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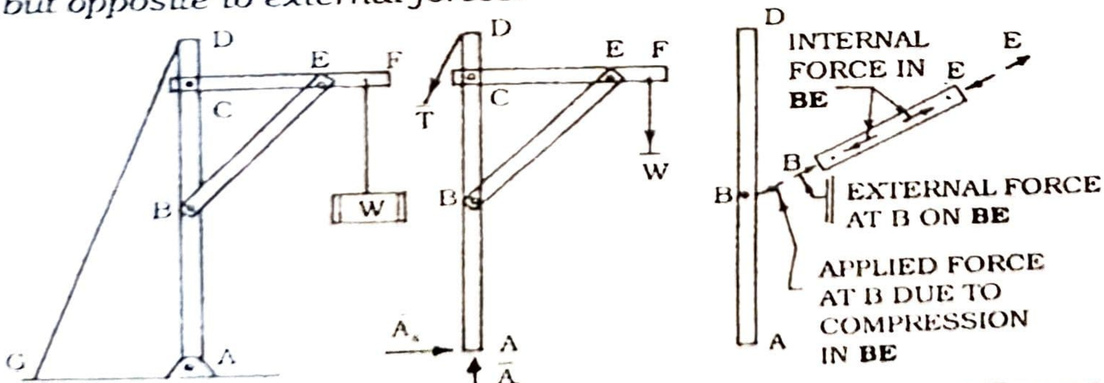
3 EQUIVALENT SYSTEMS OF FORCES OF RIGID BODIES

3·1 External and Internal Forces : Forces acting on rigid bodies may be classified as

(1) external forces (2) internal forces.

(1) External forces : *They represent the action of other bodies on the body under consideration.* They will be responsible for the *movement* or *rest* condition of the body. In Fig 3·1 (b), T, W, A_x & A_y are external forces, *as these forces are acting externally on this structure.*

(2) Internal Forces : *They are the forces which hold to gather the particles forming the rigid body. Due to action of external forces, the forces developed inside the member of a structure are called internal forces.* In Fig. 3.1(c), the member BE of the structure is shown with its *external* and *internal* forces. *Internal forces are equal to but opposite to external forces.*



(a) Structure **(b) External Forces** **(c) Internal Forces**

Fig 3·1

In Fig. 3.2, *compression* and *tension members* are indicated. The *sign conventions* for compression and tension are denoted by showing *internal forces*.

Arrows toward ends of a member indicate compression, while arrows away from the ends (toward centre) indicate tension.

(a) Compression member **(b) Tension member.**

Fig 3·2

3·2 Equivalent Forces :

The two forces $\overline{F}$ and $\overline{F'}$ are said to be equivalent if they are having the same magnitude, same direction, same line of action but acting at different points and thus having the same effect on the rigid body if acting independently (Fig 3·3). Thus the principle of transmissibility is used here.

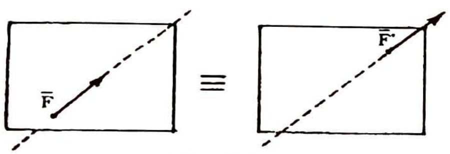


Fig 3·3

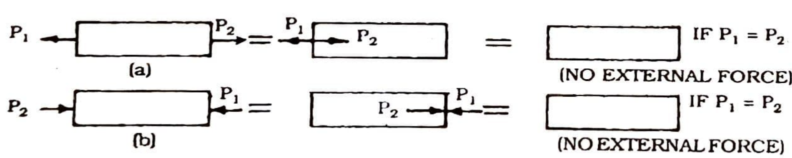


Fig 3·4

From the point of view of the mechanics of rigid bodies, the systems shown in Fig 3·4 (a) and (b) are **equivalent but the internal *forces and deformations produced by the two systems are clearly different.*** *The* **rod (a)** *is in* **tension** *whereas* **rod (b)** *is in* **compression.** Thus principle of transmissibility may be used freely to determine the *conditions of motion* or *equilibrium* of rigid bodies and to compute external fores but it should be used ***with care in determining internal forces and deformations.***

3·3 Vector Product of Two Vectors (Cross Product) :

Let, $\overline{V}$ is the *cross product* of two vectors $\overline{P}$ and $\overline{Q}$.

Mathematically,

$$\overline{V} = \overline{P} \times \overline{Q}$$

and magnitude of $\overline{V}$, $V = PQ \sin \theta$.

The line of action of $\overline{V}$ is perpendicular to the plane containing $\overline{P}$ and $\overline{Q}$.

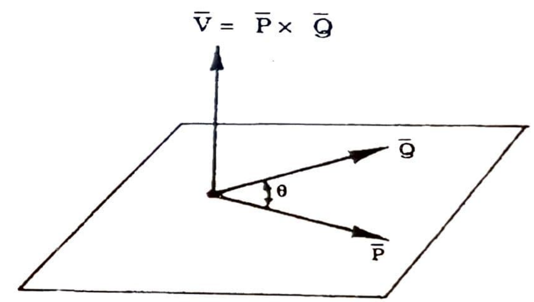


Fig 3·5

In Fig. 3.5, the cross product of vectors $\overline{P}$ and $\overline{Q}$ is a vector $\overline{V}$. *The direction of $\overline{V}$ is* ***perpendicular*** *to the plane containing $\overline{P}$ and $\overline{Q}$.*

(1) Laws of Operation for Cross Product:

(i) *The commutative law is not valid.*

i.e. $\overline{P} \times \overline{Q} - \overline{Q} \times \overline{P}$

Rather, $\overline{P} \times \overline{Q} = -(\overline{Q} \times \overline{P})$

(ii) *The distributive Law :*

$$\overline{P} \times (\overline{Q} + \overline{S}) = (\overline{P} \times \overline{Q}) + (\overline{P} \times \overline{S})$$

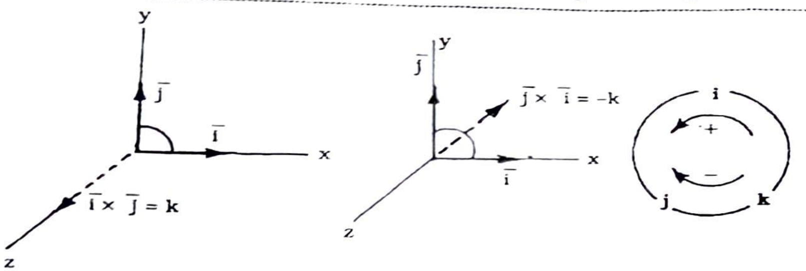
(iii) *The associative law is not valid.*

i.e. $(\overline{P} \times \overline{Q}) \times \overline{S} \neq \overline{P} \times (\overline{Q} \times \overline{S})$

(2) In terms of Rectangular Components :

The cross product depends upon the **sine** of the angle between the vectors. As sin 0°=0 and sin 90°=1,

$\overline{i} \times \overline{i} = 0$	$\overline{j} \times \overline{i} = -\overline{k}$	$\overline{k} \times \overline{i} = \overline{j}$
$\overline{i} \times \overline{j} = \overline{k}$	$\overline{j} \times \overline{j} = 0$	$\overline{k} \times \overline{j} = -\overline{i}$
$\overline{i} \times \overline{k} = -\overline{j}$	$\overline{j} \times \overline{k} = \overline{i}$	$\overline{k} \times \overline{k} = 0$



(a) (b) (c)

Fig 3.6

Using rectangular components,

$$\bar{V} = \bar{P} \times \bar{Q} = (P_x\, \bar{i} + P_y\, \bar{j} + P_z\, \bar{k}) \times (Q_x\, \bar{i} + Q_y\, \bar{j} + Q_z\, \bar{k})$$

$$\bar{V} = (P_y\, Q_z - P_z\, Q_y)\, \bar{i} + (P_z\, Q_x - P_x\, Q_z)\, \bar{j} + (P_x\, Q_y - P_y Q_x)\, \bar{k}$$

$$V_x = P_y\, Q_z - P_z Q_y$$

$$V_y = P_z Q_x - P_x Q_z$$

$$V_z = P_x Q_y - P_y Q_x$$

The Vector product $\bar{V}$ may be expressed in determinant form

$$\bar{V} = \begin{vmatrix} i & j & k \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$$

3·4 Moment of a Force about a Point :

If we *pull* or *push* a body, say cupboard, it will be shifted in the line of action of the force, provided the gravitational force (centroidal axis) and line of applied force coincide *(i.e. forces acting on it are concurrent)*. But if we apply the force at the corner, the body will be rotated. *Thus* ***concurrent forces do not cause rotation***, **but nonconcurrent forces rotate the body** *(Refer Fig. 3·8). The* ***tendency of a force to rotate the body is called moment.***

The moment of a force about a point or axis provides a measure of tendency of the force to cause a body to rotate about the point or axis.

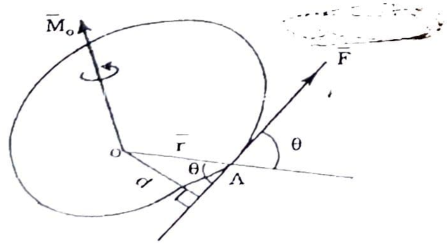
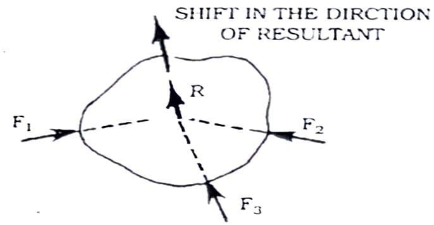
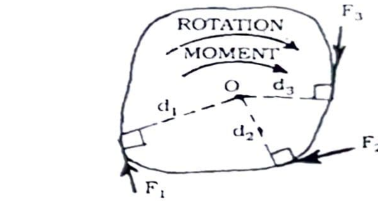


Fig 3·7



Concurrent Forces
No moment
No rotation of body
Fig 3·8 (a)



Nonconcurrent Forces
Moment = $F_1d_1 + F_2d_2 + F_3d_3$
Clockwise rotation
Fig 3·8 (b)

In fig. 3·7, Force $\bar{F}$ is acting at point A of the rigid body. The vector $\bar{r}$ is the position vector of point A from point O where the moment is to be determined. Angle θ is the angle between $\bar{F}$ and $\bar{r}$. Distance d is the perpendicular distance (shorfest distance) between line of action of $\bar{F}$ and point O.

The moment of $\bar{F}$ about O is a ***cross product*** *of* $\bar{r}$ & $\bar{F}$.

$\boxed{\bar{M}_o = \bar{r} \times \bar{F}}$ .... **Vector Formulation.**

Where $\bar{r}$ = **position vector** which joins the fixed reference point O with A lying on the line of action of $\bar{F}$.

$\bar{F}$ = **Force** applied at A which rotates the body.

The **magnitude** of M_o is

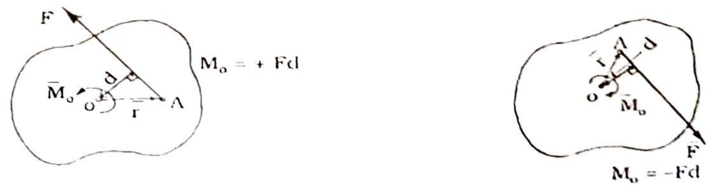
$$M_o = r\, F \sin\theta = F\,(r \sin\theta) = Fd$$

$\boxed{M_o = Fd}$ .... **Scalar Formulation.**

Where d = **perpendicular distance** of line of action of $\bar{F}$ from O.

The direction and sense of $\bar{M}_o$ are determined by the right - hand rule as it applies to the cross product. *Curl the right hand fingers from* $\bar{r}$ *toward* $\bar{F}$, " $\bar{r}$ *cross* $\bar{F}$," *the thumb is directed upward or perpendicular to the plane of* $\bar{r}$ *and* $\bar{F}$ *as* $\bar{M}_o$.

Anticlockwise moment **Clockwise moment**
Anticlockwise rotation **Clockwise rotation**



(a) **Fig 3·9** (b)

Now, if the force is in space (Fig. 3·10), the position vector.

$$\bar{r} = x\ \bar{i} + y\ \bar{j} + z\ \bar{k} \quad \text{where x, y, z are coordinates of A}$$

$$\bar{F} = F_x\ \bar{i} + F_y\ \bar{j} + F_z\ \bar{k}$$

Moment about point O, $\bar{M}_o = \bar{r} \times \bar{F}$

$$\bar{M}_o = (x\ \bar{i} + y\ \bar{j} + z\ \bar{k}) \times (F_x\ \bar{i} + F_y\ \bar{j} + F_z\ \bar{k})$$

$$\bar{M}_o = \bar{r} \times \bar{F} = M_x\ \bar{i} + M_y\ \bar{j} + M_z\ \bar{k}$$

$$M_x = y\ F_z - z\ F_y$$

$$M_y = z\ F_x - x\ F_z$$

$$M_z = x\ F_y - y\ F_x$$

We may write in **determinant form,**

$$\bar{M}_o = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix} \quad \ldots \left\{ \begin{array}{l} \textbf{Equation} \\ \textbf{for moment} \\ \textbf{about origin O} \end{array} \right.$$

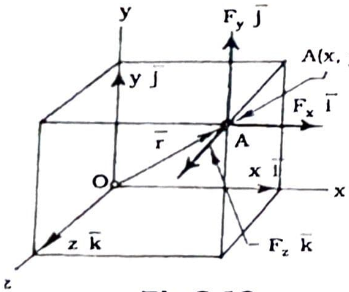


Fig 3·10

Where x, y, z are the coordinates of a point where force is applied.

Now moment about any point B (arbitrary point) of a force applied at A (Fig. 3·11) can be computed by

$$\bar{M}_B = \bar{r}_{A/B} \times \bar{F} = (\bar{r}_A - \bar{r}_B) \times \bar{F}$$

where position vector $\bar{r}_{A/B}$ = position vector of A relative to B

Let, Coordinates of A = x_A, y_A, z_A

Coordinates of B = x_B, y_B, z_B

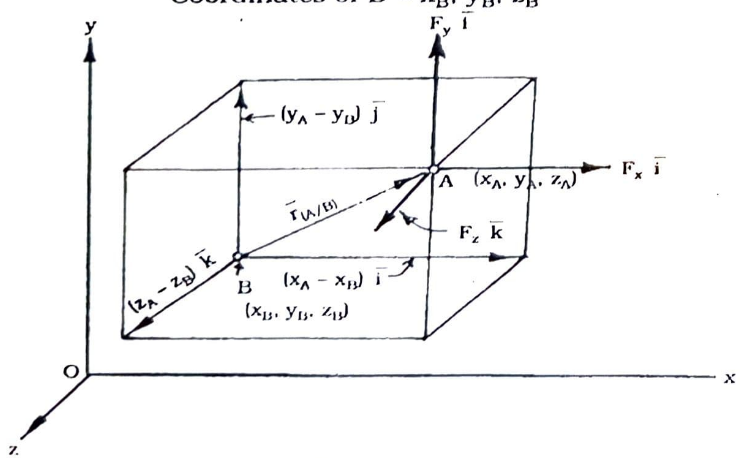


Fig 3·11

Moment about any point B

$$= \bar{M}_B = \bar{r}_{A/B} \times \bar{F}$$

$$= (\bar{r}_A - \bar{r}_B) \times \bar{F}$$

Using **determinant form,**

$$\bar{M}_B = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ x_{A/B} & y_{A/B} & z_{A/B} \\ F_x & F_y & F_z \end{vmatrix} \quad \ldots\ldots \left\{ \begin{array}{l} \textbf{Equation} \\ \textbf{for Moment} \\ \textbf{about any point B} \end{array} \right.$$

Where, $x_{A/B} = x_A - x_B$, $y_{A/B} = y_A - y_B$, $z_{A/B} = z_A - z_B$ — Coordinates of force point **minus** coordinates of moment point

3·5 Varignon's Theorem : *The moment about a given point O of the resultant of several concurrent forces is equal to the sum of the moments of the various forces about the same point O.*

$$\bar{M}_o = \bar{r} \times \bar{R} = \bar{r} \times (\bar{F}_1 + \bar{F}_2 + \bar{F}_3 + \ldots.)$$

$$= \bar{r} \times \bar{F}_1 + \bar{r} \times \bar{F}_2 + \bar{r} \times \bar{F}_3 + \ldots.$$

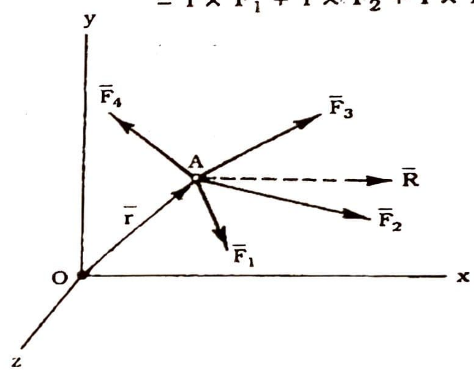


Fig 3·12

3·6 Scalar Product of Two Vectors (Dot product) :

Mathematically, " $\bar{P}$ dot $\bar{Q}$" is equal to

$$\bar{P}.\ \bar{Q} = PQ \cos\theta$$

The above expression is not a vector but is *a* **scalar.**

(1) Laws of Operation :

***(i)** Commutative law :*

$$\bar{P}.\ \bar{Q} = \bar{Q}.\ \bar{P}$$

***(ii)** Distributive law :*

$$\bar{P}.\ (\bar{Q} + \bar{S}) = (\bar{P}.\ \bar{Q}) + (\bar{P}.\ \bar{S})$$

(2) In terms of Rectangular Components :

*The dot product depends upon the **cosine of** angle between two vectors.*

As $\cos 0° = 1$, $\cos 90° = 0$,

$$\bar{i}\cdot\bar{i} = 1 \quad \bar{j}\cdot\bar{j} = 1 \quad \bar{k}\cdot\bar{k} = 1$$
$$\bar{i}\cdot\bar{j} = 0 \quad \bar{j}\cdot\bar{k} = 0 \quad \bar{k}\cdot\bar{i} = 0$$
$$\bar{i}\cdot\bar{k} = 0 \quad \bar{j}\cdot\bar{i} = 0 \quad \bar{k}\cdot\bar{j} = 0$$

$$\bar{P}\cdot\bar{Q} = (P_x\bar{i} + P_y\bar{j} + P_z\bar{k})\cdot(Q_x\bar{i} + Q_y\bar{j} + Q_z\bar{k})$$

$$\boxed{\therefore \bar{P}\cdot\bar{Q} = P_x Q_x + P_y Q_y + P_z Q_z}$$

and $\bar{P}\cdot\bar{P} = P_x^2 + P_y^2 + P_z^2 = P^2$

(3) Applications of dot products in mechanics :

(i) For determining angle formed by two given vectors :

Using the dot product we can determine the angle between two vectors if required.

Two given vectors are say :

$$\bar{P} = P_x\bar{i} + P_y\bar{j} + P_z\bar{k}$$
$$\bar{Q} = Q_x\bar{i} + Q_y\bar{j} + Q_z\bar{k}$$

and $\bar{P}\cdot\bar{Q} = PQ\cos\theta = P_x Q_x + P_y Q_y + P_z Q_z$

Solving for $\cos\theta$, we have

$$\theta = \cos^{-1}\left(\frac{\bar{P}\cdot\bar{Q}}{PQ}\right), \quad \boxed{\cos\theta = \frac{P_x Q_x + P_y Q_y + P_z Q_z}{PQ}}$$

When $\bar{P}.\bar{Q} = 0$, $\theta = \cos^{-1} 0 = 90°$

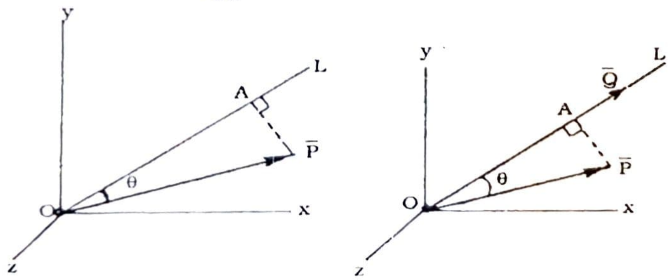
so, $\bar{P}$ is perpendicular to $\bar{Q}$

(ii) For finding projection of a vector on a given axis :

Dot product can also be used for finding the projection of a vector on a given axis.

The projection of P on the axis OL is defined as the scalar

$$P_{OL} = P\cos\theta$$



(a) **Fig 3·13** (b)

Projection P_{OL} = Length of OA

Now, $\bar{Q}$ vector is along OL

So, $\bar{P}.\bar{Q} = PQ\cos\theta = P_{OL}\,Q$

From which,

$$P_{OL} = \frac{\bar{P}.\bar{Q}}{Q} = \frac{P_x Q_x + P_y Q_y + P_z Q_z}{Q}$$

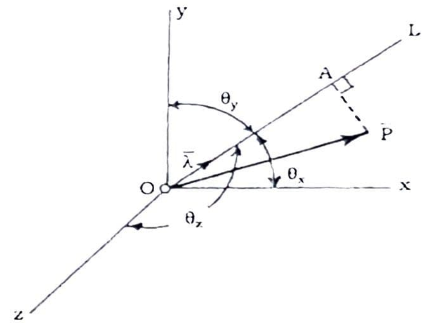


Fig 3·14

If Unit vector $\bar{\lambda}$ is acting along OL.

then $\boxed{P_{OL} = \bar{P}\,.\,\bar{\lambda}}$

$$\bar{P} = P_x\,\bar{i} + P_y\,\bar{j} + P_z\,\bar{k}$$
$$\bar{\lambda} = \cos\theta_x\,\bar{i} + \cos\theta_y\,\bar{j} + \cos\theta_z\,\bar{k}$$
$$\therefore \boxed{P_{OL} = P_x\cos\theta_x + P_y\cos\theta_y + P_z\cos\theta_z}$$

where, θ_x, θ_y and θ_z are the angles of OL with x, y and z axes respectively.

3·7 Mixed Triple Product of Three Vectors :

The mixed triple product of three vectors can be defined as scalar expression. Mathematically,

"S dot P cross Q", equal to, $S.(\bar{P}\times\bar{Q}) = \begin{vmatrix} S_x & S_y & S_z \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$

3·8 Moment of a Force about a Given Axis :

Previously, we have seen *moment about origin* ($\bar{M}_o$) and also *about any arbitrary point B* ($\bar{M}_B$) *which are* **cross products** *of position vector and force vector. Thus moment about point is a vector.*

Now let us take an example of a door. If we shut a door, the *door will be rotated about axis* (vertical hinges) due to application of force. *The tendency of the force which causes rotation about axis is called moment about axis.*

The moment M_{OL} of the force $\bar{F}$ about axis OL provides a measure of the tendency of the force $\bar{F}$ to rotate the rigid body about the fixed axis OL (Fig. 3·15).

The moment M_{OL} of force $\bar{F}$ about the axis OL is the projection OC of the moment $\bar{M}_o$ on the axis OL (Fig. 3.15).

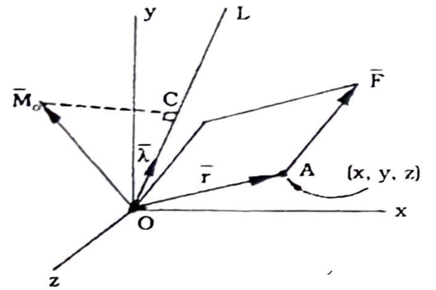


Fig. 3·15

$\bar{\lambda}$ = unit vector along OL.

$$\boxed{M_{OL} = \bar{\lambda} \cdot \bar{M}_o = \bar{\lambda} \cdot (\bar{r} \times \bar{F})}$$

where, M_{OL} is a moment of force $\bar{F}$ about axis OL and is *scalar.*

M_{OL} is equal to the mixed triple product, " **$\bar{\lambda}$ dot $\bar{r}$ cross $\bar{F}$.**"

But $\bar{\lambda} = \lambda_x \bar{i} + \lambda_y \bar{j} + \lambda_z \bar{k}$

$\bar{r} = x\,\bar{i} + y\,\bar{j} + z\,\bar{k}$

and $\bar{F} = F_x \bar{i} + F_y \bar{j} + F_z \bar{k}$.

M_{OL} in the **determinant form,**

$$M_{OL} = \begin{vmatrix} \lambda_x & \lambda_y & \lambda_z \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix} \quad \ldots\ldots\ldots \left\{ \begin{array}{l} \textbf{Equation for} \\ \textbf{Moment about} \\ \textbf{axis passing} \\ \textbf{through origin.} \end{array} \right.$$

Where : $\lambda_x, \lambda_y, \lambda_z$ = direction cosines of axis OL

x, y, z = coordinates of point of application of $\bar{F}$

F_x, F_y, F_z = components of force $\bar{F}$

Now, ***moment** of a force* $\bar{F}$ *applied at A* ***about an axis which** does **not pass through origin*** is obtained by choosing a point B on the axis and determining the projection on the axis BL of the moment $\bar{M}_B$ of $\bar{F}$ about B.

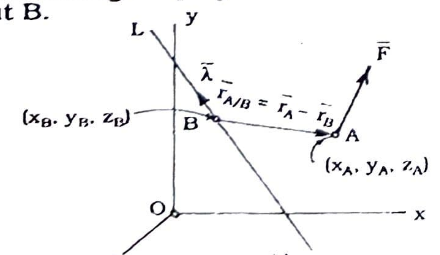


Fig 3·16

We may write

$$M_{BL} = \bar{\lambda} \cdot \bar{M}_B = \bar{\lambda} \cdot (\bar{r}_{A/B} \times \bar{F})$$

where, $\bar{r}_{A/B} = \bar{r}_A - \bar{r}_B$ = position vector drawn from B to A.

M_{BL} can be expressed in **determinant form** as

$$M_{BL} = \begin{vmatrix} \lambda_x & \lambda_y & \lambda_z \\ x_{A/B} & y_{A/B} & z_{A/B} \\ F_x & F_y & F_z \end{vmatrix} \quad \ldots\ldots \left\{ \begin{array}{l} \textbf{Equation for Moment} \\ \textbf{about axis not passing} \\ \textbf{through origin} \end{array} \right.$$

Where $\lambda_x, \lambda_y, \lambda_z$ = direction cosines of BL.

$x_{A/B} = x_A - x_B$

$y_{A/B} = y_A - y_B$ $\quad F_x, F_y, F_z$ = components of force $\bar{F}$.

$z_{A/B} = z_A - z_B$

Once the magnitude of $\bar{M}_{OL}$ *or* $\bar{M}_{BL}$ *(i.e.* M_{OL} *or* M_{BL}*) is determined, we can express it in Cartesian vector form since its direction is defined by* $\bar{\lambda}$ *(axis).*

We have $\bar{M}_{OL} = M_{OL}\ \bar{\lambda}_{OL}$

similarly, $\bar{M}_{BL} = M_{BL}\ \bar{\lambda}_{BL}$

3·9 Moment of a couple :

(1) Couple : *It is defined as two parallel forces having the same magnitude, opposite direction and are separated by a perpendicular distance d.* The forces $\bar{F}$ and $-\bar{F}$ are two paralled forces but having opposite directions and *hence* ***they will rotate the body.***

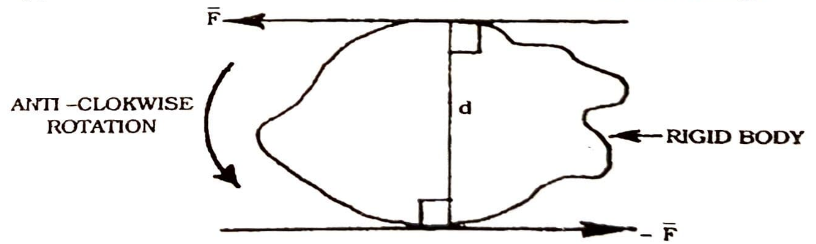


Fig 3·17

The effect of a couple is to produce a **rotation** *or tendency of rotation in a specified direction,* ***though the resultant force is zero.***

(2) Couple – Moment : *It is the moment produced by a couple which is equivalent to the sum of the moments of both couple forces, computed about any arbitrary point O in space.*

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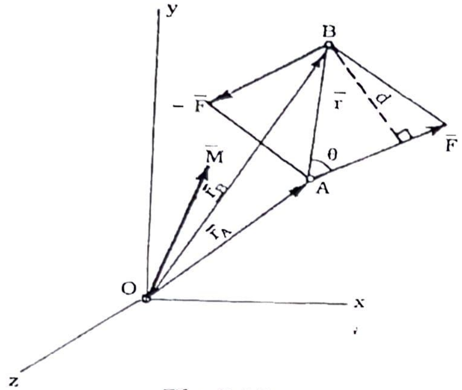


Fig 3·18

From above figure,

$$M = \bar{r}_A \times \bar{F} + \bar{r}_B \times (-\bar{F})$$
$$= (\bar{r}_A - \bar{r}_B) \times \bar{F}$$
$$= \bar{r} \times \bar{F}$$

So, $\boxed{\bar{M} = \bar{r} \times \bar{F}}$ .... **Vector Formulation.**

The magnitude of moment of a couple is

$$M = r\,F \sin\theta = F\,d, \quad (\text{as } r\sin\theta = d)$$

$\boxed{M = Fd}$ .... **Scalar Formulation.**

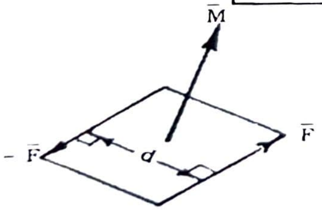


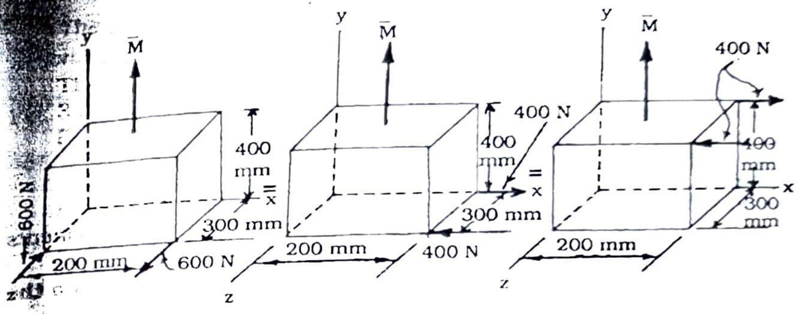
Fig 3·19

The moment of a couple is a **free vector** which may be ***applied at any point.***

This means that a couple – moment can be shifted to any point on the rigid body without changing its effects on the body.

3·10 Equivalent Couples :

The two couples having the same moment $\bar{M}$ are called equivalent. The magnitude of force couple acting on different box may be different, but the couple moment may be same as the product F.d is same as shown in Fig. 3.20.



M = Fd
= (600 N) × (0·2 m)
M = 120 Nm

M = Fd
= (400 N) × (0·3 m)
M = 120 Nm

M = Fd
= (400 N) × (0·3 m)
M = 120 Nm

Fig 3·20

Each of the three couples shown in Fig 3·20 has the same moment $\bar{M}$ (same direction and same magnitude M = 120 N.m). Each will have the same effect on the box i.e. each will rotate the box around y axis in clockwise direction.

3·11 Transformation of Equivalent Force System in to One–Another :

The two systems of forces are said to be *equivalent* (i.e. *they have same effect on a rigid body)* if we can transform one of them in to other by means of one or several of the following operations :

(1) Replacing two forces acting on the same particle by their resultant :

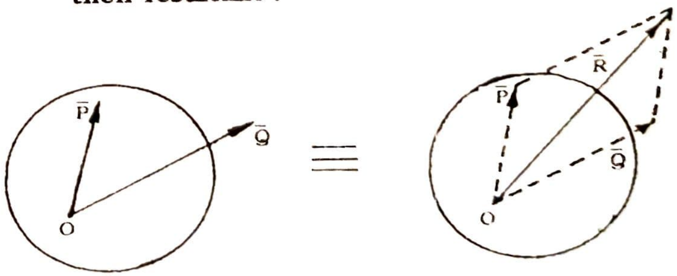


Fig. 3·21

Here, forces $\bar{P}$ and $\bar{Q}$ are replaced by the resultant $\bar{R}$.

The effect will remain unchanged either we apply $\bar{P}$ and $\bar{Q}$ or only $\bar{R}$. These two force systems are called **equivalent.**

(2) Resolving a force in to two components :

Fig. 3·22

Instead of applying only $\overline{F}$, apply $\overline{F}_1$ and $\overline{F}_2$ components of $\overline{F}$.

(3) Cancelling two equal and opposite forces acting on the same particle :

Fig. 3·23

Here, three forces say $\overline{F}$, $-\overline{F}$ and $\overline{P}$ are acting at O.

$\overline{F}$ and $-\overline{F}$ are cancelled. Thus instead of 3 forces, only single force will remain at O.

(4) Attaching to the same particle two equal and opposite forces :

Fig. 3·24

$\overline{P}$ and $-\overline{P}$ may be attached to the system of forces at a particle.

(5) Moving a force along its line of action :

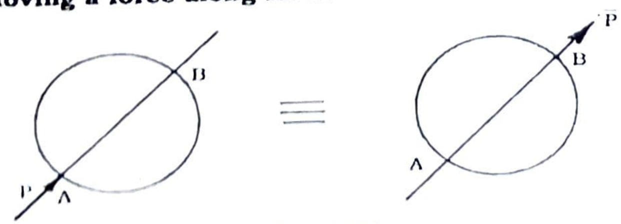


Fig. 3·25

The force $\overline{P}$ acting at A can be replaced to B on the same line of action.

The above operations can ***jointly*** be done as under (Refer Fig. 3·26).

(i) ***Replacing*** – $\overline{F}_1$ from C to B and $\overline{F}_1$ from D to A on the same line of action.
(moment remains unchanged F_1d_1)

(ii) ***Resolving*** – $\overline{F}_1$ into – $\overline{P}$ and – $\overline{Q}$ and $\overline{F}_1$ into $\overline{P}$ and $\overline{Q}$
(Moment remains unchanged $F_1d_1 = Qd_2$)

(iii) ***Cancelling*** $\overline{P}$ and – $\overline{P}$ (moment = Qd_2).

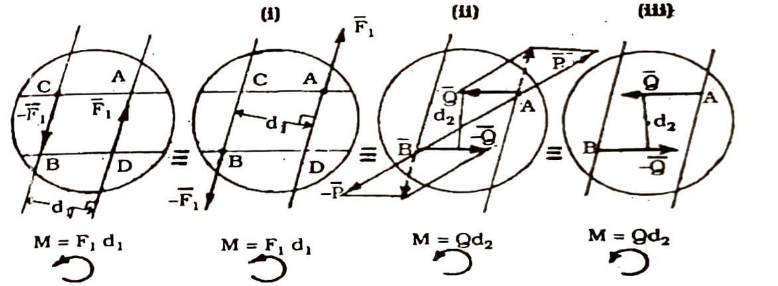
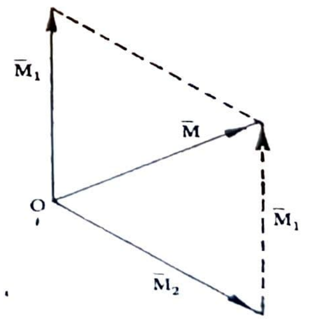


Fig 3·26

3·12 Addition of Couples :

Sum of two couples of moments $\overline{M}_1$ and $\overline{M}_2$ is a couple of moment $\overline{M}$ and is equal to the vector sum of $\overline{M}_1$ and $\overline{M}_2$. Thus the couple – moments acting at O can be added by using parallelogram law or vector triangle.



$$\overline{M} = \overline{M}_1 + \overline{M}_2$$

Fig 3·27

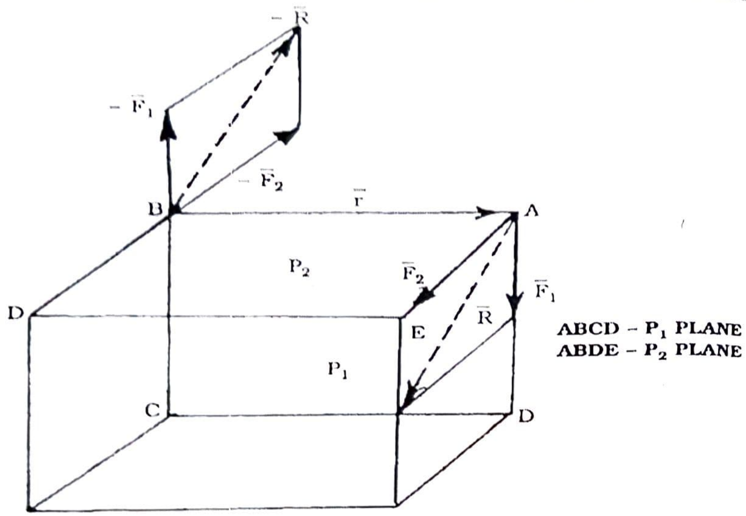


Fig 3·28

In above figure, couple $\bar{F}_1$ and $-\bar{F}_1$ is acting in P_1 plane and couple $\bar{F}_2$ and $-\bar{F}_2$ acting in P_2 plane. Resultant $\bar{R}$ of $\bar{F}_1$ and $\bar{F}_2$ and $-\bar{R}$ of $-\bar{F}_1$ and $-\bar{F}_2$ form a couple. Hence

$$\bar{M} = \bar{r} \times \bar{R} = \bar{r} \times (\bar{F}_1 + \bar{F}_2)$$
$$= \bar{r} \times \bar{F}_1 + \bar{r} \times \bar{F}_2$$
$$= \bar{M}_1 + \bar{M}_2$$

The vector representing a couple is called couple vector. Symbol ↺ is added to avoid any confusion with vectors representing forces. *The couple vector M may be resolved in to component vectors M_x, M_y and M_z directed along the axes of coordinates.*

*$\bar{M}$ couple moment vector is a **free vector** and **can be replaced at any point.***

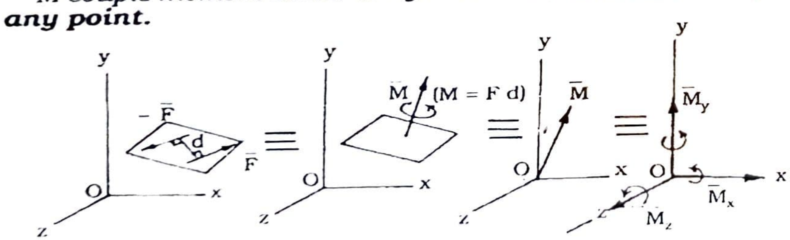


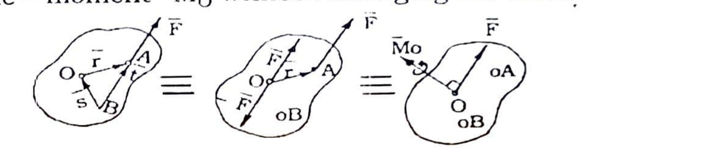
Fig. 3·29

$\bar{M}$ is resolved into M_x, M_y and M_z.

3·13 Resolution of a Given Force at A in to a Force at O and Couple :

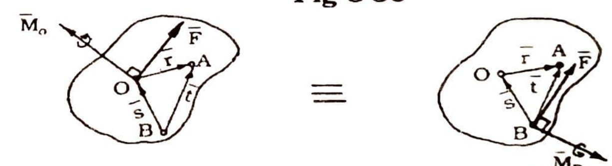
Any force $\bar{F}$ acting on a rigid body may be moved to an arbitrary point O, provided a couple is added which is equal to the moment of $\bar{F}$ about O.

So force $\bar{F}$ at A is moved to O by adding a couple $\bar{M}_o$ where $\bar{M}_o = \bar{r} \times \bar{F}$. In Fig 3.30, force $\bar{F}$ at A is replaced to O only by adding moment $\bar{M}_o$ at O. Here, force $\bar{F}$ is acting at A which is to be replaced to point O. Hence, equal & opposite forces $\bar{F}$ and $-\bar{F}$ are applied at O in addition to force $\bar{F}$ at A. Thus $\bar{F}$ at A and $-\bar{F}$ at O will form couple M_O · So at Point O, same force $\bar{F}$ is to be applied with couple – moment M_O without changing the effect.



(a) (b) (c)

Fig 3·30



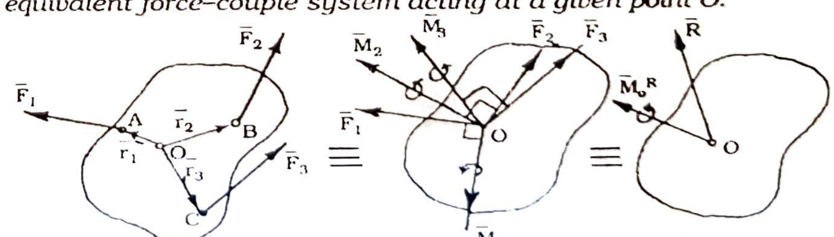
(a) Fig 3·31 (b)

Further, force $\bar{F}$ and couple $\bar{M}_o$ at O can be moved to any arbitrary point B by only replacing $\bar{M}_o$ by $\bar{M}_B$, as shown in Fig 3·31.

$$\bar{M}_B = \bar{t} \times \bar{F} = (\bar{r} + \bar{s}) \times \bar{F} = \bar{r} \times \bar{F} + \bar{s} \times \bar{F}$$
$$\bar{M}_B = \bar{M}_o + \bar{s} \times \bar{F}$$

3·14 Reduction of a System of Forces in to one Force and one Couple :

Any system of forces, however complex, may be reduced to an equivalent force–couple system acting at a given point O.



(a) (b) (c)

Fig 3·32

$\bar{F}_1$, $\bar{F}_2$ and $\bar{F}_3$ are the non concurrent forces. They can be added vectorially and replaced by a resultant $\bar{R}$ and single couple vector $\bar{M}_O^R$.

$$\bar{R} = \Sigma\bar{F} \qquad \bar{M}_O^R = \Sigma\bar{M}_O = \Sigma\bar{r} \times \bar{F}$$

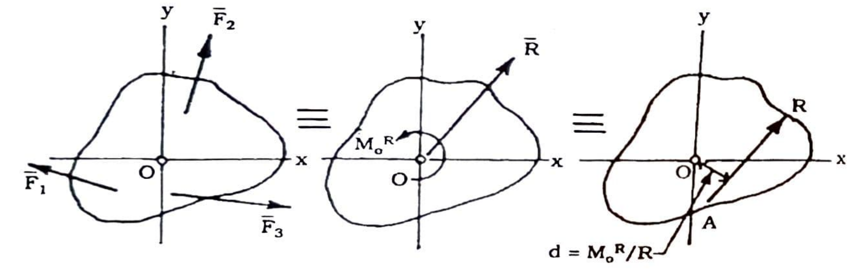
In rectangular components, we write

$$\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$$

$$\bar{F} = F_x\bar{i} + F_y\bar{j} + F_z\bar{k}$$

$$\bar{R} = R_x\bar{i} + R_y\bar{j} + R_z\bar{k}, \qquad \bar{M}_O^R = M_x^R\bar{i} + M_y^R\bar{j} + M_z^R\bar{k}$$

Further reduction of a system of forces can be made as shown in Fig 3·33 & 3·34. Resultant $\bar{R}$ can be replaced by a distance d as shown in Fig 3.33.

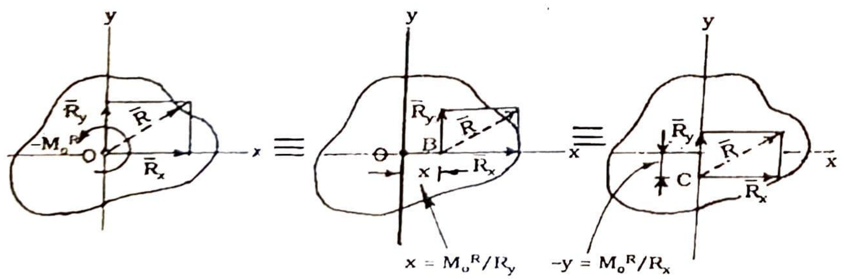


(a) (b) (c)

Fig 3·33

$\bar{R}_x$ or $\bar{R}_y$ may be moved to –y or x distance respectively as shown in Fig 3.34. Here, $\bar{R}_x$, $\bar{R}_y$ and $\bar{M}_O^R$ are acting at point O. The same resultant can be shifted to point B along x axis by a distance x. The moment at O remains same and anticlock wise.

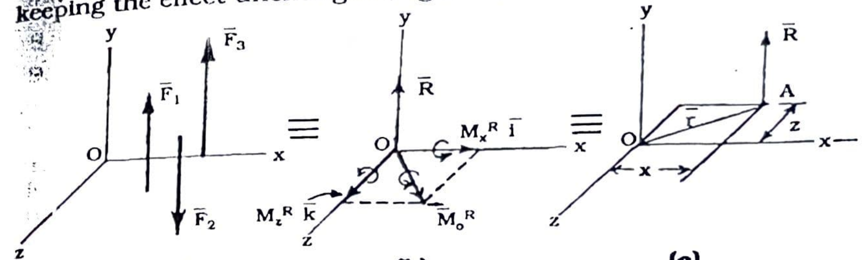
Similary it can be shifted along y axis with –y distance for keeping anticlockwise moment at point O.



(a) (b) (c)

Fig 3·34

For parallel forces, reduction can be as under. Resultant $\bar{R}$ can be applied at O with $\bar{M}_o$. The same $\bar{R}$ can be applied at A keeping the effect unchanged (Fig 3·35).



(a) (b) (c)

Fig 3·35

As the forces are parallel to y axis, their resultant $\bar{R}$ will be parallel to y and moment ($\bar{M}_O^R$) will lie in z–x plane.

$$\bar{R} = \bar{R}_y = \Sigma F_y\,\bar{j} \quad \text{and} \quad M_x^R\,\bar{i} = \Sigma M_x\,\bar{i}$$

$$M_z^R\,\bar{k} = \Sigma M_z\,\bar{k}$$

Now, if $\bar{R}$ is to be applied at A, then $\bar{r} \times \bar{R} = \bar{M}_O^R$

$$(x\bar{i} + z\bar{k}) \times R_y\,\bar{j} = M_x^R\,\bar{i} + M_z^R\,\bar{k}.$$

$$-z = \frac{M_x^R}{R_y} \quad \text{and} \quad x = \frac{M_z^R}{R_y}$$

IMPORTANT EQUATIONS

1. Vector Product (Cross Product) :

Cross product is a vector.

$$\bar{V} = \bar{P} \times \bar{Q}$$

$$\bar{P} \times \bar{Q} = -(\bar{Q} \times \bar{P})$$

$$\bar{i} \times \bar{i} = 0$$

$$\bar{j} \times \bar{j} = \bar{k}$$

$$\bar{i} \times \bar{k} = -\bar{j}$$

Magnitude $V = PQ\sin\theta$

$$\bar{V} = \bar{P} \times \bar{Q} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$$

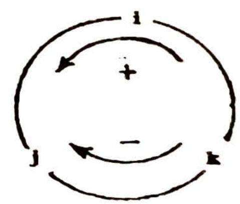


Fig 3·36

2. Moment of a Force about a Point :

$$\bar{M}_o = \bar{r} \times \bar{F}$$

$$M_o = F\,d$$

3. **Scalar Product (Dot Product) :**
Dot product is a scalar.

$\overline{P} \cdot \overline{Q} = PQ \cos\theta$

$\overline{P} \cdot \overline{Q} = \overline{Q} \cdot \overline{P}$

$\overline{i} \cdot \overline{i} = 1$

$\overline{i} \cdot \overline{j} = 0$

$\overline{i} \cdot \overline{k} = 0$

$\overline{P} \cdot \overline{Q} = P_x Q_x + P_y Q_y + P_z Q_z$

$$\overline{S} \cdot (\overline{P} \times \overline{Q}) = \begin{vmatrix} S_x & S_y & S_z \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$$

4. **Moment of a Force $\overline{F}$ acting at A :**

About Point (vector)

(i) *About Origin :*

$$M_O = \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

(ii) *About any point B :*

$$M_B = \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ x_{A/B} & y_{A/B} & z_{A/B} \\ F_x & F_y & F_z \end{vmatrix}$$

Where $\mathbf{x_{A/B} = (x_A - x_B)}$

About Axis (Scalar)

(i) *About axis passing through origin :*

$$M_{OL} = \begin{vmatrix} \lambda_x & \lambda_y & \lambda_z \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

(ii) *About axis not passing through origin (any axis)*

$$M_{BL} = \begin{vmatrix} \lambda_x & \lambda_y & \lambda_z \\ x_{A/B} & y_{A/B} & z_{A/B} \\ F_x & F_y & F_z \end{vmatrix}$$

Moment about axis can be expressed in Cartesian vector since its direction is defined by axis $\overline{\lambda}$. The magnitudes M_{OL} & M_{BL} can be expressed in Cartesian vectors as

$$\overline{\mathbf{M}}_{OL} = \mathbf{M}_{OL}\ \overline{\lambda}_{OL}$$

$$\overline{\mathbf{M}}_{BL} = \mathbf{M}_{BL}\ \overline{\lambda}_{BL}$$

5. **Couple Moment :**

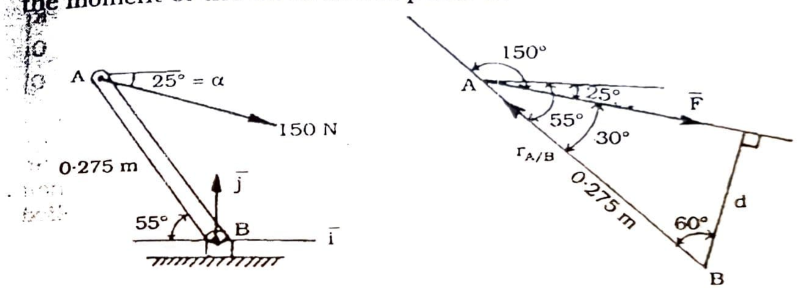
$\overline{M} = \overline{r} \times \overline{F}$

$M = Fd.$

6. By Replacing the force $\overline{F}$ at A = $\overline{F}$ at O + $\overline{M}_o$ = $\overline{F}$ at B + $\overline{M}_B$
7. $\overline{R} = \Sigma\overline{F}$, $\overline{M}_O^R = \Sigma\overline{M}_O = \Sigma\overline{r} \times \overline{F}$
8. $\overline{R} = R_x\overline{i} + R_y\overline{j} + R_z\overline{k}$
9. $\overline{M}_O^R = M_x^R\overline{i} + M_y^R\overline{j} + M_z^R\overline{k}$

SOLVED EXAMPLES

1. A 150–N force is applied to the control rod AB as shown. Knowing that the length of the rod is 275 mm and $\alpha = 25°$, determine the moment of the force about point B.



(a) **Fig. 3·37** **(b)**

The problem can be solved by **five** ways.

(a) $\overline{M}_B = \overline{r} \times \overline{F}$

$M_B = rF\sin\theta$ where θ = angle bet. lines of action of $\overline{r}$ and $\overline{F}$

here $\theta = 150°$

$M_B = 0{\cdot}275 \times 150 \times \sin 150°$

$= 20{\cdot}625$ N.m clockwise $\boxed{M_B = (20{\cdot}625 \text{ N.m})\curvearrowright}$

(b) $M_B = Fd$

$= 150 \times 0{\cdot}275 \sin 30°$

$\boxed{M_B = 20{\cdot}625 \text{ N.m Clockwise}}$ $\boxed{\overline{M}_B = -(20{\cdot}625 \text{ N.m})\overline{k}}$

(c) Force will be having two rectangular components.

$\overline{F}_x = (150 \cos 25°)\ \overline{i}$

$\overline{F}_y = (-150 \sin 25°)\ \overline{j}$

$\overline{F} = (150 \cos 25°)\ \overline{i} - (150 \sin 25°)\ \overline{j} = (135{\cdot}95 \text{ N})\ \overline{i} - (63{\cdot}4 \text{ N})\ \overline{j}$

$\overline{r}_{A/B} = -(0{\cdot}275 \cos 55°)\text{ m}\ \overline{i} + (0{\cdot}275 \sin 55°)\text{m}\ \overline{j}$

$= -(0{\cdot}158 \text{ m})\ \overline{i} + (0{\cdot}225 \text{ m})\ \overline{j}$

$\overline{M}_B = \overline{r}_{A/B} \times \overline{F}$

$= [-(0{\cdot}158\text{m})\ \overline{i} + (0{\cdot}225 \text{ m})\ \overline{j}] \times [(135{\cdot}95 \text{ N})\overline{i} - (63{\cdot}4\text{N})\ \overline{j}]$

$\boxed{\overline{M}_B = -(20{\cdot}62 \text{ N.m})\ \overline{k}}$

(d) Take the components of force 150 N along rod AB and perpendicular to rod AB. The component along the rod AB will not create moment about B as it is passing through B.

The moment will be only due to perpendicular compoment to rod AB.

$$M_B = F \cdot d$$
$$= (150 \sin 30^\circ) \times 0.275$$
$$\boxed{M_B = 20.625 \text{ N·m clockwise.}}$$

(e) $$\overline{M_B} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ x_{A/B} & y_{A/B} & z_{A/B} \\ F_x & F_y & F_z \end{vmatrix} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ -0.158 & +0.225 & 0 \\ 135.95 & -63.4 & 0 \end{vmatrix}.$$

$$\boxed{\overline{M_B} = -(20.62 \text{ N.m})\ k}$$

2. A force of 20 N is applied to the edge of a 0.8 m wide door as shown below. Find the moment of the force about the hinge when (a) a force is perpendicular to the door and (b) when force is applied at an angle of 60° to the edge of the door.

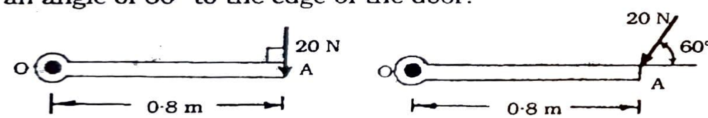


Fig 3·38

Case (a) : 20 N force is perpendicular to door.

∴ Moment of the force about hinge

$$= Fd$$
$$= 20 \text{ N} \times 0.8 \text{ m}$$
$$\boxed{M_o = 16 \text{ N.m clockwise}}$$

Case (b) : 20 N force is making 60° with the door.

There will be two rectangular components of this 20 N force

(i) $F_x = 20 \cos 60^\circ = 10$ N (←)

The moment of this F_x about O is zero as distance d is zero (F_x is passing through point O). This component will not rotate the door as it will not create moment.

(ii) $F_y = 20 \sin 60^\circ = 17.32$ N (↓)

The moment about O is equal to

$M_o = Fd = 17.32 \text{ N} \times 0.8 \text{ m}$

$$\boxed{M_o = 13.86 \text{ N.m clockwise}}$$

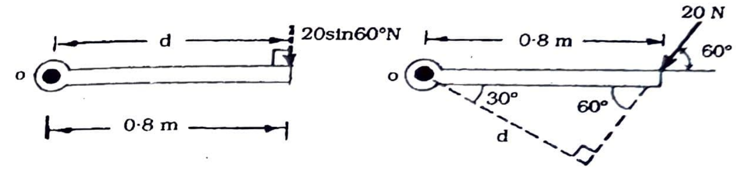


Fig 3·39

Alternatively, $M_o = Fd = 20 \text{ N} \times 0.8 \sin 60^\circ$
$$= 20 \text{ N} \times 0.693 \text{ m}$$
$$\boxed{M_o = 13.86 \text{ N.m clockwise}}$$

3. A horizontal plank of 15 m length is acted upon by four non-concurrent forces as shown below. Find the magnitude, direction and position of the resultant force.

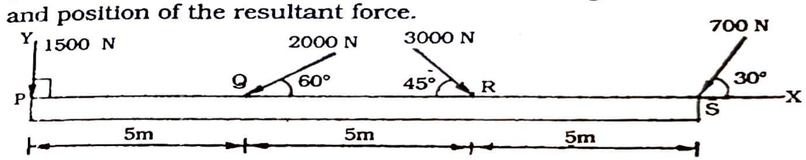


Fig 3·40

Resultant Force :

(i) Magnitude : (a) $\Sigma F_x \bar{i} = 0 - 2000 \cos 60^\circ \bar{i} + 3000 \cos 45^\circ \bar{i} - 700 \cos 30^\circ \bar{i}$

$$= -1000 \bar{i} + 2121.32 \bar{i} - 606.2 \bar{i}$$

$\therefore R_x \bar{i} = (515.12 \bar{i})$N

$$R_x = \Sigma F_x = 515.12 \text{ N } (\rightarrow)$$

(b) $\Sigma F_y \bar{j} = -1500 \bar{j} - 2000 \sin 60^\circ \bar{j} - 3000 \sin 45^\circ \bar{j} - 700 \sin 30^\circ \bar{j}$

$\therefore R_y \bar{j} = (-5703 \bar{j})$ N

$$R_y = \Sigma F_y = 5703 \text{ N } (\downarrow)$$

Hence, $\bar{R} = R_x \bar{i} + R_y \bar{j}$

$$= (515.12 \text{ N}) \bar{i} - (5703 \text{ N}) \bar{j}$$

and $R = \sqrt{(515.12)^2 + (-5703)^2}$

$$\therefore \boxed{R = 5726.22 \text{ N}}$$

(ii) Direction :

Let α = angle, which the resultant makes with PS.

$$\alpha = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{(-5703)}{515 \cdot 12} = -84 \cdot 84^\circ$$

$$\boxed{\alpha = 84 \cdot 84^\circ \searrow}$$

(iii) Position :

These four forces are nonconurrent forces. Hence, to determine position of the resultant, we have to take moment about any point, say P.

Let x = horizontal distance between point P and line of action of resultant.

Taking moment about P of the **vertical components of the forces** as well as **resultant** and equating the same,

$$\mathbf{M_p} : 5703\ x = (1500 \times 0) + (2000 \sin 60^\circ) \times 5 + (3000 \sin 45^\circ) \times 10 + (700 \sin 30^\circ) \times 15$$

$$\therefore \boxed{x = 6 \cdot 16\ \text{m}}$$

4. The lever ABC of a component of a machine is hinged at B and is subjected to a system of coplanar–non concurrent forces as shown below. Neglecting friction, determine the magnitude of $\bar{P}$ to keep the lever in equilibrium. Also determine the magnitude and direction of the reaction at B.

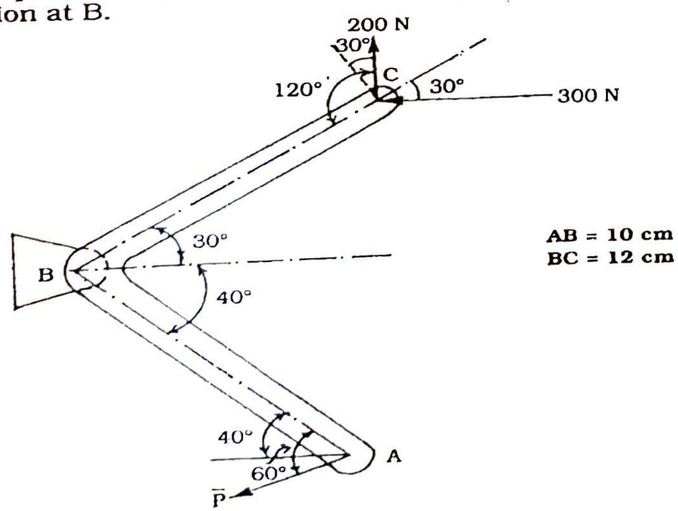


Fig 3.41

The lever ABC should be in equilibrium. Again the lever is hinged at B. For the equilibrium of non-concurrent forces, moment at B should be zero.

To find the magnitude of force $\bar{P}$, we have to take moment about hinge B as $M_B = 0$. Considering clock-wise moment positive.

$$\therefore M_B = P \sin 60^\circ \times 10 - 200 \cos 30^\circ \times 12 - 300 \sin 30^\circ \times 12 = 0$$

$$\therefore \boxed{P = 447 \cdot 8\ \text{N}}$$

To find the reaction components at B, we have to consider the all forces and reaction components to gether.

$$\Sigma \mathbf{F_x} = \mathbf{0},\ -300\ \bar{i} - P \cos 20^\circ\ \bar{i} + R_x\ \bar{i} = 0$$

$$\therefore R_x\ \bar{i} = (720 \cdot 8\ \text{N})\ \bar{i}$$

$$\boxed{R_x = (720 \cdot 8\ \text{N})\ (\rightarrow)}$$

$$\Sigma \mathbf{F_y} = \mathbf{0},\ 200\ \bar{j} - P \sin 20^\circ\ \bar{j} + R_y\ \bar{j} = 0$$

$$\therefore R_y\ \bar{j} = (-46 \cdot 85\ \text{N})\ \bar{j}$$

$$\therefore \boxed{R_y = 46 \cdot 85\ \text{N}\ (\downarrow)}$$

Reaction at B is.

$$\bar{R} = R_x\ \bar{i} + R_y\ \bar{j}$$

$$= (720 \cdot 8\ \text{N})\ \bar{i} + (-46 \cdot 85\ \text{N})\ \bar{j}$$

and magnitude of reaction at B is

$$R = \sqrt{(720 \cdot 8)^2 + (46 \cdot 85)^2}$$

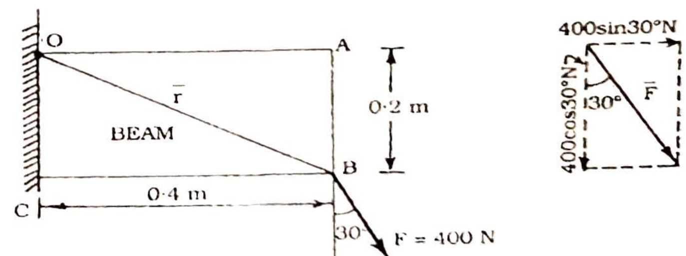
$$\boxed{R = 722 \cdot 3\ \text{N}}$$

and direction of $\bar{R}$ is

$$\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{-46 \cdot 85}{720 \cdot 8} = -3 \cdot 75^\circ$$

$$\boxed{\theta = 3 \cdot 75^\circ \searrow}$$

5. The force $\bar{F}$ acts at the end B of the beam (OABC) as shown below. Determine the moment of the force about point O.



(a) **Fig 3.42** (b)

Here, Position Vector $\bar{r} = (0{\cdot}4\ \bar{i} - 0{\cdot}2\ \bar{j})$ m

and Force $\bar{F} = (400 \sin 30°\ \bar{i} - 400 \cos 30°\ \bar{j})$ N

$= (200\ \bar{i} - 346{\cdot}4\ \bar{j})$ N.

The moment about O is

$$\bar{M}_o = \bar{r} \times \bar{F} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 0{\cdot}4 & -0{\cdot}2 & 0 \\ 200 & -346{\cdot}4 & 0 \end{vmatrix}$$

$$\boxed{\therefore\ \bar{M}_o = (-98{\cdot}6\ \bar{k})\ \text{N.m}}$$

and magnitude of the moment is

$$\boxed{M_o = 98{\cdot}6\ \text{N.m clock wise}}$$

OR, the moment can be determined by using equation F × d.

$$M_O = 400 \cos 30° \times 0{\cdot}4 - 400 \sin 30° \times 0{\cdot}2$$

$$\boxed{M_O = 98{\cdot}6\ \text{N.m. clockwise}}$$

6. The rod is supported by two brackets at A and B as shown below. Determine the magnitude of the moment $\bar{M}_{AB}$ produced by the force $\bar{F} = (-600\ \bar{i} + 200\ \bar{j} - 300\ \bar{k})$ N, which tends to rotate the rod about the AB axis.

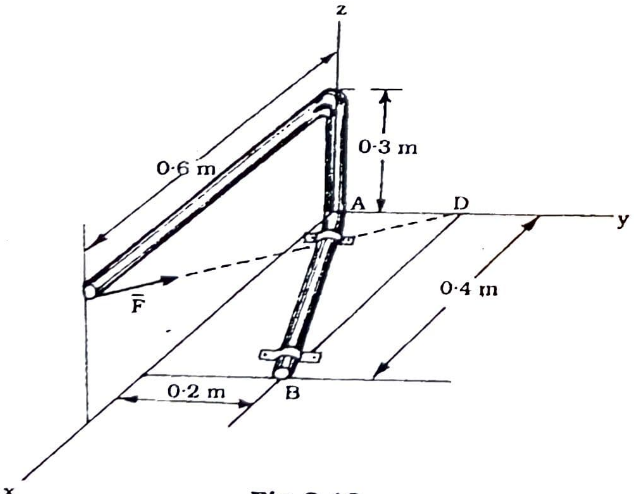


Fig 3·43

Here, coordinates of the points are A (0, 0, 0)
C (0·6, 0, 0·3)
B (0·4, 0·2, 0)
D (0, 0·2, 0)

Moment of force $\bar{F}$ about axis AB is

$$M_{AB} = \begin{vmatrix} \lambda_x & \lambda_y & \lambda_z \\ x_{C/A} & y_{C/A} & z_{C/A} \\ F_x & F_y & F_z \end{vmatrix}$$

λ_x, λ_y and λ_z are direction cosines of axis AB.

Where $\lambda_x = \dfrac{x_B - x_A}{d_{BA}} = \dfrac{0{\cdot}4}{\sqrt{0{\cdot}4^2 + 0{\cdot}2^2}} = \dfrac{0{\cdot}4}{0{\cdot}447} = 0{\cdot}894$

$\lambda_y = \dfrac{y_B - y_A}{d_{BA}} = \dfrac{0{\cdot}2}{0{\cdot}447} = 0{\cdot}447$ and $\lambda_z = 0$

$x_{C/A} = x_C - x_A = 0{\cdot}6$, $y_{C/A} = 0$, $z_{C/A} = 0{\cdot}3$

$F_x = -600$ N, $F_y = +200$ N, $F_z = -300$ N

Hence

$$M_{AB} = \begin{vmatrix} 0{\cdot}894 & 0{\cdot}447 & 0 \\ 0{\cdot}6 & 0 & 0{\cdot}3 \\ -600 & 200 & -300 \end{vmatrix}$$

$$M_{AB} = -53{\cdot}64\ \text{N.m}$$

$$\boxed{M_{AB} = 53{\cdot}64\ \text{N.m clockwise}}$$

7. A parallelopiped as shown is acted upon by a force 200 N at E Determine the moment of this force about **(i)** Origin O **(ii)** Point H **(iii)** Axis OD and **(iv)** Axis CF.

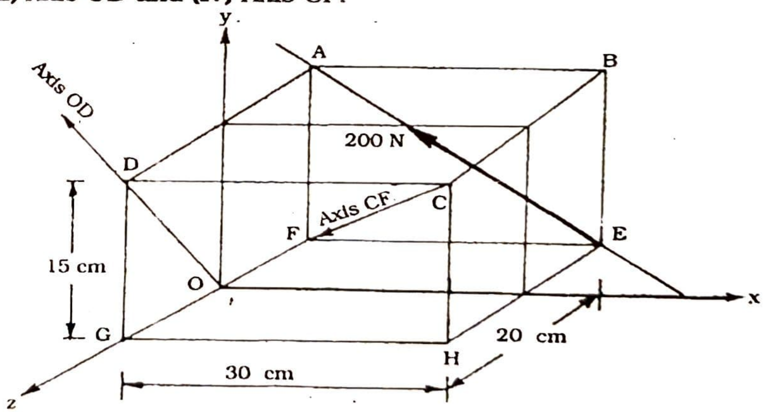


Fig 3·44

Moments about points are vectors where as moments about axis are scalar. *Force of 200 N is acting at E in the direction of E to A.*

The coordinates of points are

O = (0, 0, 0) E = (30, 0, −10)
H = (30, 0, 10) A = (0, 15, −10)
D = (0, 15, 10)
C = (30, 15, 10)
F = (0, 0, −10)

(i) Moment about origin O :

$$\bar{M}_o = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ x_E & y_E & z_E \\ F_x & F_y & F_z \end{vmatrix}$$

Here, $F_x = \dfrac{F\,d_x}{d} = \dfrac{200(x_A - x_E)}{d_{EA}}$

$= \dfrac{200\,(-30)}{33.54} = -178.9$ N

$F_y = \dfrac{F\,d_y}{d}$

$= \dfrac{200\,(15)}{33.54} = 89.45$ N

$$\bar{M}_o = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 30 & 0 & -10 \\ -178.9 & 89.45 & 0 \end{vmatrix}$$

$F_z = \dfrac{F\,d_z}{d}$

$= \dfrac{200\,(0)}{33.54} = 0$

$\bar{M}_o = (890.45 \text{ N.cm})\,\bar{i} - (1789 \text{ N.cm})\,\bar{j} + (2683.5 \text{ N.cm})\,\bar{k}$

(ii) Moment about any point H :

$$\bar{M}_H = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ x_{E/H} & y_{E/H} & z_{E/H} \\ F_x & F_y & F_z \end{vmatrix} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 0 & 0 & -20 \\ -178.9 & 89.45 & 0 \end{vmatrix}$$

$= (1789)\,\bar{i} - (3578)\,\bar{j}$

$$\boxed{\bar{M}_H = (1789 \text{ N.cm})\,\bar{i} - (3578 \text{ N.cm})\,\bar{j}}$$

(iii) Moment about axis OD passing through origin :

$$M_{OD} = \begin{vmatrix} \lambda_x & \lambda_y & \lambda_z \\ x_E & y_E & z_E \\ F_x & F_y & F_z \end{vmatrix}$$

Here, $\lambda_x = (\lambda_{OD})_x = \dfrac{d_x}{d} = \dfrac{x_D}{OD}$

$\therefore\ \lambda_x = 0$

$\lambda_y = \dfrac{15}{\sqrt{15^2 + 10^2}} = 0.83$

and

$$= \begin{vmatrix} 0 & 0.83 & 0.55 \\ 30 & 0 & -10 \\ -178.9 & 89.45 & 0 \end{vmatrix}$$

$\lambda_z = \dfrac{10}{\sqrt{15^2 + 10^2}} = 0.55$

$= -0.83\,(-1789) + 0.55\,(2683.5)$

$= 2960.8$ N.cm.

$$\boxed{M_{OD} = 2960.8 \text{ N.cm}}$$

(iv) Moment about axis CF not passing through origin :

$$M_{CF} = \begin{vmatrix} \lambda_x & \lambda_y & \lambda_z \\ x_{E/C} & y_{E/C} & z_{E/C} \\ F_x & F_y & F_z \end{vmatrix}$$

Here, $\lambda_x = (\lambda_{CF})_x = \dfrac{d_x}{d} = \dfrac{x_F - x_C}{d}$

$\lambda_x = \dfrac{-30}{\sqrt{-30^2 + -15^2 - 20^2}} = -0.77$

$\lambda_y = \dfrac{-15}{39.05} = -0.38$

$\lambda_z = \dfrac{-20}{39.05} = -0.51$

and $x_{E/C} = x_E - x_C$

$$= \begin{vmatrix} -0.77 & -0.38 & -0.51 \\ 0 & -15 & -20 \\ -178.9 & 89.45 & 0 \end{vmatrix}$$

$= -0.77\,(+1789) + 0.38\,(-3578) - 0.51\,(-2683.5)$

$= -1368.59$

$$\boxed{M_{CF} = -1368.59 \text{ N.cm}}$$

8. A frame ACD is hinged at A and D and supported by a cable which passes through a ring at B and is attached to hooks at G and H. Knowing that the tension in the cable is 1125 N, determine the moment about the diagonal AD of the force exerted on the frame by portion BH of the cable.

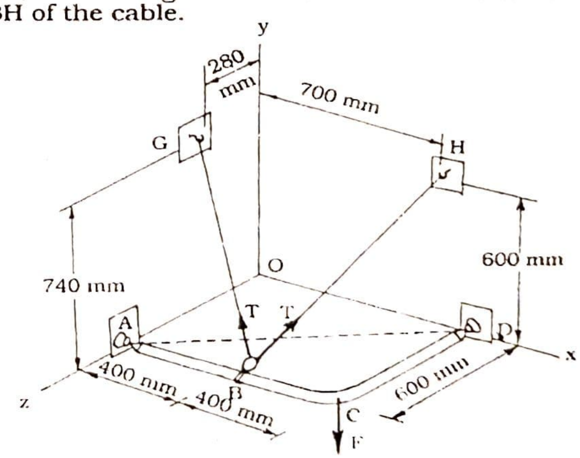


Fig. 3.45

Here, the tension in cable BG and BH remains same, 1125 N

The coordinates of points are as under (in meters).

$A = (0, 0, 0{\cdot}6)$

$D = (0{\cdot}8, 0, 0)$

$B = (0{\cdot}4, 0, 0{\cdot}6)$

$H = (0{\cdot}7, 0{\cdot}6, 0)$

Moment of BH about axis AD is

$$M_{AD} = \begin{vmatrix} \lambda_x & \lambda_y & \lambda_z \\ x_{B/A} & y_{B/A} & z_{B/A} \\ F_x & F_y & F_z \end{vmatrix}$$

Here, $\lambda_x = \dfrac{x_D - x_A}{d_{AD}} = \dfrac{0{\cdot}8}{\sqrt{0{\cdot}8^2 + 0{\cdot}6^2}} = 0{\cdot}8$

$\lambda_y = 0$ and

$\lambda_z = \dfrac{-0{\cdot}6}{1} = -0{\cdot}6$

$x_{B/A} = x_B - x_A$

$F_x = \dfrac{F\,d_x}{d} = \dfrac{1125\,(x_H - x_B)}{d_{BH}}$

$F_x = \dfrac{1125\,(0{\cdot}3)}{0{\cdot}9} = 375$

$F_y = \dfrac{1125\,(0{\cdot}6)}{0{\cdot}9} = 750$

$F_z = \dfrac{1125\,(-0{\cdot}6)}{0{\cdot}9} = -750$

$$M_{AD} = \begin{vmatrix} 0{\cdot}8 & 0 & -0{\cdot}6 \\ 0{\cdot}4 & 0 & 0 \\ 375 & 750 & -750 \end{vmatrix}$$

$= -0{\cdot}6\,(300)$

$= -180$ N.m

$$\boxed{M_{AD} = -180 \text{ N.m}}$$

9. A couple acts at the end of the beam as shown below. Replace it by an equivalent couple having a pair of forces that act through

(i) points A and B and **(ii)** point D and E

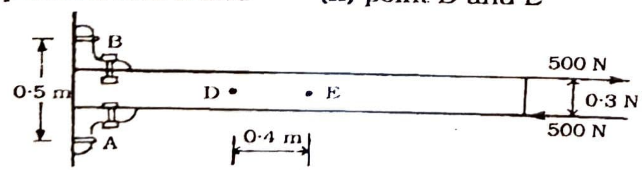
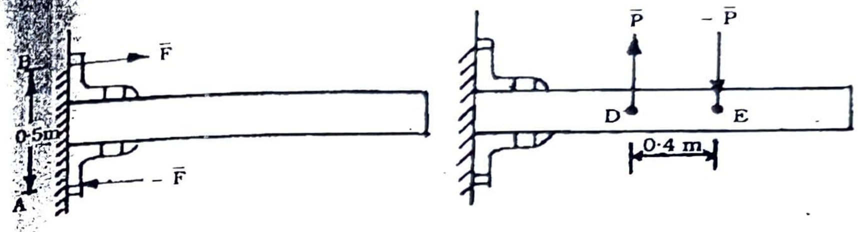


Fig 8·46 (a)



(b) (c)

Fig 8·46

The couple has a magnitude $M = F\,d$

$= 500 \times 0{\cdot}3$

$= 150$ N.m clockwise

The couple moment is a free vector and can be replaced at any point on the structure.

(i) To preserve the direction of M (i.e., clockwise), the forces as shown in fig (b) above should be applied.

$M = F\,d$

$150 = F\,(0{\cdot}5)$

$$\boxed{F = 300 \text{ N}}$$

(ii) To preserve the direction of M (clockwise) the forces as shown in fig (c) above should be applied.

$M = P\,d$

$150 = P\,(0{\cdot}4)$

$$\boxed{P = 375 \text{ N}}$$

10. Four 38 mm – diameter pegs are attached to a board as shown. Two strings are passed around the pegs and pulled with the forces indicated. **(a)** Determine the resultant couple acting on the board. **(b)** If only one string is used, around which pegs should it pass and in what directions should it be pulled to create the same couple with the minimum tension in the string ? **(c)** What is the value of that minimum tension ?

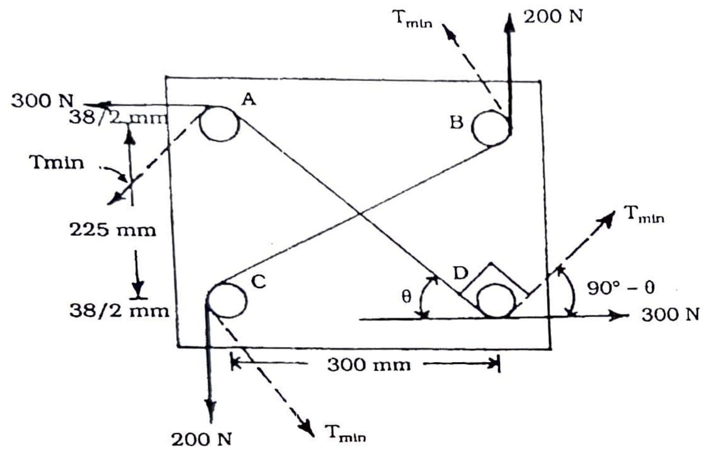


Fig. 3.47

(a) Resultant Couple :

$$M = F_1\,d_1 + F_2\,d_2$$
$$= 300\,(0.225 + 0.038) + 200\,(0.3 + 0.038)$$

$$\boxed{M = 146.5 \text{ N.m counter–clockwise}}$$

(b) To have minimum tension and same couple, there should be maximum distance d.

For maximum d either it should pass arround

$\boxed{\text{A and D or B and C}}$ and

$$\theta = \tan^{-1} = \frac{0.225}{0.3} = 36.87°$$

$\therefore$ Angle of T_{min}
$= 90° - \theta = 53.1°$

Angle of T_{min} = 53.1° at D
and 53.1° at A
OR similar angles at B and C with vertical

(c) $M = T_{min}\,d_{AD}$ **OR** d_{BC}

$$146.5 = T_{min}\,(\sqrt{0.225^2 + 0.3^2} + 0.038)$$

$$\boxed{\therefore T_{min} = 354.72 \text{ N}}$$

11. Replace the forces acting on the rod by an equivalent single resultant force and couple system acting at point A.

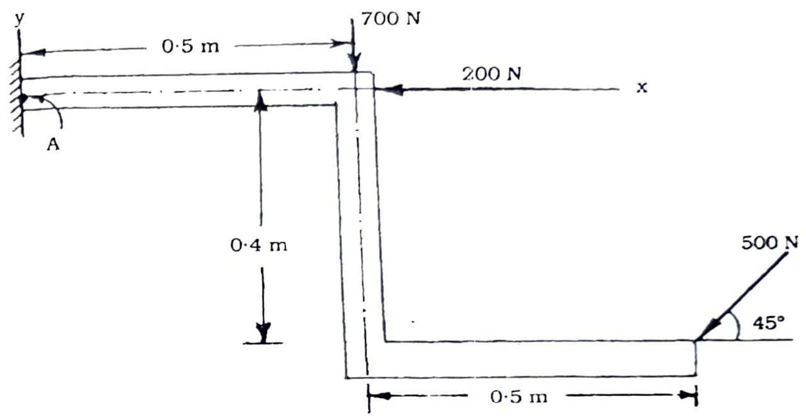


Fig 3.48

Three forces acting on the rod can be replaced at A by applying their **resultant** at A and a **couple moment** due to them at A.

Force at A :

$$\bar{F}_R = \bar{F}_1 + \bar{F}_2 + \bar{F}_3$$
$$= -500 \cos 45°\ \bar{i} - 500 \sin 45°\ \bar{j} - 200\ \bar{i} - 700\ \bar{j}$$

$\boxed{\bar{F}_A = \{-553.55\ \bar{i} - 1053.55\ \bar{j}\}\text{ N}}$ **OR** $\boxed{F_A = 1190 \text{ N}\ 62.28°}$

Couple at A :

$$\bar{M}_A = \Sigma r \times \bar{F}$$
$$= (1.0\ \bar{i} - 0.4\ \bar{j}) \times (-500 \cos 45°\ \bar{i} - 500 \sin 45°\ \bar{j})$$
$$+ (0.5\ \bar{i}) \times (-200\ \bar{i}) + (0.5\ \bar{i}) \times (-700\ \bar{j})$$
$$= \{(-353.55 - 350) - (141.42)\}\ \bar{k}$$

$$\boxed{\bar{M}_A = (-844.97 \text{ N.m})\ \bar{k} = 844.97 \text{ N. m clockwise}}$$

12. A column is subjected to a couple moment $\bar{M}$ and forces $\bar{F}_1$ and $\bar{F}_2$ as shown in Fig 3.49. Replace this system by an equivalent single resultant force and couple moment acting at point O.

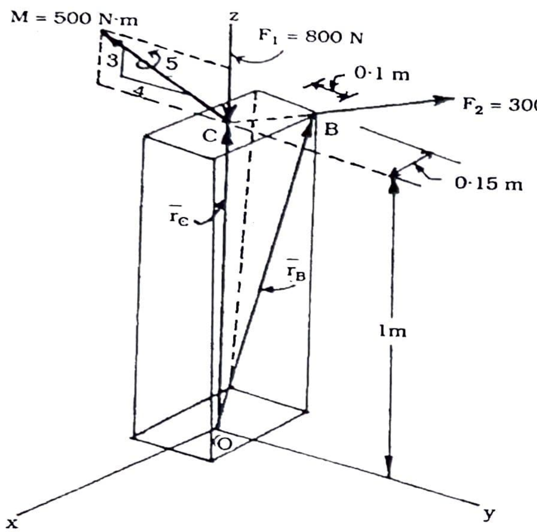


Fig 3·49

Here, $\bar{F}_1 = (-800\ \bar{k})$ N

The line of action of F_2 is passing through point C and F_2 is applied at a point B having coordinates

x = − 0·15 m
y = + 0·1 m
z = + 1·0 m.

The coordinates of point C are

x = 0
y = 0
z = + 1 m.

$$\bar{F}_2 = F_2\ \bar{\lambda} = (300\ \text{N}) \left[\frac{-0{\cdot}15\ \bar{i} + 0{\cdot}1\ \bar{j}}{\sqrt{(-0{\cdot}15)^2 + (0{\cdot}1)^2}} \right]$$

$$\therefore \quad \bar{F}_2 = (-249{\cdot}6\ \bar{i} + 166{\cdot}4\ \bar{j})\ \text{N}$$

The vector of $\bar{M}$ is in the y − z plane.

$$\text{and} \quad \bar{M} = -500\left(\frac{4}{5}\right)\bar{j} + 500\left(\frac{3}{5}\right)\bar{k}$$

$$= (-400\ \bar{j} + 300\ \bar{k})\ \text{N.m.}$$

Resultant of $\bar{F}_1$ and $\bar{F}_2$ and couple moment due to $\bar{F}_1$ & $\bar{F}_2$ and free moment $\bar{M}$ will be replaced at O.

Resultant Force at O is

$$\bar{F}_R = \Sigma F = \bar{F}_1 + \bar{F}_2 = -800\ \bar{k} - 249{\cdot}6\ \bar{i} + 166{\cdot}4\ \bar{j}$$
$$= (-249{\cdot}6\ \bar{i} + 166{\cdot}4\ \bar{j} - 800\ \bar{k})$$

$$\therefore \quad \boxed{\bar{F}_R \text{ at O} = (-249{\cdot}6\ \bar{i} + 166{\cdot}4\ \bar{j} - 800\ \bar{k})\ \text{N}}$$

Couple Moment at O is

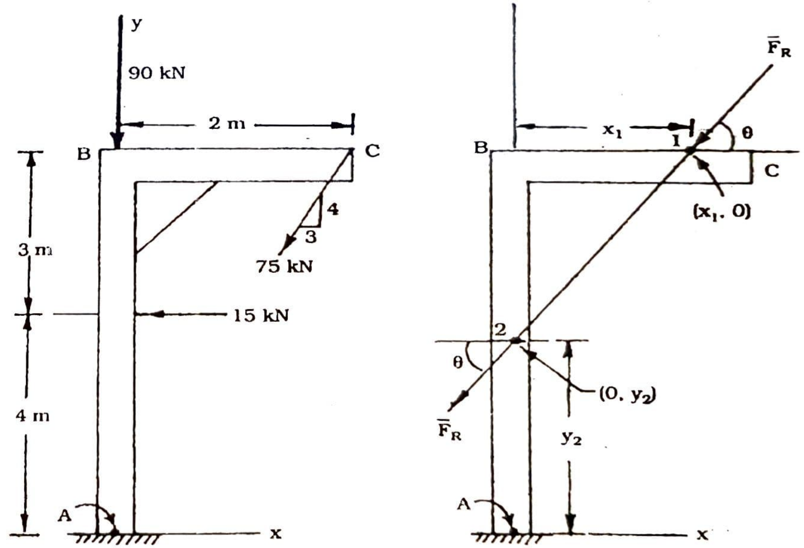
$$\bar{M}_{RO} = \Sigma M_O = \bar{M} + \bar{r}_C \times \bar{F}_1 + \bar{r}_B \times \bar{F}_2$$

$$\bar{M}_{RO} = (-400\ \bar{j} + 300\ \bar{k}) + (1\ \bar{k}) \times (-800\ \bar{k}) + \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ -0{\cdot}15 & 0{\cdot}1 & 1 \\ -249{\cdot}6 & 166{\cdot}4 & 0 \end{vmatrix}$$

$$= (-400\ \bar{j} + 300\ \bar{k}) + (0) + (-166{\cdot}4\ \bar{i} - 249{\cdot}6\ \bar{j})$$

$$\boxed{\bar{M}_{RO} = (-166{\cdot}4\ \bar{i} - 649{\cdot}6\ \bar{j} + 300\ \bar{k})\ \text{N.m}}$$

13. The frame shown in figure is subjected to three coplanar forces. Replace this loading by an equivalent single resultant force and specify where the line of action of the resultant intersects members AB and BC.



(a) Fig. 3·50 (b)

Here, three forces are acting and they can be replaced to A by their resultant and moments.

Resultant : $\bar{F}_R = F_{Rx}\,\bar{i} + F_{Ry}\,\bar{j}$

$F_{Rx} = 75\,(3/5) + 15 = 60$ kN ←

$F_{Ry} = 75\,(4/5) + 90 = 150$ kN ↓

$F_R = \sqrt{60^2 + 150^2} = 161.6$ kN

$\theta = \tan^{-1}\left(\dfrac{150}{60}\right) = 68.2°$

$\boxed{F_R = 161.6 \text{ kN}, \theta = 68.2°}$

Moment about A :

$M_A = 4\,(15) + 0\,(90) + 7\,(75)\,(3/5) - 2\,(75)\,(4/5)$

$= 255$ kN.m

The resultant force can be applied at any point on the structure and should have the same moment at A.

(1) If $\bar{F}_R$ acting at point (1) having co-ordinates $(x_1, 0)$

$7\,(60) - x_1\,(150) = 255$

$\boxed{\therefore x_1 = 1.10 \text{ m}}$

(2) If $\bar{F}_R$ acting at point (2) having co-ordinates $(0, y_2)$

$0\,(150) + y_2\,(60) = 255$

$\boxed{y_2 = 4.25 \text{ m}}$

14. Find the magnitude, direction and line of action of the resultant of the coplanar force system of four forces shown in figure.

(Pune University, April - 1996)

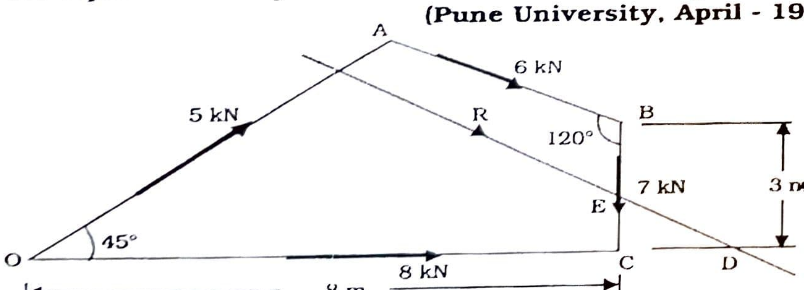


Fig. 3·51

For Resultant force, $R_x = 5\cos 45° + 8 + 6\cos 30° = 16.73$ kN

and $R_y = 5\sin 45° - 6\sin 30° - 7 = -6.46$ kN

Hence $R = \sqrt{16.73^2 + 6.46^2} = 17.93$ kN, and direction is 21·11°

$\boxed{R = 17.93 \text{ kN} \quad 21.11°}$

Let the line of action of resultant intersects line of action of 8 kN force at a distance of x_1 from O (i.e. distance OD).

Taking moment of all forces and the resultant about point O;

$6\cos 30° \times 3 + 6\sin 30° \times 8 + 7 \times 8 = 6.46 \times x_1$

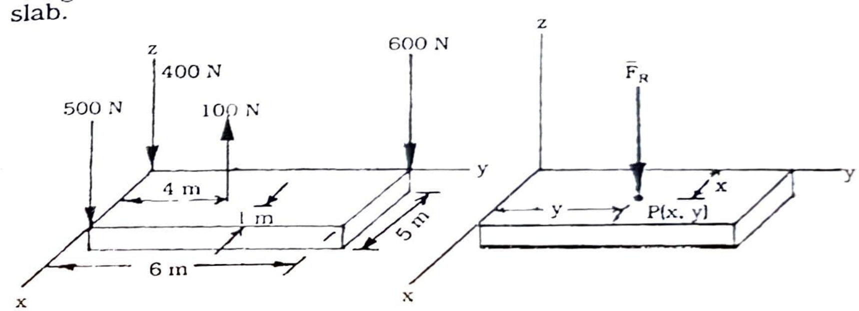
$\therefore \boxed{x_1 = 14.8 \text{ m from O i.e. distance OD}}$

And let the line of action of resultant intersects line of action of 7 kN force at a distance y_2 from point C (i.e. distance EC). Taking moment of all forces and the resultant about point O,

$6\cos 30° \times 3 + 6\sin 30° \times 8 + 7 \times 8 = 6.46 \times 8 + 16.73 \times y_2$

$\therefore \boxed{y_2 = 2.62 \text{ m from C i.e. distance EC}}$

15. The slab is subjected to four parallel forces. Determine the magnitude and direction of a single resultant force equivalent to the given force system and locate its point of application on the slab.



(a) **Fig. 3·52** (b)

Resultant force :

$F_R = 600 + 400 + 500 - 100$

$\boxed{F_R = 1400 \text{ N} \downarrow}$

Moment about x axis :

$M_x = 600 \times 6 - 100 \times 4$

$= 3200$ N m

Moment about y axis :

$M_y = 500 \times 5 - 100 \times 4$

$= 2100$ N m

Location of P Where resultant force will act. :

About x axis : $1400 \times y = 3200$

$\boxed{\therefore y = 2.29 \text{ m}}$

About y axis :

$$1400 \times x = 2100$$

$$\boxed{\therefore x = 1{\cdot}5 \text{ m.}}$$

Note : The above problem can also be solved by using vector formulation i.e. $\bar{M} = \Sigma\, \bar{r} \times \bar{F}$.

16. The following four forces act on a body at a pont (10, 20, 10)

$$\bar{F}_1 = -2000\ \bar{j}$$
$$\bar{F}_2 = -1000\ \bar{k}$$
$$\bar{F}_3 = 2002\ \bar{j} + 1001\ \bar{k}$$
$$\bar{F}_4 = -408\ \bar{i} - 816\ \bar{j} - 408\ \bar{k}$$

Reduce this force system to an equivalent system at origin.

(Pune University, Oct./Nov. - 1995)

At origin resultant plus moment should be applied. Resultant at a point (10, 20, 10) which can be applied at origin

$$\bar{R} = -408\ \bar{i} - 2000\ \bar{j} + 2002\ \bar{j} - 816\ \bar{j} - 1000\ \bar{k} + 1000\ \bar{k} - 408\ \bar{k}.$$
$$= -408\ \bar{i} - 814\ \bar{j} - 408\ \bar{k}$$

And moment at origin

$$M_O = \bar{r} \times \bar{R} = (10\ \bar{i} + 20\ \bar{j} + 10\ \bar{k}) \times (-408\ \bar{i} - 814\ \bar{j} - 408\ \bar{k})$$

OR

$$\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 10 & 20 & 10 \\ -408 & -814 & -408 \end{vmatrix}$$
$$= \bar{i}\,(4080) - \bar{j}\,(0) + \bar{k}\,(20)$$
$$= 4080\ \bar{i} + 20\ \bar{k}$$

$$\boxed{\begin{aligned} \bar{R}_O &= -408\ \bar{i} - 814\ \bar{j} - 408\ \bar{k} \\ \bar{M}_O &= 4080\ \bar{i} + 20\ \bar{k} \end{aligned}}$$

THEORY RELATED QUESTIONS

1. Distinguish between :

- **(i)** External forces and Internal forces.
- **(ii)** Dot Product and Cross Product
- **(iii)** Moment about Point and Moment about Axis.
- **(iv)** Moment about origin and Moment about any point B.
- **(v)** Moment about axis passing through origin and Moment about axis not passing through origin.

2. Explain :

- **(i)** Varignon's Theorem.
- **(ii)** Moment of a couple.
- **(iii)** Equivalent Couples.
- **(iv)** How to resolve a given force at A in to a force at O and a couple ?
- **(v)** How to reduce a system of forces in to a one force and one couple ?

EXERCISES

3·1 Determine the moment of a force at A about **(i)** point O and **(ii)** point P.

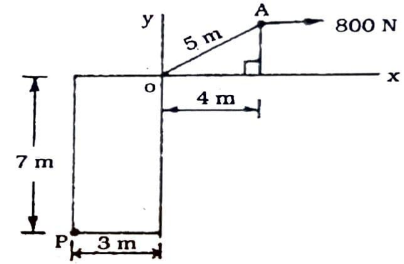


Fig. 3·53

3·2 Determine the moment of a force $\bar{F}$ at A about **(i)** point O and **(ii)** point P.

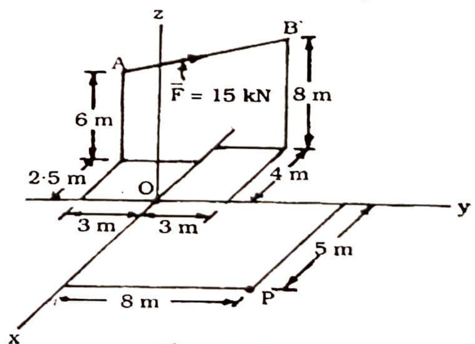


Fig. 3·54

3·3 Determine the moment about point A of each of the three forces acting on the beam.

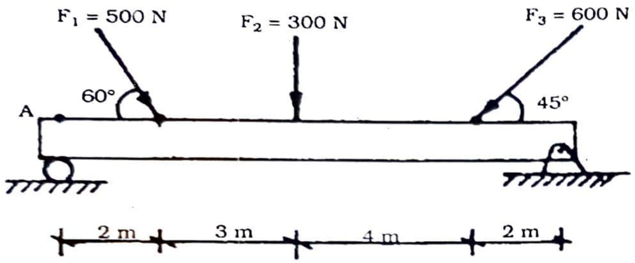


Fig 3.55

3.4 Determine the moment of each of the forces about point A. Also find the net moment at A due to all three forces acting together.

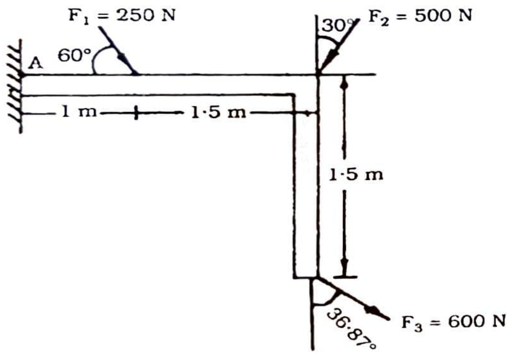


Fig 3.56

3.5 Determine the moment of the force at A about **(i)** point O and **(ii)** point P.

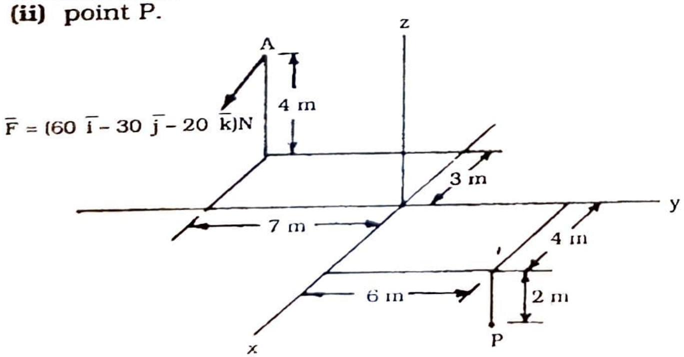


Fig 3.57

3.6 A 500N force is applied as shown to the bracket ABC. Determine the moment of the force about **(i)** point A **(ii)** point B.

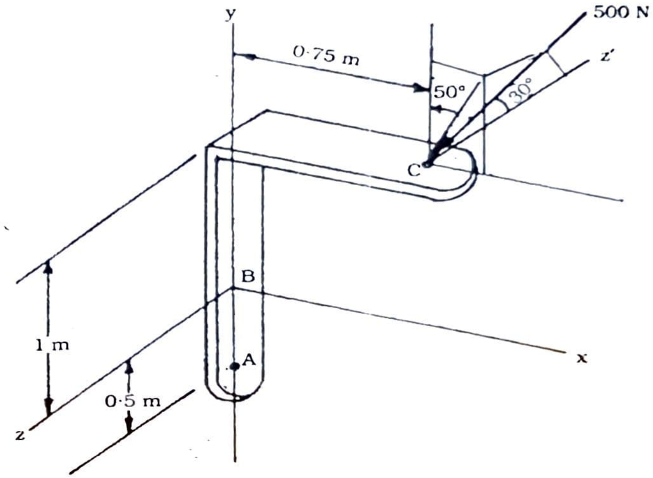


Fig 3.58

3.7 Determine the moment of the force $\bar{F}$ about **aa'** axis.

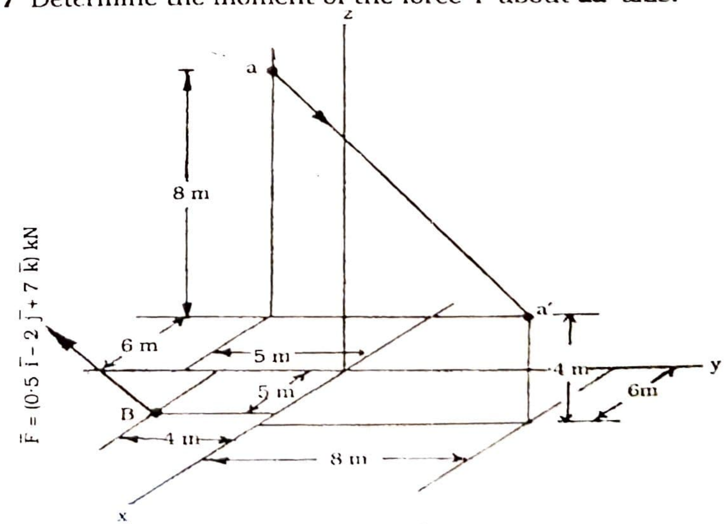


Fig. 3.59

3·8 The rectangular platform is hinged at A and B and supported by a cable which passes over a frictionless hook at E. Knowing that the tension in the cable is 1349 N, determine the moment about each of the coordinate axes of the force exerted by the cable **(i)** at C **(ii)** at D.

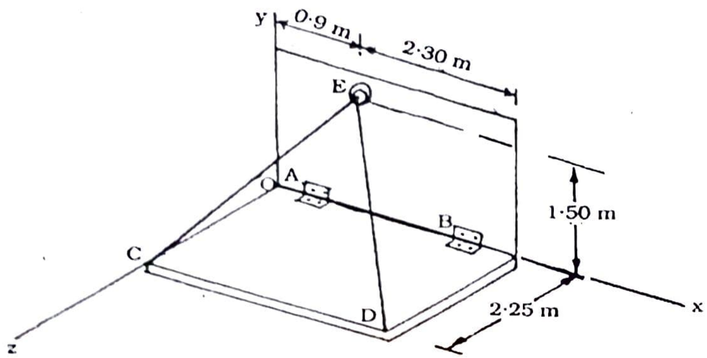


Fig. 3·60

3·9 Determine the moment of the force $\overline{F}$ about the x, y and z axes.

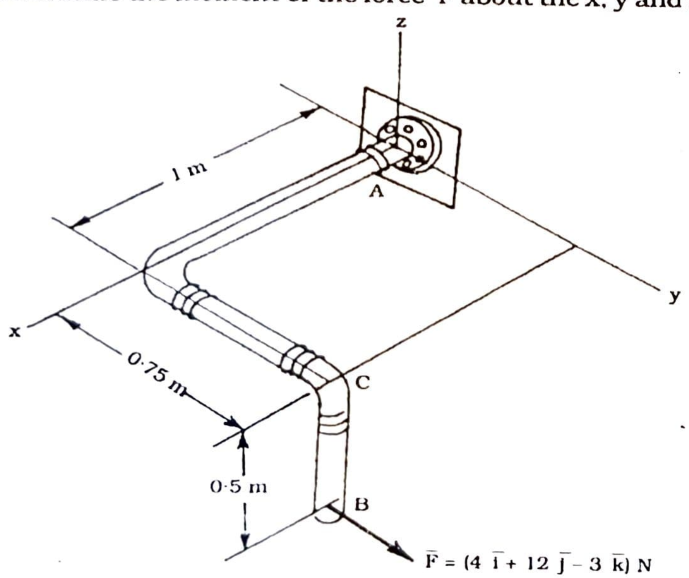


Fig 3·61

3·10 Two 100–N forces are applied as shown to the corners B and D of a rectangular plate. **(a)** Determine the moment of the couple formed by the two forces by resolving each force in to horizontal and vertical components and adding the moments of the two resulting couples. **(b)** Use the result obtained to determine the perpendicular distance between lines BE and DF.

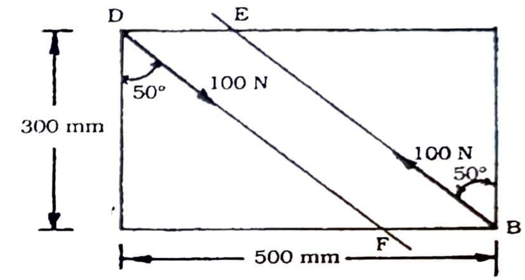
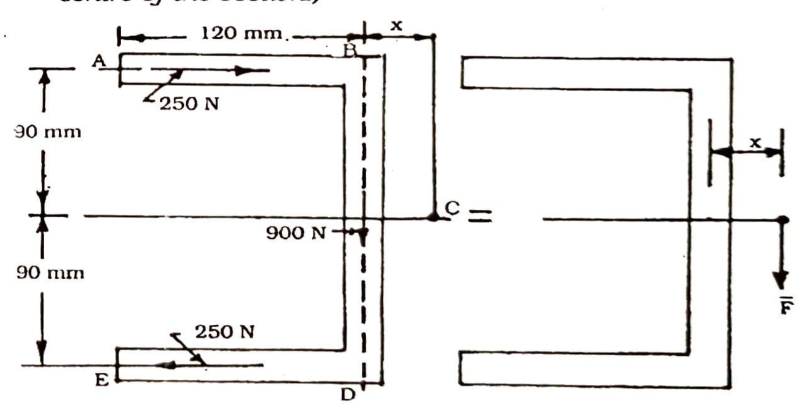


Fig 3·62

3·11 The shearing forces exerted on the cross section of a steel channel may be represented by a 900 N vertical force and two 250 N horizontal forces as shown. Replace this force and couple by a single force $\overline{F}$ applied at point C and determine the distance x from C to line BD. *(Point C is defined as the shear centre of the section.)*



(a) **Fig 3·63** **(b)**

Hint : The couple moment 250 N × 180 mm, and 900 N vertical force acting on cross section can be replaced by a single vertical force F = 900 N at a distance x.

$$\therefore \quad x = \frac{250 \text{ N} \times 180 \text{ mm}}{900 \text{ N}} = 50 \text{ mm}$$

3·12 A 1300 N force is applied at A to the rolled steel section. Replace that force by an equivalent force – couple system at the center C of the section.

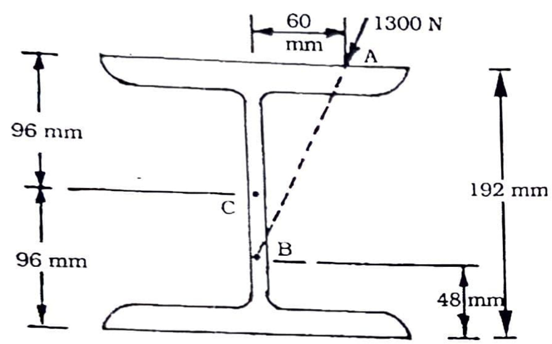


Fig. 3·64

3·13 Two couples act on the frame. If the resultant couple moment is to be zero, determine the distance d as shown in the figure.

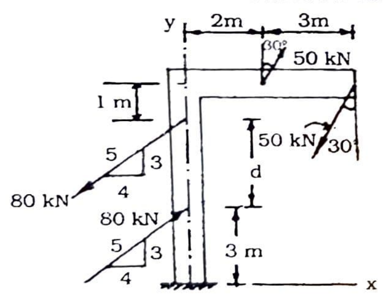


Fig. 3·65

3·14 Replace the force and couple moment system by an equivalent force and couple moment acting **(i)** at O **(ii)** at P.

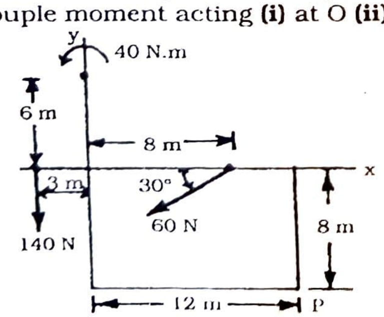


Fig. 3·66

3·15 A force and couple act on the rod assembly. Replace this system by an equivalent single resultant force. Specify the point where the line of action of the resultant force intersects the x axis. The pipe lies in the x–y plane.

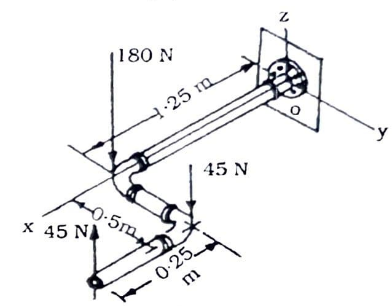


Fig. 3·67

3·16 The three forces and a couple of magnitude M = 8 N.m are applied to an angle bracket. **(a)** Find the resultant of this system of forces. **(b)** Locate the points where the line of action of the resultant intersects line AB and line BC.

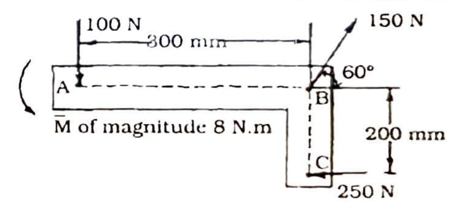


Fig. 3·68

3·17 Determine the magnitudes of the additional loads which must be applied at B and F, if the resultant of all six loads is to pass through the center of the mat which is a regular hexagon of side 3 m.

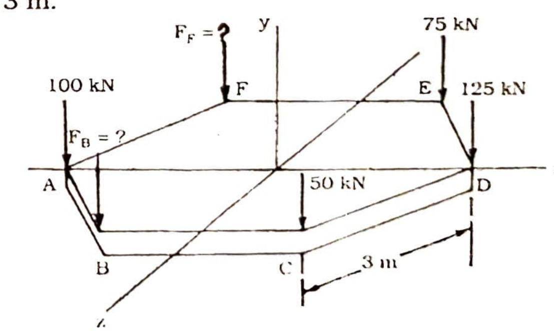


Fig. 3·69

SOUTIONS OF EXERCISES

3·1 (i) $M_o = Fd$
$= 800 \times 3$
$M_o = 2400$ N.m

(ii) $M_p = Fd$
$= 800 \times (7 + 3)$
$M_p = 8000$ N·m

Both are clockwise moments.

3·2 (i) Coordinates of

A = (−2·5, −3, 6)
B = (−4, 3, 8)
P = (5, 8, 0)

$d_x = (x_B - x_A)$

$$F_x = F\frac{d_x}{d} = 15\frac{\{-4-(-2\cdot5)\}}{\sqrt{1\cdot5^2+6^2+2^2}} = -3\cdot46$$

Similarly F_y and F_z are to be determined. The force is acting at A having coordinates (−2·5, −3, 6).

$$\bar{M}_o = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ -2\cdot5 & -3 & 6 \\ -3\cdot46 & 13\cdot85 & 4\cdot62 \end{vmatrix}$$

$\bar{M}_o = \{-96\cdot96\ \bar{i} - 9\cdot21\ \bar{j} - 45\ \bar{k}\}$ kN·m

(ii) For moment about point P.

$x_{A/P} = x_A - x_P = -2\cdot5 - 5 = -7\cdot5$
$y_{A/P} = -3 - 8 = -11$
$z_{A/P} = 6 - 0 = 6$

$$\bar{M}_p = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ -7\cdot5 & -11 & 6 \\ -3\cdot46 & 13\cdot85 & 4\cdot62 \end{vmatrix}$$

$\bar{M}_p = \{-133\cdot92\ \bar{i} + 13\cdot89\ \bar{j} - 141\cdot94\ \bar{k}\}$ kN·m

3·3 $M_{A1} = F_1 \sin 60° \times 2 = 500 \sin 60° \times 2$
$M_{A2} = F_2 \times 5 = 300 \times 5$
$M_{A3} = F_3 \sin 45° \times 9 = 600 \sin 45° \times 9$
$M_{A1} = 866\cdot03$ N·m $M_{A2} = 1500$ N·m $M_{A3} = 3818\cdot9$ N.m

3·4 $M_{A1} = 250 \sin 60° \times 1$
$M_{A2} = 500 \cos 30° \times 2\cdot5$
$M_{A3} = 600 \cos 36\cdot87° \times 2\cdot5 - 600 \sin 36\cdot87° \times 1\cdot5$
Net $M_A = M_{A1} + M_{A2} + M_{A3}$
$M_{A1} = 216\cdot5$ N·m $M_{A2} = 1082\cdot5$ N.m $M_{A3} = 660$ N·m
Net $M_A = 1959$ N·m

(**Note :** Angles of Forces F_1, F_2 and F_3 with vertical are 30°, 30° and 36·87° respectively)

3·5 Coordinates of

A = (−3, −7, 4)
P = (4, 6, −2)

$x_{A/P} = x_A - x_P = -3 - 4 = -7$, $y_{A/P} = -13$, $z_{A/P} = 6$

(i) $$\bar{M}_o = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ -3 & -7 & 4 \\ 60 & -30 & -20 \end{vmatrix}$$ (ii) $$\bar{M}_p = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ -7 & -13 & 6 \\ 60 & -30 & -20 \end{vmatrix}$$

$\bar{M}_o = \{260\ \bar{i} + 180\ \bar{j} + 510\ \bar{k}\}$ N·m, $\bar{M}_p = \{440\ \bar{i} + 220\ \bar{j} + 1200\ \bar{k}\}$ N·m

3·6 The force is making angle 30° with z axis, hence it should be first resolved along z axis and another component in x − y plane. Then the component in x − y plane is further to be resolved along y and x directions.

Coordinates of

A = (0, −0·5, 0), B = (0, 0, 0)
C = (0·75, 1, 0)

$x_{C/A} = x_C - x_A$
$F_z = 500 \cos 30° = 433$ N
$F_{xBy} = 500 \sin 30° = 250$ N
$F_y = -F_{xBy} \cos 50° = -160\cdot7$ N
$F_x = -F_{xBy} \sin 50° = -191\cdot51$ N

(i) $$\bar{M}_A = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 0\cdot75 & 1\cdot5 & 0 \\ -191\cdot51 & -160\cdot7 & 433 \end{vmatrix}$$

$\bar{M}_A = \{649\cdot5\ \bar{i} - 324\cdot75\ \bar{j} + 166\cdot74\ \bar{k}\}$ N·m

(ii) $$\bar{M}_B = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 0\cdot75 & 1 & 0 \\ -191\cdot51 & -160\cdot7 & 433 \end{vmatrix} = \{433\ \bar{i} - 324\cdot75\ \bar{j} + 70\cdot98\ \bar{k}\} \text{ N·m}$$

3·7 The coordinates of points are

a = (−6, −5, 8)
a′ = (6, 8, 4)
B = (5 −4, 0)

$$(\lambda_{aa'})_x = \frac{(x_{a'} - x_a)}{d_{aa'}} = \frac{6-(-6)}{\sqrt{12^2+13^2+4^2}} = 0\cdot66$$

$x_{B/a} = x_B - x_a = 5 - (-6) = 11$

$$M_{aa'} = \begin{vmatrix} 0\cdot66 & 0\cdot72 & -0\cdot22 \\ 11 & 1 & -8 \\ 0\cdot5 & -2 & 7 \end{vmatrix} = -59\cdot29 \text{ kNm.}$$

$\overline{M}_{aa}' = M_{aa}' \ \overline{\lambda}_{aa}'$

$\overline{M}_{aa}' = (-39{\cdot}13\ \overline{i} - 42{\cdot}69\ \overline{j} + 13{\cdot}10\ \overline{k})$ N·m

The manitude of moment (M_{aa}') can be expressed in Cartesian vector form since its direction is defined by $\lambda_{aa'}$.

3·8 The coordinates of points are

C = 0, 0, 2·25

D = 3·2, 0, 2·25

E = 0·9, 1·5, 0.

$$(F_{CE})_x = F_{CE}\frac{(x_E - x_C)}{d_{CE}} = 1349\frac{(0{\cdot}9)}{\sqrt{0{\cdot}9^2 + 1{\cdot}5^2 + 2{\cdot}25^2}} = 426 \text{ N}$$

$$(F_{CE})_y = 1349\frac{(1{\cdot}5)}{2{\cdot}85} = 710 \text{ N}$$

$$(F_{CE})_z = 1349\frac{(-2{\cdot}25)}{2{\cdot}85} = -1065 \text{ N}$$

Due to CE force, $M_x = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 2{\cdot}25 \\ 426 & 710 & -1065 \end{vmatrix} = -1597{\cdot}5$ N.m.

(i) Due to CE Force, $M_x = -1597{\cdot}5$ N·m

$M_y = 958{\cdot}5$ N·m and $M_z = 0$

$$(F_{DE})_x = 1349\frac{(0{\cdot}9 - 3{\cdot}2)}{\sqrt{2{\cdot}3^2 + 1{\cdot}5^2 + 2{\cdot}25^2}} = -874 \text{ N}$$

$$(F_{DE})_y = 1349\frac{1{\cdot}5}{3{\cdot}55} = 570 \text{ N}$$

$$(F_{DE})_z = 1349\frac{-2{\cdot}25}{3{\cdot}55} = -855 \text{ N}$$

Due to DE force, $M_x = \begin{vmatrix} 1 & 0 & 0 \\ 3{\cdot}2 & 0 & 2{\cdot}25 \\ -874 & 570 & -855 \end{vmatrix} = -1282{\cdot}5$ N.m.

(ii) Due to DE Force, $M_x = -1282{\cdot}5$ N·m

$M_y = 769{\cdot}5$ N·m

$M_z = 1824$ N·m

Note :	***For x axis,*** $\lambda_x = 1, \lambda_y = 0, \lambda_z = 0$
	For y axis, $\lambda_x = 0, \lambda_y = 1, \lambda_z = 0$
	For z axis, $\lambda_x = 0, \lambda_y = 0, \lambda_z = 1.$

3·9 The coordinates of point B are 1, 0·75, –0·5.

$M_x = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 0{\cdot}75 & -0{\cdot}5 \\ 4 & 12 & -3 \end{vmatrix} = 3{\cdot}75$ N.m.

Similarly determine moment about y and z axis.

$M_x = 3{\cdot}75$ N·m, $M_y = 1{\cdot}0$ N·m, $M_z = 9$ N·m

3·10 (a) Horizontal components = 100 sin 50° = 76·6 N

Vertical components = 100 cos 50° = 64·28 N

M due to horizontal forces = 76·6 × 0·3 = 22·98 N·m

M due to vertical forces = 64·28 × 0·5 = 32·14 N·m

Net moment = 9·16 N·m

(b) Perpendicular distance between BE and DF = $\frac{9{\cdot}16 \text{ N.m.}}{100 \text{ N.}}$

d = 0·0916 m.

3·11 Resultant force of two 250 N horizontal and 900N vertical force will be applied at C. To find the distance x, we have to apply at C equal and opposite resultant force of 900 N.

Total couple moments will be equated to zero.

250 N × 180 mm = 900 N × x mm

x = 50 mm

Force at C = 900 N· vertically downward.

x = 50 mm.

3·12 The inclination of force is not given, hence components can be determined from line of action (i.e. coordinates of point A and B)

$$F_{A(v)} = 1300\frac{(192 - 48)}{\sqrt{60^2 + (192 - 48)^2}} = 1200 \text{ N } (\downarrow)$$

$F_{A(v)} = 1200$ N (↓), $F_{A(H)} = 500$ N(←)

$M_c = 1200 \times 0{\cdot}06 - 500 \times 0{\cdot}096 = 24$ N·m

At C, single force = 1300 N ∠ 67·38°

and couple M = 24 N·m

OR apply equal and opposite force of 1300 N at C and determine couple moment by using perpendicular distance between AB and C.

3·13 There will not be effect of vertical components of 80 kN and horizontal components of 50 kN. as these components are passing through the axis and create compressions in the members of the frame.

The other components will only create moments.

$50 \cos 30° \times 3 = 80 \times \frac{4}{5} \times d \quad \therefore d = 2{\cdot}03$ m

3·14 Resultant force = $-140\ \overline{j} - 60 \sin 30°\ \overline{j} - 60 \cos 30°\ \overline{i}$

$\overline{R} = -51{\cdot}96\ \overline{i} - 170\ \overline{j}$

$R = \sqrt{51{\cdot}96^2 + 170^2} = 177{\cdot}76$ N ∠ 73°

Couple moment at O = 40 + 140 × 3 – 60 sin 30° × 8

= 220 N·m

Couple moment at P = 40 + 140 × 15 + 60 cos 30° × 8 + 60 sin 30° × 4

= 2675·7 N.m

Resultant force at O OR P = 177·76 N 73°

Couple moment at O = M_o = 220 N·m

Couple moment at P = M_p = 2675·7 N·m

3·15 Here, 45 N × 0·25 m = 11·25 N.m

is a couple moment which is a free veator and same can be replaced directly to any place say O without making any change.

F_R = (–180 $\bar{k}$) N.

This resultant force is acting at a point on x axis which is at x distance away from O.

$$x = \frac{180 \times 1{\cdot}25 - 45 \times 0{\cdot}25}{180} = 1{\cdot}188 \text{ m}$$

3·16 (a) Resultant force = 177·54 N 9·7°,

$R_x = -250 + 150 \cos 60°. = 175$ N (←),

$R_y = -100 + 150 \sin 60° = 29{\cdot}9$N (↑)

(b) If this resultant is acting on line BC at a distance y from B, then taking moment about B.

M_B, $100 \times 0{\cdot}3 + 8 - 250 \times 0{\cdot}2 = -175 \times y \quad \therefore \quad y = 68{\cdot}6$ mm

If resultant acts on line AB at a distance x from B, then taking moment about B.

M_B, $100 \times 0{\cdot}3 + 8 - 250 \times 0{\cdot}2 = -29{\cdot}9 \times x \quad \therefore \quad x = 40{\cdot}1$ mm

3·17 If resultant force is passing through origin (center of mat), then the moment of all forces about x and z axes must be balancing. The perpendicular distance of B, C, E and F from x axis is 2·6 m.

M_x, $(75 + F_F)\, 2{\cdot}6 = (50 + F_B)\, 2{\cdot}6 \qquad \therefore \; F_F = F_B - 25$

M_Z, $75\,(1{\cdot}5) + 50\,(1{\cdot}5) + 125\,(3) = F_B\,(1{\cdot}5) + F_F\,(1{\cdot}5) + 100\,(3)$

$\therefore \; F_B = 100$ kN, $F_F = 75$ kN.

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