Tutorial - III

1. The R.V. X denotes the number of trials (Bernoulli) needed to obtain the first success. S.T. pmf $f(x) = (1-p)^{x-1} p$, $0<p<1$, $x=1,2,3,\ldots$ Also S.T. $F(x) = 1-q^{(x)}$. (Geometric distribution)

2. Find the mean of R.V. X, the number of trials needed to obtain a zero when generating a series of random digits. [10]

3. Find value of c, that makes $f(x) = ce^{x}$, $x=1,2,3,\cdots$ a pdf. Find moment generating fun for X, using which find $E[X]$ & $E[X^2]$.

4. Suppose that X is Hypergeometric with $N=20$, $r=3$ & $n=5$. What are the possible values for X? What is $E[X]$ & $\text{var} X$?

5. Define gamma random variable X with parameter α & β. Find $E[X]$ variance X if $m_x(t) = (1-\beta t)^{\alpha}$, $t < 1/\beta$ is moment generating fn for X.

6. Find density fn & cumulative fn for a random variable X distributed uniformly over (30,40).

7. The joint density for (X, Y) is given by $f_{xy}(x,y) = \frac{1}{n^2}$, $x=1,2,3,\ldots,n$, $y=1,2,3,\ldots,n$.
   (i) Verify $f_{xy}(x,y)$ satisfies the conditions necessary to be a density.
   (ii) Find marginal densities for X & Y.
   (iii) Are X & Y independent?
   (iv) Find $\text{Cov}(X,Y)$

8. Economic conditions cause fluctuations in the prices of raw commodities as well as in finished products. Let X denotes the price paid for a barrel of crude oil by the initial carrier, & let Y denotes the price paid by the refinery purchasing the product from the carrier. Assume that the joint density for (X, Y) is given by $f_{xy}(x,y) = C$, $20 < x < y < 40$. Answer the following:

(i) Find the value of c that makes this a joint density for a two-dimensional random variable.

(ii) Find the value of probability that the carrier will pay at least $25 per barrel and the refinery will pay atmost $30 per barrel for the oil.

(iii) Find the probability that the price paid by the refinery exceeds that of the carrier by atleast $10 per barrel.

(iv) Find the marginal densities for X & Y.

(v) Find the probability that the price paid by the carrier is at least $25.

(vi) Find the probability that the price paid by the refinery is atmost $30.

(vii) Are X & Y independent? Explain.

(viii) From a physical standpoint, should cov(X,Y) be +ve or -ve?

(ix) Find E[X], E[Y], E[XY] & cov(X,Y)

(x) Find E[Y-X].

Tutorial - IV
Sampling of Variables.

Page No.
Date

Q.1 A machine which produces mica insulating washers for use in electric device to turn out washers having a thickness of 10 mm. A sample of 10 washers has an average thickness 9.52 mm with a standard deviation of 0.6 mm. Find out t.

Q.2 Ten individuals are chosen at random from a population and their heights are found to be in inches 63, 63, 64, 65, 66, 69, 70, 70, 71. Discuss the suggestion that the Mean height of universe is 65. [For 9 degree of freedom, t at 5% level of significance = 2.262]

Q.3 A random sample of size 16 values from a normal population showed a mean of 53 and a sum of squares of deviation from the mean equals to 150. Can this sample be regarded as taken from the population having 56 as mean ? Obtain 95% and 99% confidence limits of the mean of the population.

$\gamma = 15$, $\alpha = 0.05$, $t = 3.131$

$\alpha = 0.01$, $t = 2.947$.

Q.4 Two independent samples of 8 and 7 items resp. had the following values of the variable (weight in ounces):

Sample 1: 9 11 13 11 15 9 12 14

Sample 2: 10 12 10 14 9 8 10

Is the difference between the means of the sample significant ? [Given for $V = 13$, $t_{0.05} = 2.16$].

Q.5 Memory capacity of 9 students was tested before and

after a course of meditation for a month. State whether the course was effective or not from the data below (in same units)

Before	10	15	9	3	7	12	16	17	4
After	12	17	8	5	6	11	18	20	3

[t_{tab} at 0.05 for 8 d.o.f. is 2.31]

Q.6 A sample of 20 items has mean 42 units and S.D. 5 units. Test the hypothesis that it is a random sample from a normal population with mean 45 units.
[t_{tab} at 5% L.O.S. for 19 d.f. = 2.09]

Q.7 Two samples of sodium vapour bulbs were tested for length of life and the following results were got:

	Size	Sample mean	Sample S.D.
Type I	8	1234 hrs.	36 hrs.
Type II	7	1036 hrs.	40 hrs.

Is the difference in the means significant to generalize that type I is superior to type II regarding length of life?
[$t_{0.05}$ at 13 d.f. = 1.77]

Q.8 The following table is given:

		Eye colour in sons		
		Not light	Light	
Eye colour in Fathers	Not light	230	148	378
	Light	251	471	622
		361	619	1000

Test whether the colour of the son's eyes is associated with that of the fathers.
[Given : Value of χ^2 is 3.84 for 1 d.o.f.]

Q.9 From the following table, showing the number of plants having certain characters, test the hypothesis that the flower colour is independent of flatness of leaf.

	Flat leaves	Curled leaves	Total
White Flowers	99	36	135
Red Flowers	20	5	25
Total	119	41	160

[χ^2_{tab} = 0.0158 at 0.1 L.O.S. for 1 D.O.F.]

Q.10 A set of five similar coins is tossed 320 times and the result is :

No. of heads	0	1	2	3	4	5
Frequency	6	27	72	112	71	32

Test the hypothesis that the data follow a binomial distribution.
[χ^2_{tab} = 11.07 for 5 d.o.f.]

Q.11 Fit a poisson distribution to the following data and test the goodness of fit.

x	0	1	2	3	4	5	6
f	275	72	30	7	5	2	1

[χ^2_{tab} at 5.1 L.O.S. for 2 d.f. = 5.99]

Answers:

Q.1 $t = -2.5258$

Q.2 $t_{cal} < t_{tab}$, H_0 accepted. i.e. Mean height is 65.

Q.3 The sample cannot be regarded as taken from the population.

Q.4 $t_{cal} < t_{tab}$, the difference between the means of samples is not significant.

Q.5 $t_{cal} < t_{tab}$, H_0 accepted. i.e. The training was not effective in improving performance.

Q.6 $t_{cal} > t_{tab}$, H_0 rejected. i.e. Difference is significant

Q.7 $t_{cal} > t_{tab}$, H_0 rejected, H_1 is accepted.

Type I is definitely superior to Type II.

Q.8 $\chi^2_{cal} > \chi^2_{tab}$, H_0 rejected.

Q.9 $\chi^2_{cal} > \chi^2_{tab}$, H_0 is wrong.

Q.10 $\chi^2_{cal} > \chi^2_{tab}$, H_0 is rejected.

Q.11 $\chi^2_{cal} > \chi^2_{tab}$, P.D. is not good fit.