4

RELATIONAL ALGEBRA AND CALCULUS

Exercise 4.1 Explain the statement that relational algebra operators can be *composed*. Why is the ability to compose operators important?

Answer 4.1 Every operator in relational algebra accepts one or more relation instances as arguments and the result is always an relation instance. So the argument of one operator could be the result of another operator. This is important because, this makes it easy to write complex queries by simply composing the relational algebra operators.

Exercise 4.2 Given two relations $R1$ and $R2$, where $R1$ contains N1 tuples, $R2$ contains N2 tuples, and N2 > N1 > 0, give the minimum and maximum possible sizes (in tuples) for the resulting relation produced by each of the following relational algebra expressions. In each case, state any assumptions about the schemas for $R1$ and $R2$ needed to make the expression meaningful:

(1) $R1 \cup R2$, (2) $R1 \cap R2$, (3) $R1 - R2$, (4) $R1 \times R2$, (5) $\sigma_{a=5}(R1)$, (6) $\pi_a(R1)$, and (7) $R1/R2$

Answer 4.2 See Figure 4.1.

Exercise 4.3 Consider the following schema:

Suppliers(*sid:* `integer`, *sname:* `string`, *address:* `string`)
Parts(*pid:* `integer`, *pname:* `string`, *color:* `string`)
Catalog(*sid:* `integer`, *pid:* `integer`, *cost:* `real`)

The key fields are underlined, and the domain of each field is listed after the field name. Therefore *sid* is the key for Suppliers, *pid* is the key for Parts, and *sid* and *pid* together form the key for Catalog. The Catalog relation lists the prices charged for parts by Suppliers. Write the following queries in relational algebra, tuple relational calculus, and domain relational calculus:

Expression	Assumption	Min	Max
$R1 \cup R2$	$R1$ and $R2$ are union-compatible	N2	N1 + N2
$R1 \cap R2$	$R1$ and $R2$ are union-compatible	0	N1
$R1 - R2$	$R1$ and $R2$ are union-compatible	0	N1
$R1 \times R2$		N1 * N2	N1 * N2
$\sigma_{a=5}(R1)$	$R1$ has an attribute named a	0	N1
$\pi_a(R1)$	$R1$ has attribute a, N1>0	1	N1
$R1/R2$	The set of attributes of $R2$ is a subset of the set of attributes of $R1$	0	0
$R2/R1$	The set of attributes of $R1$ is a subset of the set of attributes of $R2$	0	$\lfloor$N2 / N1$\rfloor$

Figure 4.1 Answer to Exercise 4.2.

1. Find the *name*s of suppliers who supply some red part.
2. Find the *sid*s of suppliers who supply some red or green part.
3. Find the *sid*s of suppliers who supply some red part or are at 221 Packer Street.
4. Find the *sid*s of suppliers who supply some red part and some green part.
5. Find the *sid*s of suppliers who supply every part.
6. Find the *sid*s of suppliers who supply every red part.
7. Find the *sid*s of suppliers who supply every red or green part.
8. Find the *sid*s of suppliers who supply every red part or supply every green part.
9. Find pairs of *sid*s such that the supplier with the first *sid* charges more for some part than the supplier with the second *sid*.
10. Find the *pid*s of parts supplied by at least two different suppliers.
11. Find the *pid*s of the most expensive parts supplied by suppliers named Yosemite Sham.
12. Find the *pid*s of parts supplied by every supplier at less than $200. (If any supplier either does not supply the part or charges more than $200 for it, the part is not selected.)

Answer 4.3 In the answers below RA refers to Relational Algebra, TRC refers to Tuple Relational Calculus and DRC refers to Domain Relational Calculus.

1. ■ RA

$$\pi_{sname}(\pi_{sid}((\pi_{pid}\sigma_{color='red'}Parts) \bowtie Catalog) \bowtie Suppliers)$$

■ TRC

$$\{T \mid \exists T1 \in Suppliers(\exists X \in Parts(X.color ='red' \wedge \exists Y \in Catalog (Y.pid = X.pid \wedge Y.sid = T1.sid)) \wedge T.sname = T1.sname)\}$$

■ DRC

$$\{\langle Y \rangle \mid \langle X, Y, Z \rangle \in Suppliers \wedge \exists P, Q, R(\langle P, Q, R \rangle \in Parts \wedge R ='red' \wedge \exists I, J, K(\langle I, J, K \rangle \in Catalog \wedge J = P \wedge I = X))\}$$

■ SQL

```
SELECT S.sname
FROM   Suppliers S, Parts P, Catalog C
WHERE  P.color='red' AND C.pid=P.pid AND C.sid=S.sid
```

2. ■ RA

$$\pi_{sid}(\pi_{pid}(\sigma_{color='red' \vee color='green'}Parts) \bowtie catalog)$$

■ TRC

$$\{T \mid \exists T1 \in Catalog(\exists X \in Parts((X.color = `red' \vee X.color = `green') \wedge X.pid = T1.pid) \wedge T.sid = T1.sid)\}$$

■ DRC

$$\{\langle X \rangle \mid \langle X, Y, Z \rangle \in Catalog \wedge \exists A, B, C(\langle A, B, C \rangle \in Parts \wedge (C ='red' \vee C ='green') \wedge A = Y)\}$$

■ SQL

```
SELECT C.sid
FROM   Catalog C, Parts P
WHERE  (P.color = 'red' OR P.color = 'green')
       AND P.pid = C.pid
```

3. ■ RA

$$\rho(R1, \pi_{sid}((\pi_{pid}\sigma_{color='red'}Parts) \bowtie Catalog))$$
$$\rho(R2, \pi_{sid}\sigma_{address='221PackerStreet'}Suppliers)$$
$$R1 \cup R2$$

- TRC

$$\{T \mid \exists T1 \in Catalog(\exists X \in Parts(X.color = `red' \wedge X.pid = T1.pid) \wedge T.sid = T1.sid) \vee \exists T2 \in Suppliers(T2.address =' 221 PackerStreet' \wedge T.sid = T2.sid)\}$$

- DRC

$$\{\langle X\rangle \mid \langle X, Y, Z\rangle \in Catalog \wedge \exists A, B, C(\langle A, B, C\rangle \in Parts \wedge C =' red' \wedge A = Y) \vee \exists P, Q(\langle X, P, Q\rangle \in Suppliers \wedge Q =' 221 PackerStreet')\}$$

- SQL

```
SELECT S.sid
FROM   Suppliers S
WHERE  S.address = '221 Packer street'
       OR S.sid IN ( SELECT C.sid
                     FROM   Parts P, Catalog C
                     WHERE  P.color='red' AND P.pid = C.pid )
```

4. - RA

$$\rho(R1, \pi_{sid}((\pi_{pid}\sigma_{color='red'}Parts) \bowtie Catalog))$$
$$\rho(R2, \pi_{sid}((\pi_{pid}\sigma_{color='green'}Parts) \bowtie Catalog))$$
$$R1 \cap R2$$

- TRC

$$\{T \mid \exists T1 \in Catalog(\exists X \in Parts(X.color = `red' \wedge X.pid = T1.pid) \wedge \exists T2 \in Catalog(\exists Y \in Parts(Y.color =' green' \wedge Y.pid = T2.pid) \wedge T2.sid = T1.sid) \wedge T.sid = T1.sid)\}$$

- DRC

$$\{\langle X\rangle \mid \langle X, Y, Z\rangle \in Catalog \wedge \exists A, B, C(\langle A, B, C\rangle \in Parts \wedge C =' red' \wedge A = Y) \wedge \exists P, Q, R(\langle P, Q, R\rangle \in Catalog \wedge \exists E, F, G(\langle E, F, G\rangle \in Parts \wedge G =' green' \wedge E = Q) \wedge P = X)\}$$

- SQL

```
SELECT C.sid
FROM   Parts P, Catalog C
WHERE  P.color = 'red' AND P.pid = C.pid
       AND EXISTS ( SELECT P2.pid
                    FROM   Parts P2, Catalog C2
                    WHERE  P2.color = 'green' AND C2.sid = C.sid
                           AND P2.pid = C2.pid )
```

5. ■ RA

$$(\pi_{sid,pid}Catalog)/(\pi_{pid}Parts)$$

■ TRC

$$\{T \mid \exists T1 \in Catalog(\forall X \in Parts(\exists T2 \in Catalog \\ (T2.pid = X.pid \wedge T2.sid = T1.sid)) \wedge T.sid = T1.sid)\}$$

■ DRC

$$\{\langle X \rangle \mid \langle X, Y, Z \rangle \in Catalog \wedge \forall \langle A, B, C \rangle \in Parts \\ (\exists \langle P, Q, R \rangle \in Catalog(Q = A \wedge P = X))\}$$

■ SQL

```
SELECT C.sid
FROM   Catalog C
WHERE  NOT EXISTS (SELECT P.pid
                   FROM   Parts P
                   WHERE  NOT EXISTS (SELECT C1.sid
                                      FROM   Catalog C1
                                      WHERE  C1.sid = C.sid
                                             AND C1.pid = P.pid))
```

6. ■ RA

$$(\pi_{sid,pid}Catalog)/(\pi_{pid}\sigma_{color='red'}Parts)$$

■ TRC

$$\{T \mid \exists T1 \in Catalog(\forall X \in Parts(X.color \neq 'red' \\ \vee \exists T2 \in Catalog(T2.pid = X.pid \wedge T2.sid = T1.sid)) \\ \wedge T.sid = T1.sid)\}$$

■ DRC

$$\{\langle X \rangle \mid \langle X, Y, Z \rangle \in Catalog \wedge \forall \langle A, B, C \rangle \in Parts \\ (C \neq 'red' \vee \exists \langle P, Q, R \rangle \in Catalog(Q = A \wedge P = X))\}$$

- SQL

```
SELECT C.sid
FROM   Catalog C
WHERE  NOT EXISTS (SELECT P.pid
                   FROM   Parts P
                   WHERE  P.color = 'red'
                   AND (NOT EXISTS (SELECT C1.sid
                                    FROM   Catalog C1
                                    WHERE  C1.sid = C.sid AND
                                           C1.pid = P.pid)))
```

7. - RA

$$(\pi_{sid,pid}Catalog)/(\pi_{pid}\sigma_{color='red' \vee color='green'}Parts)$$

- TRC

$$\{T \mid \exists T1 \in Catalog(\forall X \in Parts((X.color \neq \text{`}red' \\ \wedge X.color \neq \text{`}green') \vee \exists T2 \in Catalog \\ (T2.pid = X.pid \wedge T2.sid = T1.sid)) \wedge T.sid = T1.sid)\}$$

- DRC

$$\{\langle X \rangle \mid \langle X, Y, Z \rangle \in Catalog \wedge \forall \langle A, B, C \rangle \in Parts \\ ((C \neq \text{`}red' \wedge C \neq \text{`}green') \vee \exists \langle P, Q, R \rangle \in Catalog \\ (Q = A \wedge P = X))\}$$

- SQL

```
SELECT C.sid
FROM   Catalog C
WHERE  NOT EXISTS (SELECT P.pid
                   FROM   Parts P
                   WHERE  (P.color = 'red' OR P.color = 'green')
                   AND (NOT EXISTS (SELECT C1.sid
                                    FROM   Catalog C1
                                    WHERE  C1.sid = C.sid AND
                                           C1.pid = P.pid)))
```

8. - RA

$$\rho(R1, ((\pi_{sid,pid}Catalog)/(\pi_{pid}\sigma_{color='red'}Parts))) \\ \rho(R2, ((\pi_{sid,pid}Catalog)/(\pi_{pid}\sigma_{color='green'}Parts))) \\ R1 \cup R2$$

- TRC

$$\{T \mid \exists T1 \in Catalog((\forall X \in Parts$$
$$(X.color \neq \text{`}red' \vee \exists Y \in Catalog(Y.pid = X.pid \wedge Y.sid = T1.sid))$$
$$\vee \forall Z \in Parts(Z.color \neq \text{`}green' \vee \exists P \in Catalog$$
$$(P.pid = Z.pid \wedge P.sid = T1.sid))) \wedge T.sid = T1.sid)\}$$

- DRC

$$\{\langle X \rangle \mid \langle X, Y, Z \rangle \in Catalog \wedge (\forall \langle A, B, C \rangle \in Parts$$
$$(C \neq \text{`}red' \vee \exists \langle P, Q, R \rangle \in Catalog(Q = A \wedge P = X))$$
$$\vee \forall \langle U, V, W \rangle \in Parts(W \neq \text{`}green' \vee \langle M, N, L \rangle \in Catalog$$
$$(N = U \wedge M = X)))\}$$

- SQL

```
SELECT C.sid
FROM   Catalog C
WHERE  (NOT EXISTS (SELECT P.pid
                    FROM   Parts P
                    WHERE  P.color = 'red' AND
                    (NOT EXISTS (SELECT C1.sid
                                 FROM   Catalog C1
                                 WHERE  C1.sid = C.sid AND
                                        C1.pid = P.pid))))
       OR ( NOT EXISTS (SELECT P1.pid
                        FROM   Parts P1
                        WHERE  P1.color = 'green' AND
                        (NOT EXISTS (SELECT C2.sid
                                     FROM   Catalog C2
                                     WHERE  C2.sid = C.sid AND
                                            C2.pid = P1.pid))))
```

9. - RA

$$\rho(R1, Catalog)$$
$$\rho(R2, Catalog)$$
$$\pi_{R1.sid, R2.sid}(\sigma_{R1.pid=R2.pid \wedge R1.sid \neq R2.sid \wedge R1.cost > R2.cost}(R1 \times R2))$$

- TRC

$$\{T \mid \exists T1 \in Catalog(\exists T2 \in Catalog$$
$$(T2.pid = T1.pid \wedge T2.sid \neq T1.sid$$
$$\wedge T2.cost < T1.cost \wedge T.sid2 = T2.sid)$$
$$\wedge T.sid1 = T1.sid)\}$$