Tutorial-4

Int. M-Sc 5th year – Number Theory

1. Prove the following
   (a) If $a \equiv b \pmod{n}$ and $c > 0$, then $ca \equiv cb \pmod{cn}$
   (b) If $a \equiv b \pmod{n}$ and the integers a, b, n are all divisible by $d > 0$, then $a/d \equiv b/d \pmod{n/d}$.

2. Give an example to show that $a^2 \equiv b^2 \pmod{n}$ need not imply that $a \equiv b \pmod{n}$.

3. If $a \equiv b \pmod{n}$, P.T $\gcd(a, n) = \gcd(b, n)$.

4. Find the remainder when 41^{65} is divided by 7?

5. Prove that the integer $53^{103} + 103^{53}$ is divisible by 39.

6. If $a_1, a_2, \ldots, a_n$ is a complete set of residues modulo n, and $\gcd(a, n) = 1$, prove that $aa_1, aa_2, \ldots, aa$ is also a complete set of residues modulo n.

7. Prove the following statement:
   If $\gcd(a, n) = 1$, then the integers
   $$c,\ c+a,\ c+2a,\ c+3a,\ \ldots,\ c+(n-1)a$$
   form a complete set of residues modulo n for any c.

8. Give an example to show that $a^k \equiv b^k \pmod{n}$ and $k \equiv j \pmod{n}$ need not imply that $a^j \equiv b^j \pmod{n}$.

9. Use the theory of congruences to verify

$89 \mid 2^{44} - 1$ and $97 \mid 2^{48} - 1$.

10. Find the remainder when 4444^{4444} is divided by 9

11. Find the values of $n \geq 1$ for which

$1! + 2! + 3! + \cdots + n!$ is a perfect square.

12. Use binary exponential algorithm to compute

$19^{53} \pmod{503}$.

13. Without performing the divisions, determine whether the integer 176521221 is divisible by 9 or 11.