Department of Electronics and Telecommunication Engineering

University of Moratuwa

EN2040 – Random Signals and Processes

Simulation Assignment

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Content

Question 1

A 1000 long binary sequence with the same probability for '0' and '1' is generated by using rand() MATLAB built-in function.

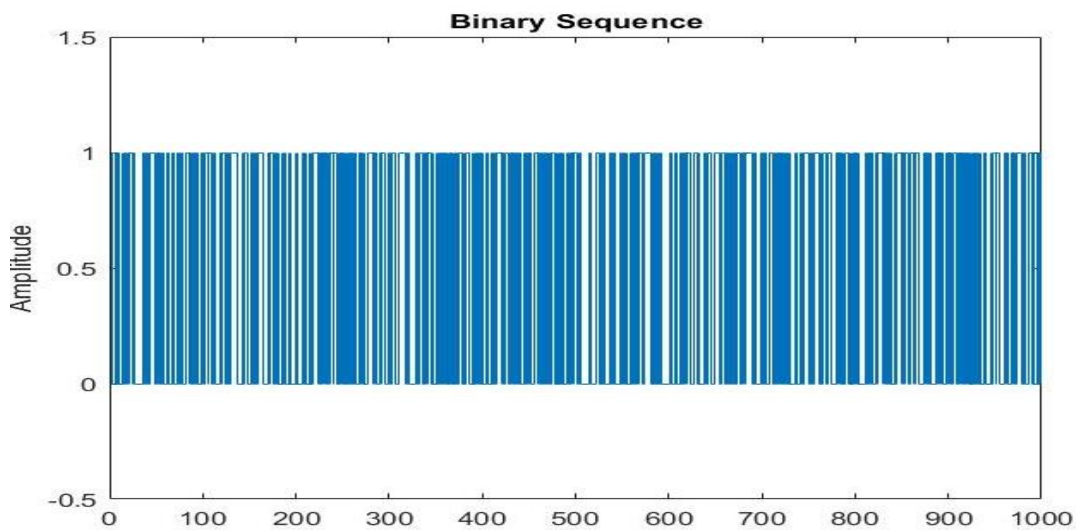

Figure 1: Binary Sequence

Then '0' and '1' bits are mapped to '–A' and 'A' respectively where A=1.

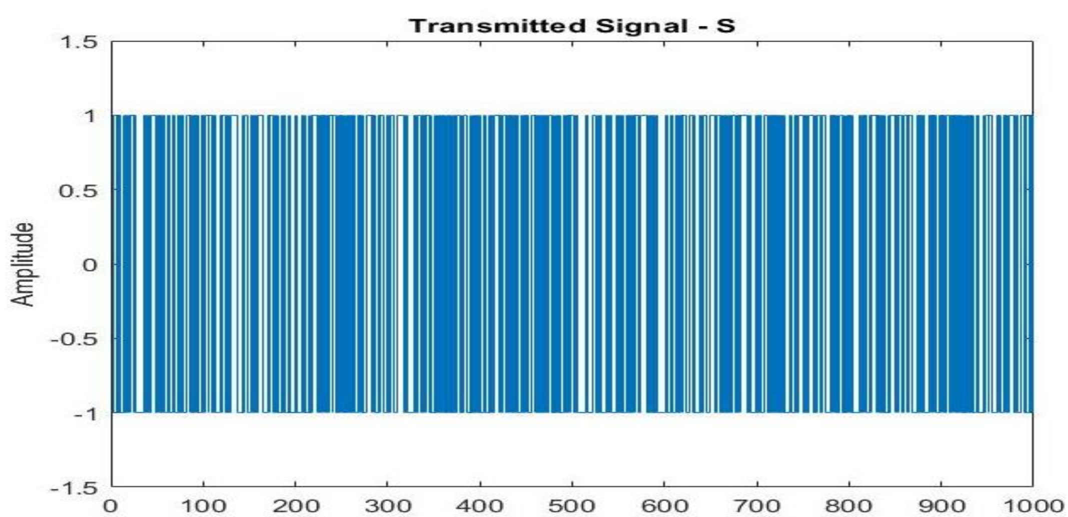

Figure 2: Transmitted Signal

Question 2

A 1000 long AWGN sequence with 0 mean and variance 1 is generated using normrnd() MATLAB built-in function.

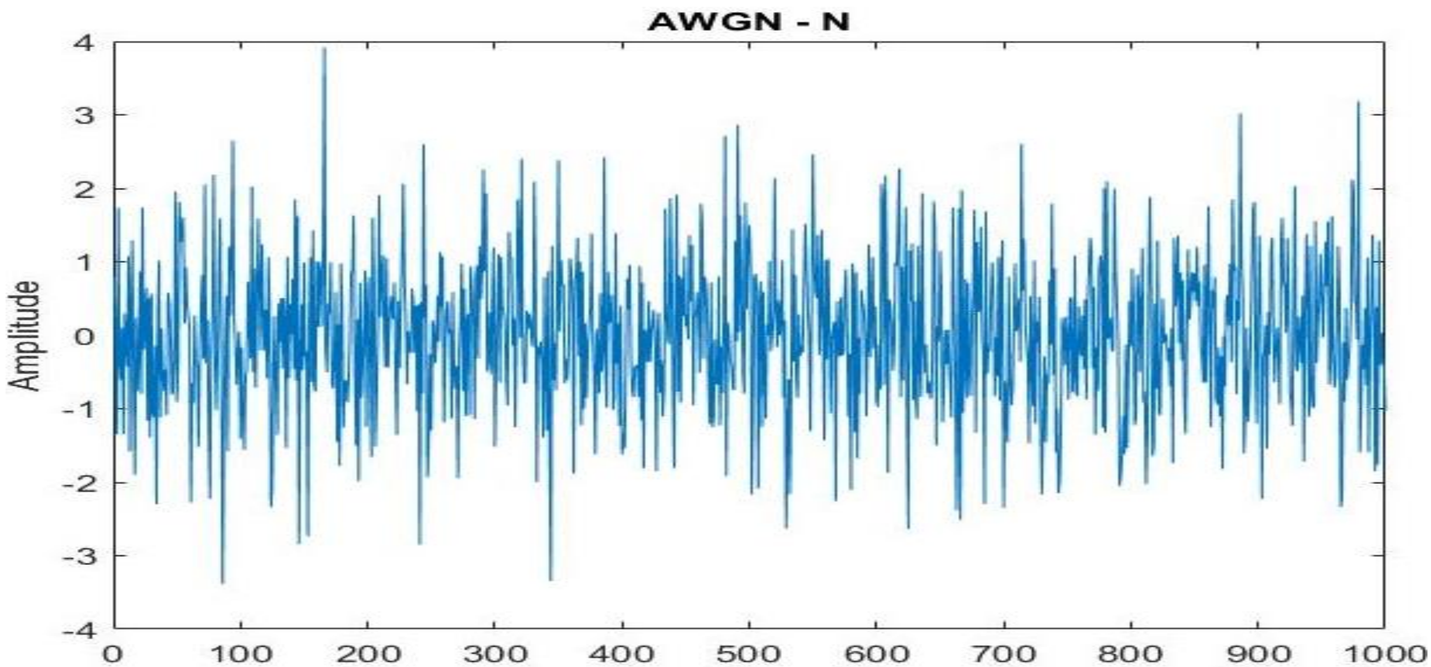

Figure 3: AWGN Signal

Question 3

When the transmitted signal (S) is transmitted through the communication channel, it is corrupted by the AWGN signal (N). Therefore, the received signal (R) is as follows.

$$R = S + N$$

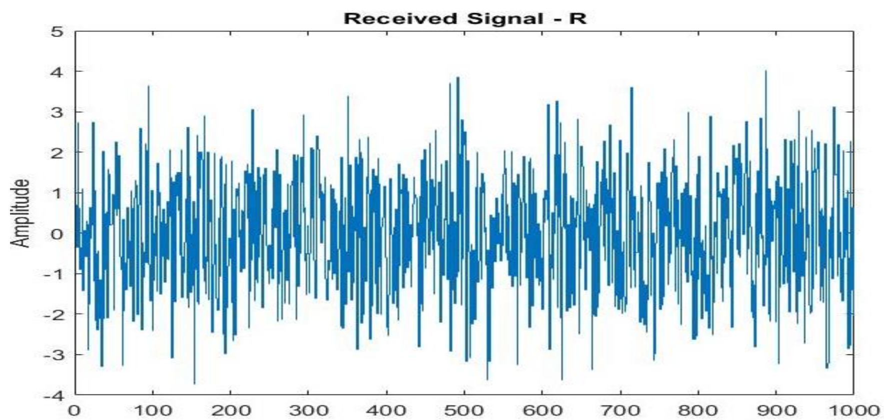

Figure 4: Received Signal with variance=1 AWGN

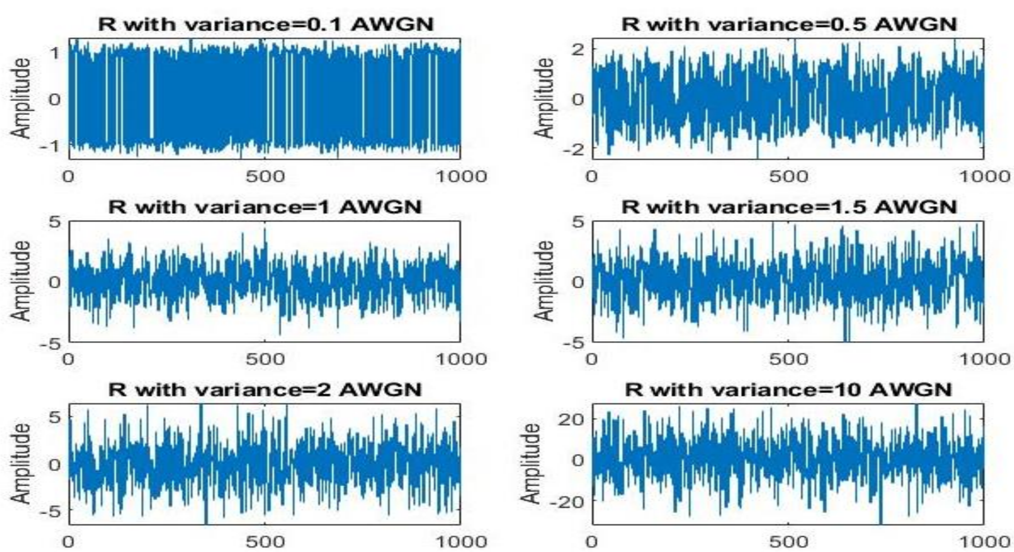

Figure 5: Received Signal comparison by changing the variance of the AWGN Signal

As shown in Figure 5, the corruption of the received signal is getting higher when the variance of the AWGN signal is increased. The power of the AWGN signal is increased when the variance is increased. Therefore, the error probability of the received signal is increased and more corruption occurs in the received signal.

Question 4

The amplitude of the received signal is compared with the threshold (tau=0) to generate the decoded signal (Y).

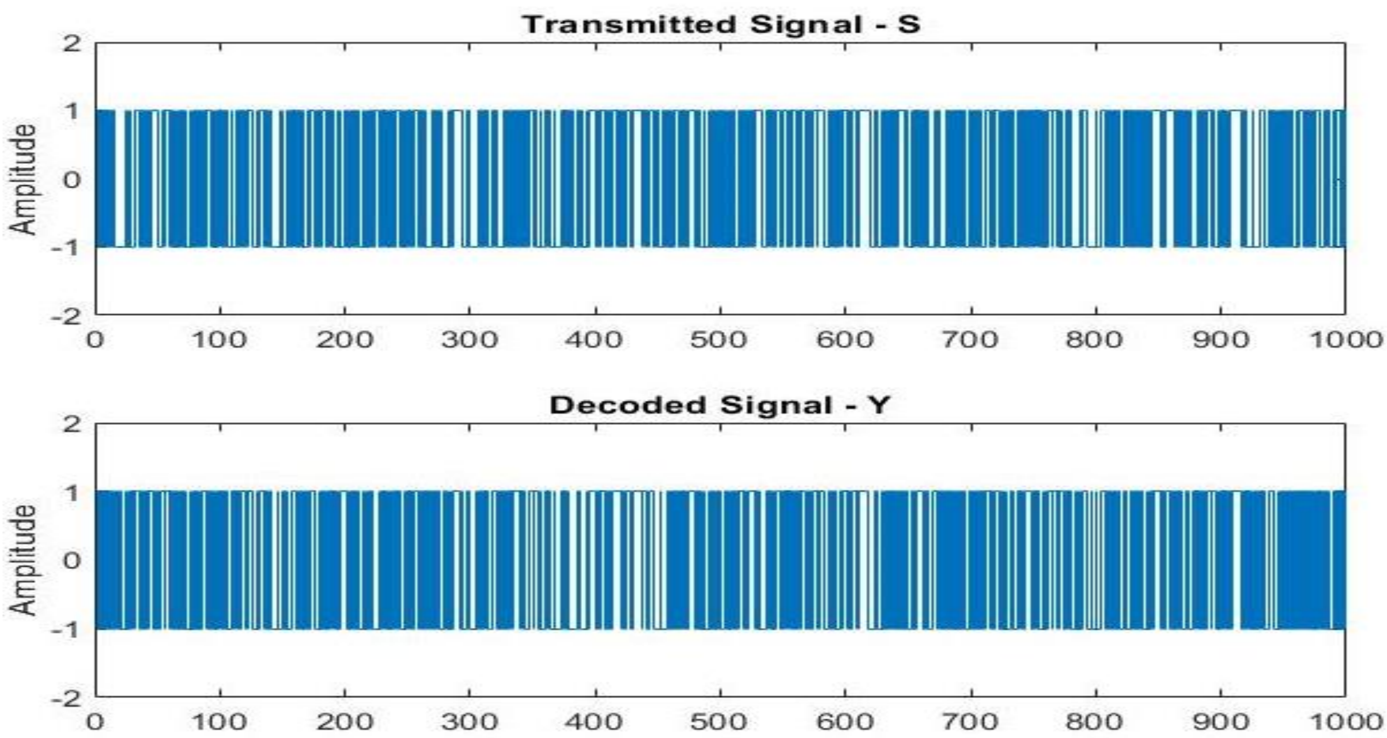

Figure 6: Transmitted Signal and Decoded Signal

To compare the transmitted signal (S) with the decoded signal (Y), the error rate is calculated for several variance values of the AWGN signal.

Variance of the AWGN signal	Error Rate
0.1	0%
0.5	2.3%
1	16.1%
1.5	23.9%
2	33.1%
10	46.8%

Table 1: Error Rates with different variances

As shown in Table 1, the error rate of the received signal is increased when the variance is increased which means that the number of incorrectly received bits is increased with the increment of the variance of the AWGN signal.

Question 5

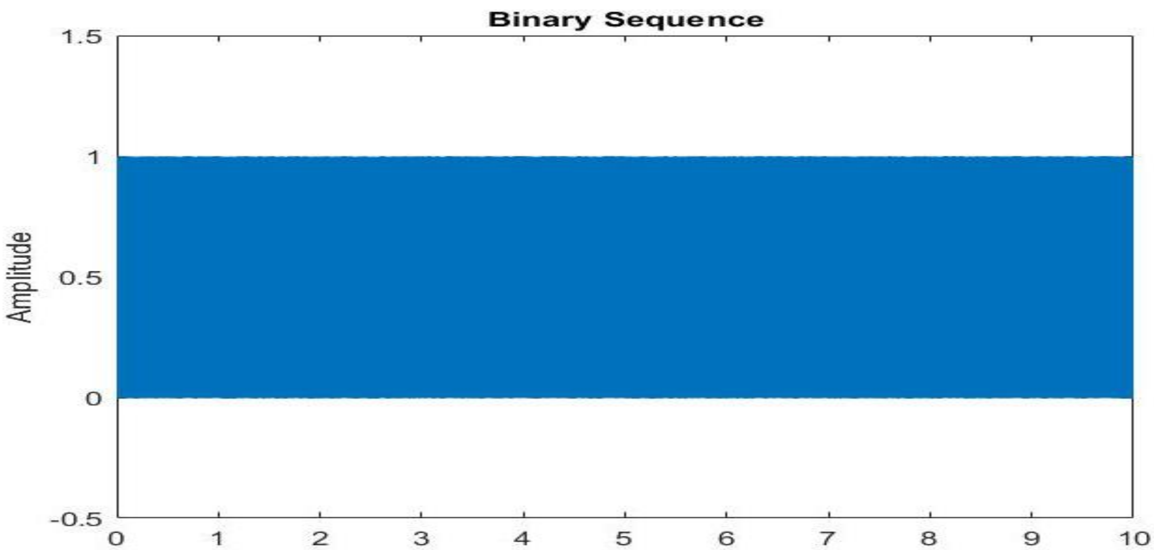

Figure 7: Binary Sequence when L=100000

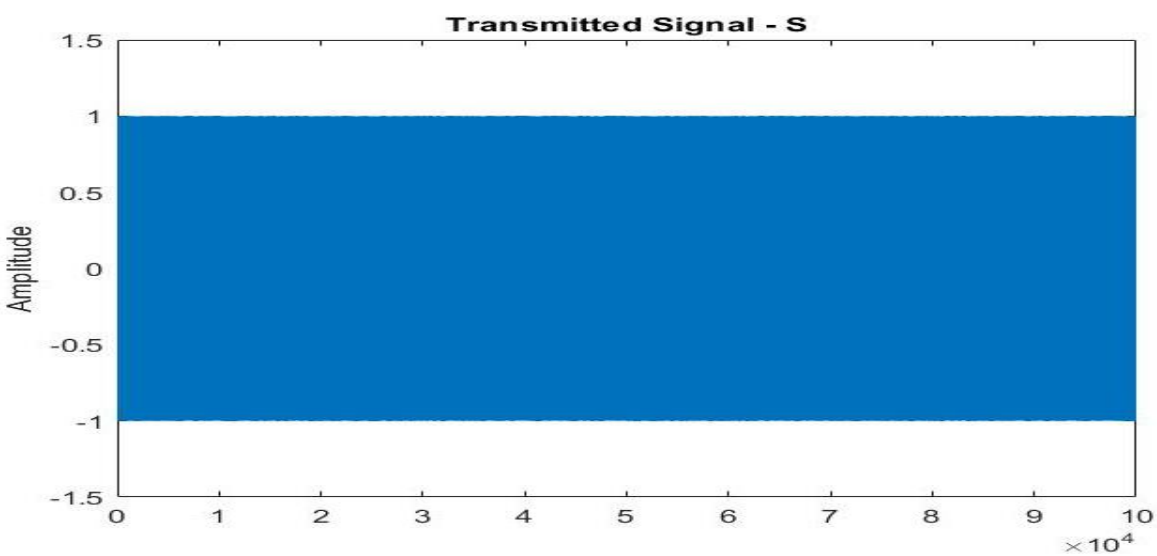

Figure 8: Transmitted Signal when L=100000

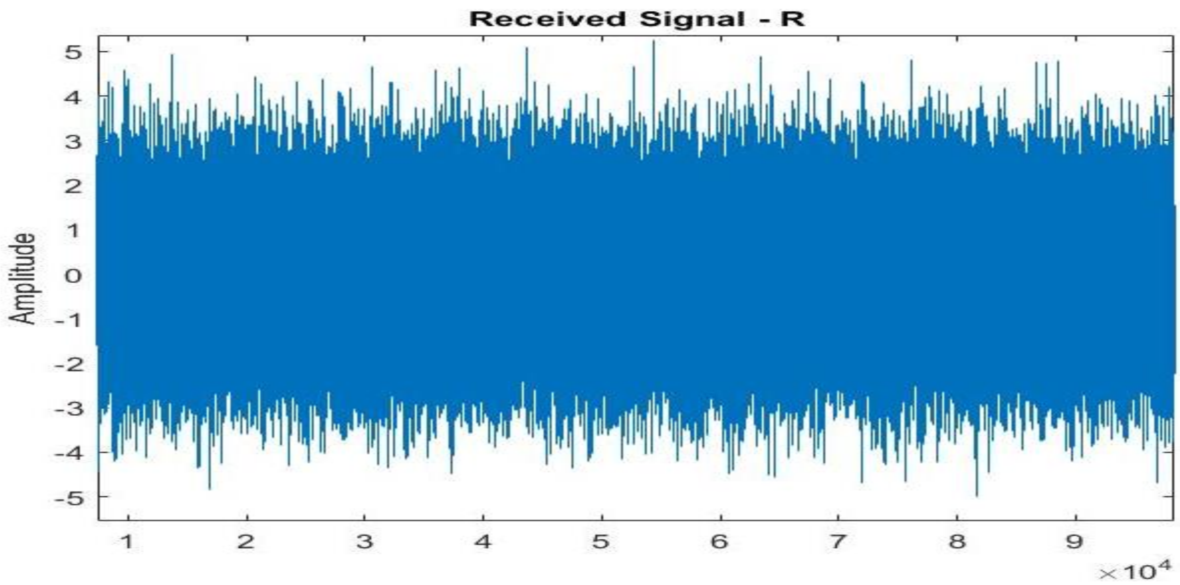

Figure 9: Received Signal when L=100000

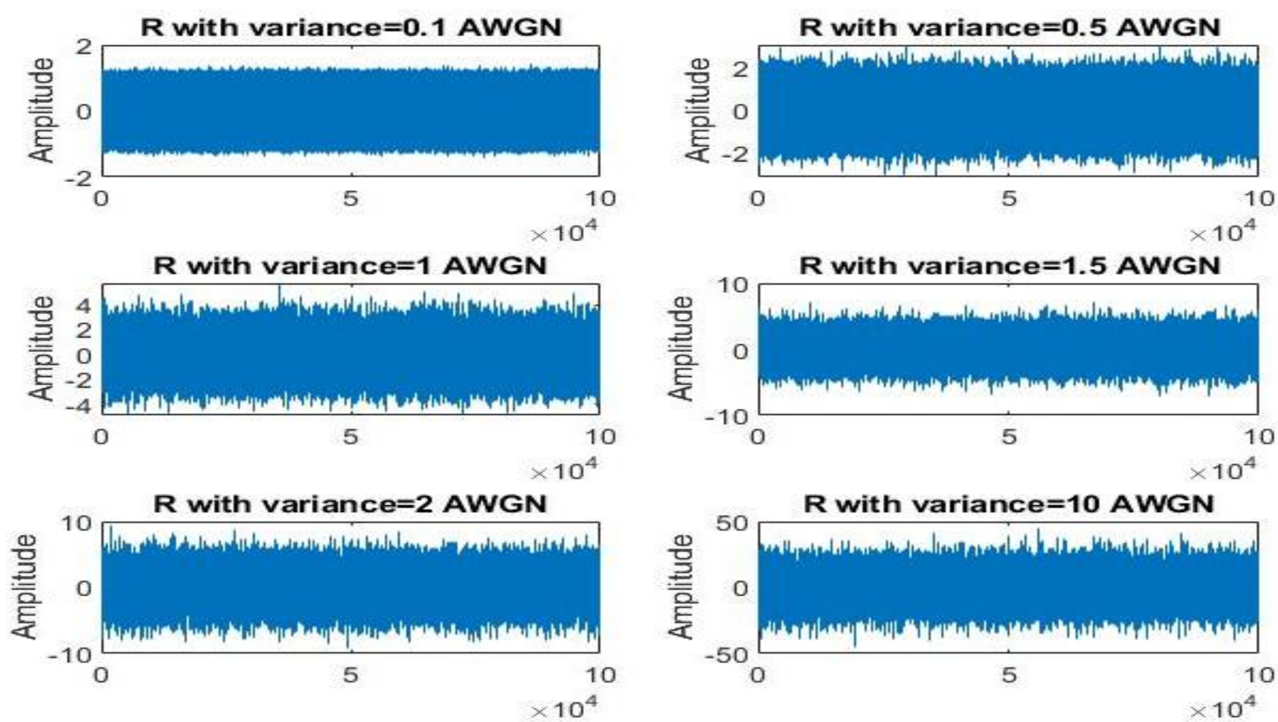

Figure 10: Received Signal comparison by changing the variance of the AWGN Signal when L=100000

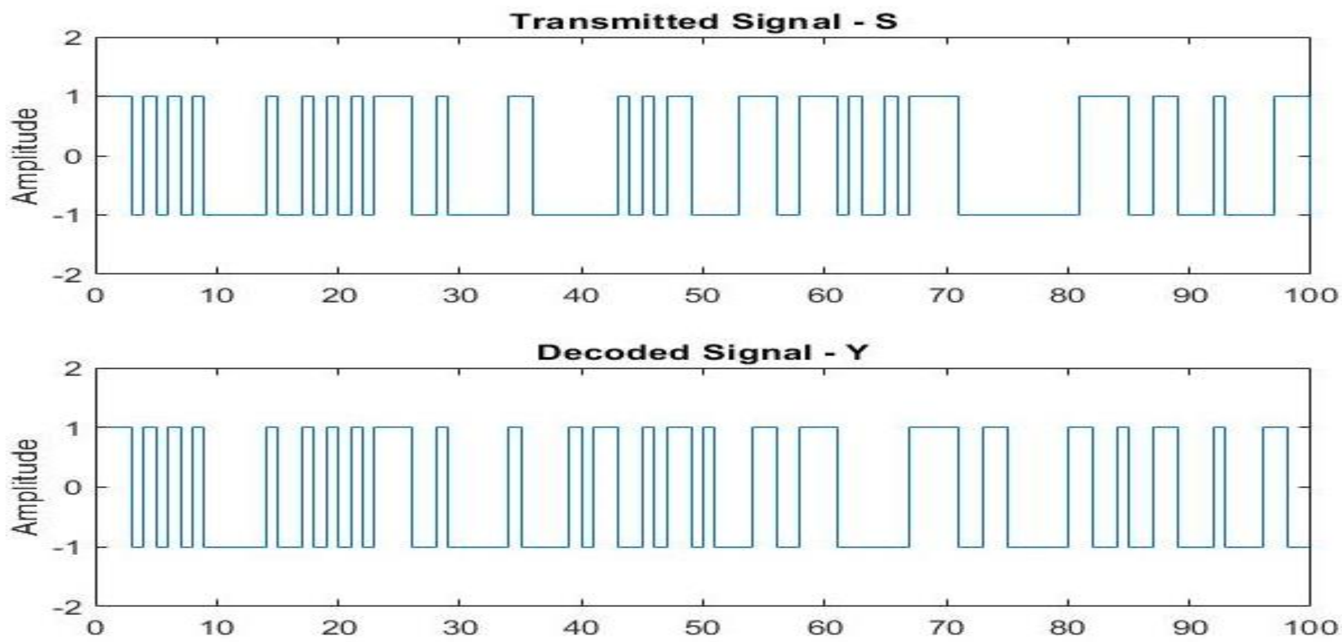

Figure 11: Transmitted Signal and Decoded Signal when L=100000 (Zoomed-In)

Variance of the AWGN signal	Error Rate
0.1	0
0..5	2.25
1	15.805
1.5	25.081
2	31.123
10	45.845

Table 2: Error Rates with different variances when L=100000

As shown in Table 1 and Table 2, error rates for corresponding variance values are quite similar which means that the length of the sequence does not affect the received signal's error rate and it depends on the AWGN signal power.

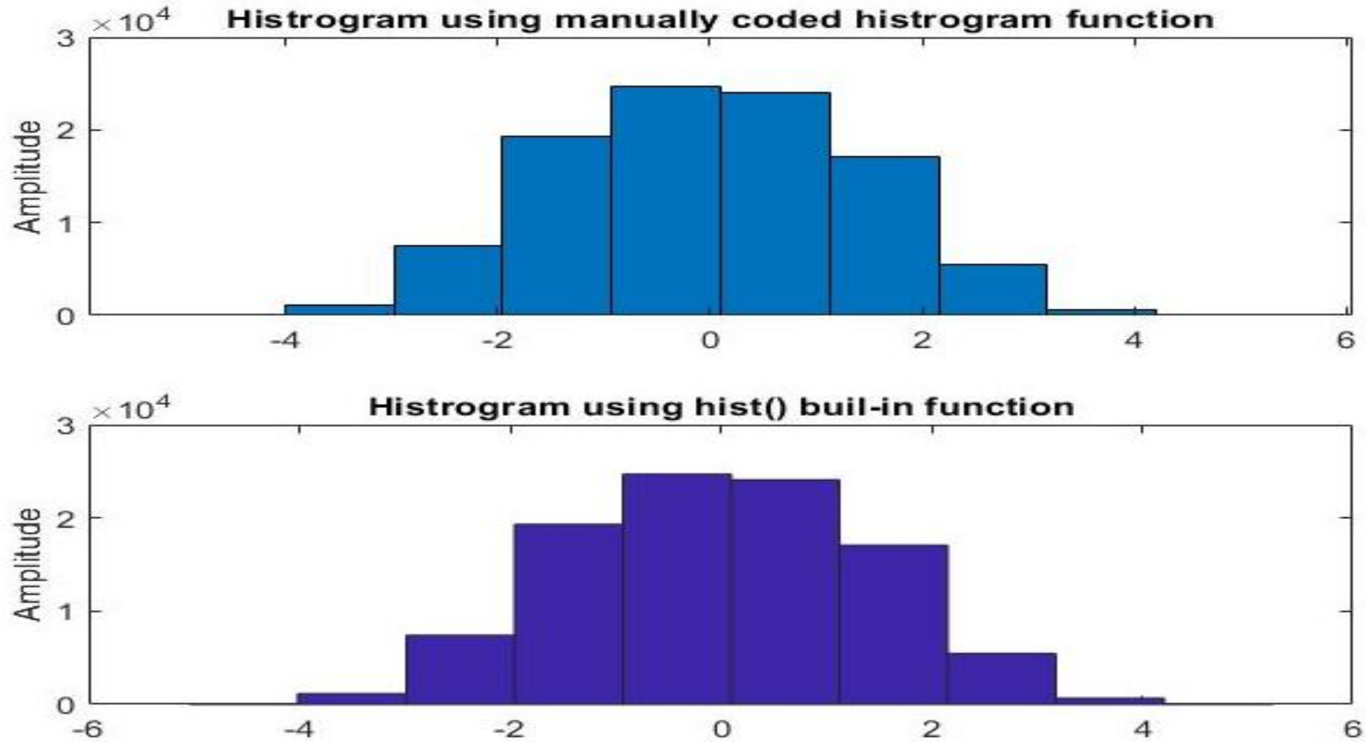

Figure 12: Histogram of the Received Signal with 10 Bins

Figure 12 shows the histograms of received signal which are gain using manually coded histrogram() function and hist() MATLAB built-in function. There is no considerable difference between those histograms.

(a).

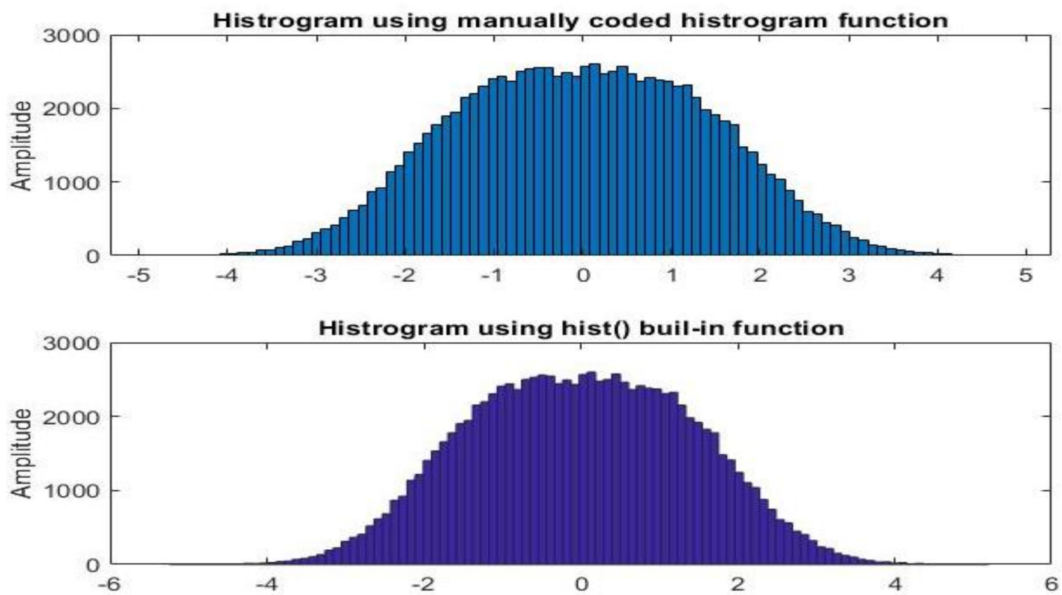

Figure 13: Histogram of the Received Signal with 100 Bins

By increasing the number of bins of the histogram, more details about the received signal can be obtained.

(b).

The following sections are continued using 200 bins for the histogram.

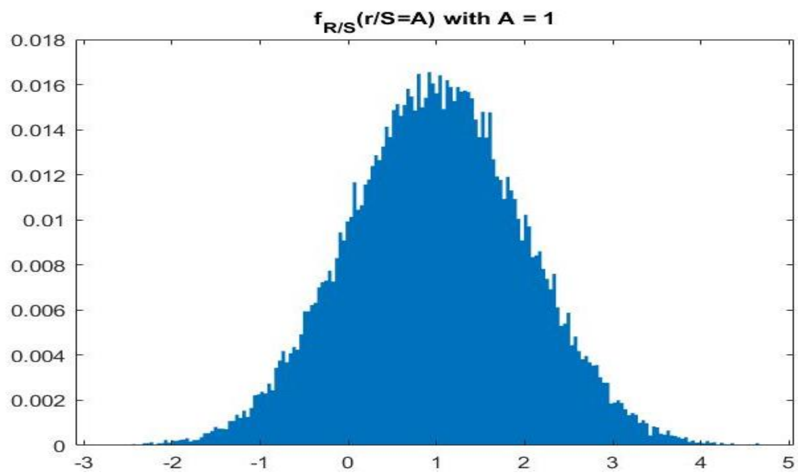

Figure 14: $f_{R/S}(r / S = A)$ with A=1

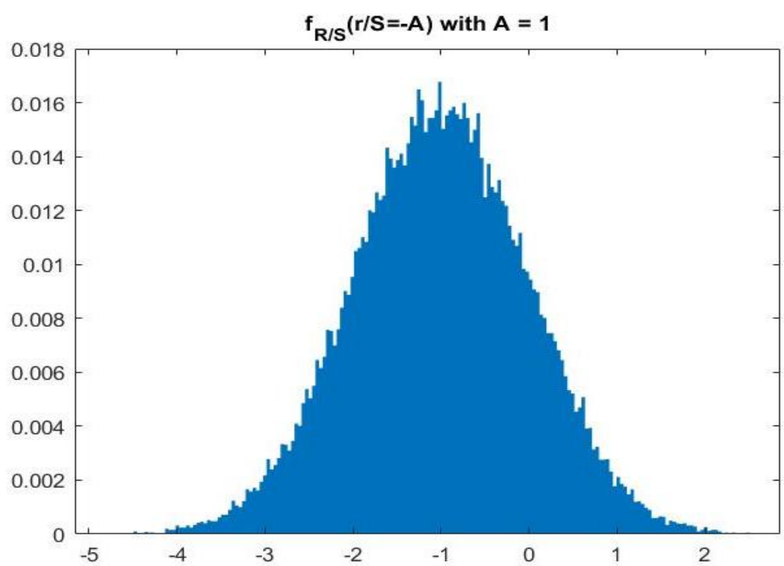

Figure 15: $f_{R/S}(r / S = -A)$ with A=1

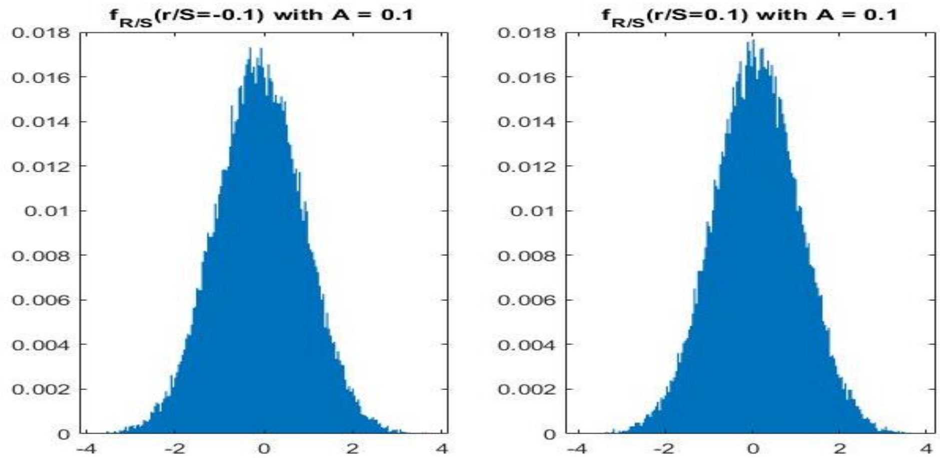

Figure 16: $f_{R/S}(r / S = -A)$ and $f_{R/S}(r / S = A)$ with A=0.1

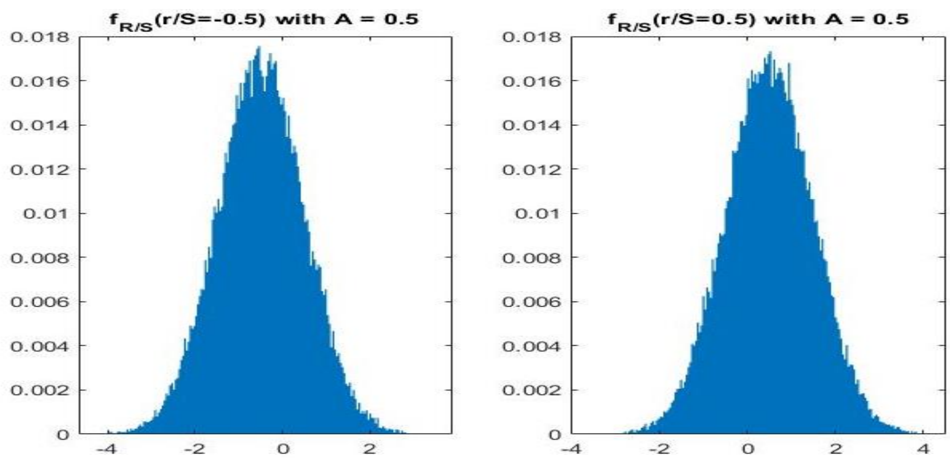

Figure 17: $f_{R/S}(r / S = -A)$ and $f_{R/S}(r / S = A)$ with A=0.5

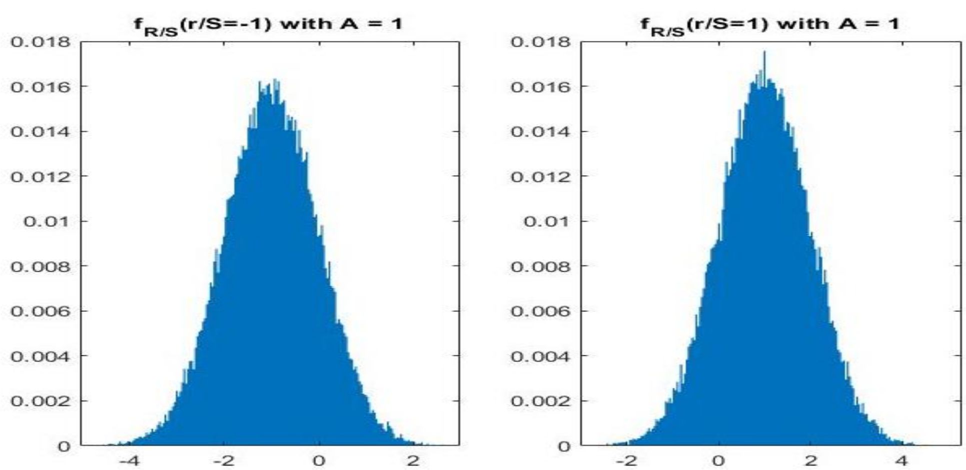

Figure 18: $f_{R/S}(r / S = -A)$ and $f_{R/S}(r / S = A)$ with A=1

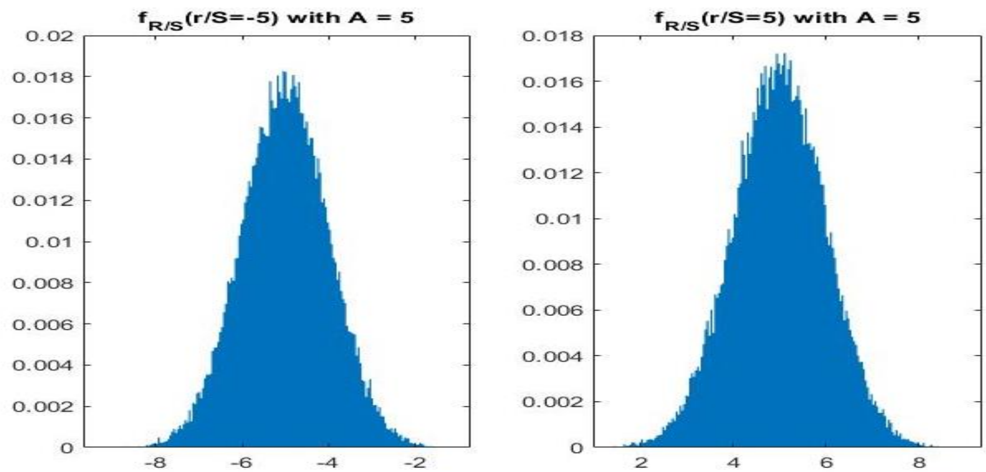

Figure 19: $f_{R/S}(r / S = -A)$ and $f_{R/S}(r / S = A)$ with A=5

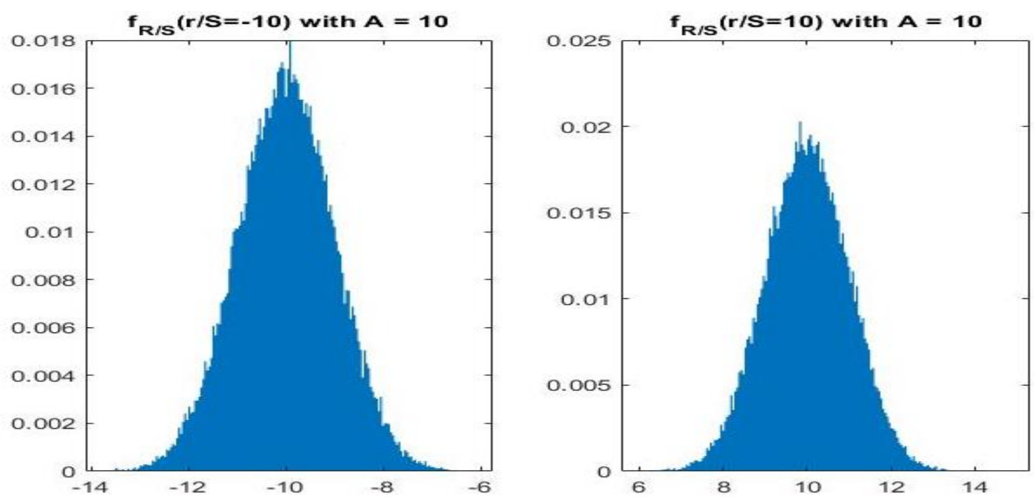

Figure 20: $f_{R/S}(r / S = -A)$ and $f_{R/S}(r / S = A)$ with A=10

As shown in Figure 16, Figure 17, Figure 18, Figure 19, and Figure 20, the shape of the conditional PDFs has not been changed when the 'A' value is changed. But the center of the PDF has been changed. When change the 'A', conditional PDFs are normally distributed around the 'A' value.

(c).

Expected values are calculated by calculating the mean of the received signal's bit array. MATLAB built-in function mean() is used to calculate the mean of the array.

A	E [R / S = A]	E [R / S = -A]	E[R]
0.1	0.0933	-0.1031	-0.0051
0.5	0.4933	-0.5031	-0.0060
1	0.9933	-1.0031	-0.0071
5	4.9933	-5.0031	-0.0161
10	9.9933	-10.0031	-0.0273

Table 3: Expected Values

(d).

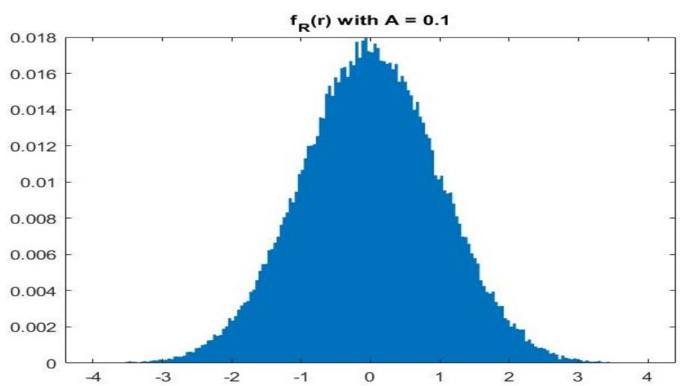

Figure 21: $f_R(r)$ with A=0.1

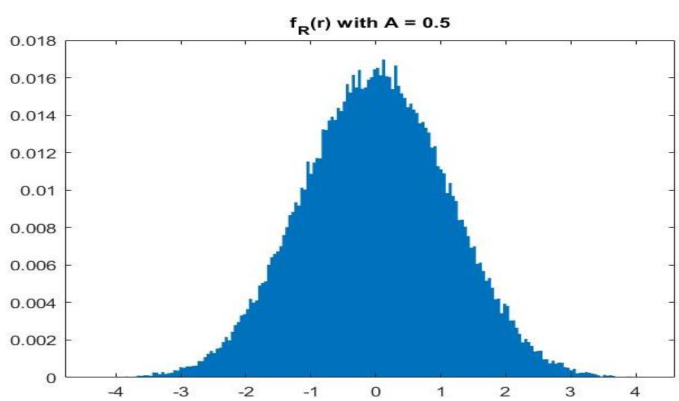

Figure 22: $f_R(r)$ with A=0.5

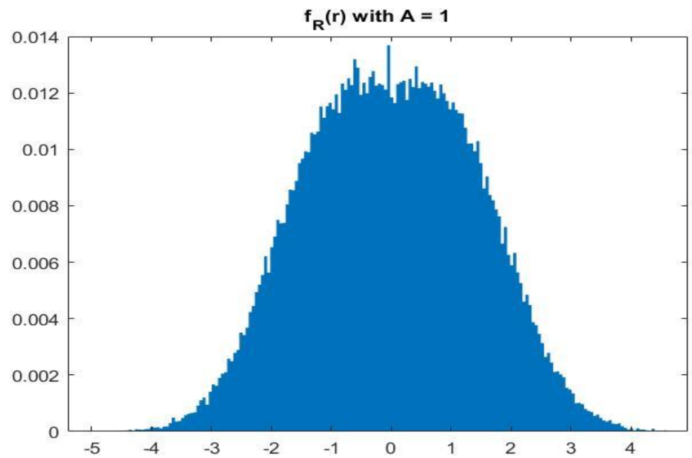

Figure 23: $f_R(r)$ with A=1

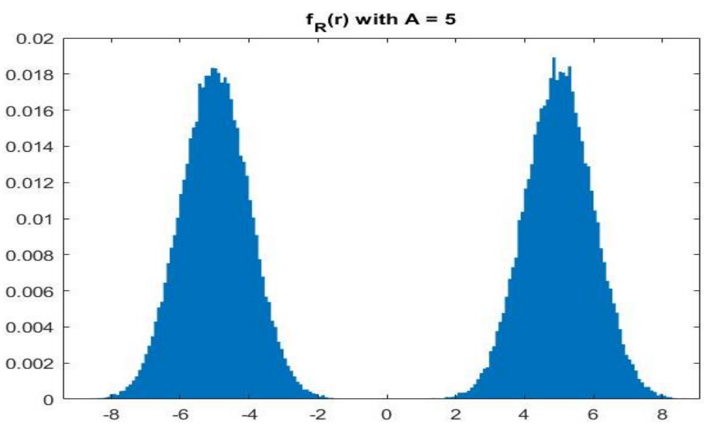

Figure 24: $f_R(r)$ with A=5

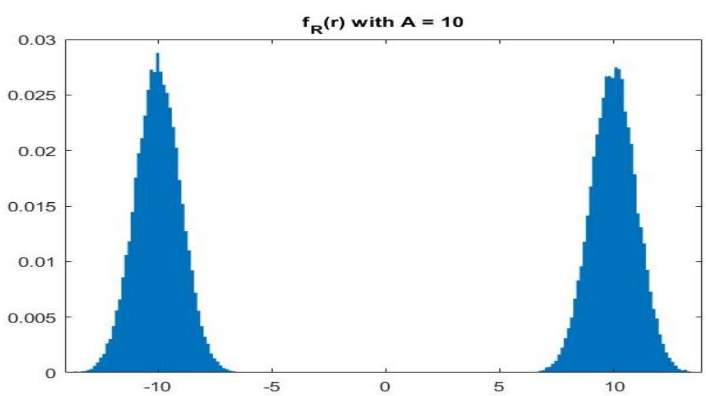

Figure 25: $f_R(r)$ with A=10

Question 6

(b).

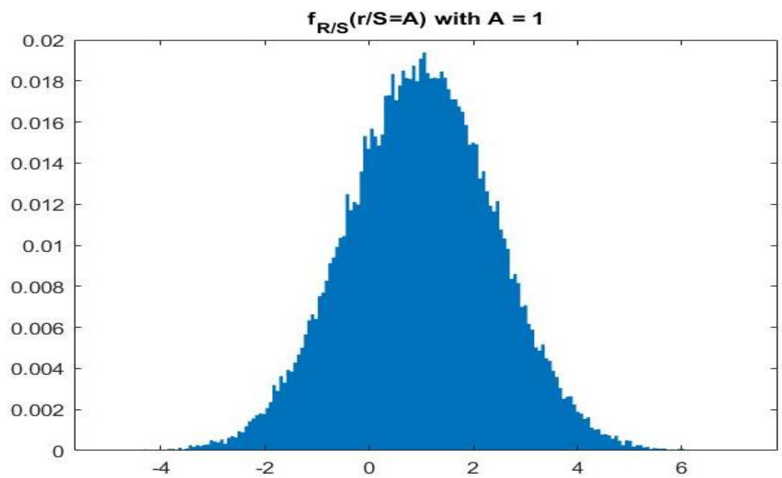

Figure 26: $f_{R/S}(r / S = A)$ with A=1

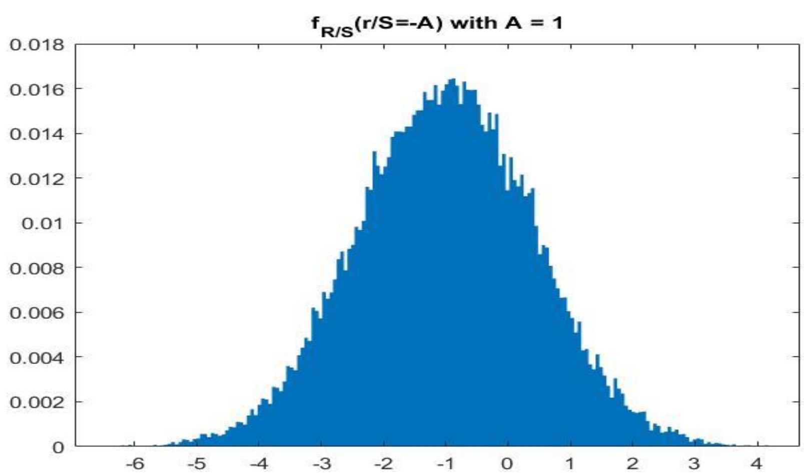

Figure 27: $f_{R/S}(r / S = -A)$ with A=1

When comparing Figure 14 and Figure 26, conditional PDFs have been more spread when an additional interference signal is added to the received signal. Therefore, the variance of the conditional PDFs is increased when an interference signal is involved.

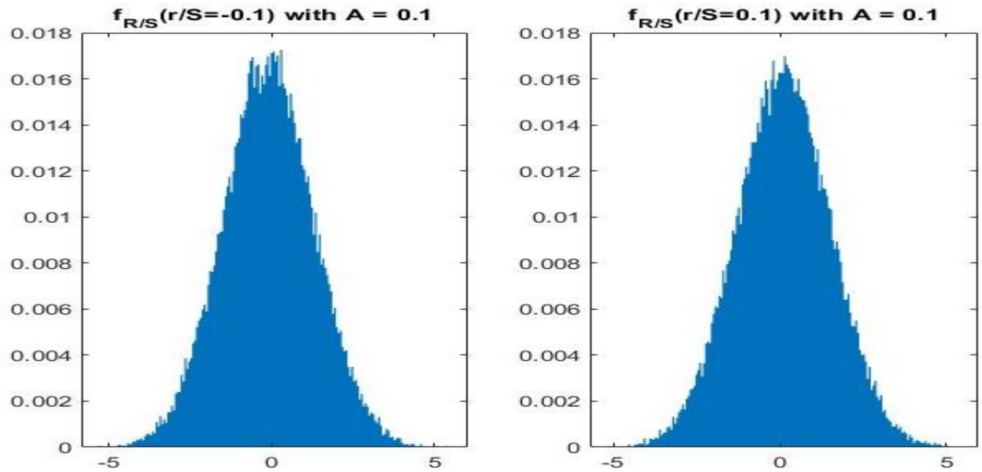

Figure 28: $f_{R/S}(r / S = -A)$ and $f_{R/S}(r / S = A)$ with A=0.1

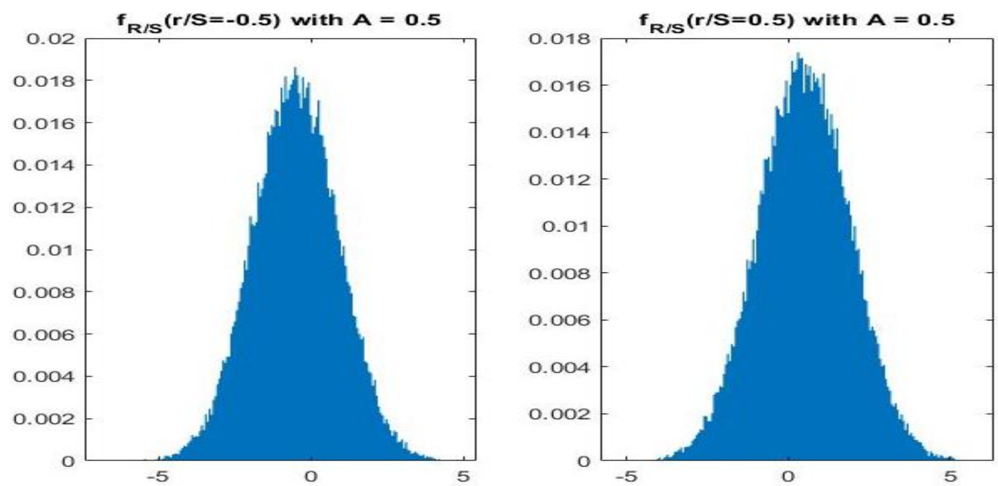

Figure 29: $f_{R/S}(r / S = -A)$ and $f_{R/S}(r / S = A)$ with A=0.5

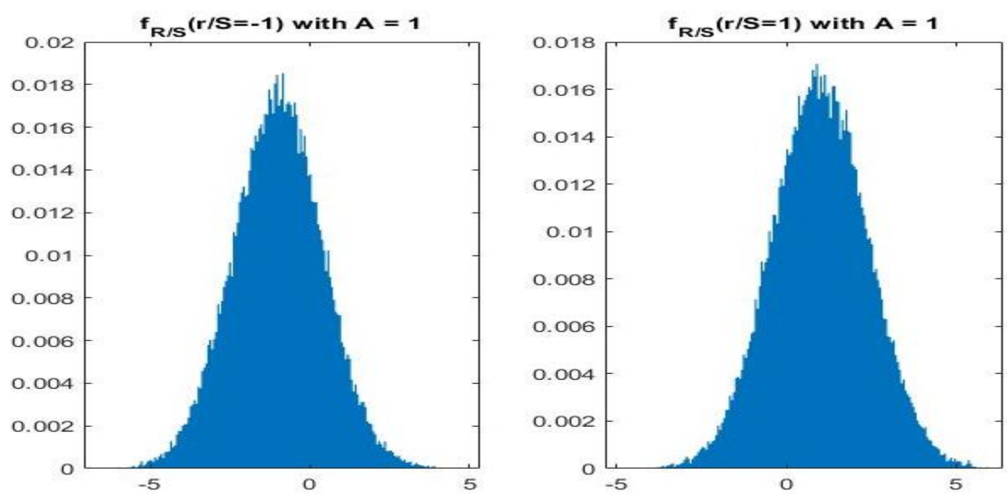

Figure 30: $f_{R/S}(r / S = -A)$ and $f_{R/S}(r / S = A)$ with A=1

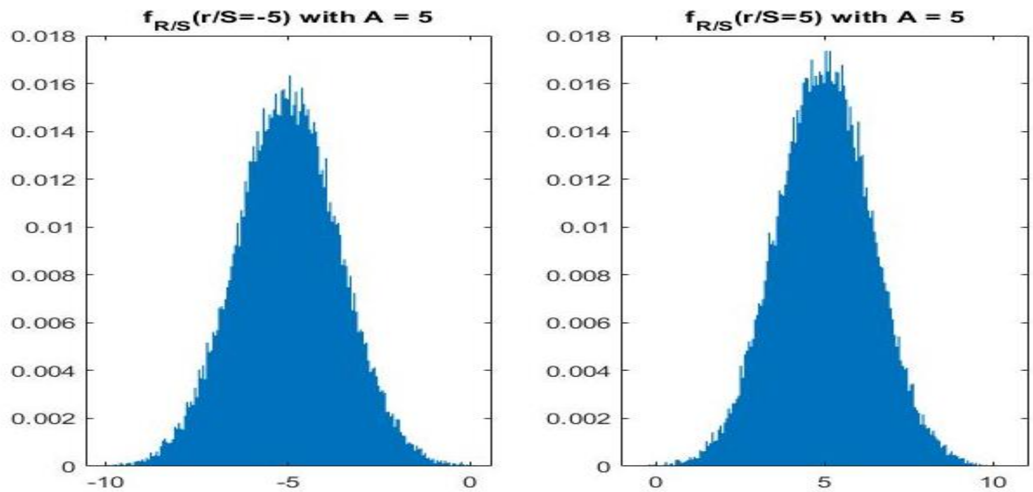

Figure 31: $f_{R/S}(r / S = -A)$ and $f_{R/S}(r / S = A)$ with A=5

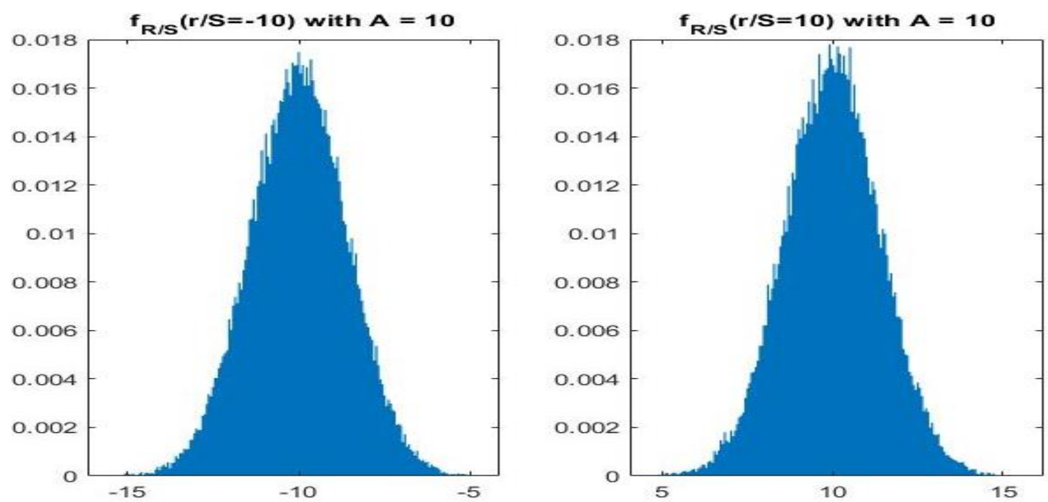

Figure 32: $f_{R/S}(r / S = -A)$ and $f_{R/S}(r / S = A)$ with A=10

(c).

A	E [R / S = A]	E [R / S = -A]	E[R]
0.1	0.1065	-0.0979	0.0042
0.5	0.5065	-0.4979	0.0042
1	1.0065	-0.9979	0.0041
5	5.0065	-4.998	0.0034
10	10.0065	-9.998	0.0025

Table 4: Expected Values

When comparing Table 3 and Table 4, we can see that there is no enormous difference between the expected values when there is an interference signal and not. This is because the mean of the interference signal is zero and the expected values of the received signal are not affected when interference signal is added to it.

(d).

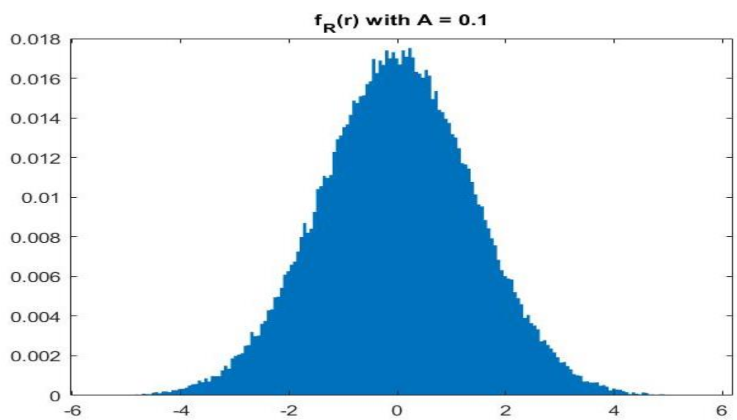

Figure 33: $f_R(r)$ with A=0.1

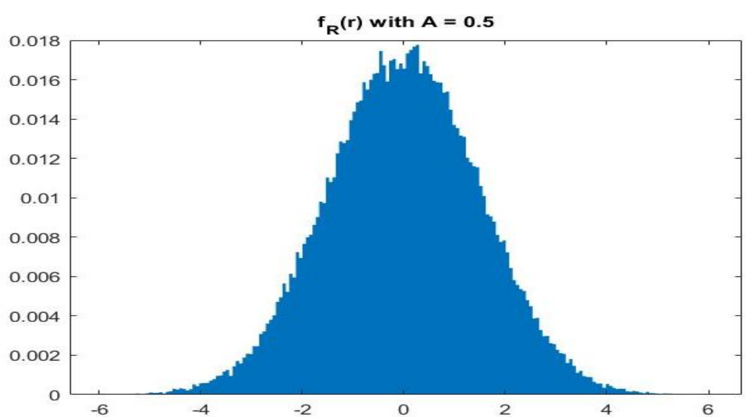

Figure 34: $f_R(r)$ with A=0.5

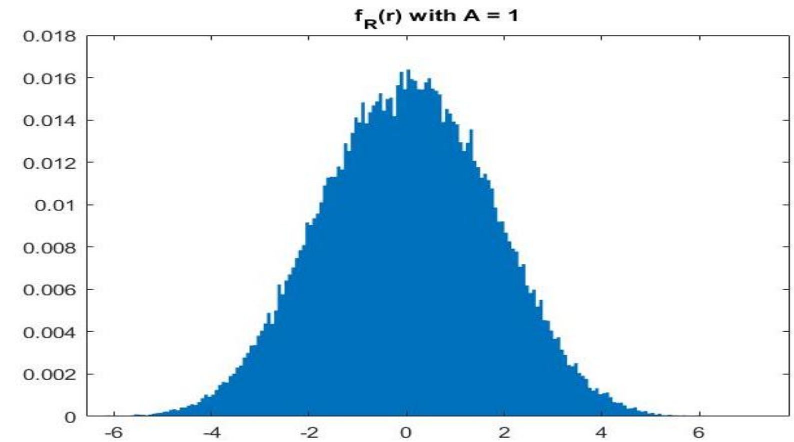

Figure 35: $f_R(r)$ with A=1

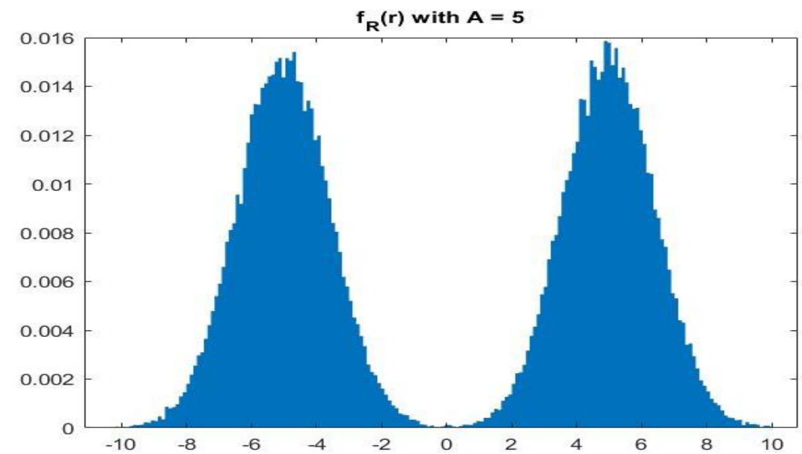

Figure 36: $f_R(r)$ with A=5

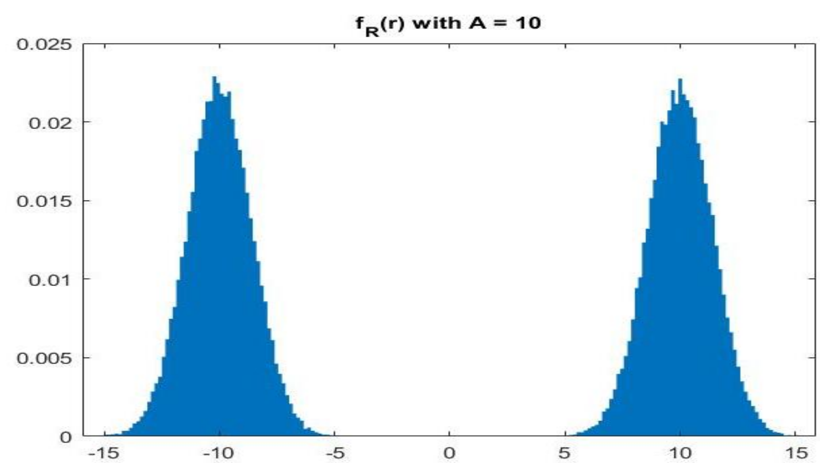

Figure 37: $f_R(r)$ with A=10

When comparing the pard d of question 5 and the part d of question 7, we can see the PDF of the received signal is more spread around 'A' and '-A' when there is an interference signal in addition to the noise signal which means the variance of the received signal is increased due to the addition of the interference signal.

Question 7

Under this section, we change the amplitude of the transmitted signal by multiplying it by a scalar (Alpha). Alpha=1 case is same as the Question 5.

Case I: Alpha=0.1

(b).

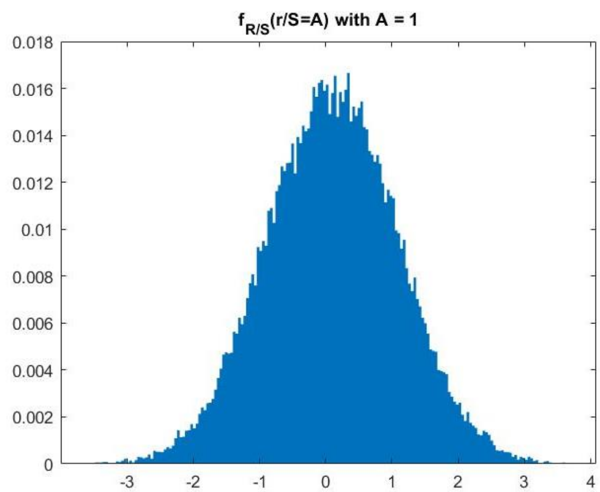

Figure 38: $f_{R/S}(r / S = A)$ with A=1 and Alpha=0.1

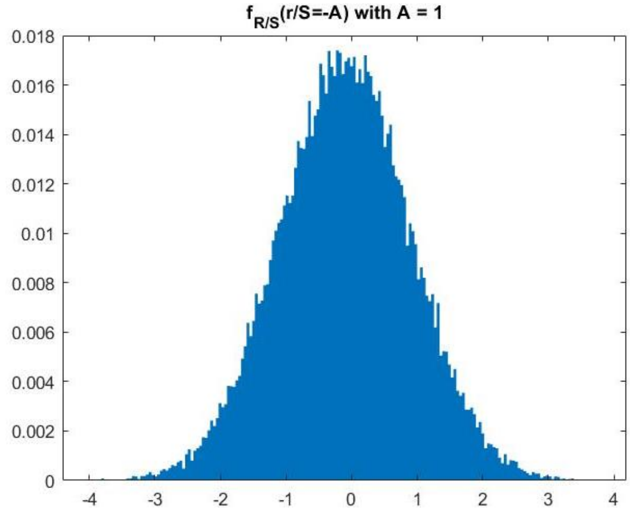

Figure 39: $f_{R/S}(r / S = -A)$ with A=1 and Alpha=0.1

(d).

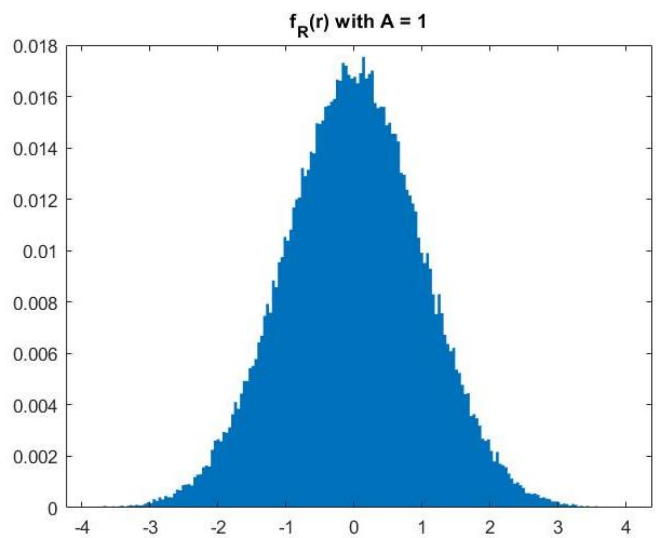

Figure 40: $f_R(r)$ with A=1 and Alpha=0.1

Case II: Alpha=0.5

(b).

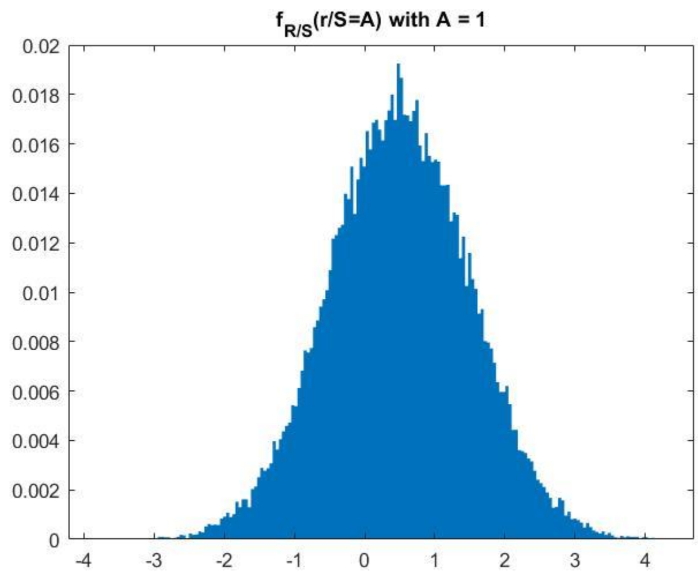

Figure 41: $f_{R/S}(r / S = A)$ with A=1 and Alpha=0.5

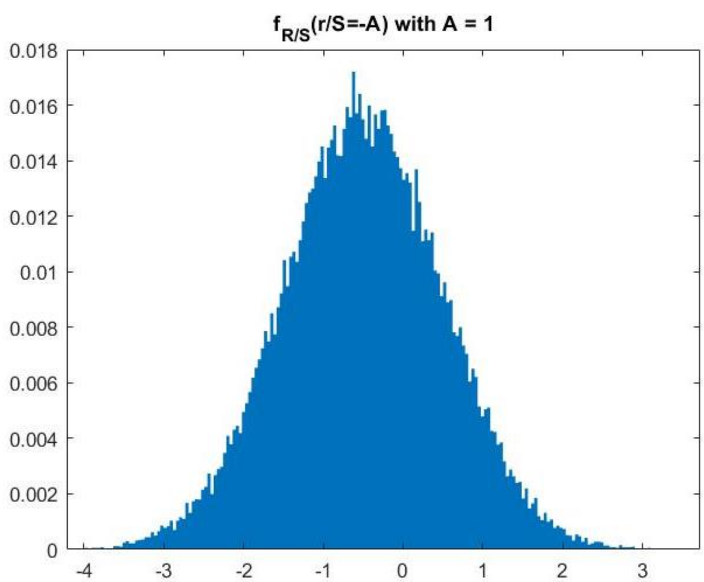

Figure 42: $f_{R/S}(r / S = -A)$ with A=1 and Alpha=0.5

(d).

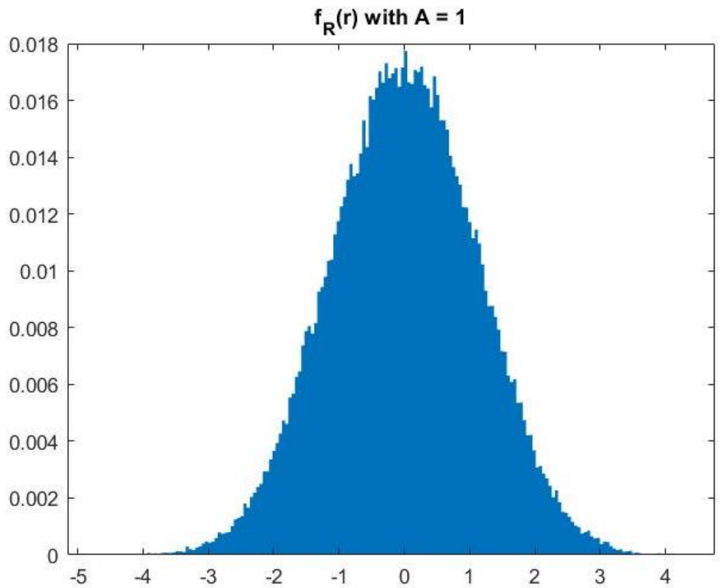

Figure 43: $f_R(r)$ with A=1 and Alpha=0.5

Case III: Alpha =2

(b).

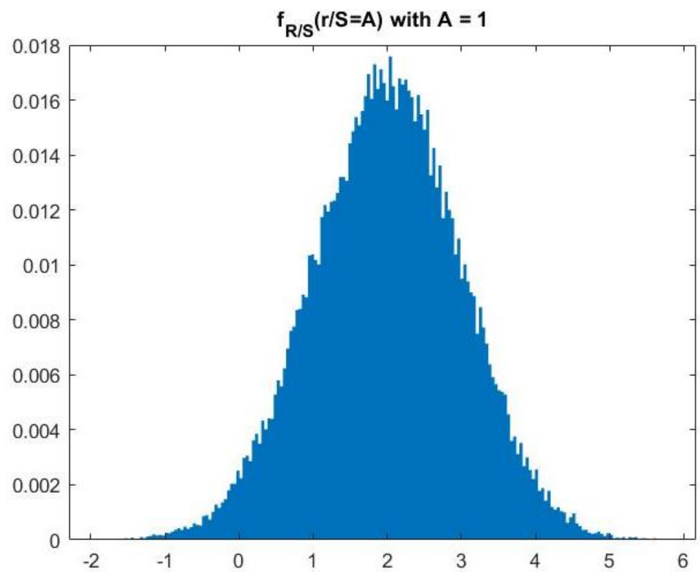

Figure 44: $f_{R/S}(r / S = A)$ with A=1 and Alpha=2

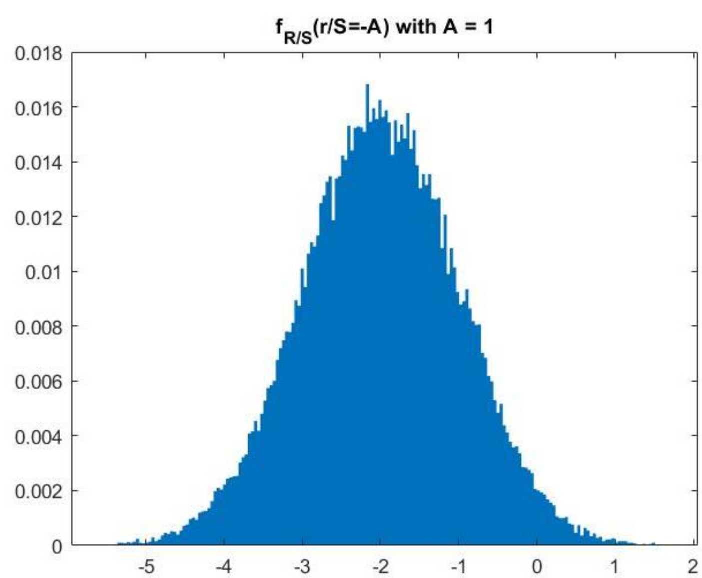

Figure 45: $f_{R/S}(r / S = A)$ with A=1 and Alpha=2

(d).

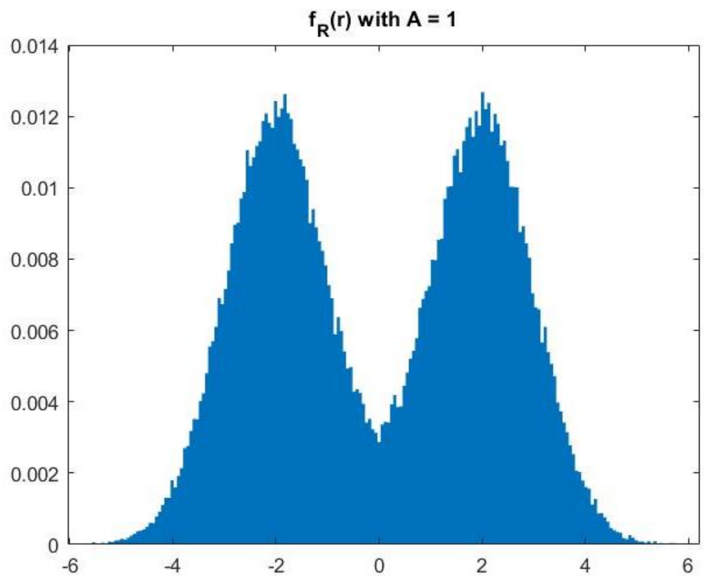

Figure 46: $f_R(r)$ with A=1 and Alpha=2

By comparing above plots with question 5, we can see that the shape of the plots does not change due to the multiplication of the transmitted signal by a scalar. But the variance of the received signal can be changed due to the change in the amplitude of the received signal.

(c).

Alpha	E [R / S = A]	E [R / S = -A]	E[R]
0.1	0.1003	-0.0969	0.00093
0.5	0.2077	-0.4992	0.00625
2	2.0015	-2.0020	-0.0031

Table 5: Expected Values