

Modelling with Proportionality - L3.

$$y = kx_i$$

$$f(x_i) = mx_i$$

$$S = \sum_{i=1}^n (y_i - f(x_i))^2$$

$$\frac{dS}{dm} = \sum_{i=1}^n 2(y_i - mx_i)(-x_i) = 0$$

$$\text{for } y = mx + b, \quad m = \frac{\sum_{i=1}^n x_i y_i}{\sum_{i=1}^n x_i^2}$$

$$\frac{dS}{dm} = \sum_{i=1}^n 2(y_i - (mx + b))(-x_i) = 0$$

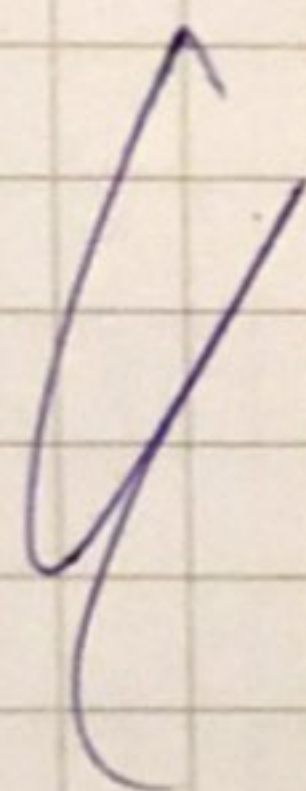
$$= \sum_{i=1}^n (y_i - (mx + b))^2$$

To find minimum function or max. func.

$\rightarrow f'' > 0 \rightarrow \text{min.}$ $f' \text{ at min} = 0$

$f'' < 0 \rightarrow \text{max.}$

$f'' < 0 \rightarrow \text{saddle point.}$



Discrete Models (discrete dynamic system)

$$a_{n+1} = a_n + r a_n = (1+r) a_n$$

$$a_{n+1} = b a_n \Rightarrow b = (1+r), \quad b \neq 0$$

$a_n = b^n a_0 \rightarrow \text{analytical solution.}$
 Long term behaviour.

$$a_{n+1} = b a_n$$

Logistic growth model

$$P_{n+1} = (1+r) P_n$$

$$\Delta P_n = P_{n+1} - P_n = r P_n$$

$r = \text{constant (hourly) growth rate.}$

Model refinement

approaching 0 as k is close.

$$P_{n+1} = P_n + b(k - P_n) P_n$$

$k = \text{carrying capacity.}$

$b = \text{growth rate that balances off other elements (limitations).}$

- intrinsic growth rate. Logistic map / constrained growth model

Logistic eqⁿ for single sp. populaⁿ growth.

$$P_{n+1} - P_n = r P_n \left(1 - \frac{P_n}{k} \right)$$

$$= P_{n+1} = P_n + \frac{rk}{b} \left(1 - \frac{P_n}{k} \right) P_n$$

Is harvesting per month.

$$\Delta P_n = r P_n \left(1 - \frac{P_n}{k} \right) - h P_n$$

$h = \text{harvesting per month.}$

Discrete Dynamical system - Non linear predator prey model.

Linear predator prey model.

$$F_{n+1} - F_n = \Delta F_n = -a F_n + b R_n$$

$$R_{n+1} - R_n = \Delta R_n = -c F_n + d R_n$$

↓ rearrange

$$F_{n+1} = (1-a) F_n + b R_n$$

Death rate of F .

$$R_{n+1} = -c F_n + (1+d) R_n$$

Birth rate of R .

Non linear predator prey model.

$$\Delta F_n = F_{n+1} - F_n = -a F_n + b R_n F_n.$$

$$\Delta R_n = R_{n+1} - R_n = -c F_n R_n + d R_n.$$

* Interaction effect. \downarrow reactivity.

$$F_{n+1} = (1-a)F_n + b R_n F_n.$$

$$R_{n+1} = -c R_n F_n + (1+d)R_n.$$

Lotka Volterra Model.