Report

Learning Phase Transitions by Confusion on the 2D Ising Model

Project Members

Bhagyaraju Akepogu

Aadarsh Namala

Contents

1 Aim

The aim of this project is to study phase transitions in the 2D Ising model using a novel machine learning approach called the *Confusion Scheme*...

2 Theory

The 2D Ising model consists of spins on a square lattice...

$$H = -J \sum_{\langle i,j \rangle} s_i s_j$$

where J is the interaction strength, and the summation is over nearest-neighbor spin pairs.

At a critical temperature T_c, the system undergoes a second-order phase transition from an ordered (magnetized) phase at low temperature to a disordered (paramagnetic) phase at high temperature.

Machine learning, specifically neural networks, is employed here not to predict phases directly, but to identify the phase transition point by exploiting the structure in data across temperatures through a *Confusion Scheme*.

3 Methodology

3.1 Data Generation

- Spin configurations are generated using the Metropolis Monte Carlo algorithm.
- A 2D lattice is initialized randomly.
- The system is equilibrated for a number of steps.
- Configurations (samples) are recorded over a temperature range (e.g., 1.0 to 3.5).
- For each temperature, multiple uncorrelated samples are collected.

3.2 Confusion Scheme

- A trial critical temperature T_c' is selected.
- Labels are assigned as follows:
  - Label 0 if $T < T_c'$
  - Label 1 if $T > T_c'$
- A neural network is trained on this mislabeled dataset.
- The prediction accuracy is measured over the entire dataset.
- This process is repeated for different trial values T_c', producing a characteristic W-shaped accuracy curve.
- The center peak of this curve corresponds to the true critical temperature.

4 Code Parameters

- **L**: The linear size of the 2D Ising lattice. Total spins = $L \times L$. Larger L gives sharper phase transitions.
- **T_values**: A list/array of temperatures (e.g., 1.0 to 3.5) covering ordered and disordered phases.
- **n_samples**: Number of spin configurations per temperature. More samples = better neural net performance.
- **beta**: Inverse temperature, $\beta = \frac{1}{T}$, used in the Metropolis algorithm.
- **Tc_trial / T_c'**: Guessed critical temperature to assign labels in the confusion scheme.
- **epochs**: Number of training iterations.
- **batch_size**: Number of samples per weight update. Smaller = faster but noisier.
- **learning_rate**: Step size in weight updates. Too high = unstable, too low = slow.
- **regularization** (e.g., L2): Prevents overfitting by penalizing large weights.
- **hidden_units**: Neurons in the hidden layers. Balances model complexity.

5 Expected Results

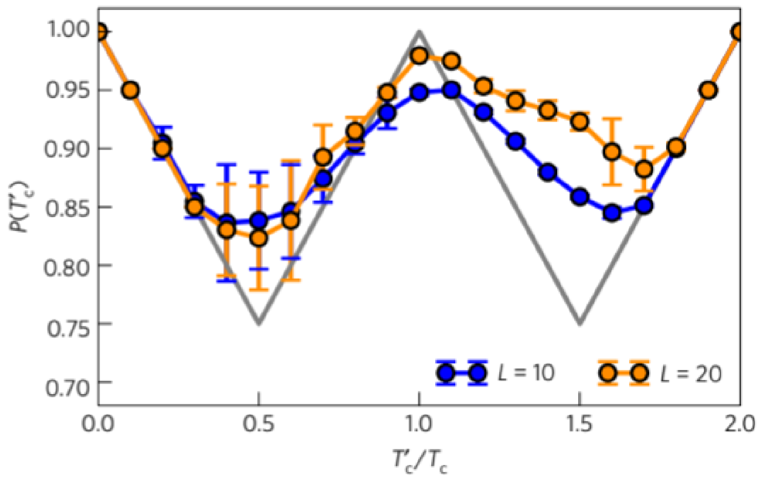

Figure 1: W-shaped accuracy curve from the Confusion Scheme.

Y-Axis: $P(T_c')$
This is the classification accuracy of the neural network.
For each guessed trial critical temperature T_c', it shows the percentage of correct predictions made by the model.
In simpler terms: How well the model separates the configurations into two groups based on the artificial labels assigned using T_c'.

X-Axis: $\frac{T_c'}{T_c}$
This is the guessed critical temperature T_c' normalized by the true critical temperature $T_c \approx 2.27$.
For example, $\frac{T_c'}{T_c} = 1$ means your guess matches the actual critical temperature.
This axis helps visualize how well different guesses of T_c' perform in classifying the data.

6 Observed Results and Conclusion

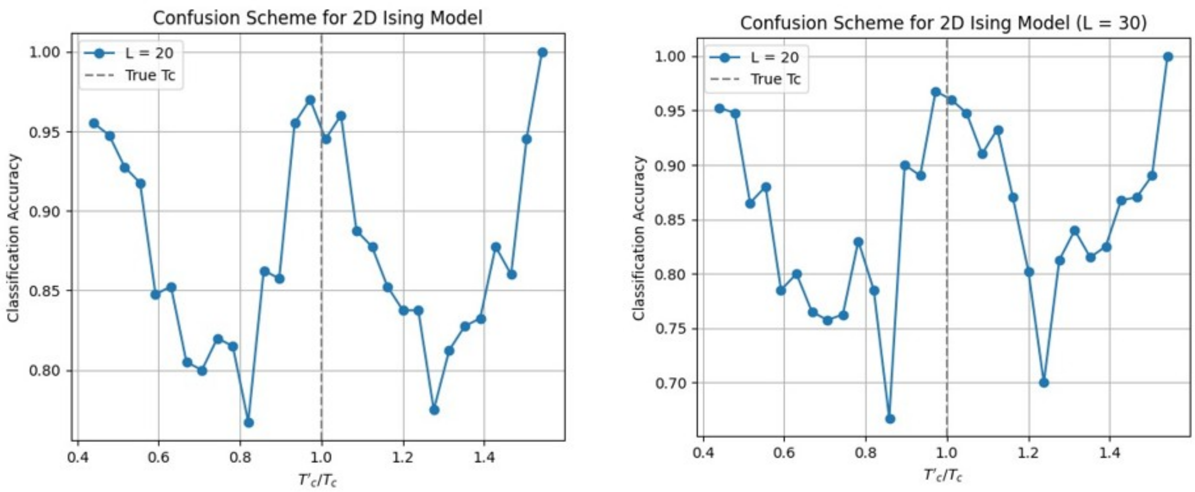

Figure 2: Two plots run for the same $L = 20$ configurations where T_c is kept at 2.27

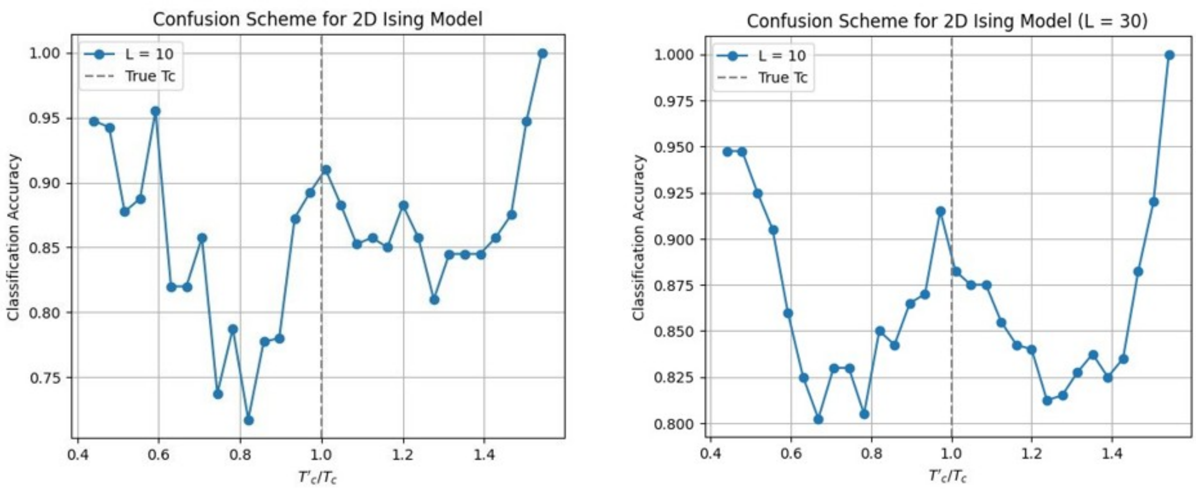

Figure 3: Two plots run for the same $L = 10$ configurations where T_c is kept at 2.27

From the observed graphs, I noticed that:

- The phase transition occurs at around 1 on the x-axis, where $T_c' = T_c$. This is done entirely without any additional information except for the reference value of $T_c = 2.27$ used for the x-axis.
- The points right next to the peak on either side fluctuate more as the value of L changes in the code. One can average the number of observations for observing the error significance. Significant error is observed when the points do not align with the ideal W-shape.

References

[1] Van Nieuwenburg, E. P., Liu, Y. H., & Huber, S. D. (2017). Learning phase transitions by confusion. *Nature Physics*, 13(5), 435–439.