U19CS012
(BHAGYA RANA)

(Q6)

(Explain Hyper plane classification using maximal-margin)
(Illustrated & derive)

① if $w \rightarrow$ weight vector realising a functional margin of 1 on the positive point $x^+$ and the negative point $x^-$, compute it Geometric margin

② functional margin of 1 implies

$$\langle w \cdot x^+ \rangle + b = +1$$
$$\langle w \cdot x^- \rangle + b = -1$$

③ while to compute Geometric margin, Normalize $w$, geometric margin $\gamma \longrightarrow$ functional margin of resulting classifier

$$\gamma = \frac{1}{2}\left( \left\langle \frac{w}{\|w\|_2} \cdot x^+ \right\rangle - \left\langle \frac{w}{\|w\|_2} \cdot x^- \right\rangle \right)$$

$$= \frac{1}{2\|w\|_2} \left( \langle w \cdot x^+ \rangle - \langle w \cdot x^- \rangle \right) = \frac{1}{\|w\|_2}$$

④ Resulting margin $= \frac{1}{\|w\|_2}$

⑤ Given linearly seperable training sample

$$S = ((x_1, y_1); (x_2, y_2) \ldots (x_\ell, y_\ell))$$

⑥ Hyperplane $(w \cdot b)$ that solves optimization problem

$$\text{minimize}_{w, b} \quad \langle w, w \rangle$$

subject to $y_i \left( \langle w \cdot x_i \rangle + b \right) \geq 1, \quad i = 1 \ldots \ell,$

$\rightarrow$ realises the maximum margin hyperplane with geometric margin $= \left[ \gamma = \frac{1}{\|w\|_2} \right]$

(transform it to dual problem