MA212 LINEAR ALGEBRA AND STATISTICAL ANAYSIS — BHAGYA RANA [U19CS012]

TUTORIAL 1

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1.> Define Random Variable. Dissuss types.

1.> A Random variable is variable whose value is unknown for given sample case or a function that assigns value to each of an experiments outcomes. Random variables are often denoted by letters.

Some of its types are :-

1> DISCRETE

It is type of random variable in which variables take only finite discrete number of values. The best example is Dice {1,2,3,4,5,6}

2> CONTINOUS

Unlike discrete ones, continous Rondom variables can take finite continous (decimal V) number of values. Eg: return on stocks is continous random variable.

2.> A drug is used to maintain steady heart rate in patients who have sufferred a mild heart attack. let X denote the number of heartbeats per minute obtained per patient. Consider a new drug with Y no.s of heartbeat per minute obtained per patient. The hypothetical density for both drugs is given as

X/Y	40	60	68	70	72	80	100
f(x)	.01	.04	0.05	0.80	0.05	0.04	0.01
f(y)	.40	.05	0.04	0.02	0.04	0.05	0.40

Find E[X], E[Y], Var X & Var Y. Which drug you think is more effecient? Which unit is associated with σ_X & σ_Y?

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2.>

$$E[X] = \sum_{i=1}^{n} f_i x_i = \boxed{70}$$

$$E[Y] = \sum_{i=1}^{n} f_i y_i = \boxed{70}$$

$$\text{Var } X = \sum f_i x_i^2 - [E[X]]^2$$
$$= 4926.4 - 4900$$
$$= \boxed{26.4}$$

$$\text{Var } Y = \sum f_i y_i^2 - [E[Y]]^2$$
$$= 5630.32 - 4900$$
$$= \boxed{730.32}$$

$$\sigma_X = \sqrt{26.4} = \boxed{5.13}$$
$$\sigma_Y = \sqrt{730.32} = \boxed{27.02}$$

Ans: (1) Since the mean of X and Y are same, hence both distributive drugs are equally effecient.

(2) σ_X / σ_Y unit will be same as that of X/Y i.e. No. of heartbeat per minute per patient.

3.> Let X be discrete Random Variable with density f. Let c be any real number. S.T. $E[c] = c$ & $E[cX] = c\,E[X]$. What will be Var c and σ_c?

3.> To Prove: $E[c] = c$ and ① $E[cX] = c\,E[X]$

Let $E[X] = x_1 p_1 + x_2 p_2 + \dots + x_n p_n$

$E[cX] = c x_1 p_1 + c x_2 p_2 + \dots + c x_n p_n$

$\therefore \boxed{E[cX] = c.E[X]}$

To Prove: ② $E[c] = c$ Let $p(x) = \begin{cases} 1, & x = c \\ 0, & \text{otherwise} \end{cases}$

$E[x] = 0 + x.1$

$\boxed{\therefore E[c] = c}$, {substituting $x = c$}

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$$\text{Var } C = \sum p(x) \times x^2 - \left(\sum p(x) \cdot x\right)^2$$
$$= c^2 \cdot 1 + 0 - c^2$$
$$= \boxed{0}$$

$$\therefore \sigma_c = \sqrt{\text{Var } c} = \sqrt{0} = \boxed{0}$$

4.) Let X & Y be independent R.V. with $E[X] = 3$, $E[X^2] = 25$, $E[Y] = 10$ & $E[Y^2] = 164$

(a) Find Var X and Var Y.

$$\text{Var } X = E[X^2] - (E[X])^2 = 25 - 9 = \boxed{16}$$

$$\text{Var } Y = E[Y^2] - (E[Y])^2 = 164 - 100 = \boxed{64}$$

(b) Find $E[3X + Y - 8]$

$$= 3\,E[X] + E[Y] - 8$$
$$= 3 \times (3) + 10 - 8$$
$$= \boxed{11}$$

(c) Find $E[2X - 3Y + 7]$

$$= 2\,E[X] - 3\,E[Y] + 7$$
$$= 2(3) - 3(10) + 7$$
$$= \boxed{-17}$$

(d) Find σ_X & σ_Y

$$\sigma_X = \sqrt{\text{Var } X} = \sqrt{16} = \boxed{4}$$
$$\sigma_Y = \sqrt{\text{Var } Y} = \sqrt{64} = \boxed{8}$$

(e) Find $\text{Var}[3X + Y - 8]$

$$= \text{Var}[3X + Y]$$
$$= \text{Var}[3x] + \text{Var}[Y] + 2 \times 3 \times \text{cov}[x, y]$$
$$= 9\,\text{Var}[X] + \text{Var}[Y] + 0$$
$$= 9 \times 16 + 64$$
$$= \boxed{208}$$

(f) Find $\text{Var}[2X - 3Y + 7]$

$$= \text{Var}[2X - 3Y]$$
$$= 4\,\text{Var}[X] + 9\,\text{Var}[Y] - 12\,\text{Cov}[X, Y]$$
$$= 4 \times 16 + 9 \times 64$$
$$= 64 + 9 \times 64$$
$$= \boxed{640}$$

(g) $E\left[\frac{X-3}{4}\right]$ & $\text{Var}\left[\frac{X-3}{4}\right]$

$$E\left[\frac{X-3}{4}\right] = E\left[\frac{X}{4}\right] - \frac{3}{4} = \frac{1}{4} \times 3 - \frac{3}{4} = \boxed{0}$$

$$\text{Var}\left[\frac{X-3}{4}\right] = \text{Var}\left[\frac{X}{4}\right] = \frac{\text{Var}[X]}{16} = \boxed{1}$$

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(b) $E\left[\frac{(Y-10)}{8}\right]$ & $Var\left[\frac{(Y-10)}{8}\right]$

$$E\left[\left(\frac{Y-10}{8}\right)\right] = \frac{E[Y]}{8} - \frac{10}{8} = \boxed{0}$$

$$Var\left[\frac{Y-10}{8}\right] = Var\left[\frac{Y}{8}\right] = \frac{Var[Y]}{64} = \boxed{1}$$

5.> Let X be a binomial R.V. with parameters $n = 15$ & $p = 0.2$ find $E[X]$, $m_X(t)$, $Var\ X$ & σ_X.

5.> $E[X] = np = 15 \times 0.2 = \boxed{3}$

$Var\ X$ = Variance $X = npq = np(1-p) = 15 \times 0.2 \times 0.8 = \boxed{2.4 = 12/5}$

$\sigma_X = \boxed{\sqrt{2.4} = 1.5491}$

$m_x(t) = [q + pe^t]^m = [(1 - 1/5) + 1/5\, e^t]^m$

$= \boxed{\left(\frac{4 + e^t}{5}\right)^m}$

6.> It has been found that 30% of all printers used on home computers operate correctly at the time of installation. The rest require some adjustment. A particular dealer sells 10 units during a given month.

(a) Find the Probability that atleast nine of the printers operate correctly upon installation.

$$P(X) = {}^{10}C_9 \left(\frac{8}{10}\right)^9 \left(\frac{2}{10}\right) + {}^{10}C_{10} \left(\frac{8}{10}\right)^{10} = 2.8 \times \left(\frac{8}{10}\right)^9$$

$= \boxed{0.3758}$

(b) Consider 5 months in which 10 units are sold per month. What is the probability that atleast 9 units operate correctly in each of 5 months

$P(X)$ for 5 months $= (0.3758) \times (0.3758) \times (0.3758) \times (0.3758) \times (0.3758)$

$= \boxed{(0.3758)^5}$

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7.) Let X be Poisson random variable with parameter k = 10

(a) find E[X] (b) find Var X (c) σ_X

Since Poisson distribution is

$$f(x, \lambda) = \frac{\lambda^x e^{-x}}{x!}$$

i) $E[X] = 10$ (ii) $Var\ X = 10$ (iii) $\sigma_X = \sqrt{10} = 3.1622$

(d) What is the expression for the density for X.

Expression is

$$\boxed{F(x; 10) = \sum_{i=0}^{x} \frac{e^{-10}\, 10^i}{i!}}$$

Also Calculate

(i) $P[X \leq 4] = \sum_{i=0}^{4} \frac{e^{-10} 10^i}{i!} = e^{-10}\left[\frac{1}{1} + \frac{10}{1} + \frac{10^2}{2} + \frac{10^3}{6} + \frac{10^4}{24}\right]$

$= \boxed{2.92 \times 10^{-4}}$

(ii) $P[X \geq 4] = 1 - P[X \leq 3]$

$= 1 - (P[X \leq 4] - P[4])$

$= 1 - (e^{-10}(227.67))$

$= 1 - 0.010$

$= \boxed{0.99}$

(iii) $P[4 \leq X \leq 9] = e^{-10}\left[\frac{10^4}{4!} + \frac{10^5}{5!} + \frac{10^6}{6!} + \frac{10^7}{7!} + \frac{10^8}{8!} + \frac{10^9}{9!}\right]$

$= \boxed{0.447}$

8.) A particular nuclear plant releases detectable amount of radioactive gases twice a month on average. (i) Find the probability that there will be atmost four such emissions during a month?

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$$P(X) = \frac{e^{-2}(2)^x}{x!}$$

P (Atmost 4 emission) = $P(X \le 4)$ = $P(0) + P(1) + P(2) + P(3) + P(4)$

= $\boxed{0.9473}$

(ii) What is expected number of emissions during a 3-month period?

$P(X) = 2$ [1 month = 2 emissions]

$P(3X) = 3 \times 2 = \boxed{6}$, 6 emissions are expected.

(iii) If, in fact 12 or more emissions are detected during 3 months period. Do you think that there is a reason to suspect the reported average figure twice a month? Explain on the basis of probability

Yes, it is matter of suspense to report figure of twice involved. twice a month since the propability of getting 12 or mor emissions is just 2%. and Hence we can't rely completely on one month data.

9.) The marks X obtained in mathematics by 1000 Students in normally distributed with mean 78% & s.d. 11%. Determine:

(a) How many students got marks above 90%?

Mean = 78%, SD = 11%, Value = 90%.

$z = \frac{90 - 78}{11} = 1.09$, for $z = 2$, value $= 0.1379$

No. of Students = $1000 \times (1 - 0.1379)$

$\approx$ $\boxed{138 \text{ students}}$

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(b) What was the highest marks obtained by lowest 10% of students?

Lowest 10% value = −1.28

$-1.28 = \frac{(x-78)}{11}$ $\boxed{x \simeq 64}$

$\therefore$ Highest marks are $\boxed{64}$

(c) Semi- inter Quarontile Range

$-0.67 = \frac{(x_L - 78)}{11}$

$0.67 = \frac{(X_H - 78)}{11}$ $\rightarrow$ $X_L = 70.6\%$, $X_H = 85.37\%$

Semi- Quarontile $= \frac{15.37}{2} = \boxed{7.37}$

(d) within what limits did the middle 90% of student's lie?

Middle of 90% $\Rightarrow$ 5 to 95%

$z = -1.645$ and $z = 1.645$

$\downarrow$ $\downarrow$

val = 60 val = 96

$\boxed{\therefore \text{ Range is } [60, 96]}$

10.) The Life time in hours of certain kind of radiotube is a random variable having a probability density function given by

$$f(x) = \begin{cases} 0 & , x \leq 100 \\ 100/x^2 & , x > 100 \end{cases}$$

(i) What is the probability that exactly 2 of 5 such tubes in radio set will have to be replaced within first 150 hours of operation? Assume that the events A_i $i = 1,2,3,--5$, i^{th} such tube will have to be replaced within this time are independent.

10.) $P(X < 150) = P(100 < X < 150)$

$$= \int_{100}^{150} \left(\frac{100}{x^2}\right) dx$$

$$= \left[\frac{-100}{x}\right]_{100}^{150}$$

$$= 100\left[\frac{1}{100} - \frac{1}{150}\right]$$

$$= \boxed{1/3}$$

$$P_0(X=2) = {}^5C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^3$$

$$= \frac{5 \times \cancel{4}^{2}}{\cancel{2}} \times \frac{8}{3^5}$$

ANS: $= \boxed{\frac{80}{243}} = \boxed{0.3292}$

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