

Student name - Bhagyashri Bhamar

Register number - 181I1712

Course code - Probability
and Statistics (PT302)

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(Q.1)

A - Event

Event A - I is transmitted

Event \bar{A} - O is transmitted

Event B - I is received

Event \bar{B} - O is received.

Given

$$P(A) = 0.5$$

$$P(B/A) = 0.95$$

$$P(B/A) = 0.90$$

$$P(\bar{A}) = 0.5$$

$$\begin{aligned} P(\bar{B}/A) &= 1 - P(B/A) \\ &= 1 - 0.9 \\ &= 0.1 \end{aligned}$$

$$\begin{aligned} P(B/\bar{A}) &= 1 - P(\bar{B}/\bar{A}) \\ &= 1 - 0.95 = 0.05 \end{aligned}$$

B

P

$$\text{a) } P(B) = P(B/A) \times P(A) + P(B/\bar{A}) \times P(\bar{A})$$

$$= 0.90 \times 0.5 + 0.05 \times 0.5$$

$$= 0.475$$

Hence the Probability Of being reciv
0.475

$$P(B) = 0.475$$

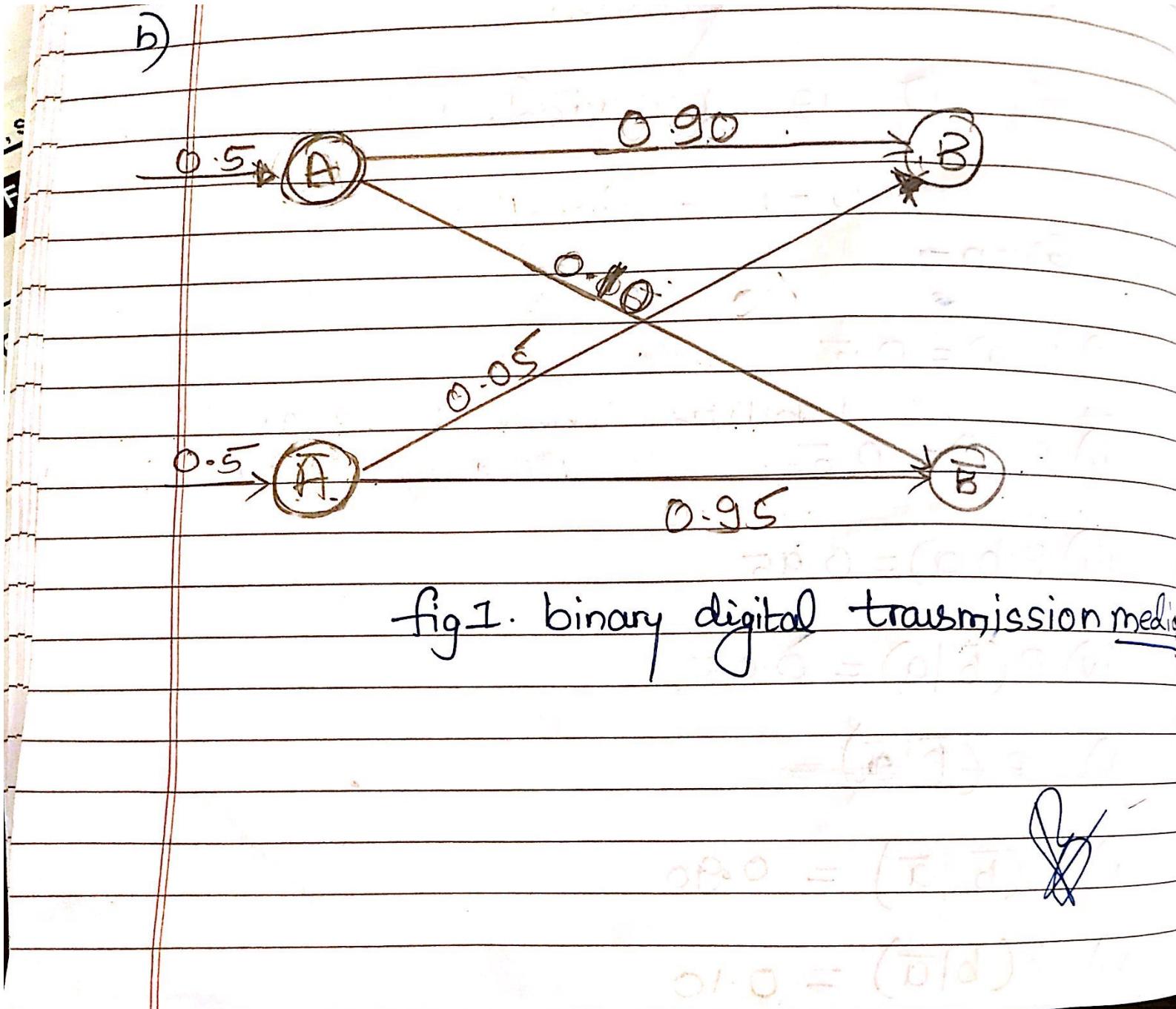


fig 1. binary digital transmission media

Q(2)

 $P(A)$ - Student own a car $P(B)$ - Student own bicycle. $P(C)$ - Student Own two wheeler.

a) Given

$$P(A) = 0.3$$

$$P(B) = 0.6$$

$$P(B|A) \doteq 0.5 \quad P(C) = 0.3$$

$$\frac{P(B \cap A)}{P(A)} = 0.5$$

$$P(B \cap A) = 0.5 \times P(A)$$

$$P(B \cap A) = 0.5 \times 0.3$$

$$P(B \cap A) = 0.15$$

The probability that random selected student own a car and bicycle is 0.15.

B/

$$\textcircled{b} \quad P(B \cap \bar{A}) = P(B) - P(A \cap B) = 0.6 - 0.15 \\ = 0.45$$

Probability that randomly selected student own a bicycle but does not own a car is 0.45

$$\textcircled{c} \quad P(C \cup B \cup A) = P(A) + P(B) + P(C) - P(B \cap A) \\ - P(C \cap B) \\ = 0.3 + 0.6 + 0.3 - 0.15 - 0.1 \\ = 0.9$$

$$1 - P(A \cup B \cup C) = 1 - 0.9 \\ = 0.1$$

Probability that randomly selected student owns a bicycle / does not own any of three is 0.1

8

$$\text{Q3. } P(B \cap \bar{C} \cap \bar{A}) = P(B) - P(B \cap A) - P(B \cap C)$$
$$= 0.6 - 0.15 - 0.15$$
$$= 0.3.$$

$$P(B \cap \bar{C} \cap \bar{A}) = 0.3$$

B

Q3

$$P = \frac{30}{100}$$

$$q = \frac{70}{100}$$

$$n = 500$$

$$\text{mean} = np$$

$$= 500 \times \frac{30}{100}$$

$$= 150$$

$$\sigma = \sqrt{npq} = \sqrt{500 \times \frac{30}{100} \times \frac{70}{100}} = \sqrt{105}$$

$$= 10.2469$$

for $\alpha = 84.5$

$$Z = \frac{84.5 - 150}{10.2469}$$

$$= -6.3921$$

$$Z = \frac{\alpha - \mu}{\sigma}$$

$$P(Z \geq -6.3921) = 1 - P(Z < -6.3921)$$

$$= 1 - 0$$

$$= 1$$

3(b) ~~P~~

$$P = \frac{60}{100} \quad q = \frac{40}{100} \quad n = 500$$

$$\mu = n \times p = 500 \times \frac{60}{100} = 300$$

$$\sigma = \sqrt{npq} = \sqrt{500 \times \frac{60}{100} \times \frac{40}{100}} = \sqrt{120}$$

$$= 10.9544$$

for $x = 84.5$

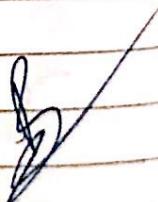
$$z = \frac{84.5 - 300}{10.9544}$$

$$= -19.6724$$

$$P(Z \geq -19.67) = 1 - P(Z < 19.6724)$$

$$= 1 - 0$$

$$= 1$$



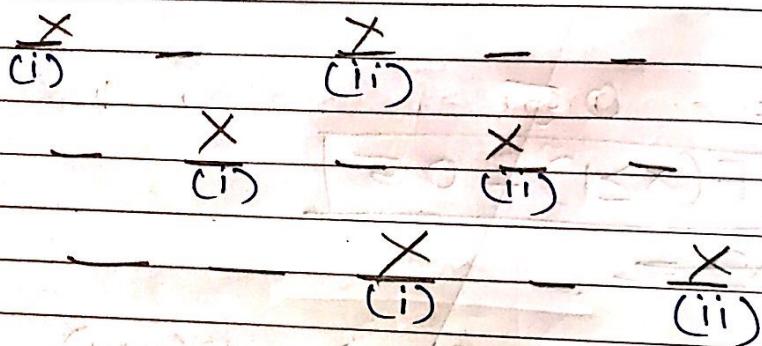
(b)

Q. 4

A-P~~re~~ Number of ways of arranging 5 digits = $5!$
 $= 120$

B-

Q) ~~P~~ There is one digit between 1 and



Number of way Rearranging 3 Number

Number of way Rearranging 1, 2 is =

~~$P(A) = \dots$~~
 ~~$P(A) = \dots$~~

$$P(A) = (3! \times 2) \times 3$$

$$\frac{5!}{6}$$

$$= \frac{6 \times 2 \times 3}{20}$$

$$= \frac{6}{20} = \frac{3}{10} = 0.3$$

$$\boxed{P(A) = 0.3}$$

(b) C - There are exactly 2 digits between them -

$\times \quad - \quad \times \quad -$

Number of ways of arranging 3 Numbers = $8!$

Number of ways Rearranging 2 Numbers = 2.

$$P(C) = \underline{(8! \times 2) \times 2}$$

$\frac{51}{0}$

$$= \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 \times 2}{120}$$

$\frac{4}{20}$

$$= \frac{2}{10}$$

$$\boxed{P(C) = 0.3}$$

(c) -D - There are exactly three digits between 1 and 2.

X — — — X

P(1)

Number of ways arranging 3 Number
= $3!$

Number of ways arranging 2 Number
= 2.

$$P(D) = \frac{(3! \times 2) \times 1}{5!} \rightarrow$$

$$= \frac{6 \times 2}{120}$$

$$= \frac{2}{20}$$

$$= 0.1$$

✓

$$\boxed{P(D) = 0.1}$$

$$(5) \int_{-\infty}^{\infty} f(x) dx = 1$$

Given $f(x) = \begin{cases} 0 & x < 0 \\ 0.5 - Ce^{-x} & 0 \leq x \leq 1 \\ 0 & x \geq 1 \end{cases}$

$$\int_{-\infty}^{\infty} f(x) dx = 1$$

$$\int_0^1 0.5 dx + \int_1^{\infty} C e^{-x} dx = 1$$

$$\left[0.5x \right]_0^1 - C \left[e^{-x} \right]_1^{\infty} = 1$$

$$0.5 - C(0 - e^{-1}) = 1$$

$$0.5 - 1 = C(e^{-1})$$

$$-0.5 = C(e^{-1})$$

$$0.5 = C(e^{-1})$$

$$e \times 0.5 = C$$

$$2.718 \times 0.5 = C$$

$$C = 1.359$$

(b) ~~$x < 1$~~ Given $x < 1$

$$E(x) = \int_0^1 xf(x)dx + \int_{-\infty}^0 xf(x)dx$$

$$= 0 + \int_0^1 0.5x^2 dx$$

$$= \left[\frac{0.5x^2}{2} \right]_0^1$$

$$= \frac{0.5}{2}$$

$$= 0.25.$$

$$\boxed{E(x < 1) = 0.25}$$

45) Given $x \geq 1$

$$E(x \geq 1) = \int_{-\infty}^{\infty} x f(x) dx.$$

$$= \int_{-\infty}^{\infty} e^x x e^{-x} dx.$$

$$= \int_1^{\infty} e^x x e^{-x} dx$$

$$= \frac{e}{2} \left[-x e^{-x} - e^{-x} \right]_1^{\infty}$$

$$= \frac{e}{2} [e^1 + e^1]$$

$$= \frac{e}{2} * \frac{2}{e}$$

$\therefore E(x \geq 1) = 1$

46) $E(x) = E(x < 1) + E(x \geq 1)$

$$= 1 + 0.25$$

$$= 1.25$$

Expected value of x is 1.25 ✓

(6)

\times - denoting no of bits received in error

We have,

Given :-

$$p = 0.00001$$

$$n = 10 \times 10^6$$

$$\lambda = 100.$$

So

Probability that more than 100 error occur

$$= \sum_{x=101}^{10 \times 10^6} P(x; 10 \times 10^6, 0.00001).$$

As $n \rightarrow \infty$ & $p \rightarrow 0$

We will perform Poisson's approximation

$$b(x; n, p) \xrightarrow{n \rightarrow \infty} P(x; \lambda)$$

$$\text{mean} = \mu = np = 10^5 \times 0.00001 = 100.$$

Required probability

$$P(x > 100) = \sum_{x=101}^{\infty} P(x; 100)$$

$$= 0.47344$$

Probability of more than 100 errors occurred = 0.47344