Tutorial - I

(1.) Define Random variable. Discuss types.

(2.) A drug is used to maintain steady heart rate in patients who have suffered a mild heart attack. Let X denotes the number of heart beats per minutes obtained per patient. Consider a new drug, with Y nos. of heart beats per minutes obtained per patient. The hypothetical density fun for both drugs is given as

X/y	40	60	68	70	72	80	100
f(x)	·01	·04	0.05	0·80	0·05	0·04	0·01
f(y)	·40	·05	0·04	0·02	0·04	0·05	0·40

Find $E[X]$, $E[Y]$, Var X & Var Y. Which drug you think is more efficient? Which unit is associated with σ_X & σ_Y?

(3) Let X be a discrete random variable with density f. Let c be any real number. S.T. $E[c]=c$ & $E[cX]=cE[X]$. What will be Var C & σ_c?

(4) Let X & Y be independent R.V. with $E[X]=3$, $E[X^2]=25$, $E[Y]=10$ & $E[Y^2]=164$.

(a) Find Var X, Var Y (b) Find $E[3X+Y-8]$ (c) Find $E[2X-3Y+7]$

(d) Find σ_X & σ_Y (e) Find $Var[3X+Y-8]$ (f) Find $Var[2X-3Y+7]$

(g) $E\left[\frac{(X-3)}{4}\right]$ & $Var\left[\frac{(X-3)}{4}\right]$ (h) $E\left[\frac{(Y-10)}{8}\right]$ & $Var\left[\frac{(Y-10)}{8}\right]$.

(5) Let X be a binomial R.V. with parameters $n=15$ & $p=0.2$. Find $E[X]$, $m_X(t)$, Var X & σ_X.

(6) It has been found that 80% of all printers used on home computers operate correctly at the time of installation. The rest require some adjustment. A particular dealer sells 10 units during a given month. (a) Find the probability that at least nine of the printers operate correctly upon installation.

(b) Consider 5 months in which 10 units are sold per month. What is the probability that at least 9 units operate correctly in each of the 5 months?

[Ans. 0.3758
$(0.3758)^5$]

⑦ Let X be a Poisson random variable with parameter $k=10$. ⓐ Find $E[X]$ ⓑ Find Var X ⓒ σ_x. ⓓ What is the expression for the density for X. Also, calculate ⓘ $P[X \le 4]$ ⓘⓘ $P[X \geq 4]$ ⓘⓘⓘ $P[4 \le X \le 9]$

⑧ A particular nuclear plant releases a detectable amount of radioactive gases twice a month on the average. Find the probability that there will be at most four such emissions during a month. What is the expected number of emissions during a 3-month period? If, in fact 12 or more emissions are detected during a 3-month period; do you think that there is a reason to suspect the reported average figure of twice a month? Explain, on the basis of the probability involved.

⑨ The marks X obained in mathematics by 1000 students in normally distributed with mean 78% & s.d. 11%. Determine
ⓐ How many students got marks above 90%?
ⓑ What was the highest marks obtained by the lowest 10% of students?
ⓒ semi-inter quartile range?
ⓓ within what limits did the middle 90% of students lie?

⑩ The life time in ~~hours~~ hours of a certain kind of radio tube is a random variable having a probability density fun given by $f(x) = \begin{cases} 0, & x \le 100 \\ \frac{100}{x^2}, & x > 100 \end{cases}$. What is the probability that exactly 2 of 5 such tubes in a radio set will have to be replaced within the first 150 hours of operation? Assume that the events $A_i, i = 1, 2, \ldots 5$, that ith, such tube will have to be replaced within this time are independent.

[80/243]