MATHS TUTORIAL-2 (Set Theory)

Questions)

1. Let $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{x \in \mathbb{Z} \mid x$ is divisible by $6\}$, and $C = \{x \in \mathbb{R} \mid x^2 = 2 \text{ or } x^3 = 1\}$. Mark the following true or false.

   a. $3 \in A$ b. $6 \in A$ c. $2 \notin A$
   d. $2 \in B$ e. $6 \in B$ f. $24 \in B$
   g. $28 \notin B$ h. $2 \in C$ i. $1 \in C$
   j. $-\sqrt{2} \in C$ k. $5 \in A \cup B$ l. $6 \in A \cap B$
   m. $1 \in A \cap C$ n. $\sqrt{2} \in B \cup C$

2. Mark the following true or false.

   a. $28 \in \mathbb{Z}$ b. $-5 \in \mathbb{N}$ c. $\sqrt{2} \notin \mathbb{Q} \cap \mathbb{R}$
   d. $\mathbb{Z} \cup \mathbb{Q} = \mathbb{R}$ e. $\mathbb{R} \cap \mathbb{C} = \mathbb{R}$

3. Let $U = \{a, b, c, d, e, f, g\}$, $A = \{a, d, e, f\}$, and $B = \{b, e, g\}$ be sets, where U acts as the universal set. Determine the following.

   a. $(A \cup B)'$ b. $A \cap B$
   c. $A - B$ d. $B - A$

4. Let U be the set of all students in a college. Let A be the set of students taking the discrete mathematics course and B be the set of students taking the calculus course. Describe the following.

   a. $A \cup B$ b. $A \cap B$ c. $A - B$
   d. $B - A$ e. A'

5. Let $P = \{x \in \mathbb{N} \mid 2 < x \leq 8\}$, $Q = \{x \in \mathbb{Z} \mid 0 \leq x < 5\}$, $R = \{x \in \mathbb{N} \mid 1 \leq x \leq 10\}$. Let $U = \{x \in \mathbb{Z} \mid -2 \leq x < 12\}$ be the universal set. Determine the following.

   a. $P \cup R$ b. $Q \cap R$
   c. $P \Delta R$ d. Q'

6. Let P, Q, R, and U be the same as in Exercise 5. Verify the following.

   a. $(P \cup Q)' = P' \cap Q'$ b. $P \cap (P \cup R) = P$
   c. $P \cup (Q \cap R) = (P \cup Q) \cap (P \cup R)$

7. Let $A = \{x \in \mathbb{R} \mid 1 < x \leq 5\}$ and $B = \{x \in \mathbb{R} \mid 3 \leq x \leq 8\}$. Find $A \cup B, A \cap B, A - B, B - A$.

8. Determine whether the following pairs of sets are equal. Justify your answer.

$$A = \left\{ n \in \mathbb{Z} \mid n = \frac{1}{n} \right\} \quad \text{and} \quad B = \{x \in \mathbb{R} \mid x^2 = 1\}.$$

9. Does every set has a subset? Give an example of a set that has only one proper subset.
10. Let X be a set with 4 elements. Find $|\mathcal{P}(X)|$.
11. Find $\mathcal{P}(\mathcal{P}(\mathcal{P}(\emptyset)))$.
12. Let $I_n = \{1, 2, \dots, n\}$, the set of first n natural numbers.
    a. Describe the set $I_{10} - I_5$.
    b. Describe the set $I_n - I_m$ if
       (i) $n > m$ (ii) $n = m$ (iii) $n < m$
13. Let A and B be subsets of the set U. Draw the Venn diagram of the following sets.
    a. $(A \cup B)'$ b. $(A \cap B)'$
    c. $A \Delta B$ d. $(A \cup B) - (A \cap B)$
14. Let A, B, and C be subsets of the set U. Draw the Venn diagram of the following sets.
    a. $(A \cup B) \cap C$ b. $(A \cap B) \cup C$
    c. $(A \cap B) - C$ d. $(A - B) - C$
    e. $(A - (B \cup C)) \cup (B - (A \cup C))$
15. Let A, B, C, and D be subsets of the set U. Draw the Venn diagram of the following sets.
    a. $A \cap B \cap C \cap D$
    b. $(A \cup B \cup C) \cap D$
    c. $(A \cup B) \cap (C \cap D)$
16. Let A and B be sets. Prove that $A \subseteq B$ if and only if $A \cap B = A$.
17. Prove those parts of Theorem 1.1.3 that are not proved in this section.
18. Suppose P and Q are two sets. Let R be a set that contains elements belonging to P or Q but not both. Let T be a set that contains elements belonging to Q or the complement of P but not both. Show that R is the complement of T.
19. Let A and B be sets. Prove that $A - (A - B) = A \cap B$.
20. Justify the following statements or else give an example to disprove the result. Let A, B, and C be subsets of a set U.
    (a) $A \Delta C = B \Delta C \Rightarrow A = B$
    (b) $(A - C) - (B - C) = (A - B) - C$
    (c) $(A - B)' = (B - A)'$

Theorem 1.1.3: Let X, Y, Z be subsets of a set U. Then the following assertions hold.

(i) If $X \subseteq Y$, then $X \cup Y = Y$ and $X \cap Y = X$.

(ii) Laws of identity: $X \cup \emptyset = X$ and $X \cap \emptyset = \emptyset$.

(iii) Laws of idempotency: $X \cup X = X$ and $X \cap X = X$.

(iv) Laws of commutativity: $X \cup Y = Y \cup X$ and $X \cap Y = Y \cap X$.

(v) Laws of associativity:

$$(X \cup Y) \cup Z = X \cup (Y \cup Z),$$
$$(X \cap Y) \cap Z = X \cap (Y \cap Z).$$

(vi) Laws of distributivity:

$$X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z),$$
$$X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z).$$

(vii) Laws of absorptivity:

$$X \cap (X \cup Y) = X, \qquad X \cup (X \cap Y) = X.$$