

TUTORIAL - 6

[BHAGYA VINOD RANA]

CFG

UACS012

- 1> What is Context Free Grammar (CFG)? List down the applications of CFG.
- 1> CFG stands for Context Free Grammar. It is a Formal grammar which is used to generate all possible patterns of strings in given formal language.

Context-Free Grammar G can be defined by four tuples as:

$G = (V, T, P, S)$, where G : Grammar, which consist of set of production rule [used to generate string]

$T =$ finite set of terminal symbol. [denoted by lower case letters]

$V =$ finite set of Non-terminal symbol [Capital Letters]

$P =$ Set of Production Rules (Substitution Rules) which is used for replacing non-terminals symbols (left side of) in string with other terminal or non-terminal symbols. (right side)

$S =$ start symbol which is used to derive the string.

Type 2 Grammar $\alpha \rightarrow \beta$ Eg: $S \rightarrow OS1 \mid \epsilon$ CFG

only one variable

It has generated
 $\{0^n 1^n, n \geq 0\}$
 language.

OS1

↓

OS1

↓

OS1

↓

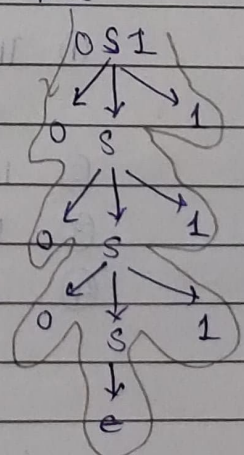
OS1

↓

ϵ

$\Rightarrow 00001111$

Tree



Applications of C.F.G.

- For construction of compilers
- For defining and translation of programming languages
- For parsing the program by constructing syntax tree
- For describing arithmetic expression.
- Development of XML (Extensible Markup Language)
(DTD (Document type Definition) \rightarrow CFG)

2. Define the following terms:

a) Derivation

The process of deriving a string is called derivation.

b) Parse Tree

(derivation tree)

The geometric representation of a derivation is called parse tree.

parse tree follows precedence of operators.

The deepest sub-tree is traversed first (i.e. operator in parent node has less precedence over the operator in sub-tree).

$$(E = a|b|c)$$

(*) Properties

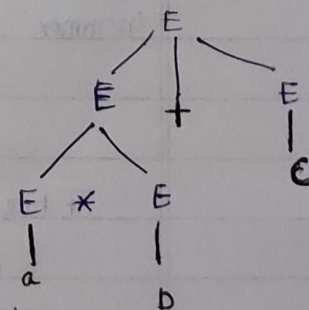
Rules: $E = E + E$, $E = E * E$, Eg: $a * b + c$

① The Root node is always a node indicating start symbol.

② The derivation is read from left to right.

③ The leaf nodes is always terminal nodes.

④ The interior nodes are always non-terminal nodes.

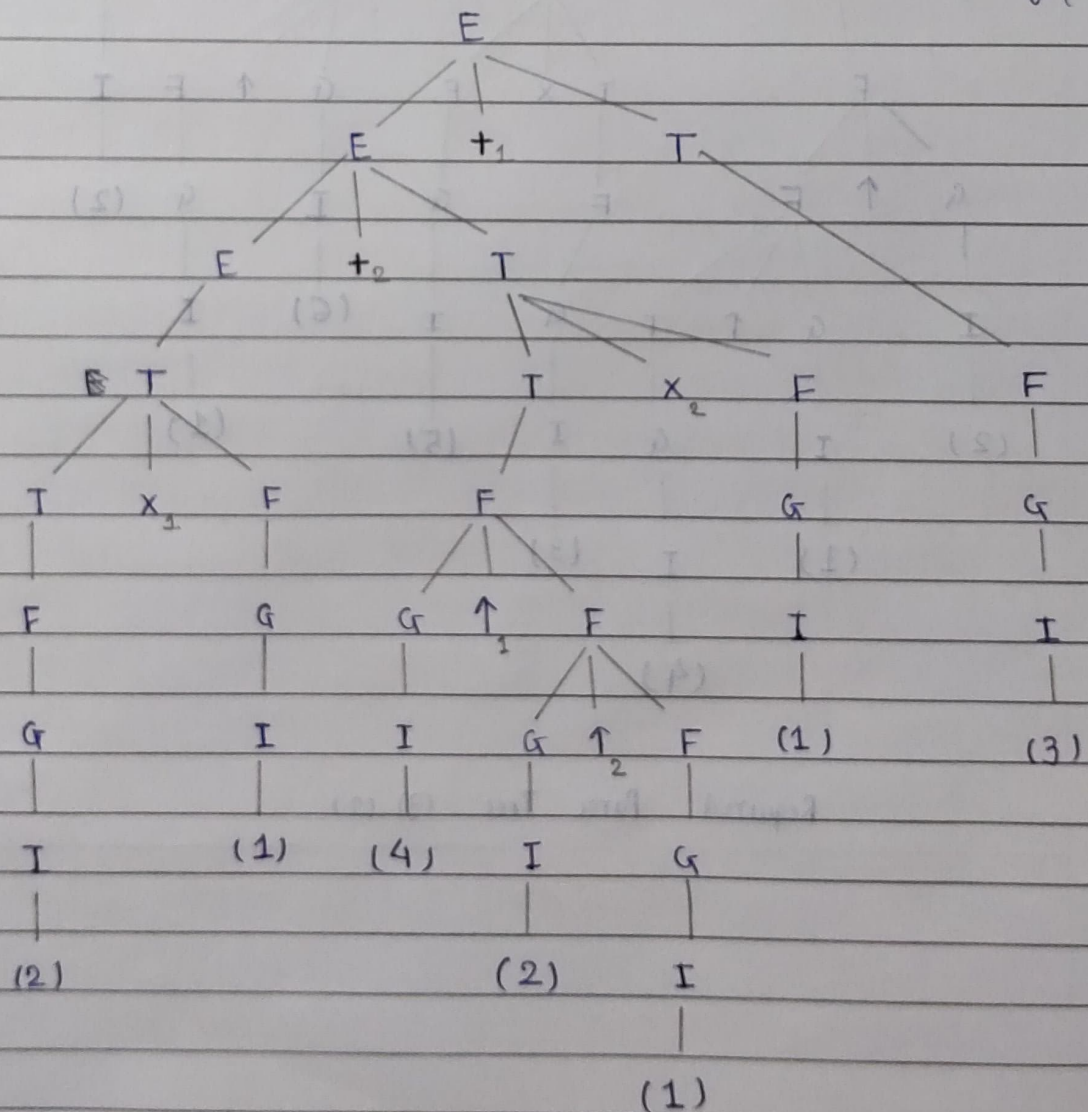


$$\begin{aligned}
 3.) \quad & E \rightarrow E + T \mid E - T \mid T \\
 & T \rightarrow T \times F \mid T \div F \mid F \\
 & F \rightarrow G \uparrow F \mid G \\
 & G \rightarrow I \\
 & I \rightarrow 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9
 \end{aligned}
 \quad \left. \vphantom{\begin{aligned} E \rightarrow E + T \mid E - T \mid T \\ T \rightarrow T \times F \mid T \div F \mid F \\ F \rightarrow G \uparrow F \mid G \\ G \rightarrow I \\ I \rightarrow 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9 \end{aligned}} \right\} \text{Grammar}$$

Draw the parse tree for the string:

$$1.) \quad ((2 \times_1 1) +_2 ((4 \uparrow_1 (2 \uparrow_2 1)) \times_2 1)) +_1 3$$

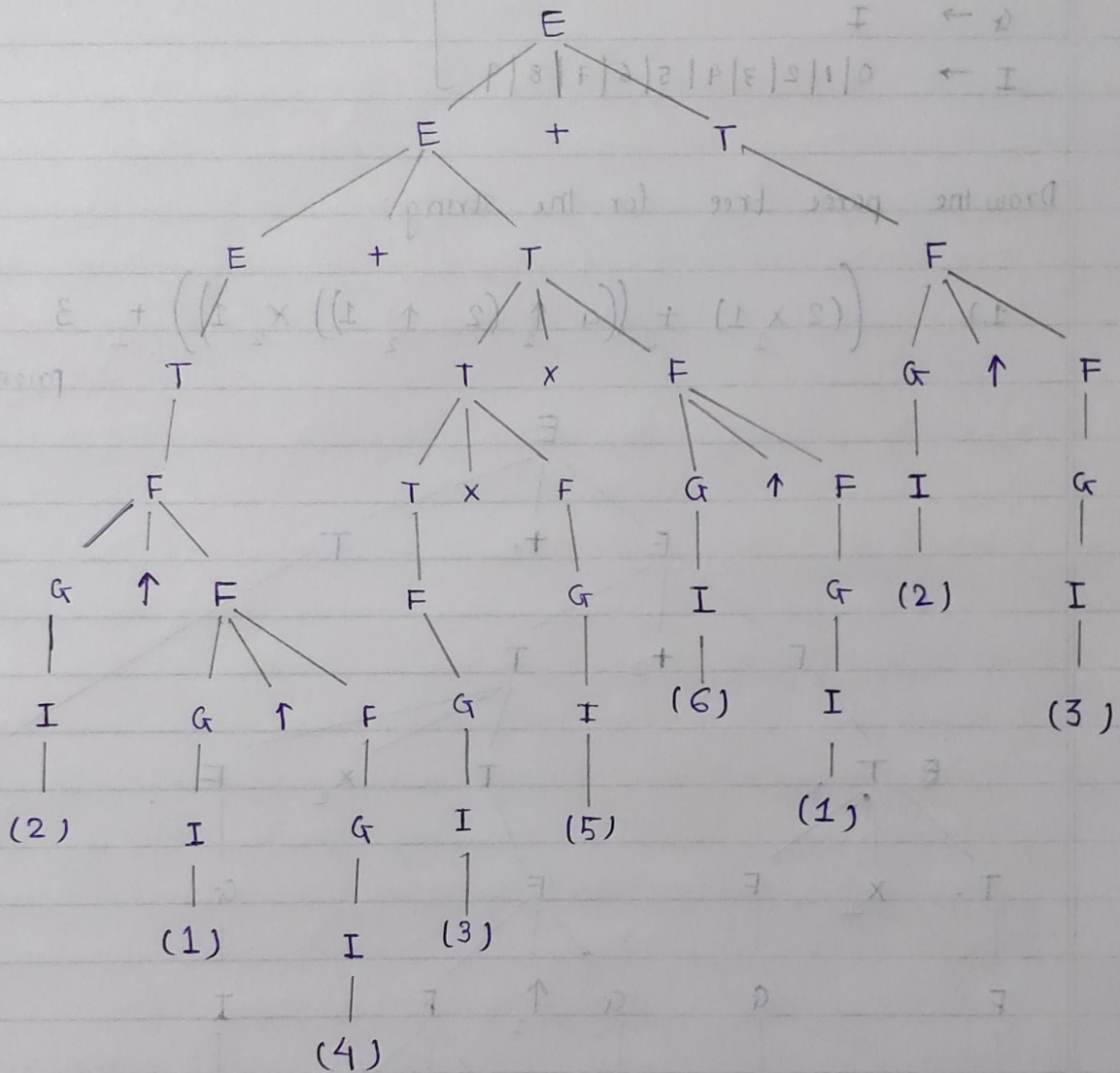
priority $\{I > \uparrow > (\times/\div) > (+/-)\}$



Required Parse Tree (3.(a))

$$2) \quad 2 \uparrow 1 \uparrow 4 + 3 \times 5 \times 6 \uparrow 1 \uparrow + 2 \uparrow 3$$

$$= ((2 \uparrow (1 \uparrow 4)) + ((3 \times (5 \times (6 \uparrow 1))) \uparrow 2)) + (2 \uparrow 3)$$



Required Parse Tree (3). (2)

$$\begin{aligned}
 4. & \quad E \rightarrow E + T \mid T \\
 & \quad T \rightarrow F \times T \mid F \\
 & \quad F \rightarrow I \\
 & \quad I \rightarrow 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9
 \end{aligned}$$

Derive the string : $2 + 3 \times 5 \times 6 + 2$

Using

$$\begin{aligned}
 E & \rightarrow E + T & E & \rightarrow E + T \\
 & \rightarrow E + T + T & E & \rightarrow E + T \\
 & \rightarrow (T) + (F \times T) + T & E & \rightarrow T, T \rightarrow F \times T \\
 & \rightarrow (F) + F \times F \times T + F & T & \rightarrow F, T \rightarrow F \times T, I \rightarrow F \\
 & \rightarrow F + F \times F \times F + F & I & \rightarrow F \\
 & \rightarrow I + I \times I \times I + I & F & \rightarrow I \\
 & \rightarrow 2 + 3 \times 5 \times 6 + 2 & I & \rightarrow 2, I \rightarrow 3, I \rightarrow 5, I \rightarrow 6
 \end{aligned}$$

Hence, the given string is derived using given grammar.

5. > What are different possible strings for the grammar

$$\begin{aligned}
 1. & \quad S \rightarrow SS \\
 & \quad S \rightarrow a \\
 & \quad S \rightarrow b
 \end{aligned}$$

$$\begin{aligned}
 S & \rightarrow SS & (\text{even length}) \\
 & \rightarrow (SS)S & (\text{odd length})
 \end{aligned}$$

Any length string can be formed
($n \geq 2$)

$S \rightarrow$ can be either a or b. $L = (a+b)^2 (a+b)^*$ (Regular Exp)

\therefore This grammar can generate every possible combination of string comprising of letters (a or b) of length greater than or equal to 2
Eg: $L = \{aa, ab, ba, bb, aba, abh, \dots\}$

```

graph TD
    S --> A
    S --> B
    A --> aAb
    A --> ab
    B --> abB
    B --> epsilon["ε"]
    aAb --> a
    aAb --> Ab
    Ab --> b
    Ab --> ab
    abB --> ab
    abB --> B
    B --> abB
    B --> epsilon
    abB --> ab
    abB --> B
    B --> abB
    B --> epsilon
    abB --> ab
    abB --> B
    B --> abB
    B --> epsilon
    
```

$$a^n b^n, n \geq 1$$

$$B \rightarrow (ab)^n, n \geq 0$$

$$[a^n b^n, (n \geq 1) \text{ OR } x^7 (ab)^n \text{ where } (n \geq 0)]$$
$$S \rightarrow AB \mid C$$

$$A \rightarrow aAb \mid ab$$

$$B \rightarrow c B d \mid c d$$

$$C \rightarrow aCd \mid aDd$$

$$D \rightarrow bDc \mid bc$$

(central derivation)

$$S \rightarrow C$$

→ \overbrace{acd}

$$\rightarrow a(a \text{ } \overbrace{D}^{\text{D}} d) d$$

$$\rightarrow a (a (\overbrace{b D c}) d) d$$

→ a a b (b c) c d d

→ aa bb cc dd

using

$$S \rightarrow C$$

$$c \rightarrow da \in d$$

$$C \rightarrow aDd$$

$$D \rightarrow bDc$$

$$D \rightarrow bc$$

Left most derivation \Rightarrow

$$S \rightarrow AB$$

$$\rightarrow aAbB$$

→ aabbB

→ aabb cBd

→ aa bb c c d d

Using

$$S \rightarrow AB$$

$$A \rightarrow aAb$$

$$A \rightarrow ab$$

$$B \rightarrow c B d$$

$$C \rightarrow cd$$