Association Rules Mining

Topics

- Basic concepts of Association Rules
- Rule strength measures
- Basic Algorithms
  - Apriori Algorithm
  - FP-Growth Algorithm
  - Other Approaches
  - Interestingness Measures
  - Sequential Pattern Mining
- Summary

Association rule mining

- Motivation: Finding inherent regularities in data
  - What products were often purchased together?— Clothes and Milk !
  - What are the subsequent purchases after buying a PC?
  - What kinds of DNA are sensitive to new drug?
  - Can we automatically recommend next web document?
- Applications
  - Basket data analysis, Cross-marketing, Rack arrangement, Sale campaign analysis
  - DNA sequence analysis
  - Web log (click stream) analysis

Association rule mining

- Frequent pattern
  - A pattern (a set of items, subsequences, substructures, etc.) that occurs frequently in a data set
- First proposed by Agrawal et al. in 1993 in the context of frequent itemsets and association rule mining
- An important data mining model studied extensively

Association rule mining

- Given a set of transactions, find rules that will predict the occurrence of an item based on the occurrences of other items in the transaction
- Initially used for Market Basket Analysis to find how items purchased by customers are related

  Bread $\rightarrow$ Milk   [Sup = 5%, Conf = 100%]

The model: Data

- $I = \{i_1, i_2, \ldots, i_m\}$: a set of *items*
- Transaction *t*: a set of items, and $t \subseteq I$
- Transaction Database *T*: a set of transactions $T = \{t_1, t_2, \ldots, t_n\}$

Transaction data: Supermarket data

- Market basket transactions:

  t1: {bread, cheese, milk}

  t2: {apple, biscuit, salt, yogurt}

  …   …

  tn: {biscuit, bread, milk}

- Concepts:
  - An *item*: an item/article in a basket
  - *I*: the set of all items sold in the store
  - A *transaction*: items purchased in a basket; it may have TID (transaction ID)
  - A *transactional dataset*: A set of transactions

Transaction data: a set of documents

- **Text document data set, each document is treated as a “bag” of keywords**

  doc1: Student, Teach, School

  doc2: Student, School

  doc3: Teach, School, City, Game

  doc4: Baseball, Basketball

  doc5: Basketball, Player, Spectator

  doc6: Baseball, Coach, Game, Team

  doc7: Basketball, Team, City, Game

- **Web page data set**

  Session1: PageA.html, PageB.html, PageC.html

  Session2: PageC.html, PageD.html, PageE.html

  Session3: PageA.html, PageC.html, PageD.html

The model: Rules

- A transaction *t* contains *X*, a set of items (itemset) in *I*, if $X \subseteq t$
- An association rule is an implication of the form:

  $$X \to Y, \text{ where } X, Y \subset I, \textit{ and } X \cap Y = \emptyset$$

- An itemset is a set of items
  - E.g., X = {milk, bread, cereal} is an itemset
- A *k*-itemset is an itemset with *k* items
  - E.g., {milk, bread} is a 2-itemset

  {milk, bread, cereal} is a 3-itemset

Rule Strength Measures

- An association rule is a pattern that states when *X* occurs, *Y* occurs with certain probability
  - Support
  - Confidence

Support and Confidence

- Support
  - The rule holds with support *sup* in *T* (the transaction data set having n transactions) if sup% of transactions contain $X \cup Y$
  - *sup* = $\Pr(X \cup Y)$

$$\sup = \frac{(X \cup Y).count}{n}$$

    - Relative Support
  - The frequency count of an itemset *X U Y*, denoted by (*XUY).count*, in a data set *T* is the number of transactions
    - Count/Absolute Support

Rule strength measures

- Confidence
  - The rule holds in *T* with confidence *conf* if % of transactions that contain *X* also contain *Y*.
  - *conf* = Pr(*Y* | *X*)

$$confidence = \frac{(X \cup Y).count}{X.count}$$

Goal and key features

- **Goal:** Find all rules that satisfy the user-specified *minimum support* (minsup) and *minimum confidence* (minconf)
- **Key Features**
  - Completeness: find all rules
  - Compute the support and confidence for each rule
  - Prune rules that fail the *minsup* and *minconf* thresholds
  - Mining with data on hard disk (not in memory)

An example

t1:	Bread, Biscuit, Milk
t2:	Bread, Cheese
t3:	Cheese, Boots
t4:	Bread, Biscuit, Cheese
t5:	Bread, Biscuit, Clothes, Cheese, Milk
t6:	Biscuit, Clothes, Milk
t7:	Biscuit, Milk, Clothes

- Transaction data
- Assume:

  minsup = 30%
  minconf = 80%

- An example frequent *itemset*: {Biscuit, Clothes, Milk} [sup = 3/7]
- Association rules from the itemset:

  Clothes $\rightarrow$ Milk, Biscuit [sup = 3/7, conf = 3/3]
  … …
  Clothes, Biscuit $\rightarrow$ Milk, [sup = 3/7, conf = 3/3]

Assumption

- A simplistic view of shopping baskets transactions
  - Some important information not considered e.g.
    - The quantity of each item purchased
    - The price paid
- Assume all data are categorical
  - Examples:
    - Item Purchased or not ?
    - ID numbers, eye color {brown, black, etc.}, zip codes
    - Height in {tall, medium, short}

Many mining algorithms

- A large number of them!!
- Use of different strategies and data structures
- Resulting sets of rules are all the same
- Computational efficiencies and memory requirements may be different

The Apriori algorithm

- **The best known algorithm**
- **Two steps**:
  - Find all itemsets that have minimum support (*frequent itemsets*, also called large itemsets)
  - Use frequent itemsets to generate rules

- E.g., a frequent itemset

  {Biscuit, Clothes, Milk}   [sup = 3/7]

  and one rule from the frequent itemset

  Clothes $\rightarrow$ Milk, Biscuit   [sup = 3/7, conf = 3/3]

Step 1: Mining all frequent itemsets

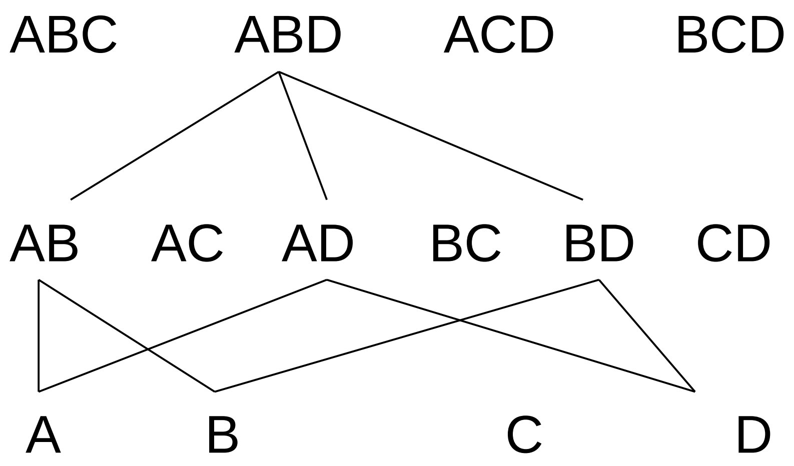
- A frequent *itemset* is an itemset whose support is ≥ minsup
- Key idea
  - The apriori property (downward closure property)
    - Any subsets of a frequent itemset are also frequent itemsets

If **{juice, glass, nuts}** is frequent, so is **{juice, glass}**

i.e., every transaction having

{juice, glass, nuts}

also contains {juice, glass}



The Algorithm

- Iterative algo. (also called level-wise search): Find all 1-item frequent itemsets; then all 2-item frequent itemsets, and so on
  - In each iteration *k*, only consider itemsets that contain some *k*-1 frequent itemset

- Find frequent itemsets of size 1: F_1
- For $k = 2$
  - C_k = candidates of size *k*: those itemsets of size *k* that could be frequent, given F_{k-1}
  - F_k = those itemsets that are actually frequent, $F_k \subseteq C_k$ (need to scan the database once)

The Apriori Algorithm—An Example

Database T

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

$Sup_{min} = 2$

The Apriori Algorithm—An Example

$Sup_{min} = 2$

Database T

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

C_1

$\xrightarrow{\text{1st scan}}$

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

The Apriori Algorithm—An Example

$Sup_{min} = 2$

Database TDB

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

$\xrightarrow{\text{1st scan}}$ C_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

$\xrightarrow{L_1}$

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3

The Apriori Algorithm—An Example

$Sup_{min} = 2$

Database TDB

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

$\xrightarrow{\text{1st scan}}$ C_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

$\longrightarrow$ L_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3

C_2

Itemset
{A, B}
{A, C}
{A, E}
{B, C}
{B, E}
{C, E}

The Apriori Algorithm—An Example

$Sup_{min} = 2$

Database TDB

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

1st scan $\longrightarrow$

C_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

$\longrightarrow$ L_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3

C_2

Itemset
{A, B}
{A, C}
{A, E}
{B, C}
{B, E}
{C, E}

2nd scan $\longleftarrow$

Itemset	sup
{A, B}	1
{A, C}	2
{A, E}	1
{B, C}	2
{B, E}	3
{C, E}	2

The Apriori Algorithm—An Example

$Sup_{min} = 2$

Database TDB

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

1st scan →

C_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

→ L_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3

C_2

Itemset
{A, B}
{A, C}
{A, E}
{B, C}
{B, E}
{C, E}

2nd scan ←

C_2

Itemset	sup
{A, B}	1
{A, C}	2
{A, E}	1
{B, C}	2
{B, E}	3
{C, E}	2

The Apriori Algorithm—An Example

$Sup_{min} = 2$

Database TDB

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

1st scan $\longrightarrow$

C_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

$\longrightarrow$

L_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3

C_2

Itemset
{A, B}
{A, C}
{A, E}
{B, C}
{B, E}
{C, E}

2nd scan $\longleftarrow$

C_2

Itemset	sup
{A, B}	1
{A, C}	2
{A, E}	1
{B, C}	2
{B, E}	3
{C, E}	2

$\longleftarrow$

L_2

Itemset	sup
{A, C}	2
{B, C}	2
{B, E}	3
{C, E}	2

The Apriori Algorithm—An Example

Database TDB

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

$Sup_{min} = 2$

1st scan $\rightarrow$

C_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

$\rightarrow$ L_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3

C_2

Itemset
{A, B}
{A, C}
{A, E}
{B, C}
{B, E}
{C, E}

2nd scan $\leftarrow$

C_2

Itemset	sup
{A, B}	1
{A, C}	2
{A, E}	1
{B, C}	2
{B, E}	3
{C, E}	2

$\leftarrow$ L_2

Itemset	sup
{A, C}	2
{B, C}	2
{B, E}	3
{C, E}	2

C_3

Itemset
{B, C, E}

The Apriori Algorithm—An Example

$Sup_{min} = 2$

Database TDB

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

1st scan $\rightarrow$ C_1

Itemset	sup
{A}	2
{B}	3
{C}	3
🟥 {D}	🟥 1
{E}	3

$\rightarrow$ L_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3

C_2

Itemset
{A, B}
{A, C}
{A, E}
{B, C}
{B, E}
{C, E}

2nd scan $\leftarrow$ C_2

Itemset	sup
🟥 {A, B}	🟥 1
{A, C}	2
🟥 {A, E}	🟥 1
{B, C}	2
{B, E}	3
{C, E}	2

$\leftarrow$ L_2

Itemset	sup
{A, C}	2
{B, C}	2
{B, E}	3
{C, E}	2

C_3

Itemset
{B, C, E}

3rd scan $\rightarrow$ L_3

Itemset	sup
{B, C, E}	2

The Apriori Algorithm

C_k: Candidate itemset of size k

F_k : frequent itemset of size k

```
Algorithm Apriori(T)
    $C_1 \leftarrow$ init-pass($T$);
    $F_1 \leftarrow \{f \mid f \in C1, f.\text{count}/n \geq minsup\}$;     // n: no. of transactions in T
    for ($k = 2$; $F_{k-1} \neq \emptyset$; $k$++) do
          $C_k \leftarrow$ candidate-gen($F_{k-1}$);
          for each transaction $t \in T$ do
              for each candidate $c \in C_k$ do
                    if $c$ is contained in $t$ then
                        $c.count$++;
              end
          end
         $F_k \leftarrow \{c \in C_k \mid c.count/n \geq minsup\}$
    end
return $F \leftarrow \bigcup_k F_k$;
```

Apriori candidate generation

- Function takes F_{k-1} and returns a superset (called the candidates) of the set of all frequent *k*-itemsets
- It has two steps
  - *join* step: Generate all possible candidate itemsets C_k of length *k*
  - *prune* step: Remove those candidates in C_k that cannot be frequent

Implementation of Apriori

- Example of Candidate-generation
  - L_3={*abc, abd, acd, ace, bcd*}
  - Self-joining: L_3*L_3
    - *abcd* from *abc* and *abd*
    - *acde* from *acd* and *ace*
  - Pruning:
    - *acde* is removed because *ade* is not in L_3
  - C_4 = {*abcd*}

Assignment example

1. **2-Itemset=**{{A, C}, {B, C}, {B, E}, {C, E}}

- **3-itemset ?**

2. **2-Itemset=**{{I1,I2}, {I1,I3}, {I1,I5}, {I2,I3}, {I2,I4}, {I2,I5}}}

- **3-itemset ?**

Candidate-gen function

```
Function candidate-gen($F_{k-1}$)
    $C_k \leftarrow \emptyset$;
    forall $f_1, f_2 \in F_{k-1}$
            with $f_1 = \{i_1, \ldots, i_{k-2}, i_{k-1}\}$
            and $f_2 = \{i_1, \ldots, i_{k-2}, i'_{k-1}\}$
            and $i_{k-1} < i'_{k-1}$ do
        $c \leftarrow \{i_1, \ldots, i_{k-1}, i'_{k-1}\}$;        // join $f_1$ and $f_2$
        $C_k \leftarrow C_k \cup \{c\}$;
        for each ($k$-1)-subset $s$ of $c$ do
            if ($s \notin F_{k-1}$) then
                delete $c$ from $C_k$;                  // prune
        end
    end
    return $C_k$;
```

Step 2: Generating rules from frequent itemsets

- Frequent itemsets ≠ association rules
- For each frequent itemset *X*,

  For each proper nonempty subset *A* of *X*,
  - Let *B* = X - *A*
  - A $\rightarrow$ B is an association rule if
    - Confidence(A $\rightarrow$ B) ≥ minconf,

      support(A $\rightarrow$ B) = support(A∪B) = support(X)

      confidence(A $\rightarrow$ B) = support(A ∪ B) / support(A)

Generating Rules: an example

- Suppose {2,3,4} is frequent, with sup=50%
  - Proper nonempty subsets: {2,3}, {2,4}, {3,4}, {2}, {3}, {4}, with sup=50%, 50%, 75%, 75%, 75%, 75% respectively
  - These generate these association rules:
    - $2,3 \rightarrow 4$ confidence=100%
    - $2,4 \rightarrow 3$ confidence=100%
    - $3,4 \rightarrow 2$ confidence=67%
    - $2 \rightarrow 3,4$ confidence=67%
    - $3 \rightarrow 2,4$ confidence=67%
    - $4 \rightarrow 2,3$ confidence=67%
    - All rules have support = 50%

Generating Rules: summary

- To recap, in order to obtain A $\rightarrow$ B, we need to have support(A $\cup$ B) and support(A)
- All the required information for confidence computation has already been recorded in itemset generation
    - No need to see the data *T* any more
- This step is not as time-consuming as frequent itemsets generation

Assignment Exercise: 1

- A database has five transactions.

Let *min sup* = 60% and *min con f* = 80%.

<u>TID</u>	***<u>items bought</u>***
T100	{M, O, N, K, E, Y}
T200	{D, O, N, K, E, Y}
T300	{M, A, K, E}
T400	{M, U, C, K, Y}
T500	{C, O, O, K, I ,E}

Find all frequent itemsets using Apriori.

Apriori Algorithm

Seems to be very expensive

- Breadth-first (Level-wise) search
- If, K = the size of the largest itemset then makes at most K passes over data
- Very simple and fast
  - Under some conditions, all rules can be found in linear time
- Scale up to large data sets

Apriori Algorithm

- Major computational challenges
  - Multiple scans of transaction database
  - Huge number of candidates
    - The number of frequent itemsets to be generated is senstive to the minsup threshold
    - When minsup is low, there exist potentially an exponential number of frequent itemsets
    - Example:
      - 10^4 frequent 1-itemsets, generate more than 10^7 candidate 2-itemsets
      - To discover a frequent pattern of size 100, such as $\{a_1, \ldots, a_{100}\}$
        - Generated candidates $2^{100} – 1$ = (Approx.) 10^{30}
  - Tedious workload of support counting for candidates

Apriori Algorithm

- Improving Apriori: general ideas
  - Reduce passes of transaction database scans
  - Shrink number of candidates
  - Facilitate support counting of candidates

Mining Frequent Patterns without Candidate Generation ???

Pattern-Growth Approach: Mining Frequent Patterns Without Candidate Generation

- The FPGrowth Approach given by J. Han, J. Pei, and Y. Yin, SIGMOD' 00
  - Depth-first search
  - Avoid explicit candidate generation

FPGrowth Approach

- Compress a large database into a compact, Frequent-Pattern tree (FP-tree) structure
  - Highly condensed, but complete for frequent pattern mining
  - Avoid costly database scans
- An efficient, FP-tree-based frequent pattern mining method
  - A divide-and-conquer methodology: decompose mining tasks into smaller ones calls conditional databases
  - Avoid candidate generation: sub-database mining only!

Example

TID	***Items bought***
100	{*f, a, c, d, g, i, m, p*}
200	{*a, b, c, f, l, m, o*}
300	{*b, f, h, j, o, w*}
400	{*b, c, k, s, p*}
500	{*a, f, c, e, l, p, m, n*}

min_support = 3

Step 1: Scan DB once, find frequent 1-itemset (single item pattern)

TID	***Items bought***
100	{*f, a, c, d, g, i, m, p*}
200	{*a, b, c, f, l, m, o*}
300	{*b, f, h, j, o, w*}
400	{*b, c, k, s, p*}
500	{*a, f, c, e, l, p, m, n*}

min_support = 3

Header Table

Item	***frequency***
f	*4*
c	*4*
a	*3*
b	*3*
m	*3*
p	*3*

Step 2: Sort frequent items in frequency descending order, f-list

TID	***Items bought***	***(ordered) frequent items***
100	{*f, a, c, d, g, i, m, p*}	{*f, c, a, m, p*}
200	{*a, b, c, f, l, m, o*}	{*f, c, a, b, m*}
300	{*b, f, h, j, o, w*}	{*f, b*}
400	{*b, c, k, s, p*}	{*c, b, p*}
500	{*a, f, c, e, l, p, m, n*}	{*f, c, a, m, p*}

min_support = 3

Header Table

Item	***frequency***
f	*4*
c	*4*
a	*3*
b	*3*
m	*3*
p	*3*

F-list = f-c-a-b-m-p

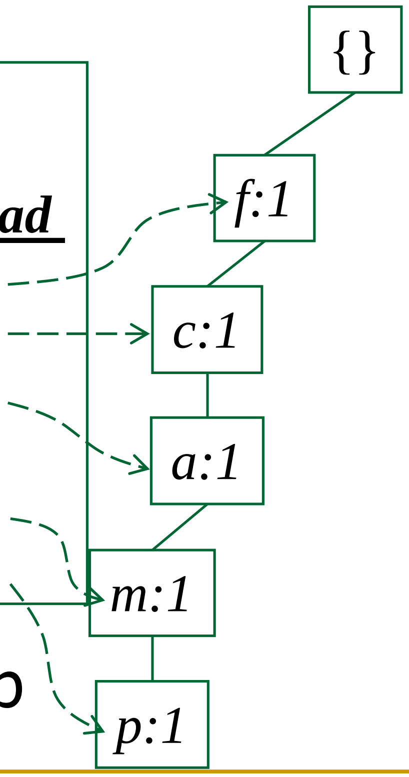
Step 3: Scan DB again, construct FP-tree

TID	***Items bought***	***(ordered) frequent items***
100	{*f, a, c, d, g, i, m, p*}	{*f, c, a, m, p*}
200	{*a, b, c, f, l, m, o*}	{*f, c, a, b, m*}
300	{*b, f, h, j, o, w*}	{*f, b*}
400	{*b, c, k, s, p*}	{*c, b, p*}
500	{*a, f, c, e, l, p, m, n*}	{*f, c, a, m, p*}

min_support = 3

Header Table

Item	***frequency***	***head***
f	*4*	
c	*4*	
a	*3*	
b	*3*	
m	*3*	
p	*3*	



- To facilitate the tree traversal, an item header table is built with a chain of node-links

F-list = f-c-a-b-m-p

Step 3: Cont…

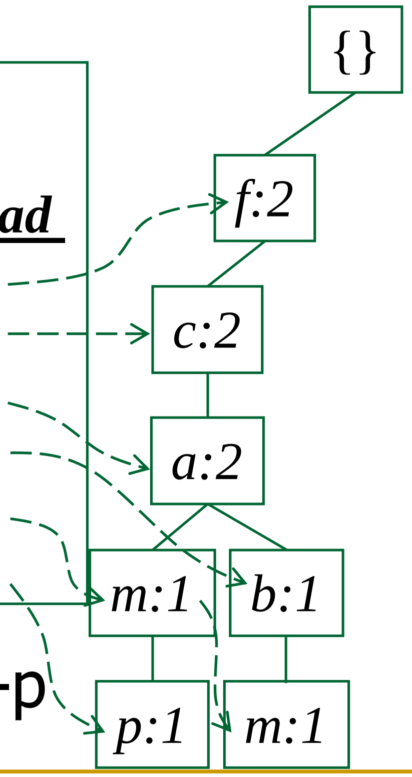
TID	***Items bought***	***(ordered) frequent items***
100	{*f, a, c, d, g, i, m, p*}	{*f, c, a, m, p*}
200	{*a, b, c, f, l, m, o*}	{*f, c, a, b, m*}
300	{*b, f, h, j, o, w*}	{*f, b*}
400	{*b, c, k, s, p*}	{*c, b, p*}
500	{*a, f, c, e, l, p, m, n*}	{*f, c, a, m, p*}

min_support = 3

- To facilitate the tree traversal, an item header table is built with a chain of node-links

Header Table

Item	***frequency***	***head***
f	*4*	
c	*4*	
a	*3*	
b	*3*	
m	*3*	
p	*3*	



F-list = f-c-a-b-m-p

Step 3: Cont…

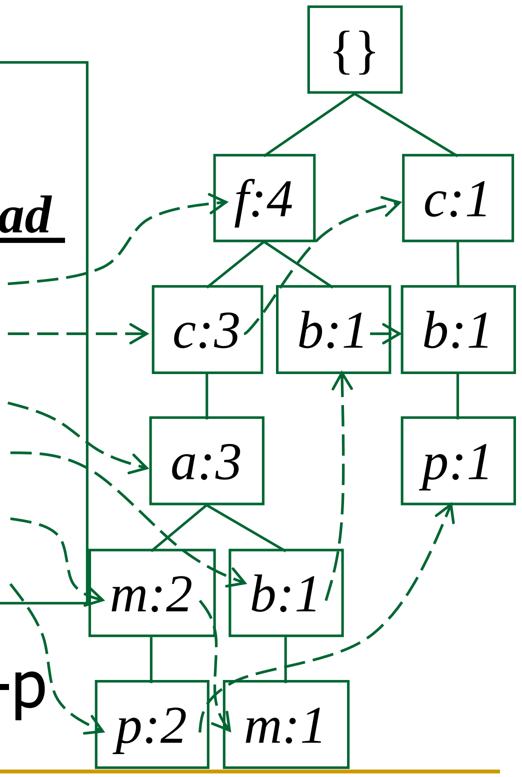
TID	***Items bought***	***(ordered) frequent items***
100	{*f, a, c, d, g, i, m, p*}	{*f, c, a, m, p*}
200	{*a, b, c, f, l, m, o*}	{*f, c, a, b, m*}
300	{*b, f, h, j, o, w*}	{*f, b*}
400	{*b, c, k, s, p*}	{*c, b, p*}
500	{*a, f, c, e, l, p, m, n*}	{*f, c, a, m, p*}

***min_support* = 3**

- To facilitate the tree traversal, an item header table is built with a chain of node-links

Header Table

Item	***frequency***	***head***
f	*4*	
c	*4*	
a	*3*	
b	*3*	
m	*3*	
p	*3*	



F-list = f-c-a-b-m-p

FPGrowth Example

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

MinSup=2

FPGrowth Assignment-2

Prepare the FP-Tree

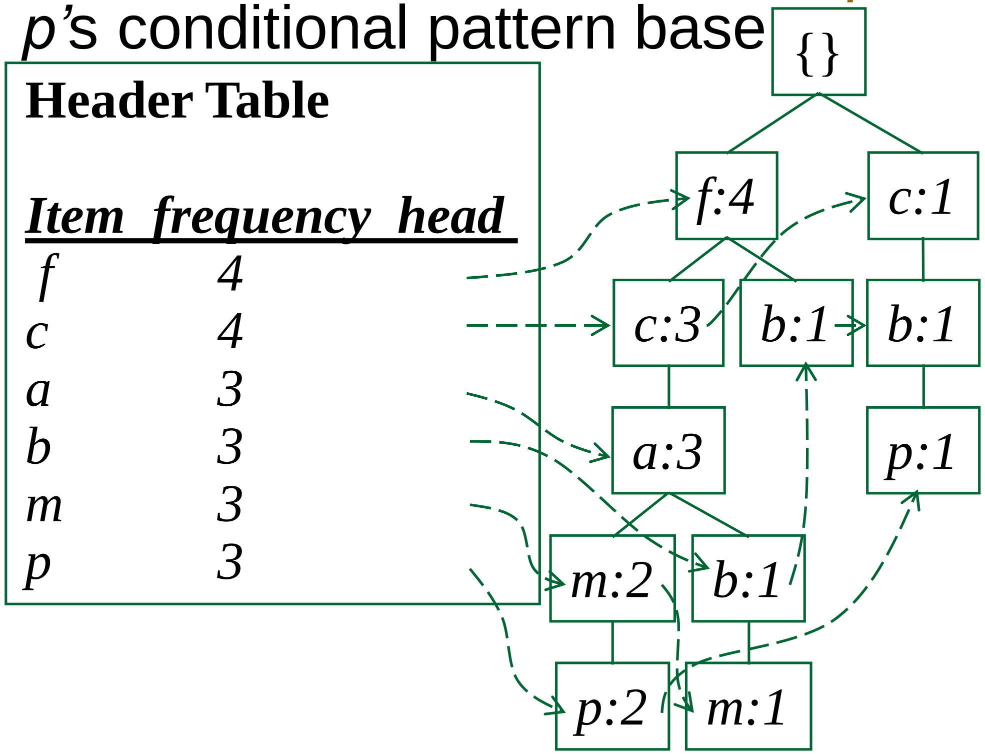
TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

Step 4: Mining of FP-Tree: Partition Patterns and Databases

- Frequent patterns can be partitioned into subsets according to f-list
  - F-list = f-c-a-b-m-p
  - Patterns containing p
  - Patterns having m but no p
  - …
  - Patterns having c but no a nor b, m, p
  - Pattern f
- Completeness and non-redundency

Find Patterns Having P From P-conditional Database

- Starting at the frequent item header table in the FP-tree
- Traverse the FP-tree by following the link of each frequent item *p*
- Accumulate all of *transformed prefix paths* of item *p* to form *p*'s conditional pattern base



Header Table

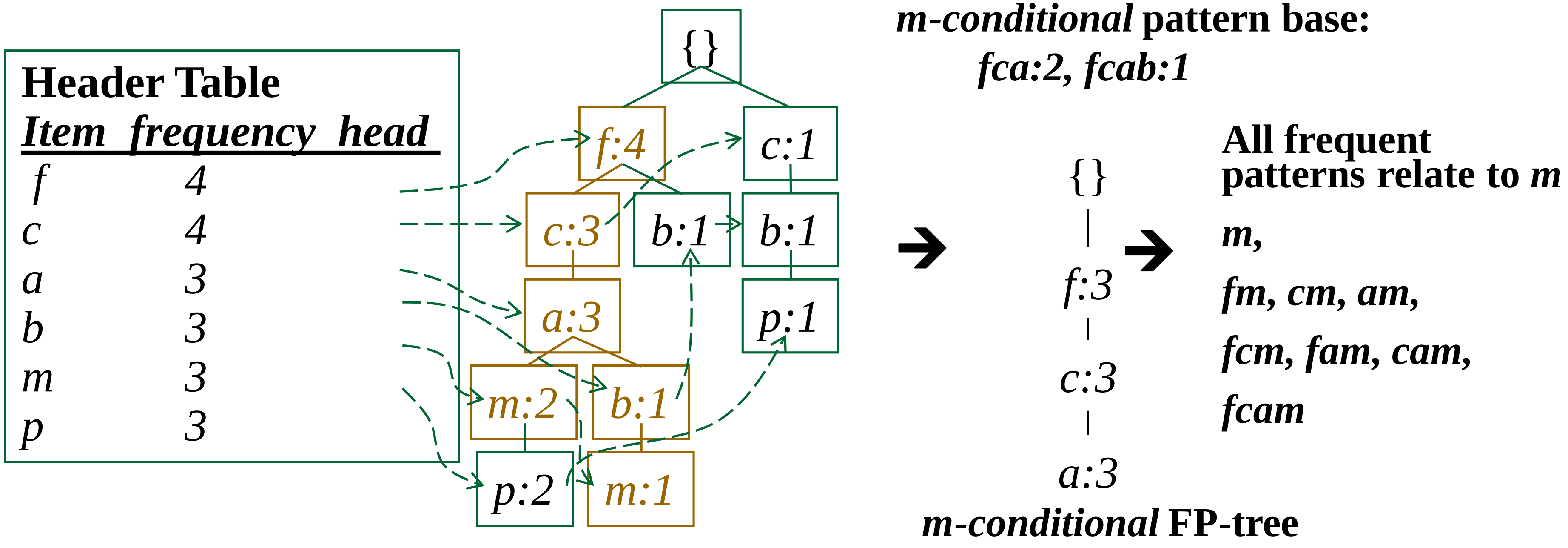
Item	***frequency***	***head***
f	4	
c	4	
a	3	
b	3	
m	3	
p	3	

Conditional **pattern bases**

item	***cond. pattern base***
c	***f:3***
a	***fc:3***
b	***fca:1, f:1, c:1***
m	***fca:2, fcab:1***
p	***fcam:2, cb:1***

From Conditional Pattern-bases to Conditional FP-trees

- For each pattern-base
  - Accumulate the count for each item in the base
  - Construct the FP-tree for the frequent items of the pattern base
    - having support count greater than the min support



- Conditional FP-Tree: Including items having support count greater than the min support

Recursion: Mining Each Conditional FP-tree

$$\begin{tikzcd}
\{\} \arrow[d, dash] \\
f{:}3 \arrow[d, dash] \\
c{:}3 \arrow[d, dash] \\
a{:}3
\end{tikzcd}$$

m-conditional **FP-tree**

Cond. pattern base of "am": (fc:3)

$$\begin{tikzcd}
\{\} \arrow[d, dash] \\
f{:}3 \arrow[d, dash] \\
c{:}3
\end{tikzcd}$$

am-conditional **FP-tree**

Cond. pattern base of "cm": (f:3)

$$\begin{tikzcd}
\{\} \arrow[d, dash] \\
f{:}3
\end{tikzcd}$$

cm-conditional **FP-tree**

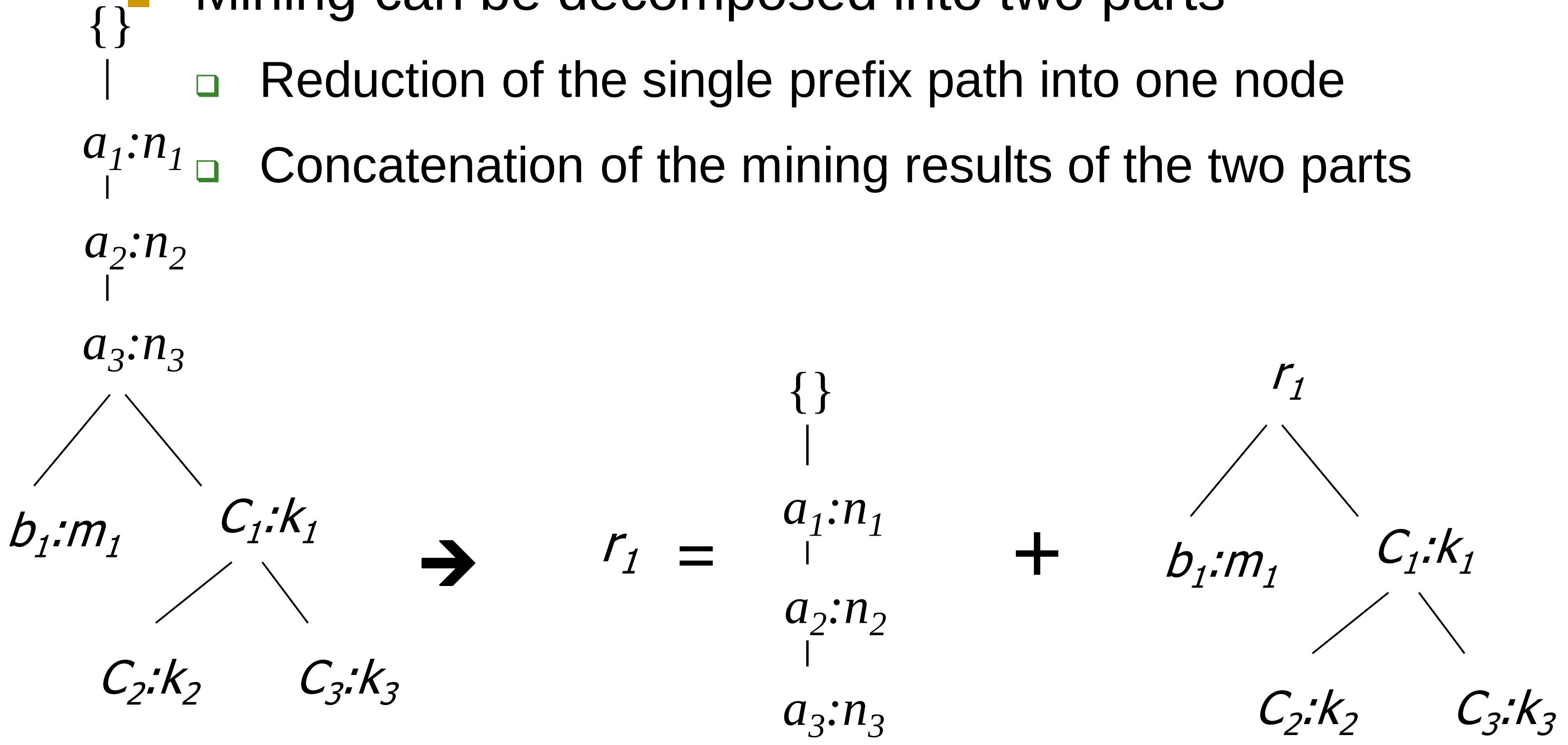
Cond. pattern base of "cam": (f:3)

$$\begin{tikzcd}
\{\} \arrow[d, dash] \\
f{:}3
\end{tikzcd}$$

cam-conditional **FP-tree**

A Special Case: Single Prefix Path in FP-tree

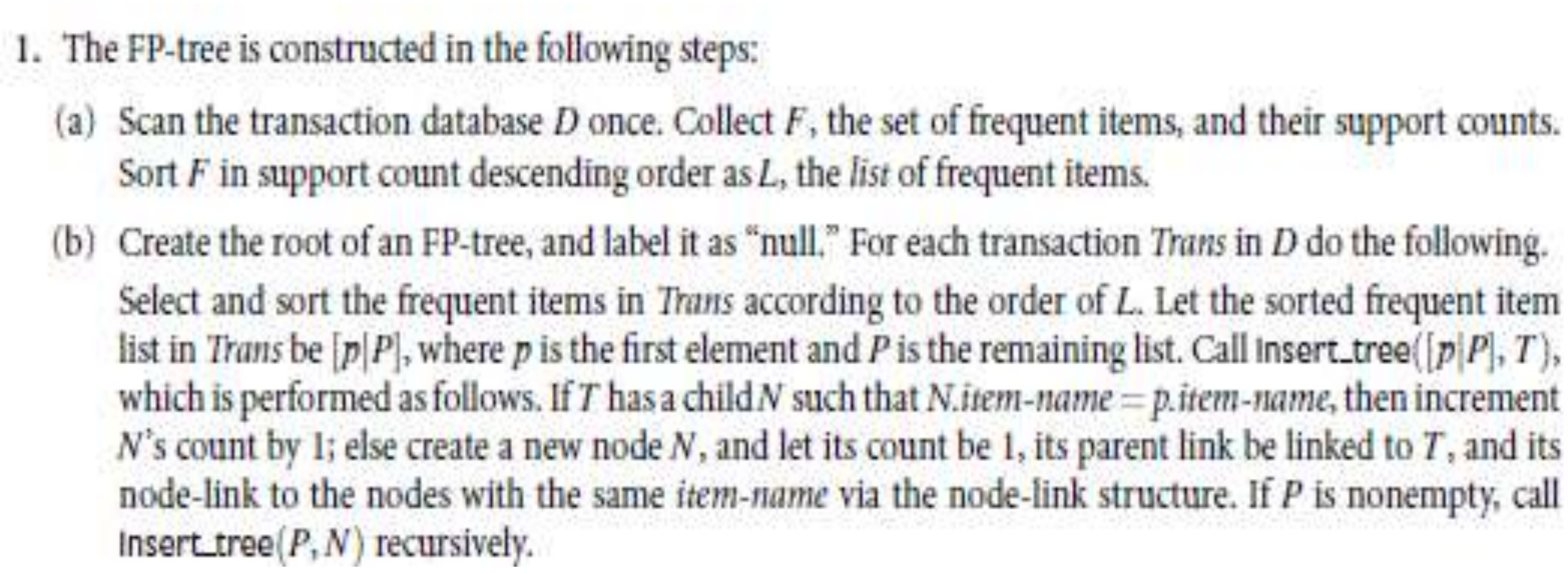
- Suppose a (conditional) FP-tree T has a shared single prefix-path P
- Mining can be decomposed into two parts
  - Reduction of the single prefix path into one node
  - Concatenation of the mining results of the two parts



The FP-Growth Mining Method

- Idea: Frequent pattern growth
  - Recursively grow frequent patterns by pattern and database partition
- Method
  - For each frequent item, construct its conditional pattern-base, and then its conditional FP-tree
  - Repeat the process on each newly created conditional FP-tree
  - Until the resulting FP-tree is empty, or it contains only one path—single path will generate all the combinations of its sub-paths, each of which is a frequent pattern

FP-Growth Algorithm



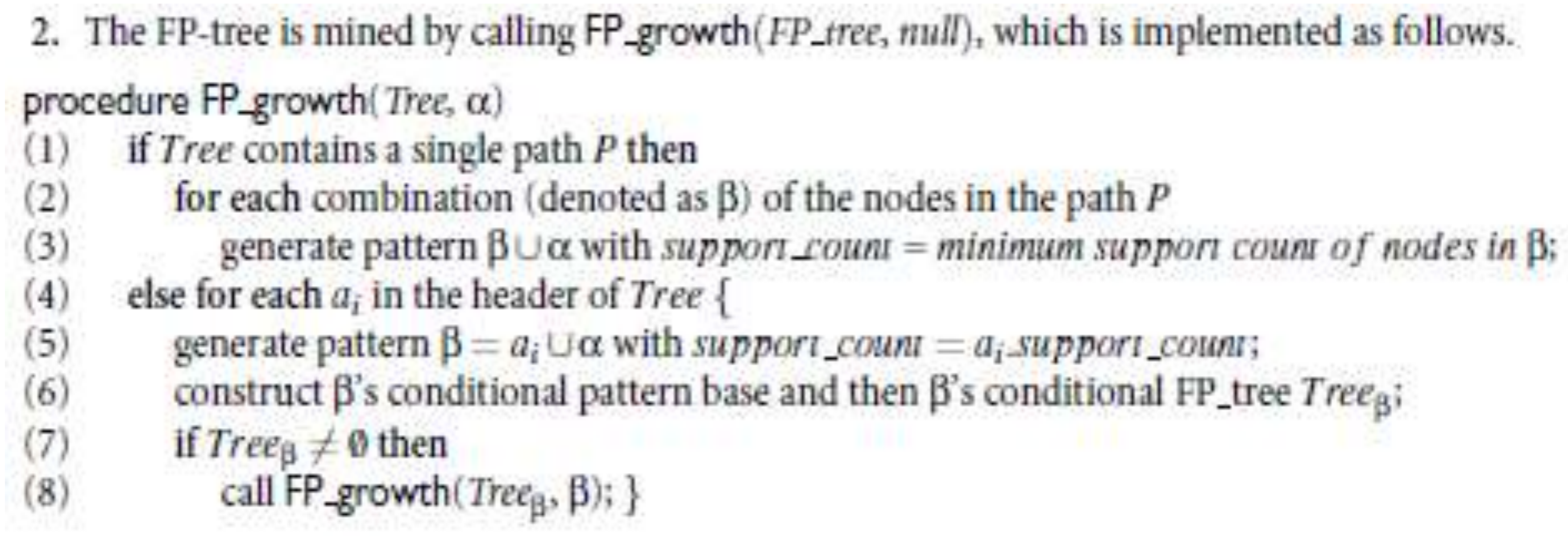
1. The FP-tree is constructed in the following steps:

   (a) Scan the transaction database D once. Collect F, the set of frequent items, and their support counts. Sort F in support count descending order as L, the *list* of frequent items.

   (b) Create the root of an FP-tree, and label it as "null." For each transaction *Trans* in D do the following.

   Select and sort the frequent items in *Trans* according to the order of L. Let the sorted frequent item list in *Trans* be $[p|P]$, where p is the first element and P is the remaining list. Call insert_tree($[p|P]$, T), which is performed as follows. If T has a child N such that N.*item-name* = p.*item-name*, then increment N's count by 1; else create a new node N, and let its count be 1, its parent link be linked to T, and its node-link to the nodes with the same *item-name* via the node-link structure. If P is nonempty, call insert_tree(P, N) recursively.

FP-Growth Algorithm Cont…



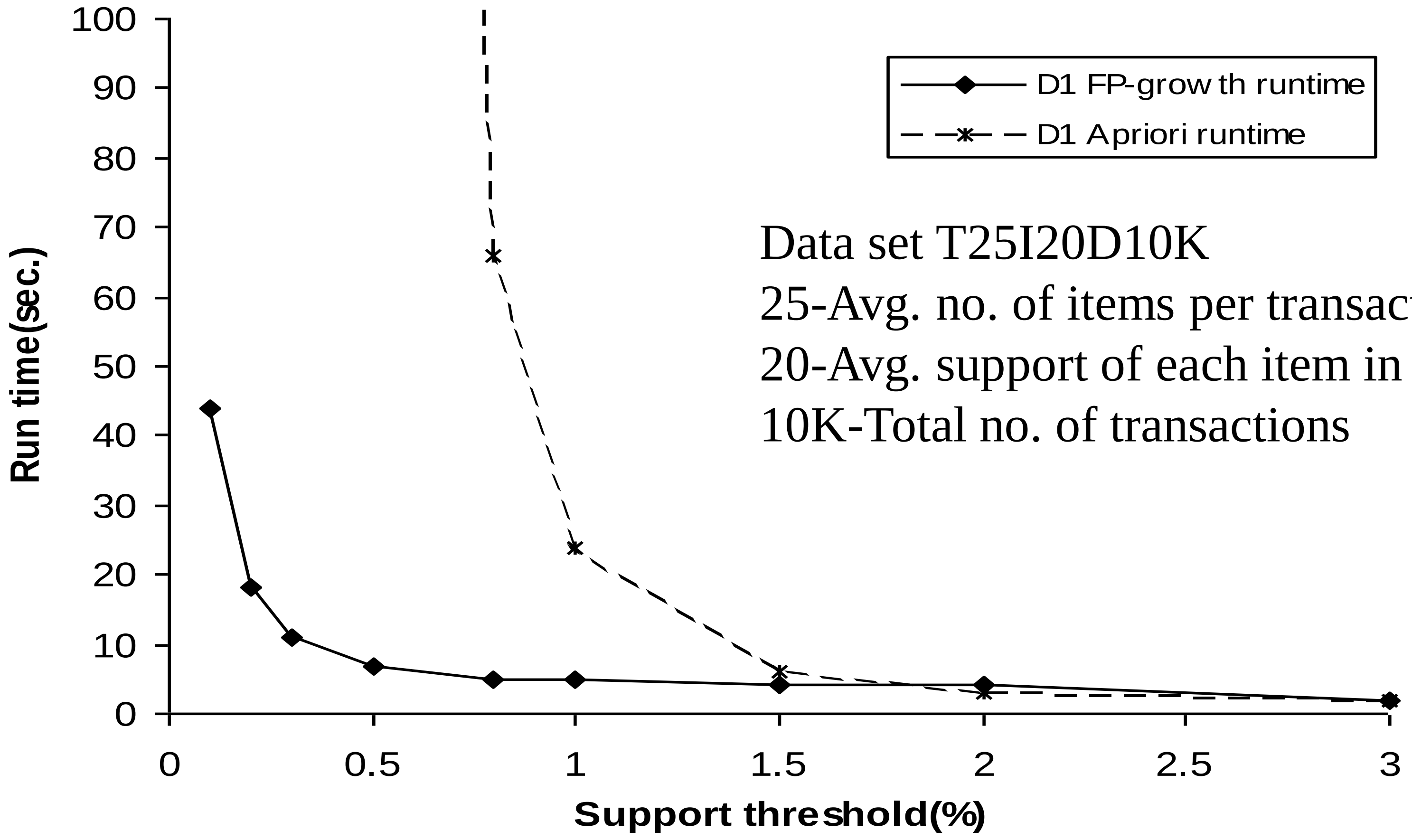
2. The FP-tree is mined by calling FP_growth(*FP_tree*, *null*), which is implemented as follows.

```
procedure FP_growth(Tree, $\alpha$)
(1)   if Tree contains a single path P then
(2)       for each combination (denoted as $\beta$) of the nodes in the path P
(3)           generate pattern $\beta \cup \alpha$ with support_count = minimum support count of nodes in $\beta$;
(4)   else for each $a_i$ in the header of Tree {
(5)       generate pattern $\beta = a_i \cup \alpha$ with support_count = $a_i$.support_count;
(6)       construct $\beta$'s conditional pattern base and then $\beta$'s conditional FP_tree $Tree_\beta$;
(7)       if $Tree_\beta \neq \emptyset$ then
(8)           call FP_growth($Tree_\beta$, $\beta$); }
```

Benefits of the FP-tree Structure

- Completeness
  - Preserve complete information for frequent pattern mining
  - Never break a long pattern of any transaction
- Compactness
  - Reduce irrelevant info—infrequent items are gone
  - Items in frequency descending order: the more frequently occurring, the more likely to be shared
  - Never be larger than the original database

FP-Growth vs. Apriori: Scalability With the Support Threshold



Data set T25I20D10K
25-Avg. no. of items per transaction
20-Avg. support of each item in dataset
10K-Total no. of transactions

FP-Growth Approach

- Divide-and-conquer
  - Decompose both the mining task and DB according to the frequent patterns obtained so far
  - Lead to focused search of smaller databases
- Performance is Faster than Apriori
  - Use compact data structure
  - No candidate generation, no candidate test
  - Eliminate repeated database scans
  - Basic operation is counting and FP-Tree building
- Problem:
  - When the database is large, sometimes unrealistic to construct a main memory based FP-Tree

Data Format

- Apriori and FP-Growth
  - { TID: itemset }
    - TID: Transaction ID
    - Itemset: set of items bought in transaction TID
  - Horizontal Data Format
- Alternative way
  - { Item: TID_set }
    - Item: item name
    - TID_set: set of transaction identifiers containing the item
  - Vertical Data Format

Data Format

Horizontal Data Layout

TID	Items
1	A,B,E
2	B,C,D
3	C,E
4	A,C,D
5	A,B,C,D
6	A,E
7	A,B
8	A,B,C
9	A,C,D
10	B

Vertical Data Layout

A	B	C	D	E
1	1	2	2	1
4	2	3	4	3
5	5	4	5	6
6	7	8	9	
7	8	9		
8	10			
9				

TID-list

Mining by Exploring Vertical Data Format

- ECLAT (Equivalence CLASS Transformation)
- Developed by Zaki

ECLAT Algorithm

- Deriving frequent patterns based on vertical intersections
  - $t(X) = t(Y)$: X and Y always happen together
  - $t(X) \subset t(Y)$: transaction having X always has Y
- To count itemset AB
  - Intersect TID-list of itemA with TID-list of itemB

ECLAT Algorithm

- Transform the horizontally formatted data to the vertical format by scanning the data set once
- Support count of an itemset
  - The length of the TID_set of the itemset

ECLAT Algorithm

- Determine support of any k-itemset by intersecting tid-lists of two of its (k-1) subsets.

A
1
4
5
6
7
8
9

$\wedge$

B
1
2
5
7
8
10

$\rightarrow$

AB
1
5
7
8

- 3 traversal approaches:
  - top-down, bottom-up and hybrid

ECLAT Algorithm

- Starting with k=1, the Frequent k-itemsets can be used to construct the candidate (k+1) itemsets based on the Apriori property
  - Done by intersection of the TID_sets of the frequent k-itemsets to compute the TID_sets of the corresponding (k+1) itemsets
- This process repeats, with k incremented by 1 each time, until no frequent itemsets or no candidate itemsets can be found

ECLAT Algorithm Summary

- Intersection is more efficient
- Pipelined counting for frequent itemsets
- Advantage
  - Less number of database scan
  - Very fast support counting
  - No need to scan the database to find the support of (k+1) itemsets (for k>=1)
    - Because the TID_set of each k-itemset carries the complete information required for counting each support

ECLAT Algorithm

- Disadvantage
  - Intermediate tid-lists may become too large for memory
  - Long computation time for intersecting the long set

- Performance improvement Idea
  - Using diffset to accelerate mining [CHARM Algorithm]
    - Only keep track of differences of tids
    - $t(X) = \{T_1, T_2, T_3\}$, $t(XY) = \{T_1, T_3\}$
    - Diffset $(XY, X) = \{T_2\}$

Problem of Frequent Item sets

- A long pattern contains a combinatorial number of sub-patterns
  - e.g., $\{a_1, \ldots, a_{100}\}$ contains

$= \binom{100}{1} + \binom{100}{2} + \ldots + \binom{100}{100}$

$= 2^{100} – 1$

$= 1.27*10^{30}$ sub-patterns!

- Solution

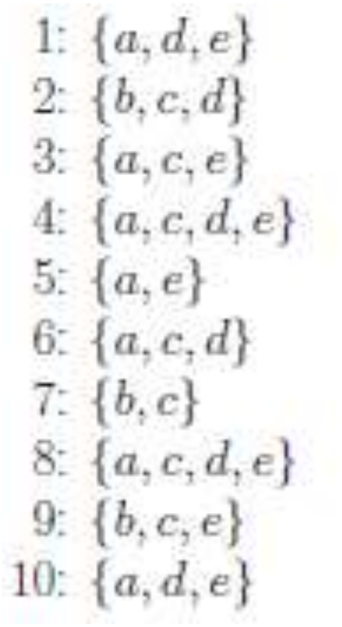
Mine closed patterns and max-patterns instead

Closed Patterns

- An itemset X is closed if X is *frequent* and there exists *no super-pattern* Y ɔ X, *with the same support* as X
- It is a lossless compression of freq. patterns
  - Reducing the # of patterns and rules

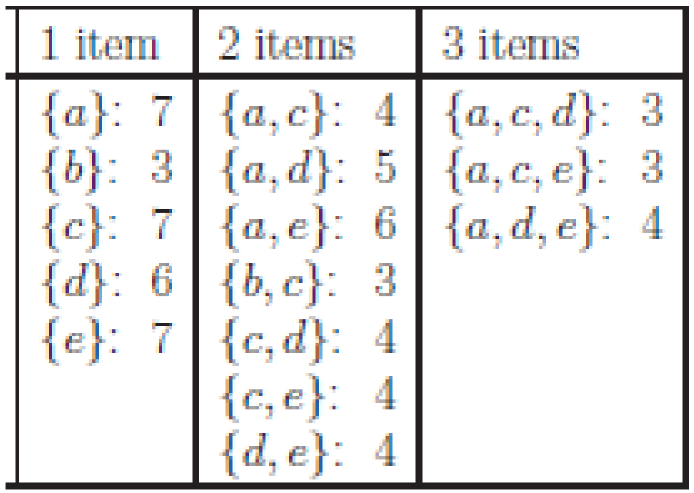
Closed Patterns - Example

Transaction Database



1:	$\{a, d, e\}$
2:	$\{b, c, d\}$
3:	$\{a, c, e\}$
4:	$\{a, c, d, e\}$
5:	$\{a, e\}$
6:	$\{a, c, d\}$
7:	$\{b, c\}$
8:	$\{a, c, d, e\}$
9:	$\{b, c, e\}$
10:	$\{a, d, e\}$

Frequent Item Set



1 item	2 items	3 items
$\{a\}$: 7	$\{a, c\}$: 4	$\{a, c, d\}$: 3
$\{b\}$: 3	$\{a, d\}$: 5	$\{a, c, e\}$: 3
$\{c\}$: 7	$\{a, e\}$: 6	$\{a, d, e\}$: 4
$\{d\}$: 6	$\{b, c\}$: 3	
$\{e\}$: 7	$\{c, d\}$: 4	
	$\{c, e\}$: 4	
	$\{d, e\}$: 4	

- {b} is a subset of {b,c} both have a support of 3
- {d,e} is a subset of {a,d,e} both have a support of 4

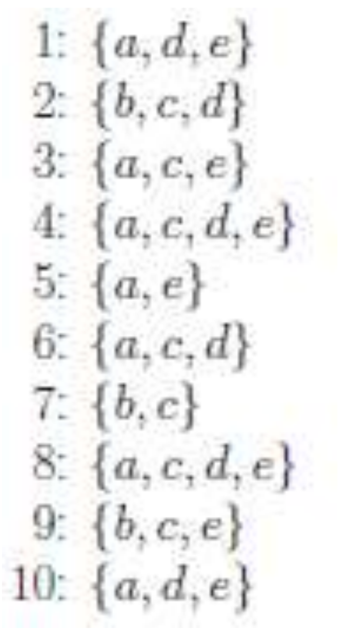
All frequent item sets are Closed **except {b} and {d, e}**

Max-Patterns

- An itemset X is a max-pattern (maximal) if X is frequent and there exists no frequent super-pattern Y ɔ X

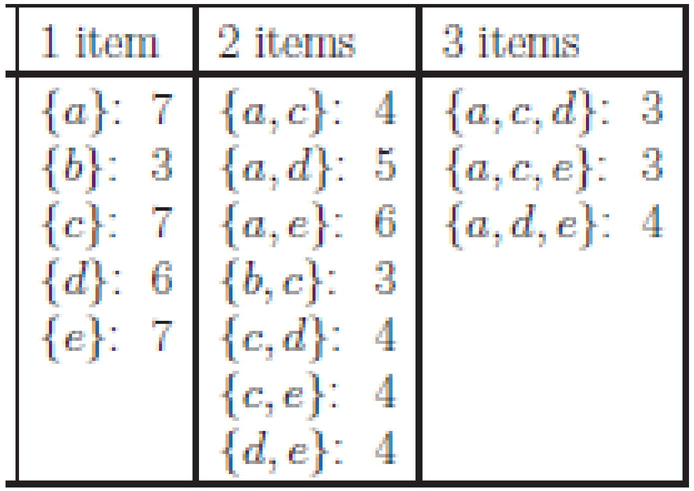
Max-Patterns - Example

Transaction Database



1: $\{a, d, e\}$
2: $\{b, c, d\}$
3: $\{a, c, e\}$
4: $\{a, c, d, e\}$
5: $\{a, e\}$
6: $\{a, c, d\}$
7: $\{b, c\}$
8: $\{a, c, d, e\}$
9: $\{b, c, e\}$
10: $\{a, d, e\}$

Frequent Item Set



1 item	2 items	3 items
$\{a\}$: 7	$\{a, c\}$: 4	$\{a, c, d\}$: 3
$\{b\}$: 3	$\{a, d\}$: 5	$\{a, c, e\}$: 3
$\{c\}$: 7	$\{a, e\}$: 6	$\{a, d, e\}$: 4
$\{d\}$: 6	$\{b, c\}$: 3	
$\{e\}$: 7	$\{c, d\}$: 4	
	$\{c, e\}$: 4	
	$\{d, e\}$: 4	

The maximal item sets are {b,c} {a,c,d} {a,c,e} {a,d,e}

- Every frequent itemset is a subset of at least one of these sets

Closed Patterns and Max-Patterns

- Exercise:
- DB = $\{<a_1, \ldots, a_{100}>, < a_1, \ldots, a_{50}>\}$
  - Min_sup = 1.
- What is the set of closed itemset?
  - $<a_1, \ldots, a_{100}>$: 1
  - $< a_1, \ldots, a_{50}>$: 2
- What is the set of max-pattern?
  - $<a_1, \ldots, a_{100}>$: 1

Mine the Closed and Max-Patterns

TID	Items
1	ABC
2	ABCD
3	BCE
4	ACDE
5	DE

Minimum support = 2