# 2 STATICS OF PARTICLES



## 1. Scalars and Vectors :

Most of the physical quantities in mechanics can be expressed athematically by means of scalars and vectors.

**Scalar :** *A quantity characterized by a positive or negative number called scalar. Mass, volume and length* are scalar quantities often ;ed in statics. The scalar quatities do not have directions.

**Vector :** *A quantity having both magnitude and direction and* ›eys *the parallelogram law of addition is called vector. Position,* rce *and moment* are vector quantities used in statics.

## 2 System of Forces :

*When two or more than two forces act on a body, they are called form a system of forces.*

**(1) Force :** It represents the *action of one body on another* as *"push"* or *"pull"*. It is completely characterized by

(i) its *point of application.*
(ii) its *magnitude.*
(iii) its *direction.*

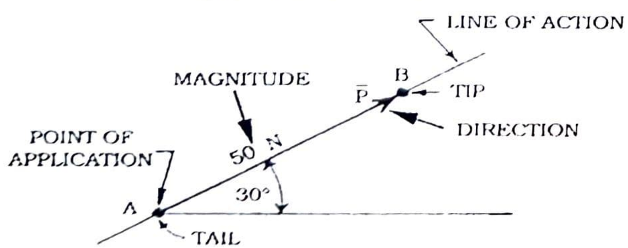



**Fig 2·1. Force.**



In Fig. 2.1, force $\bar{P}$ is acting at A having *magnitude* 50 N and *direction* E 30° N i.e. towards east-north with angle 30° from east to north. It can also be considered as pull at A or push at B. Here, B is the *tip of the force* and A is the *tail of the force.*

Rules of vector operations can be applied to forces.

**(2) Coplanar Forces :** *Forces contained in the same plane* are called *coplanar forces.* The plane in which forces lie may be vertical, horizontal or inclined. [Refer Fig. 2·3 (a), (b) and (f)].

**(3) Non-coplanar Forces :** All forces are not contained in the same plane but in different planes. [Refert Fig. 2·3 (c), (d) and (g)].

**(4) Concurrent Forces :** *Forces* are said to be *concurrent when their lines of actions pass through a single point.*

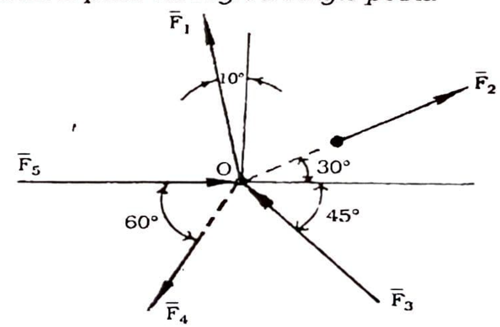
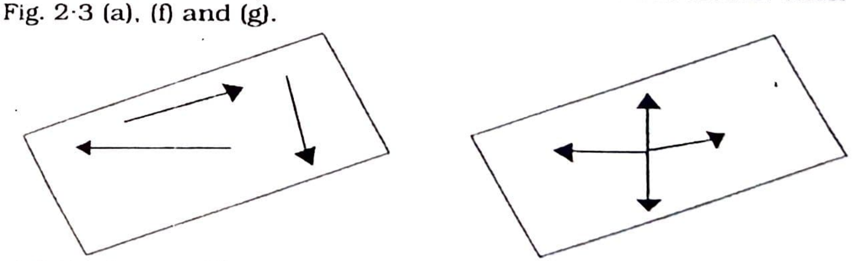


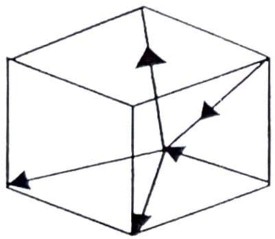

**Fig. 2.2. *Concurrent Forces***

In Fig. 2.2, all five forces are *acting in one plane of paper* hence they are *coplanar* and also *their lines of actions passing through point O hence* they are *concurrent.*

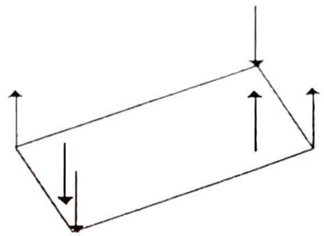
**(5) Non-concurrent Forces :** Forces, whose lines of action do not meet at one point, are called non-concurrent forces. Refer Fig. 2·3 (a), (f) and (g).



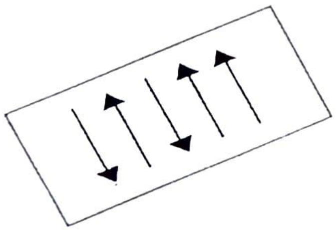
**(a) Coplanar - Non-concurrent** **(b) Coplanar - Concurrent**
**Fig. 2·3 Different Force Systems**



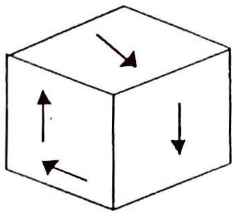
:c) Noncoplanar - Concurrent



(d) Noncoplanar-
Nonconcurrent



(f)Parallel - Coplanar
(Coplanar - Nonconcurrent)



(g) Parallel - Noncoplanar
(Noncoplanar- Nonconcurrent)

**Fig. 2·3 Different Force Systems**

**(6) Collinear Forces :** *The forces, whose lines of actions lie on e same line,* are known as *collinear forces.*

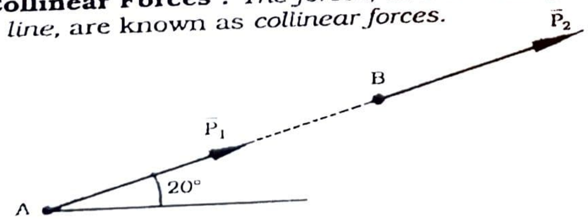



**Fig. 2·4 Collinear Forces**

In Fig. 2.4, $\bar{P}_1$ and $\bar{P}_2$ are *collinear forces* as they are *in the une line,* though applied at different points.

**(7) Space Diagram :** *A sketch showing the physical conditions the problem* is known as a *space diagram.* In practice, a problem engineering mechanics is derived from an actual physical situation. Fig. 2.5(a), three forces are acting at B. *The directions of all these rces are represented in space diagram.* The magnitudes are not :cessary to be shown according to the scale here.



**(8) Free-Body Diagram :** *A diagram showing a significan particle and forces acting on it .is called free body diagram* Fig. 2·5 (b) is the free-body diagram of the space diagram shown ir Fig. 2·4 (a). To construct a free-body diagram, following three step: are necessary (Fig. 2·5-b).

*(i) Step-1 :* Imagine the *particle* to be *isolated* or cut *"free"* fron its surroundings. Draw or sketch its outlined shape.

*(ii) Step-2 :* Indicate on this sketch *all the forces acting on th particle.* These forces can be active and reactive forces. *Active force tend to set the particle in motion,* such as those caused by *attache cables, weight etc. Reactive forces tend to prevent motion* which ar caused by the *constraints* or *supports.*

*(iii) Step-3 :* Label the *known forces* with their proper magnitud and directions. Label the *unknown forces* by using some "Letters."

The same steps are to be applied for the *rigid body* also, i which instead of particle, *body is isolated.*

**(9) Vector Diagram (Force diagram) :** *A force triangle fc three forces and force polygone for more than three forces* draw according to *triangle rule* or *polygone rule* with the *exact scale* fc the magnitude of the forces and according to *exact direction* is calle the *vector diagram or force diagram.* Fig. 2·5 (c) is the vector diagram c force diagram of the free-body diagram shown in Fig. 2·4 (b).

**(10) Bow's Notations :** They are the *capital letters placed o both sides of all the forces in space diagram or free body diagram fc conveniency of naming the forces.* In Fig 2·5(b), Bow's notations ar expressed as P, Q and R.

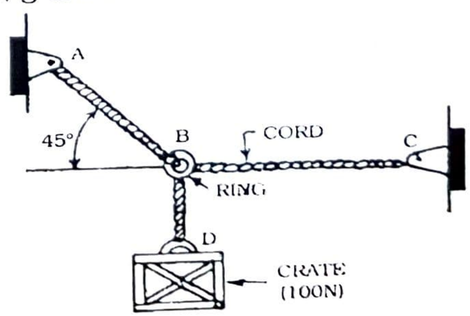
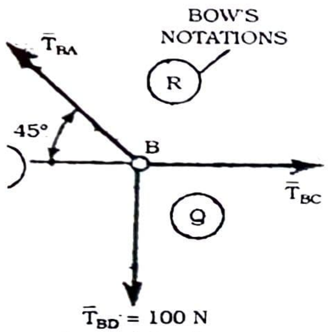
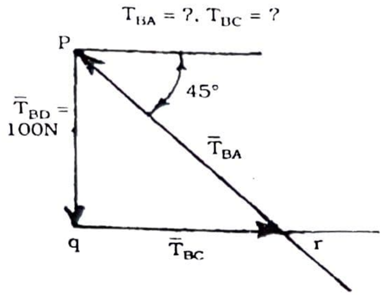



**Fig 2·5 (a) Space diagram**




SCALE : 1 cm = 25 N

**ee-body diagram** **Vector diagram (Force triangle)**

**(b)** **Fig 2.5** **(c)**

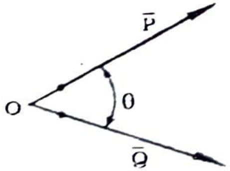
**Resultant Force :** *It is a single force which replaces the eral forces acting on the same particle without changing the effect hem. Thus without changing the effect on the body,* the number orces can be replaced by a *single force* called *resultant*. If *resultant ero*, the *body will be in rest position* called in *equilibrium* under action of forces. The point B in Fig. 2·5 is under equilibrium ans under rest conditions. Hence the resultant force of all three ces is zero. Thus, the vector diagram is a closed triangle of three ces.

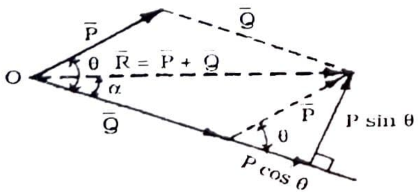
### ·1 Resultant of Two Concurrent Forces :

There are *two laws* as under :

**(1) Parallelogram law :** *"If two forces acting simultaneously on article be represented in magnitude and direction by the two acent sides of a parallelogram; their resultant may be represented nagnitude and direction by the diagonal of the parallelogram, which sses through their point of interaction."*

$\bar{P}$ and $\bar{Q}$ are the two concurrent forces whose resultant $\bar{R}$ be determined by drawing the diagonal of parallelogram having es $\bar{P}$ and $\bar{Q}$.




**(a)** **Fig 2·6** **(b)**



By summing vectors, $\bar{R} = \bar{P} + \bar{Q}$

From Fig 2·6 (b), $R^2 = (Q + P\cos\theta)^2 + (P\sin\theta)^2$

$$= Q^2 + P^2\cos^2\theta + 2PQ\cos\theta + P^2\sin^2\theta$$

$$= P^2 + Q^2 + 2PQ\cos\theta$$

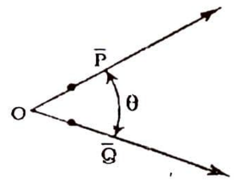
Magnitude of $\bar{R} = R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

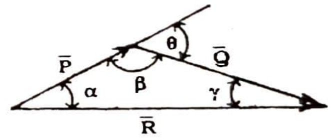
and its direction, $\tan\alpha = \dfrac{P\sin\theta}{Q + P\cos\theta}$

**(2) Triangle Rule :** *"If two forces acting simultaneously on a particle be represented in magnitude and direction by the two sides of a triangle taking in order; their* ***resultant*** *may be represented in magnitude and direction by the third side of the triangle,* ***taken in opposite order."***

Here at *tip* of $\bar{P}$ start *tail* of $\bar{Q}$, then join tail of $\bar{P}$ and *tip* of $\bar{Q}$ which will be the resultant $\bar{R}$ of $\bar{P}$ and $\bar{Q}$. Here, the **direction of $\bar{R}$ is in opposite to the order of $\bar{P}$ and $\bar{Q}$.**

If point O is under equilibrium then $\bar{R}$ will be zero and there must be a third force **(equilibrant - E)** having same magnitude of $\bar{R}$ but **opposite direction** means in the order of $\bar{P}$ and $\bar{Q}$.




**Space diagram** **Fig 2·7** **Vector diagram**

Here θ = included angle between $\bar{P}$ & $\bar{Q}$ in *space diagam*

β = included angle between $\bar{P}$ & $\bar{Q}$ in *vector diagram.*

= 180 − θ

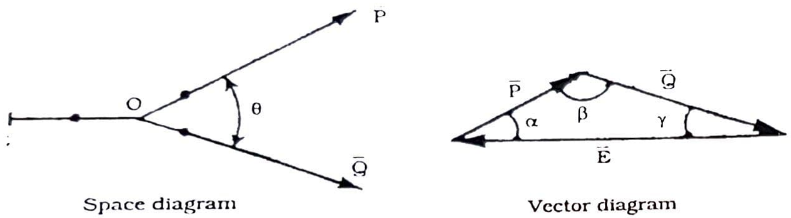
**Sine law (Lami's Theorem) :** $\dfrac{P}{\sin\gamma} = \dfrac{Q}{\sin\alpha} = \dfrac{R}{\sin\beta}$

**Cosine law :** $R = \sqrt{P^2 + Q^2 - 2PQ\cos\beta}$

It should be noted here that if angle θ between the forces $\bar{P}$ and $\bar{Q}$ are considered then,

**as used in parallelogram law,** $R = \sqrt{P^2+Q^2+2PQ\cos\theta}$

**(3) Lami's Theorem :** "If *three coplanar forces, acting on a body, are in equilibrium then each force is proportional to the sine of the angle between the other two forces."*




$= - \bar{R}$

**If point O is in equilibrium under action of $\bar{P}$, $\bar{Q}$ & $\bar{E}$.**

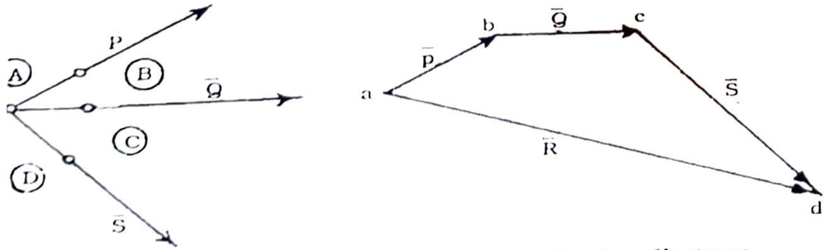
**Fig. 2·8**

Here three forces are acting at point O which is under uilibrum. Force $\bar{E}$ is equilibrant of $\bar{P}$ and $\bar{Q}$.

**Lami's Theorem :** $\dfrac{P}{\sin\gamma} = \dfrac{Q}{\sin\alpha} = \dfrac{E}{\sin\beta}$

## 3·2 Resultant of Several Concurrent Forces :

**Law of Polygone :** It is an extension of triangle law of forces for ore than two forces, which states *"If a number of forces acting multaneously on a particle be represented in magnitude and rection by the sides of a polygone taken in order, then the* ***resultant*** *all these forces may be represented in magnitude and direction by e closing side of the polygone taken in* ***opposite order."***




**Space diagram** **Fig 2·9** **Vector diagram**

**The order of directions of $\bar{P}$, $\bar{Q}$ and $\bar{S}$ and that of $\bar{R}$ aving opposite order in the polygone is to be noted carefully. Ioreover, representation of forces $\bar{P}$, $\bar{Q}$, $\bar{S}$ by bow's notations n space and vector diagrams should be noted.**

Force $\bar{P}$ is represented by vector $\overline{ab}$

$\bar{Q}$ is represented by vector $\overline{bc}$

$\bar{S}$ is represented by vector $\overline{cd}$.



## 2·4 Resolution of a Force into Components :

***The process of splitting up the given force in to a number of components, without changing its effect on body is called resolution of a force.***

In parallelogram law, $\bar{P}$ and $\bar{Q}$ are also called the ***components*** **of a single force $\bar{R}$.**

**(1) Resolution along any two directions (Not mutually perpendicular) :**

A single force $\bar{F}$ is to be resolved in to two components having *two fixed directions* X' and Y' *not perpendicular to each other.* As shown in Fig. 2·10, draw a parallelogram having diagonal $\bar{F}$, then the *sides* will represent *components* of $\bar{F}$ *along required directions.*

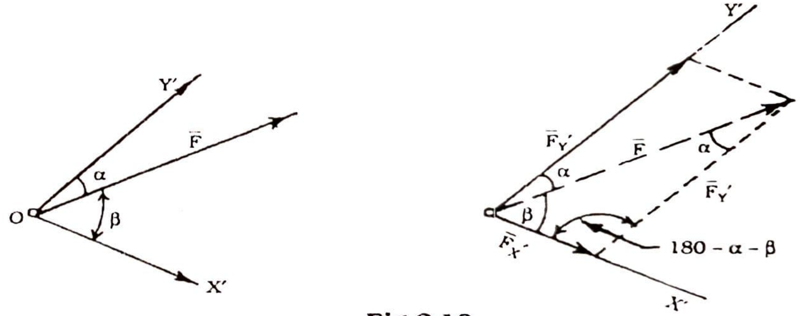



**Fig 2·10**

Here, **$\alpha + \beta \neq 90°$.**

Hence, **$F_{x'} \neq F \cos\beta$ and $F_{y'} \neq F \sin\beta$**

The components $F_{x'}$ and $F_{y'}$ of force F can be found out by applying sine law.

**Sine law :**

$$\frac{F_{x'}}{\sin\alpha} = \frac{F_{y'}}{\sin\beta} = \frac{F}{\sin(180-\alpha-\beta)}$$

**(2) Resolution into Rectangular Components (Mutually perpendicular) :**

It is desirable to resolve a force in to *two perpendicular components.* The parallelogram, drawn to obtain the two perpendicular components, is a rectangle and $\bar{F}_x$ and $\bar{F}_y$ are called rectangular components.

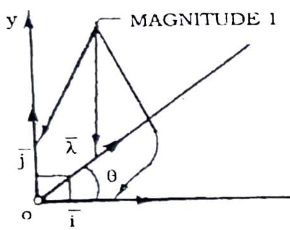
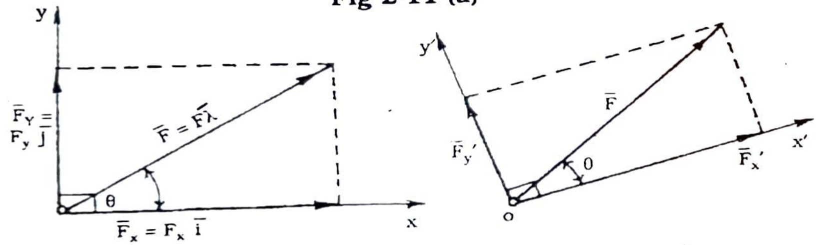



Fig 2·11 (a)




(b) (c)

Fig 2·11

Force $\bar{F}$ having magnitude F and direction $\bar{\lambda}$ will be having rectangular components $\bar{F}_x$ and $\bar{F}_y$ along x and y directions respectively. The rectangular components can also be determined along any two mutually perpendicular **desired directions** say x′ and y′ as shown in Fig. 2·11 (c).

Using Fig. 2·11 (b)

$\bar{F}_x = F_x\,\bar{i}$ $\qquad \bar{F} = F\,\bar{\lambda}$

$\bar{F}_y = F_y\,\bar{j}$ $\qquad \bar{F} = F_x\,\bar{i} + F_y\,\bar{j}$

Rectangular components, $F_x = F\cos\theta$ $\qquad F = \sqrt{F_x^2 + F_y^2}$

$F_y = F\sin\theta$ $\qquad \theta = \tan^{-1}\dfrac{F_y}{F_x}$

**Important :** While determining *components* of a force along *two* ***mutually perpendicular*** *axes*, we can take directly ***cosine of the angle with the given axis*** and ***sine for the another component.*** *But for components along* ***nonperpendicular*** *axes, we have to draw force* ***triangle*** *or parallelogram or* ***polygone*** *and* ***sine law*** *should be applied.*



## 2·5 Addition of Forces by Summing x and y Components :

The addition of forces can be made by algebraically addition of their rectangular components.

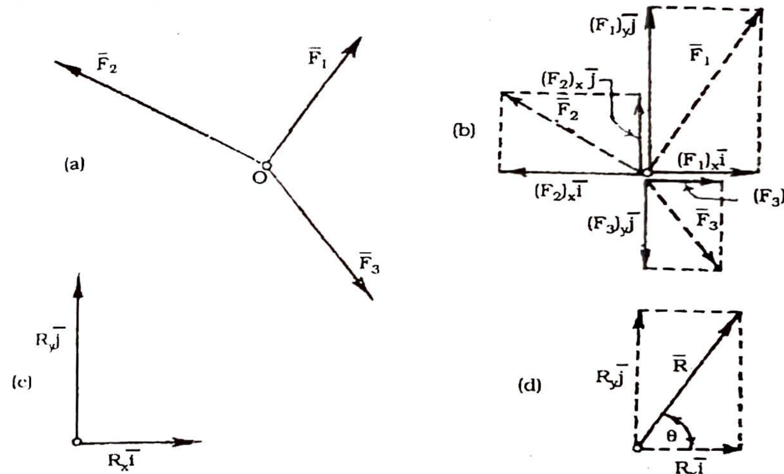



Fig 2.12

Three forces acting at O are resolved into their **x and y rectangular components.**

Here, $\bar{R} = \Sigma F = \bar{F}_1 + \bar{F}_2 + \bar{F}_3$

and $\bar{R} = R_x\,\bar{i} + R_y\,\bar{j}$ $\qquad R = \sqrt{R_x^2 + R_y^2}$

$= (\Sigma F_x)\,\bar{i} + (\Sigma F_y)\,\bar{j}$ $\qquad \tan\theta = \dfrac{R_y}{R_x}$

Where, $R_x\,\bar{i} = \Sigma F_x\,\bar{i} = (F_1)_x\,\bar{i} + (F_2)_x\,\bar{i} + (F_3)_x\,\bar{i}$

$R_y\,\bar{j} = \Sigma F_y\,\bar{j} = (F_1)_y\,\bar{j} + (F_2)_y\,\bar{j} + (F_3)_y\,\bar{j}$

It must be noted that, ***x component of resultant is equal to the sum of the x components of individual forces.***

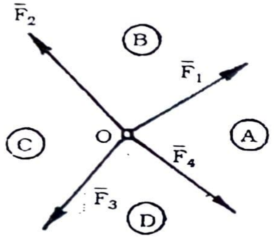
## 2·6 Equilibrium of a Particle :

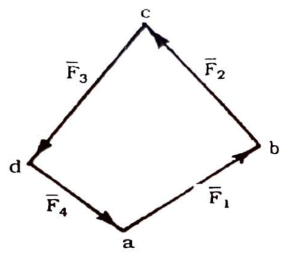
*When the resultant of all the forces acting on a particle is zero, the particle is said to be in equilibrium.*

Here, $\Sigma F_x = 0$ $\qquad R = 0 = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$

$\Sigma F_y = 0$

*The force polygone will be the closed one and having no resultant.*




**(a) Space diagram** **Fig 2·13** **(b) Force diagram**

In above figure particle O is in equilibrium under the action of four **concurrent coplanar forces** - $F_1$ to $F_4$. $F_1$ force is drawn as $\overline{ab}$ vector and similarly $F_2$ force as $\overline{bc}$ vector. Finally, $F_4$ is drawn as $\overline{da}$ vector. Here the point on which forces acting is under *equilibrium*, hence $\overline{da}$ should intersect with $\overline{ab}$ at **a** only. There should not be ***closing error*** and hence resultant should be zero.

## 2·7 Rectangular Components of a Force in Space :

**(1) For the given angles of force with axes :**

The problems involving the *three dimensions in space* will be considered in this section.

Here, force $\overline{F}$ is in space means having three coordinate system. $\lambda$ is an unit vector along $\overline{F}$. The force $\overline{F}$ making an angle $\theta_y$ with y axis. Hence, a parallelogram can be drawn with $\overline{F}$ as diagonal and $F_y$, $F_h$ as sides. First, resolve this $\overline{F}$ in xoz plane ( $F_h$) and along y direction ( $F_y$) according to the given angles. Then $F_h$ should again be resolved along x and z directions.

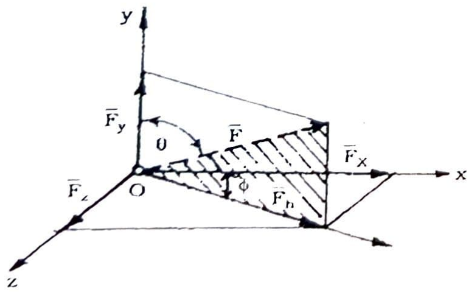



**Fig. 2·14**

$$\overline{F} = F\,\overline{\lambda}$$
$$F_y = F\cos\theta_y$$
$$F_h = F\sin\theta_y$$
$$F = \sqrt{F_y^2 + F_h^2}$$
$$F_x = F_h\cos\varnothing$$
$$= F\sin\theta_y\cos\varnothing$$
$$F_z = F_h\sin\varnothing$$
$$= F\sin\theta_y\sin\varnothing$$
$$F_h = \sqrt{F_x^2 + F_z^2}$$
$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$



If the **angles with the axes** are given then

$$F_x = F\cos\theta_x,\quad F_y = F\cos\theta_y,\quad F_z = \cos\theta_z$$
$$\overline{F} = F_x\,\overline{i} + F_y\,\overline{j} + F_z\,\overline{k} = F\,\overline{\lambda}$$

where $\theta_x$, $\theta_y$ and $\theta_z$ are the angles of $\overline{F}$ with x, y and z axes respectively.

Again,

$$\lambda_x = \cos\theta_x$$
$$\lambda_y = \cos\theta_y \qquad \overline{\lambda} = \cos\theta_x\,\overline{i} + \cos\theta_y\,\overline{j} + \cos\theta_z\,\overline{k}$$
$$\lambda_z = \cos\theta_z$$

**(2) For the given coordinates of two points on a line of action of a force :**

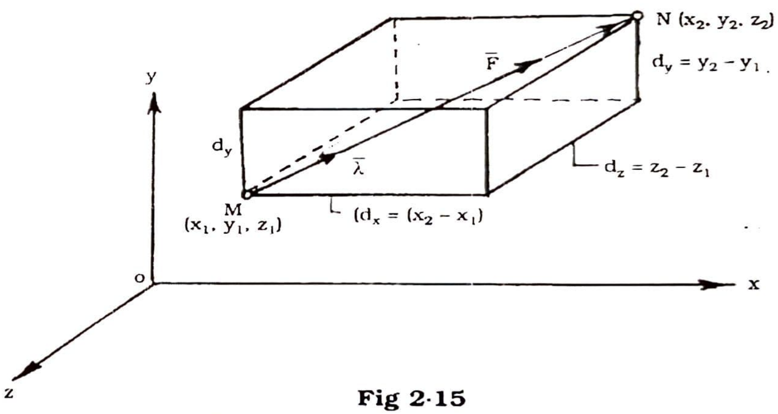



**Fig 2·15**

The force $\overline{F}$, having two points on its line of action i.e. M $(x_1, y_1, z_1)$ and N $(x_2, y_2, z_2)$, is shown in above figure.

Vector $\overline{MN} = d_x\,\overline{i} + d_y\,\overline{j} + d_z\,\overline{k}$, where $d_x$, $d_y$ and $d_z$ are components of d along three axes.

$$\text{Unit Vector } \overline{\lambda} = \frac{\overline{MN}}{MN} = \frac{1}{d}(d_x\,\overline{i} + d_y\,\overline{j} + d_z\,\overline{k})$$

where, $d_x = x_2 - x_1$ and $\lambda_x = \dfrac{d_x}{d} = \cos\theta_x$

$d_y = y_2 - y_1$ $\quad \lambda_y = \dfrac{d_y}{d} = \cos\theta_y$

$d_z = z_2 - z_1$ $\quad \lambda_z = \dfrac{d_z}{d} = \cos\theta_z$

$$d = \sqrt{d_x^2 + d_y^2 + d_z^2}$$

omponents, $F_x = \frac{F\,d_x}{d}$, $F_y = \frac{F\,d_y}{d}$, $F_z = \frac{F\,d_z}{d}$

and $\bar{F} = F_x\,\bar{i} + F_y\,\bar{j} + F_z\,\bar{k}$

## ·8 Addition of Concurrent Forces in Space :

If several concurrent forces are acting in space, then their esultant will be

$$\bar{R} = R_x\,\bar{i} + R_y\,\bar{j} + R_z\,\bar{k} = \Sigma F$$
$$= R\,\bar{\lambda}$$

$$R_x = \Sigma F_x \qquad \cos\theta_x = \frac{R_x}{R}$$

$$R_y = \Sigma F_y \qquad \cos\theta_y = \frac{R_y}{R}$$

$$R_z = \Sigma F_z \qquad \cos\theta_z = \frac{R_z}{R}$$

and $R = \sqrt{R_x^2 + R_y^2 + R_z^2}$

## ·9 Equilibrium of a Particle in Space :

*A particle is said to be in equilibrium if the resultant of all the rces acting on it is zero.*

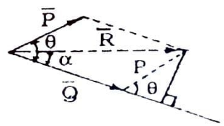
Then,
$$R_x = \Sigma F_x = 0$$
$$R_y = \Sigma F_y = 0$$
$$R_z = \Sigma F_z = 0$$

$$\bar{R} = 0,\ R_x\,\bar{i} + R_y\,\bar{j} + R_z\,\bar{k} = 0$$

## IMPORTANT EQUATIONS

. **Parallelowgram Law :**

$$R = \sqrt{P^2 + Q^2 + 2\,PQ\cos\theta}$$

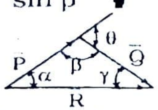



$$\tan\alpha = \frac{P\sin\theta}{Q + P\cos\theta}$$

. **Sine Law (Lami's Theorem) :** $\frac{P}{\sin\gamma} = \frac{Q}{\sin\alpha} = \frac{R}{\sin\beta}$

. **Cosine Law :** $R = \sqrt{P^2 + Q^2 - 2\,PQ\cos\beta}$

also $R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$




$\theta$ = included angle between $\bar{P}$ & $\bar{Q}$ *in Space diagram*

$\beta$ = included angle between $\bar{P}$ & $\bar{Q}$ *in Vector diagram*



**4. Rectangular Components :**

$$\bar{F} = F_x\,\bar{i} + F_y\,\bar{j} = F\,\bar{\lambda}$$
$$F_x = F\cos\theta$$
$$F_y = F\sin\theta$$
$$F = \sqrt{F_x^2 + F_y^2}$$

**5. Addition of Forces :**

$$\bar{R} = \Sigma F = \bar{F}_1 + \bar{F}_2 + \bar{F}_3$$
$$\bar{R} = R_x\,\bar{i} + R_y\,\bar{j}$$
$$= (\Sigma F_x)\,\bar{i} + (\Sigma F_y)\,\bar{j}$$
$$\Sigma F_x = (F_1)_x + (F_2)_x + (F_3)_x + \ldots$$
$$\Sigma F_y = (F_1)_y + (F_2)_y + (F_3)_y + \ldots$$

**6. Equilibrium of a Particle :**

$R = 0$; $\Sigma F_x = 0$, $\Sigma F_y = 0$ ...... (In plane)

$R = 0$; $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma F_z = 0$ ...... (In space)

**7. Force In Space :** $\bar{F} = F\lambda$

$$\bar{F} = F_x\,\bar{i} + F_y\,\bar{j} + F_z\,\bar{k}$$

$$\bar{\lambda} = \lambda_x\,\bar{i} + \lambda_y\,\bar{j} + \lambda_z\,\bar{k} \qquad F_x = \frac{F\,d_x}{d}$$

$$\lambda_x = \cos\theta_x = \frac{d_x}{d} \qquad F_y = \frac{F\,d_y}{d}$$

$$\lambda_y = \cos\theta_y = \frac{d_y}{d} \qquad F_z = \frac{F\,d_z}{d}$$

$$\lambda_z = \cos\theta_z = \frac{d_z}{d} \qquad d = \sqrt{d_x^2 + d_y^2 + d_z^2}$$

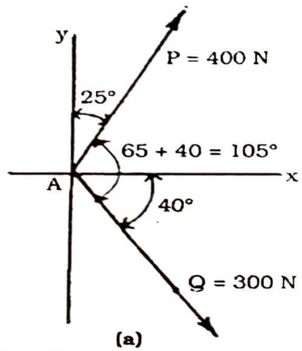
**8.** $\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi$

$\cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi.$

## SOLVED EXAMPLES

**1.** The two forces $\bar{P}$ and $\bar{Q}$ act on a point A as shown in Fig. 2·16 (a). Determine the magnitude of the resultant force and its direction measured counter clockwise from the positive x-axis.

***Trigonometric solution :*** From space diagram of two forces, a force diagram (triangle) can be drawn as shown here. The triangle rule is used. Two sides and the included angles are known. Resultant is determined by using the law of cosines.




Here θ = 105°

and β = 180 – 105

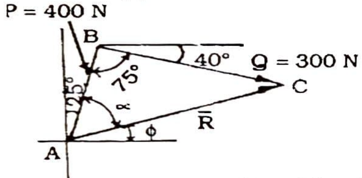
= 75°

$$R = \sqrt{P^2 + Q^2 - 2\,PQ\cos 75^\circ}$$

$$= \sqrt{(400\ N)^2 + (300\ N)^2 - 2\,(400\ N)\,(300\ N)\cos 75^\circ}$$

$$\boxed{R = 433{\cdot}46\ N}$$

The angle α is determined by applying the law of sines and using the computed value of R.




$$\frac{\sin\alpha}{Q} = \frac{\sin\beta}{R}$$

$$\frac{\sin\alpha}{300} = \frac{\sin 75^\circ}{433{\cdot}46}$$

$$\alpha = 41{\cdot}95^\circ$$

$$\varnothing = 90 - 25 - 41{\cdot}95 = 23{\cdot}05^\circ \qquad \boxed{\phi = 23{\cdot}05^\circ}$$

Here, angle φ is with positive x axis measured counter clockwise.

**Fig. 2.16**

If the force exerted on point A is 100N in the direction shown Fig 2·17, determine the force components acting along (a) the x ıd y axes, (b) the x' and y' axes, (c) the x and y' axes and (d) etermine the angle α, knowing that the component of force along e x axis is to be 80 N while resolving along the x and y' axes.

**(S.G. Uni. March '97)**

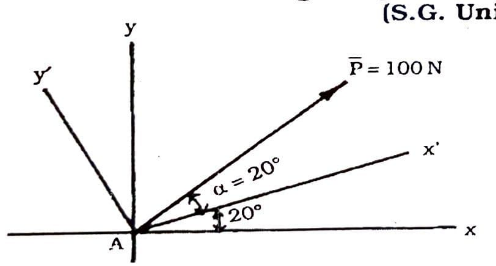


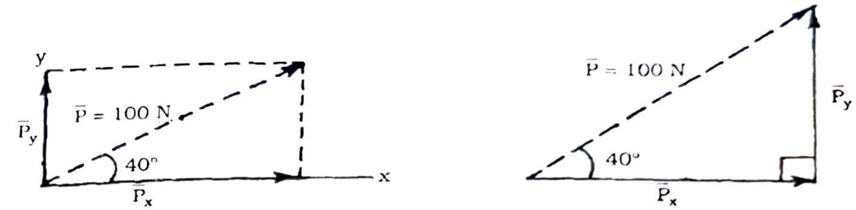

**Fig 2·17**

Here parallelogram law is used to resolve the force into two mponents, and then the sine rule is applied.

**(a) Resolving in x and y directions :**

(Two ***mutually perpendicular*** directions)






**(a) Fig. 2·18 (b)**

Using parallelogram law, the components along x and y axes are constructed.

$P_x = P\cos\theta = 100\cos 40^\circ$ $\quad\boxed{P_x = 76{\cdot}60\ N}$

$P_y = P\sin\theta = 100\sin 40^\circ$ $\quad\boxed{P_y = 64{\cdot}28\ N}$

**(b) Resolving in x' and y' directions :**

(Two ***mutually perpendicular*** directions)

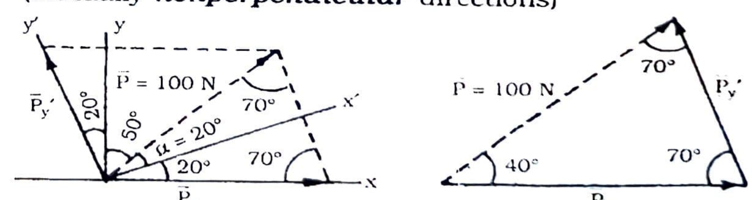

**(a) Fig 2·19 (b)**

$P_x' = P\cos 20^\circ = 100\cos 20^\circ$ $\quad\boxed{P_x' = 93{\cdot}97\ N}$

$P_y' = P\sin 20^\circ = 100\sin 20^\circ$ $\quad\boxed{P_y' = 34{\cdot}20\ N}$

**(c) Resolving in x and y' directions :**

(Mutually ***nonperpendicular*** directions)




**(a) (b)**

***Fig 2·20***

Here we need to resolve the force along x and y' axes. Construction of parallelogram needs special attention. Using vector triangle and applying law of sines.

$$\frac{P_y'}{\sin 40^\circ} = \frac{P}{\sin 70^\circ}$$

$$\frac{P_y'}{\sin 40^\circ} = \frac{100}{\sin 70^\circ}$$

$$\boxed{P_y' = 68.40 \text{ N}}$$

$$\frac{P_x}{\sin 70^\circ} = \frac{P}{\sin 70^\circ}$$

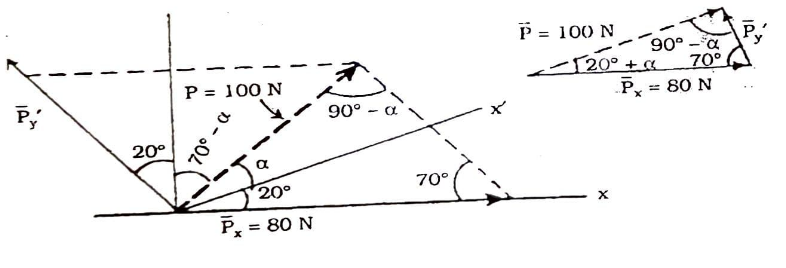
$$\frac{P_x}{\sin 70^\circ} = \frac{100}{\sin 70^\circ}$$

$$\boxed{P_x = 100 \text{ N}}$$

**Resolution along in x and y' :**

(Mutually ***non-perpendicular*** directions)

Here, angle $\alpha$ is to be determined. so that $P_x$ = 80 N while ›lving along x and y′ axes.




(a) Fig 2·21 (b)

Applying the same procedure adopted in above section (c).

$$\frac{P_x}{\sin (90^\circ - \alpha)} = \frac{P}{\sin 70^\circ}$$

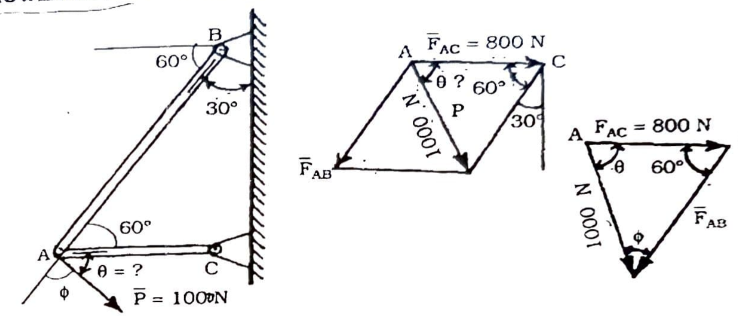
$$\frac{80}{\sin (90 - \alpha)} = \frac{100}{\sin 70^\circ}$$

$$\boxed{\alpha = 41.26^\circ}$$

**3.** The force $\bar{P}$ acting on the frame has a magnitude of 000 N and is to be resolved in to two compontents acting along truts AB and AC. Determine the angle $\theta$, so that the component $\bar{F}_{AC}$ is directed from A toward C and has a magnitude of 800 N.



Here, joint A is acted upon by three forces. Let **$\bar{P}$ is directed downward as shown below.**




(a) (b) (c)

**Fig 2·22**

By using the law of parallelogram, the vector addition of the two components is yielding the resultant as shown in above figure.The lines of action of components are known. The angle θ can be found out after finding Ø.

Using **sine rule :**

$$\frac{800}{\sin Ø} = \frac{1000}{\sin 60^\circ}$$

$$\therefore \sin Ø = \left(\frac{800}{1000}\right) \sin 60^\circ$$

$$\therefore Ø = 43.9^\circ$$

and hence

$$\theta = 180^\circ - 60^\circ - 43.9^\circ = 76.1^\circ. \qquad \boxed{\theta = 76.1^\circ}$$

Now

$$\frac{F_{AB}}{\sin 76.1^\circ} = \frac{1000}{\sin 60^\circ}$$

$$\therefore F_{AB} = (\sin 76.1^\circ)\frac{1000}{\sin 60^\circ} = 1120 \text{ N} \qquad \boxed{F_{AB} = 1120 \text{ N}}$$

**If $\bar{P}$ is directed upward as shown below,** then it will be having the θ angle with AC and will still produce the required component $\bar{F}_{AC}$.

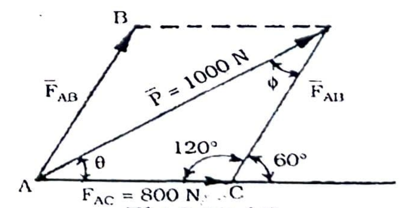



**Fig 2·22 (d)**

In this case,

$$\frac{800}{\sin \varnothing} = \frac{1000}{\sin 120^\circ}$$

$$\therefore \sin \varnothing = \left(\frac{800}{1000}\right) \sin 120^\circ$$

$$\therefore \varnothing = 43.9^\circ$$

Hence $\theta = 180^\circ - 120^\circ - 43.9^\circ = 16.1^\circ$

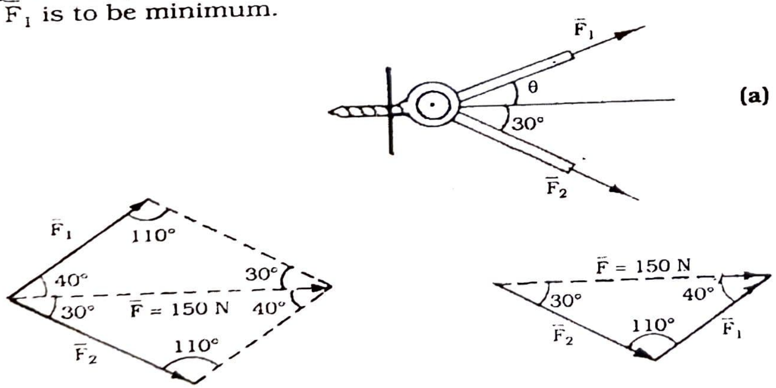
and

$$\frac{F_{AB}}{\sin 16.1^\circ} = \frac{1000}{\sin 120^\circ}$$

$$\therefore F_{AB} = (\sin 16.1^\circ)\frac{1000}{\sin 120^\circ} = 320.22 \text{ N}$$

| |
|---|
| For upward direction of $\bar{P}$ & having angle $\theta$ with AC, $\theta = 16.1^\circ$ and $F_{AB} = 320.22$ N<br>Here AC in Compression, AB in Tension |
| For downward direction of $\bar{P}$ & having angle $\theta$ with AC, $\theta = 76.1^\circ$ and $F_{AB} = 1120$ N<br>Here, AC in Compression, AB in compression |

The hook shown is subjected to two forces, $\bar{F}_1$ and $\bar{F}_2$. If it is quired that the resultant force have a magnitude of 150 N and be ected horizontal towards right, determine (a) the magnitudes of $\bar{F}_1$ and $\bar{F}_2$ provided $\theta = 40^\circ$ and (b) the magnitudes of $\bar{F}_1$ and $\bar{F}_2$ $\bar{F}_1$ is to be minimum.




(b) Fig 2.23 (c)



(a) Vector addition according to the parallelogram law is show in Fig 2.23 (b). From the vector triangle, the unknowns $F_1$ and F can be determined by using the law of sines.

$$\frac{F_1}{\sin 30^\circ} = \frac{F}{\sin 110^\circ}$$

$$\frac{F_1}{\sin 30^\circ} = \frac{150}{\sin 110^\circ}$$

$$\boxed{F_1 = 79.81 \text{ N}}$$

$$\frac{F_2}{\sin 40^\circ} = \frac{F}{\sin 110^\circ}$$

$$\frac{F_2}{\sin 40^\circ} = \frac{150}{\sin 110}$$

$$\boxed{F_2 = 102.61 \text{ N}}$$

(b) Here $F_1$ should be minimum. The direction of $F_2$ remain unchanged.

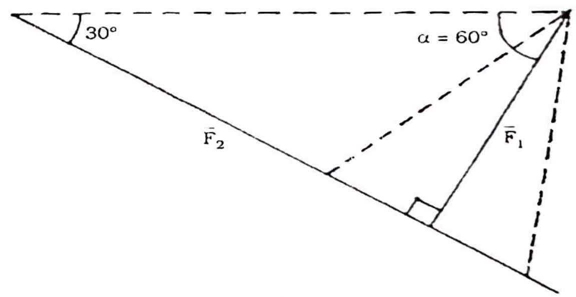



Fig 2.24

From the vector triangle, we can see that there are various ways to obtain $F_1$ and $F_2$. For the fixed direction of $F_2$, the force $F_1$ will be having minimum length when $F_1$ is perpendicular to $F_2$, i.e. $\alpha = 60^\circ$

$$\sin 30^\circ = \frac{F_1}{F} = \frac{F_1}{150}$$

$$\boxed{F_1 = 75 \text{ N}}$$

$$\cos 30^\circ = \frac{F_2}{F} = \frac{F_2}{150}$$

$$\boxed{F_2 = 129.90 \text{ N}}$$

5. Determine x and y components of $\bar{F}_1$, $\bar{F}_2$ and $\bar{F}_3$ shown in figure. Also express each force in vector form.

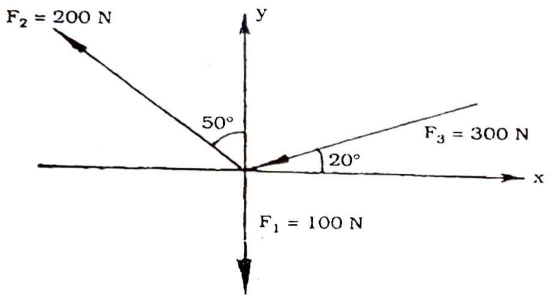



**Fig 2.25**

**Force $\bar{F}_1$ :** x component $= 0$ $\boxed{(F_1)_x = 0}$

y component $= -100$ N $\boxed{(F_1)_y = -100 N}$

**Scalar :** $\boxed{(F_1)_x = 0 \text{ and } (F_1)_y = 100\ N\downarrow}$

**Vector :** $\boxed{\bar{F}_1 = (-100\ N)\ \bar{j}}$

**Force $\bar{F}_2$ :** x component $= -200 \sin 50°$

$= -153.21$ N $\boxed{(F_2)_x = -153.21\ N}$

y component $= +200 \cos 50°$

$= 128.56$ N $\boxed{(F_2)_y = 128.56\ N}$

**Scalar :** $\boxed{(F_2)_x = 153.21\ N \leftarrow \text{and } (F_2)_y = 128.56\ N\uparrow}$

**Vector :** $\boxed{\bar{F}_2 = (-153.21\ N)\ \bar{i} + (128.56)\ \bar{j}}$

**Force $\bar{F}_3$ :** x component $= -300 \cos 20°$

$= -218.91$ N $\boxed{(F_3)_x = -218.91\ N}$

y component $= -300 \sin 20°$

$= -102.61$ N $\boxed{(F_3)_y = -102.61\ N}$

**Scalar :** $\boxed{(F_3)_x = 218.91\ N \leftarrow \text{and } (F_3)_y = 102.61\ N\downarrow}$

**Vector :** $\boxed{\bar{F}_3 = -(218.91\ N)\ \bar{i} - (102.61\ N)\ \bar{j}}$

6. Determine the magnitude and direction of the resultant of five forces. Specify its direction measured counter clockwise from the positive x axis. **(S.G. Uni, Dec. 1998)**



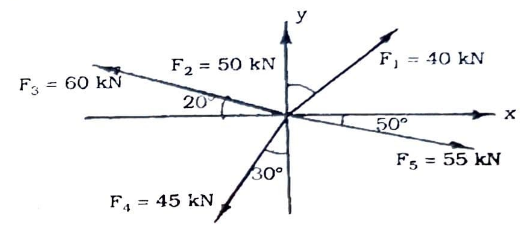



**Fig 2.26 (a)**

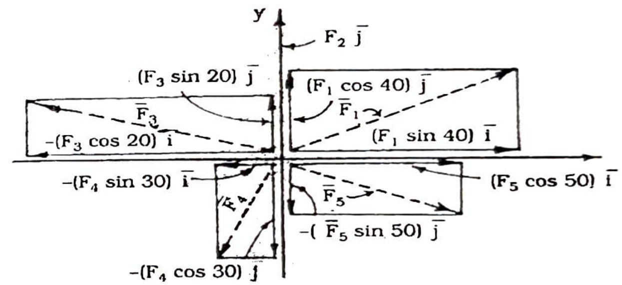



**Fig. 2.26 (b)**

| Force | x-component (N) | y-component (N) |
|---|---|---|
| $F_1$ = 40 kN | + 25.71 | + 30.64 |
| $F_2$ = 50 kN | 0 | + 50.00 |
| $F_3$ = 60 kN | - 56.38 | + 20.52 |
| $F_4$ = 45 kN | - 22.50 | - 38.97 |
| $F_5$ = 55 kN | + 35.35 | - 42.13 |
| | $R_x = -17.82$ | $R_y = +20.06$ |

Resultant $\bar{R}$ is .............

$$\bar{R} = R_x\ \bar{i} + R_y\ \bar{j}$$

$$\boxed{\bar{R} = (-17.82\ kN)\ \bar{i} + (20.06\ kN)\ \bar{j}}$$

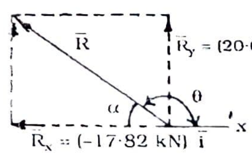



**Fig 2.27**

$$\tan \alpha = \frac{|R_y|}{|R_x|} = \frac{20.06\ kN}{17.82\ kN}$$

$$\alpha = 48.38°$$

$$\theta = 180° - \alpha = 131.62°$$

$$\boxed{\theta = 131.62°}$$

2

$$R = \sqrt{R_x^2 + R_y^2}$$

$$= \sqrt{(-17.82\ \text{kN})^2 + (20.06\ \text{kN})^2}$$

$$\boxed{R = 26.83\ \text{kN}}$$

·. Knowing that the connection is in equilibrium, determine the ıagnitudes of the forces $\bar{F}_4$ and $\bar{F}_5$.

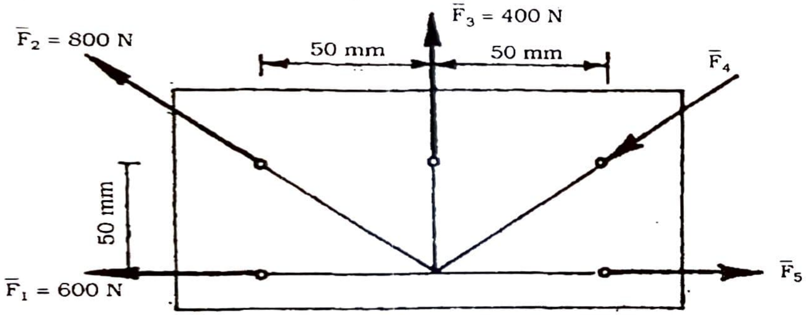
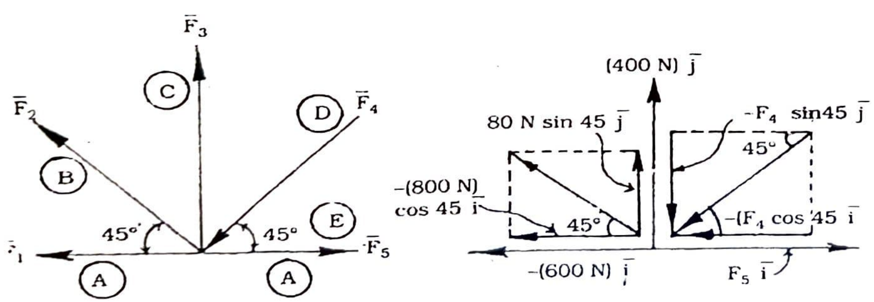



**Fig. 2·28**

The connection is considered as a particle which is subjected to ve concurrent forces. Free - body diagram shows action of various ›rces on connection.




**Free-body Diagram** **x and y components**

**Fig 2·29**

Since the connection is in equilibrium, the resultant of all forces ıust be zero i.e.

$$\bar{F}_1 + \bar{F}_2 + \bar{F}_3 + \bar{F}_4 + \bar{F}_5 = 0$$



$$-(600\ \text{N})\ \bar{i} - (800\ \text{N}) \cos 45°\ \bar{i} + (800\ \text{N}) \sin 45°\ \bar{j} + (400\ \text{N})\ \bar{j}$$
$$- F_4 \cos 45°\ \bar{i} - F_4 \sin 45°\ \bar{j} + F_5\ \bar{i} = 0$$

$$(-600\ \text{N} - 800\ \text{N} \cos 45° - F_4 \cos 45° + F_5)\ \bar{i}$$
$$+ (+800\ \text{N} \sin 45° + 400\ \text{N} - F_4 \sin 45°)\ \bar{j} = 0$$

The solution of the equation is that the coefficients of $\bar{i}$ and $\bar{j}$ must be zero, i.e. sum of force components along x and y axes is zero.

$$\Sigma F_x = 0,\ -600\ \text{N} - 800\ \text{N} \cos 45° - F_4 \cos 45° + F_5 = 0$$
$$\Sigma F_y = 0,\ +800\ \text{N} \sin 45° + 400\ \text{N} - F_4 \sin 45° = 0$$

Solving the two equations, we have

$$\boxed{\begin{aligned} F_4 &= 1365.69\ \text{N} \\ F_5 &= 2131.36\ \text{N} \end{aligned}}$$

Graphically also, we should able to draw closed polygone. Using Bow's notations indicated in free - diagram and by taking some scale, a pobygone is drawn here.

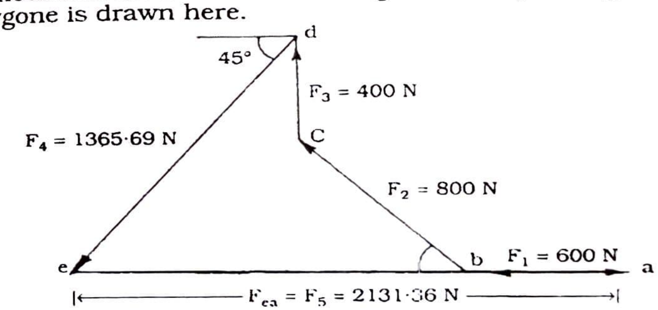
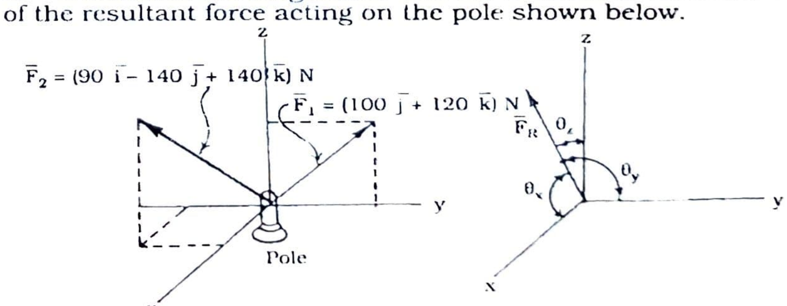



**Fig. 2·30**

Force polygon can help to check the correctness of the solution in this case.

**8.** Determine the magnitude and the coordinate direction angles of the resultant force acting on the pole shown below.




(a) **Fig 2·31** (b)

Here, force $F_2$ is in space having 3 components, where as $F_1$ is in plane having 2 components.

$$\bar{F}_R = \Sigma F = \bar{F}_1 + \bar{F}_2$$
$$= (100\,\bar{j} + 120\,\bar{k}) + (90\,\bar{i} - 140\,\bar{j} + 140\,\bar{k})$$
$$= (90\,\bar{i} - 40\,\bar{j} + 260\,\bar{k})\text{ N.}$$

The magnitude of $\bar{F}_R$ is

$$F_R = \sqrt{(90)^2 + (-40)^2 + (260)^2}$$
$$= 278\text{ N.} \qquad \boxed{F_R = 278\text{ N}}$$

Unit Vector acting along $\bar{F}_R$ is

$$\bar{\lambda}_{FR} = \frac{\bar{F}_R}{F_R} = \frac{90}{278}\,\bar{i} - \frac{40}{278}\,\bar{j} + \frac{260}{278}\,\bar{k}$$

$$\therefore\ \bar{\lambda}_{FR} = 0{\cdot}324\,\bar{i} - 0{\cdot}144\,\bar{j} + 0{\cdot}935\,\bar{k}$$

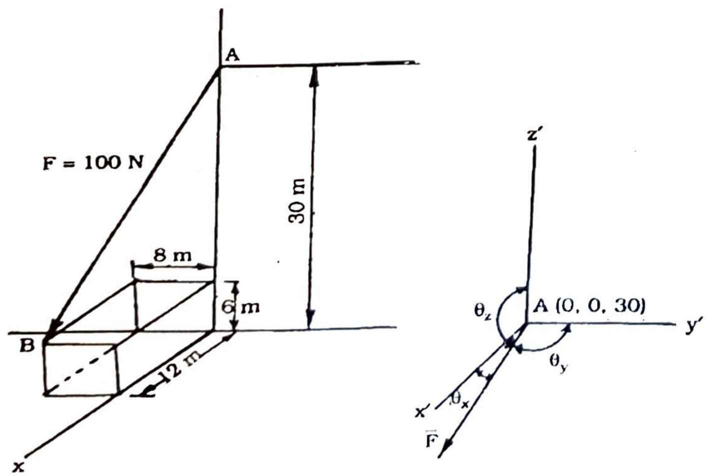
Hence three components of unit vector give direction cosines of the resultant.

$\lambda_x = \cos\theta_x = 0{\cdot}324$
$\lambda_y = \cos\theta_y = -0{\cdot}144$
$\lambda_z = \cos\theta_z = 0{\cdot}935$

$\therefore$ $\boxed{\theta_x = 71{\cdot}09^\circ,\ \theta_y = 98{\cdot}28^\circ,\ \theta_z = 20{\cdot}77^\circ}$

**9.** A man pulls the cord AB at B with a force of 100 N. Represent this force, acting on the support A, in a vector form and determine its direction.

At point A, local axes x′, y′ and z′ and Force F with its angles are shown in figure (b) below.




(a) **Fig 2.32** (b)



The coordinates of points are

$$A = (0, 0, 30)$$
$$B = (12, -8, 6)$$

Now, Unit vector along AB is

$$\bar{\lambda}_{AB} = \lambda_x\,\bar{i} + \lambda_y\,\bar{j} + \lambda_z\,\bar{k}$$

where $\lambda_x = \frac{d_x}{d} = \frac{x_B - x_A}{d_{AB}}$ Here, far end (B) minus near end (A)

Here $d_{AB} = \sqrt{(12)^2 + (-8)^2 + (-24)^2} = 28$ m.

$$\therefore\ \lambda_x = \frac{12}{28}$$
$$\lambda_y = \frac{-8}{28}$$
$$\lambda_z = \frac{-24}{28}$$

$$\therefore\ \bar{\lambda}_{AB} = \left(\frac{12}{28}\right)\bar{i} + \left(\frac{-8}{28}\right)\bar{j} + \left(\frac{-24}{28}\right)\bar{k}$$

And $\bar{F} = F\,\bar{\lambda}_{AB}$

$$= 100\text{ N}\left(\frac{12}{28}i - \frac{8}{28}j - \frac{24}{28}k\right)$$

$$\therefore\ \boxed{\bar{F} = (42{\cdot}46\text{ N})\,\bar{i} - (28{\cdot}57\text{ N})\,\bar{j} - (85{\cdot}71\text{ N})\,\bar{k}}$$

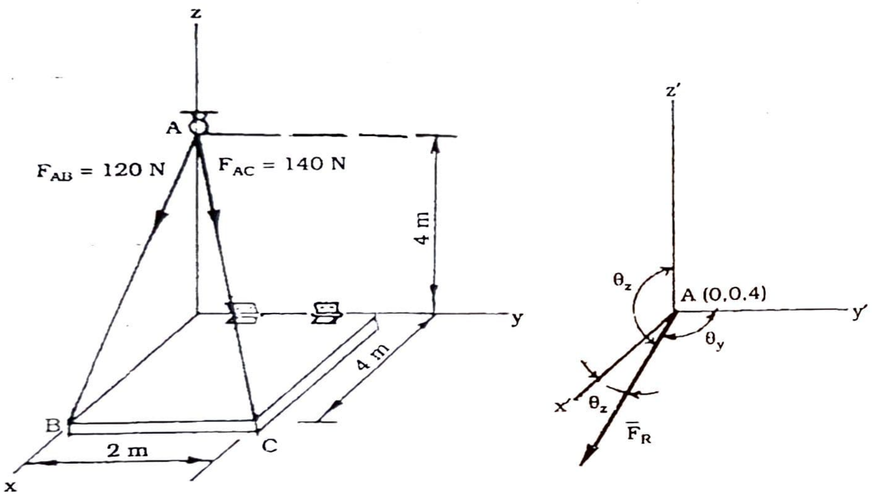
and direction of this force $\bar{F}$ is

$$\theta_x = \cos^{-1}\lambda_x = \cos^{-1}\left(\frac{12}{28}\right) = 64{\cdot}6^\circ. \qquad \boxed{\theta_x = 64{\cdot}6^\circ}$$

$$\theta_y = \cos^{-1}\lambda_y = \cos^{-1}\left(\frac{-8}{28}\right) = 107^\circ \qquad \boxed{\theta_y = 107^\circ}$$

$$\theta_z = \cos^{-1}\lambda_z = \cos^{-1}\left(\frac{-24}{28}\right) = 149^\circ \qquad \boxed{\theta_z = 149^\circ}$$

**10.** Two cables exert forces $F_{AB} = 120$ N and $F_{AC} = 140$ N on the ring at A as shown in figure. Determine the magnitude and direction of the resultant force acting at A.




(a) **Fig 2·33** (b)

The coordinates of points are

A = (0, 0, 4)

B = (4, 0, 0)

C = (4, 2, 0)

Here $\bar{F}_{AB} = F_{AB}\ \bar{\lambda}_{AB}$

[Far end (B) – Near end (A)] : $\bar{\lambda}_{AB} = (\lambda_{AB})_x\ \bar{i} + (\lambda_{AB})_y\ \bar{j} + (\lambda_{AB})_z\ \bar{k}$

$$= \frac{(4-0)}{5{\cdot}66}\ \bar{i} + \frac{(0-0)}{5{\cdot}66}\ \bar{j} + \frac{(0-4)}{5{\cdot}66}\ \bar{k}$$

$$= \frac{4}{5{\cdot}66}\ \bar{i} - \frac{4}{5{\cdot}66}\ \bar{k}$$

$$\bar{F}_{AB} = 120\ \left(\frac{4}{5{\cdot}66}\ \bar{i} - \frac{4}{5{\cdot}66}\ \bar{k}\right)$$

$$= \{84{\cdot}81\ \bar{i} - 84{\cdot}81\ \bar{k}\}\ \text{N}$$

Similarly $\bar{F}_{AC} = F_{AC}\ \bar{\lambda}_{AC} = 140\ \left(\frac{4}{6}\ \bar{i} + \frac{2}{6}\ \bar{j} - \frac{4}{6}\ \bar{k}\right)$

$$= \{\ 93{\cdot}33\ \bar{i} + 46{\cdot}67\ \bar{j} - 93{\cdot}33\ \bar{k}\}\ \text{N}$$

Resultant force

$$\bar{F}_R = \bar{F}_{AB} + \bar{F}_{AC} = \{84{\cdot}81\ \bar{i} - 84{\cdot}81\ \bar{k}\}\ \text{N} + \{93{\cdot}33\ \bar{i} + 46{\cdot}67\ \bar{j} - 93{\cdot}33\ \bar{k}\}\ \text{N}$$

$$\bar{F}_R = \{\ 177{\cdot}74\ \bar{i} + 46{\cdot}67\ \bar{j} - 178{\cdot}14\ \bar{k}\}\ \text{N}$$



The magnitude of $\bar{F}_R$ is

$$F_R = \sqrt{(177{\cdot}74)^2 + (46{\cdot}67)^2 + (-178{\cdot}14)^2} \qquad \therefore\ F_R = 256\ \text{N}$$

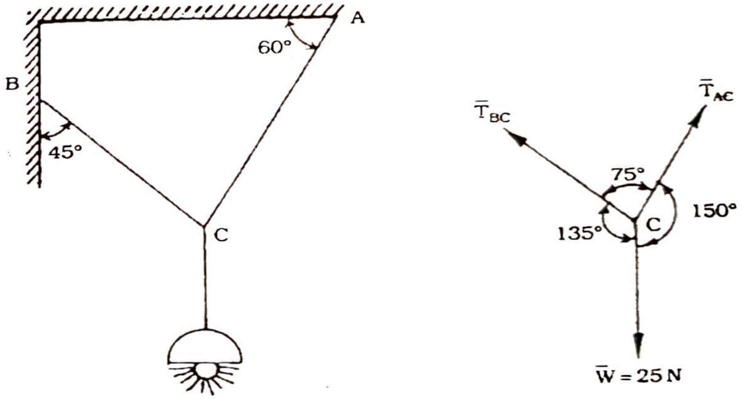
The direction of resultant $\bar{F}_R$ can be given as

$$\theta_x = \cos^{-1}\lambda_x = \cos^{-1}\frac{177{\cdot}74}{256} = 46{\cdot}03^\circ \qquad \theta_x = 46{\cdot}03^\circ$$

$$\theta_y = \cos^{-1}\lambda_y = \cos^{-1}\frac{46{\cdot}67}{256} = 79{\cdot}50^\circ \qquad \theta_y = 79{\cdot}5^\circ$$

$$\theta_z = \cos^{-1}\lambda_z = \cos^{-1}\frac{-178{\cdot}14}{256} = 134{\cdot}1^\circ \qquad \theta_z = 134{\cdot}1^\circ$$

**11.** An electric light fixture weighing 25 N hangs from a point C, by two strings AC and BC as shown in figure. Determine forces in AC and BC.




(a) **Fig 2·34** (b)

Here, point C is under equilibrium.

Two unknown forces are $\bar{T}_{BC}$ and $\bar{T}_{AC}$.

This can be solved by Lami's theorm (Sine Rule)

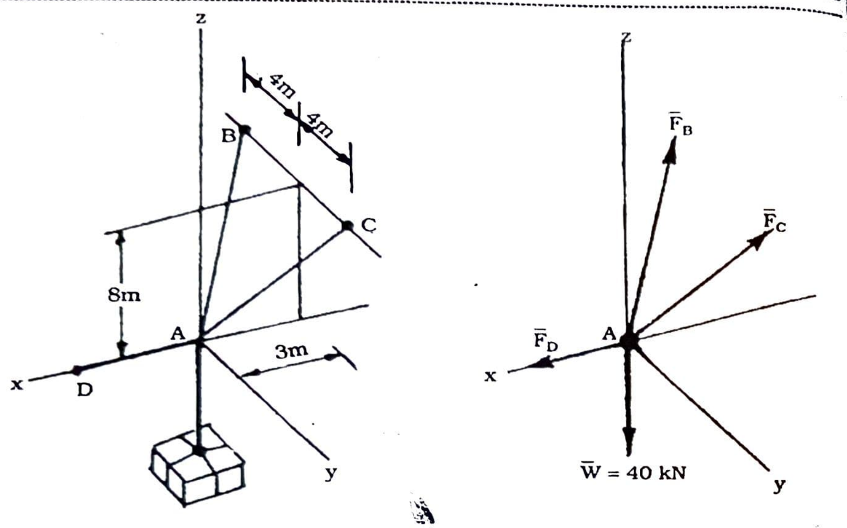
or equilibrium equation $\Sigma F = 0$

Using Lami's Theorem (Sine Rule)

$$\frac{25}{\sin 75^\circ} = \frac{T_{BC}}{\sin 150^\circ} = \frac{T_{AC}}{\sin 135^\circ}$$

$$\therefore \quad \boxed{\begin{aligned} T_{BC} &= 12{\cdot}94\ \text{N} \\ T_{AC} &= 18{\cdot}3\ \text{N} \end{aligned}}$$

**12.** Determine the force developed in each of the three cables used to support the 40 kN crate as shown in figure.




(a) Fig 2·35 (b)

The coordinates of B & C are B (−3, −4, 8) and C (−3, 4, 8).

Here, the point A is in equilibrium under the action of four forces.

Hence $\Sigma F_x = 0$, $\Sigma F_y = 0$ and $\Sigma F_z = 0$ at A

| Sr. No. | Force | x component | y component | z component |
|---|---|---|---|---|
| 1. | $\bar{F}_B$ | $\left(\frac{-3}{9.43} F_B\right) \bar{i}$ $= -0.318 F_B \bar{i}$ | $\left(\frac{-4}{9.43} F_B\right) \bar{j}$ $= -0.424 F_B \bar{j}$ | $\left(\frac{8}{9.43} F_B\right) \bar{k}$ $= +0.848 F_B \bar{k}$ |
| 2. | $\bar{F}_C$ | $\left(\frac{-3}{9.43} F_C\right) \bar{i}$ $= -0.318 F_C \bar{i}$ | $\left(\frac{+4}{9.43} F_C\right) \bar{j}$ $= +0.424 F_C \bar{j}$ | $\left(\frac{8}{9.43} F_B\right) \bar{k}$ $= +0.848 F_C \bar{k}$ |
| 3. | $\bar{F}_D$ | $F_D \bar{i}$ | 0 | 0 |
| 4. | $\bar{W}$ | 0 | 0 | $-40 \bar{k}$ |

Now $\Sigma F_x = 0;\ -0.318 F_B - 0.318 F_C + F_D = 0$

$\Sigma F_y = 0;\ -0.424 F_B + 0.424 F_C = 0$

$\Sigma F_z = 0;\ 0.848 F_B + 0.848 F_C - 40 = 0$

From above equations, we get

$$\boxed{\begin{aligned} F_B = F_C &= 23.6 \text{ kN} \\ F_D &= 15 \text{ kN} \end{aligned}}$$



13. The pole is held in place by three cables. If the force of each cable acting on the pole is as shown, determine the position (x, y, o) for fixing cable DC at C so that the resultant force exerted on the pole is directed along its axis.

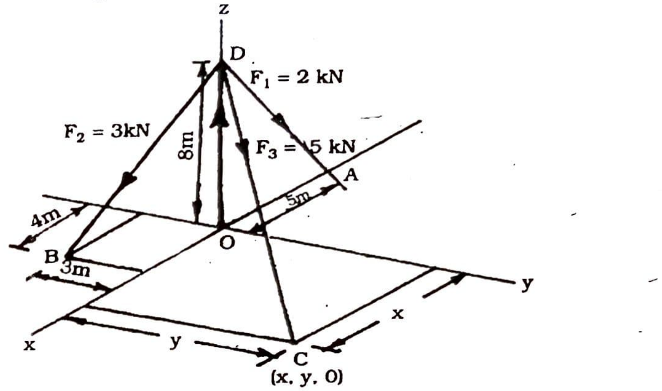



Fig 2·36

The coordinates of points are:

A = (−5, 0, 0)
B = (4, −3, 0)
C = (x, y, 0)
D = (0, 0, 8)
O = (0, 0, 0)

Point D is subjected to four forces. Resultant of three cable forces should be along DO (from D to O). But D point is in equilibrium, hence resultant force must be equal to internal force in pole OD (from O to D). Resolving each force into rectangular components :

$\overline{DA} = (-5\text{ m})\ \bar{i} + (0)\ \bar{j} + (-8\text{m})\ \bar{k}$ $\quad DA = 9.43$ m

$\overline{DB} = (+4\text{ m})\ \bar{i} + (-3\text{m})\ \bar{j} + (-8\text{m})\ \bar{k}$ $\quad DB = 9.43$ m

$\overline{DC} = (x\text{ m})\ \bar{i} + (y\text{ m})\ \bar{j} + (-8\text{m})\ \bar{k}$ $\quad DC = \sqrt{x^2 + y^2 + 64}$

$\overline{OD} = (0\text{m})\ \bar{i} + (0\text{m})\ \bar{j} + (8\text{m})\ \bar{k}$ $\quad DO = 8$ m

$\bar{F}_1 = F_1 \bar{\lambda}_{DA} = -1.06\, i - 1.70\, \bar{k}$, where $\lambda_{DA} = \frac{\overline{DA}}{DA}$ & $F_1$=2kN

$\bar{F}_2 = F_2\ \bar{\lambda}_{DB} = 1.27\ \bar{i} - 0.95\ \bar{j} - 2.54\ \bar{k}$ Here, $F_2$= 3 kN

$\bar{F}_3 = F_3\ \bar{\lambda}_{DC} = \frac{5x}{\sqrt{x^2 + y^2 + 64}}\ \bar{i} + \frac{5y}{\sqrt{x^2 + y^2 + 64}}\ \bar{j} - \frac{40}{\sqrt{x^2 + y^2 + 64}}\ \bar{k}$

$\bar{F}_4 = F_{OD}\ \bar{\lambda}_{OD} = F_{OD}\ \bar{k}$

At point D resultant force will be zero, as the point is in equilibrium. So either sum of all four forces can be equated to zero, OR sum of x and y components of three cable forces will be taken as zero as the resultant is along z axis.

$\Sigma F = 0$

$\bar{F}_{DA} + \bar{F}_{DB} + \bar{F}_{DC} + \bar{F}_{OD} = 0$

$(-1.06\ \bar{i} - 1.70\ \bar{k}) + (1.27\ \bar{i} - 0.95\ \bar{j} - 2.54\ \bar{k})$

$$+ \left( \frac{5x}{\sqrt{x^2+y^2+64}}\ \bar{i} + \frac{5y}{\sqrt{x^2+y^2+64}}\ \bar{j} - \frac{40}{\sqrt{x^2+y^2+64}}\ \bar{k} \right) + (F_{OD}\ \bar{k}) = 0$$

$$\left(-1.06 + 1.27 + \frac{5x}{\sqrt{x^2+y^2+64}}\right)\bar{i} + \left(-0.95 + \frac{5y}{\sqrt{x^2+y^2+64}}\right)\bar{j}$$

$$+ \left(-1.70 - 2.54 - \frac{40}{\sqrt{x^2+y^2+64}} + F_{OD}\right)\bar{k} = 0$$

This equation will be satisfied when coefficients of $\bar{i}$, $\bar{j}$ and $\bar{k}$ are zero, i.e. the sum of forces along x, y and z axes being zero.

$$-1.06 + 1.27 + \frac{5x}{\sqrt{x^2+y^2+64}} = 0$$

$$-0.95 + \frac{5y}{\sqrt{x^2+y^2+64}} = 0$$

$$-1.70 - 2.54 - \frac{40}{\sqrt{x^2+y^2+64}} + F_{OD} = 0$$

Solving these equations, we have

$\boxed{x = 0.38 \text{ m}, \quad y = 1.55 \text{ m}}$

**14.** Three coplanar forces act at a point on a bracket as shown in figure. Determine the value of the angle α such that the resultant of the three forces will be vertical. Also find the magnitude of the resultant. **(Pune Uni., Oct./Nov. '95)**

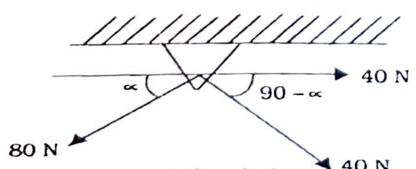



**Fig. 2·37**

Resultant is vertical, hence $\Sigma F_x = 0$

$-80 \cos \alpha + 40 + 40 \cos (90 - \alpha) = 0$

$\therefore -80 \cos \alpha + 40 + 40 \sin \alpha = 0$

$\therefore 40 (-2 \cos \alpha + \sin \alpha + 1) = 0$

$\therefore 2 \cos \alpha + \sin \alpha + 1 = 0$

$2 \cos \alpha - \sin \alpha = 1$

Putting different value of ∝ and getting

Left side = Right side, $\boxed{\alpha = 37°}$



Resultant

$\Sigma F_y = 80 \sin 37° + 40 \sin (90 - 37°)$

$\boxed{R = 80.09 \text{ N}}$

**15.** Concurrent forces of 1, 3, 5, 7, 9 and 11 N are applied at the centre of a regular hexagon acting towards vertices as shown in Fig. 2·38. Determine the resultant completely. **(Pune Uni., April '97)**

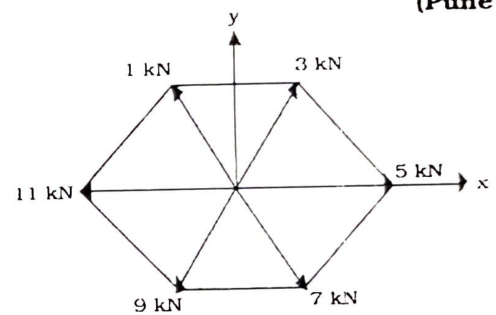



**Fig. 2·38**

Angle of 1, 3, 7, and 9 kN with x axis is 60°

$\Sigma F_x = 5 + 3 \cos 60° + 7 \cos 60° - 11 - 1 \cos 60° - 9 \cos 60° = -6$ kN

$\Sigma F_y = 1 \sin 60° + 3 \sin 60° - 7 \sin 60° - 9 \sin 60° = -10.39$ kN

$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = \sqrt{6^2 + 10.39^2}$

$= 12$ kN

$\theta = \tan^{-1}\left(\frac{-10.39}{-6}\right) = 60°$

**16.** The force $F_1$ is of 100N magnitude. The resultant of force $F_1$ and unknown force $F_2$ is 100 N in magnitude. Find the magnitude of $F_2$, if the resultant force is perpendicular to force $F_1$. **(Guj. Uni., June '90)**

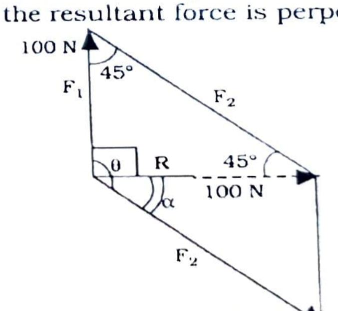



**Fig. 2·39**

$$\frac{R}{\sin 45°} = \frac{F_2}{\sin 90°} = \frac{F_1}{\sin 45°}$$

$$\frac{100}{\sin 45°} = \frac{F_2}{1} = \frac{100}{\sin 45°}$$

$\therefore \boxed{F_2 = 141.42 \text{ N}}$

$\alpha = \tan^{-1} \frac{100}{100} = 45°$

$\theta = 90° + 45° = 135°$

**17.** Four forces act on a bolt A as shown in Fig. 2·40. Determine the magnitude and direction of the resultant of the forces acting on a bolt. Comment on results. **(Guj. Uni., June '88)**

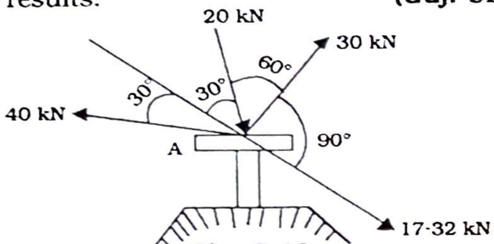



**Fig. 2·40**

Resolving along 17·32 kN force.

$$17{\cdot}32 + 20\cos 30° - 40\cos 30° = 0{\cdot}0$$

Resolving perpendicular to 17.32 kN force

$$30 - 20\cos 60° - 40\sin 30° = 0{\cdot}0$$

| Resultant is zero. |
|---|
| Bolt is under equilibrium. |

**18.** The forces 20 N, 30 N, 40 N, 50 N and 60 N are acting on one of the angular points of a *regular hexagon* towards the other five angular points taken in order. Find the magnitude and direction of the resultant force. **(AMIE Exam.)**

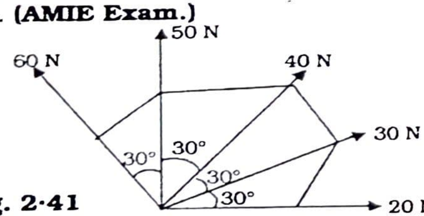



**Fig. 2·41**

$$\Sigma F_x = 20 + 30\cos 30° + 40\cos 60° - 60\sin 30° = 35{\cdot}98 \text{ N } (\rightarrow)$$

$$\Sigma F_y = 30\sin 30° + 40\sin 60° + 50 + 60\cos 30° = 151{\cdot}60 \text{ N } (\uparrow)$$

$$\therefore R = \sqrt{35{\cdot}98^2 + 151{\cdot}60^2}$$

$$\boxed{R = 155{\cdot}81 \text{ N}}$$

$$\theta = \tan^{-1}\frac{\Sigma F_y}{\Sigma F_x} = \tan^{-1}\frac{151{\cdot}6}{35{\cdot}98}$$

$$\boxed{\theta = 76{\cdot}649°}$$

## THEORY RELATED QUESTIONS

1. Define : **(i)** Force **(ii)** Coplanar forces **(iii)** Concurrent forces **(iv)** Collinear forces **(v)** Bow's Notations **(vi)** Nonconcurrent forces.



2. Draw and explain : **(i)** Space diagram **(ii)** Free – body diagram **(iii)** Vector or Force diagram.
3. Explain in brief : Equilibrium of particle in plane and space.
4. Write only, Conditions of equilibrium of particle. **(Pune Uni. April '96)**
5. State Lami's theorem. In which situation is it used ? **(Pune Uni. Oct./Nov. '95)**
6. State the law of parallelogram of forces. **(Pune Uni. Oct./Nov. '95)**

## EXERCISES

**2·1.** Determine the magnitude and direction of the resultant o the two forces shown in Fig 2·42.

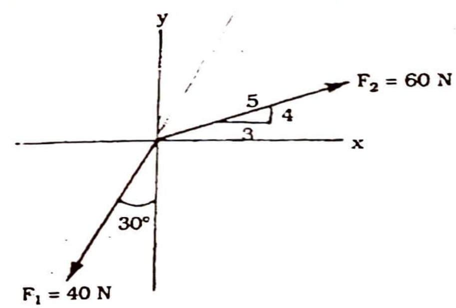



**Fig. 2·42**

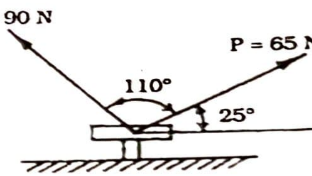



**Fig. 2·43**

**2·2.** The two Forces P and Q act on a bolt A. Determine the magnitude and direction of the resultant (Fig. 2·43). **(S.G. Uni. Dec. '98)**

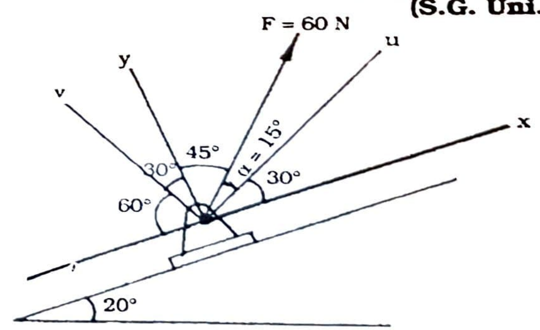



**Fig 2·44**

**2·3.** Resolve the 60 N force acting on the pin into components along (a) the x and y axes (b) the u and v axes (c) the x and v axes, and (d) the u and y axes (Fig 2·44).

**2·4.** Determine the angle α, knowing that the components of force along x axis is to be 30 N while resolving along the x and v axes (Fig 2·44).

**2·5.** The force $\overline{F}$ acting on the frame has a magnitude of 600 N and is to be resolved into two components acting along struts AB and AC. Determine the angle θ so that the component $F_{AC}$ is directed from A towards C and has a magnitude of 350 N. (Fig. 2·45).

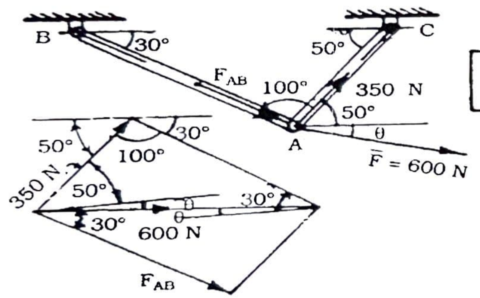



Fig 2·45

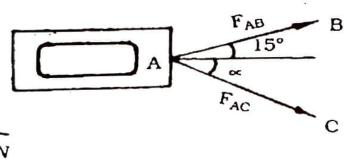



Fig 2·46

**2·6.** A disabled automobile is pulled by two ropes as shown in Fig 2·46. If the resultant of the two forces exerted by the ropes is 4 kN, which is parallel to the axis of the automobile find (a) the tension in each of the ropes knowing that α = 25° (b) the value of α such that the tension in rope AC is minimum.

**2·7.** If the resultant of the two forces acting on the barge is to be directed along the positive x axis and has a magnitude of 3 kN, determine the angle θ of the cable attached to the tag at A such that the force $F_a$ in this cable is minimum. What is the magnitude of force in each cable when this occurs ? (Fig 2·47)

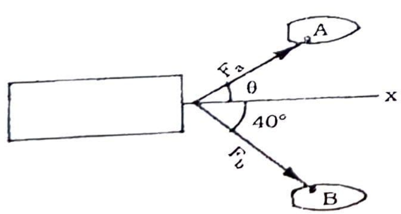



Fig 2·47

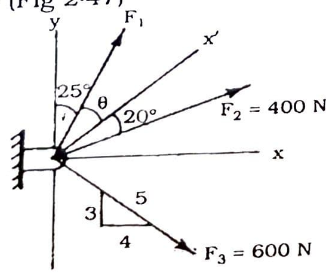



Fig 2·48



**2·8.** Three forces act on the bracket. Determine the magnitude and direction of $F_1$ so that the resultant force is directed along the positive x' axis and has a magnitude of 1 kN. (Fig. 2·48).

**2·9.** Determine the value of α for which the resultant of the forces acting at A is directed along horizontal axis. (Fig 2·49)

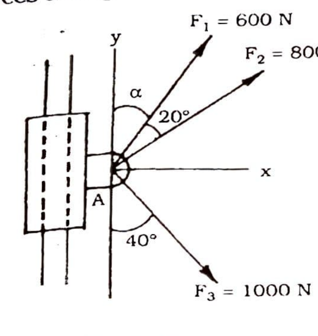



Fig 2·49

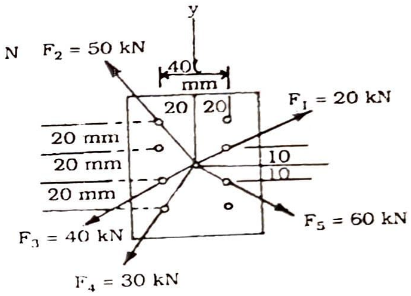



Fig 2·50

**2·10.** Five concurrent forces act on the plate. Determine the magnitude of the resultant force and its direction, measured counter clockwise from the positive x - axis (Fig 2·50).

**2·11.** Two cables which have known tensions are attached at point B. A third cable AB is used as a guy wire and is also attached at B. Determine the required tension in AB so that the resultant of the forces exerted by the three cables will be vertical (Fig 2·51).

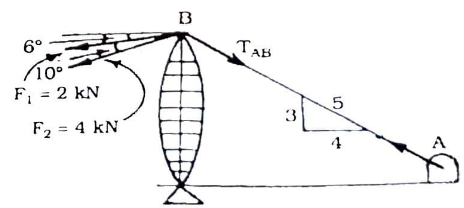



Fig 2·51

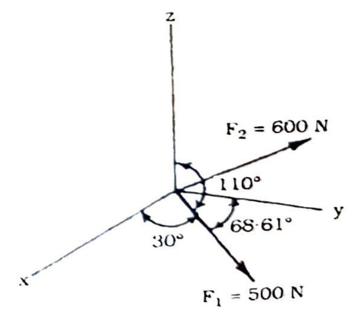
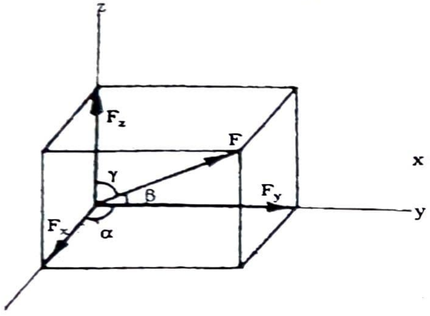



Fig 2·52

**2·12.** Two forces act as shown in Fig 2·52. Specify the direction of $F_2$ so that the resultant force acts along the positive y axis and has a magnitude of 0·7 kN.

**2·13.** Force $\vec{F}$ has components acting along the x, y and z axes
s shown in Fig 2·53. If the magnitude of F is 4 kN, and $\beta = 40°$ and
$= 80°$, determine the magnitude of its three components.



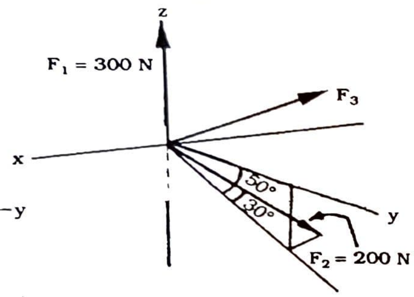
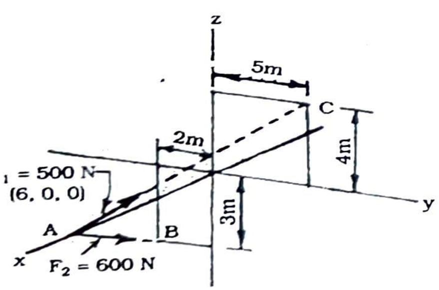




**Fig 2·53** **Fig 2·54**

**2·14.** Determine the magnitude and coordinate direction angles
f $F_3$ so that the resultant of the three forces is zero (Fig 2·54).

**2·15.** Express each of the forces in cartesian vector form and
ıen determine the magnitude and coordinate direction angles of
ıe resultant force (Fig 2·55).



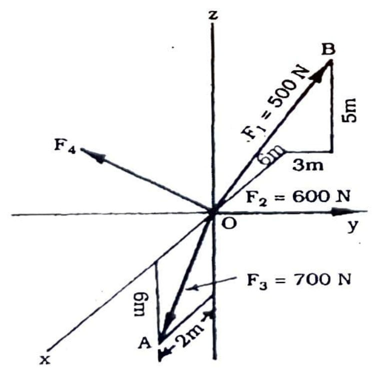
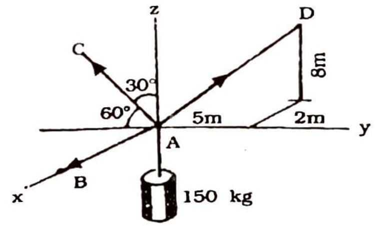




**Fig 2·55** **Fig 2·56**

**2·16.** Determine the magnitude and coordinate direction angles
f force $F_4$ for equilibrium of point O (Fig 2·56).

**2·17.** The 150 kg cylinder is supported by three cords, determine
ıe tension in each cord for equilibrium (Fig 2·57).





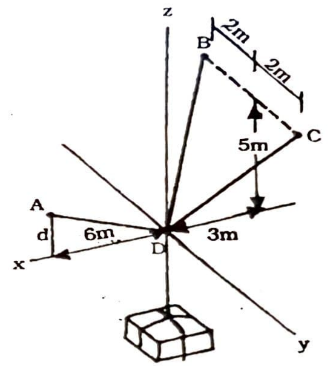




**Fig 2·57** **Fig 2·58**

**2·18.** Determine the height d of the cable AD so that the force in all cables are of equal magnitude. The crate has a mass of 60 kg (Fig 2·58).

**2·19.** Determine the force in each cable needed to support the 2kN platform shown in Fig 2·59.

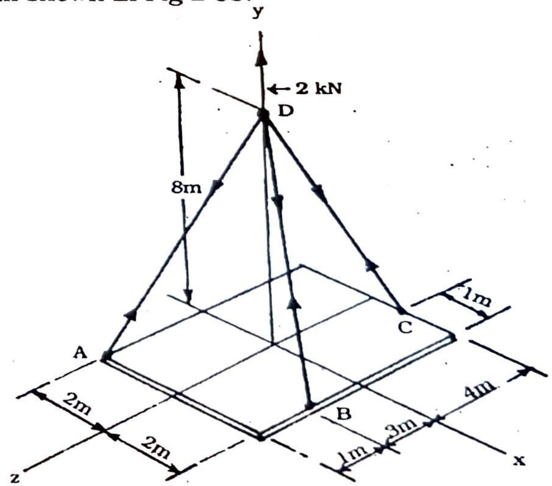



**Fig 2·59**

3

## SOLUTIONS OF EXERCISES

**2·1.** Here x and y components of $\overline{F}_1$ are negative and that of $\overline{F}_2$ are positive.

$$\Sigma F_x = -40 \sin 30° + 60 \times 3/5 = 16 \text{ N},$$

$$\Sigma F_y = -40 \cos 30° + 60 \times 4/5 = 13{\cdot}36 \text{ N}$$

$$R = \sqrt{16^2 + 13{\cdot}36^2} = 20{\cdot}84 \text{ N}, \tan\theta = \frac{13{\cdot}36}{16}$$

$$\theta = 39{\cdot}86°$$

**2·2.** $R = \sqrt{65^2 + 90^2 + 2 \times 65 \times 90 \cos 110°}$

$R = 91{\cdot}23$ N

$$\propto \text{ with } P = \tan^{-1}\left(\frac{Q \sin 110°}{P + Q \cos 110°}\right)$$

$\propto$ with $P = 67{\cdot}97°$, 92·97° (with horizontal)

**2·3.** **(a)** Here, x and y axes are ***mutually perpendicular.***

$$F_x = 60 \cos 45° = 42{\cdot}43 \text{ N},$$

$$F_y = 60 \sin 45° = 42{\cdot}43 \text{ N}$$

**(b)** Here, u and v axes are ***mutually perpendicular.***

$$F_u = 60 \cos 15° = 57{\cdot}96 \text{ N}$$

$$F_v = 60 \sin 15° = 15{\cdot}53 \text{ N}$$

**(c)** Here, x and v are ***not perpendicular*** to each other, hence draw parallelogram.

$$\frac{F}{\sin 60°} = \frac{F_x}{\sin 75°} = \frac{F_v}{\sin 45°} \quad \therefore \quad F_x = 66{\cdot}92 \text{ N}, \quad F_v = 48{\cdot}99 \text{ N}$$

**(d)** Similarly u and y axes are **mutually non – perpendicular** hence from parallelogram,

$$\frac{F}{\sin 120°} = \frac{F_y}{\sin 15°} = \frac{F_u}{\sin 45°} \quad \therefore \quad F_u = 48{\cdot}99 \text{ N}, \quad F_y = 17{\cdot}93 \text{ N}$$

**2·4.** As per above problem **2·3 (c)**, $\dfrac{F}{\sin 60°} = \dfrac{F_x}{\sin(90-\alpha)} = \dfrac{F_v}{\sin(30+\alpha)}$

$$\frac{60}{\sin 60°} = \frac{30}{\sin(90-\alpha)} = \frac{F_v}{\sin(30+\alpha)} \qquad \alpha = 64{\cdot}34°$$

**2·5.** From parallelogram shown in Fig. 2·45.

$$\frac{600}{\sin 100°} = \frac{350}{\sin(30-\theta)} \quad \therefore \quad \theta = -5{\cdot}06°$$

5·06° $\theta$

**2·6.** **(a)** By constructing parallelogram having horizontal resultant of 4 kN,



$$\frac{F_{AB}}{\sin 25°} = \frac{4000}{\sin 140°} = \frac{F_{AC}}{\sin 15°} \quad \therefore \quad F_{AB} = 2629{\cdot}9 \text{ N}, \quad F_{AC} = 1610{\cdot}6 \text{ N}$$

**(b)** By drawing the vector triangle, it can be seen that $F_{AC}$ will be minimum when it is perpendicular to $F_{AB}$, means when $\alpha = 75°$.

**2·7.** In vector triangle, $F_a$ will be minimum when $F_b$ will be perpendicular to $F_a$, means $\theta = 50°$ and

$$F_a = 3000 \cos 50° = 1928{\cdot}4 \text{ N}$$

$$F_b = 3000 \cos 40° = 2298{\cdot}1 \text{ N}$$

**2·8.** Here three forces are acting and resultant is along x′ axis having magnitude 1000N (y′ component = 0). Therefore resolving these three forces along x′ and y′ axes : (Here $F_3$ is having horizontal and vertical components which are again resolved along x′ and y′)

$$\textbf{X}'\ \textbf{comp}: 1000 = F_1 \cos\theta + 400 \cos 20° + 600 \times \frac{4}{5} \times \cos\{90° - (25° + \theta)\} - 600 \times \frac{3}{5} \cos(25° + \theta)$$

$$\textbf{Y}'\ \textbf{comp}: 0 = F_1 \sin\theta - 400 \sin 20° - 600 \times \frac{4}{5} \sin\{90° - (25° + \theta)\} - 600 \times \frac{3}{5} \sin(25° + \theta)$$

Putting, $\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi$

and $\cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi$

$$\text{we get, } 1 = 0{\cdot}219 \frac{\cos\theta}{\sin\theta} + 0{\cdot}651 \cos\theta + 0{\cdot}941 \sin\theta - 0{\cdot}453 \frac{\cos^2\theta}{\sin\theta}$$

From trial and error (i.e. by putting different value of $\theta$ say 10°, 20°, 30° and checking the sum equal to 1), $\theta = 79{\cdot}14°$

and $F_1 = 375{\cdot}36$ N

**2·9.** Resultant is along x axis hence y component of the resultant is zero.

$$600 \cos\alpha + 800 \cos(\alpha + 20°) = 1000 \cos 40°$$

Putting $\cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$

$$600 \cos\alpha + 800(\cos\alpha\cos 20° - \sin\alpha\sin 20°) = 1000 \cos 40°$$

From which, $1{\cdot}765 \cos\alpha - 0{\cdot}355 \sin\alpha = 1.$

From trial & error (i.e. by putting different value of $\alpha$), $\alpha = 44{\cdot}8°$

**2·10.** $\Sigma F_x = 20 \times \dfrac{20}{22\cdot36} - 50 \times \dfrac{20}{36\cdot06} - 40 \times \dfrac{20}{22\cdot36} - 30 \times \dfrac{20}{36\cdot06} + 60 \times \dfrac{20}{22\cdot36}$

$= -8\cdot56$ kN

$\Sigma F_y = 20 \times \dfrac{10}{22\cdot36} + 50 \times \dfrac{30}{36\cdot06} - 40 \times \dfrac{10}{22\cdot36} - 30 \times \dfrac{30}{36\cdot06} - 60 \times \dfrac{10}{22\cdot36}$

$= -19\cdot14$ kN

$R = 20\cdot98$ kN $\quad 65\cdot83°$, counter clockwise angle with positive x axis $= 245\cdot83°$

**2·11.** Here resultant is vertical hence

$\Sigma F_x = 0, \ 2\cos 6° + 4\cos 16° = T_{AB} \times \dfrac{4}{5}$

$T_{AB} = 7\cdot29$ kN

**2·12.** Here $F_1$ is in space having 3 angles with 3 axes and angles of $F_2$ is to determined.

$\Sigma F_x = 0, \ 0 = 500\cos 30° + 600\cos\theta_{x2}, \quad \theta_{x2} = 136\cdot19°$

$\Sigma F_y = 700$ N, $700 = 500\cos 68\cdot61° + 600\cos\theta_{y2}, \quad \theta_{y2} = 30\cdot37°$

$\Sigma F_z = 0, \ 0 = +500\cos 110° + 600\cos\theta_{z2}, \quad \theta_{z2} = 73\cdot44°$

**2·13.** $F_x = 4000\cos\alpha, \quad F_x^2 = F_R^2 - F_y^2 - F_z^2$

$F_x^2 = 4000^2 - 3064\cdot18^2 - 694\cdot59^2$

$F_y = 4000\cos 40° = 3064\cdot18$ N $\quad F_x = 2475\cdot55$ N

$F_z = 4000\cos 80° = 694\cdot59$ N $\quad \alpha = 51\cdot77°$

**2·14.** Force $F_2$ is making 30° with vertical yz plane, hence $F_2$ is resolved first in the yz and then xz plane. For Force $F_2$, components are,

$F_{2(yz)} = 200\cos 30° = 173\cdot21$ N

$F_{2(xz)} = 200\sin 30° = 100$ N

$F_{2y} = 173\cdot21\cos 50° = 111\cdot34$ N

$F_{2z} = -173\cdot21\sin 50° - 100\sin 50° = -209\cdot3$ N

$F_{2x} = -100\cos 50° = -64\cdot28$ N

$\Sigma F_x = 0 : -64\cdot28 + F_3\cos\theta_x = 0$

$\Sigma F_y = 0 : 111\cdot34 + F_3\cos\theta_y = 0$

$\Sigma F_z = 0 : 300 - 209\cdot3 + F_3\cos\theta_z = 0$



$F_3 = \sqrt{64\cdot28^2 + 111\cdot34^2 + 90\cdot7^2}$

$= 157\cdot35$ N

$\cos\theta_x = \dfrac{64\cdot28}{157\cdot35} \quad \therefore \theta_x = 65\cdot89°$

$\cos\theta_y = -\dfrac{111\cdot34}{157\cdot35} \quad \therefore \theta_y = 135\cdot04°$

$\cos\theta_z = -\dfrac{90\cdot7}{157\cdot35} \quad \therefore \theta_z = 125\cdot2°$

**2·15.** Coordinates of A = (6, 0, 0), B = (0, –2, –3) and C = (0, 5, 4)

$F_1$ is along AC (A to C)

$\bar{F}_1 = 500\left(\dfrac{-6}{8\cdot77}\bar{i} + \dfrac{5}{8\cdot77}\bar{j} + \dfrac{4}{8\cdot77}\bar{k}\right)$

$= -342\cdot08\,\bar{i} + 285\cdot06\,\bar{j} + 228\cdot05\,\bar{k}$

$F_2$ is along AB (A to B)

$\bar{F}_2 = 600\left(\dfrac{-6}{7}\bar{i} - \dfrac{2}{7}\bar{j} - \dfrac{3}{7}\bar{k}\right) = -514\cdot3\,\bar{i} - 171\cdot43\,\bar{j} - 257\cdot1\,\bar{k}$

$\bar{R} = -856\cdot38\,\bar{i} + 113\cdot63\,\bar{j} - 29\cdot05\,\bar{k}$, and magnitude $R = 864\cdot4$ N

$\theta_x = 172\cdot19°, \theta_y = 82\cdot45°, \theta_z = 91\cdot93°$

**2·16.** Coordinates of B = – 6, 3, 5

Coordinates of A = 2, 0, –6.

$\bar{F}_1 = 500\left(\dfrac{-6}{8\cdot37}\bar{i} + \dfrac{3}{8\cdot37}\bar{j} + \dfrac{5}{8\cdot37}\bar{k}\right)$

$\bar{F}_1 = -358\cdot4\,\bar{i} + 179\cdot2\,\bar{j} + 298\cdot7\,\bar{k}$

$\bar{F}_2 = 600\,\bar{j}$

$\bar{F}_3 = 700\left(\dfrac{2}{6\cdot33}\bar{i} - \dfrac{6}{6\cdot33}\bar{k}\right) = 221\cdot52\,\bar{i} - 664\cdot56\,\bar{k}$

$\bar{F}_4 = F_4\cos\theta_{x4}\,\bar{i} + F_4\cos\theta_{y4}\,\bar{j} + F_4\cos\theta_{z4}\,\bar{k}$

For equilibrium, $\bar{F}_1 + \bar{F}_2 + \bar{F}_3 + \bar{F}_4 = 0$

$\therefore F_{4x} = +136\cdot88, F_{4y} = -779\cdot2, F_{4z} = 365\cdot86, F_4 = 871\cdot6$ N

$\theta_{4x} = 80\cdot96°, \theta_{4y} = 153\cdot4°, \theta_{4z} = 65\cdot2°$

**2·17.** Coordinates of

D = –2, 5, 8

Force AC is in y – z plane

$$\bar{T}_{AD} = T_{AD}\left(\frac{-2}{9\cdot 64}\,\bar{i} + \frac{5}{9\cdot 64}\,\bar{j} + \frac{8}{9\cdot 64}\,\bar{k}\right)$$

$$= T_{AD}\,(-0\cdot 21\,\bar{i} + 0\cdot 52\,\bar{j} + 0\cdot 83\,\bar{k})$$

$$\Sigma F_x = 0,\ T_{AB} - 0\cdot 21\,T_{AD} = 0$$

$$\Sigma F_y = 0,\ -0\cdot 5\,T_{AC} + 0\cdot 52\,T_{AD} = 0$$

$$\Sigma F_z = 0,\ 0\cdot 866\,T_{AC} + 0\cdot 83\,T_{AD} - 150 \times 9\cdot 81 = 0$$

$$T_{AD} = 850\cdot 09\text{ N},\ T_{AB} = 178\cdot 52\text{ N},\ T_{AC} = 884\cdot 09\text{ N}$$

**2·18.** The coordinates are

$A = (6, 0, d)$
$B = (-3, -2, 5)$
$C = (-3, 2, 5)$

Here, tension in all three cables are same, say T.
Z components of three cables must be equal to weight of crate.
Y components of BD & DC cable forces are same & opposite.
X components of AD, BD and DC cables must be equal to zero.

$$\Sigma F_x = 0,\ \frac{6}{\sqrt{36 + d^2}}\,T - \frac{3}{6\cdot 16}\,T - \frac{3}{6\cdot 16}\,T = 0,\ d = 1\cdot 4\text{ m}$$

**2·19.** The coordinates of points are

$A = (-2, 0, 4)$, $C = (1, 0, -4)$
$B = (2, 0, 3)$, $D = (0, 8, 0)$

Point D is in equilibrium, The weight of platform can be applied at D in upward direction.

$$\Sigma F_x = 0,\ -0\cdot 218\,T_{DA} + 0\cdot 228\,T_{DB} + 0\cdot 11\,T_{DC} = 0 \quad \ldots (1)$$

$$\Sigma F_y = 0,\ -0\cdot 872\,T_{DA} - 0\cdot 911\,T_{DB} - 0\cdot 89\,T_{DC} + 2000 = 0 \quad \ldots (2)$$

$$\Sigma F_z = 0,\ 0\cdot 436\,T_{DA} + 0\cdot 34\,T_{DB} - 0\cdot 444\,T_{DC} = 0 \quad \ldots (3)$$

Multiplying eq. (1) with 0·436 and (3) with 0·218 we get,

$$0\cdot 091\,T_{DB} + 0\cdot 048\,T_{DC} = 0 \quad \ldots (4)$$

$$0\cdot 074\,T_{DB} - 0\cdot 097\,T_{DC} = 0 \quad \ldots (5)$$

$$\therefore\ 0\cdot 173\,T_{DB} - 0\cdot 049\,T_{DC} = 0 \quad \ldots (6)$$

Similarly from eq. (1) and (2)

$$0\cdot 199\,T_{DB} + 0\cdot 096\,T_{DC} = 0 \quad \ldots (7)$$

$$0\cdot 199\,T_{DB} + 0\cdot 194\,T_{DC} - 436 = 0 \quad \ldots (8)$$

$$\therefore\ 0\cdot 398\,T_{DB} + 0\cdot 29\,T_{DC} - 436 = 0 \quad \ldots (9)$$

From eq. (6) and (9) get the value of $T_{DC}$ & $T_{DB}$.

$$\therefore\ T_{DA} = 868\cdot 27\text{ N},\ T_{DB} = 310\cdot 3\text{ N},\ T_{DC} = 1077\cdot 57\text{ N}$$