Experiment - 02

Aim: To perform sampling and reconstruction of signal and obtain its waveforms. Also verify the Nyquist Criteria.

Apparatus: Nyquist Applet

Theory:-

A continuous-time (or analog) signal can be stored in a digital computer, in the form of equidistant discrete points or samples. The higher the sampling rate (or sampling frequency, fs), the more accurate would be the stored information and the signal reconstruction from its samples. However, high sampling rate produces a large volume of data to be stored and makes necess--ary the use of very fast analog-to-digital converter ~~It is obvious~~

Analog Signal - It has continuous time varying feature of the signal.

Digital Signal - It represents data as sequence of discrete values, at any given time it can only take on one of the finite number of values.

Technique that can be used for Analog to Digital Conversion:

Pulse Code Modulation (PCM)

It has following three processes.

Sampling

- Quantization ⇒ Digitization
- Encoding

- Sampling is the process of measuring the instantaneous values of continuous-time signal in discrete form
- Sample is a piece of data taken from the whole data which is continuous in the time domain.
- When a source generates an analog signal and if that has to be digitized, having 1s & 0s, i.e high or low, the signal has to be discretized in time
- This discretization of analog signal is called sampling

It is obvious that a much higher sampling rate is needed for sampling a signal which is rich in high frequency components, such as sound of music compared to the sampling freq needed for sampling a slowly varying signal, such as the output of a gas-chromatograph detector or the potential of a glass-electrode during acid-base titration.

The minimum sampling frequency of a signal that it will not distort its underlying information should be double the frequency of its highest frequency component. This is the Nyquist Sampling theorem

Nyquist Rate:

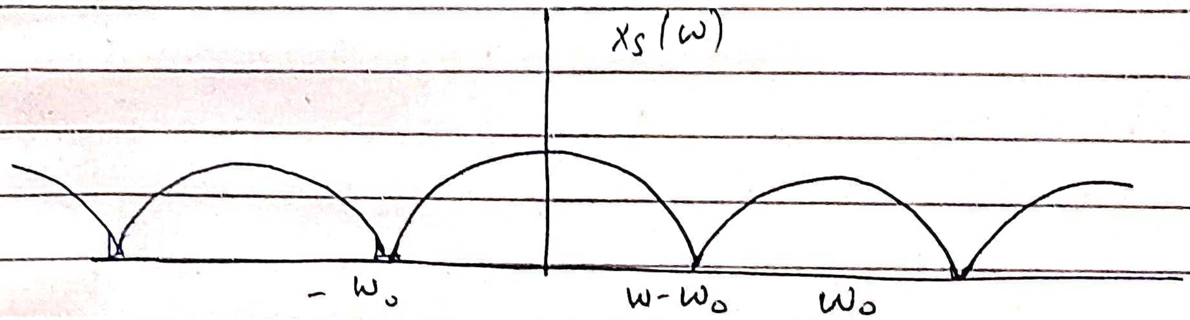
- Suppose that a signal is band-limited and W is the highest frequency
- Therefore, for effective reproduction of the original signal the sampling rate should be twice the highest frequency.

$$f_s = 2W$$

Where fs is the sampling rate and W is the highest frequency.
- This rate of sampling is called Nyquist rate
- A theorem called Sampling Theorem.

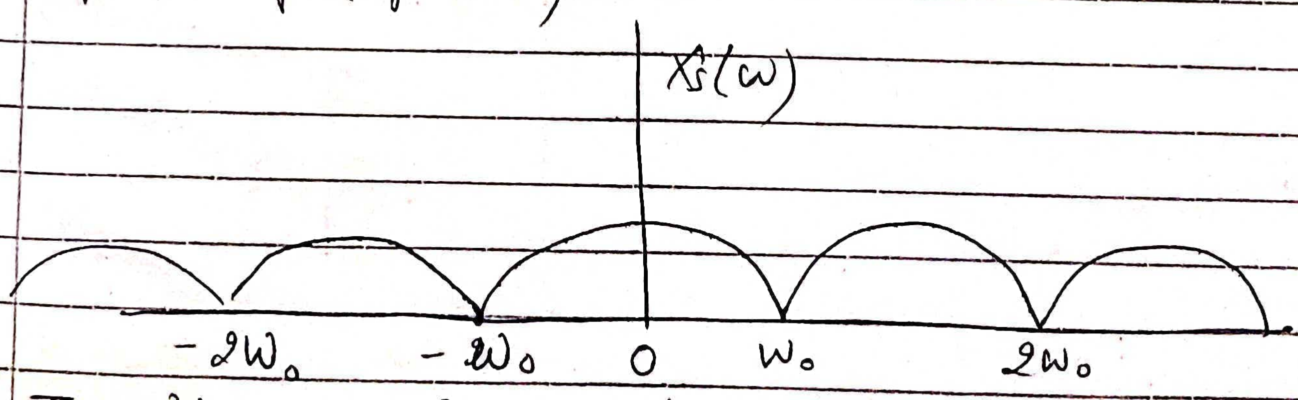
Condition 1:-
- Oversampling → If sampled at higher rate than 2W in the frequency domain ($f_s > W$)

$$X_s(\omega) = \frac{1}{T_s} \sum_{n=-\infty} X(\omega - n\omega_0)$$



- Here the information is reproduced without any loss. There is no mixing up and hence recovery is possible.

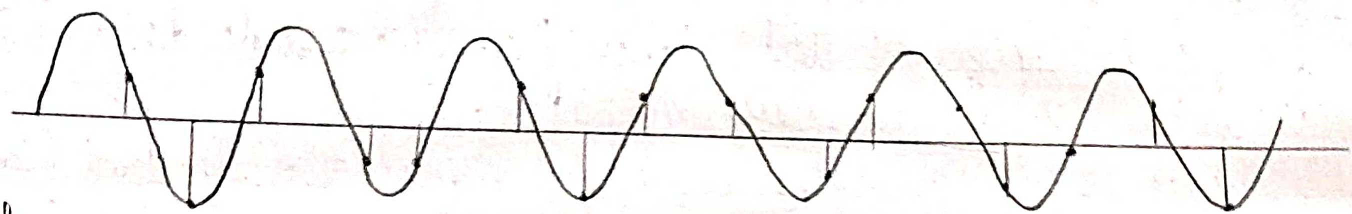
Condition-2:-
- If the sampling rate is equal to twice the highest frequency ($f_s = 2\omega$)



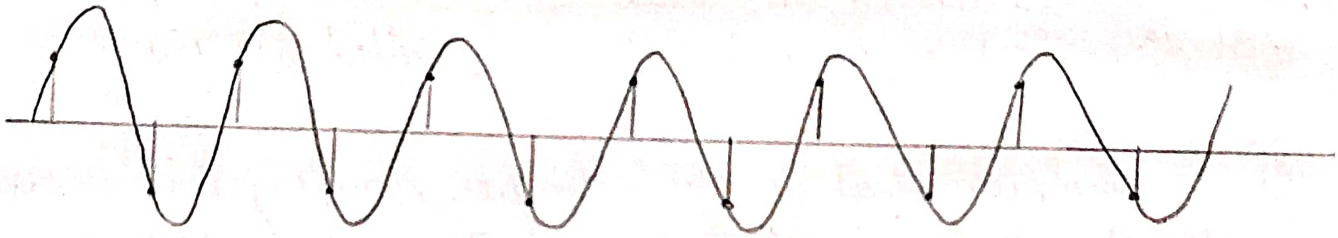
- The information is retrived without any loss. Hence this is also a good sampling rate.

Sampling of a sinusoidal signal of frequency f at different sampling rates f_s. With dashed lines are shown the alias frequencies occurring when $f_s/f < 2$

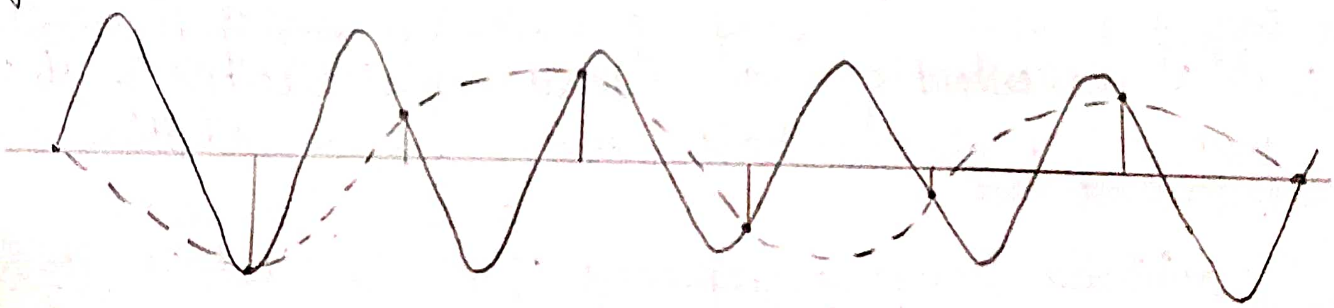
$\frac{f_s}{f} = 2.6$



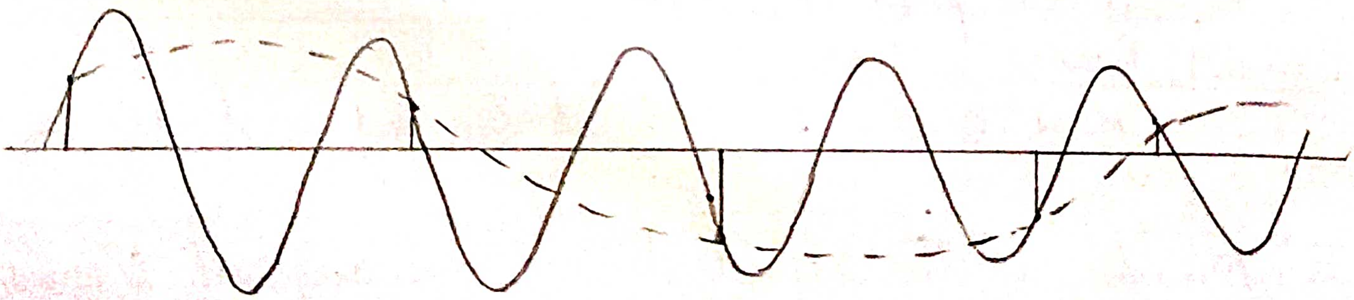
$\frac{f_s}{f} = 2.0$



$\frac{f_s}{f} = 1.4$



$\frac{f_s}{f} = 0.8$



Quantizer →

- The method of sampling chooses a few points on the analog signal and then these points are joined to round off the value to a near stabilized value. Such a process is called Quantization.
- Quantization of a signal procedures the closest representable value.

Encoding →

- The digitization of the analog signal is done by the encoder.
- After each sample is quantized, the number of bits per sample is decided.
- Each sample is changed to an n bit code
- Encoding is also used to minimize the bandwidth
- Designing this filter is to determine the bandwidth required in the acquisition system.
- The maximum frequency of the input signal should be less than or equal to half of the sampling rate
- This sets the cut off freq of the low-pass filter.

→ If fs is the sampling frequency, then the critical frequency (or Nyquist limit) f_N is defined as equal to $fs/2$.
Any sinusoidal component of the signal of frequency f' higher than f_N (eg $f' = f_N + \Delta F$) is not only lost but is reintroduced in the sampled signal by folding at frequency f_N an alias.
Sinusoidal component of frequency $f' = f_N - \Delta F$

This effect is known as aliasing (alias: false name). Aliasing is demonstrated. A sinusoidal signal (blue) of frequency f is sampled at four different sampling frequencies, $f_s = 2.6F, 2.0F, 1.4F$ & $0.8F$

In two first cases the sampling rates are adequate enough for the accurate reconstruction of the original sinusoidal signal whereas in the last two, subsampling occurs, and the collected points may be considered as belonging to signals of lower frequencies. The alias frequencies due to subsampling can be calculated by the following equation :-

Alias frequency :- $f' = |f - kf_s|$

where $k = 1, 2$

Hence when $f_s/F = 1.4$, the alias frequency is F'

$= |F - 1 \times 1.4F| = 0.4f$

whereas, when $f_s/F = 0.8$, the alias frequency is F'

$= |F - 1 \times 0.8F| = 0.2F$

Problems arising due to aliasing :- The effect of the sampling frequency on the spectrum of a signal consisting of infinite number of sinusoidal components.

In Fig the Nyquist frequency is sufficiently higher from the maximum frequency (f_{max}) component of the signal and the stored signal is not distorted.
The opposite occurs in Fig where $f_{max} > f_N$, and all frequency components higher than f_N are not only lost, but they are also folded at f_N and they are added to the other sinusoidal components, corrupting thus the stored signal due to aliasing.

A typical application :- For the digitalization of sound a sampling rate of about 6KHz is sufficient for telephony since normal human voice does not contain an appreciable amount of frequency components higher than 2-5 KHz. However, a sampling rate of about 40KHz is needed for the digitalization of music, since frequency components of about 15-20KHz are common and needed for achieving fidelity of sound reconstruction

Notes: (1): In practice, the sampling rate of f_s is commonly selected in the range $2.5 \times f_{max}$ - $3 \times f_{max}$. For digital recording of music in CD a sampling rate of 44,1KHz is commonly used.

(2): Prior to sampling the signal must pass through a low-pass filter which will remove all unnecessary components (eg noise) higher than f_{max}, preventing thus the "contamination" of the stored signal by their biased frequencies.

1)

SIGNAL FREQUENCY = 10Hz	
SAMPLING FREQ (Hz)	ALIAS FREQ (Hz)
7	3
10	0
15	5
20	–
22	–

2)

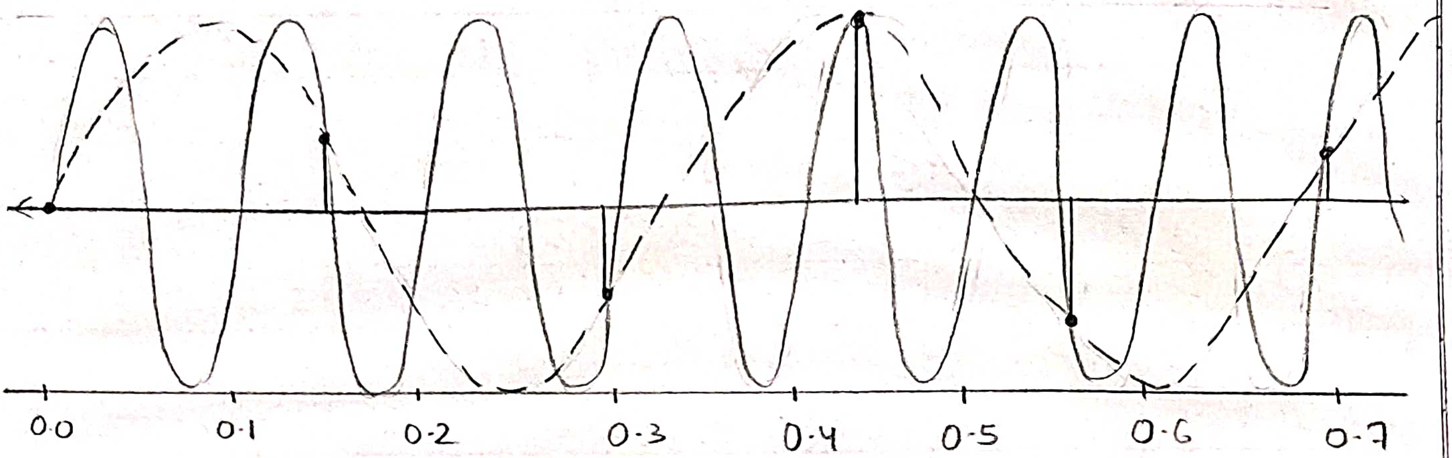
SIGNAL FREQUENCY = 20Hz	
SAMPLING FREQ (Hz)	ALIAS FREQ (Hz)
19	0
20.1	0.1
30	10
40	–
42	–

Conclusion:

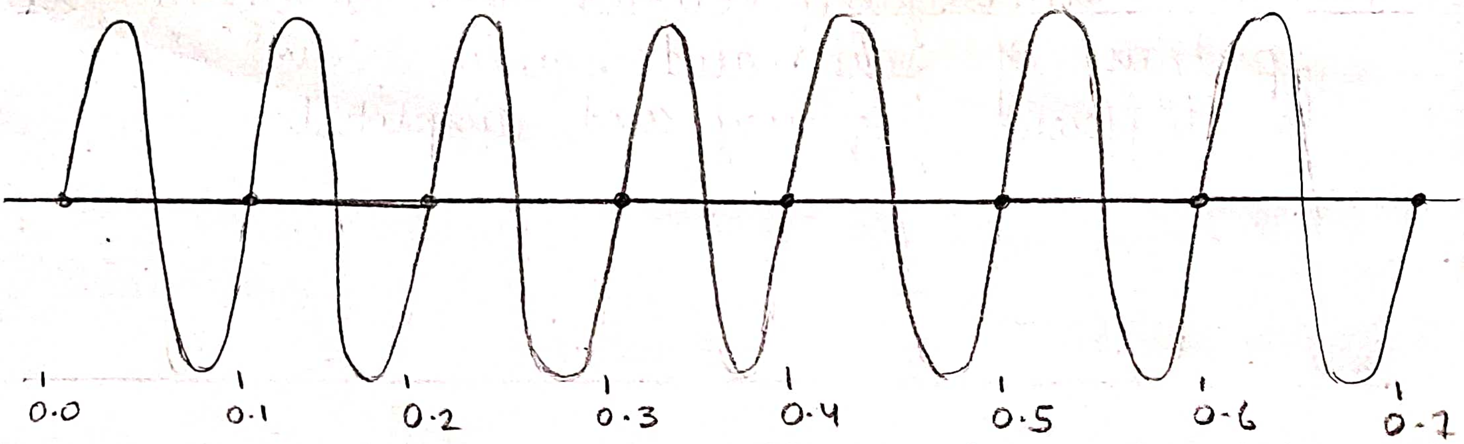
Successfully examined the sampling and reconstruction of a signal and verified the Nyquist criteria by varying the sampling frequency.

Signal Frequency : 10Hz

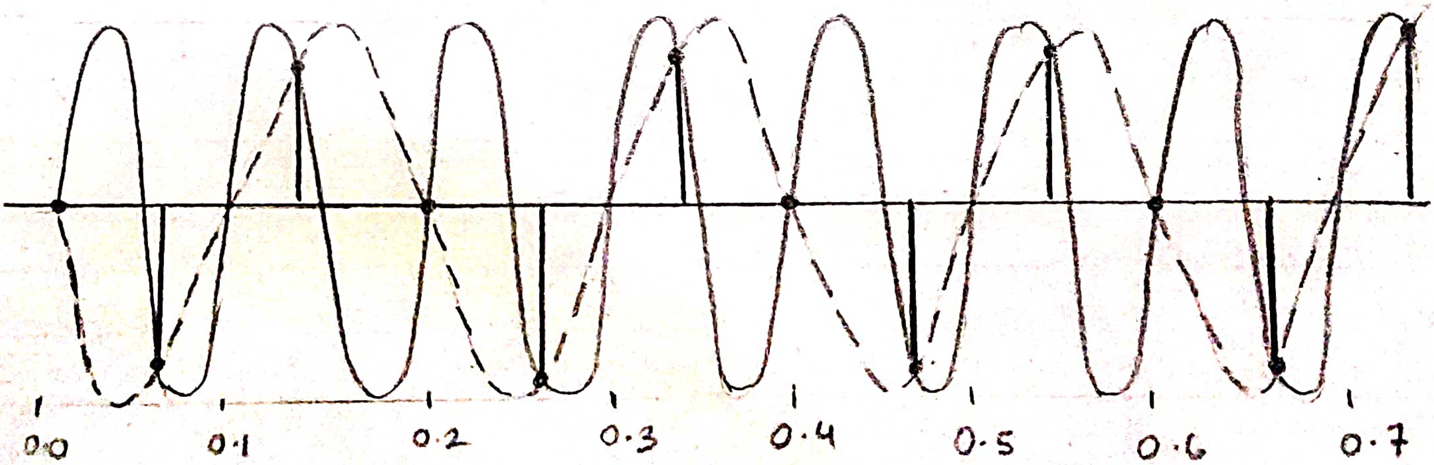
1) Sampling Frequency = 7Hz



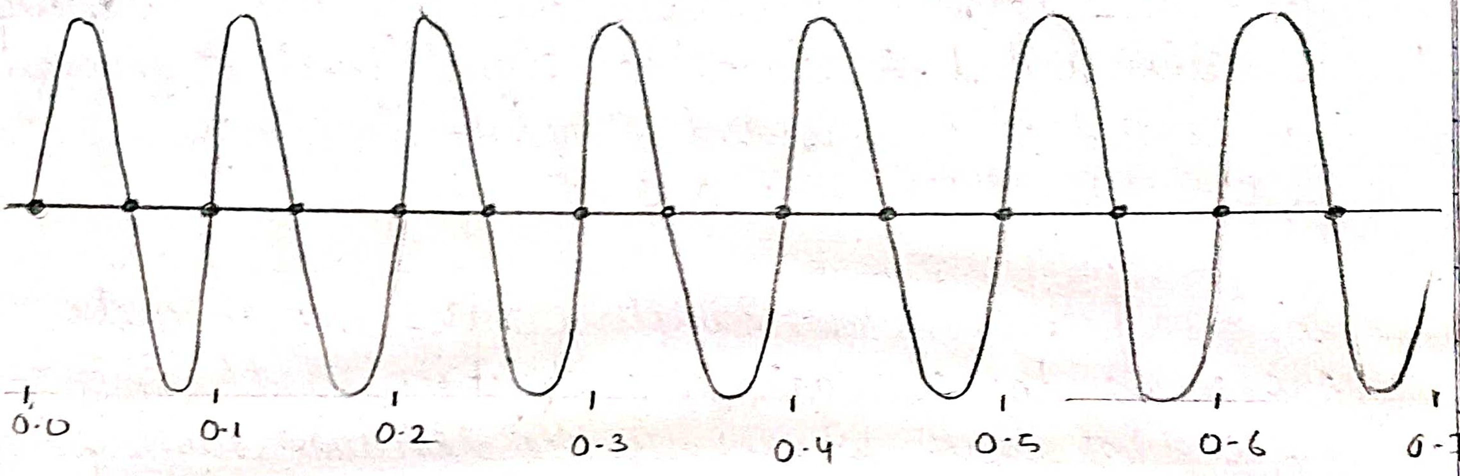
2) Sampling Frequency = 10Hz



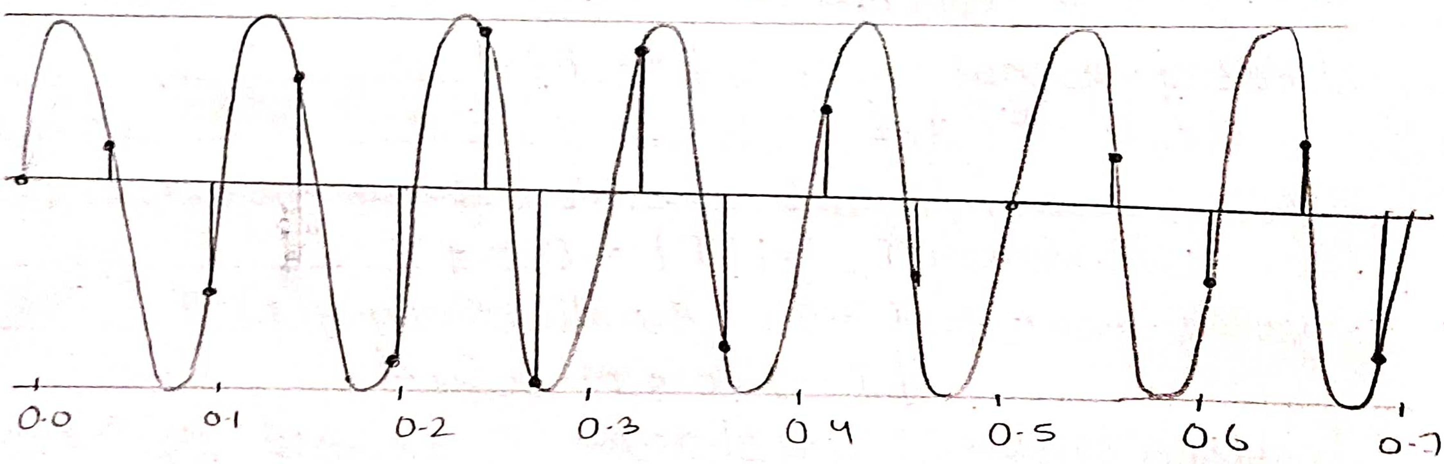
3) Sampling Frequency = 15Hz



4) Sampling Frequency = 20Hz

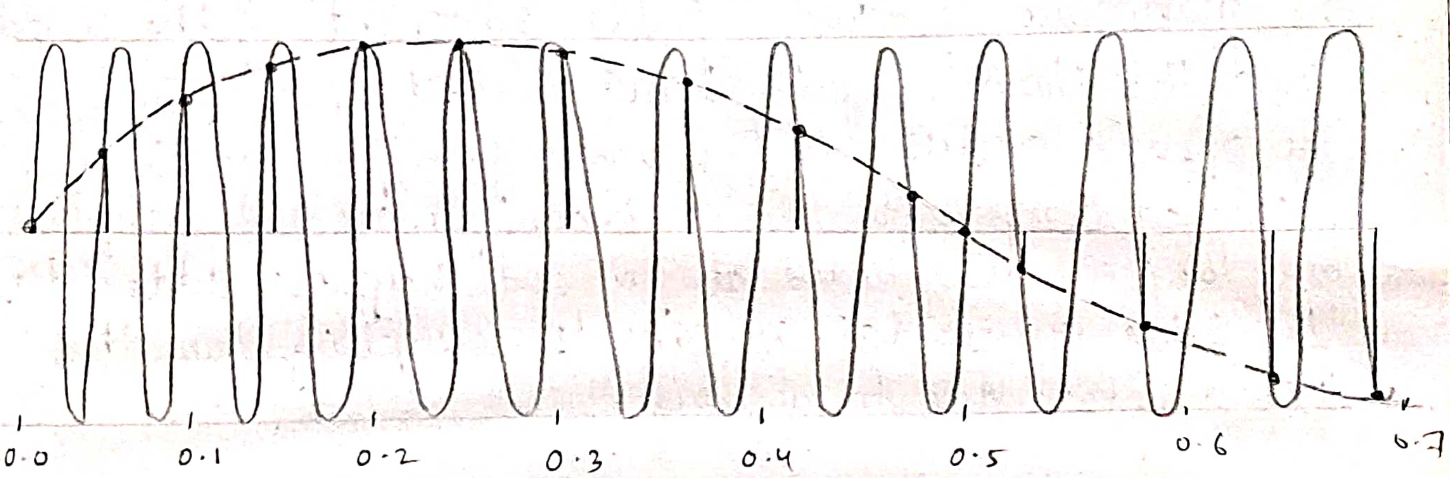


5) Sampling Frequency = 22Hz

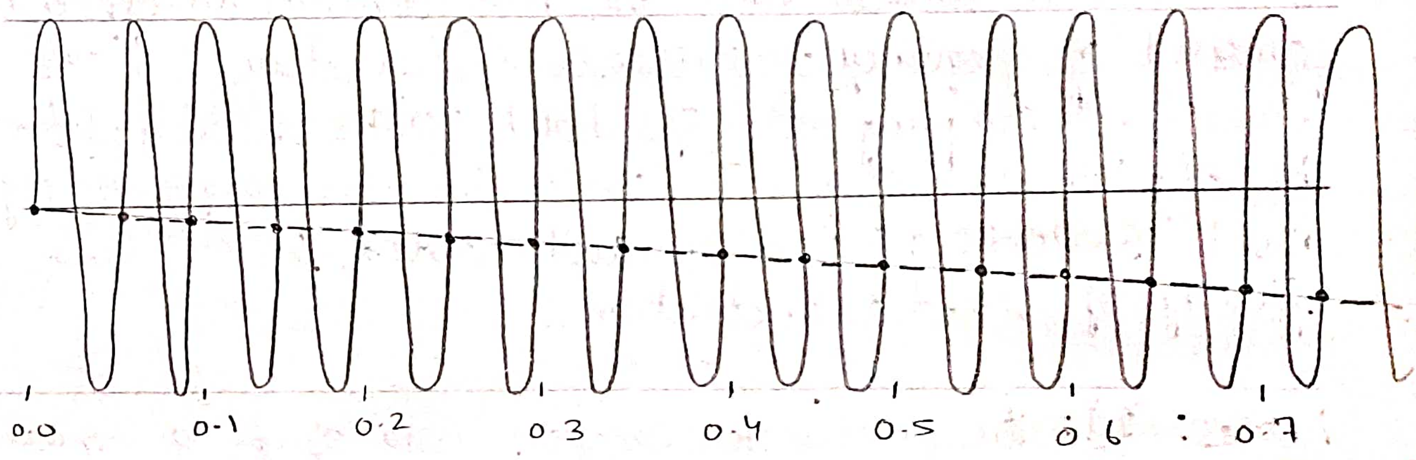


Signal Frequency = 20Hz

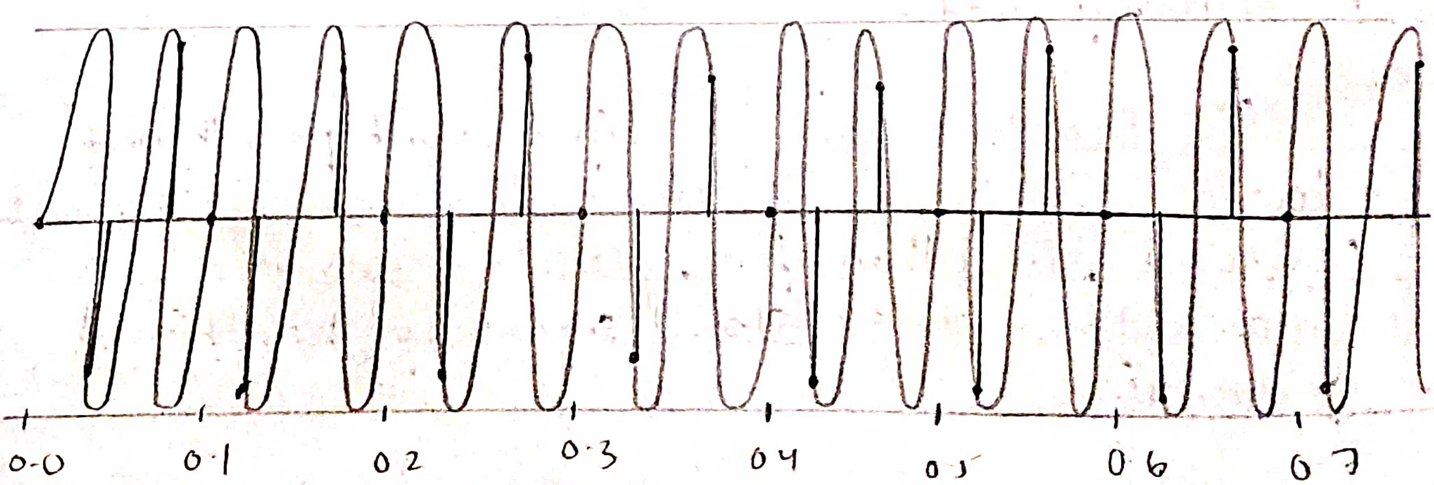
1) Sampling Frequency = 19 Hz



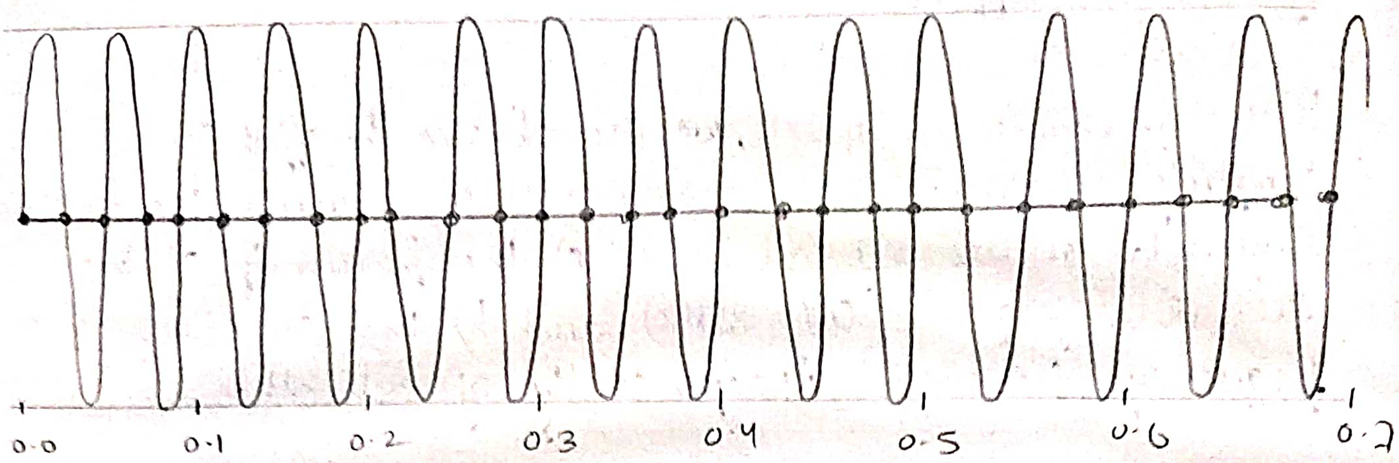
2) Sampling Frequency = 20.1 Hz



3) Sampling Frequency = 30Hz



4) Sampling Frequency = 40Hz



5) Sampling Frequency = 42 Hz