ELECTRONICS ENGINEERING DEPARTMENT
SARDAR VALLABHBHAI NATIONAL INSTITUTE OF TECHNOLOGY, SURAT

DIGITAL ELECTRONICS & LOGIC DESIGN LAB

LAB 3: 26.08.2020-27.08.2020

Objectives of Today's Lab

- Design the following circuits:
  - Half Adder
  - Full Adder
  - Half Substractor
  - Full Substractor

  using basic logic gates.
- Implement the above circuits using Multisim Online and verify the functionality.
- Assignment.

What is Adder circuit?

- Binary adder is a digital circuit that performs addition of binary numbers.
- In computers and other kinds of processors, adders are used in the Arithmetic Logic Units (ALU).
- They are also used in other parts of the processors, for example:
  - to calculate addresses
  - implementing increment and decrement operations

  and other similar applications.

Getting Started

- To design any digital circuit, we must know its Boolean expression.

- In digital circuits, Boolean expression describes how the output(s) behave according to changes in the inputs.
  - For example, y = a.b (Identify this!!)

$\Rightarrow$ We must arrive at Boolean expression for all these circuits.

- Further, Boolean expressions can be optimized using:
  - Laws of Boolean Algebra (✓)
  - Karnaugh Map

Let's Revise..

- Identify the gate from Truth-table and draw its circuit symbol.

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

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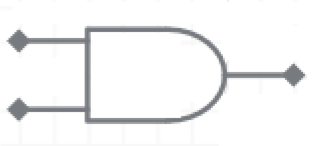
A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

AND gate : Y = A . B



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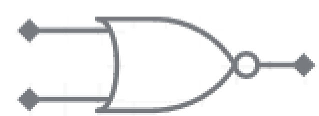
NOR gate : $Y = \overline{A + B}$



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Let's Revise..

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1	0	1
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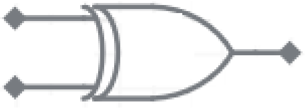
XOR gate : Y = A $\oplus$ B



Let's Revise..

- Identify the gate from Truth-table and draw its circuit symbol.

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	0

Writing Boolean Expression from Truth-table

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	0

This is not a gate. But, it does have a Boolean expression.

$$Y = \overline{A} \, . \, B$$

Writing Boolean Expression from Truth-table

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	0

This is not a gate. But, it does have a Boolean expression.

$$Y = A \cdot \overline{B}$$

Writing Boolean Expression from Truth-table

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

The Boolean expression for this truth table can be described as:

$$Y = \overline{A} . B + A . \overline{B}$$

Writing Boolean Expression from Truth-table

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

The Boolean expression for this truth table can be described as:.

$$Y = \overline{A} \,.\, \overline{B}$$

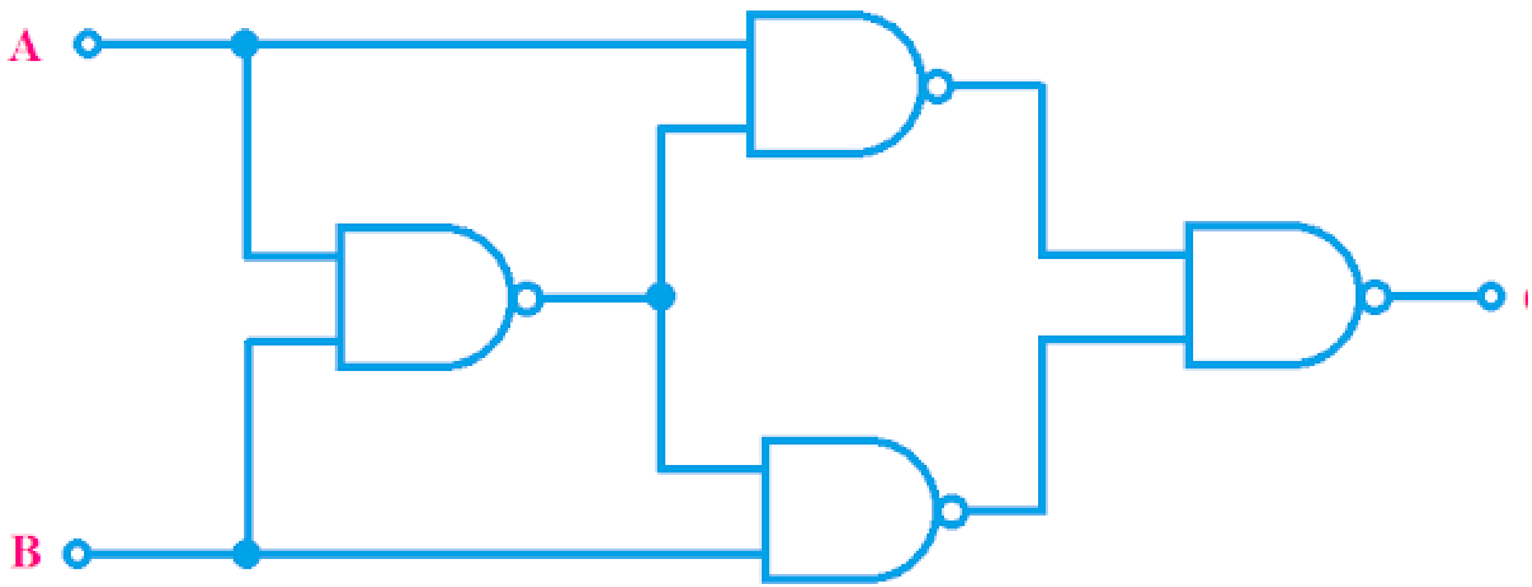
Does this mean $\overline{A} \,.\, \overline{B} = \overline{A + B}$?

Revisiting Laws of Boolean Algebra

Law of Addition	Law of Multiplication
$x + 0 = x$	$x \cdot 1 = x$
$x + x' = 1$	$x \cdot x' = 0$
$x + x = x$	$x \cdot x = x$
$x + 1 = 1$	$x \cdot 0 = 0$
$(x')' = x$	
$x + y = y + x$	$x \cdot y = y \cdot x$
$x+(y+z) = (x+y)+z$	$x \cdot (y \cdot z) = (x \cdot y) \cdot z$
$x \cdot (y+z) = x \cdot y+x \cdot z$	$x+y \cdot z = (x+y) \cdot (x+z)$
$(x+y)' = x' \cdot y'$	$(x \cdot y)' = x' + y'$
$x + (x \cdot y) = x$	$x \cdot (x + y) = x$

Exercise on Boolean algebra

What functionality is implemented by following circuit.



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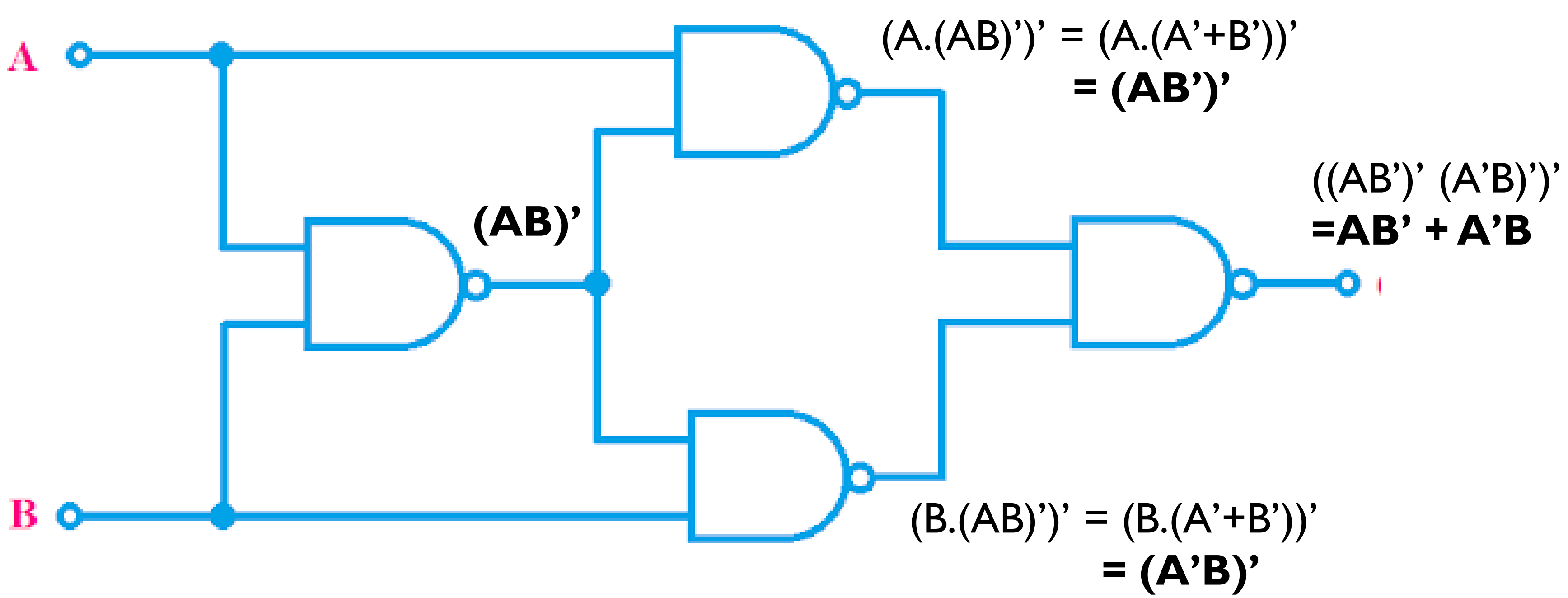

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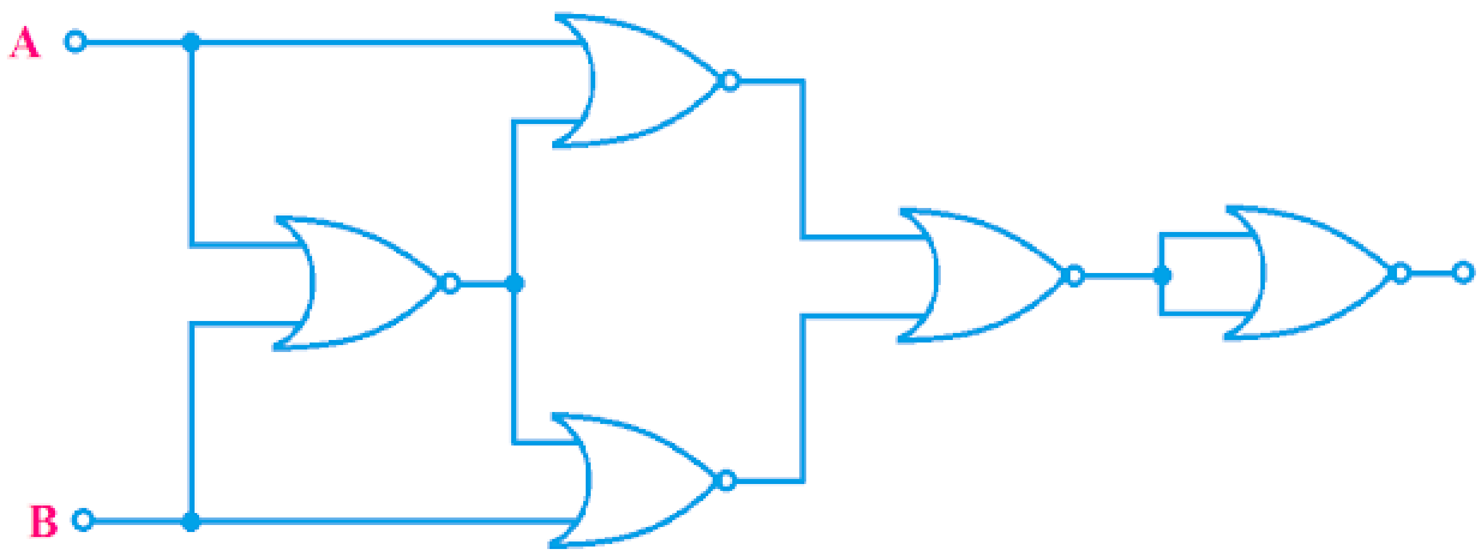

Exercise on Boolean algebra

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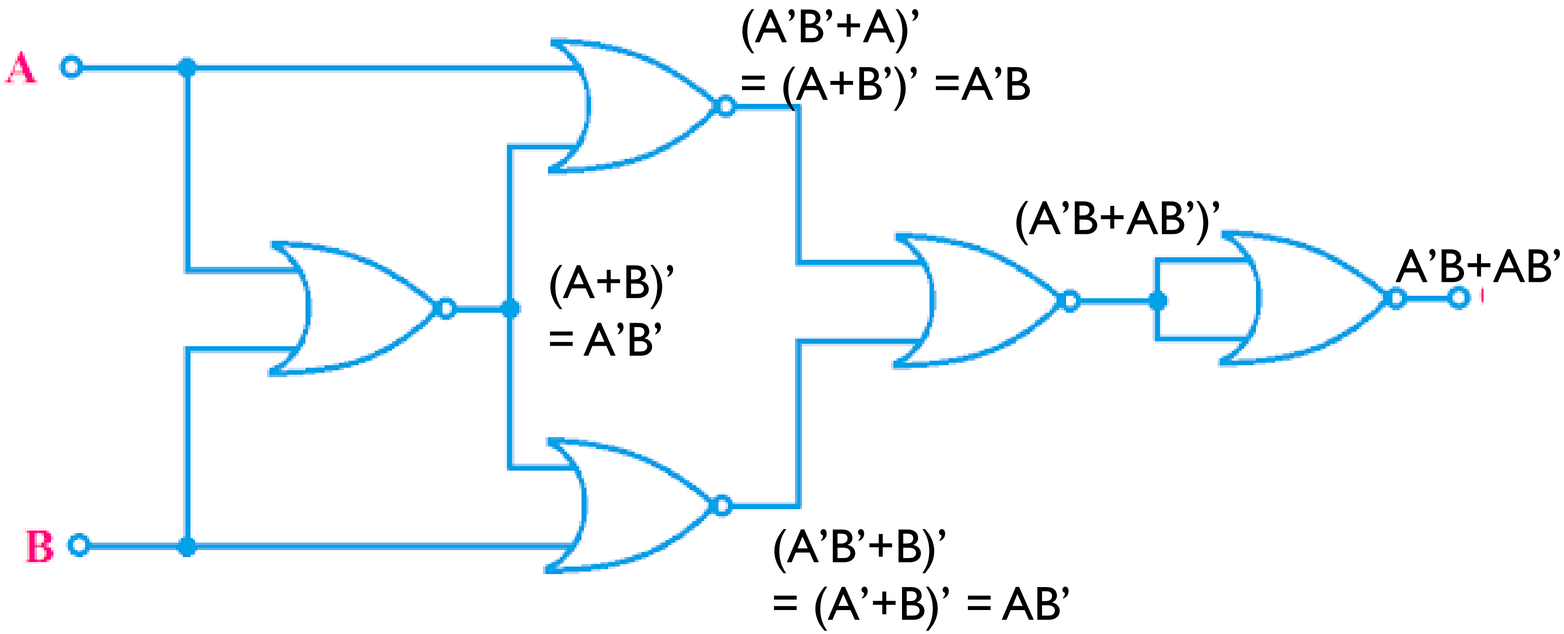

Writing Boolean Expression from Truth-table

- Write Boolean expression for this truth-table.

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Writing Boolean Expression from Truth-table

- Write Boolean expression for this truth-table.

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Y = A'BC + AB'C + ABC' + ABC

Writing Boolean Expression from Truth-table

- Write Boolean expression for this truth-table.

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Writing Boolean Expression from Truth-table

- Write Boolean expression for this truth-table.

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Y = A'B'C + A'BC' + AB'C' + ABC

What is a Half Adder?

- The half adder adds two single binary digits *A* and *B*.
- It has two outputs, sum (S) and carry (C).
- The carry represents an overflow into the next digit of a multi-digit addition.

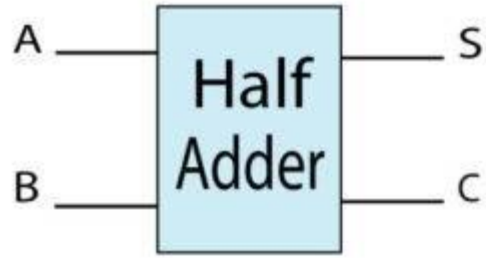

Fig.: Block Diagram Representation

Half Adder

- Truth-table:

A	B	C	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

- Write the Boolean Expression

Half Adder

- Truth-table:

A	B	C	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

- Boolean Expression

$$\text{Sum, } S = A \oplus B$$

$$\text{Carry, } C = A \, . \, B$$

Half Adder

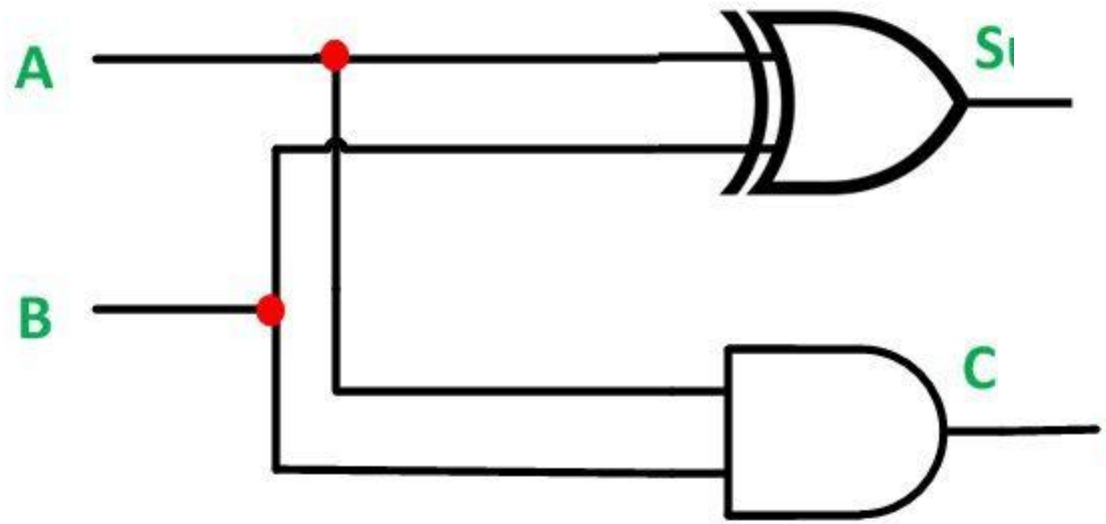

Fig. : Half Adder

What is a Full Adder?

- Full-adder adds three 1-bit numbers: A, B, and Cin.
- A and B are the operands, and C_{in} is a bit carried from the previous stage.
- The circuit produces two output bits: Output carry C_{out} and Sum S.

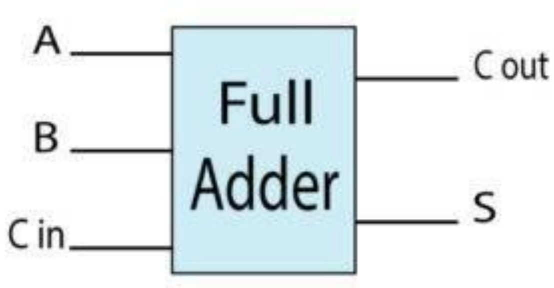

Fig.: Block Diagram Representation

Full Adder

- Truth-table

A	B	C_{in}	C_{out}	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

- Write the Boolean Expression

Full Adder

A	B	C_{in}	C_{out}	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

$$C_{out} = A'BC_{in} + AB'C_{in} + ABC_{in}' + ABC_{in}$$

$$S = A'B'C_{in} + A'BC_{in}' + AB'C_{in}' + ABC_{in}$$

Full Adder

- Lets simplify these expressions using Laws of Boolean algebra.
- $S = A'B'C_{in} + A'BC_{in}' + AB'C_{in}' + ABC_{in}$
- $S = A'B'C_{in} + ABC_{in} + A'BC_{in}' + AB'C_{in}'$
- $S = (A'B' + AB).C_{in} + (A'B + AB').C_{in}'$
- $S = (A \oplus B)'.C_{in} + (A \oplus B).C_{in}'$
- $\boxed{S = A \oplus B \oplus C}$

Full Adder

- $C_{out} = A'BC_{in} + AB'C_{in} + ABC_{in}' + ABC_{in}$
- $C_{out} = A'BC_{in} + AB'C_{in} + ABC_{in}' + ABC_{in} + ABC_{in} + ABC_{in}$
- $C_{out} = A'BC_{in} + ABC_{in} + AB'C_{in} + ABC_{in} + ABC_{in}' + ABC_{in}$
- $C_{out} = (A' + A)BC_{in} + (B' + B)AC_{in} + (C_{in}' + C_{in})AB$
- $\boxed{C_{out} = AB + BC_{in} + AC_{in}}$

Full Adder

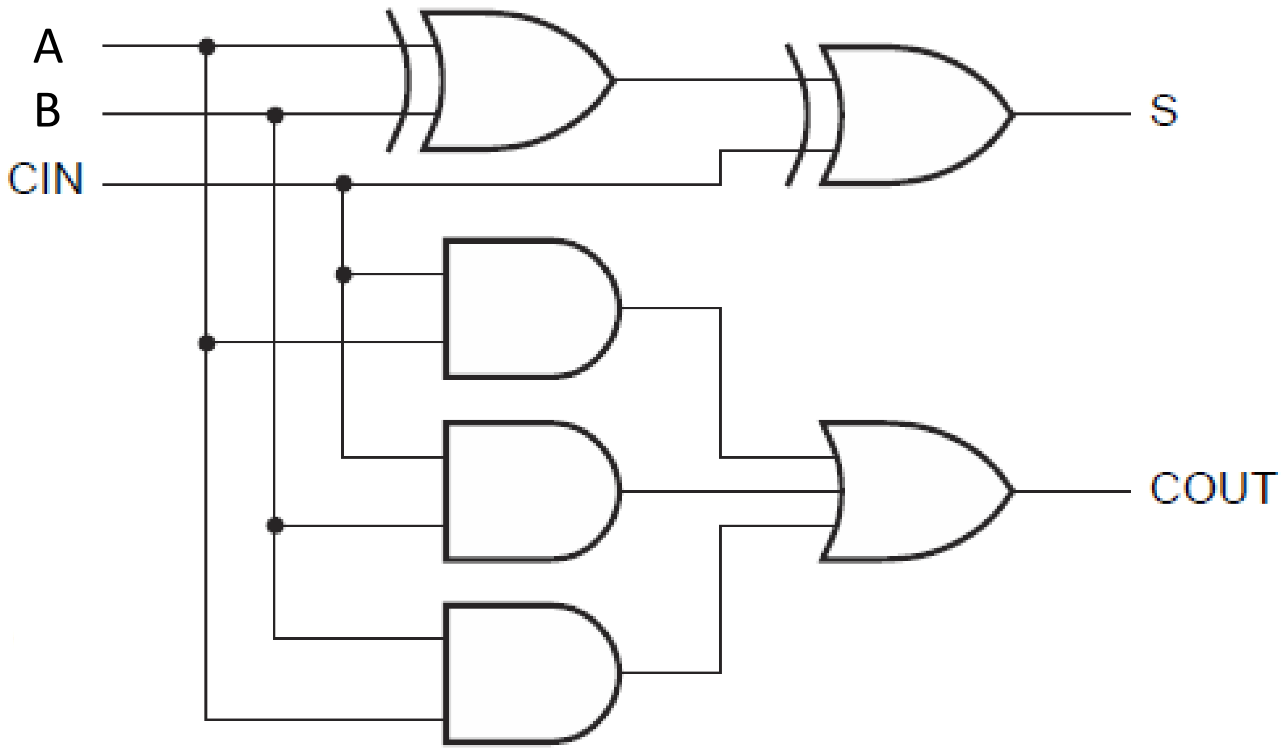

Fig. : Full Adder

Half Substractor

- A substractor can be designed using the same approach as that of an adder.
- The half substractor is a digital circuit which is used to perform subtraction of two bits.
- It has two inputs, the minuend A and subtrahend B and two outputs, the difference D and borrow out B_{out}.
- The borrow out signal is set when the substractor needs to borrow from the next digit in a multi-digit subtraction.
- $B_{out} = 1$ when $A < B$.
  - i.e. $B_{out} = 1$ if and only if $A = 0$ and $B = 1$

Half-Substractor

Truth-Table

A	B	B_{out}	D
0	0	0	0
0	1	1	1
1	0	0	1
1	1	0	0

Boolean Expressions:

$B_{out} = A' . B$

$D = A \oplus B$

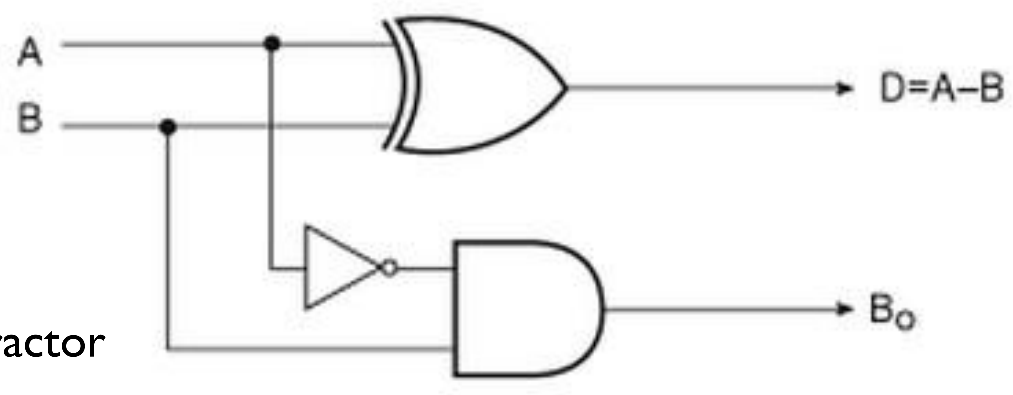

Fig. : Half Substractor

Full Substractor

- Full substractor is a digital circuit which is used to perform subtraction of three input bits: the minuend A, subtrahend B and borrow in B_{in}
- The full substractor generates two output bits: the difference D and borrow out B_{out}.
- B_{in} is set when the previous digit is borrowed from A. So, B_{in} is also subtracted from A as well as the subtrahend B.
  - $A - B - B_{in}$
- A borrow out needs to be generated when $A < (B + B_{in})$

Full Substractor

Truth-Table

A	B	B_{in}	B_{out}	D
0	0	0	0	0
0	0	I	I	I
0	I	0	I	I
0	I	I	I	0
I	0	0	0	I
I	0	I	0	0
I	I	0	0	0
I	I	I	I	I

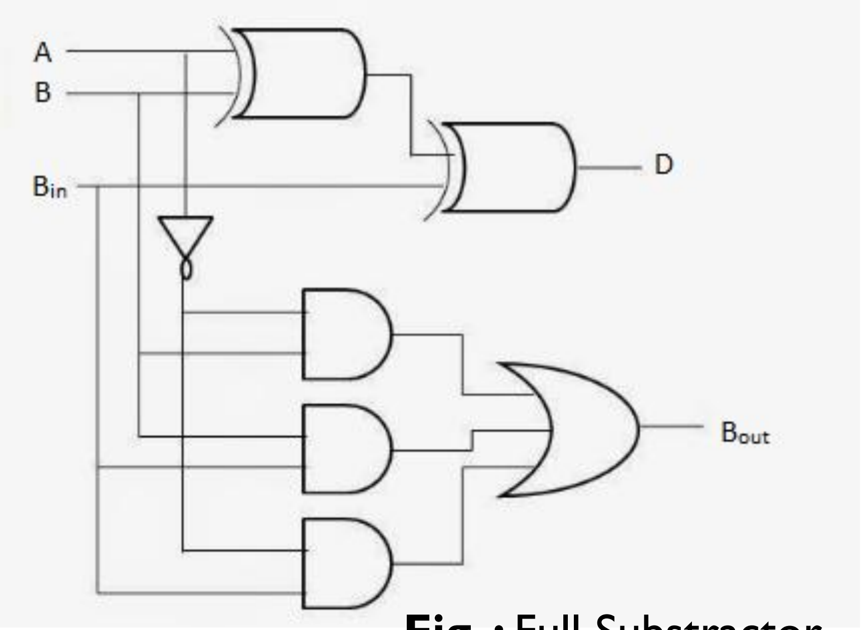

Fig. : Full Substractor

Boolean Expressions:

$$B_{out} = A'B'B_{in} + A'BB_{in}' + A'BB_{in} + ABB_{in} = A'B + A'B_{in} + BB_{in}$$

$$D = A'B'C_{in} + A'BC_{in}' + AB'C_{in}' + ABC_{in} = A \oplus B \oplus C$$

Doing it in Multisim Online

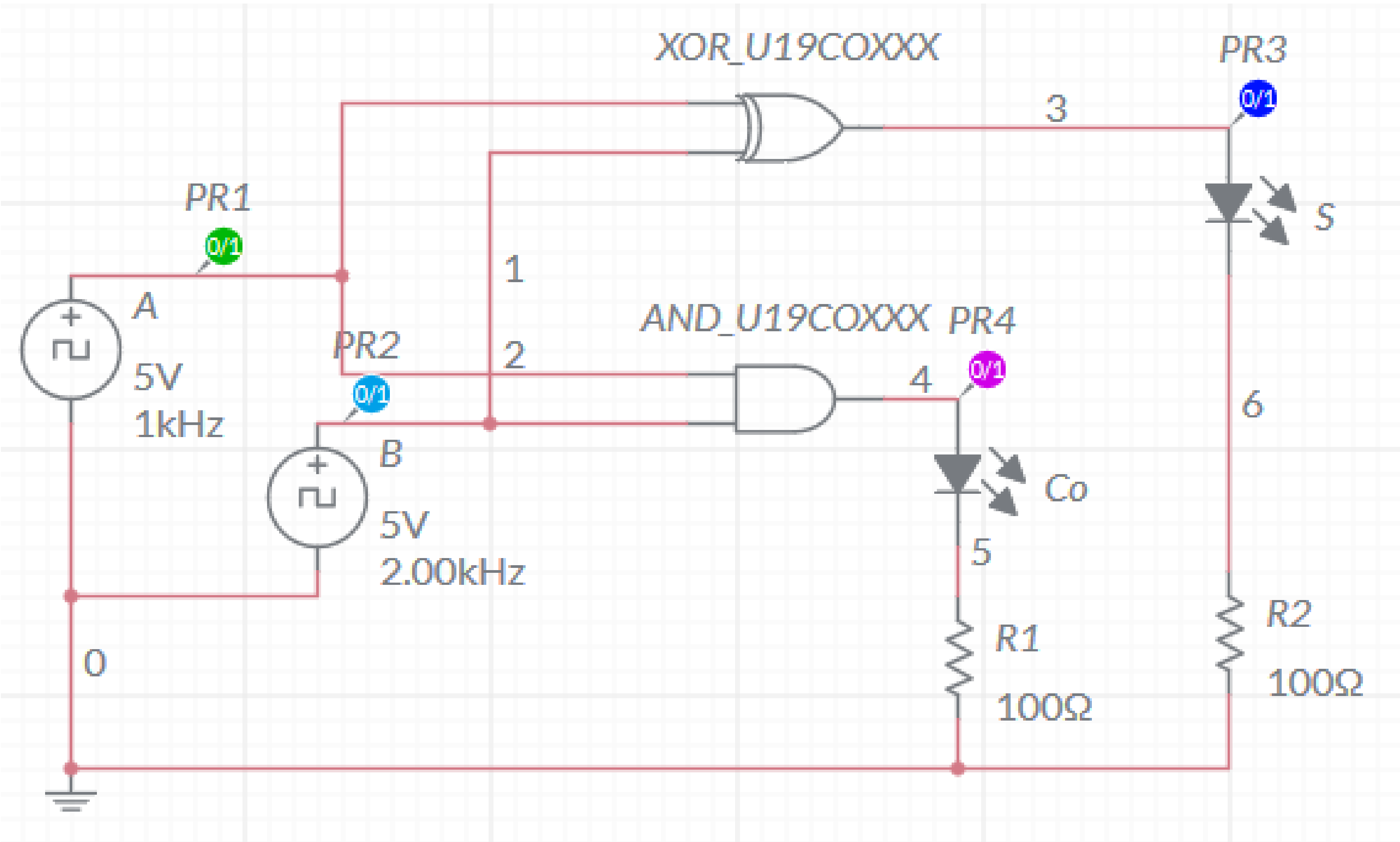

Fig.: Circuit Schematic of Half Adder

Doing it in Multisim Online

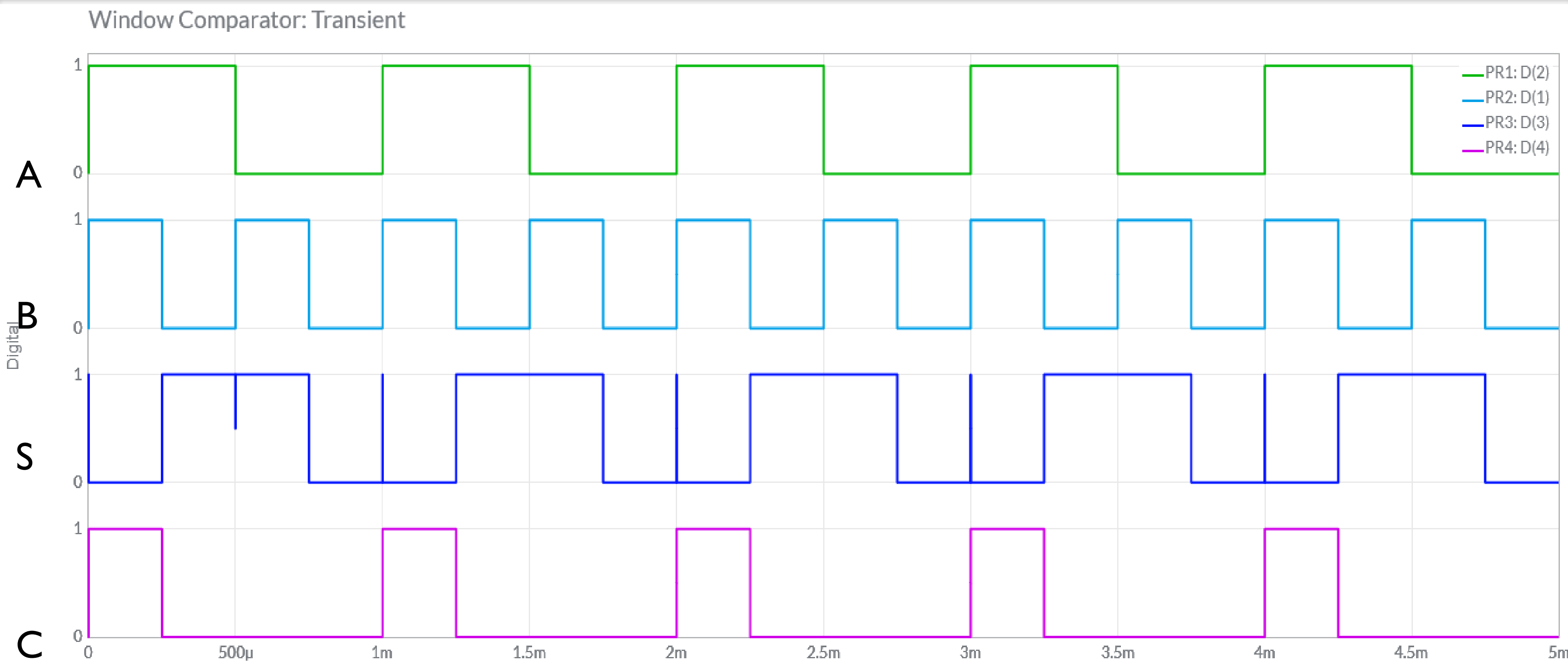

Fig.: Expected Waveform

Doing it in Multisim Online

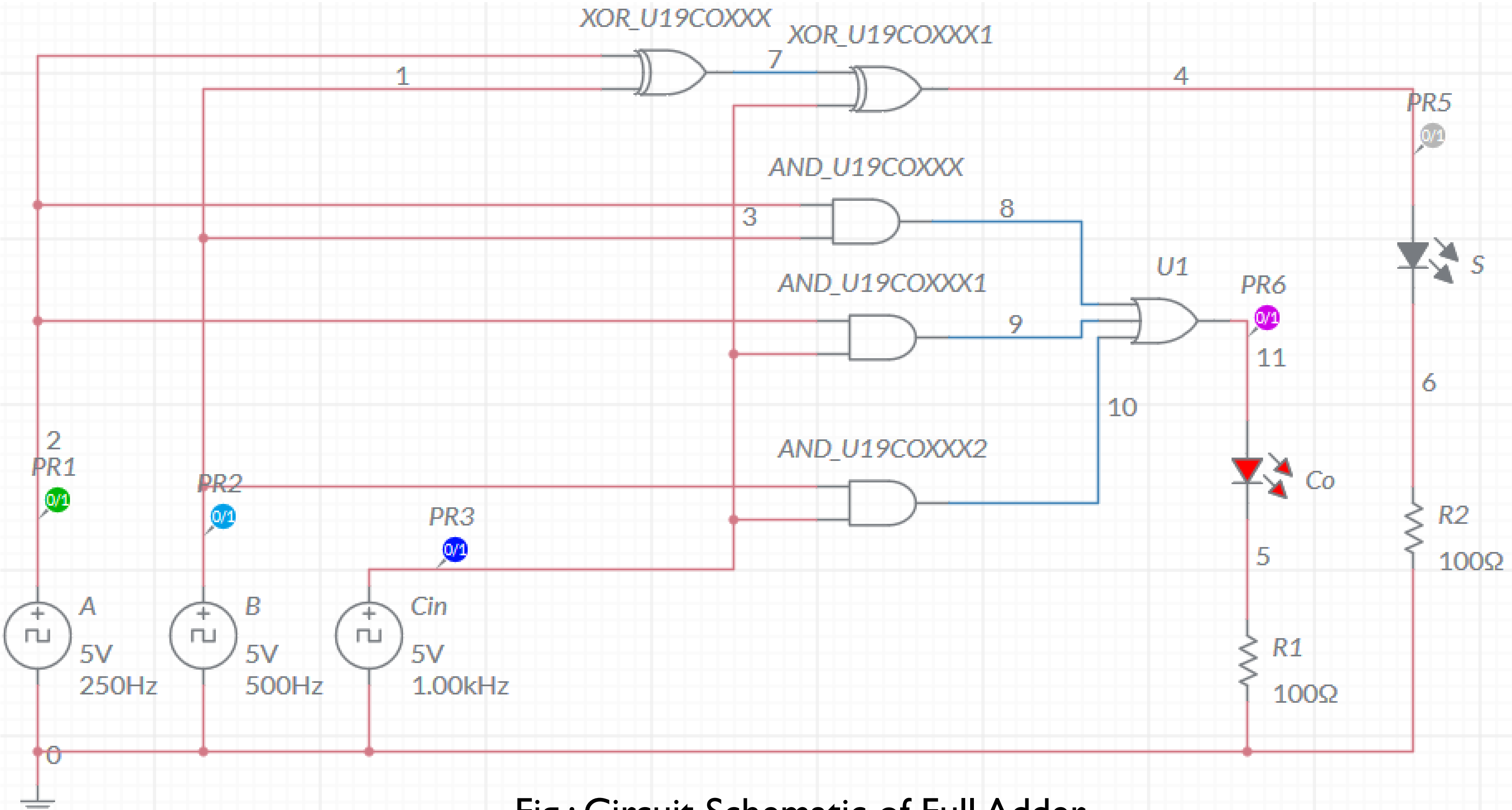

Fig.: Circuit Schematic of Full Adder

Doing it in Multisim Online

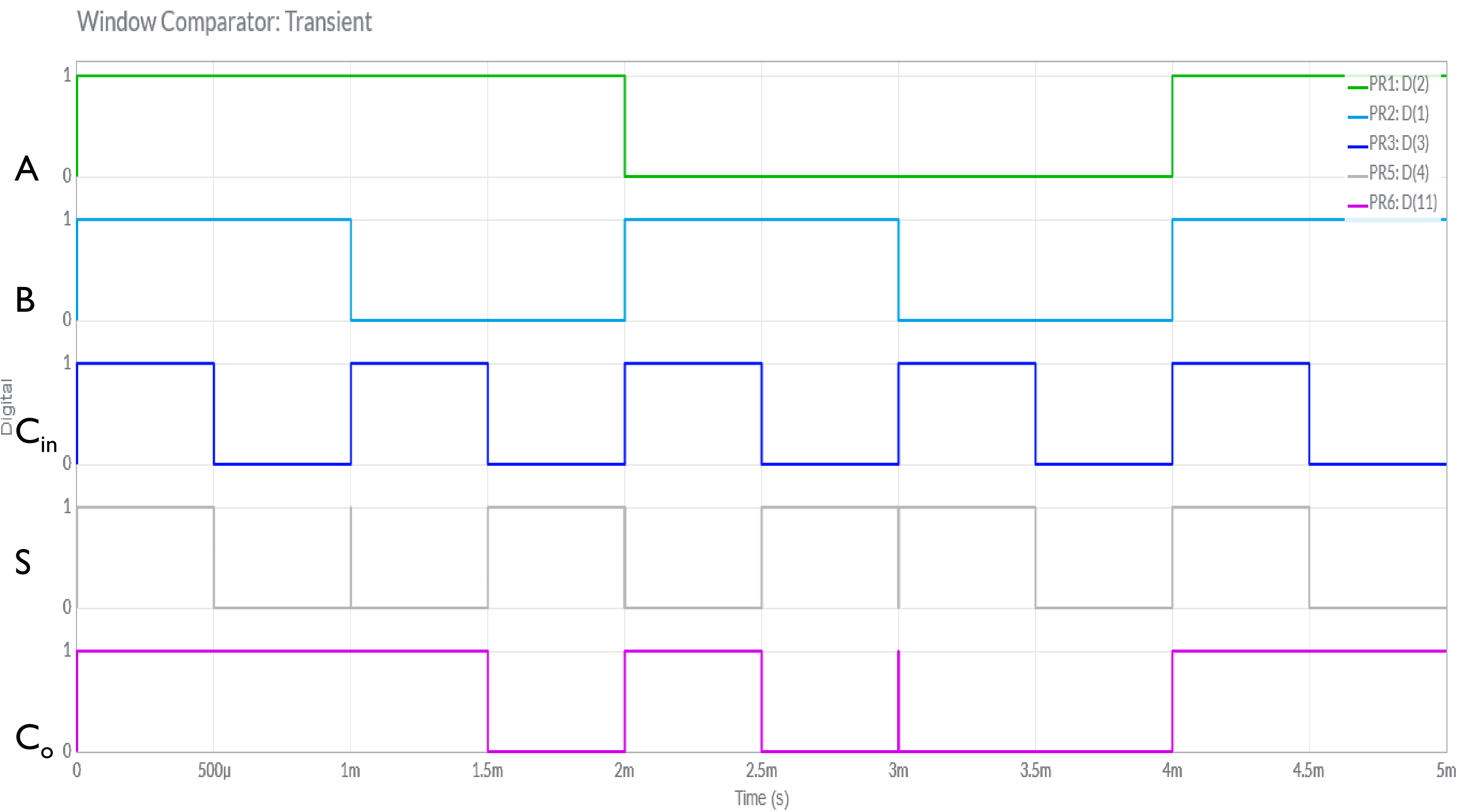

Fig.: Expected Waveform