

Tutorial-2
Number Theory (Int. HSc 5th year)

- ① Find $\gcd(306, 657)$ and $\gcd(272, 1479)$.
- ② Use Euclidean Algorithm to obtain integers x and y satisfying the following:
 - ① $\gcd(56, 72) = 56x + 72y$
 - ② $\gcd(1769, 2378) = 1769x + 2378y$.
- ③ Prove that if d is a common divisor of a and b , then $d = \gcd(a, b)$ if and only if $\gcd(a/d, b/d) = 1$.
- ④ Assuming $\gcd(a, b) = 1$ prove
 - ① $\gcd(a+b, a-b) = 1$ or 2 .
 - ② $\gcd(a+b, a^2-ab+b^2) = 1$ or 3 .
- ⑤ Prove that if $\gcd(a, b) = 1$, then $\gcd(a+b, ab) = 1$.
- ⑥ Find $\text{lcm}(143, 227)$, $\text{lcm}(306, 657)$.
- ⑦ Which of the following Diophantine equations cannot be solved?
 - ① $6x + 51y = 22$
 - ② $33x + 14y = 115$
 - ③ $14x + 35y = 93$
- ⑧ Determine all solutions in the integers of the following Diophantine equations:
 - ① $56x + 72y = 40$ ② $24x + 138y = 18$ ③ $221x + 35y = 11$.

(9) Determine all solutions in the positive integers of the following Diophantine equations:

(a) $18x + 5y = 48$.

(b) $123x + 360y = 99$

(c) $158x - 57y = 7$.

(20) The neighborhood theater charges \$1.80 for adult admissions and \$.75 for children. On a particular evening, the total receipts were \$90. Assuming that more adults than children were present, how many people attended?

(11) A certain number of sixes and nines is added to give a sum of 126; if the number of sixes and nines is interchanged, the new sum is 114. How many of each were there originally?

(12) Solve the puzzle:

Alcuin of York, 775. One hundred bushels of grain are distributed among 100 persons in such a way that each man receives 3 bushels, each woman 2 bushels, and each child $\frac{1}{2}$ bushel. How many men, women, and children are there?