29.4.__ Mathematics.

- Probability & statistics → Eng. maths
- Combinatories
- Graph theory
- Set theory & algebra
- Logic

Discrete maths

- Linear algebra
- Num. methods
- Calculus.

Engineering maths

Recommanded Book - (kenneth Rosen)

1. Lectures : Theory & WB
2. WKBook
3. PS - I & II
4. Rosen : DM
5. EM for GATE - By Sundaram Sir.

Probability & Statistics

- Basic probab.
- Probab. Distribution
- Statistics.

Basic probab. -

- Science of uncertain events.

1. Experiment
   - Random - Can't predict before.
   - Non-random - can be predicted before result comes out

2. Sample space: Set of all possible outcomes.

$$S = \{H, T\}, \quad S = \{1, 2, 3, 4, 5, 6\}$$

$$S = \{(H,H), (H,T), (T,H), (T,T)\}.$$

$$S = \{ (1,1)\ (1,2) \ - \ - \ (1,6),$$
$$(2,1)\ (2,2) \ - \ -$$
$$- \ - \ - \ - \ (6,6) \}.$$

3. Event

$$\text{Event} \subseteq \text{Sample space}$$

- event is any subset of sample space.
- Sample space is universal set.

Eg. $S = \{H, T\}$

$$E_1 = \{\ \}$$
$$E_2 = \{H\}$$
$$E_3 = \{T\}$$
$$E_4 = \{H, T\}$$

Only
i.e.
in d
like
- In p
it.
p(
Eg. in
When
prob.
Probabi
- Cla
- Fre

$$\Rightarrow \boxed{p(E) = \frac{n(E)}{n(S)}} \rightarrow \text{Classical probability eq}^n$$

Only when all outcomes are equally-likely.
i.e. all possibility has same chance to come.
in dise, (1, - - - 6) no., all have equally-likely property.

- In purely ~~class~~ lottery system, we can use it.

$$p(E_1) = \frac{0}{2} = 0 \rightarrow \text{impossible event}$$

$$p(E_2) = \frac{1}{2} = p(E_3)$$

$p(E_4)$ = either head or tail

$$= \frac{2}{2} = 1 \rightarrow \text{Sure event.}$$

Eg. in case of dice — $2^6 = 64$ events are possible.
When dice $\rightarrow$ 2^{36} events will happen.

prob. of max is 3 is $\rightarrow$

$$p(\max = 3) = \frac{5}{36}$$

(1, 2, 3, 3)
(3, 1, 2, 3)
(3, 3)
↓
should not be counted twice.

Probability Approaches —
↓ How to calculate

- Classical
- Frequency

- Classical - $p(E) = \frac{n(E)}{n(S)}$

- Frequency - $p(E) = \frac{f(E)}{\Sigma f} = r.f(E)$

Classical assumption -

- Sample space is finite.
- Outcomes must be equally-likely.

Classical - Analytical / Theoritical → Exact

⇒ Frequency - Practical → Approximate

	f	p = r.f
A	10	.10
B	20	.20
C	60	.60
D	10	.10
	100	

→ 10% chance of A grade.

- In word "BIRD", how many will start by D.

$\boxed{D \mid \ \mid \ \mid \ } \rightarrow 3! = 6$

- $p(\text{start by D}) = \frac{n(\text{start by D})}{n(S)}$

↳ permutation $= \frac{3!}{4!} = \frac{1}{4}$

- 9 Teachers

5 P | 4 Maths $\Rightarrow {}^5C_2 \times {}^4C_1$ → combinatory problem.

3

2 Physics, 1 M

Now prob. of 2 P & 1 M $\Rightarrow$

$$p(2P+1M) = \frac{{}^5C_2 \times {}^4C_1}{{}^9C_3}$$ $\rightarrow$ Conditional / Unconditional is known as probability basically.

5. Probability Axioms -

1. $0 \leq p \leq 1$
2. $p(S) = 1$
3. $p(A \cup B) = p(A) + p(B)$ $\rightarrow$ Only when A & B are mutually exclusive

Mutually exclusive - Can't happen together.

i.e., $(A \cap B) = \phi$

So, $p(A \cap B) = 0$.

Hence, $p(A \cup B) = p(A) + p(B)$.

In case of cards - kings & Hearts. They are not mutually exclusive, because hearts also have king.

This case is called - joint probability.

6. Types of events -

- equally likely
- mutually exclusive
- Collectively exhaustive
- Independent.

Eg. $A = \{1,2,3\}$
$B = \{4,5,6\}$ $\rightarrow$ equally likely

Mutually exclusive -

$A = \{1,2,3\}$
$B = \{3,4,6\}$ $\rightarrow$ eq. lik. & not mutually exclusive

1. $A \cap B = \phi$
2. $p(A \cap B) = \phi$
3. $P(A \cup B) = p(A) + p(B)$.

Collectively exhaustive -

1. $A \cup B = S$
2. $p(A \cup B) = 1$

Mutually exclusive & collectively exhaustive -

$$\Rightarrow \boxed{p(A) + p(B) = 1}$$

$$\Rightarrow p(B) = 1 - p(A).$$

Q. A & B are running race. $P(A) = 0.1$ then what is $P(B) = ?$

If "only" A & B $\rightarrow$ collectively exhaustive.

then $p(B) = 1 - p(A)$
$= 1 - 0.1 = 0.9$

Take table of dice as previously -

$$E(X) = 1\times\frac{1}{6} + 2\times\frac{1}{6} + 3\times\frac{1}{6} + \cdots 6\times\frac{1}{6}$$

$$= \frac{21}{6} = 3.5 \text{ due to symmetry}$$

Now,
$$V(X) = \sum x^2 p(x) - \left(\sum X \cdot p(x)\right)^2$$

$$= \left(40^2 \times 0.1 + 50^2 \times 0.5 + 60^2 \times 0.1 + 70^2 \times 0.3\right) - (56)^2$$

$$V(X) = \text{—}$$

Bienayme - Chemyshev Rule -

$$p(\mu - k\sigma \le X \le \mu + k\sigma) \ge 1 - \frac{1}{k^2} \quad ; k > 1$$

like
$$p(56 - 20 \le X \le 56 + 20)$$
$$p(36 \le X \le 76) \ge 1 - \frac{1}{2^2}$$
$$= \frac{3}{4} \rightarrow \text{~~7.5~~ } 75\% \text{ chance at least}$$

$$\boxed{\text{prob. of } k\sigma \text{ is always } \ge 1 - \frac{1}{k^2}}$$

$\Rightarrow$ 75% is chance of not loosing money in market.
also known as what is risk?

2. $p(x = 5) = \frac{3}{15} = \frac{1}{5}$ / $\frac{1}{6}$

3. $p(x \geq 5) = \frac{2}{6} = \frac{1}{3}$

4. $p(x \leq 5) = \frac{5}{6}$

5. $p(4 \leq x \leq 6) = \frac{3}{6} = \frac{1}{2}$

in case of dice.

6. $E(X) \rightarrow$ Expected value of X.

$$\boxed{E(X) = \mu_X = \bar{X} = \text{Avg. value of } X}$$

$$\Rightarrow \quad E(X) = \sum x \cdot p(x)$$

7. $V(X) \rightarrow$ Variable values $\rightarrow$ Variance.

$$V(X) = \sum x^2 p(x) - \left(\sum x p(x)\right)^2$$

$$\sigma_X^2 = E(X^2) - [E(X)]^2$$

$$\boxed{V(X) = \sigma_X^2 \Rightarrow \underbrace{\sigma_X}_{\text{standard deviation}} = \sqrt{V(X)}}$$

$$E(g(x)) = \sum g(x) \cdot p(x)$$

$$E(X^2) = \sum x^2 p(x)$$

$$E(X^3) = \sum x^3 \cdot p(x)$$

$$\sum (x^2 + x + 1) = \sum (x^2 + x + 1) \cdot p(x) \ldots$$

Independent -

1. $p(A/B) = p(A)$ $\longrightarrow$ Given that B already happend
conditional probab.

Unconditional probab. - marginal probab.

i.e. cond. prob is same as uncond. prob.

2. $p(B/A) = p(B)$.

3. $p(A \cap B) = p(A) \cdot p(B)$.

i.e. if B is happening, it does not effect prob. of A.

As, we know,
$$\boxed{\begin{aligned} p(A \cap B) &= p(A) * P(B/A) \\ p(A \cap B) &= P(B) * P(A/B) \end{aligned}}$$
In general

$\because$ cond. prob = uncond. prob, then,

$$p(B) = p\left(\frac{B}{A}\right)$$

Eg.

	A	B
dice	6	6
day	I	II

$\Rightarrow$ independent case.

$$\Rightarrow p\left(\frac{6}{I} \cap \frac{6}{II}\right) = p\left(\frac{6}{I}\right) \times p\left(\frac{6}{II}\right)$$
$$= \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}.$$

Again, as, if

A	A
6	6
I	II

$\rightarrow$ Not independent case.

Then $\Rightarrow \quad p\left(\frac{A}{I}\right) * p\left(\frac{A}{II} \Big/ \frac{A}{I}\right)$

$$= \frac{4}{52} \times \frac{3}{51}$$

Eg. When take 2 cards with replacement then, becomes independent.

$$\Rightarrow \quad p\left(\frac{A}{I}\right) * p\left(\frac{A}{II} \Big/ \frac{A}{I}\right)$$

$$= \frac{4}{52} \times \frac{4}{52}$$

$p(A \cup B) = A + B - (A \cap B)$

$p(A \cap B) = P(A) \times P(B/A)$

$$\boxed{\begin{aligned} &\Rightarrow p(A \cap B) \leq P(A) \\ &\Rightarrow \quad P(A \cup B) \leq p(A) + P(B) \\ &\Rightarrow p(A \cap B) \leq p(B/A) \\ &\quad P(A/B) \geq P(A \cap B) \end{aligned}}$$

Rules of probability -

1. $P(A \cup B) = p(A) + p(B) - p(A \cap B)$
2. $p(A \cap B) = P(A) \cdot p(B/A) = p(B) \cdot P(A/B)$
3. $P(A^c) = 1 - P(A) \rightarrow$ Complementary event

$$\Rightarrow \quad A \cup A^c = S$$

$$p(A^c \cap B^c) = 1 - p(A \cup B)$$

$$p(A^c \cup B^c) = 1 - p(A \cap B)$$

Q. $p(A) = 0.1$, $P(B) = 0.2$, $P(A \cap B) = 0.05$

What is neither condition case?

$$P(A^c \cap B^c) = P((A \cup B)^c)$$

$$= 1 - P(A \cup B)$$

$$= 1 - [P(A) + P(B) - P(A \cap B)]$$

$$= 1 - [0.1 + 0.2 - 0.5]$$

$= 0.75$ neither of them prob.

Q. 2 dice thrown either of them is not 6.

$$P\left(\underset{\text{I}}{6\times} \cup \underset{\text{II}}{6\times}\right) = ?$$

let us assume 1st for, $P\left(\underset{\text{I}}{6}\right)$

$$\Rightarrow P\left(\underset{\text{I}}{6^c} \cup \underset{\text{II}}{6^c}\right) = P\left(\left(\underset{\text{I}}{6} \cap \underset{\text{II}}{6}\right)^c\right)$$

$$= 1 - P\left(\underset{\text{I}}{6} \cap \underset{\text{II}}{6}\right)$$

$$= 1 - \left(\frac{1}{6} \times \frac{1}{6}\right)$$

$$= 1 - \frac{1}{36} = \frac{35}{36}$$

is prob. that neither comes with 6.

$A^c \cap B^c \rightarrow$ NOR

$A^c \cup B^c \rightarrow$ NAND

Q. Let us take all possible words from, "MISSISSIPPI"?

$$p(\text{start without } S) = 1 - p(\text{start with } S)$$

$$= 1 - \frac{n(\text{start with } S)}{n(S)}$$

M − 1
I − 4
S − 4
P − 2
―――
11

$$\Rightarrow \frac{11!}{1!\,4!\,4!\,2!} = n(S).$$

$$\Rightarrow \quad = 1 - \frac{\dfrac{10!}{4!\,3!\,2!}}{\dfrac{11!}{2!\,4!\,4!}}$$

| S | | | | | | | | | | |

$\underbrace{\qquad\qquad\qquad}$ → M − 1, I − 4, S − 3, P − 2, total 10

$$\frac{10!}{1!\,4!\,3!\,2!}$$

$$= 1 - \frac{4}{11} = \frac{7}{11}$$

4. $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$ → when event already happend

$P(B/A) = \dfrac{P(A \cap B)}{P(A)}$

<u>Q.</u> $p(\text{rain}) = 0.1$

$p(\text{humid} \;\&\; \text{rain}) = 0.05$

What is $p(\text{humid}/\text{rain}) = ?$

<u>Sol^n-</u> $p(\text{humid}/\text{rain}) = \dfrac{p(H \cap R)}{P(R)}$

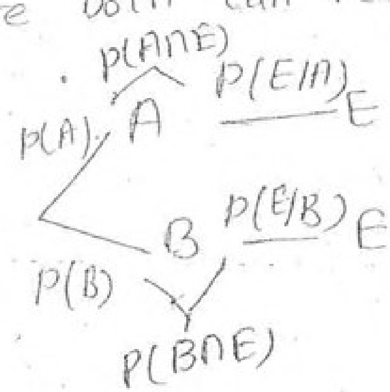
humid on day which is raining

$= \dfrac{0.05}{0.1} = \dfrac{1}{2}$

$p(\text{humid} \;\&\; \text{Rain}) = p(\text{humid}) \times P(\text{Rain}/\text{humid})$

Since $p(\text{humid})$ is not given, so, we can't use this further more.

5. Where both can happen at some particular day.



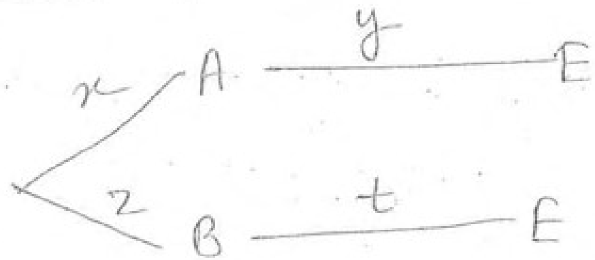
Rules of total probability -

$$p(E) = p(E \cap A) + p(E \cap B)$$

as, E can happen with. A as well as B.

$$= p(A) * p(E/A) + p(B) * P(E/B)$$

In general →



$$\Rightarrow \boxed{p(E) = xy + zt}$$

6. Baye's Theorem - Given that E is already happend. Then have to calculate P(A) or P(B) in this case. i.e. given P(A/E) & P(B/E).

As, $P(A/E) = \dfrac{P(A \cap E)}{P(E)}$

P(E) can be obtain by "Rule of total prob".

$$\Rightarrow P(A/E) = \frac{xy}{xy + zt}$$

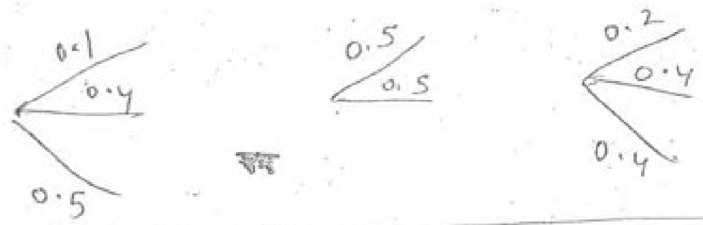
Similarly,

$$P(B/E) = \frac{P(B \cap E)}{P(E)} = \frac{zt}{\underbrace{xy + zt}_{\text{Total prob.}}}$$

probability of branch

always

$$\boxed{P(A/E) + P(B/E) + P(C/E) = 1}$$



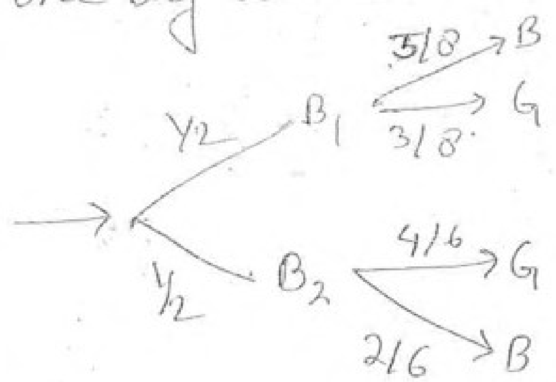
Problems – (on Rule no. 5 & 6)

Prob.

Bag 1	Bag 2
5B, 3G	2B, 4G

pick one bag at random, what is $P(G) = ?$

Sol^n –



$$\Rightarrow \quad P(G) = \frac{1}{2} \times \frac{3}{8} + \frac{1}{2} \times \frac{4}{6}$$

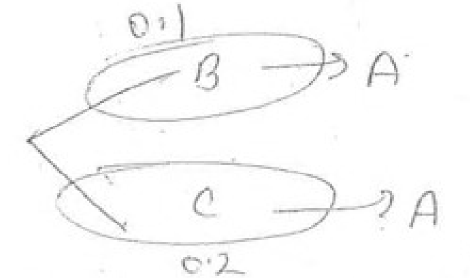
$$= \frac{1}{2}\left(\frac{3}{8} + \frac{4}{6}\right)$$

Q. he founds, ball is green, what is chance it is from bag 1.

$$\Rightarrow P\left(\frac{B_1}{G}\right) = \frac{P(B_1 \cap G)}{P(G)} = \frac{\frac{1}{2}\times\frac{3}{8}}{\frac{1}{2}\times\frac{3}{8} + \frac{1}{2}\times\frac{4}{6}}$$

$$=$$

Q.



$P(A \cap B) = 0.1$, $P(A \cap C) = 0.2$

what is, $P(B/A) = ?$

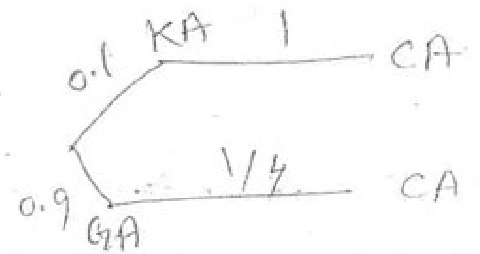
Sol^n -

$$P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{0.1}{P(A)}$$

$\because P(A) = 0.1 + 0.2 = 0.3$

$$P(B/A) = \frac{0.1}{0.3} = \frac{1}{3}$$

Prob.



0.1 Knows Ans.
0.9 Guess Ans.

what is prob. he knows correct ans? as if ques has 4 obtions.

$\Rightarrow$ Find out $P(KA/CA) = ?$

$$\Rightarrow P(KA/CA) = \frac{P(KA \cap CA)}{P(CA)}$$

$$= \frac{0.1 \times 1}{0.1 \times 1 + 0.9 \times \frac{1}{4}} = \frac{0.1}{0.1 + .225}$$

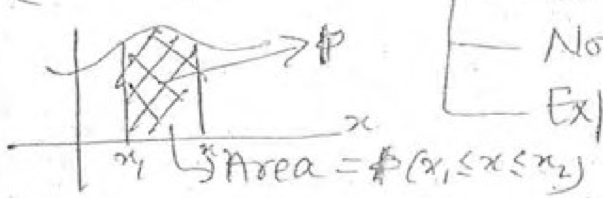
$$= \frac{0.1}{0.325} = \frac{100}{325} = \frac{40}{65} = \frac{8}{13}$$

Probability distribution - dice $\rightarrow X = \{1,2,3,4,5,6\}$

Random variables
- Discrete $\rightarrow$ One value from set of values
- Continuous $\rightarrow$ takes one value from range of values. like weight $\rightarrow 0 \leq X \leq 100$ gm

Discrete distributions (Table form)
- General
- Binomial
- Hypergeometric
- Poission

Continuous distri. (Curve form)
- General
- Uniform
- Normal, standard normal
- Exponential



Discrete distributions - We get,

x	1	2	3	4	5	6
$p(x)$	1/6	1/6	1/6	1/6	1/6	1/6

whereas in continuous prob,

$p(x=2) = \frac{1}{6}$ → distrete

$p(x=2) = 0$ → Conti. 3

4

① General Distri. -

X	40	50	60	70
p(x)	0.1	0.5	0.1	0.3

5.

$E(X) = 40 \times 0.1 + 50 \times 0.5 + \ldots$
$= 56 \quad = 4 + 25 + 6 + 21$

In dice - 6.

X	2	3	4	- - -	10	11	12
p(x)	1/36	2/36	3/36		3/36	2/26	1/36

prob. distri. table.

it is require put all values of x. 7.

1. $\sum p(x) = 1$

eg.

X	3	4	5	6	7
p(x)	k	2k	3k	4k	5k

if above is prob. distri table, then k=?

Sol^n :- $\sum p(x) = k + 2k + 3k + 4k + 5k = 1$

$15k = 1$

$\boxed{k = \frac{1}{15}}$ Ans

and $p(x=5) = \frac{3}{15} = \frac{1}{5}$

$\mu - k\sigma = 36$ $\left\{ 1 - \frac{1}{k^2} \right.$ always

$56 - k \times 10 = 36$

$k = 2$

- minimum probability we can find by this rule.

$\Sigma(x^2+1) = \Sigma(x^2+1) \cdot p(x)$

$= 2 \times \frac{1}{6} + 5 \times \frac{1}{6} + 10 \times \frac{1}{16} + 17 \times \frac{1}{6} + 27 \times \frac{1}{6}$

$+ 37 \times \frac{1}{6}$

$=$

Prob. One is keep on tossing coin. What is the expected no. of toss, that he gets 2 continuous head and stops the game?

2 - HH ✓
HT ×
TH ×
TT ×

3 - HHH ✓
HHT ✓
HTH
HTT
THH ✓
THT
TTH
TTT

$\Rightarrow$

X	2	3	4	5 ...
p(x)	1/4	1/8	1/8	

$E(X) = \Sigma\, x \cdot p(x)$ → calculate in this manner!

$= \frac{1}{4} \times 2 + \frac{1}{8} \times 3 + \frac{1}{8} \times 4 + \ldots$

$\Rightarrow$ let an example, $S = \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \frac{5}{32} + \ldots$

$\frac{1}{2}S = \frac{2}{8} + \frac{3}{16} + \frac{4}{32} + \frac{5}{64} + \ldots$

$\Rightarrow S - \frac{1}{2}S = \frac{1}{2}S = \frac{2}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \ldots$

$= \frac{1}{2} + \frac{\frac{1}{8}}{1 - \frac{1}{2}} = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

$\Rightarrow \quad S = \frac{6}{4} = \frac{3}{2} = 1.5$

So, on avg. person will play 1.5 tosses, to get 2 heads.

Prob.

x	2	3	4	5	6	- -	12
$p(x)$							

$$V(x) = \sum x^2 p(x) - \left(\sum x p(x)\right)^2$$

Variance of x = ? J.N.U.

Prob.

$$E(ax+b) = a \cdot E(x) + b$$

let $E(x) = 10$, $E(2x+5) = ?$

$\Rightarrow \quad E(2x+5) = 2 \cdot E(x) + 5$

$= 2 \times 10 + 5$

$= 25$

only in case of linear fun^n.

let if $E(ax^2+b) = \sum (ax^2+b) \cdot p(x)$ → calculate this, not direct formula.

let if $E(ax_1 + bx_2 + c) = a \cdot E(x_1) + b \cdot E(x_2) + c$

$E(x_1) = 5$ $\Rightarrow 3 \times 5 + 5 \times 7 + 3$

$E(x_2) = 2$ $= 53$ → due to linearity

What is, $E(3x_1 + 5x_2 + 3) = ?$

$$\Rightarrow \mu_{ax+b} = a \cdot \mu_x + b$$

$$\mu_{ax_1+bx_2+c} = a\mu_{x_1} + b\mu_{x_2} + c$$

$a \rightarrow$ Scaling

$b \rightarrow$ ~~Origin's~~ shifting of origin

$$\Rightarrow \left.\begin{matrix} \pm \Rightarrow \text{shifting} \\ * \Rightarrow \text{scaling} \end{matrix}\right\}$$

$$\Rightarrow V(ax+b) = a^2 \cdot V(x)$$

* Variance does not effected by shifting.

$$\sigma^2_{ax+b} = a^2 \cdot \sigma_x^2$$

$$\Rightarrow \sigma_{ax+b} = a\sigma_x$$

$\Rightarrow$ standard deviation also does not effected by shifting

let $E(X) = 50$, $V(X) = 100$

$$E(3x+5) = 3 \times 50 + 5 = 155$$

$$V(3x+5) = 3^2 \times 100 = 900$$

$$\sigma_{3x+5} = 3 \cdot \sigma_x = 30.$$

$$\Rightarrow V(ax_1 + bx_2 + c) = a^2 V(x_1) + b^2 V(x_2) + 2ab \cdot \underset{\text{covariance}}{\underbrace{Cov(X_1, X_2)}}$$

let

$V(X_1) = 100$, $V(X_2) = 200$, $Cov(X_1, X_2) = 10$

now find out, $V(3X_1 + 4X_2) = ?$

$$\Rightarrow V(3X_1 + 4X_2) = 3^2 \times 100 + 4^2 \times 200 + 2 \times 3 \times 4 \times 10$$

$$= 900 + 3200 + 240$$

$$= 4340.$$

$\boxed{\text{if } X_1 \;\&\; X_2 \text{ are indep.} \Rightarrow Cov(X_1, X_2) = 0}$

$Cov(X_1, X_2)$ measures the dependency b/w X_1 and X_2.

$\Rightarrow$ Larger value $Cov(X_1, X_2) \Rightarrow$ Larger dependency.

$\Rightarrow \boxed{-\infty \le Cov(X_1, X_2) \le +\infty}$

* if $Cov(X_1, X_2) = +100 \rightarrow$ direct dependent
$\downarrow$
both moving in same dirⁿ.

* $-ve$ value $\Rightarrow$ inverse dep. $\rightarrow$ in opposite dirⁿ

* 0 value $\rightarrow$ No connection b/w them.

$$\boxed{Cov(X_1, X_2) = E(XY) - E(X) \cdot E(Y)}$$

$$V(X) = E(X^2) - (E(X))^2$$

$\Rightarrow \boxed{Cov(X, X) = V(X)} \rightarrow$ Thats why called covariance.

Two random variable problem –

Y \ X	0	1	2	p(Y)
0	0.1	0.2	0.05	0.35
1	0.3	0.1	0.25	0.65
p(X) →	0.4	0.3	0.30	→ marginal prob.

→ Joint prob. distri table.

⇒ Ques. may be ask like –

1. $p(X=1 \cap Y=0)$
2. $p(X \geq 1 \cap Y=1)$
3. $p(X=1 \mid Y=0)$
4. $E(X)$
5. $E(Y)$
6. $V(X)$ & $V(Y)$
7. $\text{cov}(X, Y)$
8. $E(X / Y=1)$

⇒ How to answers the above –

1. $p(X=1 \cap Y=0) = 0.2$
2. $p(X \geq 1 \cap Y=1) = 0.1 + 0.25 = 0.35$
3. $p(X=1 \mid Y=0) = \dfrac{p(X=1 \cap Y=0)}{p(Y=0)}$

$$= \frac{0.2}{0.35} = \frac{4}{7}$$

(Uncondi → marginal)

$\Rightarrow$

X	0	1	2
$p(x)$	0.4	0.3	0.3

Y	0	1
$p(Y)$	0.35	0.65

$= 0 \times 0.1 + 1 \times 0.3 + 2 \times 0.03 = 0.3 + 0.6 = 0.9$

4. $E(X) = 0.9$

5. $E(Y) = 0.65$

6. $V(X) = E(X^2) - [X \cdot E(X)]^2$

$= 0.69$

$= 1.5 - (0.9)^2$

conditional $= \dfrac{\text{Inside}}{\text{Marginal}}$

7. $V(Y) = 0.65 - (0.65)^2$

$= 0.65 - 0.4 \cong 0.25$ (Approx).

8. $\text{cov}(X, Y) = E(X;Y) - E(X) \cdot E(Y)$.

$E(X, Y) = \Sigma\, XY \cdot P(X \cap Y)$

$= 1 \times 1 \times 0.1 + 2 \times 1 \times 0.25$

$= 0.1 + 0.5$

$= 0.6$

$\Rightarrow \text{cov}(X, Y) = 0.6 - 0.9 \times 0.65$

$= 0.015$.

almost indep. or dep.
we can't say this
accurately.

9. $\boxed{r(X,Y) = \dfrac{Cov(X,Y)}{\sigma_X \cdot \sigma_Y}}$ Condruence

; $-1 \leq r \leq +1$

Gives how much dependency.
measures linear dependence actually.

$$= \frac{0.015}{\sqrt{0.69} \times \sqrt{0.25}}$$

When r is close to $+1$ $\rightarrow$ highly dependent

$\Rightarrow$ When r is close to -1 $\rightarrow$ least dep.

10. $E(X/Y=1)$ $\rightarrow$ conditional expectation

X	0	1	2
$p(x)$	0.4	0.3	0.3

X	0	1	2
$p(X/Y=1)$	$\frac{0.3}{0.65}$	$\frac{0.1}{0.65}$	$\frac{0.25}{0.65}$

$$p(X=0/Y=1) = \frac{P(X=0 \wedge Y=1)}{P(Y=1)} = \frac{0.3}{0.65}$$

$$E(X/Y=1) = \sum x \cdot P(X/Y=1)$$

$$= \frac{1 \times 0.1}{0.65} + \frac{2 \times 0.25}{0.65}$$

$$=$$

if $\forall$ $p(X \cap Y) = p(X) \cdot p(Y)$

only then, we can conclude, that

X & Y are independent.

$$\boxed{\text{if } p(X \cap Y) \neq P(X) \cdot P(Y) \Rightarrow \text{Dependent.}}$$

Binomial Distribution -

n trials, x success, $p(\text{success}) = p$.

- n & p are called parameters.
- x is called random variable.

$$\boxed{p(X == x) = {}^nC_x \, p^x (1-p)^{n-x}}$$

Prob.
10 dice, 3 sixes, what prob. of success?

$n = 10, \; p(6) = \frac{1}{6}, \; X = 3,$

$$p(X=3) = {}^{10}C_3 \cdot \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^7$$

Prob- let 10 coins, 3 heads?

$$p(X=3) = {}^{10}C_3 \left(\frac{1}{2}\right)^3 \cdot \left(\frac{1}{2}\right)^7$$

Variations - always; $x = 0, 1, 2, \ldots$

$$p(x \geq 2) = p(x=2) + p(X=3) + \ldots + p(X=6)$$

10 dice
at least two sixes

$$= 1 - p(X \leq 1)$$

$$= 1 - (p(X=0) + p(X=1))$$

$$= 1 - \left[{}^{10}C_0 \cdot \left(\frac{1}{6}\right)^0 \cdot \left(\frac{5}{6}\right)^{10} + {}^{10}C_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^9 \right]$$

$$p(X \leq 2) = p(X=0) + p(X=1) + p(X=2)$$

$$\boxed{\begin{aligned} \text{In binomial,} \quad E(X) &= n \cdot p \\ V(X) &= np(1-p) \end{aligned}}$$

Prob 10 dice, what expected no. of 6.

$n = 10, \quad p(6) = \frac{1}{6}, \quad E(X) = n \cdot p = 10 \times \frac{1}{6} = 1.66$

$\Rightarrow$ 1.66 of them will be 6. This is an avg. value.

$$V(X) = np(1-p)$$

$$= 10 \times \frac{1}{6} \times \frac{5}{6} = \frac{50}{36} = \frac{25}{18} =$$

Standard deviation, $\sigma_x = \sqrt{\frac{50}{36}} = \frac{5\sqrt{2}}{6}$

* Dice & Coin always follows binomial distri.

Assumption for Binomial -

1. Success & failure is always there.

Prob $P(A) = 0.1$

$P(B) = 0.2$

$P(C) = 0.7$

What is prob. that out of 10, 4 will vote A.

$\Rightarrow \quad n = 10, \; x = 4, \; p(A) = 0.1$

So, using theorem,

$= {}^{10}C_4 \cdot (0.1)^4 \cdot (0.9)^6 \ldots$

$=$

If prob. (Not vote for A) = ?

$= {}^{10}C_4 \; (0.9)^4 \; (0.1)^6$

$=$

2. p should be same from trial to trial.

Prob 10 cards, 3 Aces, = ?

$\Rightarrow P(3 \text{ Ace}) = {}^{10}C_3 \times \left(\frac{4}{52}\right)^3 \cdot \left(\frac{48}{52}\right)^7$

is wrong.

But if with replacement, then it will be correct.

3. Should not be used, when we use sampling for a FINITE population WITHOUT replacement.

i.e., in infinite [illegible]
can use it with no problem.

4. Trial should be statistically independent, i.e. result of trial should not effect on consequent trial's result.

Prob: If $\mu = 50$, $\sigma^2 = 25$, $p(X=2) = ?$

$\Rightarrow$ $\mu = np$

$\sigma^2 = np(1-p)$

Divide both eq^ns^, $= 1-p = \frac{25}{50} = \frac{1}{2}$

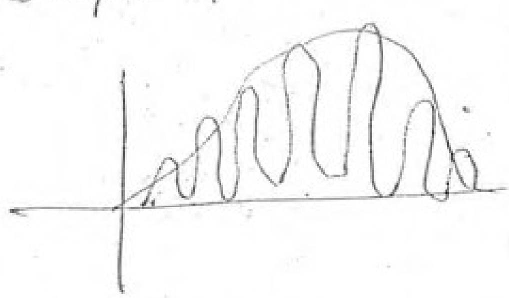
$p = \frac{1}{2}$ ⟶ it will remain same.

$50 = \frac{n}{2} \longrightarrow n = 100$

So, $= {}^{100}C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{98} = p(X=2)$

Prob: Binomial distri. for no. of 6 is obtained. What is shape of bin. distri- p(no of sixes).

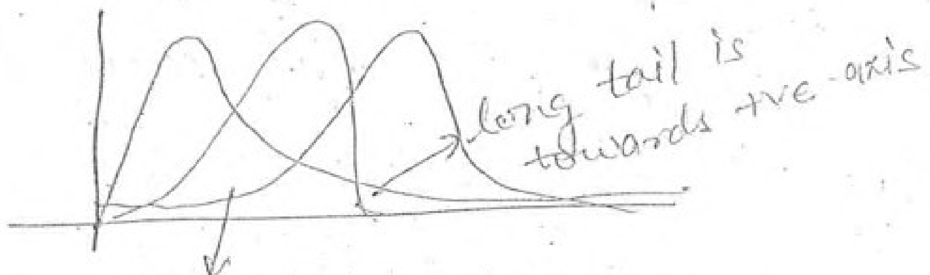
A: Symmetric
B. +ve skew
C. -ve skew
D. None.



$\Rightarrow$ Ans. It will be symmetric shape.

Prob: In case of dice, what will shape?

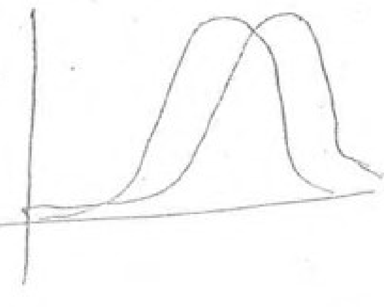
$\Rightarrow$ +vely skewed curve.



-vely skewed curve.

Prob: if getting no. ≥ 2 is success. Then prob. distri of what shape?

$$p(\geq 2) = \frac{5}{6}$$



Ans - -vely skewed.

Symmetric $\rightarrow$ $p = q = \frac{1}{2}$

+ve skew $\rightarrow$ $p < q$, $p < \frac{1}{2}$, $q > \frac{1}{2}$

-ve skew $\rightarrow$ $p > \frac{1}{2}$ $q < \frac{1}{2}$; $p > q$

$\Rightarrow$ Remember it!

-ve : mode $\geq$ Median $\geq$ Mean

+ve : Mean $\geq$ Median ~~Mode~~ $\geq$ Mode

Symmetric: Mean = Med = Mode.

$\Rightarrow$ $\boxed{\text{Max. freq} \rightarrow \text{Mode}}$

Prob- Mean = 60, Median = 50, Mode = 40,

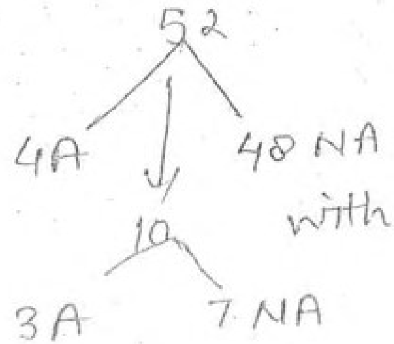
$\Rightarrow$ ∵ mode is lowest.

Hence +vely skewed distribution.

Hypergeometric Distri-

i.e; finite population without replacement.

Eg- 52 cards, out of which 4 A, 48 NA.



without replacement.

$\Rightarrow$ if $x = 3$,

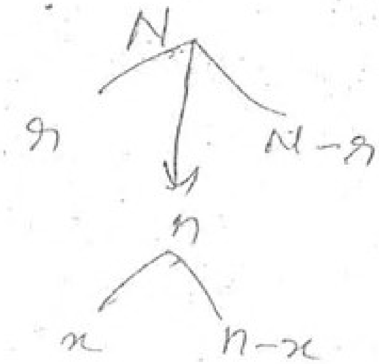
$$p(X=3) = \frac{4C_3 \times {}^{48}C_7}{{}^{52}C_{10}} = \frac{n(E)}{n(S)}$$

$$p(X \geq 3) = 1 - p(X \leq 2)$$

$$= 1 - \left[\frac{4C_0 \cdot 4C_{10}}{52C_{10}} + \frac{4C_1 \times {}^{48}C_9}{{}^{52}C_{10}} + \frac{4C_2 \; {}^{48}C_8}{{}^{52}C_{10}} \right]$$

$$\boxed{P(X=x) = \frac{{}^{r}C_x \; {}^{N-r}C_{n-x}}{{}^{N}C_n}}$$

$\Rightarrow$



$$\rightarrow \boxed{E(X) = n \cdot \frac{r}{N}}$$

expected no. of success.

Poission distri –

$$\boxed{P(X=x) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}} \; ; \; x = 0, 1, 2, \ldots \infty.$$

- Pure poission
- Binomial poission.

① Case – $\lambda = \alpha \cdot \Delta t$

② case – $\lambda = np.$

λ - is avg. no. of success in observation period.

Binomial $\boxed{n, p}$ $\boxed{x}$

Hypergeo. $\boxed{n, r, N}$, $\boxed{x}$

Poission $\boxed{\lambda}$ $\boxed{x}$

α - is avg. no. of success per unit time.
Δt - Observation period.

* In poission problem only, time factor is present.

Prob- Let $\alpha = 40$ A/hrs., $\frac{1}{2}$ hrs, $p(X=10) = ?$

$\Rightarrow$ Here, $\alpha = 40$, $\Delta t = \frac{1}{2}$

So, $\lambda = \alpha \cdot \Delta t = 40 \times \frac{1}{2} = 20$

$$\lambda = 20 \text{ A}$$

Now, $p(X=10) = \dfrac{e^{-20} \cdot 20^{10}}{10!}$

$$p(X \geq 1) = 1 - p(X=1)$$
$$= 1 - \frac{e^{-20} \cdot 20^{0}}{0!}$$
$$= 1 - e^{-20}$$

$$p(X \geq 2) = 1 - [p(X=0) + p(X=1)]$$
$$= 1 - \left[\frac{e^{-20} \cdot 20^{0}}{0!} + \frac{e^{-20} \cdot 20^{1}}{1!}\right] =$$

$$\boxed{\begin{aligned} E(X) &= \lambda \\ V(X) &= \lambda \end{aligned}}$$ → Here in case of poission distri. always holds.

the

*Whe

No

Binomial poission -

Prob. Manufacturer is 10,000 tracktors. and on avg. $\frac{1}{2000}$ tracters are defective. Then, what is prob. 4 tracters becomes defective ?

Prob if 10 tracters in year, what is P(10) is defective ?

$$P(X \geq 10) = 1 - p(X \leq 9)$$

$$=$$

Prob. $P(X=2) = ?$

$$= {}^{10,000}C_2 \left(\frac{1}{2000}\right)^2 \left(\frac{1999}{2000}\right)^{9998}$$

$$=$$

*Whenever n is large & p is less, then it creates problem in calculation.

Then we approximate binomial into poission distri.

Then, it comes out as,

$$\Rightarrow \lambda = np$$

$$= 10000 \times \frac{1}{2000} = 5$$

then, $p(X=2) = \dfrac{e^{-5} \cdot 5^{2}}{2!} =$

*When n & p ~~also~~ both large, we use

Normal distribution.

↳ continuous distri.

