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BT 3074 - Practical 04

1. Derive the best-fitted line analytically for the function $f(x_i) = mx_i + b$.

1) Best fitted line for $f(x_i) = mx_i + b$.

$$f(x_i) = mx_i + b.$$

If S = sum of squares errors,

$$S = \sum_{i=1}^{n} (y_i - f(x_i))^2$$

$$= \sum_{i=1}^{n} (y_i - (mx_i + b))^2$$

at the minimum S,

$$\frac{\partial S}{\partial m} = 0, \quad \frac{\partial S}{\partial b} = 0.$$

$$\frac{\partial S}{\partial m} = \sum_{i=1}^{n} 2(-x_i)(y_i - (mx_i + b)) = 0$$

$$-2\sum_{i=1}^{n} (y_i - (mx_i + b))x_i = 0$$

$$\sum_{i=1}^{n} (y_i - (mx_i + b))x_i = 0.$$

$$\sum_{i=1}^{n} (y_i x_i - (mx_i^2 + bx_i)) = 0.$$

$$\sum_{i=1}^{n} y_i x_i - \sum_{i=1}^{n} (mx_i^2 + bx_i) = 0.$$

$$\sum_{i=1}^{n} y_i x_i - \sum_{i=1}^{n} mx_i^2 - \sum_{i=1}^{n} bx_i = 0.$$

$$\sum_{i=1}^{n} y_i x_i = \sum_{i=1}^{n} mx_i^2 + \sum_{i=1}^{n} bx_i$$

$$\sum_{i=1}^{n} y_i x_i = m\sum_{i=1}^{n} x_i^2 + b\sum_{i=1}^{n} x_i$$

$$m = \frac{\left(\sum_{i=1}^{n} y_i x_i - b\sum_{i=1}^{n} x_i\right)}{\sum_{i=1}^{n} x_i^2}.$$

$$\frac{\partial S}{\partial b} = \sum_{i=1}^{n} 2(-1)(y_i - (mx_i + b)) = 0$$

$$-2\sum_{i=1}^{n} (y - (mx_i + b)) = 0.$$

$$\sum_{i=1}^{n} (y - (mx_i + b)) = 0.$$

$$\sum_{i=1}^{n} y - \sum_{i=1}^{n} mx_i - \sum_{i=1}^{n} b = 0.$$

$$\sum_{i=1}^{n} y = \sum_{i=1}^{n} mx_i + \sum_{i=1}^{n} b$$

$$\sum_{i=1}^{n} y = \sum_{i=1}^{n} mx_i + bn$$

$$\sum_{i=1}^{n} y = m\sum_{i=1}^{n} x_i + bn.$$

$$b = \frac{\left(\sum_{i=1}^{n} y - m\sum_{i=1}^{n} x_i\right)}{n}$$

2. Consider a data set with 20 data points that change with time and find the best-fit function for that data set.

Insert your data set.

t=0	t=1	t=2	t=3	t=4	t=5	t=6	t=7	t=8	t=9
3	4.5	5	6.5	7.9	9.8	13.3	15	14	15.6
t=10	t=11	t=12	t=13	t=14	t=15	t=16	t=17	t=18	t=19
18.9	18.5	20	21.5	25.1	28.5	29	27.5	29	32.5

Insert the fitted function.

m = 1.5568 , b = 2.4657

$$f(x_i) = 1.5568x_i + 2.4657$$

Insert the figure with data set and the fitted function.