DISCRETE MATHEMATICS MA221

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Q.1) (i) Why is f not a function from R to R if $f(x) = \frac{1}{x}$.

Function is a binary relation between two sets that associates every element of the first set to exactly one element of second set.

A function f from A to B has the property that each element of A has been assigned to one element of B.

$f(x) = \frac{1}{x}$ $x \in R$ Domain = R
Range = R

We see that at $x = 0$, the function is not defined
$x \in R$ $[\because 0 \in R]$
$f(0) = \frac{1}{0} =$ which is not defined in mathematics
[We can't divide a number by 0]

This means that there is no value for $x = 0$ in Domain,

Ans: $\therefore$ f cannot be function from R to R, since $0 \in R$

Domain should be $R - \{0\}$ to become a function

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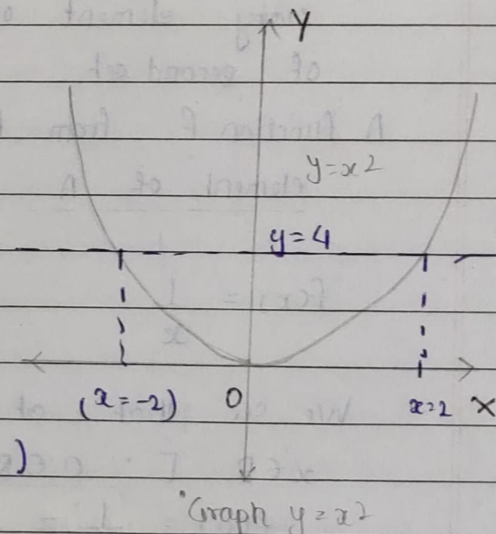
(ii) What are invertible functions?
Let f be a function from R to R defined by $f(x) = x^2$. Is it invertible?

(A)

Let us define a function $y = f(x) : X \rightarrow Y$
If we define a function $g(y)$ such that $x = g(y)$
then g is said to be inverse function of 'f'.
Invertible function = function whose inverse exist.
$\Rightarrow$ It is both $\rightarrow$ one-one (injective) } (bijective)
$\rightarrow$ onto (surjective)
[Range = Codomain]

(B) $R \rightarrow R$ $\quad f(x) = x^2$



Let us take $f(x) = 4$
$x^2 = 4$ $\quad$ (many $\rightarrow$ one)
$x = \pm 2$

function is invertible, if each input has unique output (Not one-one)

But here $\forall\ e \in R - \{0\}$
there exist $\left.\begin{matrix} e \\ -e \end{matrix}\right\} \Rightarrow (e^2)$ mapped

'Graph $y = x^2$

$f(f^{-1}(x)) = (f^{-1}(x))^2$ $\qquad [f(f^{-1}(x)) = x]$
$x = (f^{-1}(x))^2$
$[f^{-1}(x) = \sqrt{x}]$ $\otimes$ But squareroot is not defined
$\therefore$ Therefore $\quad (x \in R)$ $\quad$ for (-ve numbers)

Ans: Inverse does not exist.

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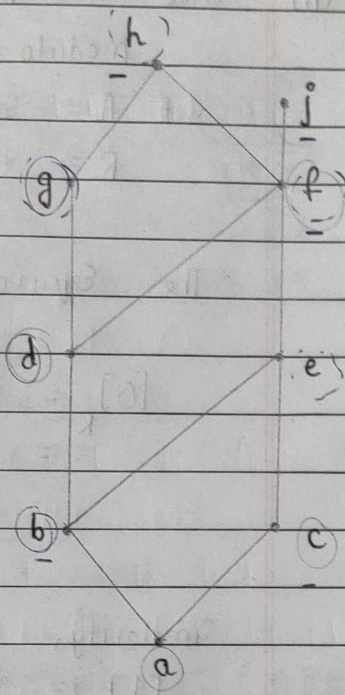
Q2.) (i) Find lower and upper bound of
Subset {a, b, c}
{j, h}
(ii) {a, c, d, f}



[Upper Bound $a \leq u \quad \forall a \in A$]

(a) for {a, b, c}

Upper Bounds = e, f, g, h
['d' is not upper bound ∵ it is not ~~connected~~ related to 'c']

Lower Bound = a
[Lower Bound (l) $l \leq a \quad \forall a \in A$]

(b) for {j, h}
Upper Bound = No upper Bound

Lower Bounds = a, b, c, d, e, f
{'g' is not lower bound, ∵ g is not related to 'j'}

(c) for {a, c, d, f}

Upper Bound = f, h, j
Lower Bound = a

(ii) Find greatest lower bound {b, d, g}
least upper bound
Upper Bounds = g, h
Lower Bounds = b, a

Greatest lower bound	= b
least upper bound	= g

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ii) Give a description of each of congruence classes modulo 6.

Let A = Set of all integers (Z)

R = { (x,y) | x mod 6 = y mod 6 }

The Equivalence class of 0 = all integers 0 (modulo 6)

$[0]_R$ = { y | y mod 6 = 0 mod 6 }
= { y | y mod 6 = 0 }
= { y | y = 6k with k ∈ Z }
= { −18, −12, −6, 0, 6, 12, ... }

Similarily,

$[1]_R$ = { y | y mod 6 = 1 mod 6 }
= { y | y mod 6 = 1 }
= { y | y = 6k + 1 with k ∈ Z }
= { −17, −11, −5, 1, 7, 13, ... }

$[2]_R$ = { y | y mod 6 = 2 mod 6 }
= { y | y mod 6 = 2 }
= { y | y = 6k + 2 with k ∈ Z }
{ −16, −10, −4, 2, 8, 14 ... }

Similarly

$[3]_R$ = { y | y = 6k + 3 with k ∈ Z }
= { −15, −9, −3, 3, 9, 15, ... }

$[4]_R$ = { y | y = 6k + 4 with k ∈ Z }
= { −14, −8, −2, 4, 10, 16, ... }

$[5]_R$ = { y | y = 6k + 5 with k ∈ Z }
= { −13, −7, −1, 5, 11, 17, ... }

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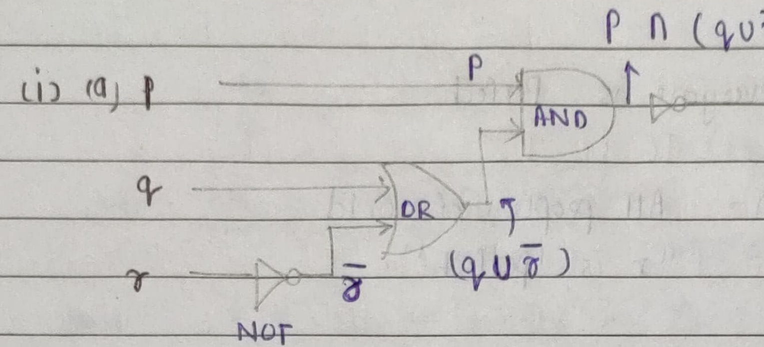
Q3.)

(i) (a)



$= \overline{p \cap (q \cup \bar{r})}$

$= \overline{p \cdot (q + \bar{r})}$

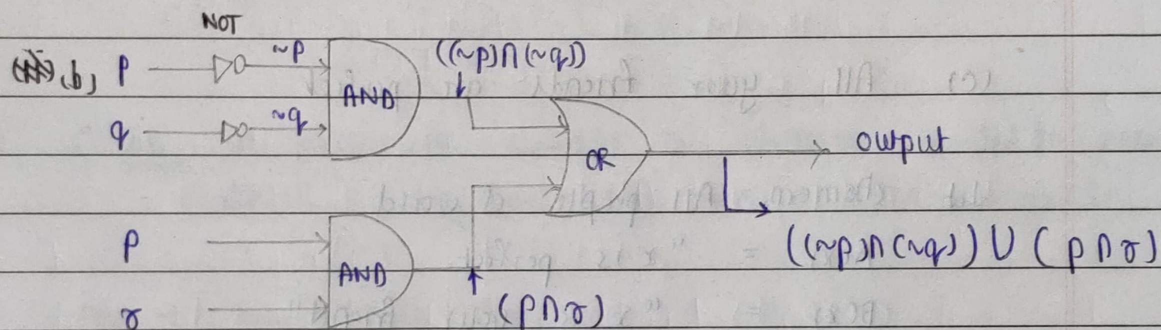
$= \bar{p} + \overline{(q+\bar{r})}$

$= \bar{p} + (\bar{q} \cdot r)$

Ans = $\sim(p \cap (q \cup \bar{r}))$ $\quad \sim$ = NOT

= $\sim p \cup ((\sim q) \cap r)$ (simplified)

(b)



$\Rightarrow \bar{p} \cdot \bar{q} + p \cdot r$

Ans: $\underline{\bar{p} \cdot \bar{q} + p \cdot r}$

(ii) Interpretation Symbols

Negation $\neg p$ = not p

Disjunction $p \vee q$ = p or q

Conjunction $p \wedge q$ = p and q

Existential quantification $\exists x\, P(x)$: There exist an element x in the domain such that P(x)

Universal quantification $\forall x\, P(x)$: P(x) for all values of x in the domain

a) No one is perfect

let domain = All people of world

A(x) = "x is perfect"

"No one" - there does not exist a person $(\neg(\exists x))$

$$\boxed{\neg(\exists x\, A(x))}$$

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(b) Not everyone is Perfect

Let Domain = All people of world
$A(x)$ = "x is perfect"

"Not Everyone" means not all people $\therefore \neg \forall x$

$$\boxed{\neg \forall x \; A(x)}$$

(c) All your friends are perfect

Let Domain = All people of world
$A(x)$ = "x is perfect"
$B(x)$ = "x is your friend"

"All" means everybody and thus $\forall x$
If x is your friend, then x has to be Perfect
($\rightarrow$)(implies)

$$\boxed{\forall x \; (B(x) \rightarrow A(x))}$$

Q4.> (i) min^m student to be sure atleast '6' receive same grade
Five possible grades
= A, B, C, D, F

Acc to Generalized Pigeonhole principle,
If we have 'N' objects,
Acc to principle, there must be atleast 'r' objects in one of the boxes as long $\lceil \frac{N}{K} \rceil \geq r$
(ceil)

The smallest integer N for which

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$$\frac{N}{k} > r-1$$

$(\because N > k(r-1))$

$$\Rightarrow \boxed{N = k(r-1)+1}$$

smallest ~~integer~~ N satisfying $\left(\left\lceil \frac{N}{k} \right\rceil \geq r\right)$

Here k = No. of grades $= 5 = k$

atleast same grade $= 6 = r$

$\therefore \left\lceil \frac{N}{5} \right\rceil = 6$ $\quad N = 5 \times (6-1) + 1$

$= 5 \times 5 + 1$

$= [26]$

Ans: $\therefore$ 26 is minimum number of students needed to ensure atleast 6 students will receive ~~a~~ same grade.

(ii) coin → flipped 10 times

Possible outcomes (a) → total

(b) Exactly two heads

(c) contains atmost three tails

(a) The coin is flipped 10 times

Each flip has 2 possible outcomes (Head or tail)

Using product rule of counting, $\left.\begin{matrix} (E_1 \to m \\ E_2 \to n \end{matrix}\right\}$ two event in sequence $(m \cdot n)$ ways

total outcomes Possible $= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$= 2^{10}$

$=$ 1024 outcomes

(b) Here, Exactly two Heads

Order of Head / tail is Not important ($\because$ no. of heads ✗ not order of heads)

We need to use combination (order not important)

$$\left[C(n,r) = \binom{n}{r} = \frac{n!}{r!(n-r)!} \right]$$

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$n = 10$ coins
$r = 2$ heads (exactly)

Ans: $C(10,2) = \frac{10!}{2!(10-2)!} = \frac{10!}{2!8!} = \frac{10\times9}{2\times1} = 45$ outcomes with exactly 2 head

(C) At most three tails
The order of head / tails is not important here
∴ Need to use combination
$n = 10$ $r \le 2$

$C(10,0) = \frac{10!}{0!(10-0)!} = \frac{10!}{0!10!} \rightarrow 1$

$C(10,1) = \frac{10!}{1!(10-1)!} = \frac{10!}{1!9!} = 10$

$C(10,2) = \frac{10!}{2!(10-2)!} = \frac{10\times9}{(2)} = 45$

$C(10,3) = \frac{10!}{3!7!} = \frac{10\times\cancel{9}^{3}\times\cancel{8}^{4}}{\cancel{3\times2\times1}} = 120$

Adding no. of outcomes for $r = 0,1,2,3$

Ans: $= 120 + 45 + 10 + 1 = 176$ outcomes with atmost 3 tails

Q5.) (i) H = Subgroup of G
$x^2 \in H$ $\forall x \in G$
To show = H is normal subgroup of G
H is normal subgroup of G $\Longleftrightarrow$

$$\left[\ \forall h \in H\ \ \forall g \in G\ :\ g^{-1}hg \in H\ \right]$$

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$g^{-1}hg = g^{-1}(g^{-1}g)hg$ [$\because aa^{-1} = e$ = identity]

[we can multiple identity with any var]

$= (g^{-1})^2 \cdot ghg$ [$a * e = e = e * a$]

$= (g^{-1})^2 (h^{-1}h) ghg$

$= (g^{-1})^2 h^{-1} (hg)^2 \in H$

[$\because (hg) \in G \rightarrow (hg)^2 \in G$]

then, $g^{-1}hg \in H$

(ii) Let $G = \{1, -1, i, -i\}$ group under multiplication

$\overline{Z_8} = \{1, 3, 5, 7\}$ a group under multiplication modulo 8

To show

G and $\overline{Z_8}$ are not isomorphic

Group G can be defined as

$(\{1, -1, i, -i\}, *) \rightarrow (\{1, 3, 5, 7\}, m_8)$

	1	-1	i	-i
1	1	-1	i	-i
-1	-1	1	-i	i
i	i	-i	-1	1
-i	-i	i		-1

	1	3	5	7
1	1	3	5	7
3	3	1	7	5
5	5	7	1	3
7	7	5	3	1

$\overline{Z_8}$ = self inverse 1

$G \rightarrow$ Not the case

There is no mapping $f : G \rightarrow \overline{Z_8}$ such to form isomorphism.

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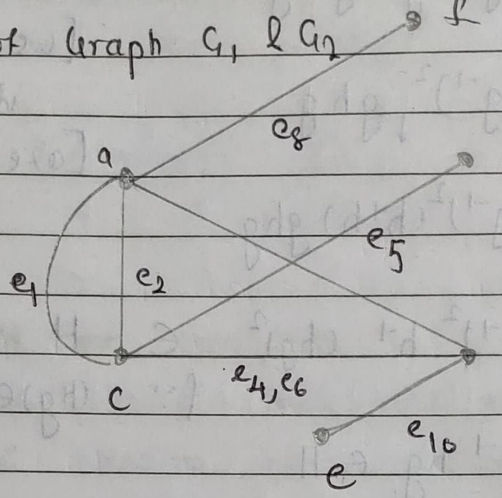
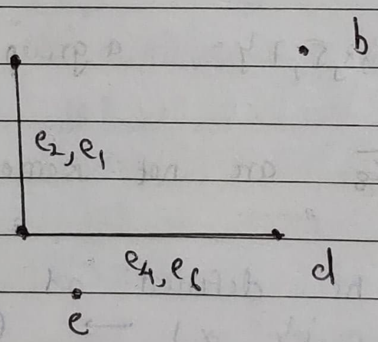
[EXTRA] (Do not consider Q6)

Q6.>

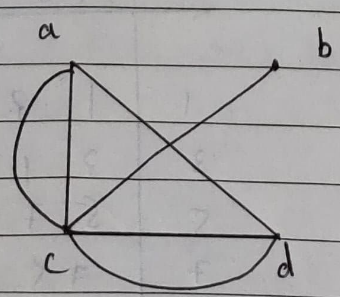
Union of Graph G_1 & G_2

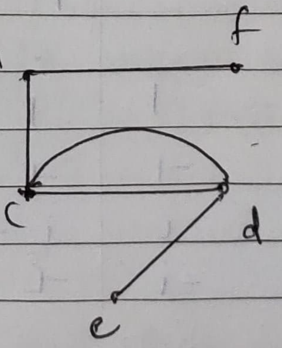


Intersection:



After short circuiting, (c & d)





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Q.7) (i) Invarient Condition for graph Isomorphism

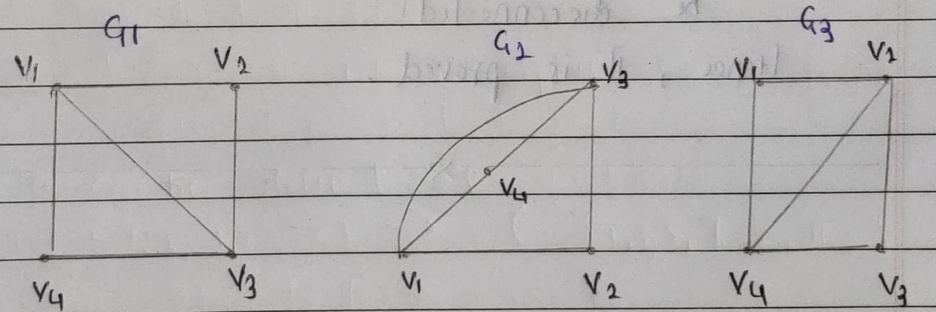
(a) Number of vertices in both the graphs must be same

(b) Number of edges in both the graphs must be same

(c) degree sequence of both the graphs must be same

(d) If a cycle of length k is formed by vertices $\{V_1, V_2, V_3, \ldots V_k\}$ in one graph

a cycle of same length must be formed by vertices $\{f(V_1), f(V_2), f(V_3), \ldots f(V_k)\}$ in other graph as well



⇒ The above four condition's are just the necessary conditions for any two graphs to be isomorphic

⇒ They are not at all sufficient to prove that the two graphs are isomorphic.

⇒ If all 4 conditions satisfy, even then it can't be said graphs are surely isomorphic.

(ii) Simple graph with n vertices must be connected if it has more than $\frac{(n-1) \times (n-2)}{2}$ edges.

To prove this,

We need Let suppose, Simple Graph is disconnected for maximum edges the graph must have $(n-1)$ edges connected (minimum)

$$\text{Total edges} = {}^{(n-1)}C_2$$

$$= \frac{(n-1)(n-2)}{2}$$

So, if edges are greater than this, then graph cannot be disconnected.

Hence, it is proved.

---x---