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Tutorial - 3

RELATIONS

1. If R is a relation from $A = \{1,2,3,4\}$ to $B = \{2,3,4,5\}$ List elements in R defined by aRb if a & b both are odd. Also, write the domain & Range of R.

$\Rightarrow$ aRb , $a \in A$, $b \in B$ and a, b are odd.

$R = \{(1,3), (1,5), (3,3), (3,5)\}$

Domain: $\{1, 3\}$
Range : $\{3, 5\}$

2. If R is a relation from $A = \{1,2,3\}$ to $B = \{4,5\}$ given by $R = \{(1,4), (2,4), (1,5), (3,5)\}$ find R^{-1}.

$R = \{(1,4), (2,4), (1,5), (3,5)\}$
$R^{-1} = \{(4,1), (4,2), (5,1), (5,3)\}$

3. Given an example of relation that is both symmetric and anti symmetric.

$\Rightarrow$ $R = \{(1,1)\}$
$R = \{(2,2), (3,3)\}$

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4. Give an example of a relation that neither symmetric nor anti symmetric.

⇒ $R = \{(1,2), (1,3), (3,1)\}$
$R = \{(1,2), (2,3), (3,2)\}$

5. Give an example of a relation that is reflexive and symmetric but not transitive.

⇒ $A = \{(4,4), (6,6), (8,8), (4,6), (6,4), (6,8), (8,6)\}$

Relation R is reflexive since for every $a \in A$, $(a,a) \in R$, i.e., $\{(4,4), (6,6), (8,8\} \in R$

Relation R is symmetric since $(a,b) \in R \Rightarrow (b,a) \in R$ $\forall$ $a,b \in R$

Relation R is not transitive since $(4,6), (6,8) \in R$ but $(4,8) \notin R$.

Hence, relation R is Reflexive and symmetric but not transitive.

6. Give an example of a relation that is reflexive and transitive but not symmetric.

⇒ $S = \{1,2,3\}$
$R = \{(1,1), (2,2), (3,3), (1,2)\}$
As $(s,s) \in R$ $s \in S$ R is reflexive.
As $(1,2) \in R$ and $(2,1) \notin R$ R is not symmetric.
$\therefore$ R is transitive.

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7. Give an example of a relation that is symmetric and transitive but not reflexive.

=) ~~$R = \{(1,2), (2,3), (2,1), (3,2), (1,3)\}$~~

$R = \{(1,1), (1,2), (2,1)\}$

$(a,b) \in R$ and $(b,a) \in R \Rightarrow$ Symmetric.

$(a,b) \in R$, $(b,c) \in R \Rightarrow$ transitive.

Since $(2,2) \notin R \Rightarrow$ not reflexive.

8. If $R_1 = \{(1,2), (2,3), (3,4)\}$

$R_2 = \{(1,1), (1,2), (2,1), (2,2), (2,3), (3,1)\}$

be the relation from $\{1,2,3\}$ to $\{1,2,3,4\}$ then find.

(i) $R_1 \cup R_2$ (iii) $R_1 - R_2$

(ii) $R_1 \cap R_2$ (iv) $R_2 - R_1$

=) $R_1 \cup R_2 = \{(1,1), (1,2), (2,1), (2,2), (2,3), (3,1), (3,4)\}$

$R_1 \cap R_2 = \{(1,2), (2,3)\}$

$R_1 - R_2 = \{(3,4)\}$

$R_2 - R_1 = \{(1,1), (2,1), (2,2), (3,1)\}$

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9. If $R = \{(x, x^2)\}$ and $S = \{(x, 2x)\}$, where x is non negative integer find

(i) $R \cap S$ (iii) $R - S$

(ii) $R \cup S$ (iv) $S - R$

$R = \{(0,0), (1,1), (2,4), (3,9) \ldots\}$

$S = \{(0,0), (1,2), (2,4), (3,6) \ldots\}$

$R \cap S = \{(0,0), (2,4)\}$

$R \cup S = \{(0,0), (1,1), (1,2), (2,4), (3,6), (3,9) \ldots\}$

$\Rightarrow a R\cup S\, b \in a \in x.$

$b \in x^2 \cup 2x$

$R - S = \{(1,1), (3,9), (4,16) \ldots\}$

$S - R = \{(1,2), (3,6), (4,8) \ldots\}$

10. If the relations R_1, R_2, R_3, R_4, R_5 is defined on set of real no's as given below:

(i) $R_1 = \{(a,b) \mid a \geq b\}$

(ii) $R_2 = \{(a,b) \mid a < b\}$

(iii) $R_3 = \{(a,b) \mid a \leq b\}$

(iv) $R_4 = \{(a,b) \mid a = b\}$

(v) $R_5 = \{(a,b) \mid a \neq b\}$

then find

(a) $R_2 \cup R_5$ (b) $R_3 \cap R_5$ (c) $R_2 - R$

(d) $R_1 . R_2$ (e) $R_2 . R_3$ (f) $R_1 . ($

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10) (a) $R_2 \cup R_5 = \{(a,b) \mid a<b\} \cup \{(a,b) \mid a \neq b\}$
$= \{(a,b) \mid a \neq b\} = R_5$

(b) ~~R_3~~ $R_3 \cap R_5 = \{(a,b) \mid a \leq b\} \cap \{(a,b) \mid a \neq b\}$
$= \{(a,b) \mid a < b\} = R_2$.

(c) Question is not completly visible

(d) $R_1 \cdot R_2 = \{(a,b) \mid a \geq b\} \cdot \{(a,b) \mid a < b\}$
$= R$

(e) $R_2 \cdot R_3 = \{(a,b) \mid a<b\} \cdot \{(a,b) \mid a \leq b\}$
$= R_3$

(f) Question is not completly visible.

11. If R, S, T be the relations on set
$A = \{0,1,2,3\}$ defined by
$R = \{(a,b) \mid a+b=3\}$, $S = \{(a,b) \mid 3 \text{ is divisible by } a+b\}$
$T = \{(a,b) \mid Max(a,b) = 3\}$

then find
(a) $R \cdot T$ (b) $T \cdot R$ (c) $S \cdot S$

$R = \{(0,3), (1,2), (2,1), (3,0)\}$
$S = \{(0,3), (0,1), (1,2), (2,1), (1,0), (3,0)\}$
$T = \{(0,3), (1,3), (2,3), (3,2), (3,1), (3,0)\}$

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(a) $R \cdot T = \{(0,2), (0,1), (0,0), (1,3), (2,3), (3,3)\}$

(b) $T \cdot R = \{(0,0), (1,0), (2,0), (3,1), (3,2), (3,3)\}$

(c) $S \cdot S = \{(0,0), (0,2), (1,1), (2,2), (1,3), (3,1), (3,3)\}$

12. Determine whether Relation R on the set of all integers is reflexive, symmetric, anti symmetric or transitive where aRb iff

(a) $a \neq b$ (b) $a \cdot b \geq 0$

(c) $ab \geq 1$ (d) a is a multiple of b

(e) modulus of $|a-b| = 1$ (f) $a = b^2$.

(g) $a \geq b$

Question is not completly Visible.

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13. Verify the following are equivalence relations or not

(i) R is the relation on set of real no's such that aRb iff a-b is an integer.

$a-a = 0 \in R$ ⇒ R is Reflexive

$(a,b) \in R$ ⇒ $a-b$ is integre and $b-a$ is also integer ⇒ $(b,a) \in R$

R is symmetric.

(a,b) & $(b,c) \in R$ ⇒ $a-b$ is integer and $c-b$ is also integer ⇒ $a-c$ is also integre. ⇒ $(a,c) \in R$

R is transitive.

⇒ R is equivalence relation

14. If R is a relation on set of Integer such that $(a,b) \in R$ iff $b = a^n$ for some positive integer n. Show that R is a partial order relation.

$b = a$ for $n = 1$

⇒ $(a,a) \in R$ ⇒ Reflexive.

$(a,b) \in R$ & $(b,a) \in R$.

⇒ $(a, a^n) \in R$. $(a^n, a) \in R$.

⇒ $n = 1$ ⇒ $b = a$ ⇒ Antisymmetric.

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$(a,b) \in R \quad a^n = b$

$(b,c) \in R \quad b^n = c$

$\Rightarrow (a^n)^n = c$

$\Rightarrow a^{2n} = c$

let $2n = m$

$\Rightarrow a^m = c$

$\Rightarrow (a,c) \in R \Rightarrow$ transitive.

$\Rightarrow$ R is partial order Relation.

15. Draw Diagraph for relation R on A. $A = \{1,2,3,4,5,6,7,8\}$ Let xRy whenever y is divisible by x. Is R equivalence relation? Is R partial ordering?

~~R = {(1,1), (2,1), (2,2), (3,1), (3,2), (3,3), (4,1), (4,2), (4,3), (4,4), (5,1), (5,2), (5,3), (5,4), (5,5), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6), (7,1), (7,2), (7,3), (7,4), (7,5), (7,6), (7,7), (8,1), (8,2), (8,3), (8,4), (8,5), (8,6), (8,7), (8,8)}~~

$R = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (1,7), (1,8), (2,2), (2,4), (2,6), (2,8), (3,3), (3,6), (4,4), (4,8), (5,5), (6,6), (7,7), (8,8)\}$

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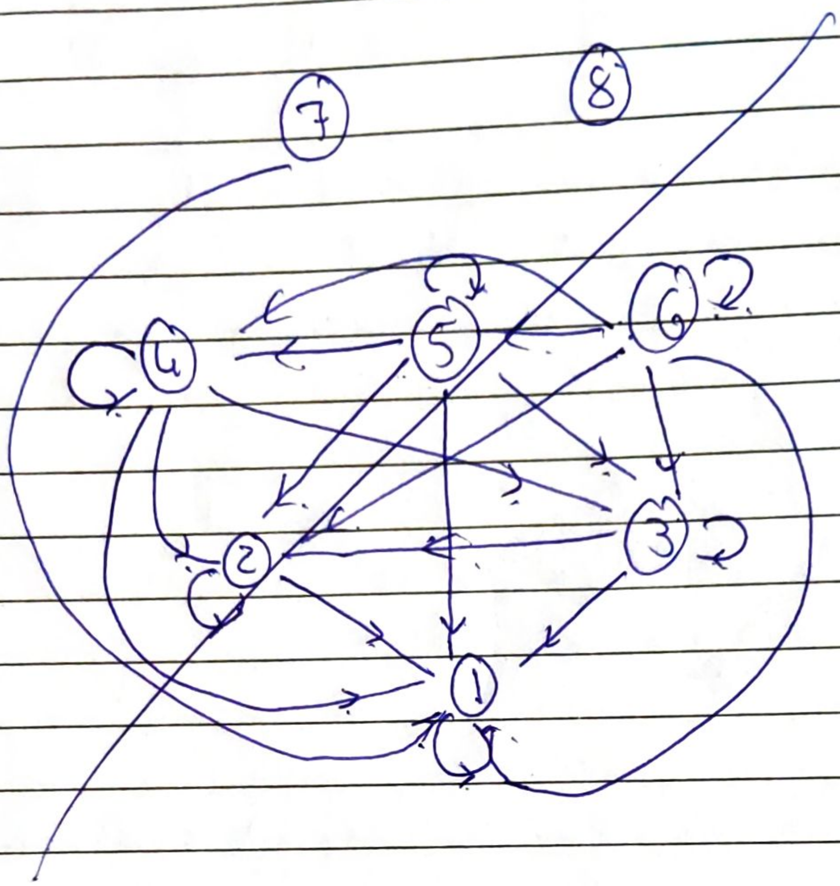

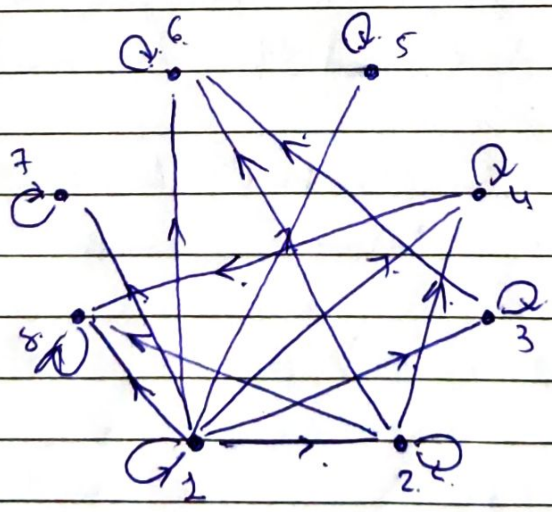

R is not symmetric and ~~not~~ anti symmetric

⇒ R is not equivalence and R is ~~not~~ partial ordering.