Department of Computer Engineering, SVNIT, Surat.
Theoretical Computer Science
Tutorial – 2
(Mathematical Induction and Method of Contradiction)

Q1 Prove the following by mathematical induction.

1. Prove by mathematical induction the statement
$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{n\cdot(n+1)}=\frac{n}{(n+1)}, n\epsilon N$$

2. Prove by mathematical induction the statement
$$1\cdot 1!+2\cdot 2!+3\cdot 3!+\cdots+n\cdot n!=(n+1)!-1, n\epsilon N$$

3. Prove by mathematical induction, that
$$\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)\ldots\left(1+\frac{2n+1}{n^2}\right)=(n+1)^2, n\epsilon N$$

4. Prove by mathematical induction, that
$$1\cdot 2+2\cdot 2^2+3\cdot 2^3+\cdots+n\cdot 2^n=(n-1)\cdot 2^{n+1}+2, n\epsilon N$$

5. Prove by mathematical induction, that $x^{2n}-y^{2n}$ is divisible by $x+y$ for all natural numbers n.

6. We have three pegs and a collection of disks of different sizes, Initially they are on top on each other according to their size on the first peg, the largest being on the bottom and the smallest on the top. A move in this game consists of moving disks from one peg to another such that larger disk can never rest on smaller one. Prove that the number of moves to transfer all disks from first peg to the last peg using second peg as intermediate is $2^n-1, n\epsilon N$.

Q2 Prove the following by contradiction

1. The square root of 7 is irrational.

2. Show that following statement is true by method of contradiction if $x^5+16x=0$, then $x=0$.

3. Using the method of contradiction, prove that sum of an irrational number and a rational number is irrational.

4. Prove that if $x>3$, then $x^2>9$ using the method of contradiction. $(x\epsilon R)$.