8 FRICTION

8·1. Introduction :

Friction may be defined as a force of resistance acting on a body which prevents or retards slipping of the body relative to a second body with which it is in contact. This force always acts tangent to the surface at points of contact with other bodies and is directed so as to oppose the possible or existing motion of the body at these points.

Uptill now we assumed that the surfaces in contact are smooth, but actually, *no perfectly frictionless surface exists. When two surfaces are in contact, tangential forces, called frictional forces, will always develop if one attempts to move one surface with respect to other.*

There are two types of friction :

Friction

Dry friction (Coulomb friction)	**Fluid friction**
occurs between *nonlubricated (dry)* surfaces of rigid bodies	develops between *layers of fluid* moving at different velocities. OR exists between the *surfaces separated by a film of fluid* (gas or liquid)
Engg. Applications	**Engg. Applications**
Wedges, Square-threaded screws, Journal bearings, Thrust bearings, Rolling resistance, and Belt friction	Flow of fluid through pipes and orifices, Bodies immersed in moving fluids.
Studied **in this chapter.**	*Studied in* **Fluid Mechanics**

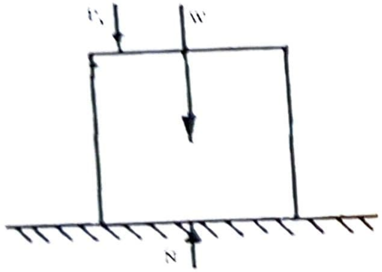
In this book only the effects of *dry friction* will be presented. his type of friction is often called *coulomb friction*, since its haracteristics were extensively studied by C.A. Coulomb in 1781. pecifically, dry friction occurs between the contacting surfaces of odies in the *absence of a lubricating fluid*.

8.2. Theory of Dry Friction :

Because of *perfectly smooth surfaces*, the *force of interaction* etween the bodies always acts normal to the surface at points of ontact. In reality, however, all surfaces are *rough*, and depending upon the nature of the problem, the ability of a body to support a *angential* as well as a normal force at its contacting surface must e considered.

The following experiments will give us the clear understanding of the dry friction.

(1) No Friction :



No Friction
$P_x = 0$

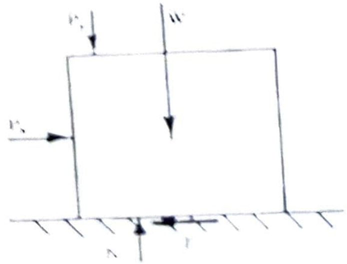
$N = P_y + W$

Fig 8.1

Here, forces acting on the block are its weight W vertically downward, external force P_y and the normal reaction of the surface N.

So $\quad N = P_y + W$
but, if $\quad P_y = 0$
then $\quad N = W$

(2) Equilibrium :



No Motion
Equilibrium exists

$\overline{F} = P_x$
$F < F_m$
$\overline{N} = P_y + \overline{W}$

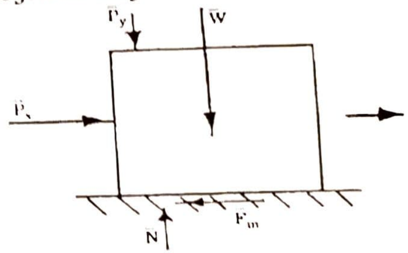
Fig 8.2

When a *small horizontal force* P_x is applied to the block, the *static - friction force* F develops which balances the P_x and the block will not move. This condition is called the *equilibrium* condition of the block.

$$\Sigma F_x = 0; \quad \overline{F} = P_x$$
$$\Sigma F_y = 0; \quad \overline{N} = P_y + \overline{W}$$
$$\Sigma M_{\text{any point}} = 0$$

(3) Impending Motion :

If the force P_x is *increased*, the friction force $\overline{F}$ also increases continuing to oppose P_x, until its magnitude reaches a certain *maximum value F_m*. At this stage, the body is *just about to slide*. This motion is called *impending motion*, and F_m *is called limiting static frictional force*.



Motion Impending

$\overline{P}_x = \overline{F}_m$
$\overline{F}_m = \mu_s \overline{N}$
$\overline{N} = \overline{P}_y + \overline{W}$

Fig 8.3

Experimental evidence shows that *the limiting (maximum) static frictional force F_m is directly proportional to the nomal reaction* N of *the surface*.

$$F_m \propto N$$

$$\boxed{F_m = \mu_s N}$$

where μ_s is a constant called the *coefficient of static friction*.

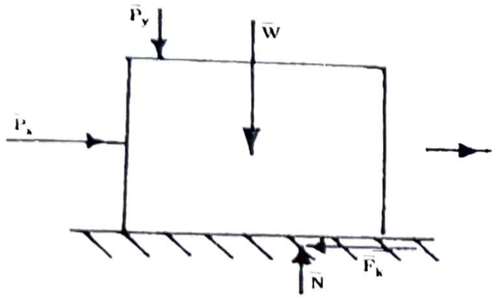
(4) Motion :

If the magnitude of P_x is *further increased so that it become greater than F_m, the frictional force at the contacting surfaces drop slightly to a smaller value of F_k*, called the *kinetic frictional force*. This is because there is *less interpenetration between the irregularities of the surfaces in contact due to motion of the block*.

The magnitude of the *kinetic frictional force* F_k may be put in the form

$$\boxed{F_k = \mu_k N}$$

where μ_k is a constant called the *coefficient of kinetic friction*



Motion

$\bar{P}_x > \bar{F}_m$

$\bar{F}_k < \bar{P}_x$

$\bar{F}_k = \mu_k \bar{N}$

$\bar{N} = \bar{P}_y + \bar{W}$

Fig 8.4

(5) Graphical Presentation :

The above effects can be summerized by reference to the graph shown below which shows the variation of the *frictional force* $\bar{F}$ *versus the applied load* $\bar{P}_x$.

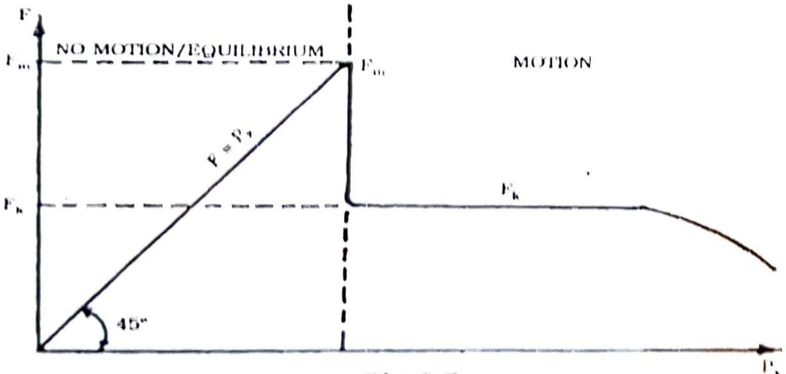


Fig 8.5

Here the frictional force is categorized in three different ways, namely,

(i) **static** *frictional force* ($\bar{F}$) if **equilibrium** is maintained.

(ii) **limiting** *(maximum) frictional force* ($\bar{F}_m$) when its magnitude reaches a *maximum* value needed to maintain **equilibrium.**

(iii) **kinetic** *frictional force* ($\bar{F}_k$) when **motion** occurs at the contacting surface.

Notice also from the graph that for very large value of P_x or for *high speeds, due to aerodynamic effects,* F_k *and likewise* μ_k *decrease somewhat.*

Typical values of coefficient of friction μ_s for Dry Surfaces

Contact Materials	μ_s
1. Metal on ice	0·03 - 0·05
2. *Metal on metal*	*0·15 - 0·60*
3. Metal on wood	0·20 - 0·60
4. Metal on stone	0·30 - 0·70
5. *Metal on leather*	*0·30 - 0·60*
6. Wood on wood	0·25 - 0·50
7. Wood on leather	0·25-0·50
8. Stone on stone	0·40 - 0·70
9. Earth on earth	0·20 - 1·00
10. *Aluminum on aluminum*	*1·10 - 1·70*
11. Rubber on concrete	0·60 - 0·90

Here it may be seen that, as in the case of *aluminum on aluminum*, it is possible for μ_s *to be greater than 1.* **Furthermore, it should be noted that** μ_s **is** *dimensionless* **and depends only upon the** *characteristics of two surfaces in contact.* ***When an exact calculation of*** F_m ***is required, the coefficient of friction should be determined directly by an experiment that involves the two materials to be used,*** as the given typical values was determined under *variable conditions of roughness and cleanliness of the contacting surfaces.*

8·3. Laws of Dry Friction :

Following **rules** are applied to the bodies subjected to dry friction.

1. The frictional force acts *tangent* to the contacting surfaces in a *direction opposite* to the *relative motion* or tendency for motion of one surface agaist another.
2. The magnitude of the *maximum static frictional force* ($\bar{F}_m$) is *independent of the area of contact.*
3. The magnitude of the *maximum static frictional force* (F_m) is generally *greater than* the magnitude of the *kinetic frictional force* (F_k) for any two surfaces of contact ($F_m > F_k$).
4. When *slipping* at the point of contact is *about to occur*, the magnitude of the *maximum static frictional force* is *proportional* to the magnitude of the *normal force* at the point of contact ($F_m \propto N$, $F_m = \mu_s N$).
5. When *slipping* at the point of contact is *occuring*, the magnitude of the *kinetic frictional force* is *proportional* to the magnitude of the *normal force* at the point of contact ($F_k \propto N$, $F_k = \mu_k N$).

8·4. Angle of Friction and Angle of Repose :

1. Angle of Friction :

It is sometimes found convenient to replace the normal force N *and the friction force* F *by their resultant* R. *The angle between the*

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normal force $\bar{N}$ and the resultant $\bar{R}$ at the time of impending motion is called the angle of static friction and is denoted by ϕ_s.

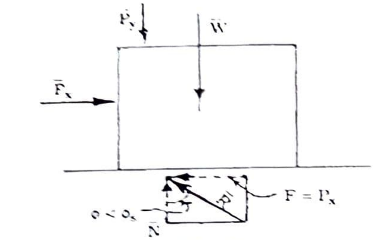


Fig 8·6

No Motion

$\phi < \phi_s$

$\bar{F} = \bar{P}_x$ and $\bar{P}_x < \bar{F}_m$

$\bar{N} = \bar{P}_y + \bar{W}$

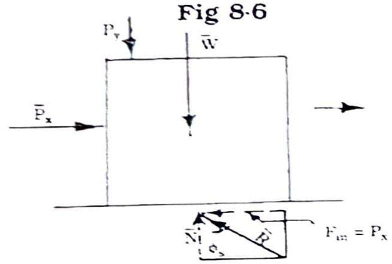


Fig 8·7

Impending Motion

$\tan \phi_s = \dfrac{\bar{F}_m}{\bar{N}}$

$\bar{F}_m = \bar{P}_x$

$\bar{N} = \bar{P}_y + \bar{W}$

Thus,

Angle of static friction $\phi_s = \tan^{-1}\left(\dfrac{F_m}{N}\right) = \tan^{-1}\left(\dfrac{\mu_s N}{N}\right) = \tan^{-1}\mu_s$

OR $\boxed{\tan \phi_s = \mu_s}$

If motion actually takes place, the magnitude of the friction force drops to F_k; similarly, the angle between R and N drops to a lower value ϕ_k, called the *angle of kinetic friction.*

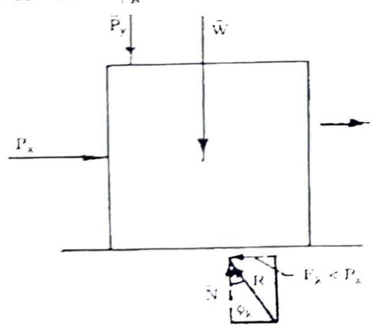


Fig 8·8

Motion

$F_k < P_x$, $F_k < F_m$

$\tan \phi_k = \dfrac{F_k}{N}$

Thus,

Angle of kinetic friction, $\phi_k = \tan^{-1}\left(\dfrac{F_k}{N}\right) = \tan^{-1}\left(\dfrac{\mu_k N}{N}\right) = \tan^{-1}\mu_k$

OR $\boxed{\tan \phi_k = \mu_k}$

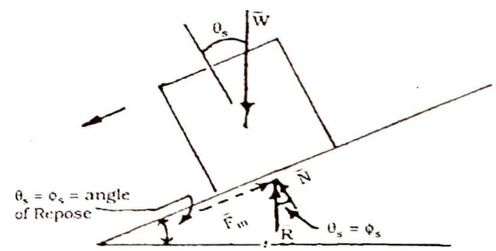
By comparision,

$$\boxed{\phi_s > \phi_k}$$

2. Angle of Repose :

Another experimental method which can be used to measure the coefficient of friction between two contacting surfaces consists of *placing a block of one material on an inclined plane of another material. If we keep increasing the angle of inclination, motion will soon become impending.* The block is on the *verge of sliding* and therefore $F_m = \mu_s N$.

The value of the angle of inclination (θ_s) corresponding to impending motion is called angle of repose.



Motion Impending

$\theta_s = \phi_s$

Fig 8·9

$\theta_s = \tan^{-1}\left(\dfrac{F_m}{N}\right) = \tan^{-1}\left(\dfrac{\mu_s N}{N}\right) = \tan^{-1}\mu_s = \phi_s$

Thus, Angle of Repose θ_s = Angle of static friction ϕ_s

Here, $\boxed{\mu_s = \tan \theta_s}$

Note that the angle of repose is independent of the weight of the block. Here the pulling or pushing force on the block for its impending motion should not be applied by external means.

8·5. Problems Involving Dry Friction :

There are *three types of mechanics- problems, involving dry friction.*

Equilibrium | **Impending Motion at all points** | **Tipping or Impending Motion at some points**

1. Equilibrium : *Equilibrium problems require the total number of unknowns to be* **equal** *to the total number of available equilibrium equations.*

Given : (1) All applied forces and
(2) The coefficient of friction.

Unknowns : (1) Check-body will remain at rest or slide
(2) Friction forces required to maintain equilibrium $(F \neq \mu_s N)$
(3) Normal forces

Solution : (1) Draw free-body diagram for each member.
(2) Solve the equations of equilibrium.

Example :

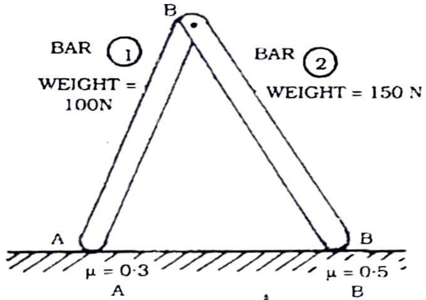
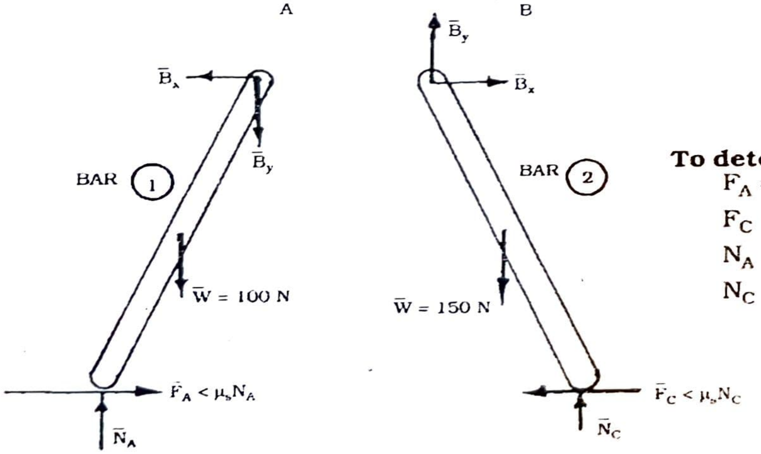


Fig 8·10



To determine
$F_A = ?$
$F_C = ?$
$N_A = ?$
$N_C = ?$

(a) **(b)**

Fig 8·11 Free-body diagrams

Here, *separate equilibrium* of bar-1 and bar-2 can be considered. But while considering the separate equilibrium, equal and opposite forces at the contact points between two bodies should be mentioned carefully. Joint B is hinged hence two reaction components are existing.

Six unknowns - B_x, B_y, F_A, N_A, F_c and N_c can be found out from *six equilibrium equations (three for each member* - $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M = 0$).

The moment can be taken about any point.

The bars will remain in equilibrium provided $F_A \leq 0{\cdot}3\, N_A$ and $F_C \leq 0{\cdot}5\, N_c$ are satisfied.

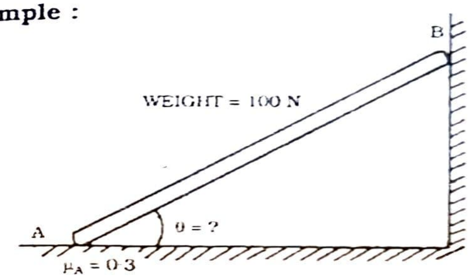
2. Impending Motion at All Points : *In this case the total number of unknowns will be* **equal** *to the total number of available equations* **plus** *the total number of available frictional equations.*

Given : (1) All applied forces
(2) The coefficient of friction. **OR** angle of plank (ladder)

Unknowns : Angle of plank (ladder) or Coefficient of friction.

Solution : (1) Draw free-body diagram.
(2) Solve equilibrium equations plus static frictional equations.

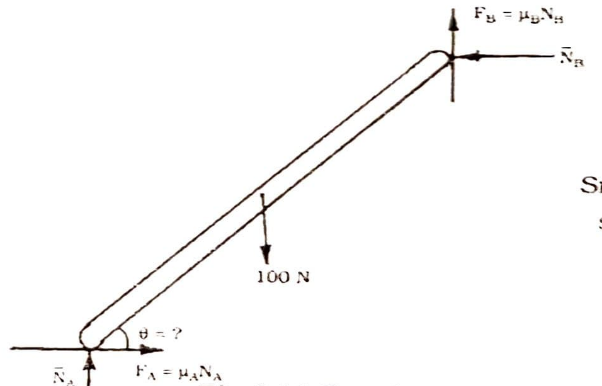
Example :



Impending Motion

$F_m = \mu_s N$

Fig 8·12



To determine
Smallest angle θ = ?
so that Bar AB is placed without slipping

Fig 8·13 Free-body diagram.

Five unknowns - F_A, N_A, F_B, N_B, θ can be found out from *three equilibrium equations* ($\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M = 0$) and *two static frictional equations* ($F_A = 0{\cdot}3\, N_A$, $F_B = 0{\cdot}4\, N_A$).

Here moment can be taken about any supporting point of plank.

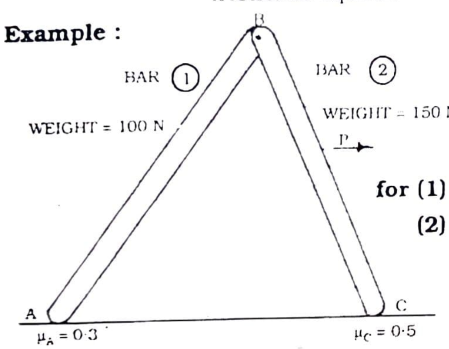
3. Tipping or Impending Motion at Some Points : *Here the total number of unknowns will be* **less** *than the number of available equilibrium equations* **plus** *the total number of frictional equations or conditional equations for tipping.*

Given : (1) Impending motion in given direction
(2) coefficients of friction.

Unknowns : (1) Applied force $\bar{P}$
(2) Frictional forces and Normal forces

Solution : (1) Draw free-body diagram of each member.
(2) Solve equations of equilibrium and static frictional equation.

Example :

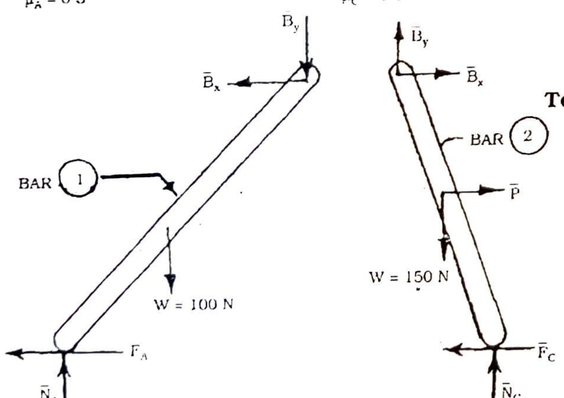


Find smallest P

for (1) $F_A = 0.3\ N_A$ & $F_C \le 0.5\ N_C$

(2) $F_A \le 0.3\ N_A$ & $F_C = 0.5\ N_C$

Fig 8.14



To determine

$N_A = ?$

$F_A = ?$

$N_C = ?$

$F_c = ?$

$B_x = ?$

$B_y = ?$

$P = ?$

(a) (b)

Fig 8.15 Free=body diagrams

Seven Unknowns - N_A, F_A, N_C, F_c, B_x, B_y, P can be found out from *six equilibrium equations* (three for each member - $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M = 0$) *and two static frictional equations* [(1) $F_A = 0.3\ N_A$ and $F_c \le 0.5\ N_c$ and then (2) $F_A \le 0.3\ N_A$ and $F_c = 0.5\ N_c$].

Find smallest P out of above two cases i.e. (1) slipping occurs at A only (2) slipping occurs at C only. If P is same for both the cases, then slipping will occur at A and B both.

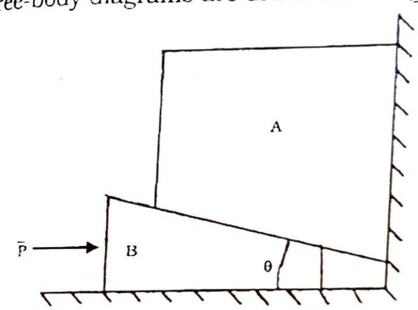
8.6. Wedges :

Wedges are simple machines which are often used to raise or displace heavy loads. They transform an applied force in to much larger force, directed at approximately right angles to the applied force.

Besides, because of the friction existing between the surfaces in contact, a wedge, if properely shaped, will remain in place after being forced under the load. Thus, wedges may be used advantageously to make small adjustments in the position of heavy pieces of machinery.

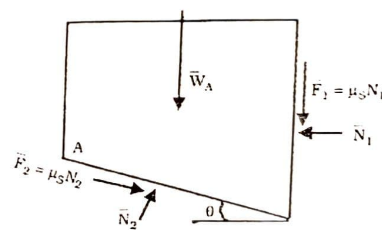
The weight of the wedge is comparatively small, hence generally neglected.

For example, to lift the block A, a force P is applied to the wedge B. Here a *small horizontal force P will be converted* through light wedge B in to a *large vertical force* to lift heavy block A. The free-body diagrams are drawn for wedge and block separately.



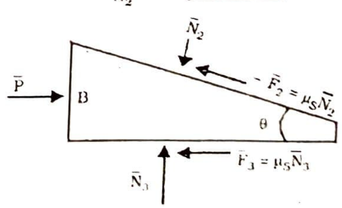
Block (A)
Wedge (B)

Fig 8.16



Free-body diagram of Block-A

Fig 8.17



Free-body diagram of Wedge-B.

Fig 8.18

First the movement of the wedge should be considered and frictional forces must be indicated in the direction opposite to its motion. Then frictional forces are to be indicated on the block opposite to the frictional forces on the wedge.

Here, wedge B is having two contact surfaces hence two frictional forces should be indicated opposite to the motion of the wedge. The normal forces in addition to frictional forces will be acting at these surfaces. Now, equal and opposite forces are denoted on block A having surface in contact with wedge B.

Friction force at the contact surface will be equal to μ_s into normal force at that particular surface. Forces considered on the Block as well as on the wedge can be determined by solving independently the equations of equilibrium (i.e. $\Sigma F_x = 0$ and $\Sigma F_y = 0$).

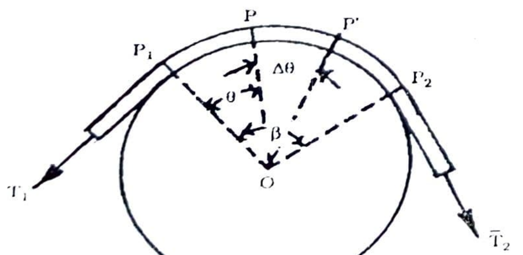
In above example; $\bar{N}_1$, $\bar{N}_2$, $\bar{N}_3$ and $\bar{P}$ are unknowns and four equilibrium equations are available (two each for block and wedge).

8·7 Belt Friction :

In the design of *belt drives* or *band brakes*, it is necessary to determine the frictional forces developed between belt and its contacting surface.

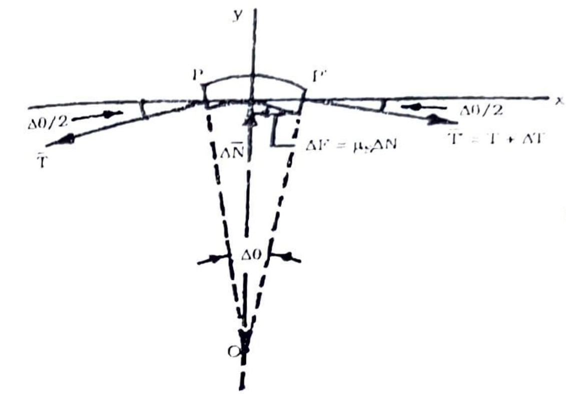
Consider a flat belt passing over a fixed drum of radius r. The *total angle* of contact of belt in radians is β and coefficient of friction between the two surfaces is μ. It is proposed to determine the relation existing between the value of T_1 and T_2 of the tension in the two parts of the belt when the belt is just about to slide toward the right. *Obviously, T_2 must be greater than T_1 since the belt must overcome T_1 and the resistance of friction at the surface of contact.*

The free-body diagram of a small element PP′ detached from the belt is shown.



Flat Belt passing over a Fixed Cylindrical Drum.

Fig 8·19



Free-body diagram of small element PP′

Fig 8·20

Writing the *equations of equilibrium* for the element PP′ :

$$\rightarrow \Sigma F_x = 0,\ (T + \Delta T)\cos\frac{\Delta\theta}{2} - T\cos\frac{\Delta\theta}{2} - \mu_s \Delta N = 0 \qquad \text{—— (1)}$$

$$+\uparrow \Sigma F_y = 0,\ \Delta N - (T + \Delta T)\sin\frac{\Delta\theta}{2} - T\sin\frac{\Delta\theta}{2} = 0 \qquad \text{—— (2)}$$

Solving eq. (2) for ΔN and substituting into eq. (1)

$$\Delta T\cos\frac{\Delta\theta}{2} - \mu_s (2T + \Delta T)\sin\frac{\Delta\theta}{2} = 0$$

Dividing by Δθ,

$$\frac{\Delta T}{\Delta\theta}\cos\frac{\Delta\theta}{2} - \mu_s \left(T + \frac{\Delta T}{2}\right)\frac{\sin(\Delta\theta/2)}{\Delta\theta/2} = 0$$

If $\Delta\theta \to 0$, $\cos\frac{\Delta\theta}{2} \to 1$,

$$\Delta T/2 \to 0, \quad \text{and} \quad \frac{\Delta T}{\Delta\theta} = \frac{dT}{d\theta}$$

and $\frac{\sin(\Delta\theta/2)}{\Delta\theta/2} \to 1$,

We write

$$\frac{dT}{d\theta} - \mu_s T = 0$$

$$\therefore \frac{dT}{T} = \mu_s\, d\theta$$

At P_1, $\theta = 0$ and $T = T_1$

At P_2, $\theta = \beta$ and $T = T_2$

Integrating from P_1 to P_2,

$$\int_{T_1}^{T_2} \frac{dT}{T} = \int_0^{\beta} \mu_s \, d\theta$$

$\therefore \quad \log_e T_2 - \log_e T_1 = \mu_s\beta$

$\therefore \quad \log_e \frac{T_2}{T_1} = \mu_s\beta \qquad$ OR $\qquad \ln \frac{T_2}{T_1} = \mu_s\beta$

OR

$$\boxed{\frac{T_2}{T_1} = e^{\mu_s\beta}}$$

Where $T_2, T_1 =$ belt tensions

T_2 in the *direction of belt impending motion*

T_1 in the *opposite direction of belt impending motion.*

$\mathbf{T_2 > T_1}$

μ_s = coefficient of static friction between belt and surface of contact

β = *angle of belt* to surface contact, measured in radians.

e = 2·718 —— base of the natural logarithm

The above derived equation is *independent of the radius of the drum*. It is also *valid* for problems of ropes wrapped around a post or capstan and even problems of band brakes in which *drum rotates while the band remains fixed. It is valid only when impending motion occurs.*

If the belt, rope, or brake is actually *slipping*, μ_s in the above equation should be *replaced* by μ_k.

The belts used in belt drives are often V - shaped. Such a belt is called a *v-belt.*

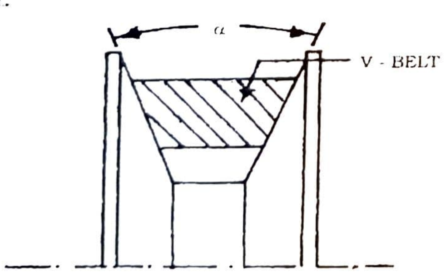


Fig 8·21 V–Belt

For the V-belt, the equation is

$$\frac{T_2}{T_1} = e^{\mu_s\beta/\sin(\alpha/2)}$$

IMPORTANT EQUATIONS

1. **No Friction,** $P_x = 0,\ F_x = 0$
2. **No Motion, Equilibrium exists,** $F = P_x,\ F < F_m$
3. **Motion Impending,** $P_x = F_m,\ F_m = \mu_s N$
4. **Motion,** $P_x > F_m,\ F_k < P_x,\ F_k = u_k N$
5. **Angle of Static Friction** $\phi_s = \tan^{-1}\mu_s = \tan^{-1}\left(\frac{F_k}{N}\right)$
6. **Angle of Kinetic Friction** $\phi_k = \tan^{-1}\mu_k = \tan^{-1}\left(\frac{F_k}{N}\right)$
7. $F_m > F_k,\ \phi_s > \phi_k$
8. **Angle of Repose** $\theta_s = \phi_s = \tan^{-1}\left(\frac{F_m}{N}\right)$
9. **Equilibrium Equations :** $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M_{\text{any point}} = 0$
10. **Static Frictional Equation :** $F = \mu_s N$
11. **Belt Friction :**

    (i) **Flat belt, ropes, band brakes :** $\frac{T_2}{T_1} = e^{\mu_s\beta}$

    (ii) **V-belt :** $\frac{T_2}{T_1} = e^{\mu_s\beta/\sin(\alpha/2)}$

SOLVED EXAMPLES

1. The coefficients of friction between the 200N block and the incline are $\mu_s = 0{\cdot}40$ and $\mu_k = 0{\cdot}30$.

(i) Determine whether the block is in equilibrium and find the magnitude and direction of the friction force when P = 480 N.

(ii) Also find the smallest value of P required **(a)** to start the block up the incline, **(b)** to keep it moving up, **(c)** to prevent it from sliding down.

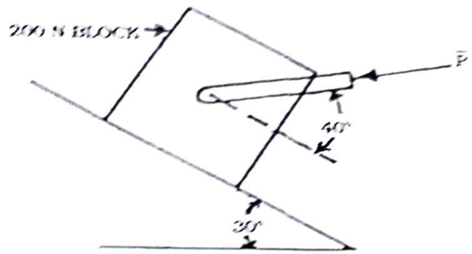
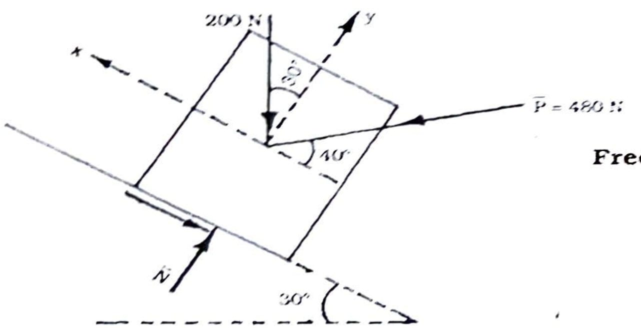


Fig. 8-22

Part (i) : To check the equilibrium, we have to find the *friction force required for equilibrium and maximum friction force.*



Free-body diagram of block

Fig. 8-23

Friction Force Required for Equilibrium :

$\Sigma F_x = 0$, $480 \cos 40° - 200 \sin 30° - F = 0$

$F = 267{\cdot}7$ N (directed down)

The friction force $\bar{F}$ required to maintain equilibrium is 267·7 N force *directed down*, the ***tendency of the block*** **is thus to *move* up the plane.**

Maximum Friction Force :

$F_m = \mu_s N$,

but N from above figure is,

$\Sigma F_y = 0$, $N - 200 \cos 30° - 480 \sin 40° = 0$

$N = 481{\cdot}74$ N

$\therefore\ F_m = \mu_s N$
$= 0{\cdot}40\,(481{\cdot}74)$
$= 192{\cdot}7$ N

Thus, the force required to maintain equilibrium (267·7 N) is *larger* than the maximum value (192·7 N), ***equilibrium will not be maintained*** and the ***block will slide up the plane*****. Block is not in equilibrium, it is moving up.**

Actual Friction Force Acting :

As the block will slide up the plane the actual friction force will be

$F_{actual} = F_k = \mu_k N$
$= 0{\cdot}3\,(481{\cdot}74)$
$= 144{\cdot}52$ N

$\boxed{F_{actual} = 144{\cdot}52 \text{ N and directed down the plane}}$

Part (ii) : Smallest $\bar{P}$ required : (P = ?)

(a) To start the block up :

Friction Force $F = F_m = \mu_s N = 0{\cdot}4$ N

$\Sigma F_y = 0$, $N - 200 \cos 30° - P\sin 40° = 0$

$\Sigma F_x = 0$, $P\cos 40° - 200 \sin 30° - 0{\cdot}4\, N = 0$

from these two equations

$\boxed{P = 332{\cdot}57 \text{ N to start the block up}}$

(b) To keep it moving up :

Friction Force $F = F_k = \mu_k N = 0{\cdot}3$ N.

$\Sigma F_y = 0$, $N - 200 \cos 30° - P\sin 40° = 0$

$\Sigma F_x = 0$, $P\cos 40° - 200 \sin 30° - 0{\cdot}3\, N = 0$

from these two equations

$\boxed{P = 265{\cdot}2 \text{ N to keep the block moving up}}$

(c) To prevent the block from sliding down :

In this case; the **F will be acting up**

and $F = F_m = \mu_s N = 0{\cdot}4$ N

$\Sigma F_y = 0$, $N - 200 \cos 30° - P \sin 40° = 0$

$\Sigma F_x = 0$, $P\cos 40° - 200 \sin 30° + 0{\cdot}4\, N = 0$

From these two equations

$\boxed{P = 30{\cdot}03 \text{ N to prevent it from sliding down}}$

2. The 100 N block A hangs from a cable as shown. Pulley C is connected by a short link to block E, which rests on a horizontal rail. Knowing that the coefficient of static friction between block E and the rail is 0·35, and neglecting the weight of the block E and the friction in the pulleys, determine the maximum allowable value of θ if the system is to remain in equilibrium.

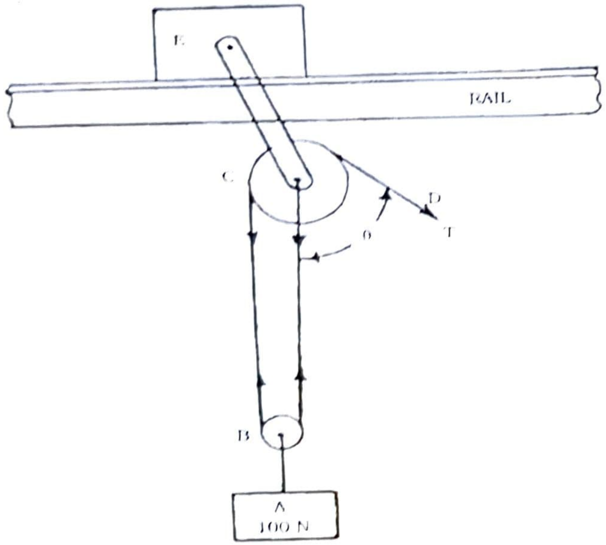
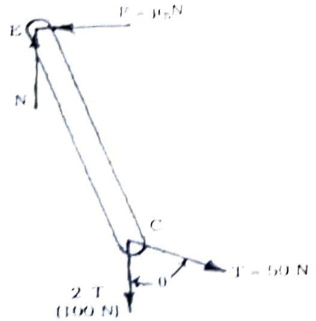


Fig. 8·24



Free-body diagram of short-link EC

Fig. 8·25

The system is under *equilibrium*, hence block E *will not move on rail, thus we can say short link EC is under equilibrium.*

The free-body diagram of link EC is shown. Here, two unknowns are N and θ.

$\overset{+}{\rightarrow}$ $\boldsymbol{\Sigma F_x = 0}$, $T \sin\theta - \mu_s N = 0$

$\uparrow +$ $\boldsymbol{\Sigma F_y = 0}$, $N - 2T - T\cos\theta = 0$

Here $T = 50N$, hence

$50 \sin\theta - 0.35\, N = 0$ ——— (i)

and, $N - 100 - 50 \cos\theta = 0$ ——— (ii)

From eq. (ii)

$N = 100 + 50 \cos\theta$ ——— (iii)

Putting this value of N in eq. (i)

$50 \sin\theta - 35 - 17.5 \cos\theta = 0$

By *trial and error*, θ (means by taking different values of θ) can be found out.

Hence,

$$\boxed{\theta = 60.644^\circ}$$

3. Two homogeneous blocks are freely resting as shown with their weights and coefficients of friction at surfaces of contact. Find the minimum value of P which will just disturb the equilibrium of the system.

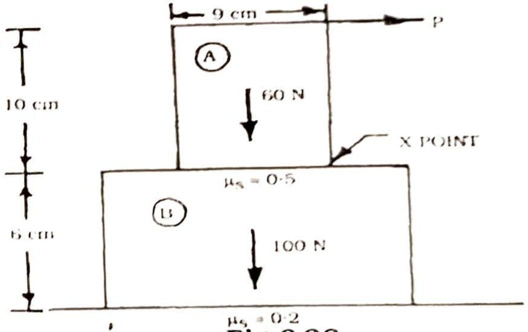


Fig 8·26

There are three ways by which the system can be disturbed.

(i) *If whole system is moving on the floor.*

$P = F_m = \mu_s N = 0.2\,(100 + 60)$

$P = 32N$ ——— (i)

(ii) *If only block A moves*

$P = F_m = \mu_s N = 0.5\,(60)$

$P = 30\ N$ ——— (ii)

(iii) *If block (A) topples about a point X* (block A rotates about X)

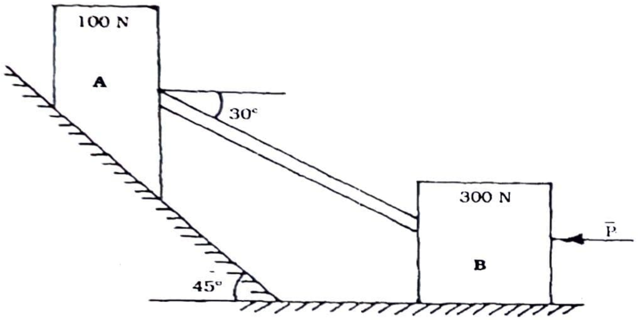
∴ Moment about X is

$P \times 10 = 60 \times 4.5$

∴ $P = 27\ N$ ——— (iii)

Out of above three cases, $\boxed{\text{minimum P is 27 N.}}$

4. A block A weighing 100 N rests on a rough inclined plane whose inclination to the horizontal is 45°. This block is connected to another block B weighing 300 N rests on a rough horizontal plane by a weightless rigid bar inclined at an angle of 30° to the horizontal. Find the horizontal force required to be applied to the block B just to move the block A in upward direction. Assume angle of limiting friction as 15° at all contact surfaces.



$$\phi_s = 15°$$
$$\mu_s = \tan 15° = 0{\cdot}268$$

Fig 8·27

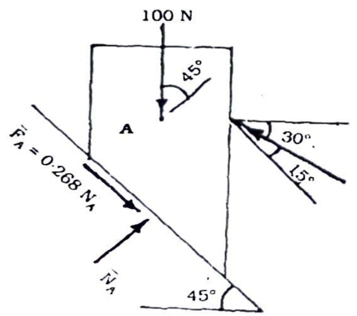


Fig 8·28

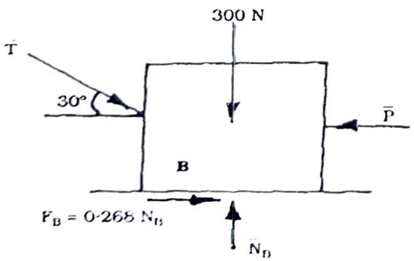


Fig 8·29

Free-body diagrams of block A and B are shown.

Equilibrium of block A :

Resolving forces along plane,

$$100 \sin 45° + 0{\cdot}268\ N_A - T \cos 15° = 0 \qquad \text{—— (i)}$$

and perpendicular to plane,

$$100 \cos 45° + T \sin 15° - N_A = 0 \qquad \text{—— (ii)}$$

From these two equations,

$$T = 99{\cdot}06 \text{ N.}$$

Equilibrium of block B :

Resolving forces along plane.

$$T \cos 30° + 0{\cdot}268\ N_B - P = 0$$

$$\therefore \quad 99{\cdot}96 \cos 30° + 0{\cdot}268\ N_B - P = 0 \qquad \text{—— (iii)}$$

and *Resolving perpendicular to plane*

$$T \sin 30° + 300 - N_B = 0$$

$$\therefore \quad 99{\cdot}96 \sin 30° + 300 - N_B = 0 \qquad \text{—— (iv)}$$

From these two equations,

$$\boxed{P = 180{\cdot}36 \text{ N}}$$

5. A uniform *ladder* of 8·0 meters rests against a vertical wall with which it makes an angle of 45°, the coefficient of friction between ladder and floor is 0·5. If a man, whose weight is one-half of that of the ladder, ascends it, how high will it be when the ladder slips ?

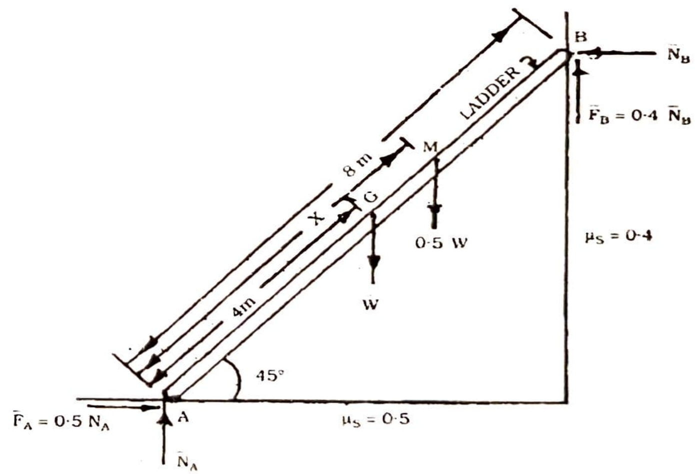


Fig 8·30

Let X be the distance between A and the man, when the *ladder is on the verge of sliding.*

At B the ladder will move downward hence frictional force will be upward, whereas at A it will move towards left hence frictional force will be towards right.

Forces acting on the ladder are shown on the sketch of the ladder itself.

Here, there are *three unknowns*, N_A, N_B and X

and We have three equilibrium equations, $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M = 0$

$\Sigma F_x = 0,\ 0.5\, N_A - N_B = 0$ ——— **(i)**

$\Sigma F_y = 0,\ N_A - W - 0.5\, W + 0.4\, N_B = 0$ ——— **(ii)**

$\Sigma M_A = 0,\ W\,(4 \cos 45°) + 0.5\, W\,(X \cos 45°) - 0.4\, N_B\,(8 \cos 45°)$

$- N_B\,(8 \sin 45°) = 0$ ——— **(iii)**

From (i) $N_B = 0.5\, N_A$ ——— **(iv)**

Putting this in (ii)

$N_A - W - 0.5\, W + 0.4\,(0.5\, N_A) = 0$

$\therefore\ N_A = 1.25\, W$ ——— **(v)**

$\therefore\ N_B = 0.625\, W$ ——— **(vi)**

Putting the value of N_B from eq. (vi) in to eq. (iii)

$$2.828\, W + 0.354\, WX - 1.414\, W - 3.536\, W = 0$$

$$\therefore\ \boxed{X = 5.99\ \text{m}}$$

6. A block B of weight 1000N is being moved slowly to the right by means of the *wedge* under the action of the vertical force P. Find the value of P so as to just move the block if the coefficient of friction at all surfaces of contact is 0.3. *Neglect weight of the wedge.*

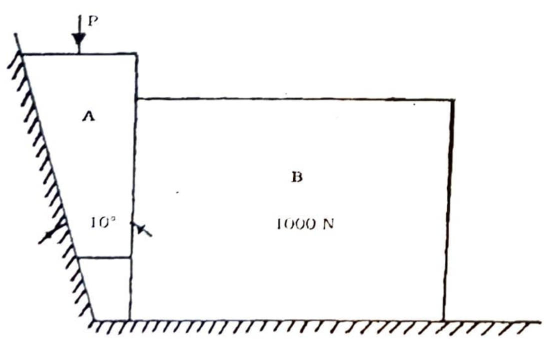
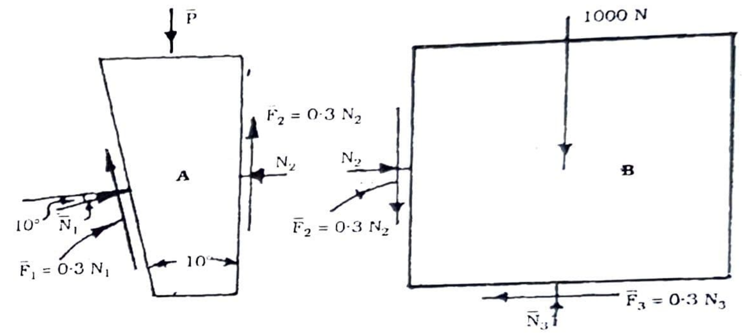


Fig 8.31



Wedge-A
(a)

Block-B
(b)

Fig. 8.32 Free-body diagrams

Wedge A will move down hence frictional forces at two contact surfaces will be towards up. Normal reactions will be perpendicular to these surfaces. On block B, equal and opposite frictional force as well as normal reaction will act at contact surface with wedge. This block B will move towards right hence frictional force will be towards left at bottom surface.

Block B :

$\Sigma F_x = 0,\ N_2 - 0.3\, N_3 = 0 \qquad \therefore\ N_3 = N_2/0.3$

$\Sigma F_y = 0,\ N_3 - 0.3\, N_2 - 1000 = 0.$

$\therefore\ N_2/\text{Ō}\ 0.3 - 0.3\, N_2 - 1000 = 0$

$\therefore\ N_2 = 329.67\ \text{N}$

Wedge A :

$\Sigma F_x = 0,\ N_1 \cos 10° - 0.3\, N_1 \sin 10° - N_2 = 0$

but $N_2 = 329.67$ N

$\therefore\ 0.985\, N1 - 0.052\, N_1 - 329.67 = 0$

$\therefore\ N_1 = 353.34$ N

$\Sigma F_y = 0,\ 0.3\, N_1 \cos 10° + N_1 \sin 10° + 0.3\, N_2 - P = 0$

substituting the values of N_1 and N_2

$$\boxed{P = 264.65\ \text{N}}$$

7. A *hawser* thrown from a ship to a pier is *wrapped two full turns around a capstan.* The tension in the hawser is 7.5 kN, by exerting a force of 150 N on its free end, a long shoreman can just keep the hawser from slipping. **(i)** Determine the coefficient between the hawser and the capstan. **(ii)** Determine the tension in the hawser that could be resisted by the 150 N force if the hawser were wrapped three full turns around the capstan.

(i) Coefficient of Friction :

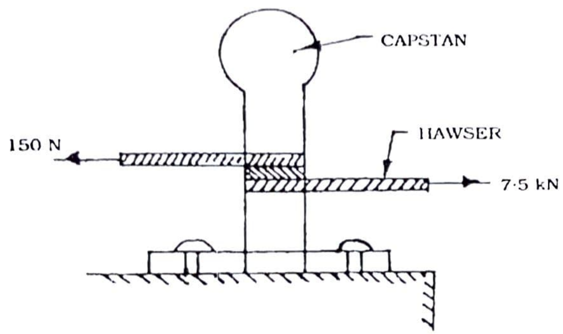


Fig 8·33

Since the slipping of the hawser is impending

$$\log_e \frac{T_2}{T_1} = \mu_s \beta$$

where β = angle at the centre of the capstan of two turns of hawser

$= 2 \times 2^1$ rad.

$= 12{\cdot}566$ rad

$T_1 = 150$ N

and, $T_2 = 7500$ N.

$$\mu_s = \log_e \frac{T_2}{T_1} / \beta$$

$$= \log_e \frac{7500}{150} / 12{\cdot}566$$

$$\boxed{\mu_s = 0{\cdot}311}$$

(ii) Hawser wrapped three turns around capstan :

Using $\mu_s = 0{\cdot}311$

$T_1 = 150$ N

$\beta = 3 \times 2^1 = 18{\cdot}85$ rad.

$$\frac{T_2}{T_1} = e^{\mu_s \beta}$$

$$T_2 = 150 \times e^{0{\cdot}31 \times 18{\cdot}85}$$

$$\boxed{T_2 = 51744{\cdot}7 \text{ N}}$$

8. *A band brake* is consisting of a leather belt on the cast iron drum 70 cm diameter, with a coefficient of friction of 0·3 between them. With a force of 100 N applied on the lever and with clockwise rotation of the drum, find the braking torque acting on the drum. $\beta = 270°$.

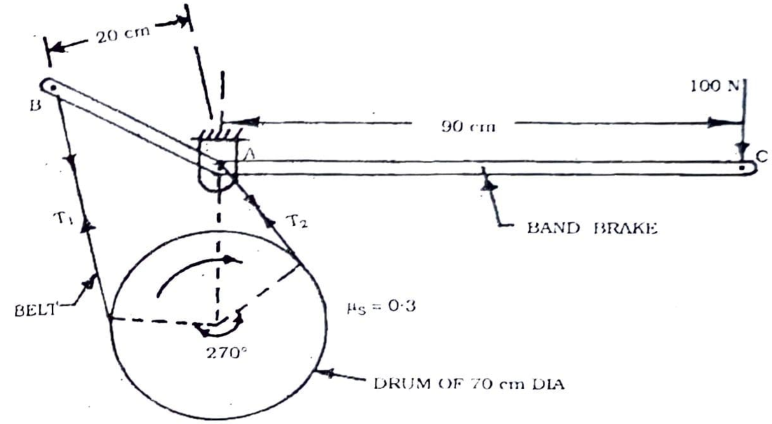


Fig. 8·34

Band brake acts as lever. By applying force at C, point B will be lifted up which will induce tension in belt and brake will be applied to drum. At hinge support A, about which points C and B rotate, moment is zero.

Taking moment about support A,

$\Sigma M_A = 0$, $T_1 \times 20 - 100 \times 90 = 0$

$\therefore \quad T_1 = 450$ N

Note that *here drum is rotating and belt (band) is fixed.*

Now,

$$\frac{T_2}{T_1} = e^{\mu_s \beta} \qquad \text{Where, } \beta = 270° = 4{\cdot}712 \text{ radians.}$$

$$\therefore \quad T_2 = T_1 \times e^{\mu_s \beta}$$

$$= 450 \times e^{0{\cdot}3 \times 4{\cdot}712}$$

$$= 1849{\cdot}8 \text{ N}$$

Therefore, the torque T on the drum

$$= (T_2 - T_1) \times \text{radius}$$

$$= (1849{\cdot}8 - 450) \times \frac{70}{2}$$

$$= 48993 \text{ N.cm}$$

$$\boxed{\text{Torque, T} = 489{\cdot}93 \text{ N.m}}$$

9. The maximum tension that can be developed in the belt is 700 N. If the *pulley* at A is *free to rotate* and the coefficient of static friction at the *fixed drums* B and C is $\mu_s = 0{\cdot}25$, determine the largest mass of the cylinder that can be lifted by the belt. Assume that the force $\bar{T}$ applied at the end of the belt is directed vertically downward as shown.

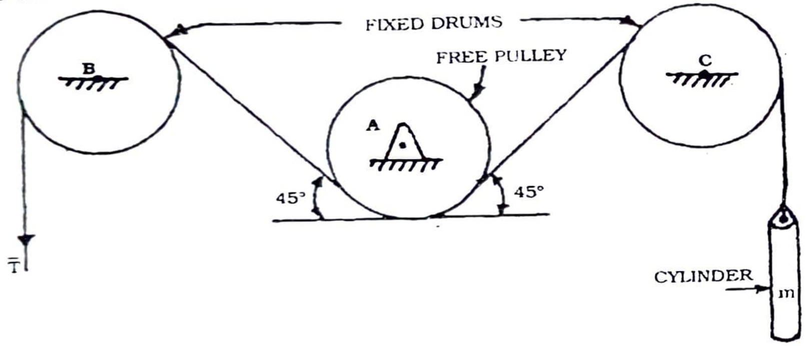


Fig 8·35

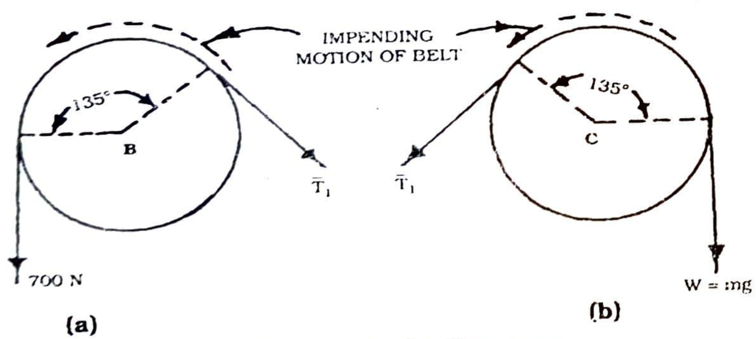


Fig 8·36 Free-body diagrams

From the free body-diagrams of drums B and C, T_1 and m can be found out.

Drum B : $T_2 = 700N$

$\mu_s = 0{\cdot}25$

$\beta = 135° = 2{\cdot}356$ radians

$T_1 = ?$

Hence, $T_1 = \dfrac{T_2}{e^{\mu_s\beta}} = \dfrac{700}{e^{0{\cdot}25 \times 2{\cdot}356}} = 388{\cdot}4$ N.

Since the pulley at A is free to rotate, equilibrium requires that the tension in the belt remains the same on both sides of the pulley.

Drum C : $T_2 = 388{\cdot}4$ N

$T_1 = W = ?$

$\beta = 2{\cdot}356$ radians

$\mu_s = 0{\cdot}25$

$\therefore \quad T_2 = T_1\, e^{\mu_s\beta}$

$\therefore \quad 388{\cdot}4 = We^{0{\cdot}25 \times 2{\cdot}356}$

$\therefore \quad W = 215{\cdot}5$ N

So, $m = \dfrac{W}{g} = \dfrac{215{\cdot}5}{9{\cdot}81}$

$\boxed{m = 21{\cdot}97 \text{ kg}}$

10. Determine force $\bar{P}$ required to move the block up the plane. The coefficient of static friction between block and inclined plane is 0·2 while between rope and fixed pulley is 0·1.

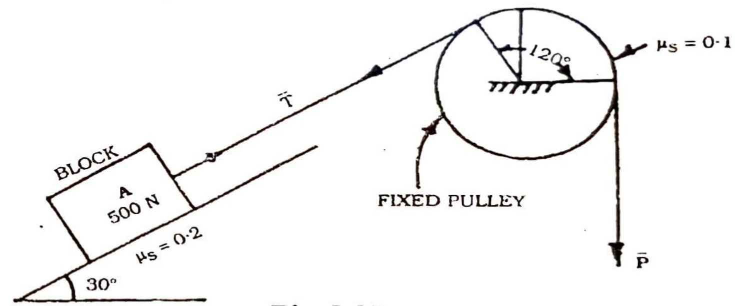


Fig 8·37

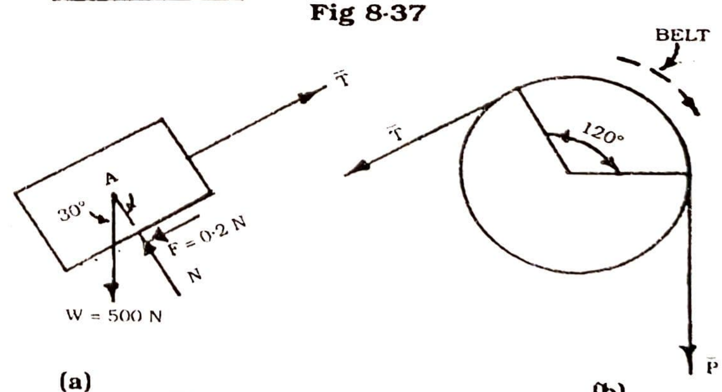


Fig 8·38 Free-body diagrams

Block A will move up hence frictional force will be acting down in addition to normal reaction on inclined surface.

Block A :

$$N - W\cos 30^\circ = 0$$
$$N - 500 \cos 30^\circ = 0$$
$$\therefore \quad N = 433{\cdot}01 \text{ N}$$
$$T - W \sin 30^\circ - 0{\cdot}2\, N = 0$$
$$T - 500 \sin 30^\circ - 0{\cdot}2\,(443{\cdot}01) = 0$$
$$T = 336{\cdot}6 \text{ N}$$

Now, $\dfrac{T_2}{T_1} = e^{\mu_s \beta}$

Where,
$$T_2 = P$$
$$T_1 = T = 336{\cdot}6 \text{ N}$$
$$\beta = 120^\circ = 2{\cdot}09 \text{ radians}$$
$$\mu_s = 0{\cdot}1$$
$$P = 336{\cdot}6\, e^{0\cdot 1 \times 2\cdot 09}$$

$$\boxed{P = 414{\cdot}84 \text{ N}}$$

11. A uniform rod AB hinged at A. Length and weight of rod are 1 m and 400 N respectively. End B rests on a rectangular block of 100 N weight. Calculate the minimum force P required to move the block to the right. Assume $\mu = 0{\cdot}3$ for all surfaces in contact.

(Pune University) (Marks : 8)

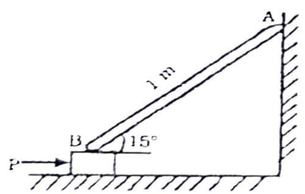
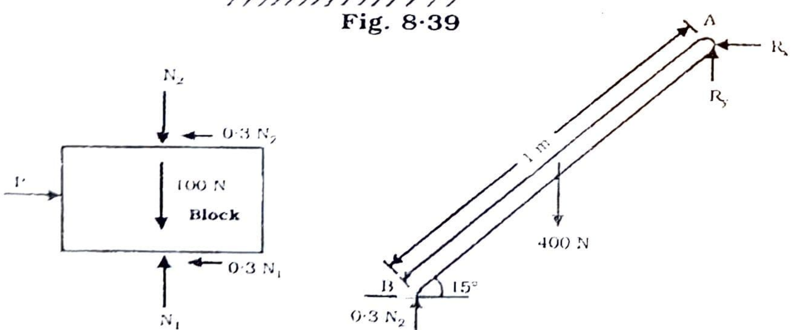


Fig. 8·39



(a) Block **(b) Rod**

Fig. 8·40

Rod AB is hinged at A, hence two reaction components R_x and R_y will be acting at A. It rests on block at B. The contact surfaces may be rough at B, hence N_2 and $0{\cdot}32\, N_2$ will be acting at B as shown in free - body diagrams.

Rod : Taking moment about A

$$N_2 \times 1 \cos 15^\circ - 400 \times 1 \cos 15^\circ/2 - 0{\cdot}3\, N_2 \times 1 \sin 15^\circ = 0.$$
$$\therefore N_2 = 217{\cdot}48 \text{ N}$$

Block : $\Sigma F_y = 0,$ $\quad -N_2 - 100 + N_1 = 0$

Putting the value of N_2

$$N_1 = 317{\cdot}48 \text{ N}$$

$\Sigma F_x = 0,$ $\quad P - 0{\cdot}3\, N_2 - 0.3\, N_1 = 0$

Putting the value of N_1 & N_2

$$\therefore P - 0{\cdot}3 \times 217{\cdot}48 - 0{\cdot}3 \times 317{\cdot}48 = 0$$

$$\boxed{P = 160{\cdot}5 \text{ N}}$$

12. A rope is wound around a pulley for one and quarter turns as shown. Determine the range of values of W if angle of friction is 18° between rope & pulley.

(Pune University) (Marks : 6)

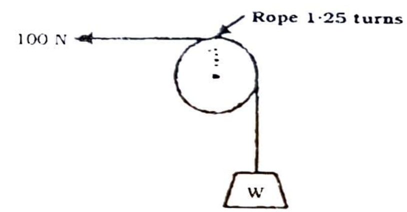


Fig. 8·41

Angle friction = 18° $\therefore \mu_s = \tan 18^\circ = 0{\cdot}325$

Angle of contact for one and quarter turns

$$= 360^\circ + 90^\circ = 2\pi + \frac{\pi}{2} = 2{\cdot}5\,\pi$$

If W is maximum : (Rope moves towards W)

$$\frac{W}{100} = e^{0\cdot 325\,(2\cdot 5\,\pi)}$$
$$\therefore W = 100 \times e^{2\cdot 553}$$
$$\therefore \boxed{W = 1284{\cdot}56 \text{ N}}$$

If W in mimmum : (Rope moves towards 100 N)

$$\frac{100}{W} = e^{0\cdot 325\,(2\cdot 5\,\pi)}$$
$$\therefore \boxed{W = 7{\cdot}79 \text{ N}}$$

13. Two blocks, weighing W_1 and W_2 are connected by a string passing over a small smooth pulley as shown in figure. If co-efficient of friction is 0·3 for both the planes, find the minimum ratio W_1/W_2 required to maintain equilibrium. **(Pune University) (Marks : 8)**

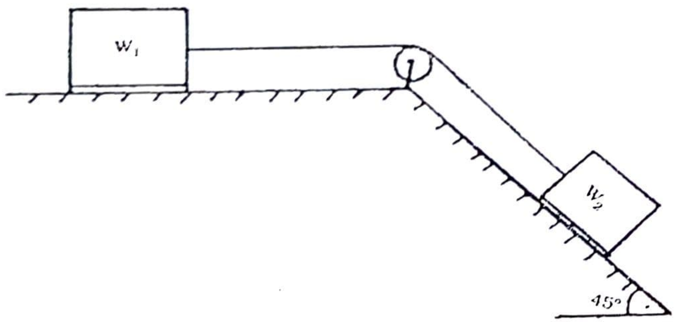
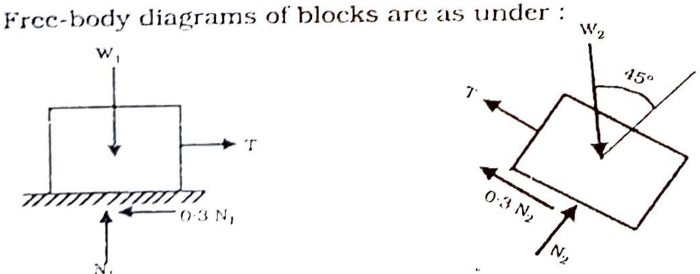


Fig. 8·42

Free-body diagrams of blocks are as under :



(a) Block W_1 **(b) Block W_2**

Fig. 8·43

Let Block W_1 moves towards ***right.***

Resolving forces acting on W_1 Block :

y axis : $W_1 = N_1$ ... (i)

x axis : $T = 0{\cdot}3\ N_1$ ... (ii)

$\therefore T = 0{\cdot}3\ W_1$ ... (A)

Resolving forces acting on W_2 Block :

Perpendicular to plane : $N_2 = W_2 \cos 45°$ ... (iii)

Along the plane : $T + 0{\cdot}3\ N_2 = W_2 \sin 45°$ ... (iv)

$\therefore\ T + 0{\cdot}3\ (W_2 \cos 45°) = W_2 \sin 45°$

$\therefore\ T = W_2 \sin 45° - 0{\cdot}3\ W_2 \cos 45°$

$\therefore\ T = 0{\cdot}5\ W_2$ ... (B)

From equations **(A)** and **(B)**

$0{\cdot}3\ W_1 = 0{\cdot}5\ W_2$

$\therefore\ W_1/W_2 = 0{\cdot}5/0{\cdot}3$ $\therefore$ $\boxed{W_1/W_2 = 1{\cdot}67}$

14. Blocks A and B are placed on each other, as well as on an inclined plane as shown. Weight of A is 800 N and angle of friction is 12° for all surfaces in contact including the fixed drum at O. Determine weight of block B. **(Pune University) (Marks : 8)**

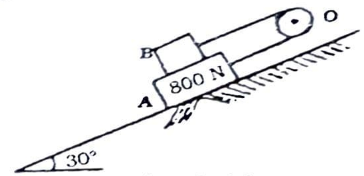
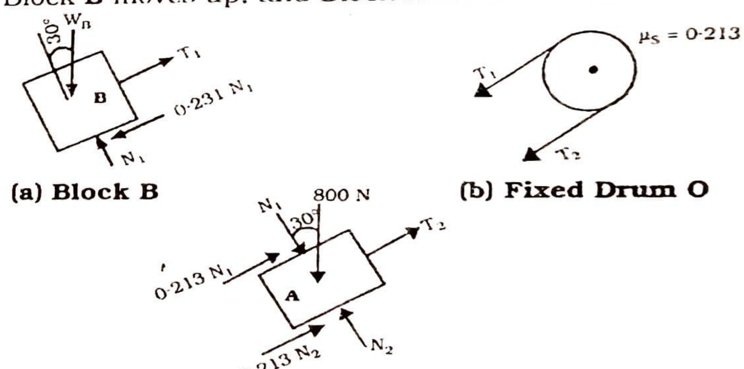


Fig. 8·44

$\mu_s = \tan \phi = \tan 12° = 0{\cdot}213$

Let Block **B** moves up, and Block **A** moves down.



(a) Block B **(b) Fixed Drum O**

(c) Block A

Fig. 8·45

Free-body diagrams are shown here.

Block A :

⊥er to plane : $N_1 + 800 \cos 30° = N_2$ ... (i)

Along plane : $T_2 + 0{\cdot}213\ N_1 + 0{\cdot}213\ N_2 = 800 \sin 30°$ ... (ii)

Putting the value of N_2 from (i) into (ii)

$T_2 + 0{\cdot}213\ N_1 + 0{\cdot}213\ (N_1 + 800 \cos 30°) = 800 \sin 30°$

$T_2 + 0{\cdot}213\ N_1 + 0{\cdot}213\ N_1 + 147{\cdot}57 = 400$

$T_2 + 0{\cdot}426\ N_1 = 252{\cdot}43$

$T_2 = 252{\cdot}43 - 0{\cdot}426\ N_1$ ... (A)

Fixed Drum - O : If Block B moves up and A moves down then T_2 will be higher than T_1

$$T_2/T_1 = e^{0.213\beta}, \quad \text{where } \beta = \pi$$

$$\therefore T_2/T_1 = 1.953$$

$$\therefore T_1 = 0.512\, T_2$$

Now putting the value of T_2 from eq. (A)

$$T_1 = 0.512\,(252.43 - 0.426\, N_1)$$

$$\therefore T_1 = 129.24 - 0.218\, N_1 \qquad \ldots \textbf{(B)}$$

Block B :

Along the plane : $T_1 - 0.213\, N_1 - W_B \sin 30° = 0$

Putting the value of T_1 from eq. (B)

$$129.24 - 0.218\, N_1 - 0.213\, N_1 - W_B \sin 30° = 0$$

$$\therefore 129.24 - 0.431\, N_1 - 0.5\, W_B = 0$$

$$\therefore N_1 = 299.86 - 1.16\, W_B \qquad \ldots \textbf{(C)}$$

$\perp^{er}$ to the plane : $N_1 = W_B \cos 30°$

$$N_1 = 0.866\, W_B \qquad \ldots \textbf{(D)}$$

From eq. **(C)** and **(D)**

$$299.86 - 1.16\, W_B = 0.866\, W_B$$

$$\therefore \boxed{W_B = 148\text{ N}}$$

15. Two blocks are connected by a horizontal link AB and rest on two planes as shown. What is the smallest weight W_A of the block A for which the equilibrium can exist ?

(Pune University, April 98) (Marks : 8)

Take : μ at A = 0.4

μ at B = 0.36

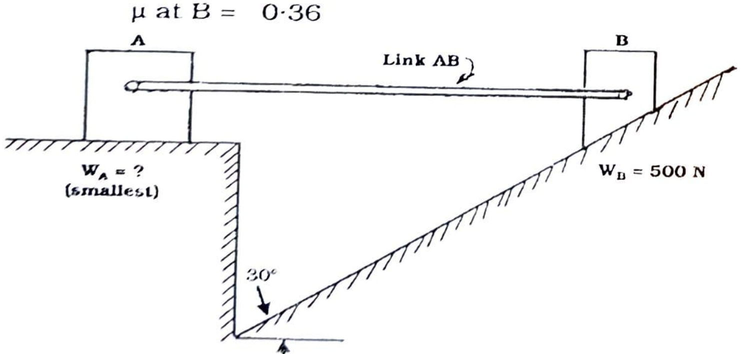


Fig. 8.46

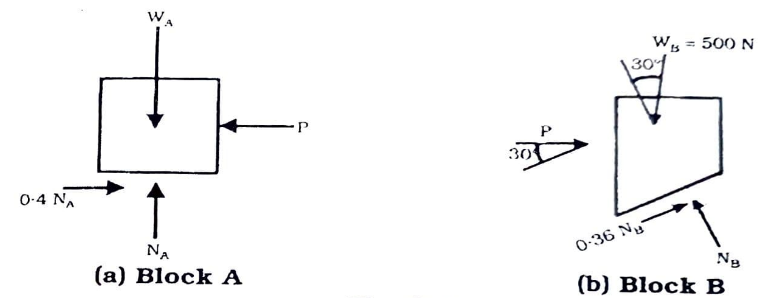


Fig. 8.47

Free-body diagrams of Block A and B are shown here. We have to determine smallest force W_A, hence start the calculation from block B. Let *Block B moves* ***down***.

Block B :

$\perp^{er}$ to the plane : $W_B \cos 30° - N_B + P \sin 30° = 0$

$$433 - N_B + 0.5\, P = 0$$

$$\therefore N_B = 433 + 0.5\, P \qquad \ldots \textbf{(i)}$$

Along the plane : $0.36\, N_B + P \cos 30° - W_B \sin 30° = 0$

$$0.36\, N_B + 0.866\, P - 250 = 0$$

$$\therefore N_B = 694.44 - 2.41\, P \qquad \ldots \textbf{(ii)}$$

From eq. (i) and eq. (ii)

$$433 + 0.5\, P = 694.44 - 2.41\, P$$

$$\therefore 2.91\, P = 261.44$$

$$\therefore P = 89.84\text{ N} \qquad \ldots \textbf{(A)}$$

Block A :

x axis : $P = 0.4\, N_A$

putting the value of P from eq. **(A)**

$$N_A = 224.6\text{ N} \qquad \ldots \textbf{(B)}$$

y axis : $W_A = N_A$

$$\therefore \boxed{W_A = 224.6\text{ N}}$$

16. A weight of 10 kN is to be lifted up as shown by a wedge of weight 1 kN. Coefficient of static frictions have been shown in the figure for all suraces in contact. Calculate the magnitude of force P to be applied hoizontally on the wedge.

(Pune University) (Marks : 6)

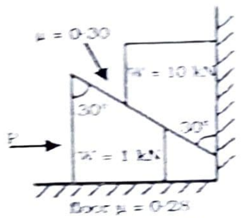
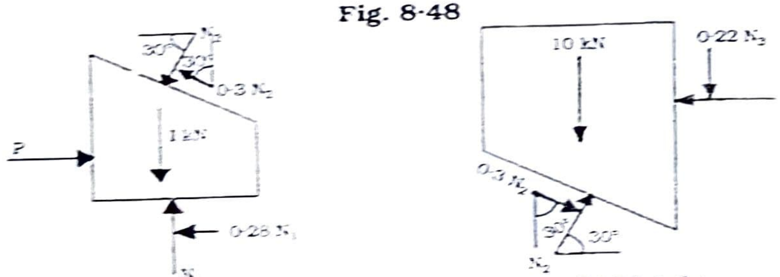


Fig. 8·48



(a) Wedge (b) Weight

Fig. 8·49

Free - body diagrams are shown above.

Weight :

$\Sigma F_x = 0$, $-N_3 + 0{\cdot}3\ N_2 \sin 30° + N_2 \cos 30° = 0$

$N_3 = 1{\cdot}02\ N_2$ ... (i)

$\Sigma F_y = 0$, $-10 - 0{\cdot}22\ N_3 + N_2 \sin 30° - 0{\cdot}3\ N_2 \cos 30° = 0$

Putting the value of N_3 from eq (i)

$N_2 = 633{\cdot}22$ kN

Wedge :

$\Sigma F_y = 0$, $-1 + N_1 - N_2 \sin 30° + 0{\cdot}3\ N_2 \cos 30° = 0$

Putting the value of N_2

$N_1 = 153{\cdot}09$ kN

$\Sigma F_x = 0$, $P - 0{\cdot}28\ N_1 - 0{\cdot}3\ N_2 \sin 30° - N_2 \cos 30° = 0$

Putting the value of N_1 & N_2

$\boxed{P = 686{\cdot}23 \text{ kN}}$

Here, it should be noted that for lifting 10 kN weight, we have to apply 686·23 kN horizontal force on wedge. ***This is very high as the angle of wedge is more i.e. 30° and wedge is also having weight.***

17. Referring to the figure, calculate the force P required to just raise the block 'B' of weight 1000 N. The wedge may be assumed to be of negligible weight. **(Pune university, Oct. 96) (Marks : 6)**

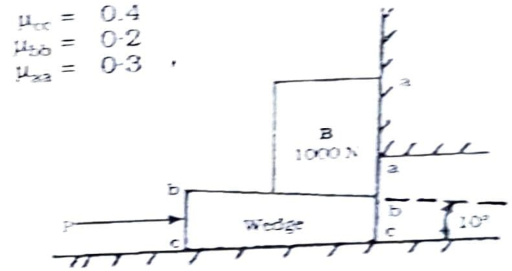
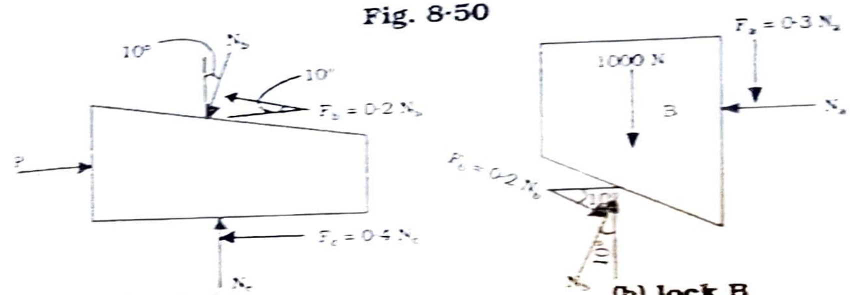


Fig. 8·50



(a) Wedge (b) lock B

Fig. 8·51

The free-body diagrams of wedge and block - B are shown above.

Block B :

$\Sigma F_x = 0$, $-N_a + N_b \sin 10° + 0{\cdot}2\ N_b \cos 10° = 0$

$\therefore N_a = 0{\cdot}37\ N_b$ ... (i)

$\Sigma F_y = 0$. $-1000 - 0{\cdot}3\ N_a + N_b \cos 10° - 0{\cdot}2\ N_b \sin 10° = 0.$

Putting the value of N_a from eq. (i)

$-1000 - 0{\cdot}3\ (0{\cdot}37\ N_b) + N_b \cos 10° - 0{\cdot}2\ N_b \sin 10° = 0.$

$N_b = 1191{\cdot}8$ N

& $N_a = 441$ N

Wedge :

$\Sigma F_y = 0$, $-N_b \cos 10° + 0{\cdot}2\ N_b \sin 10° + N_c = 0$

Putting the value of $N_b = 1191{\cdot}8$ N

$N_c = 1132{\cdot}3$ N

$\Sigma F_x = 0$. $P - 0{\cdot}4\ N_c - 0{\cdot}2\ N_b \cos 10° - N_b \sin 10° = 0$

Putting the value of N_b and N_c

$\boxed{P = 894{\cdot}6 \text{ N}}$

18. A uniform ladder of length 15 m rests against a vertical wall making an angle 60° with horizontal. Coefficient of friction between wall and ladder is 0·3 and between ground and ladder is 0·25. A man weighing 500 N ascends the ladder. How long will he be able to go before the Ladder slips ? Find the weight that is necessary to put at the bottom of the ladder so as to be just sufficient to permit the man to go to the top. Assume weight of the ladder 850 N.

(Pune University, Oct. 96) (Marks : 9)

Case I :

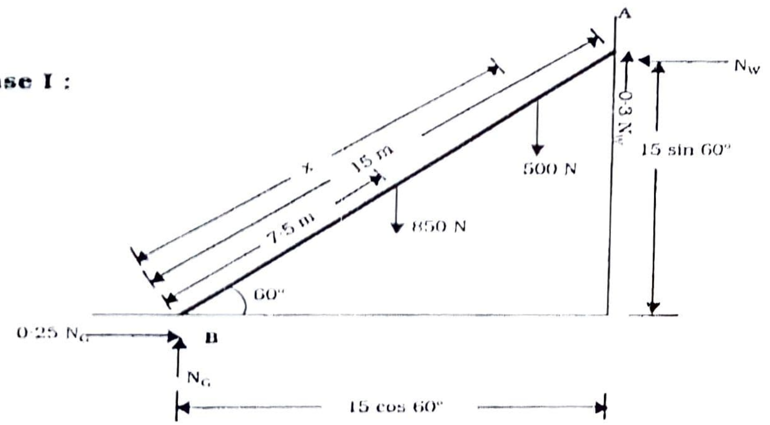


Fig. 8·52

Case II :

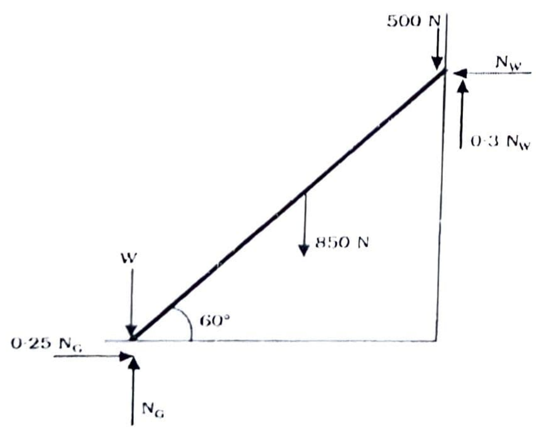


Fig. 8·53

Case I :

Let man can go upto a point x m from bottom. Taking moment about point B

$\Sigma M_B = 0,\quad + \, 850 \times 7{\cdot}5 \cos 60^\circ + 500 \times x \cos 60^\circ$

$- 0{\cdot}3\ N_w \times 15 \cos 60^\circ - N_w \times 15 \sin 60^\circ = 0$

$\therefore\ 3000 + 250\ x - 15{\cdot}24\ N_w = 0$ ... (i)

$\Sigma F_x = 0,\quad 0{\cdot}25\ N_G = N_w \quad \therefore\ N_G = 4\ N_w$ ... (ii)

$\Sigma F_y = 0,\quad N_G - 850 - 500 + 0{\cdot}3\ N_w = 0$

Putting the value of N_G from eq. (ii)

$\therefore\ 4\ N_w + 0{\cdot}3\ N_w = 1350 \quad \therefore\ N_w = 313{\cdot}95\ N$ ... (iii)

Putting the value of N_w in eq. (i)

$3000 + 250\ x - 15{\cdot}24 \times 313{\cdot}95 = 0$

$\boxed{x = 7{\cdot}14\ m}$

Case II :

$\Sigma F_x = 0,\quad 0{\cdot}25\ N_G = N_w$ **OR** $N_G = 4\ N_w$ ... (iv)

$\Sigma F_y = 0,\quad N_G - W - 850 - 500 + 0{\cdot}3\ N_w = 0$

$\therefore\ 4{\cdot}3\ N_w - W = 1350$ ... (v)

Taking moment @ point B

$\Sigma M_B = 0,\quad 850 \times 7{\cdot}5 \cos 60^\circ + 500 \times 15 \cos 60^\circ$

$- 0{\cdot}3\ N_w \times 15 \cos 60^\circ - N_w \times 15 \sin 60^\circ = 0$

$\therefore\ 6937{\cdot}5 = 15{\cdot}24\ N_w$

$\therefore\ N_w = 455{\cdot}22\ N$ ... (vi)

Putting the value of N_w in eq. (v)

$4{\cdot}3 \times 455.22 - W = 1350$

$\therefore\ \boxed{W = 607{\cdot}45\ N}$

19. A rigid U-pin bracket ABC is hinged at B. Its end C rests on a 15° wedge as shown in figure. If the vertical load acting on the bracket at A is 500 N downwards, determine the minimum horizontal force P required to push the wedge to the left. Neglect the weight of the wedge and the bracket. Assume coefficient of friction at all surfaces of contact as 0·2. **(Pune University, Oct./Nov. 94)**

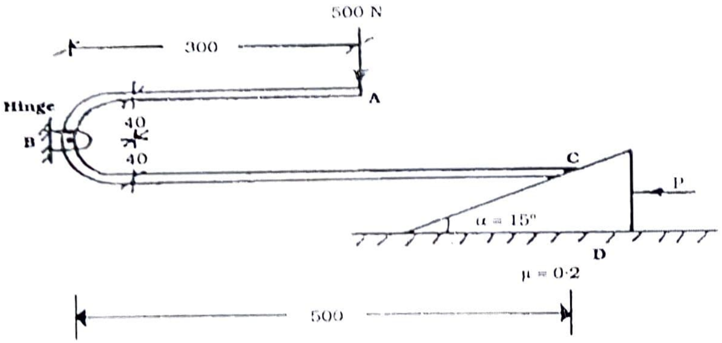
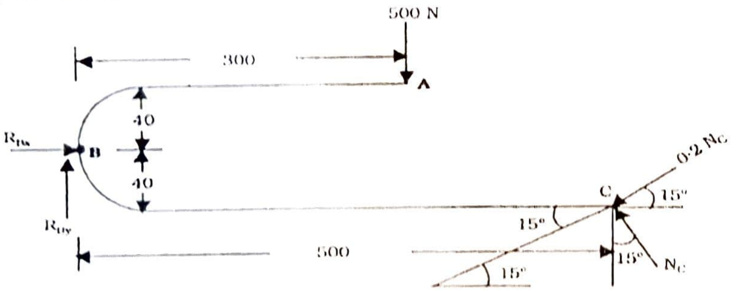
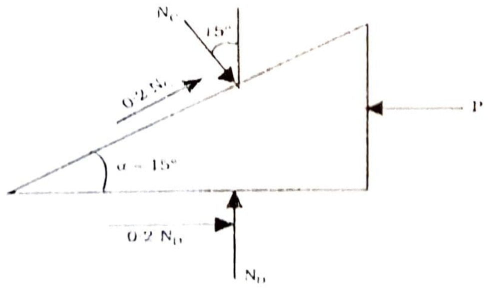


Fig. 8·54

The bracket is hinged at B. Hence net moment about this hinge support is zero.



(a) Bracket



(b) Wedge

Fig. 8·55

The free - body diagrams of bracket and wedge are shown here. **First of all, we should draw free - body diagram of wedge then of the bracket.**

BRACKET : Taking moment about hinge support B,

$$\Sigma M_B = 0, \quad +\circlearrowright,\ 500 \times 300 - N_C \cos 15° \times 500 + N_C \sin 15° \times 40$$
$$+ 0{\cdot}2\ N_C \cos 15° \times 40 + 0{\cdot}2\ N_C \sin 15° \times 500 = 0$$
$$N_C = 341{\cdot}68 \text{ N}$$

WEDGE :

$$\Sigma F_y = 0, \quad -N_C \cos 15° + 0{\cdot}2\ N_C \sin 15° + N_D = 0$$
$$N_D = 0{\cdot}91\ N_C \qquad \therefore\ N_D = 310{\cdot}93 \text{ N}$$

$$\Sigma F_x = 0, \quad 0{\cdot}2\ N_C \cos 15° + N_C \sin 15° - P + 0{\cdot}2\ N_D = 0$$

$$\boxed{P = 216{\cdot}63 \text{ N}}$$

THEORY RELATED QUESTIONS

1. **Explain** with diagrams :
   - **(i)** No friction condition.
   - **(ii)** Equilibrium condition.
   - **(iii)** Impending motion condition.
   - **(iv)** Motion condition.
2. **Explain** the relation between P_x and F with *graph*.
3. **State** the *laws of dry friction*.
4. **Distinguish** clearly the *angle of repose* and *angle of friction*.
5. **Draw** *free body diagrams* of the problems of
   - **(i)** Ladder
   - **(ii)** Wedge
   - **(iii)** Belt friction
   - **(iv)** Band brake
   - **(v)** Ropes wrapped around a capstan.

EXERCISES

8·1 The coefficients of friction between the block and the rail are $\mu_s = 0{\cdot}30$ and $\mu_k = 0{\cdot}25$. Find the magnitude and direction of the smallest force $\bar{P}$ required **(a)** to start the block up the rail, **(b)** to keep it from moving down.

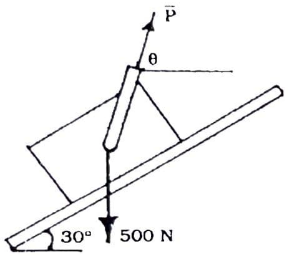


Fig 8·56

8·2 A uniform ladder 3m long weighs 200N. It is placed against a wall making an angle of 60° with the floor as shown. The ladder has also to support a man of 700 N at its top at B. Determine

(i) the horizontal force $\bar{P}$ to be applied to ladder at the floor level to prevent slipping.

(ii) If the force $\bar{P}$ is not applied, what should be the minimum inclination of the ladder with horizontal so that there is *no slipping of it, with a man at its top.*

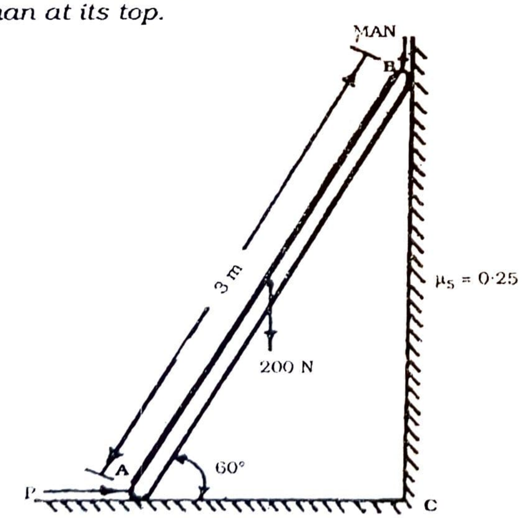


Fig 8·57

8·3 A 15° wedge A has to be driven for tightening a body B having weight 1500N as shown. If the angle of friction for all surfaces is 14°, find the force require to be applied to the wedge.

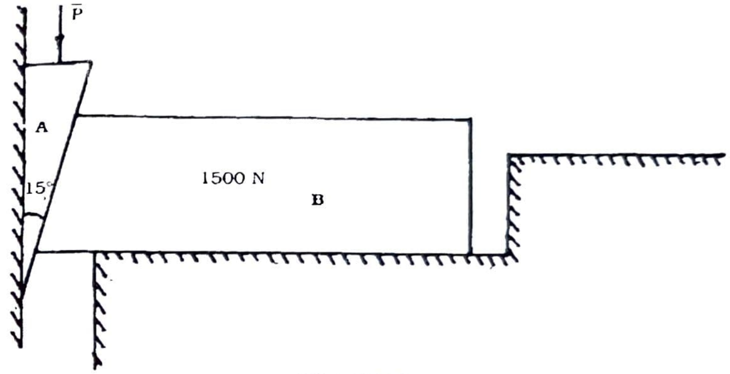


Fig 8·58

8·4 A block A weighing 10kN is to be raised agaist a surface, which is inclined at 60° with the horizontal by means of a 15° wedge B as shown. Find the smallest force $\bar{P}$ required to raise the block A. $\mu_s = 0{\cdot}2$ at all contact surfaces.

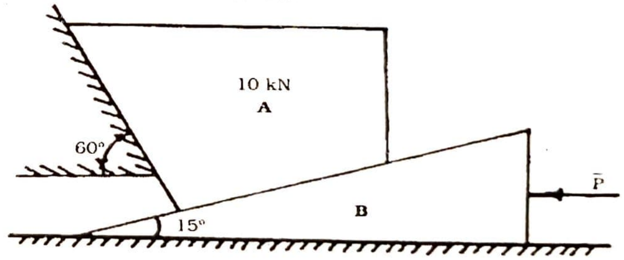


Fig 8·59

8·5 Determine minimum and maximum tension in the rope at points A and B that is necessary to maintain equilibrium. Take $\mu_s = 0{\cdot}3$ between the rope and the fixed post D.

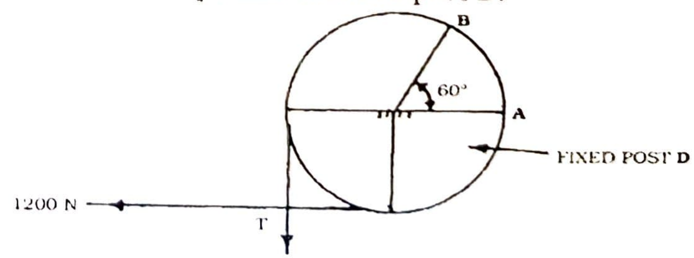


Fig 8·60

8·6 A *band brake* is used to control the speed of a flywheel as shown. The coefficients of friction are $\mu_s = 0{\cdot}30$ and $\mu_k = 0{\cdot}25$. What couple should be applied to the flywheel to keep it rotating counter clockwise at a constant speed when P = 50 N ?

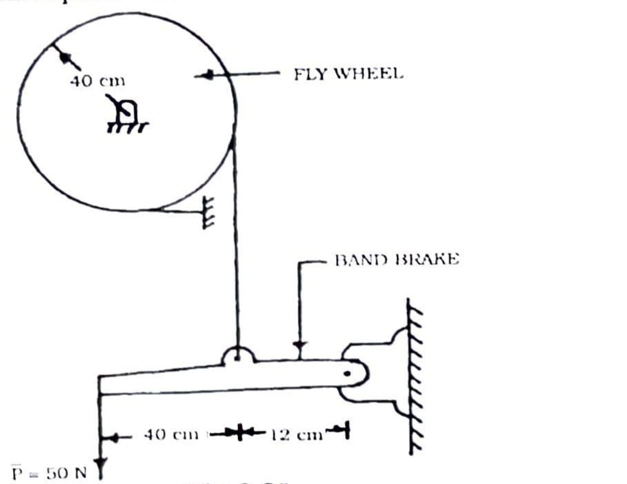


Fig 8·61

8·7 A brake drum of radius r = 150 mm is rotating counterclockwise when a force $\bar{P}$ of magnitude 60N is applied at A. Knowing that the coefficient of kinetic friction is 0·40, determine the moment about O of the friction forces applied to the drum when a = 250 mm and b = 300 mm.

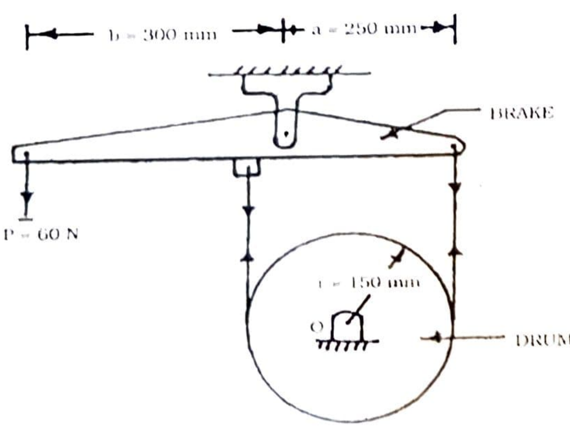


Fig 8·62

8·8 Two 8° wedges of negligible weight are used to move and position the 8 kN block. Knowing that the coefficient of static friction is 0·30 at all contact surfaces, determine the smallest force $\bar{P}$ which should be applied as shown to one of the wedges.

(i)

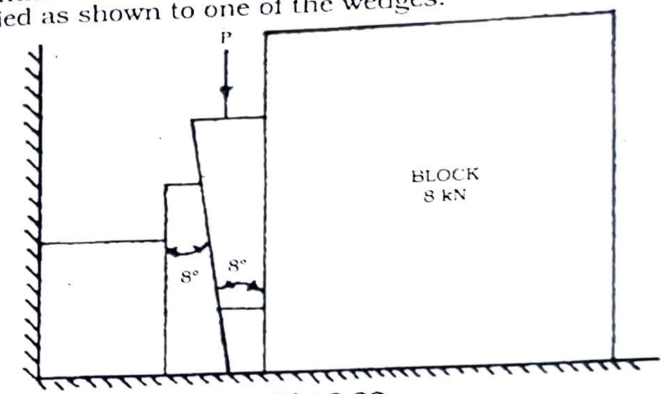


Fig 8·63

(ii)

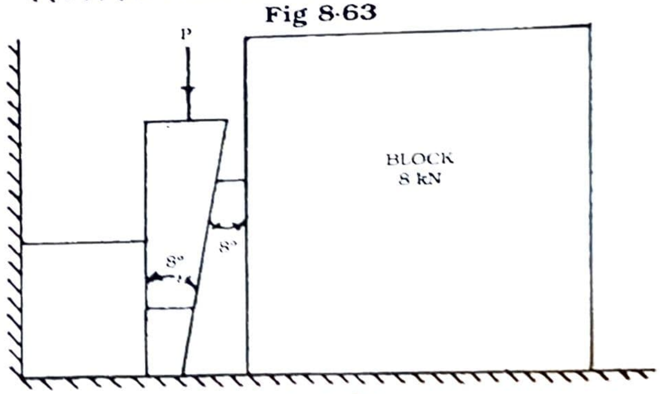


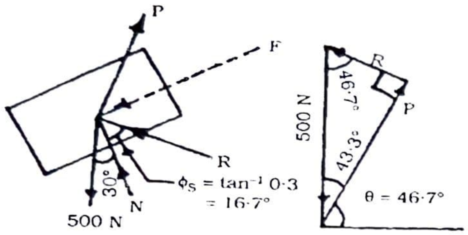
Fig 8·64

SOLUTIONS OF EXERCISES

8·1 (a) To Start the block up the rail :

Frictional force will be acting towards down as the block moves up. Here, four forces are acting: P, F, N and 500 N. Forces F and N can be replaced by resultant R. Force triangle is also shown here. *At the time of impending motion, the angle between normal reaction and the resultant will be the angle of repose* ϕ_s ($\phi_s = \tan^{-1} \mu_s$).

Here, angle between R and P will be 90°.

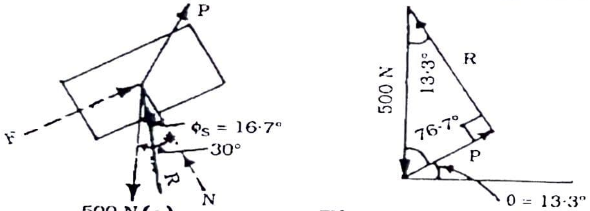


$P = 500 \sin 46.7°$
$P = 363.89$ N
$\theta = 46.7°$

(a) **Fig. 8·65** (b)

(b) To keep the block from moving down :

Frictional force will be acting upward as the block moves down. The free-body diagram and force triangle are shown here.



$P = 500 \sin 13.3°$
$P = 115$ N
$\theta = 13.3°$

500 N (a) **Fig. 8·66** (b)

8·2 When ladder slides down, the top end will move down and the bottom will move towards left. The frictional forces and normal reactions are shown accordingly in free - body diagram. Taking moment about *top end*,

$\Sigma M = 0$, $P (3 \sin 60°) + 0.25\ N_2 (3 \sin 60°) + 200 (1.5 \cos 60°) - N_2 (3 \cos 60°) = 0$ **(i)**

$\Sigma F_x = 0$, $P + 0.25\ N_2 - N_1 = 0$ **(ii)**

$\Sigma F_y = 0$, $N_2 + 0.25\ N_1 - 200 - 700 = 0$ **(iii)**

From which N_1, N_2 and P can be determined.

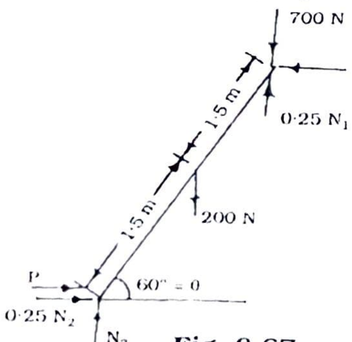


Fig. 8·67

(i) P = 203·86 N

(ii) If P is absent, $\theta = 74.17°$

8·3 The forces acting on wedge and block are shown in free - body diagrams.

Block :

$\Sigma F_x = 0$, $N_2 \cos 15° - 0.25\ N_2 \sin 15° - 0.25\ N_3 = 0$. **(i)**

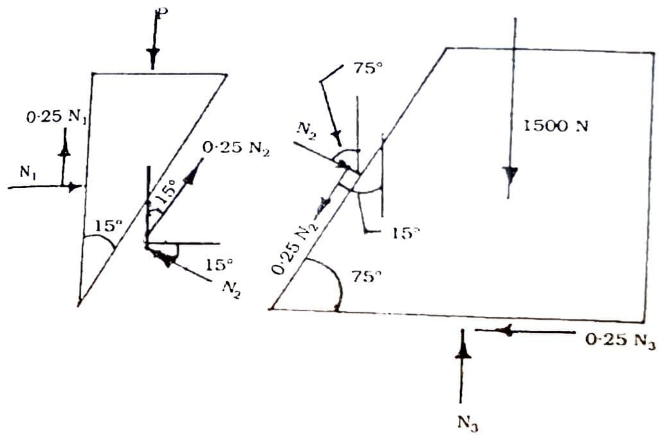
$\Sigma F_y = 0$, $N_2 \sin 15° + 0.25\ N_2 \cos 15° + 1500 - N_3 = 0$ **(ii)**

Wedge :

$\Sigma F_x = 0$, $N_1 + 0.25\ N_2 \sin 15° - N_2 \cos 15° = 0$ **(iii)**

$\Sigma F_y = 0$, $P - 0.25\ N_1 - 0.25\ N_2 \cos 15° - N_2 \sin 15° = 0$ **(iv)**

From above four equations, four unknowns can be determined.



P = 349·98 N

(a) **Fig. 8·68** (b)

8·4 Free - body diagrams are drawn separately for wedge and block.

Block :

$\Sigma F_x = 0$, $0.2\ N_3 \cos 60° + N_3 \sin 60° - 0.2\ N_2 \cos 15° - N_2 \sin 15° = 0$ **(i)**

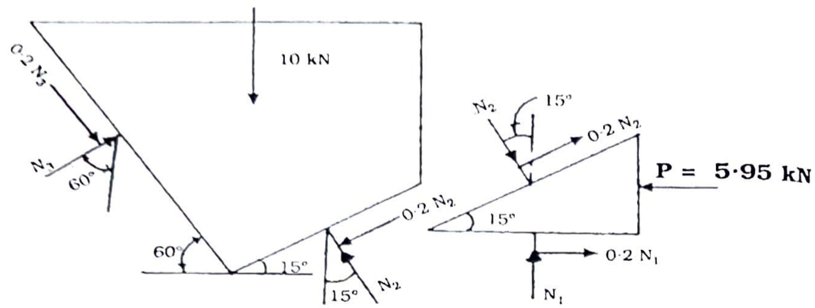
$\Sigma F_y = 0$, $0.2\ N_3 \sin 60° + 10 - N_3 \cos 60° + 0.2\ N_2 \sin 15° - N_2 \cos 15° = 0$ **(ii)**

Wedge :

$\Sigma F_x = 0$, $0.2\ N_2 \cos 15° + N_2 \sin 15° + 0.2\ N_1 - P = 0$ **(iii)**

$\Sigma F_y = 0$, $0.2\ N_2 \sin 15° - N_2 \cos 15° + N_1 = 0$ **(iv)**

From above four equations, four unknowns can be determined.



(a) **Fig. 8·69** (b)

8·5 (i) *When* $\bar{T}$ *is minimum, rope will move towards 1200 N force Here.* T < 1200 N and $\beta = 3\pi/2$

$$\frac{T_2}{T_1} = e^{\mu_s \beta}$$

$$\therefore \quad \frac{1200}{T_{min}} = e^{0.3(3\pi/2)} \quad \therefore T_{min} = 291.9 \text{ N.}$$

$$\text{Similarly,} \quad \frac{1200}{T_{A(min)}} = e^{0.3(\pi/2)} \quad \therefore T_{A(min)} = 749.06 \text{ N}$$

$$\text{and} \quad \frac{T_{A(min)}}{T_{B(min)}} = e^{0.3\,(60° \times \pi/180)}$$

$$\therefore \quad \frac{749.06}{T_{B(min)}} = e^{0.3(1.047)} \quad \therefore T_{B(min)} = 547.16 \text{ N}$$

When $\bar{T}$ is maximum, rope will move to wards $\bar{T}$.

Here T_{max} > 1200 N.

$$\frac{T_{max}}{1200} = e^{0.3(3\pi/2)} \quad \therefore \quad T_{max} = 4933.2 \text{ N}$$

$$\text{Similarly,} \quad \frac{4933.2}{T_{B(max)}} = e^{0.3\,(120° \times \pi/180)} \quad \therefore T_{B(max)} = 2631.74 \text{ N}$$

$$\text{and} \quad \frac{2631.74}{T_{A(max)}} = e^{0.3(60° \times \pi/180)} \quad \therefore T_{A(max)} = 1922.4 \text{ N}$$

$T_{A\,(min)}$ = 749·06 N, $T_{B(min)}$ = 547·16 N, $T_{(min)}$ = 291·9 N

$T_{A\,(max)}$ = 1922·4 N, $T_{B\,(max)}$ = 2631·74 N, $T_{(max)}$ = 4933·2 N

8·6 On band brake two forces are acting; P and T_1.

This T_1 should be more than T_2 so to stop the flywheel. which is rotating counterclockwise. After finding T_1 & T_2 a couple M should be determined so that the net moment will be zero.

Taking moment about hinge support,

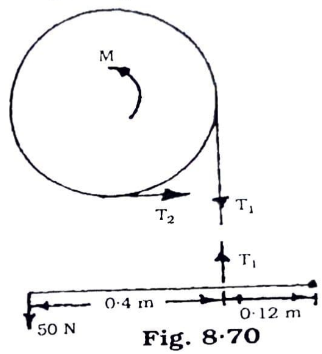


Fig. 8·70

$$50 \times 0.52 = T_1 \times 0.12$$

$$T_1 = 216.67 \text{ N}$$

$$\text{Now,} \quad \frac{216.67}{T_2} = e^{0.25 \times 1.5 \times \pi}$$

$$T_2 = 66.71 \text{ N}$$

Net moment should be zero.

$$M - 216.67 \times 0.4 + 66.71 \times 0.4 = 0$$

$$M = 59.98 \text{ N·m}$$

8·7 Taking moment about hinge support C,

$$60 \times 0.3 + T_2 \times 0.05 = T_1 \times 0.25$$

and

$$\frac{T_2}{T_1} = e^{0.4 \times \pi}$$

$$\therefore \quad T_1 = 242.26 \text{ N}$$

$$T_2 = 851.20 \text{ N}$$

M_o due to Friction Force

$$= (T_2 - T_1) \times 0.15$$

$$M_o = 91.34 \text{ N·m}$$

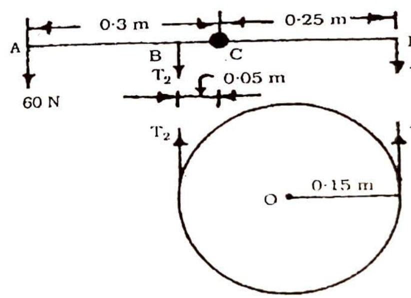


Fig. 8·71

8·8 Free - body diagrams are shown separately.

(i) Block :

$\Sigma F_x = 0$, $N_2 - 0.3\,N_3 = 0$ (1)

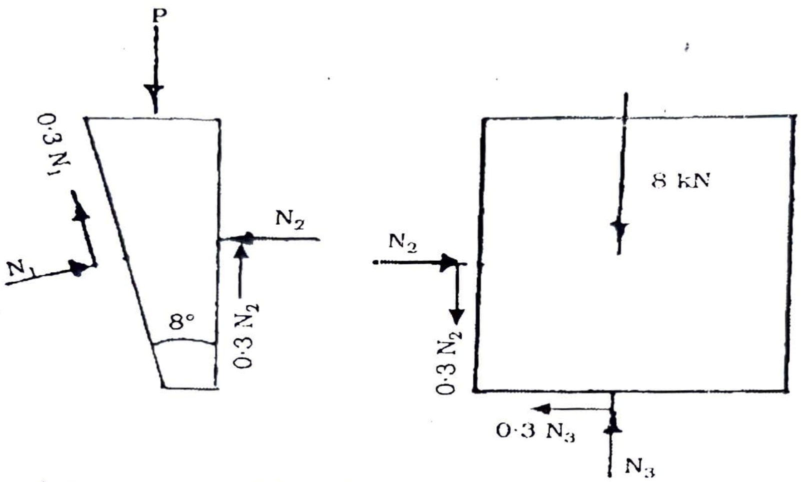
$\Sigma F_y = 0$, $N_3 - 0.3\,N_2 - 8 = 0$ (2)

Wedge :

$\Sigma F_x = 0$, $N_1 \cos 8° - 0.3\,N_1 \sin 8° - N_2 = 0$ (3)

$\Sigma F_y = 0$, $P - 0.3\,N_2 - N_1 \sin 8° - 0.3\,N_1 \cos 8°$ (4)

From above four equations, four unknowns are to be determined.



P = 2·005 kN

(a) **Fig. 8·72** (b)

(ii) Block :

$$\Sigma F_x = 0, \quad N_2 \cos 8° - 0{\cdot}3\, N_2 \sin 8° - 0{\cdot}3\, N_3 = 0 \qquad (1)$$

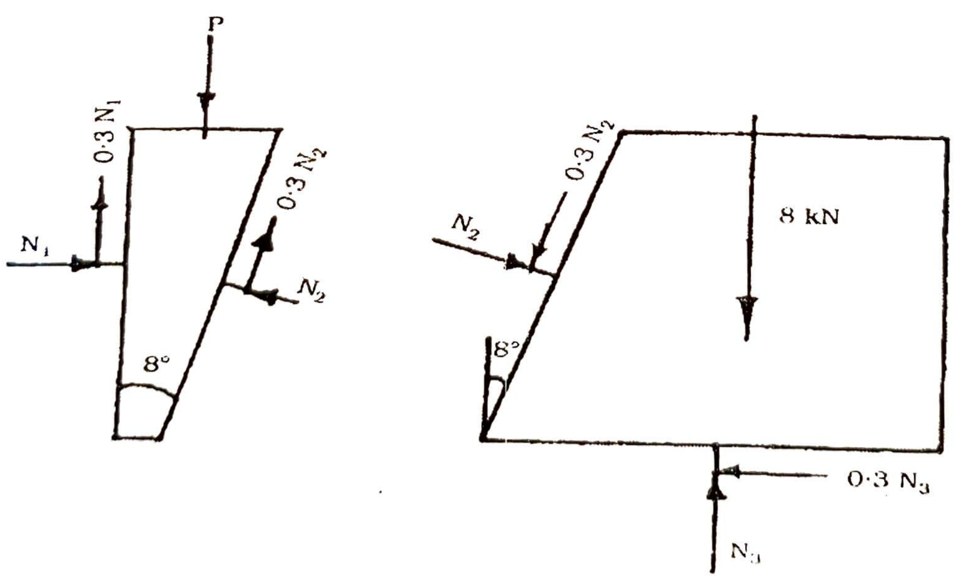
$$\Sigma F_y = 0, \quad N_2 \sin 8° + 8 + 0{\cdot}3\, N_2 \cos 8° - N_3 = 0 \qquad (2)$$

Wedge :

$$\Sigma F_x = 0, \quad N_1 - N_2 \cos 8° + 0{\cdot}3\, N_2 \sin 8° = 0 \qquad (3)$$

$$\Sigma F_y = 0, \quad P - 0{\cdot}3\, N_1 - 0{\cdot}3\, N_2 \cos 8° - N_2 \sin 8° = 0 \qquad (4)$$

From above four equations, four unknowns are to be determined.



P = 2·1166 kN

(a) **Fig. 8·73** (b)

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