MATHS TUTORIAL-2 (Group Theory)

Questions) [1-12] + [1-7]

1. An operation $*$ is defined on the set of positive rational numbers $\mathbb{Q}^+$ by $a*b = ab/2$ for $a, b \in \mathbb{Q}^+$. Show that (i) $*$ is a binary operation on $\mathbb{Q}^+$ (ii) $*$ is commutative (iii) $*$ is associative.
2. An $*$ defined on the set $\mathbb{Z}$ of integers as $(a,b) * (c,d) = (a+c, b+d)$. Show that $*$ is commutative as well as associative, $(-a,-b)$ is an inverse of (a,b).
3. Define Group. If M_2 is the set of 2×2 non-singular matrices over R. $M_2 = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mid a,b,c,d \in R \text{ \& } ad-bc \neq 0 \right\}$, prove that M_2 is a group under the operation of usual matrix multiplication. Is it abelian?
4. If $\{U_n\}$ is the set of n^{th} roots of unity, show that $\{U_n, \times\}$ is a cyclic group. Is it abelian?
5. S.T. every group of order 3 is cyclic & every group of order 4 is abelian.
6. Show that the group $\{G, +_6\}$ is cyclic group under where $G = \{0,1,2,3,4,5\}$. What are its generators?
7. S.T. the identity element of a group is the only element whose order is 1.
8. Find the order of every element of the multiplication group $G = \{a, a^2, a^3, a^4, a^5, a^6 = e\}$
9. State the basic properties of a group. Define subgroup with example.
10. If every element of a group $(G, *)$ is its own inverse prove that G is abelian.
11. If G is a group, then show that $C = \{c \mid cx = xc \text{ for all } x \in G\}$ is a subgroup of G.
12. Show by means of ~~exp~~ ex. that ~~the~~ union of two subgroups may or may not be subgroup.

Tutorial : I

1. Let G be the set of all real 2×2 matrices $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, where $ad-bc \neq 0$ is a rational no. Prove that G forms a group under matrix mult.

2. Let G be the set of all real 2×2 matrices $\begin{pmatrix} a & b \\ 0 & d \end{pmatrix}$ where $ad \neq 0$. P.T. G forms a group under matrix multiplication. Is G abelian?

~~3. (a) Let G be the group of all 2×2 matrices (a b / c d) where ad − bc ≠ 0 & a, b, c, d are integers modulo 3 relative to matrix multiplication. S.T. o(G) = 48.~~
~~(b) If we modify~~

3. Let G be a finite group whose order is not divisible by 3. Suppose that $(ab)^3 = a^3b^3$ for all $a, b \in G$. P.T. G is abelian.

4. P.T. any subgroup of a cyclic group is itself a cyclic group.

5. How many generators does a cyclic group of order n have?

6. If $a \in G$. & $a^m = e$, P.T. $o(a) \mid m$.

7. If G has no nontrivial subgroups S.T. G must be finite of prime order.

8. Let G be a group s.t. the intersection of all its subgroups which are diff. from (e) is a subgroup diff. from (e). P.T. every element in G has finite order.