LIC
भारतीय जीवन बीमा निगम
LIFE INSURANCE CORPORATION OF INDIA

Tutorial-5 (Number Theory)
Linear congruences and Chinese Remainder Theorem

Q1. Solve the following linear congruences.

(a) $25x \equiv 15 \pmod{29}$
(b) $5x \equiv 2 \pmod{26}$
(c) $34x \equiv 60 \pmod{98}$
(d) $140x \equiv 133 \pmod{301}$

Q.2. Using congruences, solve the Diophantine equations:

(a) $4x + 51y = 9$.
(b) $5x - 53y = 17$.

Q.3 Solve the following sets of simultaneous congruences:

(a) $x \equiv 5 \pmod{11}$, $x \equiv 14 \pmod{29}$, $x \equiv 15 \pmod{31}$
(b) $2x \equiv 1 \pmod{5}$, $3x \equiv 9 \pmod{6}$, $4x \equiv 1 \pmod{7}$, $5x \equiv 9 \pmod{11}$.

Q.4. Find the smallest integer $a > 2$ such that

$2 \mid a$, $3 \mid a+1$, $4 \mid a+2$, $5 \mid a+3$, $6 \mid a+4$.

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Q.5. A band of 17 pirates stole a sack of gold coins. When they tried to divide the fortune into equal portions, 3 coins remained. In the ensuing brawl over who should get the extra coins, one pirate was killed. The wealth was redistributed, but this time an equal division left 10 coins. Again an argument developed in which another pirate was killed. But now the total fortune was evenly distributed among the survivors. What was the least number of coins that could have been stolen?

Q.6. A certain integer between 1 and 1200 leaves the remainders 1, 2, 6 when divided by 9, 11, 13, respectively. What is the integer?.

Q.7. Find an integer having the remainders 1, 2, 5, 5 when divided by 2, 3, 6, 12, respectively.

Q.8. Let t_n denote the n^{th} triangular number. For which values of n does t_n divide

$$t_1^2 + t_2^2 + \cdots + t_n^2.$$

Q.9. Find the solutions of the system of congruences:

$$3x + 4y \equiv 5 \pmod{13}$$

$$2x + 5y \equiv 7 \pmod{13}.$$