U19CS012

Page No. 1

Date

TUTORIAL - 1

BHAGYA VINOD RANA (D-12)

Q1) Let $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{x \in \mathbb{Z} \mid x \text{ is divisible by } 6\}$, and $C = \{x \in \mathbb{R} \mid x^2 = 2 \text{ or } x^3 = 1\}$.

Mark the Following True or False

a.) $3 \in A$ True $A = \{1, 2, 3, 4, 5, 6\}$

b.) $6 \in A$ True

c.) $2 \notin A$ False $B = \{ -12, -6, 0, 6, 12, 18, ..., 24 ... \}$

d.) $2 \in B$ False

e.) $6 \in B$ True $C = \{-\sqrt{2}, +\sqrt{2}, 1\}$

f.) $24 \in B$ True

g.) $28 \notin B$ True [$\because 28 \neq 6(x)$ form] // {28 is not divisible by 6}

h.) $2 \in C$ False

i.) $1 \in C$ True

j.) $-\sqrt{2} \in C$ True

k.) $5 \in A \cup B$ True [$\because 5 \in A, \therefore 5 \in A \cup B$]

l.) $6 \in A \cap B$ True [$\because 6 \in A, 6 \in B$ & $A \cap B = \{6\}$]

m.) $1 \in A \cap C$ True [$A \cap C = \{1\}$]

n.) $\sqrt{2} \in B \cup C$ True [$\because \sqrt{2} \in B, \therefore \sqrt{2} \in B \cup C$]

Q2) Mark the following True or False

a.) $28 \in \mathbb{Z}$ True integers $\mathbb{Z} = \{..., -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, ...\}$

b.) $-5 \in \mathbb{N}$ False $\mathbb{N} = \{1, 2, 3, 4, 5, 6, ...\}$ Natural No.

c.) $\sqrt{2} \in \mathbb{Q} \cap \mathbb{R}$ True

[$\sqrt{2} \in \mathbb{Q} \cap \mathbb{R}$] $\mathbb{Q} \cap \mathbb{R} = \mathbb{Q}$ $\because \boxed{\mathbb{Q} \subseteq \mathbb{R}}$

d.) $\mathbb{Z} \cup \mathbb{Q} = \mathbb{R}$ False $\mathbb{R} = \mathbb{Z} + \mathbb{Q} +$ irrational

integer Rational

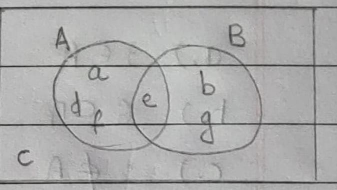
e.) $\mathbb{R} \cap \mathbb{C} = \mathbb{R}$ True

[$\because \mathbb{R} \subseteq \mathbb{C}$]

U19CS012

3.) Let $U = \{a, b, c, d, e, f, g\}$, $A = \{a, d, e, f\}$, and $B = \{b, e, g\}$ be sets, where U = acts as Universal set.

Determine the following:



a.) $(A \cup B)'$

$A \cup B = \{a, d, e, f, b, g\}$

$(A \cup B)' = U - \{A \cup B\}$

$= \{c\}$

b.) $A \cap B = \{a, d, e, f\} \cap \{b, e, g\} = \{e\}$

c.) $A - B = \{a, d, e, f\} - \{b, e, g\} = \{a, d, f\}$

d.) $B - A = \{b, e, g\} - \{a, d, e, f\} = \{b, g\}$

4.) Let U be the set of all students in a college. Let A be the set of students taking the discrete mathematics course and B be the set of students taking Calculus.

Describe the following.

a.) $A \cup B$: Students that took Discrete mathematics OR Calculus ~~Discrete ma~~

$(A + B - A \cap B)$

b.) $A \cap B$: Students taking both subjects (Discrete math & Calculus)

c.) $A - B$: Students taking Discrete mathematics but not calculus

d.) $B - A$: Student who took Calculus but not Discrete ~~Dt~~ Mathematics

(CU19CS012)

e) A' = Students who haven't taken Discrete mathematics

5.) Let $P = \{x \in N, 2 < x \le 8\}$, $Q = \{x \in Z \mid 0 \le x < 5\}$
$R = \{x \in N, 1 \le x \le 10\}$, Let $U = \{x \in Z, -2 \le x < 12\}$
be the universal set. Determine the following.

$P = \{3, 4, 5, 6, 7, 8\}$
$Q = \{0, 1, 2, 3, 4\}$
$R = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

a) $P \cup R = \{3,4,5,6,7,8\} \cup \{1,2,3,4,5,6,7,8,9,10\}$
$= \{1,2,3,4,5,6,7,8,9,10\} = R$

b) $Q \cap R = \{0,1,2,3,4\} \cap \{1,2,3,4,5,6,7,8,9,10\}$
$= \{1,2,3,4\}$

c) $P \Delta R = \{3,4,5,6,7,8\} \Delta \{1,2,3,4,5,6,7,8,9,10\}$
$= (P-R) \cup (R-P)$
Symmetric Difference of two sets $A \Delta B = (B-A) \cup (A-B)$
$= \{\} \cup \{1,2,9,10\}$
$= \{1,2,9,10\}$

d) $Q' = \{-2,-1,5,6,7,8,9,10,11\}$

6.) Let P, Q, R, U be same as in Exercise 5. Verify the following.

a.) $(P \cup Q)' = P' \cap Q'$
LHS = $(P \cup Q)'$
$= (\{3,4,5,6,7,8\} \cup \{0,1,2,3,4\})'$
$= (\{0,1,2,3,4,5,6,7,8\})'$
$= \{-2,-1,9,10,11\}$

U19CS012

RHS = $P' \cap Q' = \{3,4,5,6,7,8\}' \cap \{0,1,2,3,4\}'$

$= \{-2,-1,0,1,2,9,10,11\} \cap \{-2,-1,5,6,7,8,9,10,11\}$

$= \{-2,-1,9,10,11\}$

[LHS = RHS], Hence verified.

(b) $P \cap (P \cup R) = P$

LHS = $P \cap (P \cup R)$

$= (P \cap P) \cup (P \cap R)$

$= P \cup \underbrace{(P \cap R)}$

$= P \cup P$ $(\because P \subseteq R)$

$= P = \{3,4,5,6,7,8\}$

[LHS = RHS]

(c) $P \cup (Q \cap R) = (P \cup Q) \cap (P \cup R)$

LHS = $P \cup (Q \cap R) = \{3,4,5,6,7,8\} \cup (\{0,1,2,3,4\} \cap \{1,2,3,4,\ldots 10\})$

$= \{3,4,5,6,7,8\} \cup (\{1,2,3,4\})$

$= \{1,2,3,4,5,6,7,8\}$

RHS = $(P \cup Q) \cap (P \cup R)$

$= (\{3,4,5,6,7,8\} \cup \{0,1,2,3,4\}) \cap (\{3,4,5,6,7,8\} \cup \{1,2,3,\ldots 10\})$

$= (\{0,1,2,\ldots 8\}) \cap (\{1,2,3,\ldots 10\})$

$= \{1,2,3,\ldots 8\}$

[LHS = RHS, Hence verified]

U19CS012

7.) Let $A = \{ x \in R \mid 1 < x \leq 5 \}$ and $B = \{ x \in R \mid 3 \leq x \leq 8 \}$

Find $A \cup B$, $A \cap B$, $A - B$, $B - A$

a) $A \cup B$

$x \in A \cup B \Rightarrow x \in A$ or $x \in B$

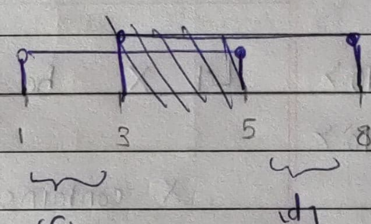
$\Rightarrow 1 < x \leq 5$ or $3 \leq x \leq 8$

$1 < x \leq 8$

$\therefore A \cup B = \{ x \in R \mid 1 < x \leq 8 \}$

b) $A \cap B$

$\{ x \in R \mid 3 \leq x \leq 5 \}$



c) $A - B$

$= \{ x \in R \mid 1 < x < 3 \}$

d) $B - A$

$= \{ x \in R \mid 5 < x \leq 8 \}$

8.) Determine whether the following set of pair are equal. Justify your ans.

$$A = \{ n \in Z \mid n = \frac{1}{n} \}$$

$$B = \{ x \in R \mid x^2 = 1 \}$$

For A, $n = \frac{1}{n} \Rightarrow n^2 = 1$

$n = +1$ or -1

$A = \{-1, 1\}$ $[n \in Z]$

For B, $x^2 = 1$ $x = +1$ or -1, $x \in R$

$B = \{-1, 1\}$

$A \subseteq B$ and $B \subseteq A$

$\therefore A = B$

U19CS012

9.> Does every set has a subset? Give an example of a set that has only one proper subset.

9.>

→ As every set is a subset of itself, so every set has a subset

→ Ex: $A = \{\}$ or ϕ null set

⇒ . Subset of $A = \{\}$ or (ϕ)

10.> Let X be a set with 4 elements. Find $|P(x)|$

10.>

X contains 4 elements, $n[x] = 4 = m$

$P(X)$ = Power set

$n[P(x)] = 2^m = 2^4$ [No. of subset of $X = 2^n$]

$= 16$

Ans: $[|P(x)| = 16]$

11.> Find $P(P(P(\phi)))$

$P(X)$ = the set of all subsets of X

$P(\phi) = \{\phi\}$

$P(P(\phi)) = \{\phi, \{\phi\}\}$

$P(P(P(\phi))) = \{\phi, \{\phi\}, \{\{\phi\}\}, \{\phi, \{\phi\}\}\}$

12.> Let $I_n = \{1, 2, 3, \ldots, n\}$, the set of first n natural numbers.

a) Describe set of $I_{10} - I_5$ = $I_{10} - I_5 = \{6, 7, 8, 9, 10\}$

b) Describe $I_n - I_m$ ⇒

3 cases

U19CS012

i) $n > m$

$I_n - I_m = \{m+1, m+2, \ldots, n\}$

ii) $n = m$

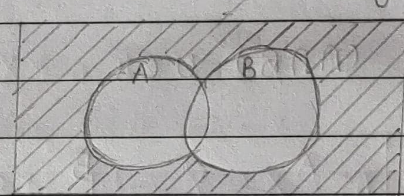
$I_n - I_m = \phi$

iii) $n < m$

$I_n - I_m = \phi$

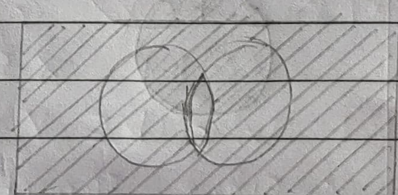
13.) Let A and B be subset of set U. Draw Venn diagram of followwing.

(a) $(A \cup B)'$



$(A \cup B)' = U - (A \cup B)$

(b) $(A \cap B)'$

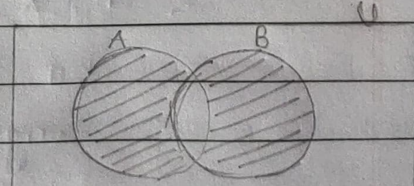


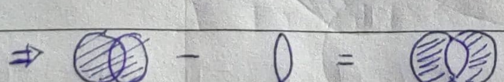
$(A \cap B)' = U - (A \cap B)$

(c) $A \Delta B$

symmetric diff. $(A - B) \cup (B - A)$

(d) $(A \cup B) - (A \cap B)$

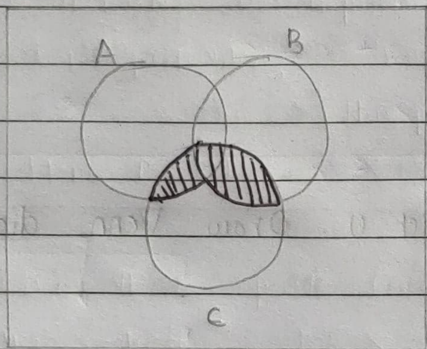




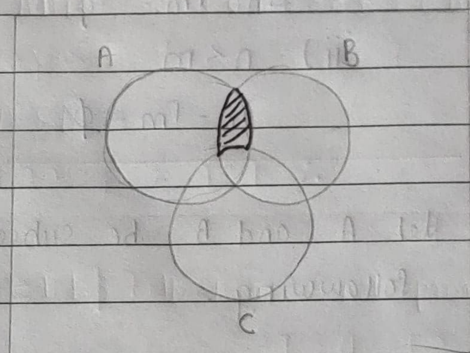
U19CS012

14> Let A, B & C be subset of U. Draw Venn Diagram of following:

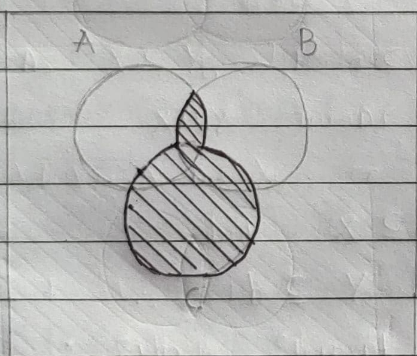
a) $(A \cup B) \cap C$



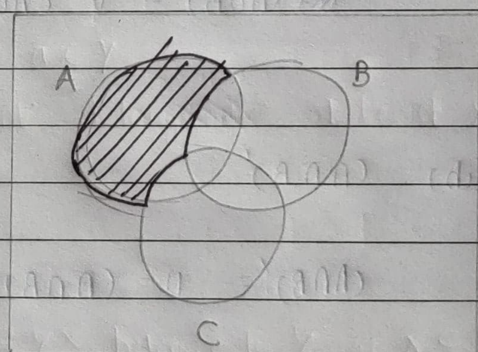
c) $(A \cap B) - C$



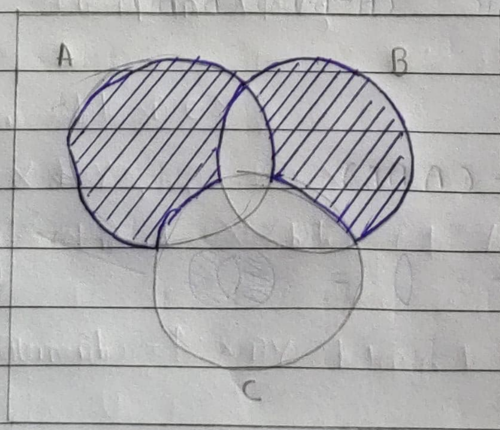
b) $(A \cap B) \cup C$



d) $(A - B) - C$



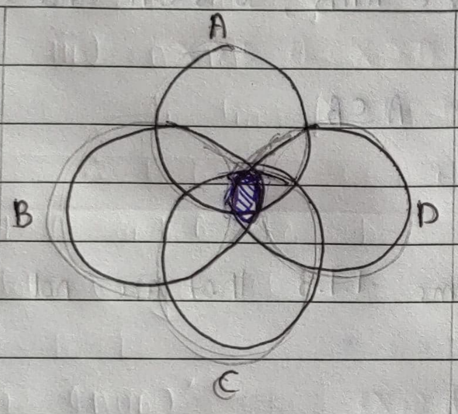
(e) $(A - (B \cup C)) \cup (B - (A \cup C))$

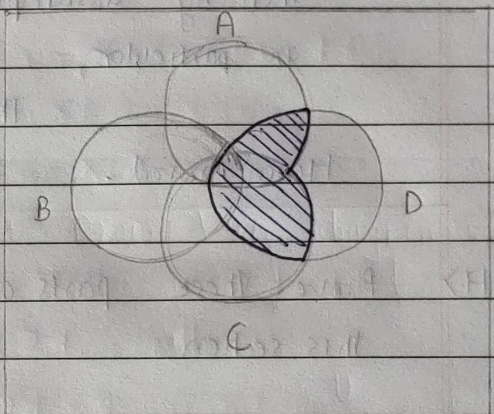


U19CS012

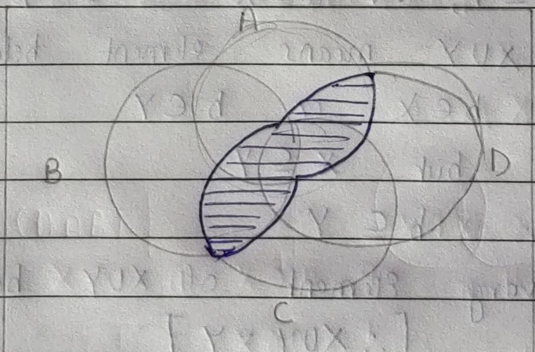
15.> Let A, B, C and D be subset of set U. Draw the venn diagram of followwing:

a) $A \cap B \cap C \cap D$ b) $(A \cup B \cup C) \cap D$





(C) $(A \cup B) \cap (C \cap D)$



16.> Let A and B be sets. Prove that $A \subseteq B$ if and only if $A \cap B = A$.

16.> Let Assume $A \subseteq B$

If $x \in A \cap B$, then $x \in A$ and $x \in B$, by definition, so in particular $x \in A$.

This proves $A \cap B \subseteq A$ —(i)

Now, if $x \in A$, then by assumption $x \in B$, too.

So, $x \in A \cap B$.

U19CS012

This proves $A \subseteq A \cap B$ ---(ii)
From (i) & (ii), toghether it implies $A = A \cap B$.

⇒ Now, we assume that $A \cap B = A$, if $x \in A$,
then by assumption, $x \in A \cap B$, So $x \in A$ and $x \in B$.
In particular, $x \in B$,
This proves $A \subseteq B$.
Hence Proved.

Q17.) Prove those parts of theorams 1.1.3 that are not proved in this section.

i) If $X \subseteq Y$, then $X \cup Y = Y$ and $X \cap Y = X$.

→ As $X \subseteq Y$, let $a \in X$ ∴ $a \in Y$
$X \cup Y$ means element belongs to at least X or Y
∴ $b \in X$ or $b \in Y$
but $X \subseteq Y$
∴ $b \subset Y$
∴ Every Element of $X \cup Y$ belongs to Y as $X \subseteq Y$.
[∴ $X \cup Y = Y$]

Let B be any element in $X \cap Y$;
∴ $B \in X$, and $B \in Y$,
but $X \subseteq Y$,
Every element X belongs to Y.
∴ Every Element of $X \cap Y$ belongs to X.
∴ $X \cap Y = X$.

U19CS012

(ii) Laws of identity: $A \cup \phi = A$ and $A \cap \phi = \phi$

a) $A \cup \phi = A$

$\rightarrow$ Let $x \in A \cup \phi \Rightarrow x \in A \cup x \in \phi$

$\Rightarrow x \in A$

then we have

$x \in A \cup \phi \Rightarrow x \in A$

$A \cup \phi \subset A$ —(i)

$\rightarrow$ Let $x \in A \Rightarrow x \in A \cup x \in \phi$

$\Rightarrow x \in (A \cup \phi)$

then we have,

$x \in A \Rightarrow x \in (A \cup \phi)$

$A \subset (A \cup \phi)$ —(ii)

From (i) and (ii),

$\boxed{A \cup \phi = A}$

b) $A \cap \phi = \phi$

Let $x \in A \cap \phi$

$\Rightarrow x \in A \cap x \in \phi$

but there is no element in ϕ

$\therefore A \cap \phi$ must be empty.

$\therefore$ By uniqueness of Empty set, then we have

$[A \cap \phi = \phi]$

U19CS012

(iii) Laws of idempotency : $X \cup X = X$ and $X \cap X = X$

(a) $X \cup X = X$

let $a \in X \cup X$ then, $a \in X \cup a \in X$
$\Rightarrow a \in X$
$X \cup X \subseteq X$ — (i)

let $b \in X$
then $b \in X$
$\Rightarrow b \in X \cup b \in X$
$\therefore X \subseteq X \cup X$ — (ii)

From (i) & (ii), $\boxed{X \cup X = X}$

(b) $X \cap X = X$

let $a \in X \cap X$
then $\Rightarrow a \in X \cap a \in X$
$\Rightarrow a \in X$
$\therefore X \cap X \subseteq X$ — (i)

let $b \in X$
then $b \in X$
$\Rightarrow b \in X \cap b \in X$
$\therefore X \subseteq (X \cap X)$

$\therefore$ From (i) & (ii),

$\boxed{X \cap X = X}$

U19CS012

(iv) Laws of commutativity

a) $X \cup Y = Y \cup X$

$X \cup Y = \{a \mid a \in X \cup a \in Y\}$
$= \{a \mid a \in Y \cup a \in X\}$
$= Y \cup X$

b) $X \cap Y = Y \cap X$

$X \cap Y = \{a \mid a \in X \cap a \in Y\}$
$= \{a \mid a \in Y \cap a \in X\}$
$= Y \cap X$

Hence Proved

(v) Laws of Associativity

a) $(X \cup Y) \cup Z = X \cup (Y \cup Z)$

Let $a \in (X \cup Y) \cup Z$
$\Rightarrow a \in (X \cup Y) \cup Z$
$a \in (X \cup Y) \cup a \in Z$
$\Rightarrow (a \in X \cup a \in Y) \cup a \in Z$
$\Rightarrow a \in X \cup a \in Y \cup a \in Z$
$\Rightarrow a \in X \cup (a \in Y \cup a \in Z)$
$\Rightarrow a \in X \cup (a \in (Y \cup Z))$
$\Rightarrow a \in X \cup (Y \cup Z)$

$\therefore (X \cup Y) \cup Z \subseteq X \cup (Y \cup Z)$ —(i)

Similarly, it can be shown, $X \cup (Y \cup Z) \subseteq (X \cup Y) \cup Z$ —(ii)

$\therefore$ From (i) & (ii),

$[(X \cup Y) \cup Z = X \cup (Y \cup Z)]$

U19CS012

(b) $(X \cap Y) \cap Z = X \cap (Y \cap Z)$

Let $a \in (X \cap Y) \cap Z$

$\Rightarrow a \in (X \cap Y) \cap a \in Z$

$\Rightarrow (a \in X \cap a \in Y) \cap a \in Z$

$\Rightarrow a \in X \cap a \in Y \cap a \in Z$

$\Rightarrow a \in X \cap (a \in Y \cap Z)$

$\Rightarrow a \in (X \cap (Y \cup Z))$

$\therefore (X \cap Y) \cap Z \subseteq X \cap (Y \cap Z)$ — (i)

similarily, we can prove that

$X \cap (Y \cap Z) \subseteq (X \cap Y) \cap Z$ — (ii)

$\therefore$ From (i) & (ii),

$[(X \cap Y) \cap Z = X \cap (Y \cap Z)]$

vi) Laws of Distributivity

(a) $X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z)$

+ Let $a \in X \cup (Y \cap Z)$

$\Rightarrow a \in X \cup a \in (Y \cap Z)$

$\Rightarrow a \in X \cup (a \in Y \cap a \in Z)$

$\Rightarrow (a \in X \cup a \in Y) \cap (a \in X \cup a \in Z)$

$\Rightarrow (a \in X \cup Y) \cap (a \in X \cup Z)$

$\Rightarrow a \in (X \cup Y) \cap (X \cup Z)$

$\therefore X \cup (Y \cup Z) \subseteq (X \cup Y) \cap (X \cup Z)$ — i)

+ Let $a \in (X \cup Y) \cap (X \cup Z)$

$\Rightarrow (a \in X \cup a \in Y) \cap (a \in X \cup a \in Z)$

$\Rightarrow a \in X \cup (a \in Y \cap a \in Z)$

U19CS012

$a \in X \cup a \in (Y \cap Z)$

$\Rightarrow a \in X \cup (Y \cap Z)$

$\therefore (X \cup Y) \cap (X \cup Z) \subseteq X \cup (Y \cap Z)$ —(ii)

From (i) & (ii),

$$\boxed{X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z)}$$

b) $X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$

Let $a \in X \cap (Y \cup Z)$

$\Rightarrow a \in X \cap (a \in Y \cup a \in Z)$

$\Rightarrow (a \in X \cap a \in Y) \cup (a \in X \cap a \in Z)$

$\Rightarrow a \in (X \cap Y) \cup a \in (X \cap Z)$

$\Rightarrow a \in (X \cap Y) \cup (X \cap Z)$

$\therefore X \cap (Y \cup Z) \subseteq (X \cap Y) \cup (X \cap Z)$ —(i)

Let $a \in (X \cap Y) \cup (X \cap Z)$

$\Rightarrow (a \in X \cap a \in Y) \cup (a \in X \cap a \in Z)$

$\Rightarrow a \in X \cap (a \in Y \cup a \in Z)$

$\Rightarrow a \in X \cap (a \in (Y \cup Z))$

$\Rightarrow a \in X \cap (Y \cup Z)$

$\therefore (X \cap Y) \cup (X \cap Z) \subseteq X \cap (Y \cup Z)$ —(ii)

From (i) & (ii),

$$\boxed{X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)}$$

U19CS012

iii) Laws of absorptivity

a) $X \cap (X \cup Y) = X$ b) $X \cup (X \cap Y) = X$

From the Law of distributivity, $X \cap (X \cup Y) = (X \cap X) \cup (X \cap Y)$

We also know that $X \cap X = X$

$\therefore X \cap (X \cup Y) = X \cup (X \cap Y)$ — (i)

Now, we need to prove that any one of them is equal to X.

$\therefore$ Let $a \in X \cap (X \cup Y)$

$\therefore a \in X \cap a \in (X \cup Y)$

$\Rightarrow a \in X \cap (a \in X \cup a \in Y)$

$\Rightarrow a \in X$

$\therefore X \cap (X \cup Y) \subseteq X$ — (ii)

Let $a \in X$

$\therefore a \in X$

$\Rightarrow a \in X \cap a \in X$

$\Rightarrow a \in X \cap (a \in X \cup a \in Y)$

$\Rightarrow a \in X \cap (X \cup Y)$

$\therefore X \subseteq X \cap (X \cup Y)$ — (iii)

From (ii) & (iii),

$X \cap (X \cup Y) = X$ — (iv)

and from (i) & (iv),

$\boxed{X \cup (X \cap Y) = X}$

U19CS012

Q18.) Suppose P and Q are two sets. Let R be a set that contains elements belonging to P or Q, but not both. Let T be a set that contains elements belonging to Q or the complement of P but not both. Show that R is complement of T.

18.) Given: Set P and Q

Acc. to Problem, $\left[T = P' \Delta Q = (P' - Q) \cup (Q - P') \right]$

Set R such that it contains element from P or Q but not both

$\therefore R = P \Delta Q$

Now, we know, $R = P \Delta Q$

$= (P - Q) \cup (Q - P)$

$R = (P \cap Q') \cup (Q \cap P')$

$\bar{R} = \overline{(P \cap Q') \cup (Q \cap P')}$ [complement on both side]

$\bar{R} = \overline{P \cap \bar{Q}} \cap \overline{(Q \cap P')}$ [Using De-morgan's Law]

$\bar{R} = (\bar{P} \cup \bar{\bar{Q}}) \cap (\bar{Q} \cup \bar{\bar{P}})$ $[\bar{P} = P']$

$\bar{R} = (\bar{P} \cup Q) \cap (\bar{Q} \cup P)$ $[\bar{\bar{P}} = P \quad \bar{\bar{Q}} = Q]$

$\therefore \bar{R} = ((\bar{P} \cup Q) \cap \bar{Q}) \cup ((\bar{P} \cup Q) \cap P)$

$\left[A \cup (B \cup C) = (A \cap B) \cup (A \cap C) \right]$

$\bar{R} = ((\bar{P} \cap \bar{Q}) \cup (Q \cap \bar{Q})) \cup ((\bar{P} \cap P) \cup (Q \cap P))$

$[\because A \cap \bar{A} = \phi]$

$\therefore \bar{R} = (\bar{P} \cap \bar{Q}) \cup (Q \cap P)$

$\therefore \bar{R} = (\bar{P} - Q) \cup (Q - \bar{P})$ $\left[\because P - Q = P \cap \bar{Q} \quad \therefore P \cap \bar{Q} = P - Q \right]$

$\bar{R} = \bar{P} \Delta Q$ $[\because T = \bar{P} \Delta Q = (\bar{P} - Q) \cup (Q - \bar{P})]$

$\bar{R} = T \xrightarrow{\text{Compl.}} \bar{R} = \bar{T} \Rightarrow \boxed{\bar{T} = R}$ [Hence Proved !]

U19CS012

(Q19) Let A and B be sets. Prove that $A-(A-B) = A \cap B$.

Let $x \in A \cap B$, the $x \in A$ and $x \in B$

$\therefore$ $x \notin A-B$ [because $x \in A-B$ would imply $x \notin B$]

So, $x \in A-(A-B)$

This shows,

$A \cap B \subseteq A-(A-B)$ — (i)

Now, let $x \in A-(A-B)$, then $x \in A$ and $x \notin A-B$,

$\therefore$ $x \notin A$ or $x \in B$ (or negation of $x \in A$ and $x \notin B$)

$\because$ we know $x \in A$, this implies $x \in B$, so $x \in A \cap B$.

This shows $A-(A-B) \subseteq A \cap B$ — (ii)

$\therefore$ From (i) & (ii), we can conclude $[A-(A-B) = A \cap B]$

, Hence Proved,

Q20.) Justify the following statement or else give an example to disprove the result. Let A, B and C be subsets of a set U.

a) $A \Delta C = B \Delta C \Rightarrow A = B$

Suppose, $x \in A$, then $x \in A-C$ or $x \in A \cap C$

Case 1: $x \in A-C$

$\because$ $A \Delta C = B \Delta C = (B-C) \cup (C-B)$

$\therefore$ $x \in B-C$ as $x \notin C$

$\therefore$ In particular, $x \in B$

U19CS012

Case 2: $x \in A \cap C$

$\because A \Delta C = B \Delta C$

$\therefore x \notin B \Delta C$

ie $x \in B \cap C$ or $x \notin B \cup C$

$\because x \in C$

$\therefore x \in B \cap C$

$\therefore$ In particular, $x \in B$

Hence, $A \subseteq B$ — (i) ; Similarly we can prove for

$B \subseteq A$ — ii

$\therefore \boxed{A = B}$

(b) $(A-C) - (B-C) = (A-B) - C$

Given: $U = A \cup B \cup C$

We know that, $X - Y = (X \cap Y')$

$\therefore (A-C) - (B-C) = (A \cap C') - (B \cap C')$

$= (A \cap C') \cap (B \cap C')'$

$= (A \cap C') \cap (B' \cap C)$ [Demorgan's theorem]

$= (A \cap C' \cap B') \cup (A \cap C' \cap C)$

$= (A \cap C' \cap B') \cup \phi$ $\quad (C' \cap C = \phi)$

$= A \cap C' \cap B'$ — (i)

Also,

$(A-B) - C = (A \cap B') - C$

$= (A \cap B') \cap C'$

$= A \cap B' \cap C'$

$= A \cap C' \cap B'$ — (ii)

$\therefore$ From (i) & (ii)

$(A-C) - (B-C) = (A-B) - C$

$\therefore$ Hence Proved

U19CS012

(C) $(A-B)' = (B-A)'$

We know that, $A-B = A \cap B'$

$\therefore$ LHS $= (A-B)'$

$= (A \cap B')'$

$= (A' \cup B)$ [Demorgan's Law]

RHS $= (B-A)' = (B \cap A')' = B' \cup A$ [Demorgan's Law]

$\therefore$ Let $x \in (A-B)'$

$\Rightarrow x \in A' \cup B$

$\Rightarrow x \in A'$ or $x \in B$

ie $x \notin A$ or $x \in B$

But if $x \in (B-A)'$, then $x \in B'$ or $x \in A$

but we already proved $x \notin A$ or $x \in B$.

$\therefore$ It contradicts.

Eg: Let U = set of all students

A = Students who play Basketball

B = Set of students who play cricket

$(A-B)'$ = Set of student either do not play Basketball or play Cricket

but

$(B-A)'$ = Set of student who either do not play Cricket or play Basketball

$\therefore$ there exist a student which is in $(A-B)'$ set but not in $(B-A)'$ and vice versa.

Hence, the above example is sufficient for Disapproval.

—×—