Discrete Mathematics and Its Applications

Seventh Edition

Chapter 2

Sets

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Sets

German mathematician **G. Cantor** introduced the concept of sets. He had defined a set as a collection of definite and distinguishable objects selected by the means of certain rules or description.

Set theory forms the basis of several other fields of study like counting theory, relations, graph theory and finite state machines. In this chapter, we will cover the different aspects of **Set Theory**.

Set Theory

A well defined collection of {distinct} objects is called a set.

- The objects are called the elements or members of the set.

- Sets are denoted by capital letters

  A, B, C …, X, Y, Z.

- The elements of a set are represented by lower case letters

  a, b, c, … , x, y, z.

Set Theory

- Set: Collection of objects ("elements")
- $a \in A$ "a is an element of A" "a is a member of A"
- $a \notin A$ "a is not an element of A"
- $A = \{a_1, a_2, \ldots, a_n\}$ "A contains…"
- Order of elements is meaningless
- It does not matter how often the same element is listed.

Set Theory

A set is an unordered collection of different elements. A set can be written explicitly by listing its elements using set bracket. If the order of the elements is changed or any element of a set is repeated, it does not make any changes in the set.

Some Example of Sets

A set of all positive integers

A set of all the planets in the solar system

A set of all the states in India

A set of all the lowercase letters of the alphabet

Sets

Definition:

- A set is an unordered collection of objects referred to as elements.
- A set is said to contain its elements.
- We write $a \in A$ to denote that a is an element of the set A.
- The notation $a \sim\in A$ denotes that a is not an element of the set A.

Examples of Set

- **EXAMPLE 1:**

The set V of all vowels in the English alphabet can be written as V={a, e, i, o, u}.

- **EXAMPLE 2:**

The set O of odd positive integers less than 10 can be expressed by O={1,3,5,7,9}.

Representation of a Set

Sets can be represented in two ways :

- Roster or Tabular Form
- Descriptive Form
- Set Builder Notation

Set Theory

TABULAR FORM

- Listing all the elements of a set, separated by commas and enclosed within braces or curly brackets{}.

EXAMPLES

- In the following examples we write the sets in Tabular Form.
- A = {1, 2, 3, 4, 5} is the set of first five **Natural Numbers**.
- B = {2, 4, 6, 8, …, 50} is the set of **Even numbers** up to 50
- C = {1, 3, 5, 7, 9, …} is the set of **positive odd numbers.**

Set Theory

Descriptive Form:

– Stating in words the elements of a set.

EXAMPLES

– Now we will write the same examples which we write in Tabular Form ,in the Descriptive Form.

– A = set of first five Natural Numbers.(is the Descriptive Form)

– B = set of positive even integers less or equal to fifty. (is the Descriptive Form)

– C = {1, 3, 5, 7, 9, …} (is the Tabular Form)

– C = set of positive odd integers. (is the Descriptive Form)

Set Theory

Set Builder Form

- Writing in symbolic form the common characteristics shared by all the elements of the set.

EXAMPLES

- Now we will write the same examples which we write in Tabular as well as Descriptive Form ,in Set Builder Form .
- $A = \{x \in N \mid x <= 5\}$ (is the Set Builder Form)
- $B = \{x \in E \mid 0 < x <= 50\}$ (is the Set Builder Form)
- $C = \{x \in O \mid 0 < x \}$ (is the Set Builder Form)

Symbol

{ }	Represent the set
|	Such that
$\in$	Is a member of

Example

{x | x < 5}

The set of all real numbers less than 5

{ k member of real's | k > 5 }

The set of all k's that are a member of the Integers, such that k is greater than 5"

Famous Sets in Math

N =Set of natural numbers

Z =Set of integers.

Z+ = Set of positive integers.

Q = {p/q | p $\in$ Z, q $\in$ Z, and q$\neq$0}, set of rational numbers.

Q+ =The set of positive rational number.

R = The set of real numbers.

R+ =The set of positive real numbers.

C =The set of complex numbers

Examples for Sets

"Standard" Sets:

- Natural numbers **N** = {0, 1, 2, 3, …}
- Integers **Z** = {…, -2, -1, 0, 1, 2, …}
- Positive Integers $\mathbf{Z}^{+}$ = {1, 2, 3, 4, …}
- Real Numbers **R** = {47.3, -12, π, …}
- Rational Numbers **Q** = {1.5, 2.6, -3.8, 15, …}

NOTE:

- The symbol "…" is called an ellipsis. It is a short for "and so forth."

Sets

- **When a and b are real numbers with $a<b$, we write**

$[a, b]=\{x|a\leq x\leq b\}$

$[a, b)=\{x|a\leq x<b\}$

$(a, b]=\{x|a<x\leq b\}$

$(a, b)=\{x|a<x<b\}$

Note that $[a, b]$ is called the closed interval from a to b.

(a, b) is called the open interval from a to b.

$(a, b]$is called half-open interval from a to b.

Equal Sets

- Two sets are equal if and only if they have the same elements.
- If A and B are sets, then A and B are equal if and only if $\forall x(x \in A \leftrightarrow x \in B)$.
- We write A=B if A and B are equal sets.
- **Example**
- A={1,5,7} and B={7,5,1}are equal, because they have the same elements.

Example

For Example;

A = {2, 3, 5}
B = {5, 2, 3}
Here, set A and set B are equal sets

Your task

Let

A = {1, 2, 3, 6} B = the set of positive divisors of 6

C = {3, 1, 6, 2} D = {1, 2, 2, 3, 6, 6, 6}

Is set A , B , C , D are Equal sets?

Solution:

Yes A, B, C, and D are all equal sets.

Other Task

Is this is equal set?

{2,3,5,7} , {2,2,3,5,3,7}

Equal

And

{2,3,5,7} , {2,3}

Not Equal

Equivalent Sets

- Two finite sets A and B are said to be equivalent (written A =~ B) if they have the same number of elements: that is, n(A) = n(B).
- **Example**
- {p, q, r, s}; {a, b, c, d}

Examples

For Example;

$A = \{p, q, r\}$

$B = \{2, 3, 4\}$

Here, we observe that both the sets contain three elements.

Example: If $A = \{1, 2, 6\}$ and $B = \{16, 17, 22\}$, they are equivalent

Note that

Equal sets are always equivalent.

Equivalent sets may not be equal

Empty Set or NULL Set

- The empty set is a set which has no elements.
- It is also called null set.
- It is denoted by $\varnothing$ or by $\{\}$.

$\{\ \}$ **or Ø**

Empty Set

Note the subtlety in $\varnothing \neq \{\varnothing\}$

- The left-hand side is the empty set
- The right hand-side is a singleton set, and a set containing a set

$$\varnothing = \{\} \qquad \varnothing \neq \{\varnothing\}$$

Singleton Set or Unit Set

- A singleton is a set that contains exactly one element.
- Sometimes, it is known as unit set.
- The singleton containing only the element a can be written {a}.
- Note that φ is empty set and {φ} is not empty set but it is a singleton set.

Singleton Set or Unit Set

Singleton set or unit set contains only one element. A singleton set is denoted by {s}.

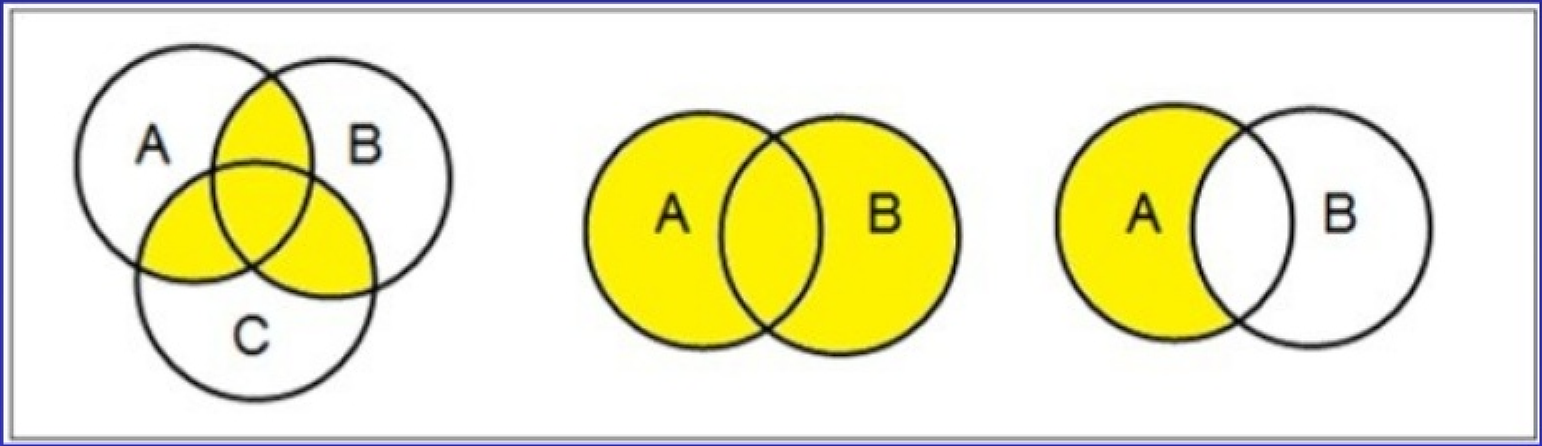
Example : $S = \{x \mid x \in N, 7 < x < 9\}$

Universal Set

- A Universal Set is the set of all elements under consideration, is represented by a rectangle.
- It is denoted by capital U.
- Note that the universal set varies depending on which objects are of interest.
- Inside this rectangle, circles or other geometrical figures are used to represent sets.

Venn Diagrams

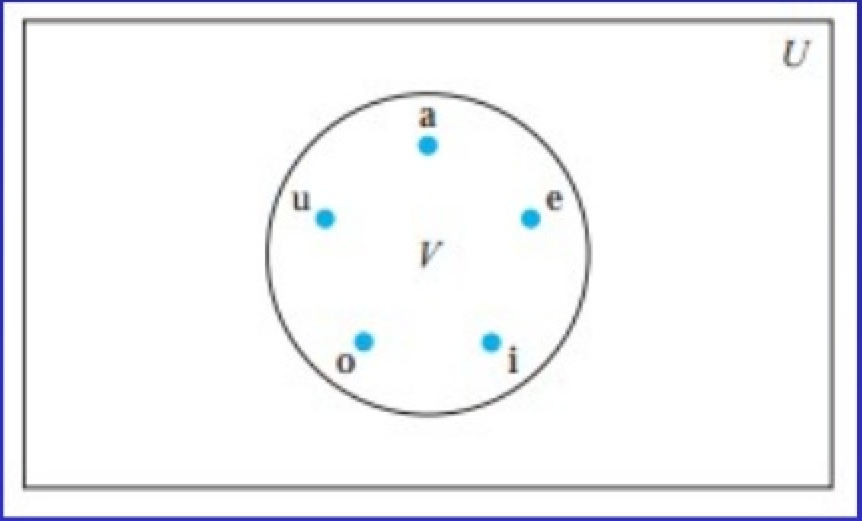
Venn diagram, invented in 1880 by John Venn, is a schematic diagram that shows all possible logical relations between different mathematical sets.



Venn Diagrams

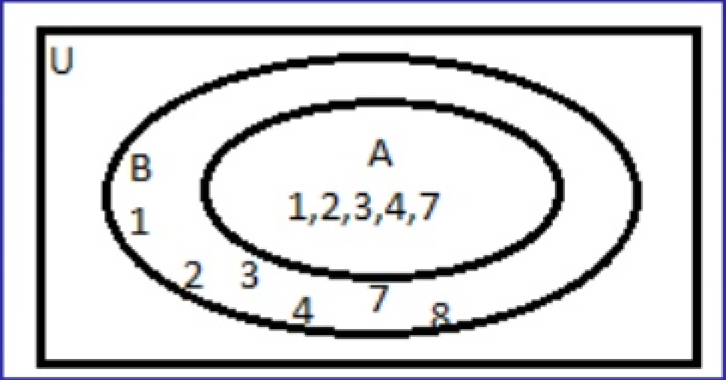
- Venn diagrams are often used to indicate the relationships between sets.
- **Example**

Venn diagram that represents V, the set of vowels in the English alphabet.



Subset

- The set A is a subset of B if and only if every element of A is also an element of B.
- We use the notation $A \subseteq B$ to indicate that A is a subset of the set B.
- The empty set({ } or φ) is a subset of every set.
- Example
- $A = \{1,2,3,4,7\}$
- $B = \{1,2,3,4,7,8\}$
- Here, A is said to be the subset.



Subsets

$A \subseteq B$ "A is a subset of B"

$A \subseteq B$ if and only if every element of A is also an element of B.

We can completely formalize this:

$A \subseteq B \Leftrightarrow \forall x\, (x \in A \rightarrow x \in B)$

Examples:

A = {3, 9}, B = {5, 9, 1, 3}, A □ B ? true

A = {3, 3, 3, 9}, B = {5, 9, 1, 3}, A □ B ? true

A = {1, 2, 3}, B = {2, 3, 4}, A □ B ? false

Proper Subset

- A set B is said to be the proper subset of any given set A, if B has some elements form the set A but B is not equal to the set A (B≠ A).
- **Example**
- A = {1, 2, 3,4 ,5, 6}
- B = { 1,2,3,4}
- B is proper subset of the set A.(B ⊂ A)

Your Task

Determine whether each of the following statements is true or false.

- $x \in \{x\}$
- $\{x\} \subseteq \{x\}$
- $\{x\} \in \{x\}$
- $\{x\} \in \{\{x\}\}$

$\varnothing \subseteq \{x\}$

$\varnothing \in \{x\}$

Solution

Determine whether each of the following statements is true or false.

- $x \in \{x\}$ TRUE
  - (Because x is the member of the singleton set { x })
- $\{x\} \subseteq \{x\}$ TRUE
  - (Because Every set is the subset of itself. Note that every Set has necessarily two subsets ∅ and the Set itself, these two subset are known as Improper subsets and any other subset is called Proper Subset)
- $\{x\} \in \{x\}$ FALSE
  - (Because { x} is not the member of {x}) Similarly other
- $\{x\} \in \{\{x\}\}$ TRUE

  $\varnothing \subseteq \{x\}$ TRUE

  $\varnothing \in \{x\}$ FALSE

Set Theory

Finite Sets

- A set **S** is said to be **finite** if it contains exactly m distinct elements where m denotes some non negative integer.
- In such case we write $|S| = m$ or $n(S) = m$

Infinite Sets

- A set is said to be **infinite** if it is not finite.

Examples

- The set S of letters of English alphabets is finite and $|S| = 26$
- The null set $\varnothing$ has no elements, is finite and $|\varnothing| = 0$
- The set of positive integers $\{1, 2, 3, \ldots\}$ is infinite.

Your Task

Determine which of the following sets are finite/infinite.

– A = {month in the year}	FINITE
– B = {even integers}	INFINITE
– C = {positive integers less than 1}	FINITE
– D = {animals living on the earth}	FINITE
– E = {lines parallel to x-axis}	INFINITE

Cardinality of Sets

- The cardinality of a set S, denoted |S|, is the number of elements in S. If the set has an infinite number of elements, then its cardinality is ∞.
- The cardinality of a set A is denoted by $|A|$.
- 1: If $A = \varphi$, then $|A| = 0$.
- 2: If A has exactly n elements, then $|A| = n$.
- Note that n is a nonnegative number.
- 3: If A is an infinite set, then $|A| = \infty$.

Examples

- Let A be the set of odd positive integers less than 10. Then $|A|=5$.
- Let S be the set of letters in the English alphabet. Then $|S|=26$.
- Let P be the set of infinite numbers. Then $|P|=\infty$.
  - The cardinality of the empty set is $|\emptyset| =0$
  - The sets $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$ are all infinite

Cardinality of Sets

Your Task!

Examples:

$A = \{\text{Mercedes, BMW, Porsche}\}$,	$\lvert A \rvert = 3$
$B = \{1, \{2, 3\}, \{4, 5\}, 6\}$	$\lvert B \rvert = 4$
$C = \emptyset$	$\lvert C \rvert = 0$
$D = \{ x \in N \mid x \leq 7000 \}$	$\lvert D \rvert = 7001$
$E = \{ x \in N \mid x \geq 7000 \}$	E is infinite!

Power Sets

- A Power Set is a set of all the subsets of a set.
- The power set of S is denoted by P(S).
- **Notation**
- The number of members of a set is often written as |S|, so we can write

$$|P(S)| = 2^n$$

Examples

- **A= { a,b,c,d}**

The power set of A is $2^4 = 16$

P(A)={}, {a}, {b}, {a, b}, {c}, {a, c}, {b, c}, {a, b, c}, {d}, {a, d}, {b, d}, {a, b, d}, {c, d}, {a, c, d}, {b, c, d}, {a, b, c, d}.

- **B={1,2,3}**

The power set of B is $2^3=8$

P(B)={}, {1}, {2}, {1, 2}, {3}, {1, 3}, {2, 3}, {1, 2, 3}

Your Task Now

- ***C*={a,1,b,2,c}**

P(*C*)={}, {a}, {1}, {a, 1}, {b}, {a, b}, {1, b}, {a, 1, b}, {2}, {a, 2}, {1, 2}, {a, 1, 2}, {b, 2}, {a, b, 2}, {1, b, 2}, {a, 1, b, 2}, {c}, {a, c}, {1, c}, {a, 1, c}, {b, c}, {a, b, c}, {1, b, c}, {a, 1, b, c}, {2, c}, {a, 2, c}, {1, 2, c}, {a, 1, 2, c}, {b, 2, c}, {a, b, 2, c}, {1, b, 2, c}, {a, 1, b, 2, c}.

Your Task

a. Find $P(\varnothing)$
b. Find $P(P(\varnothing))$
c. Find $P(P(P(\varnothing)))$

Solution:

- Since $\varnothing$ contains no element, therefore $P(\varnothing)$ will contain $2^0=1$ element.

$$P(\varnothing) = \{\varnothing\}$$

- Since $P(\varnothing)$ contains one element, namely ϕ, therefore $P(\varnothing)$ will contain $2^1 = 2$ elements

$$P(P(\varnothing)) = \{\varnothing,\{\varnothing\}\}$$

Cartesian Products

- Let A and B be sets. The Cartesian product of A and B, denoted by $A \times B$, is the set of all ordered pairs
- (a, b), where $a \in A$ and $b \in B$.
- 2-tuples are called ordered pairs.
- The ordered pairs (a, b) and (c, d) are equal if and only if $a=c$ and $b=d$. Note that (a, b) and (b, a) are not equal unless $a=b$.

Examples

- Cartesian product of set A and B is not equal to Cartesian product of set B and A.
- Cartesian product of sets “A” and “B” is denoted by $A \times B$.
- And Cartesian product of sets “B” and “A” is denoted by $B \times A$.
- **Example**
- $A=\{1,2\}$ and set $B=\{4,5\}$
- $A \times B=[\{1,4\} , \{1,5\} , \{2,4\} , \{2,5\}]$
- $B \times A=[\{4,1\} , \{4,2\} , \{5,1\} , \{5,2\}]$

Cartesian Product

The Cartesian product of two sets is defined as:

$A \times B = \{(a, b) \mid a \in A \wedge b \in B\}$

Example:

$A = \{good, bad\}$, $B = \{student, prof\}$

$A \times B = \{(good, student), (good, prof), (bad, student), (bad, prof)\}$

$B \times A = \{(student, good), (prof, good), (student, bad), (prof, bad)\}$

Examples

- What is the Cartesian product $A \times B \times C$, where $A=\{0,1\}$ $B=\{1,2\}$, and $C=\{0,1,2\}$?

Ans:

$A \times B \times C=\{(0,1,0), (0,1,1), (0,1,2), (0,2,0), (0,2,1), (0,2,2), (1,1,0), (1,1,1),(1,1,2), (1,2,0), (1,2,1), (1,2,2)\}$.

- Let $A=\{1,2\}$ and $B= \{3,4\}$

$A \times B= \{(1,3), (1,4), (2,3), (2,4)\}$.

Cartesian Product

Note that:

- $A\times\emptyset = \emptyset$
- $\emptyset\times A = \emptyset$
- For non-empty sets A and B: $A\neq B \Leftrightarrow A\times B \neq B\times A$
- $|A\times B| = |A|\cdot|B|$

The Cartesian product of two or more sets is defined as:

$A_1\times A_2\times\ldots\times A_n = \{(a_1, a_2, \ldots, a_n) \mid a_i\in A \text{ for } 1 \leq i \leq n\}$

Note

- We use the notation A^2 to denote $A \times A$, the Cartesian product of the set A with itself.
- Similarly, $A^3 = A \times A \times A$, $A^4 = A \times A \times A \times A$, and so on….
- More generally
- $A^N = \{(a_1, a_2, \ldots, a_n) \mid a_i \in A \text{ for } i = 1, 2, \ldots, n\}$
- **Example**
- Suppose that $A = \{1,2\}$
- $A^2 = (1,1),(1,2),(2,2),(2,1)$

Overlapping Set

- Two sets that have at least one common element are called overlapping sets.
- **In case of overlapping sets :**

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$

$n(A) = n(A - B) + n(A \cap B)$

$n(B) = n(B - A) + n(A \cap B)$

Example: Let, A = {1, 2, 6} and B = {6, 12, 42}. There is a common element '6', hence these sets are overlapping sets.

Disjoint Set

If two sets C and D are disjoint sets as they do not have even one element in common. Therefore, $n(A \cup B) = n(A) + n(B)$

Example: Let, A = {1, 2, 6} and B = {7, 9, 14}, there is no common element, hence these sets are overlapping sets.

Exercise

1. **List the members of these sets.**

a) $\{x \mid x$ is a real number such that $x^2 = 1\}$

Ans: $\{-1, 1\}$

b) $\{x \mid x$ is a positive integer less than $12\}$

Ans: $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

c) $\{x \mid x$ is the square of an integer and $x < 100\}$

Ans: $\{0, 1, 4, 9, 16, 25, 36, 49, 64, 81\}$

d) $\{x \mid x$ is an integer such that $x^2 = 2\}$

Ans: $\varnothing$

Exercise

Use set builder notation to give a description of each of these sets.

a) {0, 3, 6, 9, 12}

Ans:

$$\{0,3,6,9,12\} = \{x \in \mathbb{N} \mid 3 \text{ divides } x \text{ and } x \leq 12\}$$

b) {–3,–2,–1, 0, 1, 2, 3}

Ans:

$$\{-3,-2,-1,0,1,2,3\} = \{x \in \mathbb{Z} \mid |x| \leq 3\}$$

c) {*m, n, o, p*}

Ans:

$$\{m,n,o,p\} = \{x \mid x \text{ is a letter in the English alphabet between } m \text{ and } p\}$$

Exercise

For each of these pairs of sets, determine whether the first is a subset of the second, the second is a subset of the first, or neither is a subset of the other.

a) the set of airline flights from New York to New Delhi,
the set of nonstop airline flights from New York to New Delhi

Ans:

The first is a subset of the second, but the second is not a subset of the first.

Exercise

b) the set of people who speak English, the set of people who speak Chinese

Ans:

Neither is a subset of the other.

c) the set of flying squirrels, the set of living creatures that can fly

Ans:

The first is a subset of the second, but the second is not a subset of the first.

Exercise

a) the set of people who speak English, the set of people who speak English with an Australian accent

Ans:

{ people who speak English } $\supseteq$ { people who speak English with an Australian accent }

b) the set of fruits, the set of citrus fruits

Ans:

{ fruits } $\supseteq$ { citrus fruits }

c) the set of students studying discrete mathematics, the set of students studying data structures

Ans:

Normally, neither is a subset of the other.

Exercise

Determine whether each of these pairs of sets are equal.

a) $\{1, 3, 3, 3, 5, 5, 5, 5, 5\}, \{5, 3, 1\}$

Ans:

Yes

b) $\{\{1\}\}, \{1, \{1\}\}$

Ans:

No

c) $\varnothing, \{\varnothing\}$

Ans:

No

Exercise

Suppose that $A = \{2, 4, 6\}$, $B = \{2, 6\}$, $C = \{4, 6\}$, and $D = \{4, 6, 8\}$. Determine which of these sets are subsets of which other of these sets.

Ans:

Every set is a subset of itself

A and D are not subsets of any other set listed.

B is a subset of A.

C is a subset of both A and D.

Exercise

For each of the following sets, determine whether 2 is an element of that set.

a) $\{ x \in R | x$ is an integer greater than 1$\}$

Ans :

Yes

b) $\{ x \in R | x$ is the square of an integer$\}$

Ans :

No

Exercise

c) $\{2,\{2\}\}$

Ans :

Yes

d) $\{\{2\},\{\{2\}\}\}$

Ans :

No

e) $\{\{2\},\{2,\{2\}\}\}$

Ans :

No

Exercise

f) $\{\{\{2\}\}$

Ans :

No

Determine whether each of these statements is true or false.

a) $0 \in \varnothing$

Ans :

False (The empty set has no elements)

b) $\varnothing \in \{0\}$

Ans :

False (The empty set is a subset of $\{0\}$, but is not an element of it).

Exercise

c) $\{0\} \subset \varnothing$

Ans :

False (No set can be a proper subset of the empty set since, by definition, that would require the empty set to contain at least one element).

d) $\varnothing \subset \{0\}$

Ans :

True (The empty set is a subset of every set, $\varnothing \subset \{0\}$. In addition the set $\{0\}$ has one element, which is not contained in the empty set).

Exercise

e) $\{0\} \in \{0\}$

Ans :

False (The set {0} is a subset of itself, but is not an element of itself).

f) $\{0\} \subset \{0\}$

Ans:

False (0} Í {0} since {0} = {0}. But, as noted above, the proper subset relation requires the larger set to have at least one element not in the other one).

g) $\{\varnothing\} \subseteq \{\varnothing\}$

Ans :

True (Every set is a subset of itself)

Exercise

- **Determine whether these statements are true or false.**

a) $\varnothing \in \{\varnothing\}$

Ans :

True (the set $\{\varnothing\}$ has exactly one element, namely $\varnothing$).

b) $\varnothing \in \{\varnothing, \{\varnothing\}$

Ans :

True (The empty set is one of the two elements of that set).

Exercise

c) $\{\emptyset\} \in \{\emptyset\}$

Ans :

False (A set is a subset of itself, but is not an element of itself).

d) $\{\emptyset\} \in \{\{\emptyset\}\}$

Ans

True (Here the set which has the empty set as its only element is, in turn, the only element of the set on the right).

e) $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$

Ans :

True (The set on the left has the empty set as its only element and the empty set occurs as an element of the set on the right, which also contains a second element, namely $\{\emptyset\}$.

Exercise

f) $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$

Ans :

True (the set on the left has $\{\emptyset\}$ as its only element, which occurs as one of the two elements of the set on the right).

g) $\{\{\emptyset\}\} \subset \{\{\emptyset\}, \{\emptyset\}\}$

Ans :

False

Exercise

Determine whether each of these statements is true or false.

a) $x \in \{x\}$

Ans :

True (The set $\{x\}$ has exactly one element, namely x).

b) $\{x\} \subseteq \{x\}$.

Ans :

True (every set is a subset of itself).

c) $\{x\} \in \{x\}$

Ans :

False (A set is a subset of itself, but is not an element of itself).

Exercise

$\{x\} \in \{\{x\}\}$

Ans :

True (Here the set which has x as its only element is, in turn, the only element of the set on the right).

e) $\varnothing \subseteq \{x\}$

Ans :

True (The empty set is a subset of every set).

f) $\varnothing \in \{x\}$

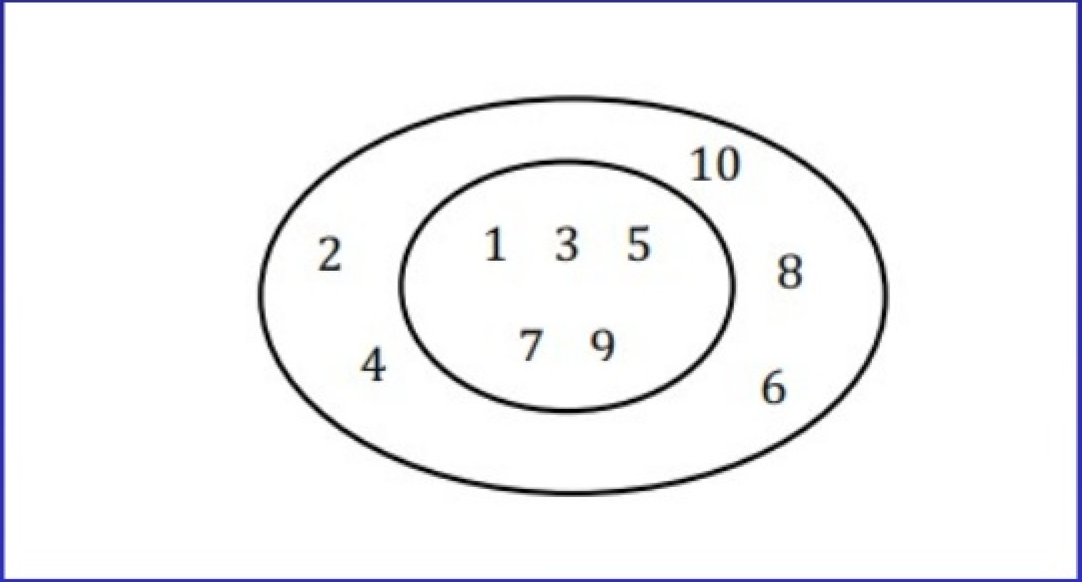
Ans :

False (The only element of the set $\{x\}$ is x).

Exercise

Use a Venn diagram to illustrate the subset of odd integers in the set of all positive integers not exceeding 10.

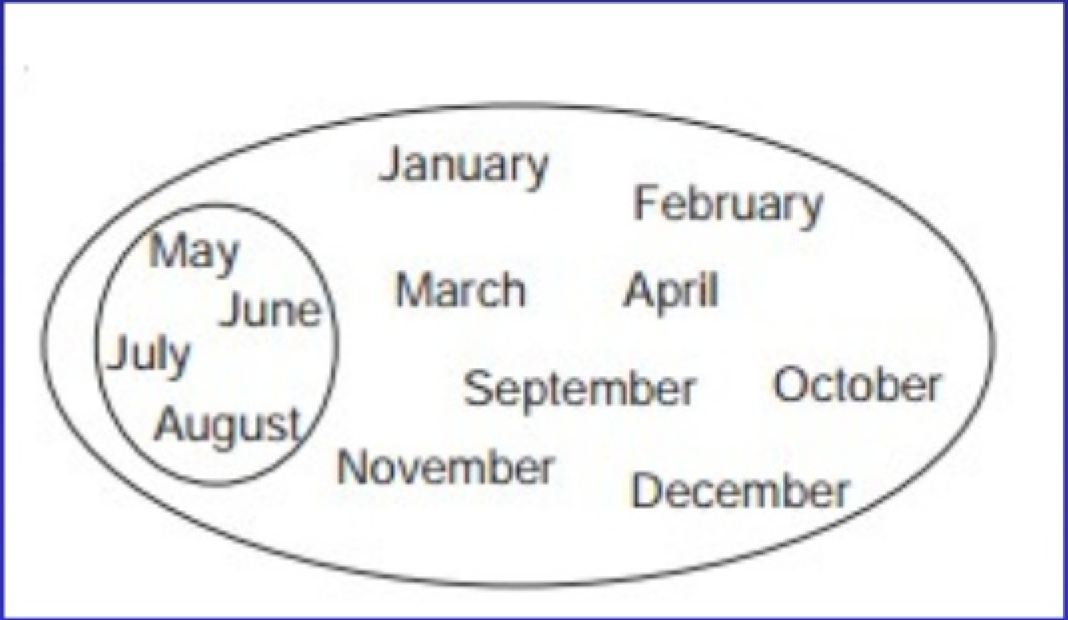
Ans:



Exercise

Use a Venn diagram to illustrate the set of all months of the year whose names do not contain the letter R in the set of all months of the year.

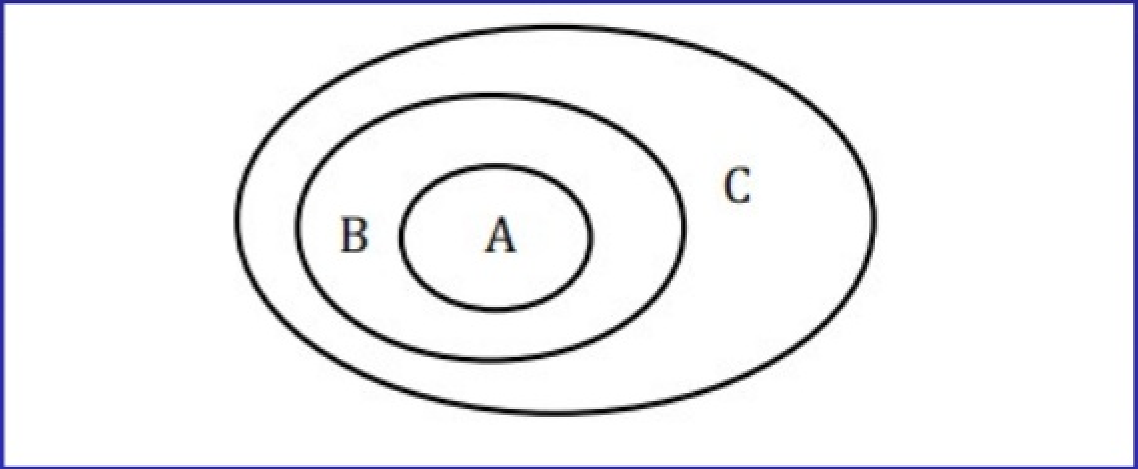
Ans :



Exercise

Use a Venn diagram to illustrate the relationship $A \subseteq B$ and $B \subseteq C$.

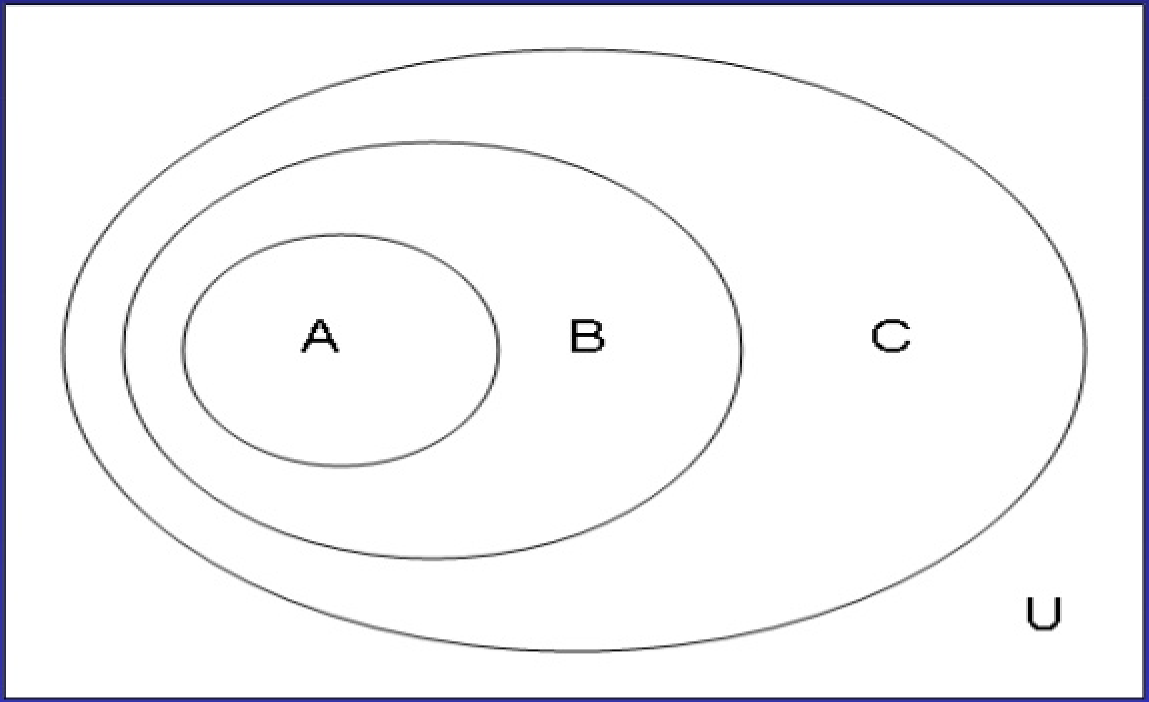
Ans:



Exercise

Use a Venn diagram to illustrate the relationships $A \subset B$ and $B \subset C$.

Ans:



Exercise

Find two sets A and B such that $A \in B$ and $A \subseteq B$

Ans:

$A = \varnothing$ and $B = \{\varnothing\}$, such that $\varnothing \in \{\varnothing\}$ and $\varnothing \subseteq \{\varnothing\}$, As we previously know that $\varnothing$ is a subset of any set.

Exercise

- **What is the cardinality of each of these sets?**

a) {a}

Ans :

$|a| = 1$

b) {{a}}

Ans :

$|b| = 1$

c) {a,{a}}

Ans :

$|c| = 2$

Exercise

d)$\{a,\{a\},\{a,\{a\}\}\}$

Ans :

$|d| = 3$

- **What is the cardinality of each of these sets?**

a) $\varnothing$

Ans :

$|a| = 0$

b)$\{\varnothing\}$

Ans :

$|b| = 1$

Exercise

c) $\{\varnothing,\{\varnothing\}\}$

Ans :

$|c| = 2$

d) $\{\varnothing,\{\varnothing\},\{\varnothing,\{\varnothing\}\}\}$

Ans :

$|d| = 3$

- **Find the power set of each of these sets, where a and b are distinct elements.**

a) $\{a\}$

Ans :

$P(a) = \{\varnothing, \{a\}\}$

Exercise

b) $\{a, b\}$

Ans :

$$P(b) = \{\varnothing, \{a\}, \{b\}, \{a, b\}\}$$

c) $\{\varnothing,\{\varnothing\}\}$

Ans :

$$P(c) = \{\varnothing,\{\varnothing\},\{\{\varnothing\}\},\{\varnothing,\{\varnothing\}\}\}$$

Exercise

Can you conclude that A=B if A and B are two sets with the same power set?

Ans :

Yes. By definition, P(A) is the set of all subsets that can be generated from A, if A and B generate the exact same collection of valid subsets, then it must be that A and B contain the same elements and are therefore equal.

Exercise

How many elements does each of these sets have

Where *a* and *b* are distinct elements?

a) $P(\{a, b, \{a, b\}\})$

Ans:

$|\{a, b, \{a, b\}\}| = 3$, so $|P(\{a, b, \{a, b\}\})| = 2^3 = 8$.

b) $P(\{\varnothing, a, \{a\}, \{\{a\}\}\})$

Ans:

$|\{\varnothing, a, \{a\}, \{\{a\}\}\}| = 4$, so $|P(\{\varnothing, a, \{a\}, \{\{a\}\}\})| = 2^4 = 16$.

c) $P(P(\varnothing))$

Ans:

$|P(\varnothing)| = 1$, so $|P(\varnothing)| = 2$.

Exercise

Determine whether each of these sets is the power set of a set, where *a* and *b* are distinct elements.

a) $\emptyset$

Ans:

No set

b) $\{\emptyset, \{a\}\}$

Ans:

This is a power set of the set { a }

Exercise

c) $\{\varnothing, \{a\}, \{\varnothing, a\}\}$

Ans:

This is not a power set, since it has 3 elements and the number of elements in a power set is 2^n, where n is the number of elements in the set.

d) $\{\varnothing, \{a\}, \{b\}, \{a, b\}\}$

Ans:

Power set of {a, b}

Exercise

Let $A = \{a, b, c, d\}$ and $B = \{y, z\}$. Find

a) $A \times B$

Ans:

There will be 4 x 2 = 8 sets.
{(a,y), (a,z), (b,y), (b,z), (c,y), (c,z), (d,y), (d,z)}

b) $B \times A$

Ans:

{(y,a), (y,b), (y,c), (y,d), (z,a), (z,b), (z,c), (z,d)}

Exercise

What is the Cartesian product $A \times B$, where Ai s the set of courses offered by the mathematics department at a university and B is the set of mathematics professors at this university? Give an example of how this Cartesian product can be used.

Ans:

It is the set of all possible combinations of math courses and possible instructors.

Exercise

- **What is the Cartesian product $A \times B \times C$, where A is the set of all airlines and B and $C$are both the set of all cities in the United States? Give an example of how this Cartesian product can be used.**

Ans :

The set of triples (a,b,c), where a is an airline and b and c are cities. A useful subset of this set is the set of triples(a,b,c) for which a flies between b and c.

Exercise

Suppose that $A \times B = \emptyset$, where A and B are sets. What can you conclude?

Ans:

If both A and B are nonempty, then this isn't possible. So at least one of A or B must be empty. And in fact, if A is empty, then so is $A \times B$; similarly so when B is empty.

Exercise

Let A be a set. Show that $\varnothing \times A = A \times \varnothing = \varnothing$.

Ans:

$\varnothing$ x A = {(x, y) | x $\in \varnothing$ and y $\in$ A} = $\varnothing$ = {(x, y) | x $\in$ A and y $\in \varnothing$} = A x $\varnothing$

OR

$\varnothing \times A=\{(x, y) \mid x \in \varnothing$ and $y \in A\} = \varnothing = \{(x, y) \mid x \in A$ and $y \in \varnothing\} = A \times \varnothing$

Exercise

Let $A=\{a, b, c\}$, $B=\{x,y\}$, and $C=\{0,1\}$. Find

a) $A\times B\times C$

Ans :

$A\times B\times C$ = (a, x, 0) ,(a, y, 0) ,(a, x, 1) ,(a, y, 1) ,(b, x, 0) ,(b, y, 0) (b, x, 1),(b ,y, 1), (c, x, 0), (c, y, 0), (c, x, 1) ,(c, y, 1)

b) $C\times B\times A$

Ans :

$C\times B\times A$ ={(0,x,a),(0,x,b),(0,x,c),(1,x,a),(1,x,b), (1,x,c),(0,y,a),
(0,y,b),(0,y,c),(1,y,a),(1,y,b),(1,y,c)}.

Exercise

d) $B \times B \times B$

Ans : $B \times B \times B = \{(x,x,x),(x,x,y),(x,y,y),(x,y,x),$
$(y,y,y),$
$(y,x,x),(y,y,x),(y,x,y)\}$

Find A^2 if

a) $A=\{0,1,3\}$

Ans : $A^2 = \{(0,0),(0,1),(0,3),(1,0),(1,1),$
$(1,3),(3,0),(3,1),(3,3)\}$

Exercise

b) $A=\{1,2,a,b\}$

Ans :

$\{(1,1), (1,2), (1,a), (1,b), (2,1), (2,2), (2,a), (2,b), (a,1), (a,2),(a, a),(a, b),(b,1),(b,2),(b, a),(b, b)\}$

Find A^3 if

a) $A=\{a\}$

Ans :

$A^3 = \{(a, a, a)\}$

b) $A=\{0,a\}$

Ans :

$A^3 = \{(0, 0, 0), (0, 0,1), (0,1, 0), (1, 0, 0), (1, 1,0),(1,0, 1), (0, 1, 1), (1, 1, 1)\}$

Exercise

How many different elements does $A \times B$ have if A has m elements and B has n elements?

Ans :

mn

How many different elements does $A \times B \times C$ have if A Has m elements has n elements, and C has p elements?

Ans :

mnp

Exercise

How many different elements does A^n have when A has m elements and n is a positive integer?

Ans :

m^n

Show that $A \times B \neq B \times A$, when A and B are nonempty, unless A=B.

Ans :

Lets us suppose that A= {a,b} B= {1,2}

$A \times B = \{(a,1),(a,2),(b,1),(b,2)\}$

$B \times A = \{(1,a),(1,b),(2,a),(2,b)\}$

Hence proved .

Exercise

Explain why $A \times B \times C$ and $(A \times B) \times C$ are not the same.

Ans :

The elements of $A \times B \times C$ consist of 3-tuples (a,b,c), where $a \in A$, $b \in B$, and $c \in C$, whereas the elements of $(A \times B) \times C$ look like $((a, b), c)$—ordered pairs, the first coordinate of which is again an ordered pair.

Any Question

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Thank You ☺