4 EQUILIBRIUM OF RIGID BODIES

4·1 Conditions For Rigid Body Equilibrium :

In previous chapters, it was stated that *a particle is in equilibrium if it remains at rest or moves with constant velocity. For equilibrium of a particle, it is both necessary and sufficient to require the resultant force acting on the particle be equal to zero.*

Equation of Equilibrium of particle, $\overline{R} = \Sigma\overline{F} = 0$

Equilibrium requires both a ***balance of forces*** to prevent the body from ***translating*** with accelerated motion, and a ***balance of moments*** to prevent the body from ***rotating.***

Thus for equilibrium of rigid body, made up of number of particles, it is necessary that all the particles should **not have *translation*** as well as ***rotation.***

The necessary and sufficient conditions for the equilibrium of a rigid body are

$$\Sigma\overline{F} = 0 \qquad \text{and} \qquad \Sigma\overline{M} = \Sigma(\overline{r} \times \overline{F}) = 0$$

Equations of equilibrium of rigid body, · $\overline{F}=0$ and $\Sigma\overline{M}=0$

In **rectangular components,** following ***six scalar equations*** are needed which are *sufficient* for ***equilibrium.***

$$\Sigma F_x = 0 \qquad \Sigma M_x = 0$$
$$\Sigma F_y = 0 \qquad \text{and} \qquad \Sigma M_y = 0$$
$$\Sigma F_z = 0 \qquad \Sigma M_z = 0$$

4·2 Free-Body Diagram :

To construct a free-body diagram for *a rigid body* or *group of bodies considered as a single system*, the following steps should be performed.

(1) Step 1 : Imagine the body to be *isolated* or cut "free" from its constraints and connections, and draw (sketch) its outlined shape.

(2) Step 2 : Identify all the external forces and couples that act on the body. Forces and couples generally encountered are those due to :

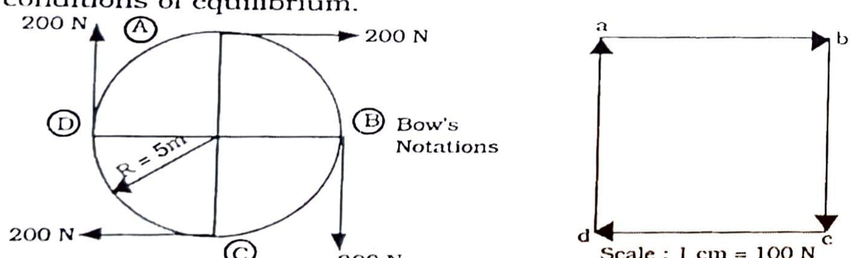
(i) *applied* **loadings,**

(ii) ***reactions*** occuring at the supports or at points of contact with other bodies, and

(iii) the ***weight*** of the body.

(3) Step 3 : Indicate the *dimensions* of the body necessary for computing the moments of forces. *Label* the known and unknown forces and couples.

In problem concerning the equilibrium of a rigid body, it is essential to consider *all* the forces and moments acting on the body. Omitting a force or additing an extraneous one would destroy the conditions of equilibrium.



(a) Space diagram **(b) Force diagram**

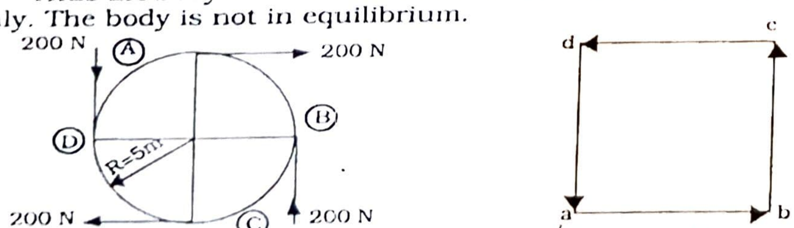
Fig. 4·1

In the above figure, the Resultant Force = 0

but the Resultant Moment = 4 (200 × 5)

= 4000 N·m clockwise

Thus the body will have **no translation,** but it will **have rotation** only. The body is not in equilibrium.



(a) Space diagram **(b) Force diagram**

Fig. 4·2

Here, Resultant Force = 0
Resultant Moment = 0

Therefore, the body has **No translation** and **No rotation.**

The body is in **equilibrium.**

4·3 Reactions at Supports and Connections :

If the support prevents translation in a given direction, then a force is developed on the member in that direction; called ***reaction.*** *Likewise, if rotation is prevented, a* ***couple moment*** *is exerted on the member.* The magnitude of force or couple represents ***unknowns.***

Reactions can be divided in to three groups.

Reactions

equivalent to

a force with known line of action	**a force of unknown direction**	**a force and couple (moment)**
One Unknown	***Two*** Unknowns	***Three*** Unknowns
(i) Rollers (ii) Rockers (iii) Frictionless Surfaces (iv) Short links & Cables (v) Collars on frictionless rods (vi) Frictionless Pins in Slots	(i) Hinges (ii) Rough Surfaces (iii) Frictionless Pins in fitted holes	(i) Fixed Supports (ii) Three force components in Ball & Socket.

Examples :

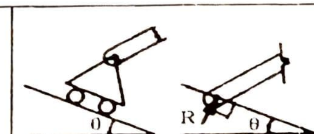
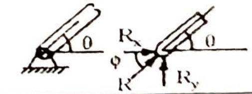
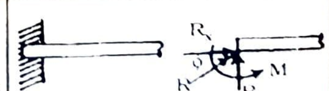
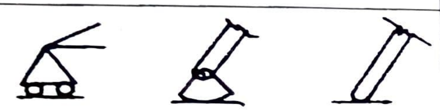
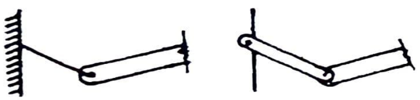
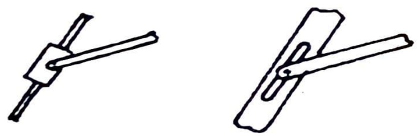
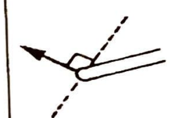
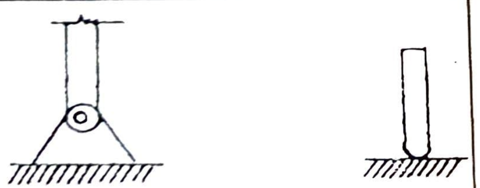
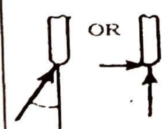
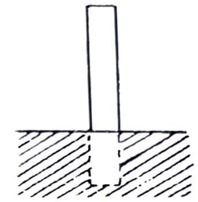
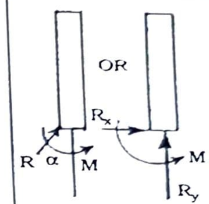
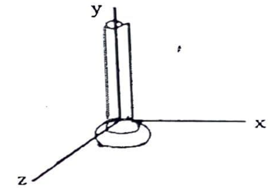
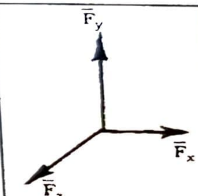
(1) Roller Support (One Unknown)		Reaction is a force which acts perpendicular to the surface on which roller moves. (If surface is horizontal, reaction is vertical).
(2) Hinge Support (Two Unknowns)		Reactions are two components of force (R). Note that ϕ and θ are not necessarily equal.
(3) Fixed Support (Three Unknowns)		The reactions are the couple (moment) and the two force components.

Table showing Reactions at Supports – Connections and Unknowns

Support OR Connection	Reaction	Number of Unknowns
 Rollers, Rocker, Leader Frictionless Surface	 Force with known line of action	1
 Short cable, Short link	 Force with known line of action	1
 Collar on frictionless rod, Frictionless pin in shot	 Force with known line of action	1
 Frictionless pin or Hinge, Rough surface	 Force of unknown direction	2

 Fixed support	 Force and Couple	3
 Ball and Socket	 Three components	3

4.4 Equilibrium of a Rigid Body in Two Dimensions :

In more general form, the equations of equilibrium for a two dimensional structure are

$$\boxed{\Sigma F_x = 0, \quad \Sigma F_y = 0, \quad \Sigma M = 0}$$

ΣM is the sum of the moments about *any point* in the plane of the structure.

(1) Equilibrium of Two – Force Body :

When body is subjected to two forces, it is commonly called a *two – force body*. The Forces should be acting at *only two points*.

*If a two – force body is in **equilibrium**, the two forces must have the same magnitude, same line of action and opposite sense.*

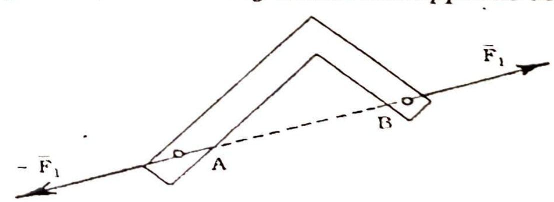


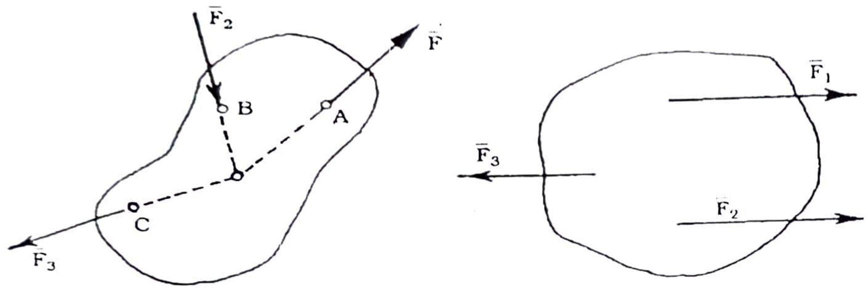
Fig. 4·3

Here, the Resultant Force = 0
and the Resultant Moment = 0

(2) Equilibrium of Three – Force Body :

A rigid body, subjected to *three forces* or a rigid body subjected to forces acting at only *three points*, is called three– force body.

If this body is in equilibrium, the lines of action of the three forces must be either concurrent or parallel.



Concurrent forces, $\Sigma F = 0,\ \Sigma M = 0$ **(a)**

Parallel forces, $\bar{F}_1 + \bar{F}_2 = \bar{F}_3,\ \Sigma M = 0$ **(b)**

Fig. 4·4

4·5 Equilibrium of a Rigid Body in Three Dimensions:

Six scalar equations are required to express the conditions of equilibrium of a rigid body in three dimensional case.

$\Sigma F_x = 0$ and $\Sigma M_x = 0$

$\Sigma F_y = 0$ and $\Sigma M_y = 0$

$\Sigma F_z = 0$ and $\Sigma M_z = 0$

In vector form,

$\Sigma \bar{F} = 0$ and $\Sigma M = \Sigma(\bar{r} \times \bar{F}) = 0$

IMPORTANT EQUATIONS

(1) Equilibrium Equations :

$R = \Sigma F = 0$ and $\Sigma M = \Sigma(\bar{r} \times \bar{F}) = 0$

(2) Six Scalar Equations of Equilibrium (in rectangular components) :

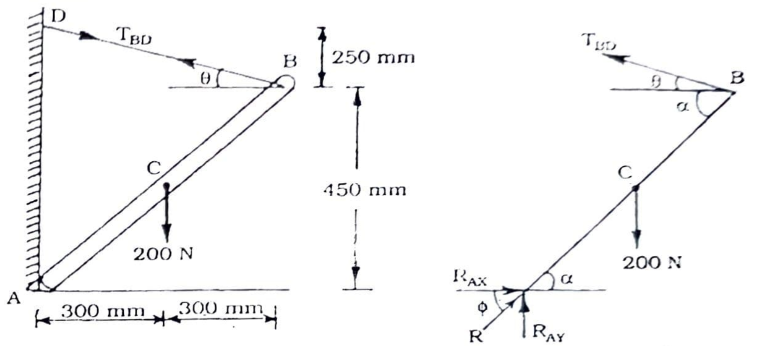
$\Sigma F_x = 0$ and $\Sigma M_x = 0$

$\Sigma F_y = 0$ and $\Sigma M_y = 0$

$\Sigma F_z = 0$ and $\Sigma M_z = 0$

SOLVED EXAMPLES

1. One end of rod AB rests in the corner A and the other is attached to cord BD. If the rod supports 200–N load at its midpoint C, find the reaction at A and the tension in the cord.



Space Diagram (a) **Fig. 4·5** **Free – body Diagram (b)**

A free–body diagram for this structure is shown in fig. 4.5 (b)

Here the corner A will become the hinge support at A for rod – AB as the end A of the rod AB is placed in the corner and there will be two perpendicular reactions at two walls. The rod AB can be rotated about support A.

Now, $\theta = \tan^{-1}\dfrac{0{\cdot}25}{0{\cdot}6}$ $\therefore$ $\theta = 22{\cdot}62°$

If rod AB is in equilibrium, then the moment about support A must be zero.

Taking moment about A,

$$M_A = 0 = 200 \times 0{\cdot}3 - T_{BD}\cos\theta \times 0{\cdot}45 - T_{BD}\sin\theta \times 0{\cdot}6$$
$$= 60 - T_{BD} \times 0{\cdot}92 \times 0{\cdot}45 - T_{BD} \times 0{\cdot}38 \times 0{\cdot}6$$

$$\boxed{\therefore T_{BD} = 93{\cdot}46\ N}$$

Reactions at A :

$\Sigma F_x = 0,\ T_{BD}\cos\theta = R_{AX}$

$\therefore R_{AX} = 86{\cdot}27\ N$

$\Sigma F_y = 0,\ R_{AY} = 200 - T_{BD}\sin\theta$

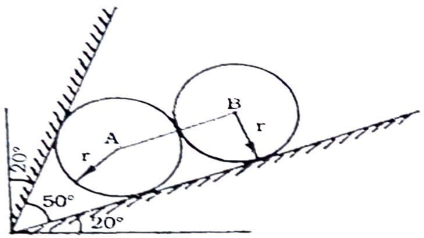
$\therefore R_{AY} = 164{\cdot}05\ N$

$$R = \sqrt{R_{AX}^2 + R_{AY}^2}$$
$$= \sqrt{164{\cdot}05^2 + 86{\cdot}27^2} = 185{\cdot}35\ N.$$

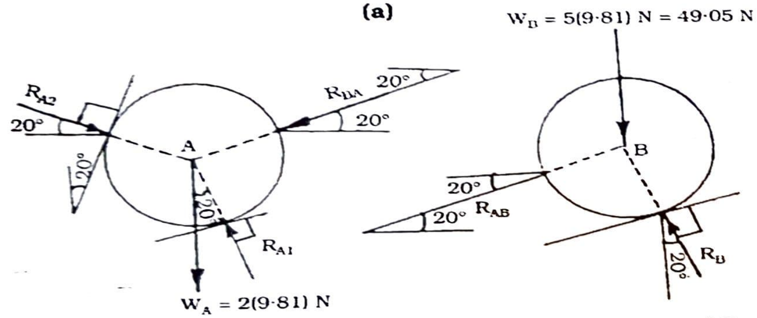
$$\phi = \tan^{-1}\frac{164{\cdot}05}{86{\cdot}27} = 62{\cdot}26°$$

$$\boxed{R_A = 185{\cdot}35\ N \angle 62{\cdot}26°}$$

2. Two smooth balls A and B, having a respective mass of 2 kg and 5 kg rest between the inclined plane. Determine the reactions of planes on balls. The radius of both balls is same.



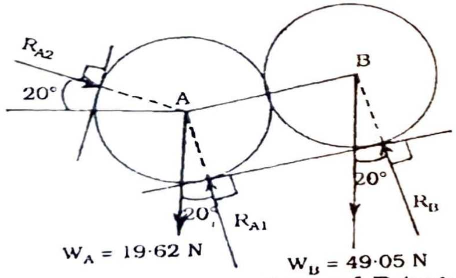
(a)



Free-body diagram of A (b)

Free-body diagram of B (c)

Fig 4.6



Free body diagram of A and B together

(d)

Fig. 4·6

Method 1 (Separate equilibrium) : Free-body diagrams of balls A and B should be drawn separately for considering the equilibrium of A and B separately. The **reactions** of planes **will act perpendicular to planes.** The balls A and B are in contact hence there will be a contact force equal in magnitude but opposite in sense for considering the equilibrium separately ($\mathbf{R_{BA} = \bar{R}_{AB}}$).

Considering first the **equilibrium of B** as it has two unknowns only.

$\Sigma F_x = 0$, $R_{AB} \cos 20° = R_B \sin 20°$

$\therefore R_{AB} = 0{\cdot}36\, R_B$ ... (i)

$\Sigma F_y = 0$, $49{\cdot}05 = R_B \cos 20° + R_{AB} \sin 20°$

Putting the value of R_{AB} from (i)

$49{\cdot}05 = 0{\cdot}94\, R_B + 0{\cdot}12\, R_B$

$\therefore \boxed{R_B = 46{\cdot}27 \text{ N}}$

Now considering the **equilibrium of A**

$\Sigma F_x = 0$, $R_{A2} \cos 20° = R_{A1} \sin 20° + R_{BA} \cos 20°$

but $R_{BA} = 0{\cdot}36 \times 46{\cdot}27 = 16{\cdot}66$ N ... (ii)

$\therefore R_{A2} = 0{\cdot}36\, R_{A1} + 16{\cdot}66$ ... (iii)

$\Sigma F_y = 0$, $19{\cdot}62 + R_{A2} \sin 20° + R_{BA} \sin 20° = R_{A1} \cos 20°$

Putting the value of R_{BA} from (ii)

$R_{A2} = 2{\cdot}75\, R_{A1} - 74{\cdot}03$ ... (iv)

From eq. (iii) & (iv)

$$\boxed{\begin{aligned} R_{A1} &= 37{\cdot}94 \text{ N} \\ R_{A2} &= 30{\cdot}32 \text{ N} \end{aligned}}$$

Method–2 (Combined equilibrium) : Using the three equations of equilibrium, the three unknowns shown in the free body–diagram of A and B balls together also can be obtained (Fig. 4.6.(d)).

$\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M_{\text{any point}} = 0$

$\Sigma F_y = 0$, $19{\cdot}62 + 49{\cdot}05 + R_{A2} \sin 20° = R_{A1} \cos 20° + R_B \cos 20°$

$R_{A1} + R_B - R_{A2}(0{\cdot}36) = 73.08$ ...(1)

$\Sigma F_x = 0$, $R_{A2} \cos 20° = R_{A1} \sin 20° + R_B \sin 20°$

$R_{A1} + R_B - R_{A2}\,(2{\cdot}75) = 0$ ...(2)

$\Sigma M_A = 0$, $R_B(2r) = 49{\cdot}05 \cos 20°\ (2r)$ $\therefore \boxed{R_B = 46{\cdot}09 \text{ N}}$

While taking moment about A, only R_B and W_B will create moments as other forces passing through A.

$\boxed{R_{A2} = 30{\cdot}58 \text{ N and } R_{A1} = 38{\cdot}0 \text{ N}}$

3. A string ABCD attached to two fixed points A and D is carrying two equal weights of 800 N as shown. Determine tensions is AB, BC and CD of the string.

7

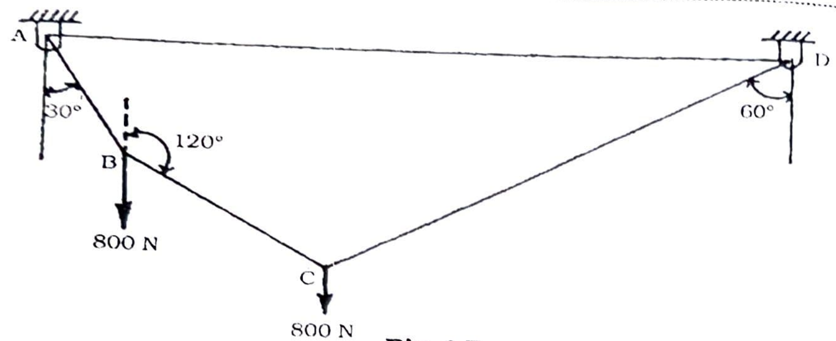
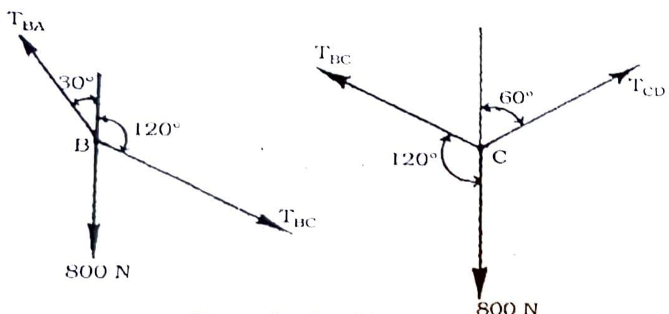


Fig 4.7

Here, we can say that the system of forces acting at joints B and C are in equilibrium, which can be shown independently in free–body diagrams.



Free–body diagrams

(a) Joint B **Fig 4·8** **(b) Joint C**

Joint B : Using Lami's theorem (sine rule).

$$\frac{T_{BA}}{\sin 60^\circ} = \frac{800}{\sin 150^\circ} = \frac{T_{BC}}{\sin 150^\circ}$$

$\therefore$ $\boxed{T_{BA} = 1385{\cdot}6 \text{ N}}$ and $\boxed{T_{BC} = 800 \text{ N}}$

Joint C : Using Lami's theorem.

$$\frac{T_{BC}}{\sin 120^\circ} = \frac{800}{\sin 120^\circ} = \frac{T_{CD}}{\sin 120^\circ}$$

$\therefore$ $\boxed{T_{BC} = 800 \text{ N}}$ and $\boxed{T_{CD} = 800 \text{ N}}$

4. The lever AB is hinged at C and attached to a control cable AD at A. If the lever is subjected at B to a 500–N horizontal force, determine **(a)** the tension in the cable, **(b)** the reaction at C.

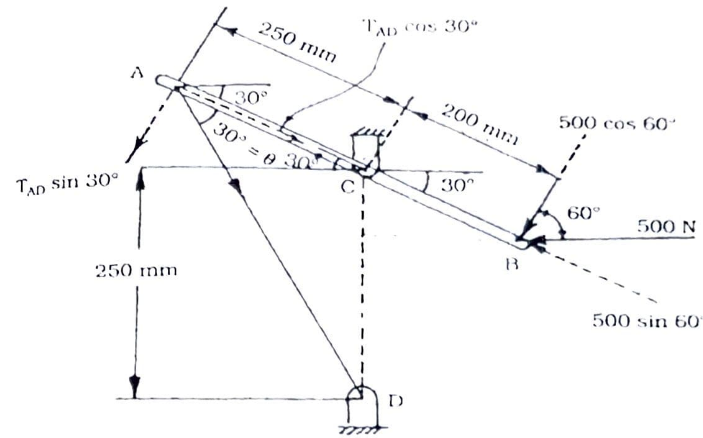


Fig 4·9 (a)

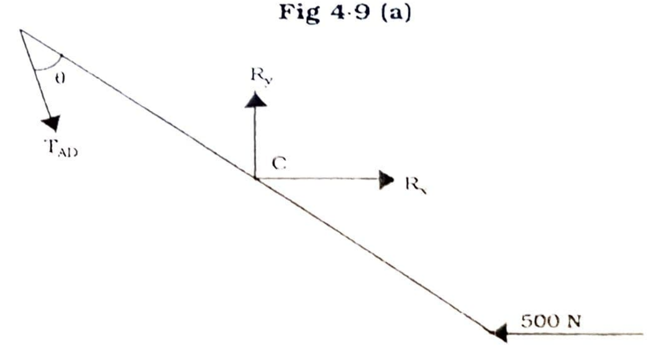


Fig 4·9 (b)

At Hinge Support C the unknown reaction components are two and moment at hinge is always zero.

The lever AB is acted upon by two forces, 500 N at B and T_{AD} at A.

The perpendicular components of these forces to the lever will create moment. The components of forces along the lever will pass through C and hence will not create moment.

Let angle between lever AB and cable AD is θ, which is not given.

Horizontal component of AC $= 250 \sin 60^\circ$

$= 216{\cdot}5$ mm

Vertical component of AC $= 250 \cos 60^\circ$

$= 125$ mm

$$\therefore \theta = \left(\tan^{-1} \frac{250 + 125}{216.5}\right) - 30^\circ$$

$= 30^\circ$

(a) Now, moment at C, $\mathbf{M_C = 0}$

$\therefore T_{AD} \sin \theta \times 250 = 500 \cos 60^\circ \times 200$

$\therefore T_{AD} \sin 30^\circ \times 250 = 500 \cos 60^\circ \times 200$

$\therefore \boxed{T_{AD} = 400 \text{ N}}$

(b) At hinge support C, the two reaction components acting, are acting, say R_x and R_y. Lever AB is in equilibrium under the action of R_x, R_y, T_{AD} and 500 N.

$\mathbf{\Sigma F_x = 0}$, $T_{AD} \cos (\theta + 30^\circ)\ \bar{i} + R_x\ \bar{i} - 500\ \bar{i} = 0$

$400 \cos 60^\circ\ \bar{i} + R_x\ \bar{i} - 500\ \bar{i} = 0$

$\therefore R_x\ \bar{i} = 300\ \bar{i} \therefore R_x = 300$ N $(\rightarrow)$

$\mathbf{\Sigma F_y = 0}$, $-T_{AD} \sin (\theta + 30^\circ)\ \bar{j} + R_y\ \bar{j} = 0$

$\therefore -400 \sin 60^\circ\ \bar{j} + R_y\ \bar{j} = 0$

$\therefore R_y\ \bar{j} = 346.4\ \bar{j}$ $\therefore R_y = 346.4$ N $(\uparrow)$

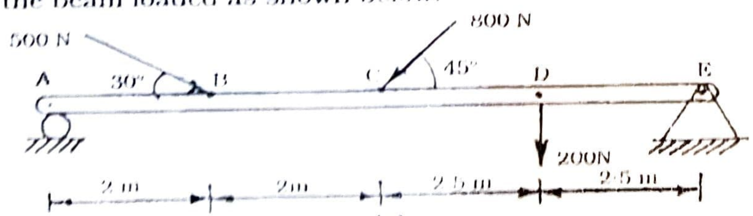
Hence, $\boxed{R = 300\ \bar{i} + 346.4\ \bar{j}}$

and $R = \sqrt{300^2 + 346.4^2} = 458.25$ N

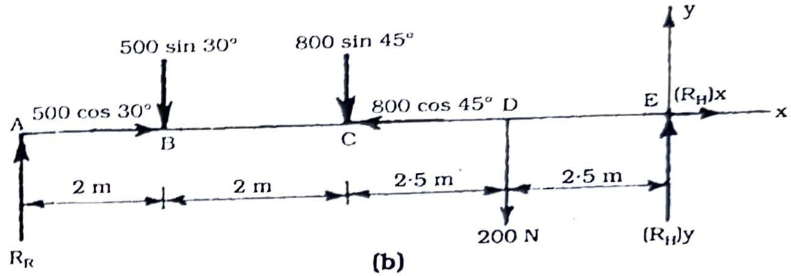
$\boxed{R = 458.25 \text{ N}}$

and $\alpha = \tan^{-1} \frac{R_y}{R_x} = \tan^{-1} \frac{346.4}{300}$ $\therefore \boxed{\alpha = 49.1^\circ}$

5. Determine the horizontal and vertical components of reaction for the beam loaded as shown below.



(a)
Fig 4.10



(b)
Fig 4.10

The free–body diagram is shown in above figure (b)

Reactions : Roller support **A – One** only – R_R (↑)

Hinge support **E – Two** – $(R_H)_x$ and $(R_H)_y$

At both roller and hinge support, the sum of moments will be zero.

Moment at Hinge support = $M_E = 0$

Taking moment about E,

$200 \times 2.5 + 800 \sin 45^\circ \times 5 + 500 \sin 30^\circ \times 7 - R_R \times 9 = 0$

$\therefore \boxed{R_R = 564.3 \text{ N } (\uparrow)}$

Taking moment about A,

$(R_H)_y \times 9 - 500 \sin 30^\circ \times 2 - 800 \sin 45^\circ \times 4 - 200 \times 6.5 = 0$

$\boxed{(R_H)_y = 451.4 \text{ N } (\uparrow)}$

Now,

$\mathbf{\Sigma F_x = 0}$, $500 \cos 30^\circ - 800 \cos 45^\circ + (R_H)_x = 0$

$(R_H)_x = -132.7$ N.

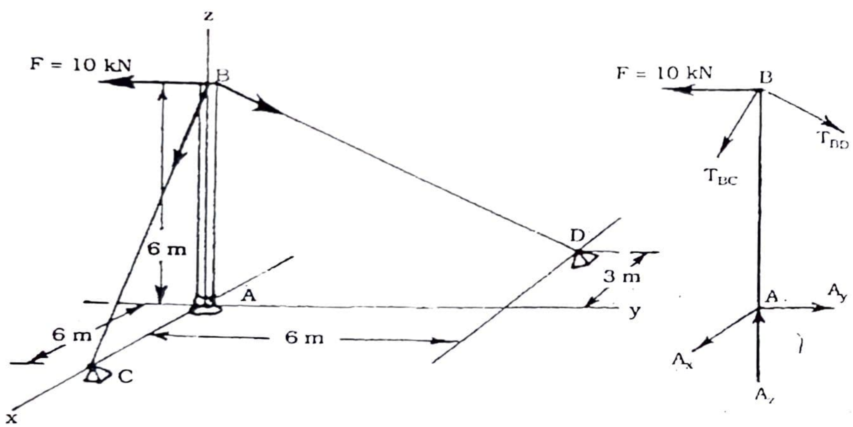
$\therefore \boxed{(R_H)_x = 132.7 \text{ N } (\leftarrow)}$

$\mathbf{\Sigma F_y = 0}$, $R_R - 500 \sin 30^\circ - 800 \sin 45^\circ - 200 + (R_H)_y = 0$

$564.3 - 250 - 565.7 - 200 + (R_H)_y = 0$

$\therefore \boxed{(R_H)_y = 451.4 \text{ N } (\uparrow)}$

6. Determine the tension in cables BC and BD and the reactions at the ball – and – socket joint A for the mast shown in figure given below.



(a) Space Diagram **Fig 4·11** **(b) Free–body Diagram**

At ball–and–socket joint, there will be only three reaction components.

The coordinates of points are as under :

C = 6, 0, 0

D = –3, 6, 0

B = 0, 0, 6

$(BC)_x = \dfrac{X_C - X_B}{\sqrt{36+36}}$, Similarly after determining $(BC)_y$ and $(BC)_z$

$\overline{BC} = 0.707\,\bar{i} - 0.707\,\bar{k}$

Similarly,

$(BD)_x = \dfrac{X_D - X_B}{\sqrt{9+36+36}}$, $BD = -0.333\,\bar{i} + 0.667\,\bar{j} - 0.667\,\bar{k}$

Here, five unknowns are to be determined, three reaction components and tensions in cables BD and BC.

Forces are, $\bar{F} = (-10\,\bar{j})$ kN

$F_A = A_x\,\bar{i} + A_y\,\bar{j} + A_z\,\bar{k}$

$T_{BC} = 0.707\,T_{BC}\,\bar{i} - 0.707\,T_{BC}\,\bar{k}$

$T_{BD} = T_{BD}\left(\dfrac{r_{BD}}{r_{BD}}\right)$

$= -0.333\,T_{BD}\,\bar{i} + 0.667\,T_{BD}\,\bar{j} - 0.667\,T_{BD}\,\bar{k}$

Equilibrium equations for the whole body give

$\Sigma F = 0,\quad \bar{F} + \bar{F}_A + \bar{T}_{BC} + \bar{T}_{BD} = 0$

$\Sigma F_x = 0;\ A_x + 0.707\,T_{BC} - 0.333\,T_{BD} = 0$ ...(1)

$\Sigma F_y = 0;\ A_y + 0.667\,T_{BD} - 10 = 0$ ...(2)

$\Sigma F_z = 0;\ A_z - 0.707\,T_{BC} - 0.667\,T_{BD} = 0$ ...(3)

Summing moment about A.

$\Sigma M_A = 0;\ \bar{r}_B \times (\bar{F} + \bar{T}_{BC} + \bar{T}_{BD}) = 0$

$6k \times (-10\,\bar{j} + 0.707\,T_{BC}\,\bar{i} - 0.707\,T_{BC}\,\bar{k}$

$-0.333\,T_{BD}\,\bar{i} + 0.667\,T_{BD}\,\bar{j} - 0.667\,T_{BD}\,\bar{k}) = 0$

Evaluating cross product and combining terms

$(-4\,T_{BD} + 60)\,\bar{i} + (4.24\,T_{BC} - 2T_{BD})\,\bar{j} = 0$

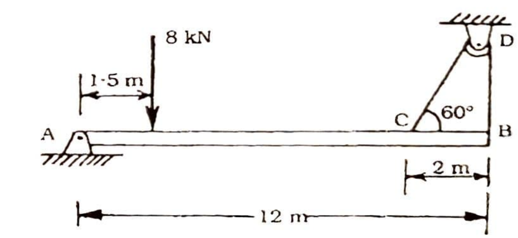
$\Sigma M_x = 0,\ -4\,T_{BD} + 60 = 0$ ... (4)

$\Sigma M_y = 0,\ 4.24\,T_{BC} - 2T_{BD} = 0$ ... (5)

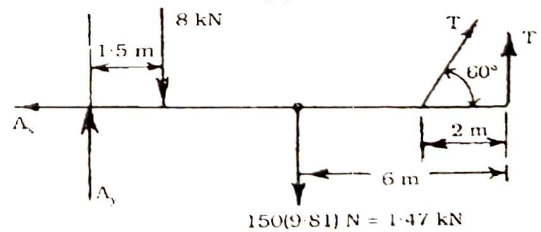
$\therefore\ \boxed{T_{BC} = 7.07 \text{ kN.}}$ $\boxed{T_{BD} = 15 \text{ kN}}$

$\boxed{A_x = 0.0 \text{ kN},\quad A_y = 0.0 \text{ kN},\quad A_z = 15 \text{ kN}}$

7. The 150 kg uniform beam AB is supported at A by pin, and at B and C by a continuous cable which wraps around a frictionless pulley located at D. Determine reactions at A and tension in cable.



(a)



(b) Free–body Diagram
Fig. 4·12

The tension in a continuous cable will remain unchanged if it is passing over a frictionless pulley.

Here, $T_{CD} = T_{BD}$. Say equal to T

The free body diagram is shown in Fig 4.12 (b).

Moment about A : $\Sigma M_A = 0$ (Anti-clockwise positive)

$$-(8\text{ kN})(1.5\text{ m}) - (1.47\text{ kN})(6\text{m}) + (T \sin 60°\text{ kN})(10\text{ m}) + T\text{ kN}(12\text{ m}) = 0$$

$$-20.82 + 20.66\,T = 0$$

$$\boxed{\therefore T = 1.008\text{ kN}}$$

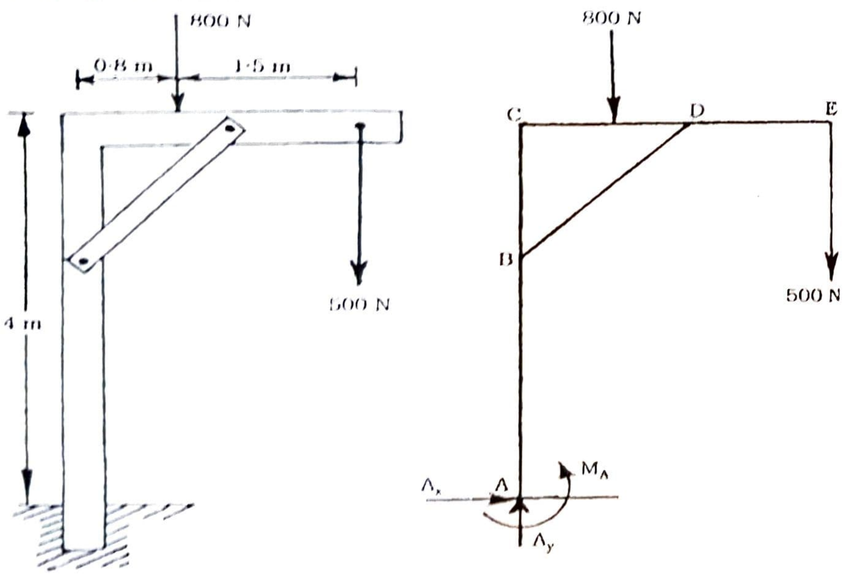
$\Sigma F_x = 0$; $-A_x + 1.008 \cos 60° = 0$

$$\boxed{\therefore A_x = 0.504\text{ kN}\leftarrow}$$

$\Sigma F_y = 0$; $A_y - 8\text{ kN} - 1.47\text{ kN} + 1.008 \sin 60°\text{ kN} + 1.008\text{ kN} = 0$

$$\boxed{\therefore A_y = 7.59\text{ kN}\uparrow}$$

8. Determine the reactions at the fixed support A for the loaded frame shown here.



Space diagram **(a)** **Fig. 4.13** **Free-body diagram** **(b)**

The support A is fixed. There are three unknowns at this fixed support, two reaction components and one fixed end moment. Here it should be noted that the moment about fixed support can not be zero without considering fixed end moment. The frame is under *equilibrium* under the *external forces* as shown in freebody diagram. There is no need to consider *internal forces* at pin joints B, C and D.

$\Sigma F_x = 0$; $\boxed{A_x = 0}$

$\Sigma F_y = 0$; $A_y - 800\text{N} - 500\text{ N} = 0$

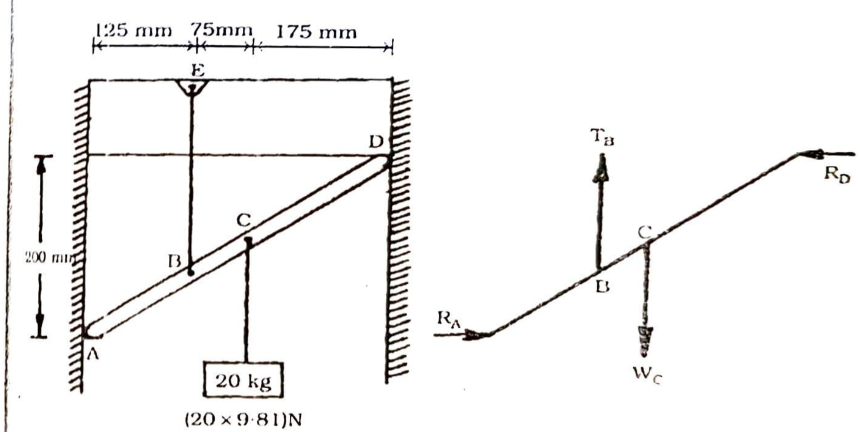
$$\boxed{A_y = 1300\text{ N}\,(\uparrow)}$$

With considering fixed end moment at fixed support, we can equate moment about fixed end support to zero.

$\Sigma M_A = 0$; $M_A - 800\text{ N}\,(0.8\text{m}) - 500\text{ N}\,(2.3\text{m}) = 0$

$$\boxed{M_A = 1790\text{ N.m, Counter clock-wise}}$$

9. A light bar AD is suspended from a cable BE and supports a 20 kg block at C. The extremities A and D of the bar are in contact with frictionless vertical walls. Determine the tension in the cable BE and the reactions at A and D.



(a) **Fig. 4.14** **(b)**

At frictionless surfaces, ends A and D will be having reactions R_A and R_D as shown in freebody diagram. ***The reaction on the rod or bar supported at frictionless surface will be perpendicular to the surface on which it rests.***

$\Sigma F_x = 0;\ R_A = R_D$
$\Sigma F_y = 0;\ T_B = W_C$

$\therefore\ T_B = 20\ (9.81)\ N$

$$\boxed{T_B = 196.2\ N}$$

As the rod AD is in equilibrium, the moment acting on the rod should also be zero. T_B and W_C will develop a couple which will be equal to a couple formed by R_A and R_D

$$R_A \times 0.2 = T_B \times 0.075$$

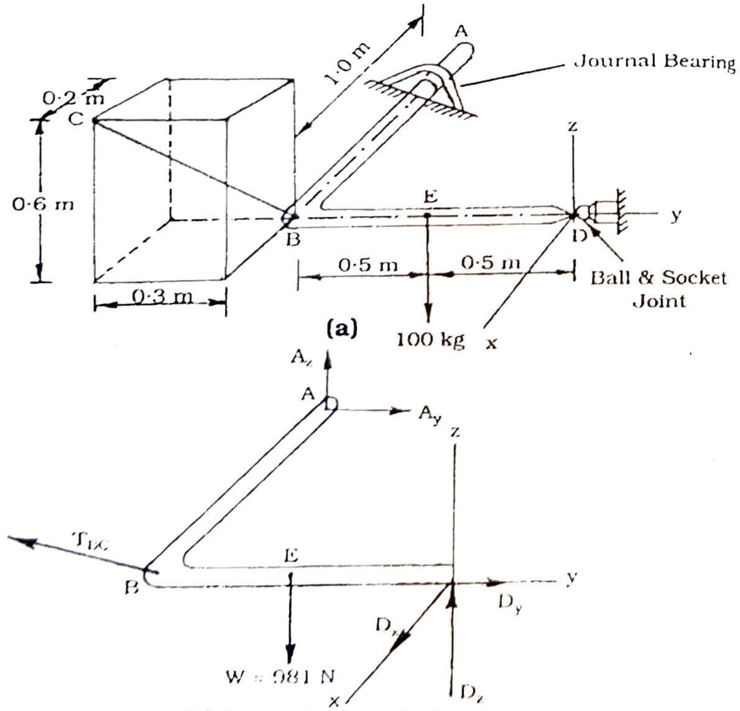
$$\therefore\ \boxed{R_D = R_A = 73.57\ N}$$

OR, Taking moment about A *(moment about frictionless simple supports will be zero).*

$$\Sigma M_A = 0;\ R_D \times 0.2 + T_B \times 0.125 - W_C \times 0.2 = 0$$

$$\therefore\ \boxed{R_D = 73.57\ N}$$

10. A bent rod is supported at A by a journal bearing, at D by a ball– and – socket joint, and at B by means of cable BC. Determine tension in cable BC.



(a)

(b) Free–body Diagram

Fig. 4.15

Reaction components at **Journal Bearing** and at **Ball - and Socket Joint** are **two** and **three** respectively.

Coordinates of Points C = (0.2, −1.3, 0.6)
A = (−1.0, −1.0, 0)
E = (0, −0.5, 0)
D = (0, 0, 0)
and B = (0, −1.0, 0)

The cable tension T_{BC} can be obtained directly by *summing moments about axis passing through points D and A* **(moment about axis DA)** thus *reaction components at A and D will be avoided in calculation.*

$$\bar{\lambda}_{DA} = \frac{\bar{r}_{DA}}{r_{DA}} = -\frac{1}{\sqrt{2}}\bar{i} - \frac{1}{\sqrt{2}}\bar{j}$$

$$= -0.707\ \bar{i} - 0.707\ \bar{j}$$

Method 1 :

Now,

$$\Sigma M_{DA} = \bar{\lambda}_{DA} \cdot \Sigma(\bar{r} \times \bar{F}) = 0$$

$$\therefore\ \bar{\lambda}_{DA} \cdot (\bar{r}_B \times \bar{T}_{BC} + \bar{r}_E \times \bar{W}) = 0$$

$$(-0.707\ \bar{i} - 0.707\ \bar{j}) \cdot [-1\ \bar{j}) \times \left(\frac{0.2}{0.7} T_{BC}\ \bar{i} - \frac{0.3}{0.7} T_{BC}\ \bar{j} + \frac{0.6}{0.7} T_{BC}\ \bar{k}\right) + (-0.5\ \bar{j}) \times (-981\ \bar{k})\ ] = 0$$

$$(-0.707\ \bar{i} - 0.707\ \bar{j}) \cdot [(-0.857\ T_{BC} + 490.5)\ \bar{i} + 0.286\ T_{BC}\ \bar{k}] = 0$$

$$-0.707\ (-0.857\ T_{BC} + 490.5) + 0 + 0 = 0$$

$$\boxed{\therefore\ T_{BC} = 572\ N}$$

Method 2 :

M_{DA}=0=moment of F_{BC} about axis DA+moment of W about axis DA

$$0 = \begin{vmatrix} (\lambda_{DA})_x & (\lambda_{DA})_y & (\lambda_{DA})_z \\ x_B & y_B & z_B \\ (F_{BC})_x & (F_{BC})_y & (F_{BC})_z \end{vmatrix} + \begin{vmatrix} (\lambda_{DA})_x & (\lambda_{DA})_y & (\lambda_{DA})_z \\ x_E & y_E & z_E \\ 0 & 0 & -W \end{vmatrix}$$

$$0 = \begin{vmatrix} -0.707 & -0.707 & 0 \\ 0 & -1.0 & 0 \\ \frac{0.2}{0.7}T_{BC} & \frac{-0.3}{0.7}T_{BC} & \frac{0.6}{0.7}T_{BC} \end{vmatrix} + \begin{vmatrix} -0.707 & -0.707 & 0 \\ 0 & -0.5 & 0 \\ 0 & 0 & -981 \end{vmatrix}$$

$$0 = -0.707\left(-\frac{0.6}{0.7}T_{BC}\right) + (-0.707)(-0.5 \times -981)$$

$$0 = 0.606\,T_{BC} - 346.78$$

$$\therefore \boxed{T_{BC} = 572.25 \text{ N}}$$

11. The 35 kg rectangular plate shown is supported by three parallel wires. Determine the tension in each wire.

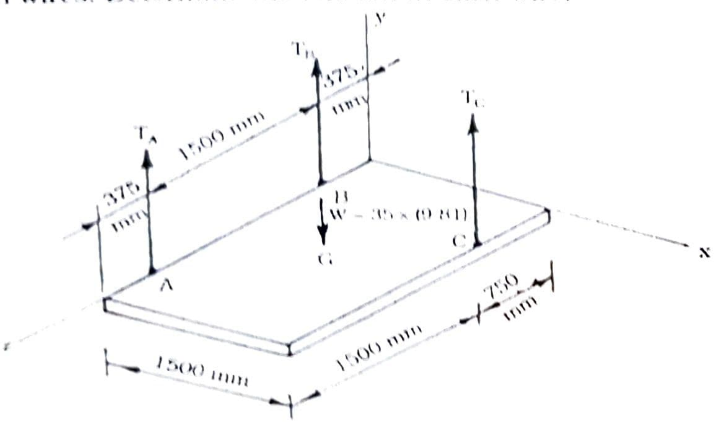


Fig 4.16

Weight of plate = 35 × 9.81 = 343.35 N which acts at the centre of plate. Three unknowns (T_A, T_B, T_C) can be determined by three equilibrium equations; $\Sigma M_z = 0$, $\Sigma M_x = 0$ and $\Sigma F_y = 0$.

Taking moment about z axis.

$\boldsymbol{\Sigma M_z = 0.}$ $T_C \times 1.5 \text{ m} - W \times 0.75 \text{ m} = 0$

$$T_C \times 1.5 - 343.35 \times 0.75 = 0$$

$$\therefore \boxed{T_C = 171.68 \text{ N}}$$

Taking moment about x axis.

$\boldsymbol{\Sigma M_x = 0.}$ $T_B \times 0.375 + T_A \times 1.875 + T_C \times 0.75 - W \times 1.125 = 0$

$\therefore T_B \times 0.375 + T_A \times 1.875 + 171.68 \times 0.75 - 343.35 \times 1.125 = 0.$

$\therefore T_B \times 0.375 + T_A \times 1.875 = 257.51$ ... (i)

$\boldsymbol{\Sigma F_y = 0.}$ But $T_A + T_B + T_C = W$

$$T_A + T_B = 171.67 \text{ N}$$

$\therefore T_A = 171.67 - T_B$ ...... (ii)

Putting the value of T_A from (ii) in to (i)

$T_B \times 0.375 + (171.67 - T_B) \times 1.875 = 257.51$

$-1.5\,T_B = -64.37$

$$\boxed{\therefore T_B = 42.91 \text{ N}}$$

and hence $\boxed{T_A = 128.76 \text{ N}}$

12. The 650 × 700 mm plate ABCD is supported by hinges along edge AB and by wire CE. Knowing that the 40 kg plate is uniform, determine the tension in the wire.

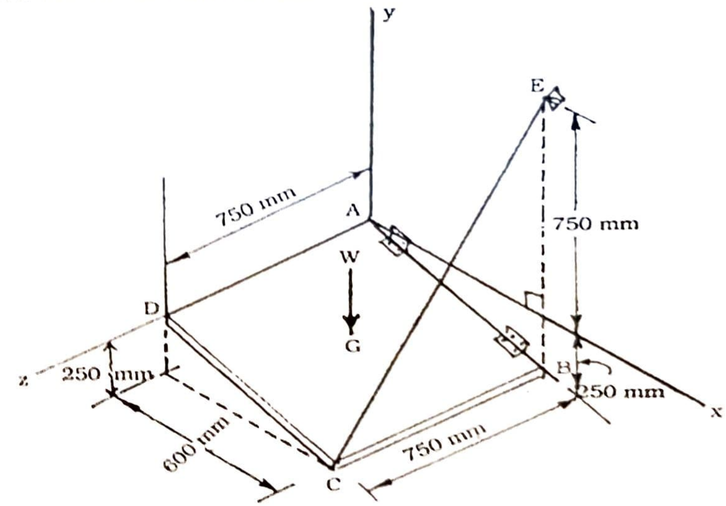


Fig 4.17

The coordinates of points are

A (0, 0, 0)
B (600, − 250, 0)
C (600, − 250, 750)
D (0, 0, 750)
E (600, 750, 0)
and G (300, − 125, 375)

Taking moment about axis AB which is hinged.

So $M_{AB} = 0$, **Moment of force CE about AB axis + Moment of weight W about AB axis** = 0

$$\begin{vmatrix} \lambda_x & \lambda_y & \lambda_z \\ x_c & y_c & z_c \\ F_x & F_y & F_z \end{vmatrix} + \begin{vmatrix} \lambda_x & \lambda_y & \lambda_z \\ x_G & y_G & z_G \\ W_x & W_y & W_z \end{vmatrix} = 0$$

Here, components of axis AB are

$$\lambda_x = \frac{x_B - x_A}{\sqrt{600^2 + 250^2}} = \frac{600}{650} = 0{\cdot}92$$

$$\lambda_y = \frac{-250}{650} = -0{\cdot}385$$

$$\lambda_z = 0$$

Force components of F_{CE} are

$$F_x = \frac{F_{CE}\,(x_E - x_C)}{\sqrt{0 + 1000^2 + 750^2}} = \frac{F_{CE} \times 0}{1250} = 0$$

$$F_y = \frac{F_{CE}\,(y_E - y_C)}{1250} = 0{\cdot}8\ F_{CE}$$

$$F_z = \frac{F_{CE}\,(z_E - z_C)}{1250} = -\,0{\cdot}6\ F_{CE}$$

Putting the above values in moment equation,

$$\begin{vmatrix} 0{\cdot}92 & -0{\cdot}385 & 0 \\ 600 & -250 & 750 \\ 0 & 0{\cdot}8\ F_{CE} & -0{\cdot}6\ F_{CE} \end{vmatrix} + \begin{vmatrix} 0{\cdot}92 & -0{\cdot}385 & 0 \\ 300 & -125 & 375 \\ 0 & -40 \times 9{\cdot}81 & 0 \end{vmatrix} = 0$$

$$\therefore \begin{matrix} 0{\cdot}92\ (150\ F_{CE} - 600\ F_{CE}) \\ +\ 0{\cdot}385\ (-\ 360\ F_{CE}) \end{matrix} + \begin{matrix} 0{\cdot}92\ (147150) \\ +\ 0{\cdot}385\ (0) \end{matrix} = 0$$

$$\therefore -\ 414\ F_{CE} - 138{\cdot}6\ F_{CE} + 135378 = 0$$

$$\therefore 552{\cdot}6\ F_{CE} = 135378$$

$$\therefore \boxed{F_{CE} = 245\ \text{N}}$$

and the force F_{CE} can be represented in vector form as

$$F_{CE} = 0{\cdot}8 \times 245\ \bar{j} - 0{\cdot}6 \times 245\ \bar{k}$$

$$\boxed{F_{CE} = 196\ \bar{j} - 147\ \bar{k}}$$

13. Determine the intensity of the distributed load w at the end C of the beam ABC for which the reaction at C is zero. Also calculate the reaction at B. **(Pune University, April - 1998)**

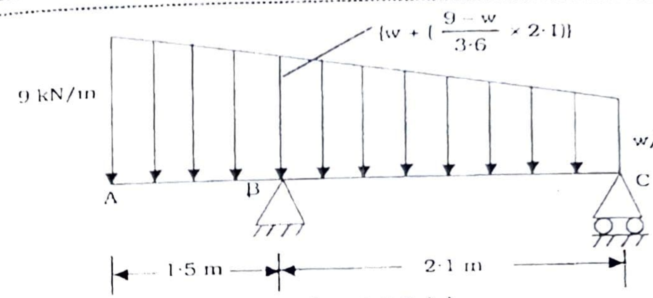


Fig. 4·18 (a)

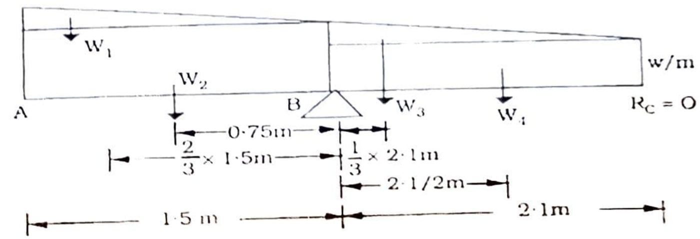


Fig. 4·18 (b)

Here the load is trapezoidal uniformly varying load having intensity 9 kN/m at A and w/m at C. **Also the reaction at C is zero.** Intensity of the load just above B is $\{w + (\frac{9 - w}{3{\cdot}6} \times 2{\cdot}1)\}$

Taking moment about B :

ΣM_B = 0, i.e., Moment of load over AB = Moment of load over BC

$$W_1 \times \frac{2}{3} \times 1{\cdot}5 + W_2 \times 0{\cdot}75 = W_3 \times \frac{1}{3} \times 2{\cdot}1 + W_4 \times 2{\cdot}1/2$$

$$\frac{1}{2} \times 1{\cdot}5 \times [9 - \{w + (\frac{9-w}{3{\cdot}6} \times 2{\cdot}1)\}] \times \frac{2}{3} \times 1{\cdot}5$$

$$+\ 1{\cdot}5 \times \{w + (\frac{9-w}{3{\cdot}6} \times 2{\cdot}1)\} \times 0{\cdot}75$$

$$= \frac{1}{2} \times 2{\cdot}1 \times [\{w + (\frac{9-w}{3{\cdot}6} \times 2{\cdot}1)\} - w] \times \frac{1}{3} \times 2{\cdot}1$$

$$+\ 2.1 \times w \times \frac{1}{2} \times 2{\cdot}1$$

$$\therefore \{0{\cdot}75 \times (9 - w - 5{\cdot}25 + 0{\cdot}58\ w) \times 1\} + \{1{\cdot}5\ (w + 5{\cdot}25 - 0{\cdot}58 \times 0{\cdot}75\}$$

$$= \{1{\cdot}05\ (w + 5{\cdot}25 - 0{\cdot}58\ w - w) \times 0{\cdot}7\} + 2{\cdot}205\ w$$

$$\{2{\cdot}8125 - 0{\cdot}315\ w\} + 0{\cdot}4725\ w + 5{\cdot}91\} = \{\ 0{\cdot}4263\ w + 3{\cdot}86\} + 2{\cdot}205\ w$$

$$0{\cdot}1575\ w + 8{\cdot}723 = 1{\cdot}7787\ w + 3{\cdot}86$$

$$\boxed{w = 3\ \text{kN/m}}$$

Also, the Reaction at B = Total load = $\frac{1}{2} \times (9 + 3) \times 3{\cdot}6$

$\therefore$ $\boxed{\text{Reaction at B} = 21{\cdot}6 \text{ kN}}$

14. The beam AB supports two concentrated loads and rests on the soil which exerts a linearly distributed reaction as shown in figure. If W_A = 18 kN/m, determine,

(i) distance 'a'

(ii) the corresponding value of 'W_B' in kN/m.

(Pune University, October 1998)

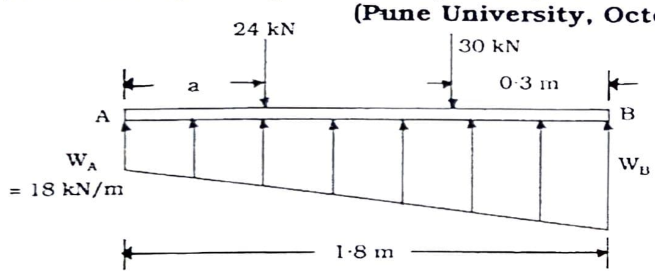


Fig 4·19

The beam is under equilibrium. The moment about any point is zero. Let's take moment about A :

$$\Sigma M_A = 0;\ 24 \times a + 30 \times 1{\cdot}5 - \{\frac{1}{2} \times 1{\cdot}8 \times (W_B - 18) \times \frac{2}{3} \times 1{\cdot}8\}$$
$$- 18 \times 1{\cdot}8 \times 0{\cdot}9 = 0$$
$$24\ a + 45 - 1{\cdot}08\ W_B + 19{\cdot}44 - 29{\cdot}16 = 0$$
$$24\ a - 1{\cdot}08\ W_B + 35{\cdot}28 = 0 \qquad \dots \text{(i)}$$

Applying another equilibrium equation

$$\Sigma F_y = 0;\ - 24 - 30 + \frac{1}{2}(18 + W_B) \times 1{\cdot}8 = 0$$
$$- 37{\cdot}8 + 0{\cdot}9\ W_B = 0$$
$$\therefore \boxed{W_B = 42 \text{ kN/m}}$$

Putting the value of W_B in equation **(i)**,

$$\boxed{a = 0{\cdot}42 \text{ m}}$$

15. A compound beam is loaded and supported as shown in figure, find the reactions at the supports A, C and D.

(Pune University, April - 1996)

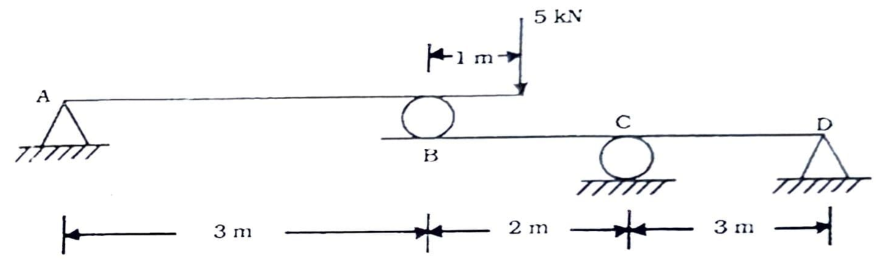


Fig 4·20 (a)

The above compound beam is under equilibrium and can be divided into **two** beams as shown below.

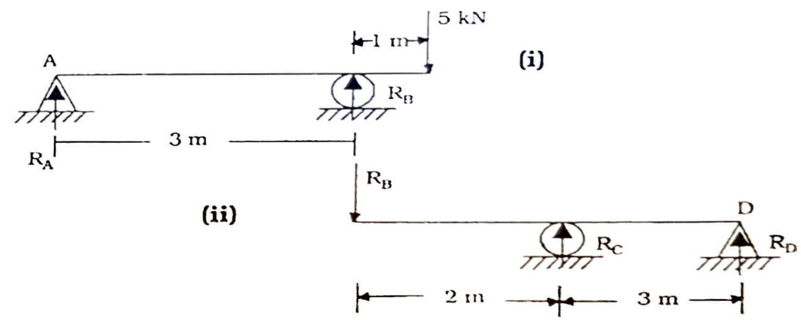


Fig. 4·20 (b)

Beam (i) : $\Sigma M_A = 0$, $R_B \times 3 = 5 \times 4$

$R_B = 6{\cdot}67$ kN

$\Sigma F_y = 0$, $R_A + R_B = 5$

$R_A = 5 - 6{\cdot}67$, $R_A = - 1{\cdot}67$ kN

$\boxed{R_A = 1{\cdot}67 \text{ kN } (\downarrow)}$

Beam (ii) : $\Sigma M_D = 0$, $R_B \times 5 = R_C \times 3$

$R_C = 6{\cdot}67 \times 5/3$

$\boxed{R_C = 11{\cdot}12 \text{ kN } (\uparrow)}$

$\Sigma F_y = 0$, $R_B = R_C + R_D$

$6{\cdot}67 = 11{\cdot}12 + R_D$

$R_D = - 4{\cdot}45$

$\boxed{R_D = 4{\cdot}45 \text{ kN } (\downarrow)}$

16. Loads P and W are suspended at points B and C on a cable ABCD supported at A and D. If the cable assumes the configuration shown in figure, and if the maximum tension in the cable is 10 kN, find the value of load P.

(Pune University, April - 1994)

8

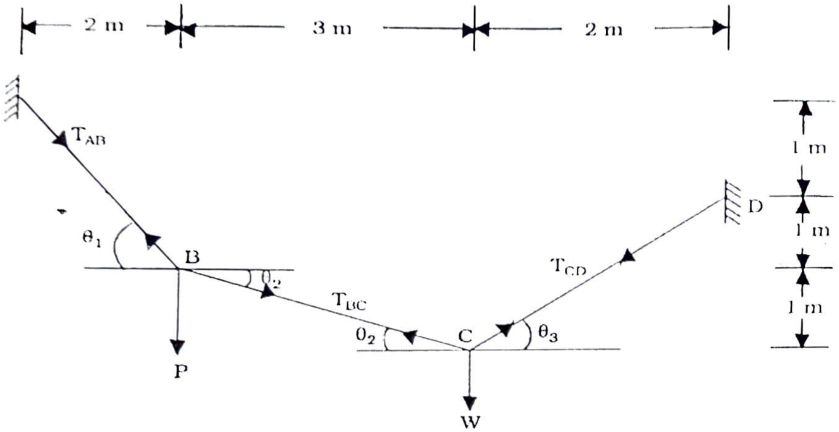


Fig. 4·21

Here, if we see the joint B and C, there are total **five** unknowns : T_{AB}, T_{BC}, T_{CD}, P and W.

The equilibrium equations at each joint are two :

$\Sigma F_x = 0$ and $\Sigma F_y = 0$. Thus total **four** equilibrium equations **plus** maximum tension 10 kN.

Joint B : $\mathbf{\Sigma F_x = 0,}$

$$T_{AB} \cos \theta_1 = T_{BC} \cos \theta_2$$
$$T_{AB} \cos 45° = T_{BC} \times 3/3{\cdot}16$$
$$T_{AB} = 1{\cdot}343\ T_{BC} \qquad \text{... (i)}$$

$\mathbf{\Sigma F_y = 0,}$

$$T_{AB} \sin \theta_1 = P + T_{BC} \sin \theta_2$$
$$T_{AB} \sin 45° = P + T_{BC} \times 1/3{\cdot}16$$
$$1{\cdot}343\ T_{BC} \times \sin 45° = P + 0{\cdot}316\ T_{BC}$$
$$\therefore\ 0{\cdot}634\ T_{BC} = P \qquad \text{... (ii)}$$

Joint C : $\mathbf{\Sigma F_x = 0,}$

$$T_{BC} \cos \theta_2 = T_{CD} \cos \theta_3$$
$$T_{BC} \times \frac{3}{3{\cdot}316} = T_{CD} \times 2/2{\cdot}83$$
$$T_{BC} = 0{\cdot}78\ T_{CD} \qquad \text{... (iii)}$$

From eq. (i) it can be concluded that $T_{AB} > T_{BC}$. Similarly from eq. (iii), $T_{CD} > T_{BC}$

Assuming $T_{AB} = 10$ kN

then $T_{BC} = 7{\cdot}446$ kN

and $T_{CD} = 9{\cdot}55$ kN

Thus maximum tension will be in AB cable.

$\therefore$ From equation (ii), $\boxed{P = 4{\cdot}72 \text{ kN}}$

17. Two rollers A and B weighing 6 kN and 4 kN respectively are supported by mutually perpendicular smooth inclined planes and are connected by a string as shown in figure. Find the angle ϕ that the string makes with the horizontal when the system is in equilibrium.

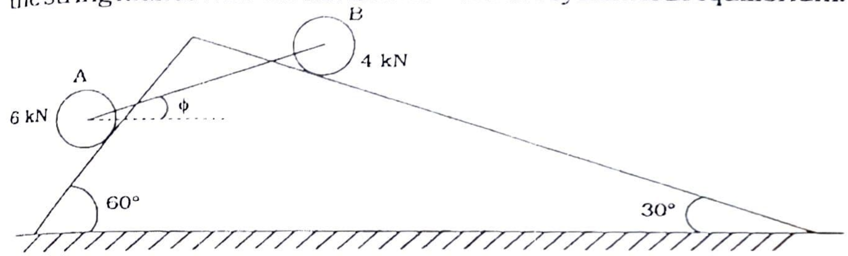
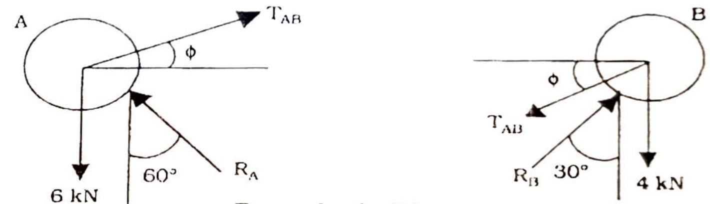


Fig. 4·22 (a)



Free - body Diagrams

Fig. 4·22 (b)

Roller A : $\mathbf{\Sigma F_x = 0,}$ $T_{AB} \cos \phi - R_A \sin 60° = 0$

$$T_{AB} \cos \phi = 0{\cdot}866\ R_A \qquad \text{... (i)}$$

$\mathbf{\Sigma F_y = 0,}$ $-6 + T_{AB} \sin \phi + R_A \cos 60° = 0$

$$T_{AB} \sin \phi = -\,0{\cdot}5\ R_A + 6 \qquad \text{... (ii)}$$

Roller B : $\mathbf{\Sigma F_x = 0,}$ $T_{AB} \cos \phi = R_B \sin 30°$

$$T_{AB} \cos \phi = 0{\cdot}5\ R_B \qquad \text{... (iii)}$$

$\mathbf{\Sigma F_y = 0,}$ $-4 - T_{AB} \sin \phi + R_B \cos 30° = 0$

$$T_{AB} \sin \phi = +\,0{\cdot}866\ R_B - 4 \qquad \text{... (iv)}$$

From eq. (i) and (iii), $R_A = 0{\cdot}577\ R_B$ ... (v)

and from eq. (ii) and (iv), $R_A = -\,1{\cdot}732\ R_B + 20$ ... (vi)

From eq. (v) and (vi), $R_B = 8{\cdot}66$ kN

From eq. (v) $\therefore$ $R_A = 5$ kN

Putting the value of R_A in eq. (i) & (ii)

$T_{AB} \cos\phi = 4{\cdot}33$ ...(vi) Dividing eq. (vii) by (vi)

$T_{AB} \sin\phi = 3{\cdot}5$ ...(vii) $\therefore \tan\phi = 0{\cdot}808$, $\boxed{\phi = 39^\circ}$

THEORY RELATED QUESTIONS

1. What is called equilibrium ?

2. What are the conditions of equilibrium ?

3. **Explain** with sketches :

(i) Space diagram **(ii)** Free body diagram. **(iii)** Reactions at Roller, Hinge and Fixed Supports. **(iv)** Equilibrium of two force body. **(v)** Equilibrium of three force body.

EXERCISES

4·1 Draw free – body diagrams only.

(i)

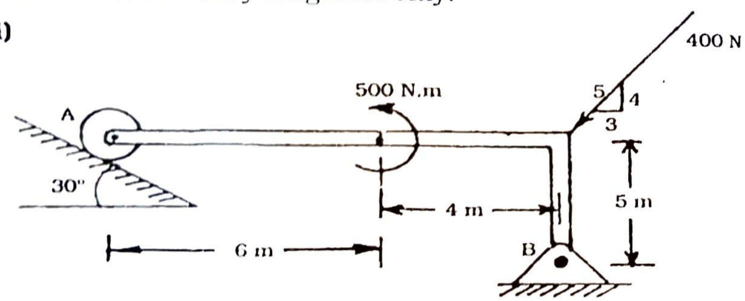


Fig. 4·23

(ii)

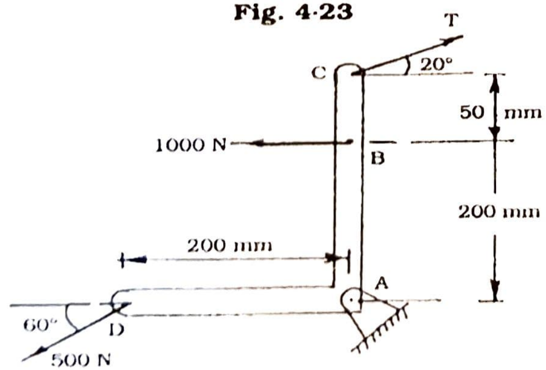


Fig. 4·24

(iii)

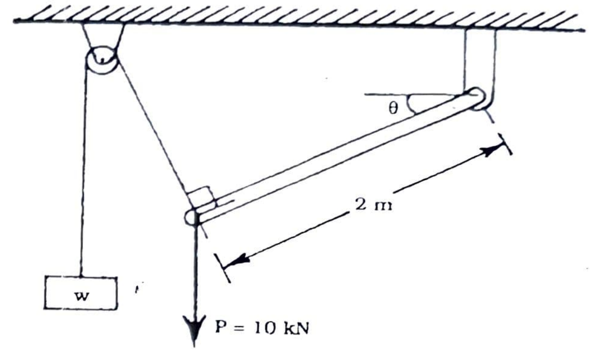


Fig. 4·25

4·2 Determine the tension in cable ABD and the reaction at the support C.

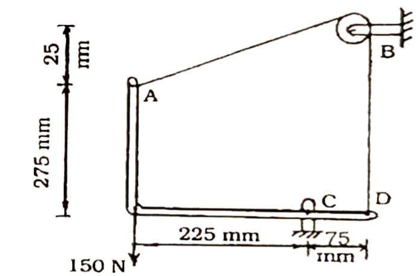


Fig. 4·26

4·3 Determine the reactions at A and B when **(a)** $\alpha = 0$ **(b)** $\alpha = 90^\circ$ **(c)** $\alpha = 30^\circ$

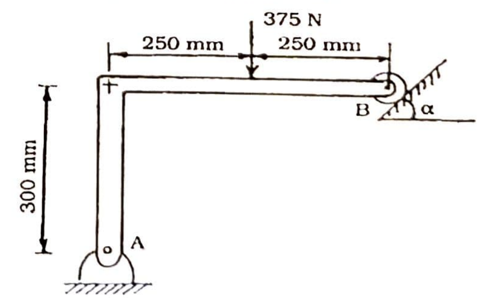


Fig. 4·27

4·4 A string ABCDE, whose extremity A is fixed, has weights W_1 and W_2 attached to it at B and C as shown. It passes round a small smooth peg at D and carrying a weight of 400 N at the free end E. If in the position of equilibrium, BC is horizontal and angles ABC and BCD are 150° and 120° respectively, find (i) Tensions in the portion AB, BC and CD of the string and (ii) Magnitudes of W_1 and W_2

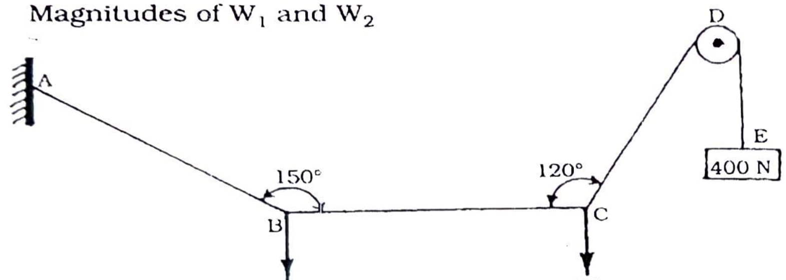


Fig 4·28

4·5 Two cylinders P and Q rest in a channel as shown. The cylinder P has diameter of 100 mm and weighs 200 N, whereas the cylinder Q has diameter of 180 mm and weighs 500 N.

If the bottom width of the box is 180 mm, with one side vertical and the other inclined at 60°, determine the reactions at all four points of contact.

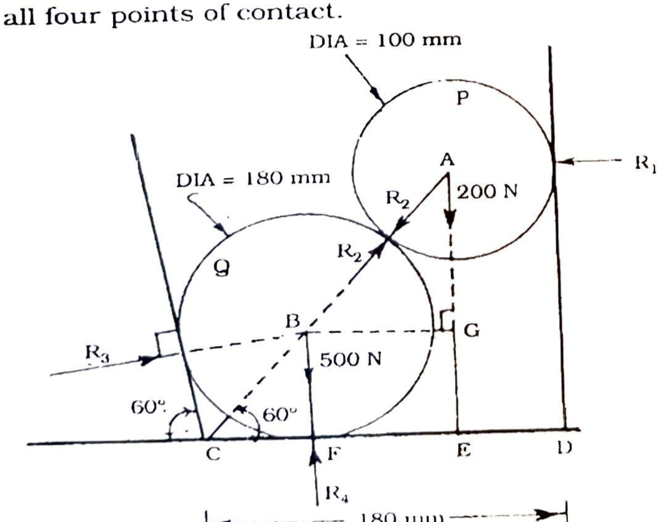


Fig 4·29

4·6 Three loads are applied to a beam as shown. The beam is supported by a roller at A and by a pin at B. Neglecting the weight of the beam, determine the reaction at A and B when P = 75 kN.

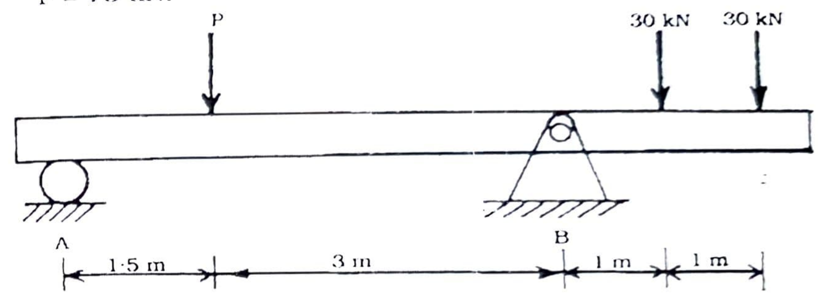


Fig 4·30

4·7 Three loads are applied as shown to a light beam supported by cables attached at B and D. Knowing that the maximum allowable tension in each cable is 12 kN and neglecting the weight of the beam, determine the range of values of Q for which the loading is safe when P = 0

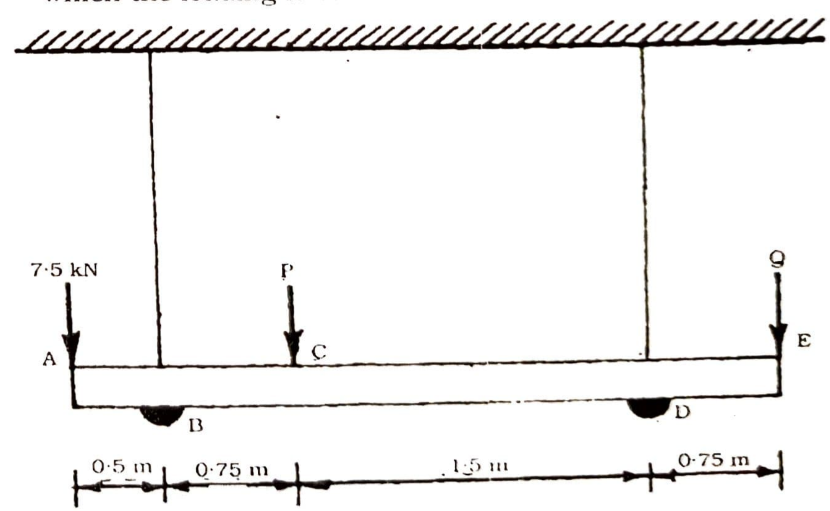


Fig 4·31

4·8 Determine the x, y, z components of reaction at A and the tension in the cable BC.

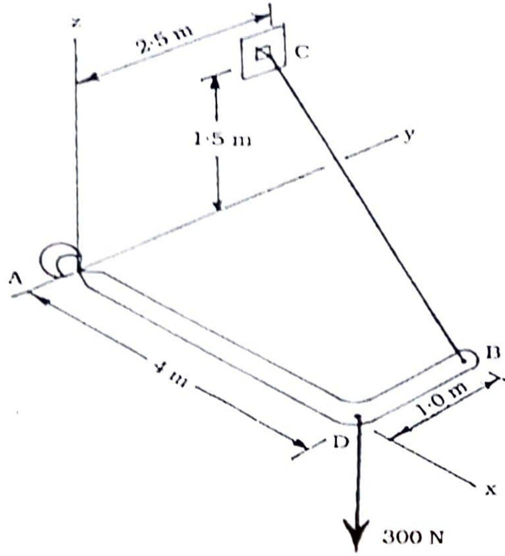


Fig. 4.32

4·9 A slender rod of length L is lodged between peg C and the vertical wall. It supports a load P at end A. Neglecting friction and weight of the rod, determine the angle θ corresponding to equilibrium.

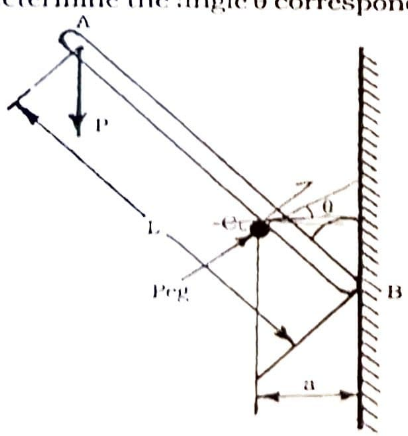


Fig 4·33

4·10 A 20 kg square plate is supported by three wires. Determine the tension in each wire.

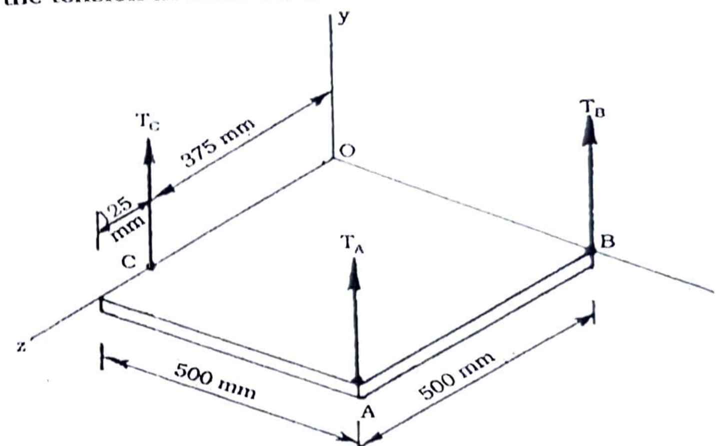


Fig. 4·34

4·11 The frame ACD is supported by ball and socket joints at A and D and by a cable which passes through a ring at B and is attached to hooks at G and H. Determine the tension in the cable.

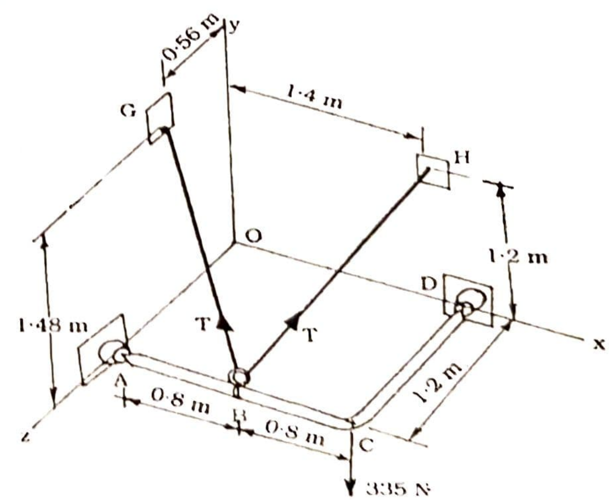


Fig. 4·35

4·12 A 120 kg sign board of uniform density measures $1{\cdot}5 \times 2{\cdot}4$ m and is supported by a ball and socket at A and by two cables as shown. Determine the tension in each cable and the reaction at A.

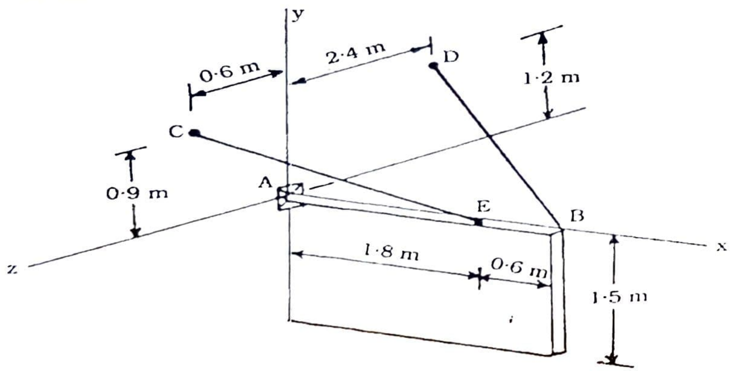


Fig 4·36

4·13 Determine the x, y, z components of reaction at the pin A and the tension in the cable BC necessary for equilibrium of the rod.

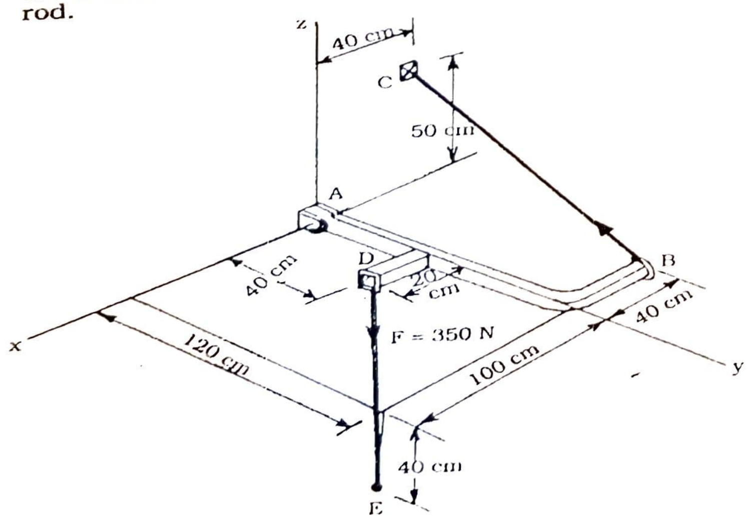


Fig 4·37

SOLUTIONS OF EXERCISES

4·1 Free – body diagrams :

(i)

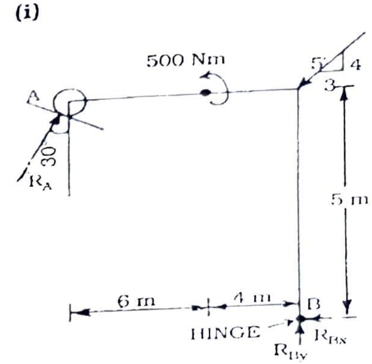


Fig. 4·38

(ii)

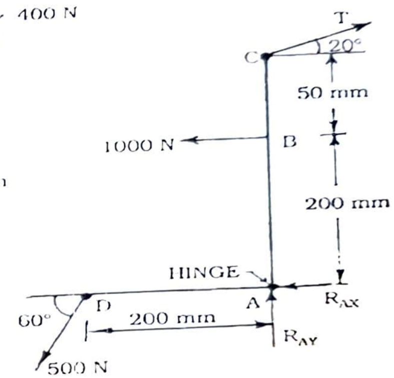


Fig. 4·39

(iii)

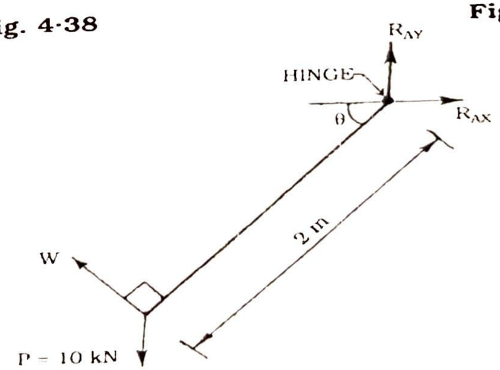


Fig. 4·40

4·2 Here three unknowns (T, R_{cx}, R_{cy}) can be determined by three equations, i.e. $\Sigma M_c = 0$, $\Sigma F_x = 0$, and $\Sigma F_y = 0$. Moment at hinge support is zero.

$$\Sigma M_c = 0, \; T \sin 4{\cdot}76° \times 0{\cdot}225 + T \cos 4{\cdot}76° \times 0{\cdot}275 - 150 \times 0{\cdot}225 - T \times 0{\cdot}075 = 0$$

$\therefore$ T = 154·82 N

R_{cy} = − 17·67 N = 17·67 N (↓)

R_{cx} = 154·29 N (←)

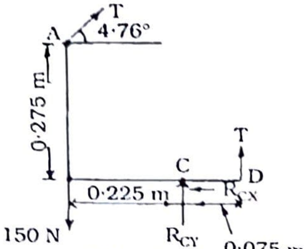


Fig. 4·41

4·3 (a) When α = 0, R_B will be vertical.

Taking moment about hinge support A

$375 \times 250 - R_B \times 500 = 0 \quad \therefore R_B = 187{\cdot}5$ N (↑)

$R_A = 375 - 187{\cdot}5 = 187{\cdot}5$ N (↑)

Horizontal reaction at A is zero as there is no horizontal load.

R_A = 187·5 N (↑), R_B = 187·5 N (↑)

(b) When α = 90°, R_B will be horizontal.

Taking moment about A

$375 \times 250 - R_B \times 300 = 0 \qquad \therefore R_B = 312{\cdot}5$ N (←)

$\Sigma F_x = 0, \; R_{Ax} - 312{\cdot}5 = 0 \qquad \therefore R_{Ax} = 312{\cdot}5$ N (→)

$\Sigma F_y = 0, \; R_{Ay} - 375 = 0 \qquad \therefore R_{Ay} = 375$ N (↑)

R_{Ay} = 375 N (↑), R_{Ax} = 312·5 N (→), R_B = 312·5 N (←)

(c) When α = 30°, R_B will be perpendicular to 30° plane, means R_B will be making 30° with vertical.

$\Sigma M_A = 0,$

$375 \times 250 - R_B \cos 30° \times 500 - R_B \sin 30° \times 300 = 0$

R_B = 160·80 N 60°

$\Sigma F_x = 0, \; R_{Ax} = R_B \sin 30° \qquad \therefore R_{Ax} = 80{\cdot}4$ N (→)

$\Sigma F_y = 0, \; R_{Ay} + R_B \cos 30° = 375 \qquad \therefore R_{Ay} = 235{\cdot}7$ N (↑)

R_{Ay} = 235·7 N (↑), R_{Ax} = 80·4 N (→), R_B = 160·80 N 60°

Note : $R_A = \sqrt{R_{Ax}^2 + R_{Ay}^2} \qquad \theta = \tan^{-1} \dfrac{R_{Ay}}{R_{Ax}}$

4·4 T_{CD} = 400 N = T_{DE} as it passes round a smooth peg.

Considering the equilibrium of point B and C separately, and using two equilibrium equations, $\Sigma F_x = 0$ & $\Sigma F_y = 0$.

$T_{AB} \cos 60° = W_1$. $T_{AB} \sin 60° = T_{BC}$. $T_{BC} = 400 \sin 30°$. T_{BC} = 200 N

$400 \cos 30° = W_2 \quad \therefore W_1 = 115{\cdot}5$ N, $W_2 = 346{\cdot}41$ N, $T_{AB} = 230{\cdot}95$ N

4·5 ∠CBF = 30°, tan 30° × BF = CF ∴ CF = 52 mm

ED = 50 mm

FE = 180 − CF − ED = 78 mm = BG

cos ∠ABG = BG/AB ∴ ∠ABG = 56·14°

Equilibrium of Cylinder Q :

On this cylinder four forces are acting: **R_3, R_4, 500 N** and **R_2 from cylinder P.**

$\Sigma F_x = 0, \; R_3 \sin 60° - R_2 \cos 56{\cdot}14° = 0 \quad \therefore R_3 = 0{\cdot}64 R_2$ **(i)**

$\Sigma F_y = 0, \; R_3 \cos 60° + R_4 - 500 - R_2 \sin 56{\cdot}14° = 0$

Putting value of R_3 in terms of R_2

$0{\cdot}32 R_2 - 0{\cdot}83 R_2 + R_4 - 500 = 0$

$R_4 - 0{\cdot}51 R_2 - 500 = 0$ **(ii)**

Equilibrium of Cylinder P :

Three forces, **R_1, 200 N and R_2 from cylinder Q** are acting.

$\Sigma F_y = 0, \; R_2 \sin 56{\cdot}14° = 200$ N $\qquad \therefore R_2 = 241$ N

$\Sigma F_x = 0, \; R_2 \cos 56{\cdot}14° = R_1 \qquad \therefore R_1 = 134{\cdot}28$ N

Putting value of R_1 and R_2 in equation **(i)** & **(ii)**

R_3 = 155 N and R_4 = 622·6 N

Inclination of R_3 with vertical is 60° and R_2 with horizontal is 56·14°.

R_1 = 134·3 N, R_2 = 241 N, R_3 = 155 N, R_4 = 622·6 N.

4·6 As the loadings are vertical, the reactions at A and B will also be vertical and upward. The moments at supports A and B will be zero.

$\Sigma M_B = 0, \; 30 \times 1 + 30 \times 2 + R_A \times 4{\cdot}5 - 75 \times 3 = 0$

$\therefore R_A$ = 30 kN (↑), R_B = 30 + 30 + 75 − 30 = 135 kN (↑)

4·7 Let reaction (tension) in wires at B and D is R_B and R_D respectively. Maximum allowable tension in wire is 12 kN. P = 0.

Q is very near to R_D, hence R_D may be 12 kN (↑) **OR** zero.

$\Sigma M_B = 0, \; -7{\cdot}5 \times 0{\cdot}5 - 12 \times 2{\cdot}25 + 3 \times Q = 0$

$\therefore Q = 10{\cdot}25$ kN (max)

When $R_D = 0$, $R_B = 7{\cdot}5 + Q$

$\Sigma \mathbf{M_D} = \mathbf{0}$, $7{\cdot}5 \times 2{\cdot}75 - R_B \times 2{\cdot}25 - Q \times 0{\cdot}75 = 0$, but $R_B = 7{\cdot}5 + Q$

$\therefore \; Q = 1{\cdot}25$ kN (min)

4·8 Support A is Ball - and - Socket joint hence only three components of the reaction will act. The coordinates of points are

D = 4, 0, 0

B = 4, 1, 0

C = 0, 2·5, 1·5

$$(BC)_x = \frac{x_C - x_B}{(d)_{BC}} = \frac{-4}{\sqrt{1{\cdot}5^2 + 1{\cdot}5^2 + 4^2}} = \frac{-4}{4{\cdot}53} = -0{\cdot}88$$

$$(BC)_y = \frac{1{\cdot}5}{4{\cdot}53} = 0{\cdot}33$$

$$(BC)_z = 0{\cdot}33$$

$\overline{T}_{BC} = T_{BC}(-0{\cdot}88\ \overline{i} + 0{\cdot}33\ \overline{j} + 0{\cdot}33\ \overline{k})$, $\overline{W} = -300\ \overline{k}$

$\Sigma \mathbf{F_x} = \mathbf{0}$, $A_x - 0{\cdot}88\, T_{BC} = 0$, $\Sigma \mathbf{F_y} = \mathbf{0}$, $A_y + 0{\cdot}33\, T_{BC} = 0$,

$\Sigma \mathbf{F_z} = \mathbf{0}$, $A_z + 0{\cdot}33\, T_{BC} - 300 = 0$

$\Sigma M_A = 0$, $\Sigma(\overline{r} \times \overline{F}) = 0$, $(4\ \overline{i} + 1\ \overline{j}) \times (-0{\cdot}88\, T_{BC}\ \overline{i} + 0{\cdot}33\, T_{BC}\ \overline{j} + 0{\cdot}33\, T_{BC}\ \overline{k}) + (4\ \overline{i}) \times (-300\ \overline{k}) = 0$

$\therefore \; T_{BC} = 909{\cdot}1$ N, $A_x = 800$ N, $A_y = -300$ N, $A_z = 0$

4·9 Reaction at the wall B will be perpendicular to the wall.

The peg C will act as roller support and hence reaction will be perpendicular to rod AB.

Reaction at B is horizontal = Q

Reaction R at C will be perpendicular to rod and making angle θ with the horizontal.

$\Sigma \mathbf{F_x} = \mathbf{0}$, $Q = R\cos\theta$ and $\Sigma \mathbf{F_y} = \mathbf{0}$, $P = R\sin\theta$.

Taking moment about B.

$$\Sigma \mathbf{M_B} = \mathbf{0} : R \times \frac{a}{\sin\theta} - P \times L\sin\theta = 0$$

$$R \times \frac{a}{\sin\theta} = R\sin\theta \times L\sin\theta \quad \therefore \quad \theta = \sin^{1/3}(a/L)$$

4·10 The coordinates of points are

B = 0·5, 0, 0

A = 0·5, 0, 0·5

C = 0, 0, 0·375

and centre of plate G = 0·25, 0, 0·25

Weight of the plate = 20 × 9·81

= 196·2 N

$\Sigma \mathbf{M_o} = \mathbf{0}$, $(0{\cdot}25\ \overline{i} + 0{\cdot}25\ \overline{k}) \times (-196{\cdot}2\ \overline{j}) + (0{\cdot}375\ \overline{k}) \times (T_c\ \overline{j}) + (0{\cdot}5\ \overline{i} + 0{\cdot}5\ \overline{k}) \times (T_A\ \overline{j}) + (0{\cdot}5\ \overline{i}) \times (T_B\ \overline{j}) = 0$

and $\Sigma \mathbf{F_y} = \mathbf{0}$, $T_A + T_B + T_C = 196{\cdot}2$.

$\therefore \; T_A = 24{\cdot}525$ N, $T_B = 73{\cdot}575$ N, $T_C = 98{\cdot}1$ N.

4·11 The coordinate of points are

A = 0, 0, 1·2

B = 0·8, 0, 1·2

G = 0, 1·48, 0·56

H = 1·4, 1·2, 0

C = 1·6, 0, 1·2

D = 1·6, 0, 0

$$(BG)_x = \frac{x_G - x_B}{\sqrt{0{\cdot}8^2 + 0{\cdot}64^2 + 1{\cdot}48^2}} = \frac{-0{\cdot}8}{1{\cdot}8} = -0{\cdot}44$$

$$(BG)_y = \frac{1{\cdot}48}{1{\cdot}8} = 0{\cdot}82$$

$$(BG)_z = \frac{0{\cdot}56 - 1{\cdot}2}{1{\cdot}8} = -0{\cdot}36$$

$$(\lambda_{AD})_x = (AD)_x = \frac{x_D - x_A}{\sqrt{1{\cdot}6^2 + 1{\cdot}2^2}} = \frac{1{\cdot}6}{2} = 0{\cdot}8$$

$$(AD)_y = 0 \text{ and } (AD)_z = \frac{-1{\cdot}2}{2} = -0{\cdot}6$$

$$(BH)_x = \frac{x_H - x_B}{\sqrt{0{\cdot}6^2 + 1{\cdot}2^2 + 1{\cdot}2^2}} = \frac{0{\cdot}6}{1{\cdot}8} = 0{\cdot}33$$

$$(BH)_y = \frac{1{\cdot}2}{1{\cdot}8} = 0{\cdot}67, \text{ and } (BH)_z = \frac{-1{\cdot}2}{1{\cdot}8} = -0{\cdot}67$$

$x_{B/A} = 0$

$x_{B/A} = x_B - x_A = 0{\cdot}8$, $y_{B/A} = 0$, $z_{B/A} = 0$, $\boxed{x_{C/A} = 1{\cdot}6}$

In this problem, the frame ACD can rotate about axis AD if it is not in equilibrium. But the frame is under equilibrium hence sum of moments about axis AD must be zero.

$$\Sigma M_{AD} = 0, \quad \text{Moment of force BG about AD} + \text{Moment of force BH about AD} + \text{Moment of 335 N about AD} = 0$$

Moment of T_{BG}, T_{BH} and 335 N force about axis AD will be zero. Here $T_{BG} = T_{BH} = T$.

$$\Sigma M_{AD} = 0, \quad \begin{vmatrix} 0{\cdot}8 & 0 & -0{\cdot}6 \\ 0{\cdot}8 & 0 & 0 \\ -0{\cdot}44\,T & 0{\cdot}82\,T & -0{\cdot}36\,T \end{vmatrix} + \begin{vmatrix} 0{\cdot}8 & 0 & -0{\cdot}6 \\ 0{\cdot}8 & 0 & 0 \\ 0{\cdot}33\,T & 0{\cdot}67\,T & -0{\cdot}67\,T \end{vmatrix}$$

$$+ \begin{vmatrix} 0{\cdot}8 & 0 & -0{\cdot}6 \\ 1{\cdot}6 & 0 & 0 \\ 0 & -335 & 0 \end{vmatrix} = 0$$

$\therefore \quad T = 449{\cdot}2$ N

4·12 The coordinates of points are.

B = 2·4, 0, 0

E = 1·8, 0, 0.

C = 0, 0·9, 0·6

D = 0, 1·2, –2·4

and center of plate, G = 1·2, –0·75, 0.

$$(EC)_x = \frac{x_C - x_E}{\sqrt{1{\cdot}8^2 + 0{\cdot}9^2 + 0{\cdot}6^2}} = \frac{-1{\cdot}8}{2{\cdot}1} = -0{\cdot}86$$

$$(EC)_y = \frac{0{\cdot}9}{2{\cdot}1} = 0{\cdot}43, \text{ and } (EC)_z = \frac{0{\cdot}6}{2{\cdot}1} = 0{\cdot}29$$

$$(BD)_x = \frac{-2{\cdot}4}{\sqrt{2{\cdot}4^2 + 1{\cdot}2^2 + 2{\cdot}4^2}} = \frac{-2{\cdot}4}{3{\cdot}6} = -0{\cdot}67$$

$$(BD)_y = \frac{1{\cdot}2}{3{\cdot}6} = 0{\cdot}33, \ (BD)_z = \frac{-2{\cdot}4}{3{\cdot}6} = -0{\cdot}67$$

Here the Sign Board can rotate about Ball - and - Socket joint A, but it is under equilibrium.

$$\Sigma M_A = 0, \quad \text{Moment of } T_{EC} \text{ about A} + \text{Moment of } T_{BD} \text{ about A} + \text{Moment of W about A} = 0$$

$$\Sigma M_A = 0,$$

$$\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1{\cdot}8 & 0 & 0 \\ -0{\cdot}86\,T_{EC} & 0{\cdot}43T_{EC} & 0{\cdot}29T_{EC} \end{vmatrix} + \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2{\cdot}4 & 0 & 0 \\ -0{\cdot}67T_{BD} & 0{\cdot}33T_{BD} & -0{\cdot}67T_{BD} \end{vmatrix}$$

$$+ \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1{\cdot}2 & -0{\cdot}75 & 0 \\ 0 & -1177{\cdot}2 & 0 \end{vmatrix} = 0 \qquad \therefore \ T_{EC} = 1375{\cdot}5 \text{ N}, \quad T_{BD} = 440{\cdot}15 \text{ N}$$

and the structure as a whole is in equilibrium under the action of external forces and reactions.

$\Sigma F_x = 0$, $-0{\cdot}86\,T_{EC} - 0{\cdot}67\,T_{BD} + A_x = 0$, $A_x = 1478$ N

$\Sigma F_y = 0$, $0{\cdot}43\,T_{EC} + 0{\cdot}33\,T_{BD} + A_y - 1177{\cdot}2 = 0$, $A_y = 440{\cdot}5$ N

$\Sigma F_z = 0$, $0{\cdot}29\,T_{EC} - 0{\cdot}67\,T_{BD} + A_z = 0$, $A_z = -104$ N

4·13 The coordinates of points are

B = –0·4, 1·2, 0

C = –0·4, 0, 0·5

D = 0·2, 0·4, 0

E = 1·0, 1·2, –0·4

$$(BC)_x = \frac{x_C - x_B}{\sqrt{1{\cdot}2^2 + 0{\cdot}5^2}} = \frac{-0{\cdot}4 - (-0{\cdot}4)}{1{\cdot}3} = 0$$

$$(BC)_y = \frac{-1{\cdot}2}{1{\cdot}3} = -0{\cdot}92$$

$$(BC)_z = \frac{0{\cdot}5}{1{\cdot}3} = 0{\cdot}38$$

$$(DE)_x = \frac{x_E - x_D}{\sqrt{0{\cdot}8^2 + 0{\cdot}8^2 + 0{\cdot}4^2}} = \frac{1 - 0{\cdot}2}{1{\cdot}2} = 0{\cdot}67$$

$$(F_{DE})_x = 350 \times 0{\cdot}67 = 234{\cdot}5 \text{ N}$$

$$(DE)_y = \frac{0{\cdot}8}{1{\cdot}2} = 0{\cdot}67, \quad (F_{DE})_y = 350 \times 0{\cdot}67 = 234{\cdot}5 \text{ N}$$

$$(DE)_z = \frac{-0{\cdot}4}{1{\cdot}2} = 0{\cdot}33, \quad (F_{DE})_z = 350 \times 0{\cdot}33 = 115{\cdot}5 \text{ N}$$

The rod can rotate about pin joint A, but it is under equilibrium, therefore sum of moments about A is zero.

$$\Sigma M_A = 0, \quad \text{Moment of } \mathbf{F_{BC}} \text{ about x axis} + \text{Moment of } \mathbf{F_{DE}} \text{ about x axis} = 0$$

Here, $\lambda_x = 1$, $\lambda_y = 0$, $\lambda_z = 0$

$$\Sigma M_A = 0,$$

$$\begin{vmatrix} 1 & 0 & 0 \\ -0{\cdot}4 & 1{\cdot}2 & 0 \\ 0 & -0{\cdot}92\,T_{BC} & 0{\cdot}38\,T_{BC} \end{vmatrix} + \begin{vmatrix} 1 & 0 & 0 \\ 0{\cdot}2 & 0{\cdot}4 & 0 \\ 234{\cdot}5 & 234{\cdot}5 & -115{\cdot}5 \end{vmatrix} = 0$$

$\therefore \quad T_{BC} = 100{\cdot}4$ N

$\Sigma F_x = 0$, $234{\cdot}5 + A_x = 0$, $\quad A_x = -234{\cdot}5$ N

$\Sigma F_y = 0$, $-0{\cdot}92\,T_{BC} + 234{\cdot}5 + A_y = 0$, $\quad A_y = -142{\cdot}13$ N

$\Sigma F_z = 0$, $0{\cdot}38\,T_{BC} - 115{\cdot}5 + A_z = 0$, $\quad A_z = 77{\cdot}35$ N

9