Tutorial-6 (Number Theory)

Int. MSc 5th year

Q.1 Use Fermat's theorem to verify that 17 divides $11^{104}+1$.

Q.2 (a) If $\gcd(a,35)=1$, show that $a^{12}\equiv 1 \pmod{35}$

(b) If $\gcd(a,42)=1$, show that $168=3\cdot 7\cdot 8$ divides a^6-1.

~~(c) If~~

Q.3 Derive the following congruences:

(a) $a^{21}\equiv a \pmod{15}$ $\forall a$.

(b) $a^{9}\equiv a \pmod{30}$ $\forall a$.

Q.4 If $\gcd(a,30)=1$, show that 60 divides a^4+59.

Q.5 (a) Find the units digits of 3^{100} by Fermat's theorem.

Q.6 If $7\nmid a$, prove that either a^3+1 or a^3-1 is divisible by 7.

Q.7 Assuming that a and b are integers not divisible by prime p, establish the following.

(a) If $a^p\equiv b^p \pmod{p}$, then $a\equiv b \pmod{p}$

(b) If $a^p\equiv b^p \pmod{p}$, then $a^p\equiv b^p \pmod{p^2}$.

Q.8 Use Fermat's theorem to prove that, if p is an odd prime, then

(a) $1^{p-1} + 2^{p-1} + 3^{p-1} + \cdots + (p-1)^{(p-1)} \equiv -1 \pmod{p}$.

(b) $1^p + 2^p + 3^p + \cdots + (p-1)^p \equiv 0 \pmod{p}$.

Q.9 Confirm the following integers are absolute pseudo primes.

(a) $1105 = 5 \cdot 13 \cdot 17$

(b) $2465 = 5 \cdot 17 \cdot 29$.

Q.10 Find the remainder when $15!$ is divided by 17.

Q.11 Arrange the integers $2, 3, 4, \ldots, 21$ in pairs a and b that satisfy $ab \equiv 1 \pmod{23}$.

Q.12 Show that $18! \equiv -1 \pmod{437}$.

Q.13 Given a prime number p, establish the congruence

$$(p-1)! \equiv p-1 \pmod{1+2+3+\cdots+(p-1)}.$$