Modelling with Proportionality - L3.

$y = kx_i$

$S = \sum_{i=1}^{n} (y_i - f(x_i))^2$.

$f(x_i) = mx_i$.

$\frac{dS}{dm} = \sum_{i=1}^{n} 2(y_i - mx_i)(-x_i) = 0$.

$\boxed{m = \frac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2}}$

for $y = mx + b$.

$\cancel{\frac{dS}{dm} = \sum_{i=1}^{n}} = S_1 = \sum_{i=1}^{n} (y_i - f(x))^2 =$

$= \sum_{i=1}^{n} (y_i - (mx + b))^2$.

To find minimum function or max func.

→ $f'' > 0 \rightarrow$ min. f' at min/max $= 0$.

$f'' < 0 \rightarrow$ max.

f'' $6x > 0 \rightarrow$ saddle point.

Discrete Models (discrete dynamic system)

$a_{n+1} = a_n + ra_n = (1+r)a_n$

$a_{n+1} = ba_n \Rightarrow b = (1+r)$. $b \neq 0$.

$a_n = b^n a_0 \rightarrow$ analytical solution.

↳ Long term behaviour.

$a_{n+1} = ba_n$

Logistic growth model

$P_{n+1} = (1+r)P_n$ ← Difference eq^n

$\Delta P_n = P_{n+1} - P_n = rP_n$

r = constant (hourly) growth rate.

Model refinement

approaching 0 as k is close.

$P_{n+1} = P_n + b(k - P_n)P_n$. ← Discrete logistic eq^n

k = carrying capacity.

b = growth rate that balances off other elements. (limitation).

- intrinsic growth rate. / logistic map / constrained growth model

Logistic eq^n for single sp. population growth.

$= P_{n+1} - P_n = rP_n\left(1 - \frac{P_n}{K}\right)$

$= P_{n+1} = P_n + \frac{rk}{b}\left(1 - \frac{P_n}{K}\right)P_n$

If harvesting per month.

$\Delta P_n = rP_n\left(1 - \frac{P_n}{K}\right) - hP_n$.

h = harvesting per month.

Discrete Dynamical system - Non linear predator prey model.

Linear predator prey model.

$F_{n+1} - F_n = \Delta F_n = -aF_n + bR_n$

$R_{n+1} - R_n = \Delta R_n = -cF_n + dR_n$.

↓ rearrange.

$F_{n+1} = \underbrace{(1-a)}_{\text{Death rate of F}} F_n + bR_n$.

$R_{n+1} = -cF_n + \underbrace{(1+d)}_{\text{Birth rate of rabbits}} R_n$.

Non linear Predator prey model.

$$\Delta F_n = F_{n+1} - F_n = -aF_n + bR_nF_n.$$

$$\Delta R_n = R_{n+1} - R_n = -cF_nR_n + dR_n.$$

* Interaction effect.

rewrite.

$$F_{n+1} = (1-a)F_n + bR_nF_n.$$

$$R_{n+1} = -cR_nF_n + (1+d)R_n.$$

Lotka Volterra Model.