Closure set of FDs

Let F be a set of functional dependencies on a relational schema R, the closure of F denoted by F+ is the set of all functional dependencies logically implied by F. Given F, we can define F+ directly from the formal definition of functional dependency. We can use Armstrong' axioms and other rules of inference to find logically implied functional dependencies (F+) from a given set of FDs (F).

Closure Of Functional Dependency : Introduction

- **The Closure Of Functional Dependency means the complete set of all possible attributes that can be functionally derived from given functional dependency using the inference rules known as Armstrong's Rules.**
- **If "F" is a functional dependency then closure of functional dependency can be denoted using "$\{F\}^+$".**
- **There are three steps to calculate closure of functional dependency. These are:**

  Step-1 : Add the attributes which are present on Left Hand Side in the original functional dependency.

  Step-2 : Now, add the attributes present on the Right Hand Side of the functional dependency.

  Step-3 : With the help of attributes present on Right Hand Side, check the other attributes that can be derived from the other given functional dependencies. Repeat this process until all the possible attributes which can be derived are added in the closure.

To compute F^+, we can use some rules of inference called **Armstrong's Axioms**:

- **Reflexivity rule:** if α is a set of attributes and $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$ holds.
- **Augmentation rule:** if $\alpha \rightarrow \beta$ holds, and γ is a set of attributes, then $\gamma\alpha \rightarrow \gamma\beta$ holds.
- **Transitivity rule:** if $\alpha \rightarrow \beta$ holds, and $\beta \rightarrow \gamma$ holds, then $\alpha \rightarrow \gamma$ holds.

These rules are **sound** because they do not generate any incorrect functional dependencies. They are also **complete** as they generate all of F^+.

To make life easier we can use some additional rules, derivable from Armstrong's Axioms:

- **Union rule:** if $\alpha \rightarrow \beta$ and $\alpha \rightarrow \gamma$, then $\alpha \rightarrow \beta\gamma$ holds.
- **Decomposition rule:** if $\alpha \rightarrow \beta\gamma$ holds, then $\alpha \rightarrow \beta$ and $\alpha \rightarrow \gamma$ both hold.
- **Pseudo Transitivity rule:** if $\alpha \rightarrow \beta$ holds, and $\gamma\beta \rightarrow \delta$ holds, then $\alpha\gamma \rightarrow \delta$ holds.

For given set of FDs $F = \{$

$A \rightarrow B$

$A \rightarrow C$

$CG \rightarrow H$

$CG \rightarrow I$

$B \rightarrow H \}$

Find F+

Applying these rules to the scheme and set F mentioned above, we can derive the following:

- $A \rightarrow H$, as we saw by the transitivity rule.
- $CG \rightarrow HI$ by the union rule.
- $AG \rightarrow I$ by several steps:
    - Note that $A \rightarrow C$ holds.
    - Then $AG \rightarrow CG$, by the augmentation rule.
    - Now by transitivity, $AG \rightarrow I$.
- (You might notice that this is actually pseudo transitivity if done in one step.)

Therefore F+ { $A \rightarrow H$, $CG \rightarrow HI$, $AG \rightarrow I$, $AG \rightarrow H$ }

Compute the closure (F+) of the following set F of functional dependencies for relational schema r (A, B, C, D, E):

A→ BC

CD→ E

B→ D

E→ A

E→BC, CD→A,

A→B………, so A→D

A→ BC……., so A→C

CD → E………, so AD→E

Ans: A→D

E→BC

As

CD→A