

AUTOMATA AND FORMAL LANGUAGES - (CS-208)

TUTORIAL - 9

- Q. 1. Design push down automata for $\{a^n b^n + m c^m \mid n, m \geq 1\}$ and give 6 specifications for PDA.

Ans:- $M = \{ \{q_0, q_1, q_2, q_3\}, \{a, b, c\}, \{N, M\}, \delta, q_0, \epsilon, q_3 \}$

where $\delta =$

$$\delta(q_0, a, \epsilon) = (q_0, N)$$

$$\delta(q_2, \$, \epsilon) = (q_3, \epsilon)$$

$$\delta(q_0, a, N) = (q_0, NN)$$

$$\delta(q_0, b, N) = (q_1, \epsilon)$$

$$\delta(q_1, b, N) = (q_1, \epsilon)$$

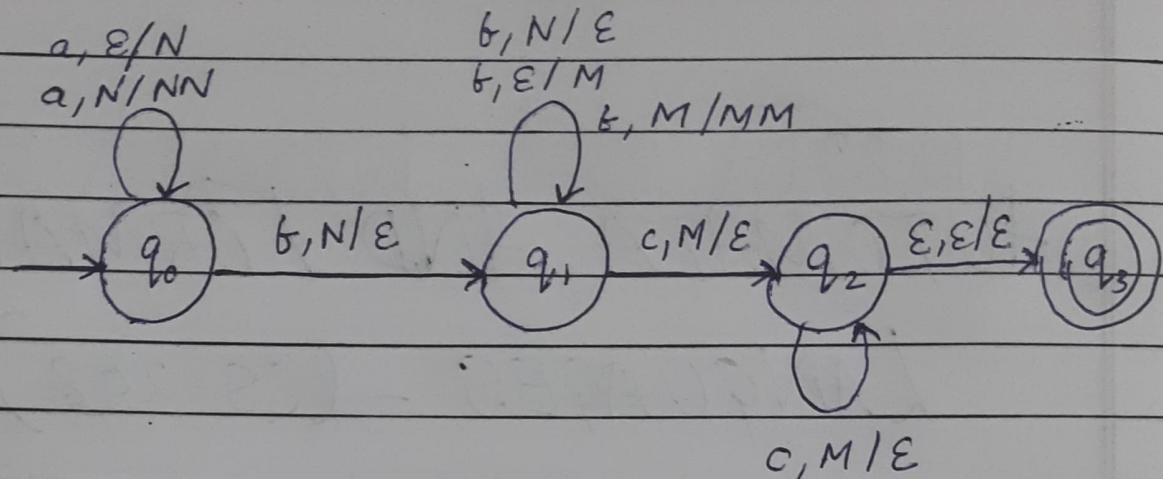
$$\delta(q_1, b, \epsilon) = (q_1, M)$$

$$\delta(q_1, b, M) = (q_1, MM)$$

$$\delta(q_1, c, M) = (q_2, \epsilon)$$

$$\delta(q_2, c, M) = (q_2, \epsilon)$$

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Q-2. Find a push down automata that recognises the following languages and give 6 tuple specifications for PDA.

$$L = \{ a^i b^* c^{i+2k} \mid i, k \geq 0 \}$$

Ans:- $M = (\{q_0, q_1, q_2, q_3\}, \{a, b, c\}, \{I, K\}, S, q_0, q_3)$

where S is defined by -

$$S(q_0, a, \epsilon) = (q_0, I)$$

$$S(q_2, c, K) = (q_2, \epsilon)$$

$$S(q_0, a, I) = (q_0, II)$$

$$S(q_2, \epsilon, \epsilon) = (q_3, \epsilon)$$

$$S(q_0, b, \epsilon) = (q_1, KK)$$

$$S(q_0, b, I) = (q_1, KKII)$$

$$S(q_1, b, K) = (q_1, KKK)$$

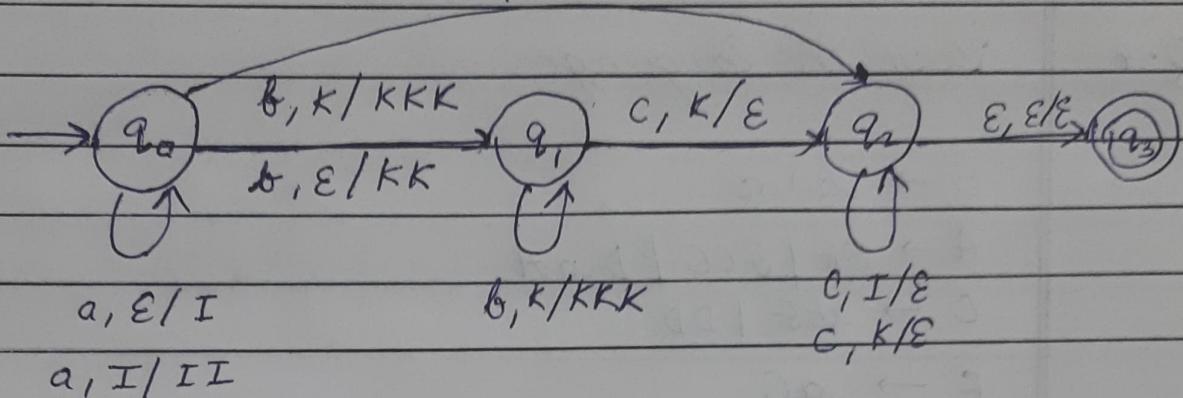
$$S(q_0, c, I) = (q_2, \epsilon)$$

$$S(q_1, c, K) = (q_2, \epsilon)$$

$$S(q_2, c, I) = (q_2, \epsilon)$$

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c, I / 8



Q.3. Find the language :-

$$S \rightarrow aABBB \mid aAA$$

$$A \rightarrow aBBB \mid a$$

$$B \rightarrow BBB \mid A$$

Ans :- $S \rightarrow \{aaa, aabaaa, aaabaa, \dots\}$

S is set of all strings of a and b such that string starts with 2 a 's and end with 2 a 's ~~except~~ and length of string is even and also string includes "aaa".

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Q.4. Find the language :-

$$S \rightarrow aAa$$

$$A \rightarrow SB / bCC / DAa$$

$$C \rightarrow abb / DD$$

$$E \rightarrow aC$$

$$D \rightarrow aDA$$

Ans:- E and D states are unnecessary as E state is unreachable and D is Ambiguous.

Reduced grammar is :-

$$S \rightarrow aAa, A \rightarrow SB / bCC, C \rightarrow abb$$

Language of S : { ab, ababbba, ababbbabba ... }

S is set of strings starting and ending with a such that

$$S = ab(ab)^2 b^* a$$

Ans,

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Q. 5

Find a push down automata with 2 states that accepts language :-

$$(A) L = \{ a^n b^{2n} \mid n \geq 1 \}$$

Ans :-

$$M = \{ (q_0, q_1, \beta), (q_1, \epsilon, N), \{N\}, \delta, q_0, \epsilon, q_1 \}$$

where δ is defined as :-

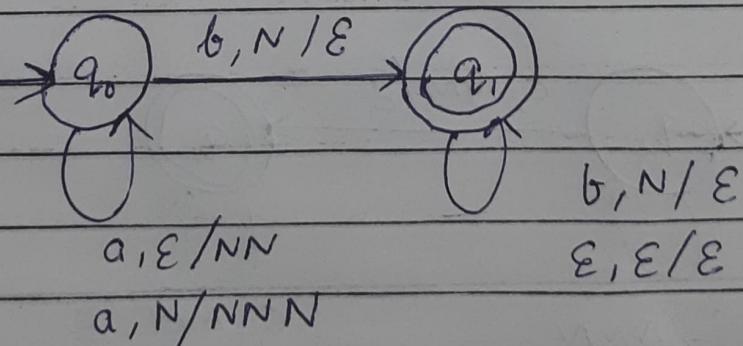
$$\delta(q_0, a, \epsilon) = (q_0, NN)$$

$$\delta(q_0, b, N) = (q_1, \epsilon)$$

$$\delta(q_1, b, N) = (q_1, \epsilon)$$

$$\delta(q_1, \epsilon, \epsilon) = (q_1, \epsilon)$$

$$\delta(q_0, a, N) = (q_0, NNN)$$



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$$(B) - L = \{ a^n b^m c^m d^{3n} \mid n \geq 0, m \geq 0 \}$$

Ans :- $M = \{ \{ q_0, A, 3 \}, \{ a, b, c, d \}, \{ N, M \}, \emptyset, q_0, 19, 3 \}$

where S is defined by :-

$$S(q_0, a, \epsilon) = (q_0, NNN)$$

$$S(q_0, a, N) = (q_0, NNNN)$$

$$S(q_0, b, N) = (q_0, MN)$$

$$S(q_0, b, M) = (q_0, MM)$$

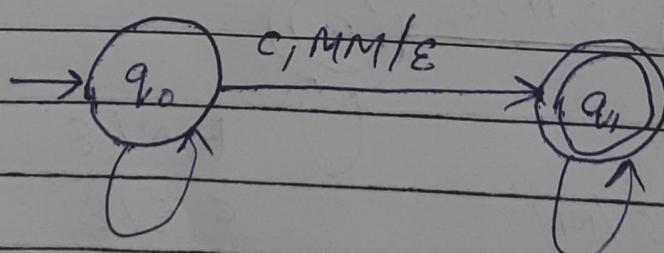
$$S(q_0, b, \epsilon) = (q_0, M)$$

$$S(q_0, c, MM) = (q_1, \epsilon)$$

$$S(q_1, c, MM) = (q_1, \epsilon)$$

$$S(q_1, d, MN) = (q_1, \epsilon)$$

$$S(q_1, \epsilon, \epsilon) = (q_1, \epsilon)$$



$a, \epsilon / NNN$

$a, N / NNNN$

$b, \epsilon / M$

$b, N / MN$

$b, M / MM$

$c, MM / \epsilon$

$d, NN / \epsilon$

$\epsilon, \epsilon / \epsilon$

Q.6. Consider a push-down automata

$$M = \{ \{q_1, q_2\}, \{0, 1, C\}, \{R, G, B\}, S, q_1, R, \phi \}$$

where δ is defined as :-

$$\delta(q_1, 0, R) = (q_1, BR)$$

$$\delta(q_2, 0, B) = (q_2, \epsilon)$$

$$\delta(q_1, 0, B) = (q_1, BB)$$

$$\delta(q_2, \epsilon, R) = (q_2, \epsilon)$$

$$\delta(q_1, 0, G) = (q_1, BG)$$

$$\delta(q_2, 1, R) = (q_2, GR)$$

$$\delta(q_1, C, R) = (q_2, R)$$

$$\delta(q_2, 1, B) = (q_1, GB)$$

$$\delta(q_1, C, B) = (q_2, B)$$

$$\delta(q_2, 1, G) = (q_1, GG)$$

$$\delta(q_1, C, G) = (q_2, G)$$

$$\delta(q_2, 1, G) = (q_2, \epsilon)$$

Ans - Give an execution trace of PDA M showing input string 001C100 is in N(M)

Input Symbol	Stack	State	Next-State
Initial	R	q_1	q_1
0	BR	q_1	q_1
0	BBR	q_1	q_1
1	GBBR	q_1	q_1
C	GBBR	q_1	q_2
1	BBR	q_2	q_2
0	BR	q_2	q_2
0	R	q_2	q_2

Input-Symbol	Stack	State	Next-State
ϵ	ϵ	q_2	q_2

Hence '001c100' is in N(m)

Q. 7. Design PDA for pallindrome with middle symbol c and trace string abba

Ans:- $M = \{q_0, q_1, q_2, \{a, b, c\}, \{A, B\}, S, q_0, \epsilon, q_1\}$

where S is defined as

$$S(q_0, a, \epsilon) = (q_0, A)$$

$$S(q_0, b, \epsilon) = (q_0, B)$$

$$S(q_0, a, A) = (q_0, AA)$$

$$S(q_0, b, A) = (q_0, BA)$$

$$S(q_1, a, \epsilon) = (q_1, \epsilon)$$

$$S(q_0, a, B) = (q_0, AB)$$

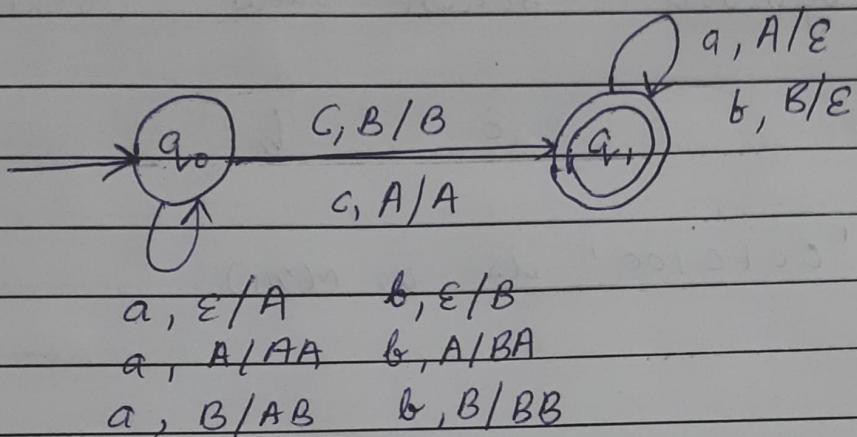
$$S(q_0, b, B) = (q_0, BB)$$

$$S(q_0, G, A) = (q_1, A)$$

$$S(q_0, c, B) = (q_1, B)$$

$$S(q_1, b, B) = (q_1, \epsilon)$$

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Tracing abcba

Input Symbol	Stack	State	next-State
a	A	q0	q0
b	BA	q0	q0
c	BA	q0	q1
ε	A	q1	q1
a	ε	q1	q1

Hence String abcba is accepted.

Q. 8. Design a PDA for $L = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i = j \text{ or } j = k\}$

Ans.: $M = \{ (q_0, a, q_1, q_2, q_3) \mid \delta(a, b, c) \in \{I, J\}, \delta, q_0, \epsilon, q_3 \}$

only non deterministic PDA possible

δ is defined as :-

$$\delta(q_0, a, \epsilon) = (q_0, I)$$

$$\delta(q_0, a, I) = (q_0, II)$$

$$\delta(q_0, b, I) = \{(q_1, JI), (q_1, \epsilon)\}$$

$$\delta(q_1, b, I) = (q_1, \epsilon)$$

$$\delta(q_1, b, J) = (q_1, JJ)$$

$$\delta(q_1, c, \epsilon) = (q_3, \epsilon)$$

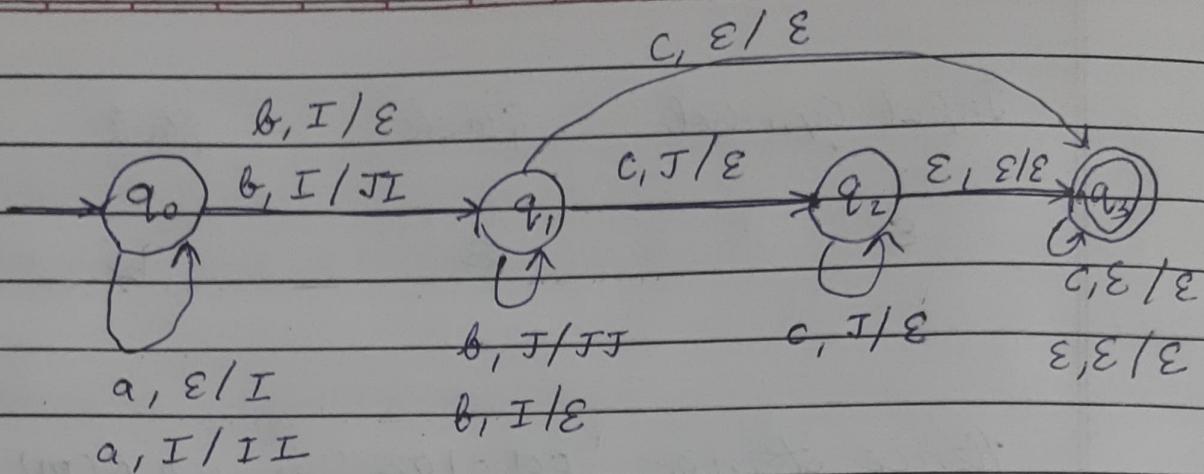
$$\delta(q_1, c, J) = (q_2, \epsilon)$$

$$\delta(q_2, c, J) = (q_2, \epsilon)$$

$$\delta(q_3, c, \epsilon) = (q_2, \epsilon)$$

$$\delta(q_1, \epsilon, \epsilon) = (q_3, \epsilon)$$

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Q.9- Construct a PDA that accepts $L = \{WWR \mid W = (a+b)^*\}$

Ans:-

$$\text{as } L = WWR$$

$$[W = (a+b)^*]$$

$$\begin{aligned} WW &= (a+b)^* (a+b)^* \\ &= (a+b)^*] \end{aligned}$$

$$L = (a+b)^* R$$

$$M = \{ (q_0, q_1, 3, (q_1, b, R) 3, 1R3, S, q_0, R, q_3) \}$$

where S is defined as

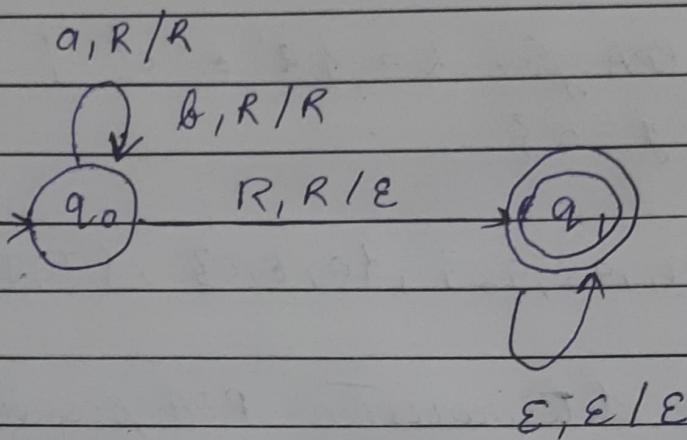
$$S(q_0, a, R) = (q_0, R)$$

$$S(q_0, b, R) = (q_0, R)$$

$$S(q_0, R, R) = (q_1, \epsilon)$$

$$S(q_1, \epsilon, \epsilon) = (q_1, \epsilon)$$

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any