mu N_A + N_B - 100 - 600 = 0$$

$$0.25 N_A + 4 N_A = 700$$

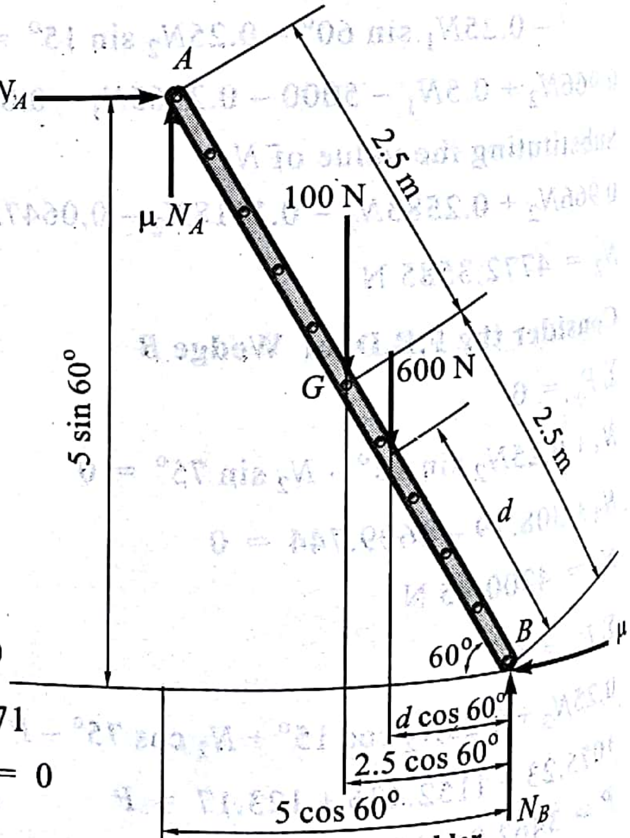
$$N_A = 164.71$$

$$\Sigma M_B = 0$$

$$100 \times 2.5 \cos 60^\circ + 600 \times d \cos 60^\circ - N_A \times 5 \sin 60^\circ - \mu N_A \times 5 \cos 60^\circ = 0$$

$$100 \times 2.5 \cos 60^\circ + 600 \times d \cos 60^\circ - 164.71 \times 5 \sin 60^\circ - 0.25 \times 164.71 \times 5 \cos 60^\circ = 0$$

$d = 2.304$ m ***Ans.***



F.B.D. of the Ladder

Problem 27

A 100 N uniform rod AB is held in the position as shown in Fig. 8.27. If coefficient of friction is 0.15 at A and B. Calculate range of values of P for which equilibrium is maintained.

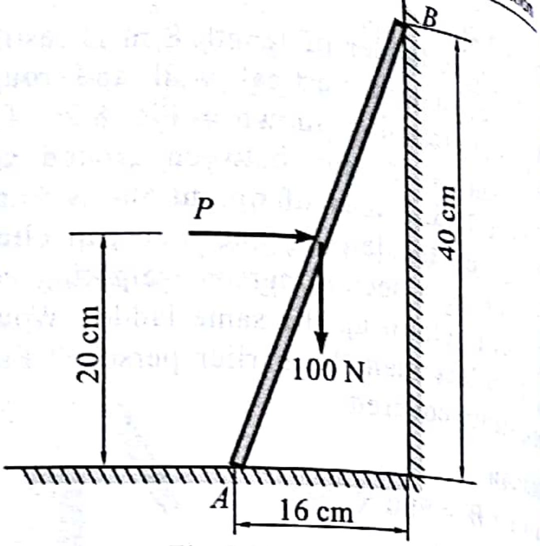


Fig. 8.27

Solution

Case I : For P_{minimum}

Consider F.B.D. of rod AB when P is minimum and in limiting equilibrium condition the tendency of rod will be to slip in downward direction.

$$\Sigma F_x = 0$$

$$P_{\min} + \mu N_A - N_B = 0$$

$$P_{\min} = N_B - 0.15\, N_A \qquad \text{... (I)}$$

$$\Sigma F_y = 0$$

$$N_A + \mu N_B - 100 = 0$$

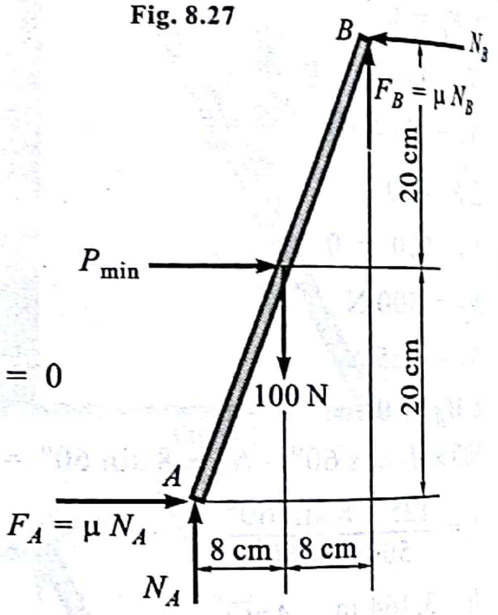
$$N_A + 0.15 N_B = 100 \qquad \text{... (II)}$$

$$\Sigma M_A = 0$$

$$\mu N_B \times 16 + N_B \times 40 - 100 \times 8 - P_{\min} \times 20 = 0$$

$$0.15\, N_B \times 16 + N_B \times 40 - 800 - (N_A - 0.15\, N_A) \times 20 = 0$$

$$3\, N_A + 22.4\, N_B = 800 \qquad \text{.... (III)}$$



F.B.D. of Rod AB for $P_{\min}$

Solving Eqs. (II) and (III),

$$N_A = 96.58 \text{ N}$$

$$N_B = 22.78 \text{ N}$$

From Eq. (I),

$$P_{\min} = 22.78 - 0.15 \times 96.58$$

$$P_{\min} = 8.29 \text{ N} \quad \textit{Ans.}$$

Case II : For P_{maximum}

Consider F.B.D. of rod AB when P is maximum and in limiting equilibrium condition the tendency of rod will be to slip in upward direction.

$$\Sigma F_x = 0$$

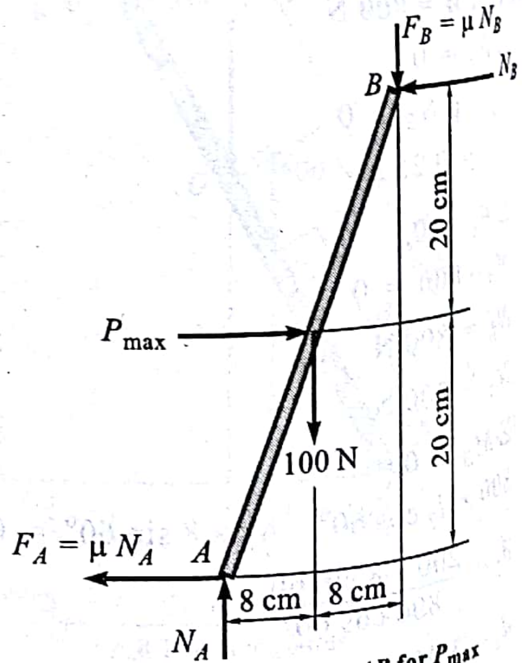
$$P_{\max} - \mu N_A - N_B = 0$$

$$P_{\max} = 0.15\, N_A + N_B \qquad \text{... (IV)}$$

$$\Sigma F_y = 0$$

$$N_A - \mu N_B - 100 = 0$$

$$N_A - 0.15\, N_B = 100 \qquad \text{... (V)}$$



F.B.D. of Rod AB for $P_{\max}$