MA212 End Semester Examination May 2021. Time: 2 pm - 4 pm (Date: 04/05/2021)

* Required

Instructions

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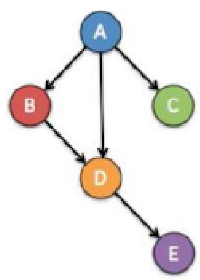


Bayesian network structure presented here, consisting of 5 binary random variables A, B, C, D, E. Each variable corresponds to a gene, whose expression can be either “ON” or “OFF”.
The probabilities values are as follows: P(A=ON) = 0.6, P(B=ON | A=OFF) = 0.1, P(B=ON | A=ON) = 0.95, P(C=ON | A=OFF) =0.8, P(C=ON | A=ON) = 0.5, P(E=ON|D=OFF) = 0.8, P(E=ON|D=ON) = 0.1, P(D=ON|A=ON, B=ON) = 0.95, P(D=ON|A=OFF, B=OFF) = 0.1, P(D=ON|A=ON, B=OFF) = 0.9, P(D=ON|A=OFF, B=ON) = 0.3. Find the P(E=ON | A= ON).

0.13675

"For a Markov process X(t), the second-order distribution is sufficient to characterize X(t)." Statement is _____ *

- (selected) True
- False

*

Using the above Bayesian network, find out the posterior conditional probability P [E = yes | C = yes] using the answer of P [C = yes] as a prior probability of computer failure. That is how the probability distribution of electricity failure E is changed given the observed evidence.

0.53

*

Congruence $15\,x \equiv 6\ (mod\ 21)$ has solution:

$x \equiv 6\ (mod\ 21)$

- [x] Option 1

$x \equiv 13\ (mod\ 21)$

- [x] Option 2
- [x] All of these

$x \equiv 20\ (mod\ 21)$

- [x] Option 3

Let X be binomial R.V. with parameters n=15 and p=0.2. The expected value of X= *

- ○ 12
- ◉ 3
- ○ 30
- ○ 0.3

Let X be a R.V. with p.d.f. f(x)=c exp(-x) for x=1, 2, 3, ... The value of c must be ____ *

- ○ 1/e
- ○ e
- ◉ e-1
- ○ 1-e
- ○ -e

The Geometric random variable X denotes *

- ○ number of trials needed for r success
- ○ number of trials needed for r failure
- ○ number of trials needed for any fixed number of success
- ◉ number of trials needed for first success

If X is Hypergeometric R. V. with N=20, r=3, and n=5. Which of the following are the possible values for X? *

- [x] {1, 2, 3}
- [x] {0, 1, 2, 3}
- [ ] {0, 1, 2, 3, 4, 5}
- [ ] {1, 2, 3, 4, 5}

Suppose that a random sample of size 4 from Poisson population yields these data: x1=12, x2=15, x3=16, x4=17. The "Maximum likelihood estimate" for the parameter k of Poisson random variable is ___ *

15

*

The unit digit of 2^{100} is

6

*

Using Bayesian network answer the following: suppose that electricity failure, denoted by E, occurs with probability 0.1, and computer malfunction, denoted by M, occurs with probability 0.2. It is reasonable to assume electricity failure and computer malfunction as independent. It is assumed that if there is no problem with the electricity and the computer has no malfunction, the computer works fine. The C denotes the computer failure. If there is no problem with electricity, but the computer has a malfunction, the probability of computer failure is 0.5. If the electricity is shut down, the computer will not start regardless its potential malfunction. In this setting, the probability of computer failure P [C = yes] can be calculated as

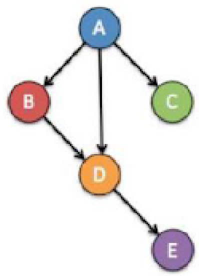
0.19

*

Which of the following is eigen value of *Adjoint* of $\begin{pmatrix} 2 & 3 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 4 \end{pmatrix}$

- ○ 16
- ◉ 4
- ○ 12
- ○ 11

*



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0.0271

*

If $ca \equiv cb \ (mod\ m)$, then $a \not\equiv \mathrm{b}$ (mod m) if c and m are:

- (x) Relative prime
- ( ) Twin Prime
- ( ) Primes and equal
- ( ) Distinct primes

For WSS random process X(t), following are always true *

- [x] Autocorrelation function at zero has maximum absolute value.
- [ ] Autocorrelation function is always non-negative
- [ ] Autocorrelation function is odd
- [x] Autocorrelation function is even

*

Markov model of weather is defined as follows: the weather is observed using three states S_1 (R): rainy, S_2 (C): cloudy, S_3 (S): sunny.

The state transition probabilities are $A=\begin{bmatrix} 0.4 & 0.3 & 0.3 \\ 0.2 & 0.6 & 0.2 \\ 0.1 & 0.1 & 0.8 \end{bmatrix}$

Given that the weather on day 1 (t = 1) is sunny (state S_3), what is the probability that the weather for the next 7 days will be "sunny, sunny, rainy, rainy, sunny, cloudy, sunny"? (Answer should be a fractional number, for example, 0.00021, do not write 2.1×10^{-4})

0.0001536

*

Measurements of the diameters of a random sample of 200 ball bearings made by a certain machine during one week showed a mean of 0.824 inch and a standard deviation of 0.042 inch. _______ inches gives 99% confidence limits for the mean diameter of all the ball bearings. Provided $Z_c = 2.58$

(a) 0.824 ± 0.006 (b) $0.824 + 0.006$ (c) $0.824 - 0.006$ (d) 0.824 ± 0.008

- (d) (selected)
- (b)
- (c)
- (a)

*

The clique potential function is designed with a smoothness term that
(a) discourage smaller differences in the labels of neighbouring sites by assigning a low probability to these configurations
(b) discourage large differences in the labels of neighbouring sites by assigning a low probability to these configurations
(c) discourage smaller differences in the labels of neighbouring sites by assigning a high probability to these configurations
(d) discourage large differences in the labels of neighbouring sites by assigning a high probability to these configurations

- ( ) a
- ( ) b
- (•) c
- ( ) d

*

Find the curve of best fit $y = a\,e^{bx}$ to the following data by using method of least square

x	1	5	7	9	12
y	10	15	12	15	21

$y = 9.5\,e^{1.06\,x}$

Option 1

$y = 9\,e^{1.07\,x}$

Option 2

$y = 5.9\,e^{1.09\,x}$

Option 3

$y = 5\,e^{1.05\,x}$

Option 4

*

The solution of the congruence $7x \equiv 2 \ (mod\ 9)$ is of the form, where $t = 0, \pm 1, \pm 2, \ldots$

- (selected) 9t+8
- 8t-9
- -(9t+8)
- 5t+8

*

The set $\{(x_1, x_2, x_3, x_4) : x_i \in \mathbb{R}\}$. This set of vectors is a vector space over $\mathbb{R}$ if

$x_1 < 0$

Option 1

$x_1 > 0$

Option 2

$x_3 = 0$

Option 3 (selected)

None

*

Which of the following primes satisfy the congruence $a^{24} \equiv 6a + 2 \pmod{13}$?

- (•) 41
- ( ) 47
- ( ) 67
- ( ) 83

The estimate of sample variance for the sample values 5, 8.5, 12, 15, 7, 9, 7.5, 6.5, 10.5 is ___ *

- ( ) 8.44
- (•) 3.082
- ( ) 9.5
- ( ) 4

*

Which of the following subset is not a subspace of $\mathbb{R}^3$?

$S = \{(x, y, z) \in \mathbb{R}^3 : y = z = 0\}$

Option 1

$S = \{(x, y, z) \in \mathbb{R}^3 : z = 0\}$

Option 2

$S = \{(x, y, z) \in \mathbb{R}^3 : x = y = 0\}$

Option 3

$S = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = z^2\}$

Option 4

*

The clique potential function is designed with discontinuity adaptive model that
(a) allow edges to form while still providing smoothing away from the regions
(b) allow regions to form while still providing smoothing away from the edges
(c) allow edges to form while still providing smoothing away from the edges
(d) none of these

- ( ) a
- ( ) b
- (x) c
- ( ) d

*

If $a \equiv b\ (mod\ m)$ and $c \equiv d\ (mod\ m)$, then which of the following is false-

$a + c \equiv\ b + d\ (mod\ m)$

- [ ] Option 1

$ka \equiv\ kb\ (mod\ m)$

- [ ] Option 2

- [x] None of these

$ac \equiv\ bd\ (mod\ m)$

- [ ] Option 4

Consider a random process X(t) defined by X(t)= (U cos wt + V sin wt), (t is entire real line) where w is a constant and U and V are r.v.'s. For X(t) to stationary random process, Expected value of U and V is ___ and ____ respectively. *

- [ ] 0, 1
- [x] 0, 0
- [ ] can be any real constant
- [ ] 1,1
- [ ] 1, 0

*

Remainder obtained when (1! + 2! + 3! + 4! + …) is divided by 8 is

1

Is the set of Natural numbers (N) forms a ring? Justify your answer. *

No, Set is Natural Numbers (N) is not a Ring, S

*

Using the above Bayesian network, find out the posterior conditional probability P [M = yes | C = yes] using the answer of P [C = yes] as a prior probability of computer failure. That is how the probability distribution of computer malfunction M is changed given the observed evidence.

0.58

*

Gibbs distribution $P(f) = Z^{-1} \times e^{-U(f)/T}$, Z is
(a) free energy of the system
(b) energy function
(c) potential function
(d) none of these

- [ ] a
- [ ] b
- [ ] C
- [x] d

*

The sum of eigen values of a matrix $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$ is

18

*

$T: U \rightarrow V$ be a linear transformation. Then

$T(u - v) = T(u) - T(v)$ for all $u, v \in U$

$T(u + v) = T(u) + T(v)$ for all $u, v \in U$

- ○ Option 1
- ○ Option 2
- ◉ Both Option 1 and Option 2
- ○ Neither Option 1 nor Option 2

*

$(n^7 - n)$ is divisible by

- ○ 1
- ◉ 7
- ○ 30
- ○ 42

*

Which of the following set is not linearly dependent with respect to vector space $\mathbb{R}$ ($\mathbb{R}$) ?

- [x] {0}
- [ ] {1}
- [ ] {1, 2, 3}
- [ ] {1, 2}

*

How many solution does the system of linear equations $\begin{pmatrix} 2 & 1 & 3 \\ 3 & 0 & 1 \\ 1 & 2 & 5 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 5 \\ -4 \\ 14 \end{pmatrix}$ has ?

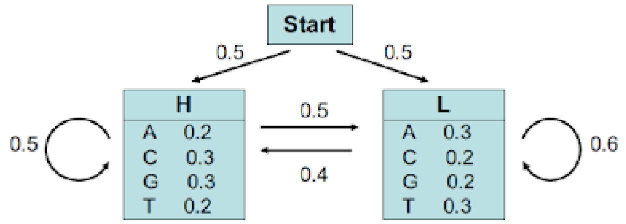
INFINITE

If Z is normally distributed with mean 0 and variance 1, then P[z>1]=____. (provided Area 0 to 1=0.3413) *

0.1587

*

Let's consider the following Hidden Markov Model. This model is composed of 2 states, H (high GC content) and L (low GC content). The state H characterizes coding DNA while L characterizes non-coding DNA. The model can then be used to predict the region of coding DNA from a given sequence. DNA sequence is observed using symbol A, C, G, T.



Consider the sequence S = GGCA. There are several paths through the hidden states (H and L) that lead to the given sequence S. For example, one of such paths, P = LLHH. The probability of the HMM to produce sequence S through the path P is (Answer should be a fractional number, for example, 0.01, do not write 1 x 10^{-2})

0.0038432

The correlation coefficient between x & y is _____, when lines of regression are 2x-9y+6=0 and x-2y+1=0. *

- ○ 3/2
- ○ 1/2
- ○ -3/2
- ◉ 2/3

The smallest Fermat composite number is *

F_0

Option 1

F_5

Option 2

F_4

Option 3

F_3

Option 4

*

The least positive common solution of the linear congruences

$x \equiv 2 \ (mod\ 3), \quad x \equiv 3 \ (mod\ 5), \quad x \equiv 5 \ (mod\ 2)$ is

23

*

A mapping $T: \mathbb{R}^3 \to \mathbb{R}^3$ is not a linear transformation if

$$T(x, y, z) = (x - y,\ y - z,\ z - x)$$

- [x] Option 1

$$T(x, y, z) = (x + y,\ 3z,\ 0)$$

- [ ] Option 2

$$T(x, y, z) = (x + 2y - 3z,\ y + z,\ x - z)$$

- [ ] Option 3

$$T(x, y, z) = (x + y,\ x - y,\ z + 1)$$

- [ ] Option 4

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