i of	UTORIAL - 1 22/01/2020
	AUTOMATA AND FORMAL LANGUAGES
	MATHEMATICAL THOUCTION
	U19CSO12 [BHAGYA VINOD RANA]
0.17	had book (19) Aponder off and and
	1.) Sch) = 1+2+3+ + n, NEN [Natural Number Set]
	Prove by mathematical induction the statement
	P(n): S(n) = n*(n+1), \forall integers n \forall 1
	2
	For any integer n ≥ 1, dot PCn) be the statement
	$P(n)$: $\Re 1 + 2 + 3 + \dots + n = n \times (n+1)$
	(n) = (1+10) + (2+81) 2
	(A) Base case: The statement P(1) says that
	SULHS = 11/HIA COUR of SALARON IT
	$RHS = 1 \times (n+1) = 1 \times 2 = 1$
	1 to 3 (1 - (1 - (1 + (1 + (1 + (1 + (1 + (1 +
	LHS = RHS, Hence P(1) is true
	(1+30) + 1+30 + +278+14 = 2841
	B) Inductive Step: Fix K 21, and suppose P(K) holds brue,
	1+2+3++k=k*(k+1)-1
	2
	To show: P(k+1) is also true, i.e.
	1+2+3++k+(k+1)=(k+1)((k+1)+1)-
	LHS = $\{1+2+3++k\}+(k+1)$ RHS = $(k+1)$ (k+2)
	$= \begin{cases} \frac{k \times (k+1)}{2} + (k+1) \end{cases}$
	- (V+1) (V 1) . 110-1112
	= (k+1) (k + 1) ; LHS = RHS PCK+1) hold's true
	= (k+1) (k+2) Thus, by the Principle of mathematical 2 induction for all n > 1, p(n) holds true
vision	induction, for all h2 1, per motor war

Q17 27 det Scn = 1+3+5+...+(2n-1). Prove by mothemotical induction, the statement PCD : Scn = n^2 , for all integers $n \ge 1$ For any integer $n \ge 1$, det PCD be the Statement PCD : 1+3+5+...+(2n+1) = n^2

Base Case: The statement P(1) says that

LHS = 1

LHS = RHS,

Hence P(1) is true.

(B) Inductive Step: Fix K Z 1, and suppose PCKI holds,

$$[1+3+5+...+(2k-1)] = k^2$$

It remains To show: PCKHII is also true,

1.e.
$$\{1+3+5+...+(2k-1)+\}$$
 + $(2(k+1)-1)$ = $(k+1)^2$

LHS = $\{1+3+5+...+2k+1\}+(2k+1)$

 $(2k+1) \qquad (acN)$

 $(k+1)^2$ [: $(a+1)^2 = a^2 + 2a + 1$]

= RHS

.. LHS = RHS

Hena, PCK+1) hold's true,

Thus, by the principle of mathematical induction, for all n Z 1, Pcns holds true.

$\begin{array}{c} \text{(C)} \\ (C)$		UIACSOIS
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	0.1.>	
for all natural number 0 18 [V integer n 2.1] For any integer n 2.1, still from he the statement P(n) \$\frac{1}{2} \((n+1)^2 + (n+2)^2 + + (2n)^2 = n \times \((2n+1) \) \((3n+1) \) \$\frac{1}{2} \\ \text{Rese (ase : The statement P(1) says that } \\ \text{LHS = \$\frac{1}{2} \text{ (2(1)}^2 = 4} \\ \text{RHS = \$\frac{1}{2} \text{ (2(2)} \text{ (2(1)}^2 = 4} \\ \text{RHS = \$\frac{1}{2} \text{ (2(2)} \text{ (2(1)} \text{ (3)} \text{ 2.4} = 4} \\ \text{B} \text{Toducher Size : Fix K21 and suppose that P(1) is true \text{B} \text{Toducher Size : Fix K21 and suppose that P(1) holds} \text{C(k+1)^2 + (k+2)^2 + (k+3)^2 + + (2k)^2 = k \times (2k+1) \text{ (3(k+1)} \text{ (3(k+1)} \) \\ \text{To stoom P(k+1) is also fine } \\ \text{In + ((k+1)+1)^2 + (k+3)^2 + + (2(k+1))^2 = (k+1) \text{ (2(k+1) + 1) \text{ (3(k+1) + 1)} \\ \text{LHS = \$\text{ (k+2)^2 + (k+3)^2 + + (2(k+1))^2 = (k+1) \text{ (2k+1) + 1) \text{ (3(k+1) + 1} \\ \text{ (2k+1) \text{ (3k+1)} - (k+1)^2 \\ \text{ (2k+1) \text{ (3k+1)} + \text{ (2k+1)^2 + (2k+2)^2} \\ \text{ = \$\text{ (2k+1) \text{ (3k+1)} + \text{ (3k} \text{ 4k+1} \\ \text{ = \$\text{ (2k+1) \text{ (3k+1)} + \text{ (3k+2)^2} \\ \text{ (2k+1) \text{ (3k+1)} + \text{ (3k+2)^2} \\ \text{ (2k+1) \text{ (3k+1)} + \text{ (2k+2)^2} \\ \text{ = \$\text{ (2k+1) \text{ (3k+1)} + \text{ (3k+2)^2} \\ \text{ = \$\text{ (2k+1) \text{ (3k+1)} + \text{ (3k+2)^2} \\ \text{ = \$\text{ (2k+1) \text{ (3k+1)} + \text{ (3k+2)^2} \\ \text{ = \$\text{ (2k+1) \text{ (3k+1)} + \text{ (3k+2)^2} \\ \text{ = \$\text{ (2k+1) \text{ (3k+1)} + \text{ (3k+2)^2} \\ \text{ = \$\text{ (2k+1) \text{ (3k+1)} + \text{ (2k+2)^2} \\ \text{ = \$\text{ (2k+1) \text{ (3k+1)} + \text{ (2k+2)^2} \\ \text{ = \$\text{ (2k+1) \text{ (3k+1)} + \text{ (2k+2)^2} \\ \text{ = \$\text{ (2k+1) \text{ (3k+1)} + \text{ (2k+2)^2} \\ \text{ = \$\text{ (2k+1) \text{ (3k+1)} + \text{ (2k+2)^2} \\ \text{ = \$\text{ (2k+1) \text{ (3k+1)} + \text{ (2k+2)^2} \\ \text{ = \$\text{ (2k+1) \text{ (3k+1)} + (3k+2)		
For any integer $n \ge 1$, solve the statement $e^{-1} = e^{-1} + e$		
For any integer $n \ge 1$, sold $P(n)$ be the slatement $P(n) \le 1$ ($n+1$) ² + ($n+2$) ² + + ($2n$) ² = $n \times (2n+1)(3n+1)$ ($n+1$) ² + ($n+2$) ² + + ($2n$) ² = $n \times (2n+1)(3n+1)$ ($n+1$) (A) Base Case: The slatement $P(n)$ says that $P(n) \ge 1$ ($n+1$) ² = $(2(1))^2 = 4$ $P(n+1)^2 = (2(1))^2 = 4$ $P(n+1)^2 = 4$		
P(n) $\frac{1}{5} (n+1)^2 + (n+2)^2 + + (2n)^2 = n \times (2n+1)(3n+1)$ (A) Rose (ast: The statement P(1) Soys that		
Rose Case: The statement PC11 Says that		
Base (ase: The statement PC1) sours that		
Binse (all : The statement PC1) says that $IHS = (I+1)^2 = (2(1))^2 = 4$ $RHS = n \times (2n+1) \times (3n+1) = 4(3)(8) = 24 = 4$ $6 $		
$ \begin{array}{c} \text{LHS} = & (1+1)^2 = & (2(1)^2 = 4) \\ \text{RHS} = & n \times (2n+1) \times (3n+1) = 1 \times (31(8) = 24 = 4) \\ \text{G} = & \text{G} = & \text{G} \\ \end{array} $ $ \begin{array}{c} \text{LHS} = & R \text{HS} \\ \text{HS} =$		Base Case: The statement PC1) says that
RHS = $0 \times (2n+1) \times (3n+1) = 1 \cdot (31 \cdot 8) = 24 = 4$ (6) (6) (8) LHS = RHS, Hapic P(1) is true (8) Toduchy Step: Fix K Z 1, and Suppose that Rk1 holds (k+1)^2 + (k+2)^2 + (k+3)^2 + + (2k)^2 = k \times (2k+1) \times (3k+1) (k+1)^2 + (k+3)^2 + + (2(k+1))^2 = (k+1) (2(k+1)+1) (3(k+1)+1) (4(k+1)+1) (4(k+	0	
$ \begin{array}{c} \text{LHS} = \text{RHS}, \underline{\text{Hence}} \text{P(1)} \text{is true} \\ \text{B} \\ \\ \text{Toduchva} \text{Step} : \text{Fix } k \text{Z1}, \text{and} \text{Suppose} \text{final} \text{P(k)} \text{holds} \\ \\ (k+1)^2 + (k+2)^2 + (k+3)^2 + \dots + (2k)^2 = k \times (2k+1) \times (3k+1) \\ \\ \text{G} \\ \\ \text{Io.} \text{((k+1)+1)}^2 + (k+3)^2 + \dots + (2(k+1))^2 = (k+1) \left(2(k+1)+1 \right) \left(3(k+1)+1 \right) \\ \\ \text{G} \\ \\ \text{IHS} = \left\{ (k+2)^2 + (k+3)^2 + \dots + (2k)^2 + (2k+1)^2 + (2k+2)^2 \\ \\ = \left\{ (k\times(2k+1)\times(3k+1) - (k+1)^2 \right\} + (2k+1)^2 + (2k+2)^2 \right\} \\ \\ \text{Fram} \left\{ (k+2)^2 + (k+3)^2 + \dots + (2k)^2 + (2k+1)^2 + (2k+2)^2 \right\} \\ \\ \text{Fram} \left\{ (k+2)^2 + (k+3)^2 + \dots + (2k)^2 + (2k+1)^2 + (2k+2)^2 \right\} \\ \\ \text{Fram} \left\{ (k+2)^2 + (k+3)^2 + \dots + (2k)^2 + (2k+1)^2 + (2k+2)^2 \right\} \\ \\ \text{Fram} \left\{ (k+2)^2 + (k+3)^2 + \dots + (2k)^2 + (2k+1)^2 + (2k+2)^2 \right\} \\ \\ \text{Fram} \left\{ (k+2)^2 + (k+3)^2 + \dots + (2k)^2 + (2k+1)^2 + (2k+2)^2 \right\} \\ \\ \text{Fram} \left\{ (k+2)^2 + (k+3)^2 + \dots + (2k)^2 + (2k+1)^2 + (2k+2)^2 + (2k$		
B To Aduction Step: Fix $K \ge 1$ and Suppose that $P(k)$ hold s $(k+1)^{2} + (k+2)^{2} + (k+3)^{2} + + (2k)^{2} = k \times (2k+1) \times (3k+1)$ $(k+1)^{2} + (k+2)^{2} + (k+3)^{2} + + (2(k+1))^{2} = (k+1) (2(k+1)+1) (3(k+1)+1)$ $(k+1)^{2} + (k+3)^{2} + + (2(k+1))^{2} = (k+1) (2(k+1)+1) (3(k+1)+1)$ $(k+1)^{2} + (k+3)^{2} + + (2k)^{2} + (2k+1)^{2} + (2k+2)^{2}$ $= (k+1)^{2} + (k+3)^{2} + + (2k)^{2} + (2k+1)^{2} + (2k+2)^{2}$ $= (k+1)^{2} + (k+3)^{2} + + (2k)^{2} + (2k+1)^{2} + (2k+2)^{2}$ $= (k+1)^{2} + (k+3)^{2} + + (2k)^{2} + (2k+1)^{2} + (2k+2)^{2}$ $= (k+1)^{2} + (k+3)^{2} + + (2k)^{2} + (2k+1)^{2} + (2k+2)^{2}$ $= (k+1)^{2} + (k+3)^{2} + + (2k)^{2} + (2k+1)^{2} + (2k+2)^{2}$ $= (k+1)^{2} + (k+1)^{2$		
B Toducher Skp: Fix KZ1, and Suppose that P(x) holds $(k+1)^{2} + (k+2)^{2} + (k+3)^{2} + + (2k)^{2} = k \times (2k+1) \times (3k+1)$ To show P(k+1) is also true, $(k+1)+1)^{2} + (k+3)^{2} + + (2(k+1))^{2} = (k+1) (2(k+1)+1) (3(k+1)+1)$ $(k+1)+1)^{2} + (k+3)^{2} + + (2(k+1))^{2} = (k+1) (2(k+1)+1) (3(k+1)+1)$ $(k+1)+1)^{2} + (k+3)^{2} + + (2k)^{2} + (2k+1)^{2} + (2k+2)^{2}$ $= (k+1) \times (3k+1) + (k+1)^{2} + (2k+1)^{2} + (2k+2)^{2} + (2k+1)^{2} + (2k+2)^{2}$ $= (k+1) \times (3k+1) + (k+1)^{2} + (k+1)^{2} + (2k+1)^{2} + (2k+2)^{2}$ $= (k+1) \times (3k+1) + (k+1)^{2} + (k+1)$		LHS = RHS Have P(1) is true
$ (k+1)^{2} + (k+2)^{2} + (k+3)^{2} + + (2k)^{2} = k \times (2k+1) \times (3k+1) $ $ (k+1)^{2} + (k+3)^{2} + + (2k+1)^{2} = (k+1) (2(k+1)+1) (3(k+1)+1) $ $ (k+1)^{2} + (k+3)^{2} + + (2(k+1))^{2} = (k+1) (2(k+1)+1) (3(k+1)+1) $ $ (k+2)^{2} + (k+3)^{2} + + (2k)^{3} + (2k+1)^{2} + (2k+2)^{2} $ $ = (k \times (2k+1) \times (3k+1) - (k+1)^{2} + (2k+1)^{2} + (2k+2)^{2} $ $ = (k \times (2k+1) \times (3k+1) - (k+1)^{2} + (2k+1)^{2} + (2k+2)^{2} $ $ = k \times (2k+1) (3k+1) + 3k^{2} + (2k+1)^{2} + (2k+2)^{2} $ $ = k \times (2k+1) (3k+1) + 3k^{2} + (2k+1)^{2} + (2k+2)^{2} $ $ = k \times (2k+1) (3k+1) + 3k^{2} + (2k+1)^{2} + (2k+2)^{2} $ $ = k \times (2k+1) (3k+1) + 3k^{2} + (2k+1)^{2} + (2k+2)^{2} $ $ = k \times (2k+1) (3k+1) + 3k^{2} + (2k+1)^{2} + (2k+2)^{2} $ $ = k \times (2k+1) \times (3k+1) + 3k^{2} + (2k+1)^{2} + (2k+2)^{2} + (2k+2)^{2} $ $ = k \times (2k+1) \times (3k+1) + 3k^{2} + (2k+1)^{2} + (2k+2)^{2} + (2k+$		
$ (k+1)^{2} + (k+2)^{2} + (k+3)^{2} + + (2k)^{2} = k \times (2k+1) \times (3k+1) $ $ (k+1)^{2} + (k+3)^{2} + + (2k+1)^{2} = (k+1) (2(k+1)+1) (3(k+1)+1) $ $ (k+1)^{2} + (k+3)^{2} + + (2(k+1))^{2} = (k+1) (2(k+1)+1) (3(k+1)+1) $ $ (k+2)^{2} + (k+3)^{2} + + (2k)^{2} + (2k+1)^{2} + (2k+2)^{2} $ $ = (k+2)^{2} + (k+3)^{2} + + (2k)^{2} + (2k+1)^{2} + (2k+2)^{2} $ $ = (k+2)^{2} + (k+3)^{2} + + (2k)^{2} + (2k+1)^{2} + (2k+2)^{2} $ $ = (k+2)^{2} + (k+3)^{2} + + (2k)^{2} + (2k+1)^{2} + (2k+2)^{2} $ $ = (k+2)^{2} + (k+3)^{2} + + (2k)^{2} + (2k+1)^{2} + (2k+2)^{2} $ $ = (k+2)^{2} + (k+3)^{2} + + (2k)^{2} + (2k+1)^{2} + (2k+2)^{2} $ $ = (k+2)^{2} + (k+3)^{2} + (k+2)^{2} + (k+2)^{2$		Inductive Step: Fix KZ1 and Suppose that PCKI holds
To show: $P(k+1)$ is also $\{ \pm \pi i e \}$, i.e. $((k+1)+1)^2 + (k+3)^2 + \dots + (2(k+1))^2 = (k+1) (2(k+1)+1) (7(k+1)+1) (6$ $1 + C = \{ (k+2)^2 + (k+3)^2 + \dots + (2k)^3 + (2k+1)^2 + (2k+2)^2 \}$ $= \{ (k+2)^2 + (k+3)^2 + \dots + (2k)^3 + (2k+1)^2 + (2k+2)^2 \}$ $= \{ (k+2)^2 + (k+3)^2 + \dots + (2k)^3 + (2k+1)^2 + (2k+2)^2 \}$ $= \{ (k+2)^2 + (k+3)^2 + \dots + (2k)^3 + (2k+1)^2 + (2k+2)^2 \}$ $= \{ (k+2)^2 + (k+3)^2 + \dots + (2k)^3 + (2k+1)^2 + (2k+2)^2 \}$ $= \{ (2k+1) + (k+3) + (k+2) $		
To show P(k+1) is also $\{\pm ue\}$ 18. $((k+1)+1)^2 + (k+3)^2 + \dots + (2(k+1))^2 = (k+1)(2(k+1)+1)(3(k+1)+1)($		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		dout contribut torpopartion und mark (A (19-1)
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	10	
$ HS = \begin{cases} (k+2)^{2} + (k+3)^{2} + + (2k)^{3} + (2k+1)^{2} + (2k+2)^{2} \\ = \left(\frac{k \times (2k+1) \times (3k+1)}{6} \right) - (k+1)^{2} + (2k+1)^{2} + (2k+2)^{2} + From 1 $ $ = k(2k+1)(3k+1) - [k^{2} + 2k+1] + [4k^{2} + 2k+1] + [4k^{2} + 8k+4] $ $ = k(2k+1)(3k+1) + 3k^{2} + 10k + 4 $ $ = k(2k+1)(3k+1) + (3k^{2} + 10k + 4) $ $ = k(2k+1)(3k+1) + (3k^{2} + 10k + 4) $ $ = (4k^{3} + 9k^{2} + k) + (2k^{2} + 60k + 24) - (4k^{3} + 51k^{2} + 61k + 24) $		
$= \frac{\left(\frac{k \times (2k+1) \times (3k+1)}{6} - (k+1)^{2}\right) + (2k+1)^{2} + (2k+2)^{2}}{6}$ $= \frac{k(2k+1)(3k+1)}{6} - \left[\frac{k^{2} + 2k + 1}{7} + \frac{4k^{2} + 2k + 1}{7} + \frac{4k^{2} + 8k + 4}{7}\right]$ $= \frac{k(2k+1)(3k+1)}{6} + \frac{3k^{2} + 10k + 4}{7}$ $= \frac{k(2k+1)(3k+1)}{6} + \frac{3k^{2} + 10k + 4}{7}$ $= \frac{k(2k+1)(3k+1)}{6} + \frac{k(3k+2)(3k+1)}{7} + \frac{k(3k+3)(3k+1)}{7} + \frac{k(3k+3)(3k+1)}{7}$		
$= \frac{k(2k+1)(3k+1)}{6} - \left[\frac{k^2+2k+1}{7} + \left[\frac{4k^2+2k+1}{4k+1}\right] + \left[\frac{4k^2+8k+4}{4k+1}\right] + \left[\frac{4k^2+8k+4}{4k+1}\right] + \left[\frac{4k^2+8k+4}{4k+1}\right] + \left[\frac{4k^2+8k+4}{4k+1}\right] + \left[\frac{4k^2+4k+1}{4k+1}\right] + \left$		
$= \frac{k(2k+1)(3k+1) - [k^2+2k+1] + [4k^2+2k+1] + [4k^2+8k+4]}{6}$ $= \frac{k(2k+1)(3k+1) + 3k^2 + 10k + 4}{6}$ $= \frac{k(2k+1)(3k+1) + (3k^2+10k+4)}{6}$ $= \frac{k(2k+1)(3k+1) + (3k^2+10k+4)}{6}$ $= \frac{k(2k+1)(3k+1) + (3k^2+10k+4)}{6}$ $= \frac{k(2k+1)(3k+1) + 3k^2+6k+4}{6}$ $= \frac{k(2k+1)(3k+1) + 3k^2+6k+4}{6}$ $= \frac{k(2k+1)(3k+1) + 4k^2+6k+4}{6}$		
$= \frac{K(2k+1)(3k+1) + 3k^2 + 10k + 4}{6}$ $= \frac{K(2k+1)(3k+1) + (3k^2 + 10k + 4)}{6}$ $= \frac{(4k^3 + 9k^2 + k) + 42k^2 + 60k + 24}{6} = \frac{(4k^3 + 51k^2 + 61k + 24)}{6}$		
$= \frac{K(2K+1)(3K+1) + 3K^{2} + 10K + 4}{6}$ $= \frac{K(14k^{2} + 9k + 1) + ((3k^{2} + 10k + 4))}{6}$ $= \frac{(14k^{3} + 9k^{2} + 10k + 4)}{6} + \frac{(14k^{3} + 51k^{2} + 61k + 24)}{6}$		
$= \frac{K(14k^2 + 9k + 1) + ((7k^2 + 10k + 4))}{6}$ $= (14k^3 + 9k^2 + k) + (42k^2 + 60k + 24) - (14k^3 + 51k^2 + 61k + 24)$		
$= (14k^{3} + 9k^{2} + k) + (42k^{2} + 60k + 24) - (14k^{3} + 51k^{2} + 61k + 24)$		
$= (14k^{3} + 9k^{2} + k) + (42k^{2} + 60k + 24) - (14k^{3} + 51k^{2} + 61k + 24)$		$= KC 4k^2 + 9k + 1] + (C + 10k + 4)$
$= (14k^{3} + 9k^{2} + k) + 42k^{2} + 60k + 24) - (14k^{3} + 51k^{2} + 61k + 24)$ $= 6$ Vision		6
Vision		$= (14k^3 + 9k^2 + k) + 42k^2 + 60k + 24) - (14k^3 + 51k^2 + 61k + 24)$
	vision	6

continued. LHS = KIK3 + 151K2 + 161K+24 days days days days

RHS = (K+1) (2K+3) (7K+8) = (K+1) (14K3+ 16K+ 21K+ 24)

(K+1) C 14K2+ 37K+ 24)

K(19k2+ 37k+24)+ (14k2+ 37k+24)

KIK3+ 37K2+ 24K+ 14K2+ 37K+24

 $(14k^3 + 51k^2 + 61k + 24) = LHS$

LHS = RHS

Thus, PCK+1) hold's true.

Thus, by the principle of mathematical induction, for all h I 1, P(n) holds

Q.1.> 4.7 Prove by mathematical induction, that

(1.n)+(2.(n-1))+(3.(n-2))+...+((n-1)2)+(n.1)=

is true for all natural numbers n. nen+1)(n+2)

For any integer n > 1, det Pens be the statement P(n): $(1,n) + (2,(n-1)) + ... + ((n-1)2) + (n.1) = h \times (n+1)(n+2)$

A Base case: The statement P(1) says that

LHS = 1.1 = 1

RHS = $\frac{1}{(n)(n+1)(n+2)} = \frac{(1)(2)(3)}{(1)(2)(3)} = \frac{1}{(1)(2)(3)}$

LHS-RHS, Hence P(1) is true.

	The contraction of the contract of the contrac
0.1>	4. > continued
	Triductive Step : Fix K 7 1, and suppose pcks holds true
	Lating and also todaying all hard
	P(K): 1.K + 2.(K-1) + 3.(K-2) + + (K-1).2 + K.1 = K(K+1)(K+2)
	pringing 10 1 10 1 6
	-(1)
	To show: PCK+1) is also true, i.e.
	(1.(k+1)) + (2.(k+1-1)) + + ((k+1-1).2) + ((k+1).1) = 1(k+1)((k+1)+1)((k+1)+2)
0	LHS = \[\(\(\text{(K+1)} \) + \(\(\text{2.K} \) \) + \(\(\text{K.2} \) + \(\(\text{K+1} \) \) \(\text{K.2} \)
	1 (add +1-1=0)
	= [1.(k+1+1-1)] + [2.(k+1-1)] + [k.(2+1-1)] + [(k+1).(1+1-1)]
	= [1.(K+1-1) + 1.(1)] + [2(K-1) + 2.1] + [K.(2-1) + K] + [(K+1)(1-1) + (K+1)]
	= [(1.k) + (1)] + [2.(k-1) + (2)] + [k(1) + (k)] + [(k+1)(0) + (k+1)]
	1 (1 5 m ()
(8	$= \int (1.k + 2.(k-1) + + k(1)) + \int (1+2+ + k+(k+1)) + \int (from \Omega)$ $= \int (1.k + 2.(k-1) + + k(1)) + \int (1+2+ + k+(k+1)) + \int (from \Omega) + \int (from \Omega)$
0	
	$= \int \frac{k(k+1)(k+2)}{6} + \frac{(k+1)(k+2)}{2} $
	- (K+1)(K+1)(K+2)
	$\frac{-\left(\frac{k+1}{3}+1\right)\left(\frac{k+1}{(k+1)(k+2)}\right)}{2}$
	= (K+1) (K+2) (K+3)
	be a second of the second of t
(0)	= (K+1) ((K+1)+1) ((K+1)+2) = RHS
(8)	Mark Third 6 4 1 7 1 1
	LHS = RHS, PCK+1) hold's true
Line	Thus, by the poinciple of mathematical induction, for all $n \geq 1$,
1 1111	Pens holds true.
Seigi and	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
vision	

Q.1.7 5.7 Poore, by Mathematical Induction, that non+1)cn+2)cn+3) is divisible by 24, Y neN.

Emathematical induction cannot be applied directly. Here we break the proposition into three pasts. I

det Pin be the proposition:

- 1.) hcn+1) is divisible by 21=2
 - 2.) n(n+1)(n+2) is divisible by 31 = 6
- 3.) (n(n+1)(n+2)(n+3) is divisible by 41=24

For P(1), [Base Case]

- 1.) IX2 = 2 is divisible by 2
- [11-1+12 11+12] + [12.] 1x2x3 = 6 is divisible by 3 1-1+1111
- 3.) 1x2x3x4 = 24 is divisible by 24 5. P(1) is true

Assume that PCK) is true for some natural number K, that is

- 1.1 k(k+1) is divisible by 2, that is k(k+1) = 2a 1
- 2.1 K(K+1)(K+2) is divisible by 6, ie K(K+1)(K+2) = 6b -2
- 3.) K(K+1)(K+2)(K+3) is divisible by 24,

 ie k(K+1)(K+2)(K+3) = 240 (3) (a,b,c)e N

For PCK+1),

1.) (k+1)(k+2) = k(k+1) + 2(k+1) = 2a + 2(k+1), by (1)= 2[a+k+1], which is divisible by 2-4

2.) (k+1)(k+2)(k+3) = K(k+1)(k+2) + 3(k+1)(k+2)

 $= \frac{6b}{6[b+a+k+1]}, \text{ divisible by } 6 -6$

3.3 (k+1) (k+2) (k+3) (k+4) = k((k+1)(k+2)(k+3)) + 4((k+1)(k+2)(k+3)) = 24c + 4(6[b+a+k+1]) = 24(c+ b+a+k+1), divisible by 24

U19CS012 27 The cube root of 2 is irrahonal. det assume 352 is rahand number 3/2 = a, where an a and b are whole numbers b and it is the simplest form so a and b can't be even at same time. they both can be odd or one of them can be odd. $3\sqrt{2} = \frac{a}{b}$ Cubing both $2 = \frac{a^3}{b^3}$ — ① $a^3 = 2b^3$ a3 is even so a will also be even. det a = 2k putting in eqn \oplus , $2 = \frac{(2k)^3}{b^3}$ (125 at 1 = 59°C) b3 is also even, so b also will be even. but a and b can't be even at same time. . It is contradiction, cube root of 2 is irrational 3.7 For every n, if n > 2 and n is prime, the n is odd. if nr2 and n is prime, Prove: n is odd det's assume n is even,

From the defination of prime,

Prime number, it has only two factors one of them is 1 and other is number itself.

But from Eq. 1 1

n is divisible by 1, (2), and n

: n has 3 or more than 3 factors, which contradicts the Prime Number defination.

.. It is contradiction and every prime number (>2) greater than 2 is odd.

	UI9CSO12
0.27	4> If n is perfect square, the n+2 is not
	Assume for a perfect square
	$\int \Omega = \alpha^2 Y$
	so, (n+2) is also a perfect square (a, b \in N)
	$(0+2) = b^2$
	$a^2 + 2 = b^2$
	$2 = b^2 - a^2$
	$2 = (b-a)(b+a) - \boxed{1}$
	(n) and (n+2) is perfect square, so a and b will be integer
9	From eq (1)
	2 = (b-a)(b+a)
	(b+a) and (b-a) will be 2 and 1 respectively
	$b+a=2$ adding $\rightarrow b=3$
	b-a=1]
	By contradiction, But, bis on integer, so our assumption is false
	". If n is perfect square, then (n+2) is not.
	5.> If m.p is even, where m and n are integers, then either
	m is even or n is even.
	det assume m, n are odd m=2k+1
	$m \cdot n = (2k+1)(2l+1)$ $n = 2l+1$ [k, $l \in \mathbb{Z}$]
	= 4kl+2k+2l+ 1
	= 2(2k(+k+1)+1 = 2(C)+1
	m.n = odd which is false,
	So our Assumption is false,
	Either morn or both con be even, so that m. n
	$m = 2k+1$ } $m \cdot n = 2(2k+1)$ l $m = 2k$ } $m \cdot n = 4k$ is even $n = 2l$ $= 2(2k+1) = 2(p)$
	(2p) $n=2l-1=2(2kl)=2(p)$

. By contradiction, we have proved,

if min is even, either of morn needs to be even.

vision