Discrete Dynamical Systems - A Nonlinear Predator-Prey Model

Let's consider a similar population of foxes and rabbits along with the same set of assumptions as in previous practical, but we will model the assumptions differently.

Recall the previous assumptions:

1. The only source of food for the foxes is rabbits and the only predator of the rabbits is foxes.
2. Without rabbits present, foxes would die out.
3. Without foxes present, the population of rabbits would grow.
4. The presence of rabbits increases the rate at which the population of foxes grows.
5. The presence of foxes decreases the rate at which the population of rabbits grows.

We will start with modeling assumptions 2 and 3 the same way:

$$\Delta F_n = F_{n+1} - F_n = -aF_n \tag{4.13}$$
$$\Delta R_n = R_{n+1} - R_n = dR_n \tag{4.14}$$

where $0 < a \leq 1$ and $0 < d \leq 1$. In practical 06 , the coefficients of F_n and R_n were kept constant. In this section we will model them as increasing or decreasing in the presence of the other population. Assumption 4 says that the presence of rabbits increases the rate of growth of foxes, so we write

$$F_{n+1} - F_n = (-a + bR_n)F_n \tag{4.15}$$

where $b \geq 0$. Likewise, assumption 5 says that the presence of foxes decreases the rate of growth of rabbits, so we have

$$R_{n+1} - R_n = (d - cF_n)R_n \tag{4.16}$$

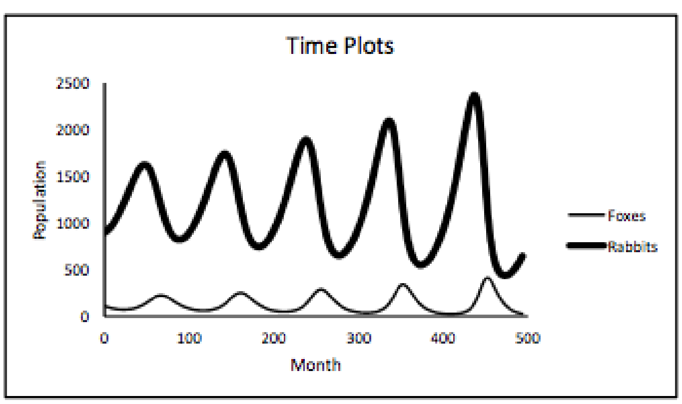
where $c \geq 0$. Rewriting (4.15) and (4.16) we get our model:

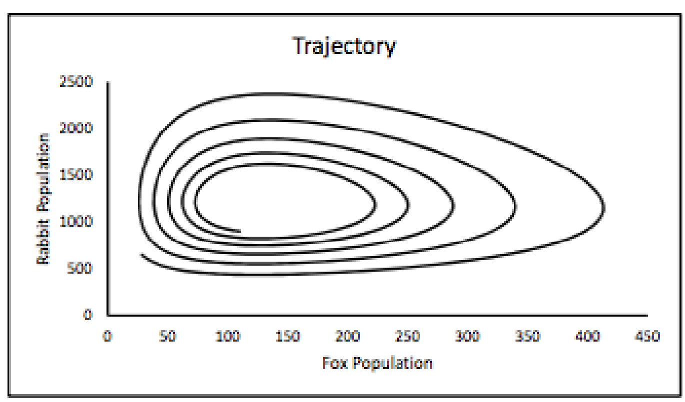
$$F_{n+1} = (1-a)F_n + bR_nF_n \tag{4.17}$$
$$R_{n+1} = -cR_nF_n + (1+d)R_n \tag{4.18}$$

This type of model is called a *Lotka-Volterra* model, named after the researchers that first devised it in the 1920s and 1930s.

Note that both equations have a term involving R_nF_n; thus the model in nonlinear. This term can be interpreted as modeling the number of *interactions* of the two species. These interactions increase the number of foxes while decreasing the number of rabbits. Also note the similarities between this nonlinear model and the linear model in (4.10). We will refer to the parameters in this model using the same names as in the linear model.

The following figures show two time plots and the trajectory of the system with the initial population $(110, 900)$, death and birth factors of foxes 0.88 and 0.0001 and death and birth factors of rabbits 0.0003 and 1.039 respectively.





This model predicts that the populations oscillate with the same period of oscillation, but with a phase shift, meaning they don't reach their peaks at the same time. These oscillations cause the spiraling nature of the trajectories in the graph of rabbits versus foxes. Oscillations such as this are actually observed in nature; thus this model appears to be more reasonable than the linear model.