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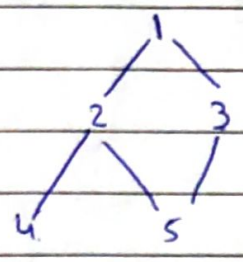
Tutorial - 4. Lattice, Boolean Algebra.

Q.1. Let A: {1, 2, 3, 4, 5} be ordered by the following Hasse diagram. Insert the correct symbol <, > or || (not comparable) between each pair of elements.



1) 1 < 5
2) 2 || 3
3) 4 < 1
4) 3 ~~>~~ 4

Q.2. Consider the ordered set A in the previous Hasse diagram.

1. Find all minimal and maximal elements of A.
2. Does A have a lower bound and an upper bound? Also, discuss glb and lub for the set A.

1) Maximal element → 1
Minimal element → 4, 5

2) ~~Lub~~ = ~~Sup~~ Sup(A) = 1
glb = Inf(A) = ∅
lower bound of A = ∅
4 does not relate to 5.
upper bound of A = 1

least upper bound = 1
greatest lower bound = ∅

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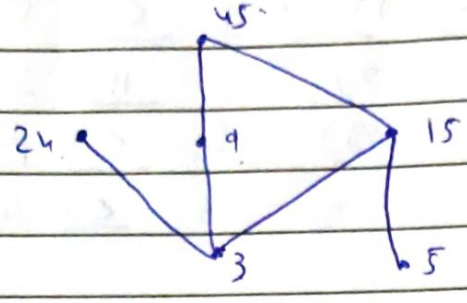
Q.3 For the poset ({3, 5, 9, 15, 24, 45}, divisor of I, find

1. the maximal and minimal elements
2. the greatest and the least elements
3. the upper bounds and the lub at {3, 5}
4. the lower bounds and glb at {15, 45}



1) Maximal elements 24, 45, minimal elements : 3, 5

2)

2) Greatest element = Does not exists
   least element : Does not exists

3) Upper bound : 15, 45, LUB = 15

4) lower bound : 3, 5, 15, GLB : 15

Q.4 If R and S are relations on A = {1, 2, 3} represented by the matrices

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad M_S = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Find the matrices that represent

1. R ∪ S 2) R ∩ S 3) R · S 4) S · R 5) R ⊕ S

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1) $M_{RUS} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

2) $M_{RNS} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

3) $M_{R \cdot S} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

4) $M_{S \cdot R} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

5) $M_{R \oplus S} = M_{RUS} - M_{RNS}$

$= \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

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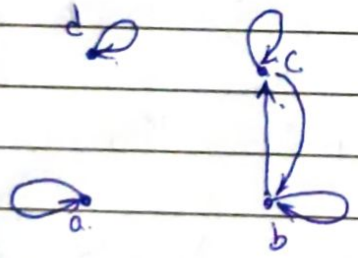
Q.5 List the ordered pairs in the relations R and S whose matrix representations are given as follows:

1) $M_R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

2) $M_S = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$

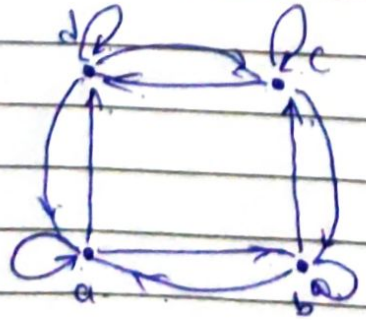
Also draw the directed graphs representing R and S. Use the graphs to find if R and S are equivalence relations.

$R = \{(a,a), (b,b), (b,c), (c,b), (c,c), (d,d)\}$



⇒ Self loop → reflexive
⇒ Symetric
⇒ transitive.
⇒ equivalence

$S = \{(a,a), (a,b), (a,d), (b,a), (b,b), (b,c), (c,b), (c,c), (c,d), (d,a), (d,c), (d,d)\}$



self loop → reflexive.
symmetric
Not transitive
⇒ Not equivalence

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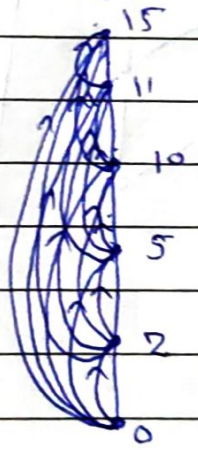
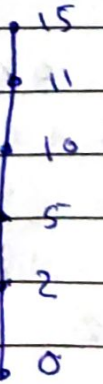
Q.6 Draw the Hasse diagram for the "less than or equal to" relation on {0, 2, 5, 10, 11, 15} starting from the diagraph.

→ diagraph.

After removing self loop.



After removing transitive relation.



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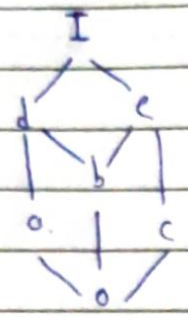
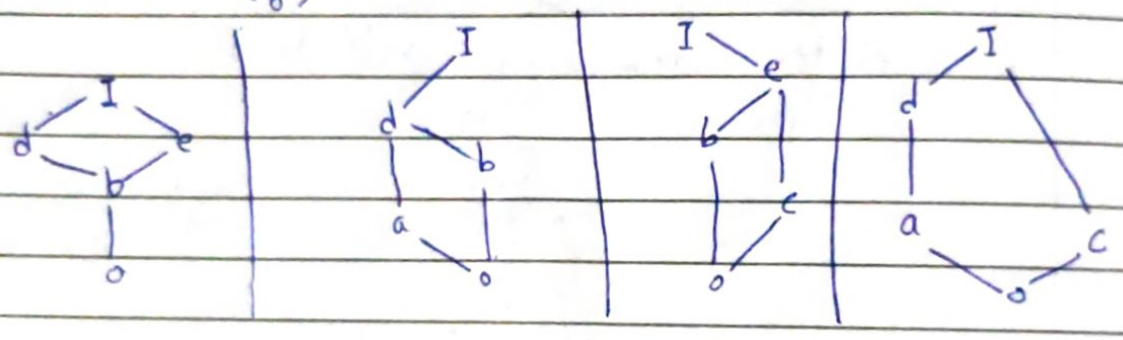
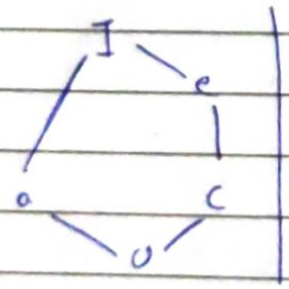
Q.7. Consider the lattice L in the following figure.

1. Find all sublattice with five elements
2. Find complements of a and b, if they exists.
3. Is L distributive ? complimented ?



1)





2) $a \vee c = I \quad a \wedge c = 0$, $a \vee e = I$, $a \wedge e = 0$

$\Rightarrow a^c = c, e$

∀ elements in L ~~is~~ no element satisfy compliment of b

$\Rightarrow b^c \neq \emptyset$ b^c does not exist.

3) Since a has multiple complement so L is not distributive. And b has no compliment so L is not complimented.

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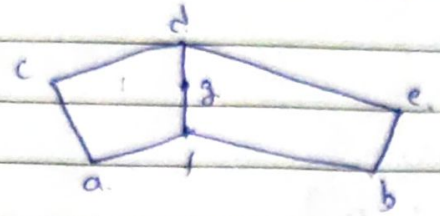
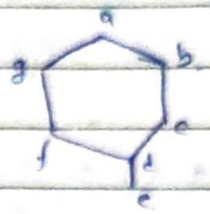
Q.8. Decide which of the following Hasse diagrams define a lattice on $\{a, b, c, d, e, f, g\}$



(i)



(ii)

For diagram (i) for every elements Join semi lattice exists and meet semi lattice also exists.

For diagram (ii) for every elements join semi lattice exists and meet semi lattice also exists.

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Q. 9. Write the duals of each Boolean eqn.

1. $(a * 1) * (0 + a')$
2. $a + a'b = a + b$

1) $(a * 1) * (0 + a')$ dual.

dual. : $(a + 0) + (0 * a')$

2) $a + a'b = a + b$

dual. : $a * a'b = a * b$

Q. 10 Given the set D_m of divisors of m is a bounded, distributive lattice with $a + b = a \vee b = lcm(a, b)$ and $a * b = a \wedge b = gcd(a, b)$

1) S.T D_m is a B.A if m is square free i.e. if m is a product of distinct prime.
2) Find the atoms of D_m.