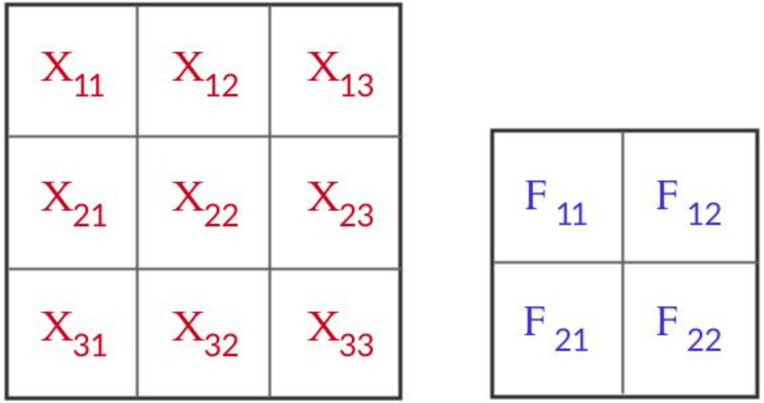
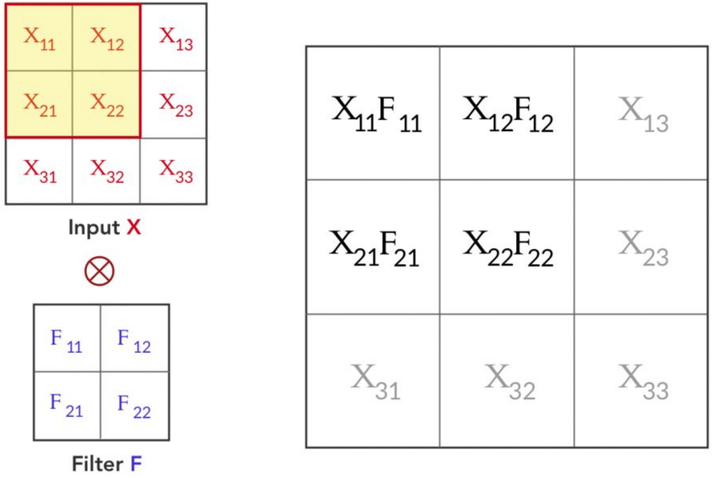
Why Convolutional NN?

1. Sharing of weights ends up **reducing the overall number of trainable weights hence introducing sparsity**
2. After several convolutional and pooling layers, the image size (feature map size) is **reduced and more complex features are extracted**.
3. The usage of CNNs are motivated by the fact that they can capture / are able to **learn relevant features** from an image at different levels similar to a human brain.

Convolution?

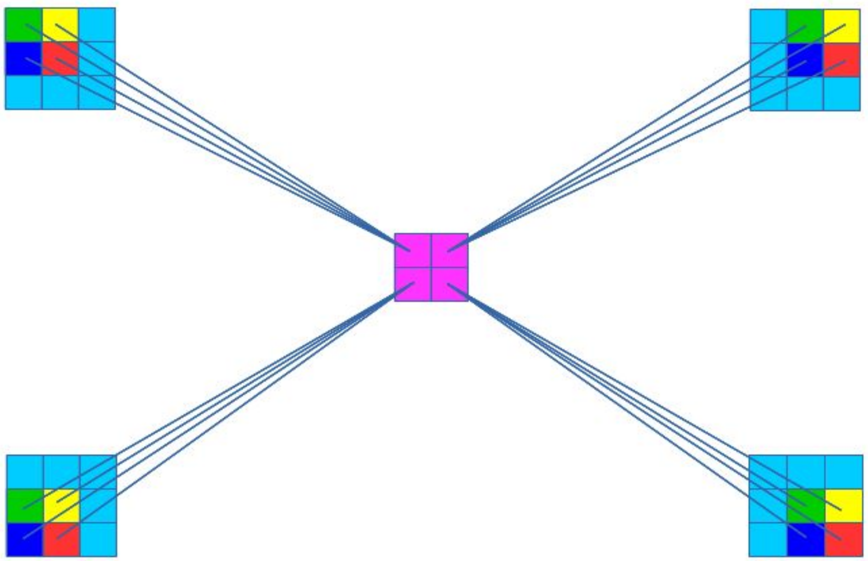


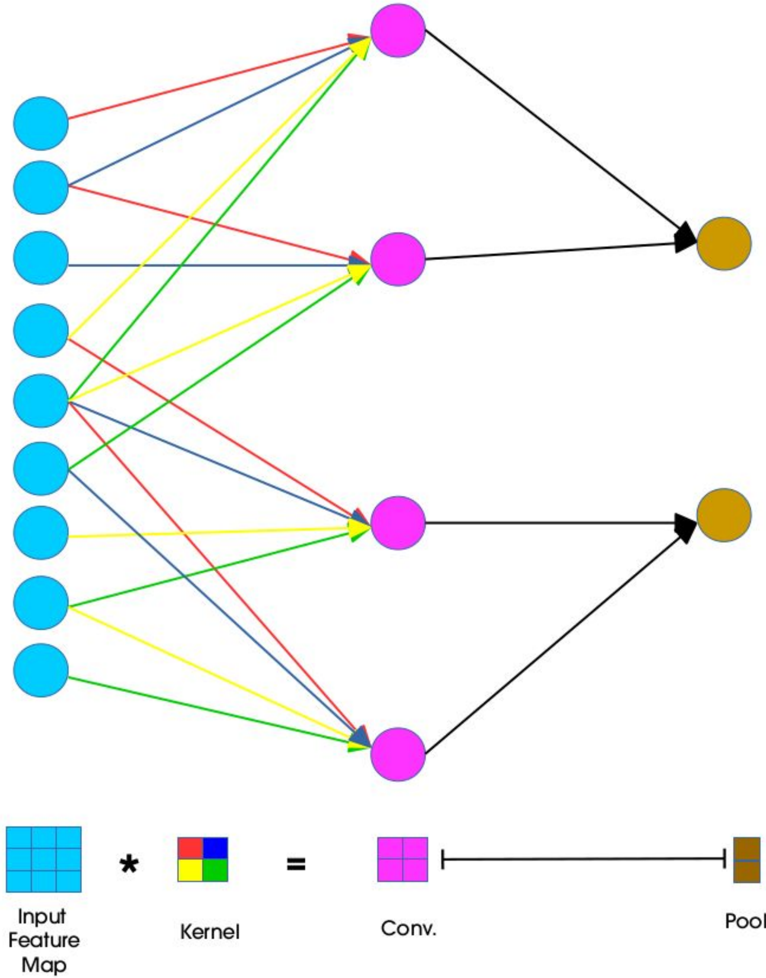
Input **X** Filter **F**



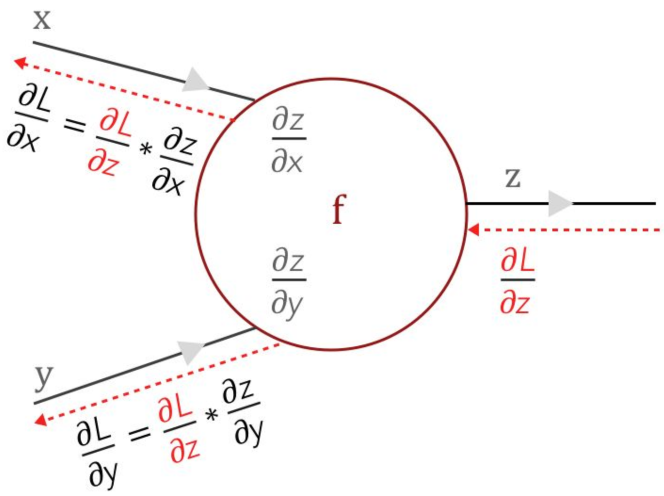
$$O_{11} = X_{11}F_{11} + X_{12}F_{12} + X_{21}F_{21} + X_{22}F_{22}$$

How Convolutional NN better?





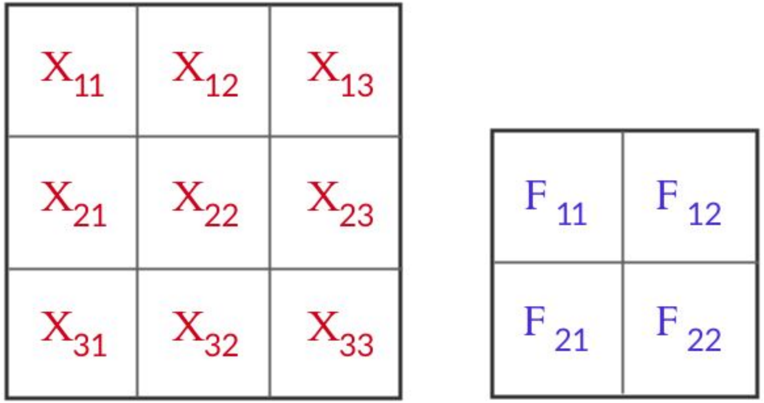
Computational Graph - Convolution Operation



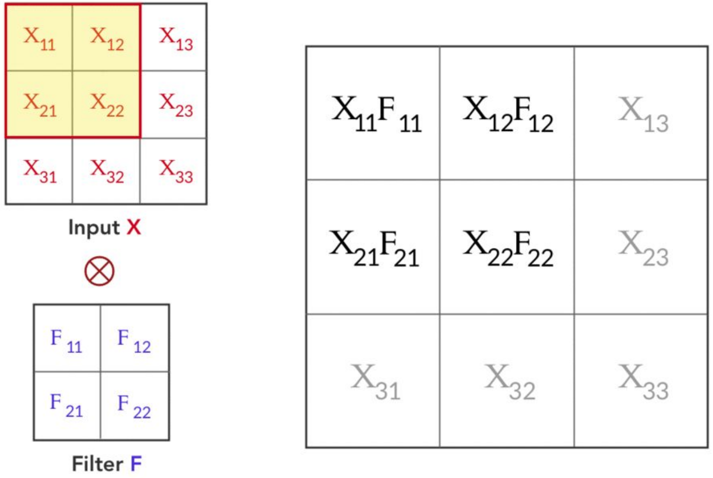
$\frac{\partial z}{\partial x}$ & $\frac{\partial z}{\partial y}$ *are local gradients*

$\frac{\partial L}{\partial z}$ *is the loss from the previous layer which has to be backpropagated to other layers*

Convolution Operation

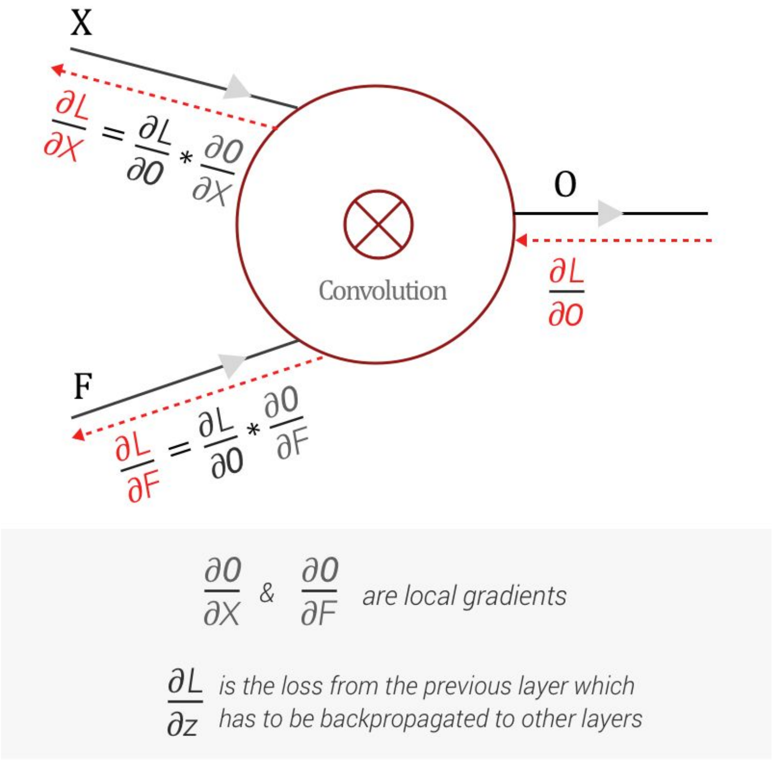


Input **X** Filter **F**



$$O_{11} = X_{11}F_{11} + X_{12}F_{12} + X_{21}F_{21} + X_{22}F_{22}$$

Computational Graph - Convolution Operation



Finding **∂L/∂F:**

Chain Rule.

$$\frac{\partial L}{\partial F} = \frac{\partial L}{\partial O} * \frac{\partial O}{\partial F}$$

Gradient to update Filter F

Loss Gradient from previous layer

Local Gradients

Finding ∂L/∂F:

Local Gradients

Local Gradients ⟶ A

$$O_{11} = X_{11}F_{11} + X_{12}F_{12} + X_{21}F_{21} + X_{22}F_{22}$$

Finding derivatives with respect to F_{11} , F_{12} , F_{21} *and* F_{22}

$$\frac{\partial O_{11}}{\partial F_{11}} = X_{11} \qquad \frac{\partial O_{11}}{\partial F_{12}} = X_{12} \qquad \frac{\partial O_{11}}{\partial F_{21}} = X_{21} \qquad \frac{\partial O_{11}}{\partial F_{22}} = X_{22}$$

Similarly, we can find the local gradients for O_{12} , O_{21} *and* O_{22}

Finding ∂L/∂F:

Applying chain rule

$$\frac{\partial L}{\partial F_{11}} = \frac{\partial L}{\partial O_{11}} * \frac{\partial O_{11}}{\partial F_{11}} + \frac{\partial L}{\partial O_{12}} * \frac{\partial O_{12}}{\partial F_{11}} + \frac{\partial L}{\partial O_{21}} * \frac{\partial O_{21}}{\partial F_{11}} + \frac{\partial L}{\partial O_{22}} * \frac{\partial O_{22}}{\partial F_{11}}$$

$$\frac{\partial L}{\partial F_{12}} = \frac{\partial L}{\partial O_{11}} * \frac{\partial O_{11}}{\partial F_{12}} + \frac{\partial L}{\partial O_{12}} * \frac{\partial O_{12}}{\partial F_{12}} + \frac{\partial L}{\partial O_{21}} * \frac{\partial O_{21}}{\partial F_{12}} + \frac{\partial L}{\partial O_{22}} * \frac{\partial O_{22}}{\partial F_{12}}$$

$$\frac{\partial L}{\partial F_{21}} = \frac{\partial L}{\partial O_{11}} * \frac{\partial O_{11}}{\partial F_{21}} + \frac{\partial L}{\partial O_{12}} * \frac{\partial O_{12}}{\partial F_{21}} + \frac{\partial L}{\partial O_{21}} * \frac{\partial O_{21}}{\partial F_{21}} + \frac{\partial L}{\partial O_{22}} * \frac{\partial O_{22}}{\partial F_{21}}$$

$$\frac{\partial L}{\partial F_{22}} = \frac{\partial L}{\partial O_{11}} * \frac{\partial O_{11}}{\partial F_{22}} + \frac{\partial L}{\partial O_{12}} * \frac{\partial O_{12}}{\partial F_{22}} + \frac{\partial L}{\partial O_{21}} * \frac{\partial O_{21}}{\partial F_{22}} + \frac{\partial L}{\partial O_{22}} * \frac{\partial O_{22}}{\partial F_{22}}$$

Finding ∂L/∂F:

Applying chain rule

$$\frac{\partial L}{\partial F_{11}} = \frac{\partial L}{\partial O_{11}} * X_{11} + \frac{\partial L}{\partial O_{12}} * X_{12} + \frac{\partial L}{\partial O_{21}} * X_{21} + \frac{\partial L}{\partial O_{22}} * X_{22}$$

$$\frac{\partial L}{\partial F_{12}} = \frac{\partial L}{\partial O_{11}} * X_{12} + \frac{\partial L}{\partial O_{12}} * X_{13} + \frac{\partial L}{\partial O_{21}} * X_{22} + \frac{\partial L}{\partial O_{22}} * X_{23}$$

$$\frac{\partial L}{\partial F_{21}} = \frac{\partial L}{\partial O_{11}} * X_{21} + \frac{\partial L}{\partial O_{12}} * X_{22} + \frac{\partial L}{\partial O_{21}} * X_{31} + \frac{\partial L}{\partial O_{22}} * X_{32}$$

$$\frac{\partial L}{\partial F_{22}} = \frac{\partial L}{\partial O_{11}} * X_{22} + \frac{\partial L}{\partial O_{12}} * X_{23} + \frac{\partial L}{\partial O_{21}} * X_{32} + \frac{\partial L}{\partial O_{22}} * X_{33}$$

Finding ∂L/∂F:

Convolution Representation

$$\begin{bmatrix} \frac{\partial L}{\partial F_{11}} & \frac{\partial L}{\partial F_{12}} \\ \frac{\partial L}{\partial F_{21}} & \frac{\partial L}{\partial F_{22}} \end{bmatrix} = \text{Convolution}\left(\begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{bmatrix}, \begin{bmatrix} \frac{\partial L}{\partial O_{11}} & \frac{\partial L}{\partial O_{12}} \\ \frac{\partial L}{\partial O_{21}} & \frac{\partial L}{\partial O_{22}} \end{bmatrix} \right)$$

where

$$\begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{bmatrix} = \text{Input } X \qquad \begin{bmatrix} \frac{\partial L}{\partial O_{11}} & \frac{\partial L}{\partial O_{12}} \\ \frac{\partial L}{\partial O_{21}} & \frac{\partial L}{\partial O_{22}} \end{bmatrix} = \frac{\partial L}{\partial O}$$

Loss gradient from previous layer

Finding **∂L/∂X**

Chain rule

For every element of X_i

$$\frac{\partial L}{\partial X_i} = \sum_{k=1}^{M} \frac{\partial L}{\partial O_k} * \frac{\partial O_k}{\partial X_i}$$

Finding ∂L/∂X

Local Gradients

Local Gradients: → B

$$O_{11} = X_{11}F_{11} + X_{12}F_{12} + X_{21}F_{21} + X_{22}F_{22}$$

Differentiating with respect to X_{11}, X_{12}, X_{21} *and* X_{22}

$$\frac{\partial O_{11}}{\partial X_{11}} = F_{11} \quad \frac{\partial O_{11}}{\partial X_{12}} = F_{12} \quad \frac{\partial O_{11}}{\partial X_{21}} = F_{21} \quad \frac{\partial O_{11}}{\partial X_{22}} = F_{22}$$

Similarly, we can find local gradients for O_{12}, O_{21} *and* O_{22}

Finding ∂L/∂X

Representation

$$\frac{\partial L}{\partial X_{11}} = \frac{\partial L}{\partial O_{11}} * F_{11}$$

$$\frac{\partial L}{\partial X_{12}} = \frac{\partial L}{\partial O_{11}} * F_{12} + \frac{\partial L}{\partial O_{12}} * F_{11}$$

$$\frac{\partial L}{\partial X_{13}} = \frac{\partial L}{\partial O_{12}} * F_{12}$$

$$\frac{\partial L}{\partial X_{21}} = \frac{\partial L}{\partial O_{11}} * F_{21} + \frac{\partial L}{\partial O_{21}} * F_{11}$$

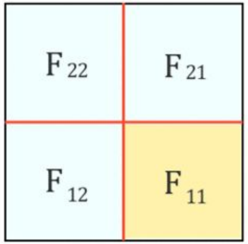
$$\frac{\partial L}{\partial X_{22}} = \frac{\partial L}{\partial O_{11}} * F_{22} + \frac{\partial L}{\partial O_{12}} * F_{21} + \frac{\partial L}{\partial O_{21}} * F_{12} + \frac{\partial L}{\partial O_{22}} * F_{11}$$

$$\frac{\partial L}{\partial X_{23}} = \frac{\partial L}{\partial O_{12}} * F_{22} + \frac{\partial L}{\partial O_{22}} * F_{12}$$

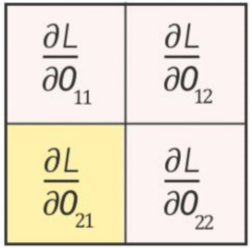
$$\frac{\partial L}{\partial X_{31}} = \frac{\partial L}{\partial O_{21}} * F_{21}$$

$$\frac{\partial L}{\partial X_{32}} = \frac{\partial L}{\partial O_{21}} * F_{22} + \frac{\partial L}{\partial O_{22}} * F_{21}$$

$$\frac{\partial L}{\partial X_{33}} = \frac{\partial L}{\partial O_{22}} * F_{22}$$

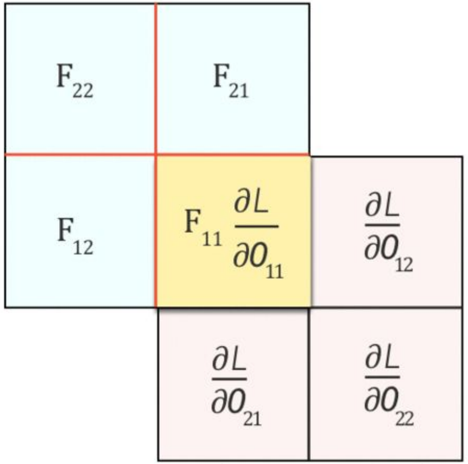


Filter F



Loss Gradient $\frac{\partial L}{\partial O}$

$$\boxed{\frac{\partial L}{\partial X_{11}} = F_{11} * \frac{\partial L}{\partial O_{11}}}$$



@pavisj

Finding ∂L/∂X

Representation

$$\underbrace{\begin{bmatrix} \frac{\partial L}{\partial X_{11}} & \frac{\partial L}{\partial X_{12}} & \frac{\partial L}{\partial X_{13}} \\ \frac{\partial L}{\partial X_{21}} & \frac{\partial L}{\partial X_{22}} & \frac{\partial L}{\partial X_{23}} \\ \frac{\partial L}{\partial X_{31}} & \frac{\partial L}{\partial X_{32}} & \frac{\partial L}{\partial X_{33}} \end{bmatrix}}_{\frac{\partial L}{\partial X}} = \text{Full Convolution}\left(\underbrace{\begin{bmatrix} F_{22} & F_{21} \\ F_{12} & F_{11} \end{bmatrix}}_{\text{Filter F}}, \underbrace{\begin{bmatrix} \frac{\partial L}{\partial O_{11}} & \frac{\partial L}{\partial O_{12}} \\ \frac{\partial L}{\partial O_{21}} & \frac{\partial L}{\partial O_{22}} \end{bmatrix}}_{\text{Loss Gradient } \frac{\partial L}{\partial O}} \right)$$

Result ∂L/∂F and ∂L/∂X

Representation

Backpropagation in a Convolutional Layer of a CNN

Finding the gradients:

$$\frac{\partial L}{\partial F} = \text{Convolution}\left(\text{Input } X \text{, Loss gradient } \frac{\partial L}{\partial O}\right)$$

$$\frac{\partial L}{\partial X} = \begin{array}{c}\text{Full}\\ \text{Convolution}\end{array}\left(\begin{array}{c}180^{\circ}\text{rotated}\\ \text{Filter } F\end{array}, \begin{array}{c}\text{Loss}\\ \text{Gradient}\end{array} \frac{\partial L}{\partial O}\right)$$

Convolution Output

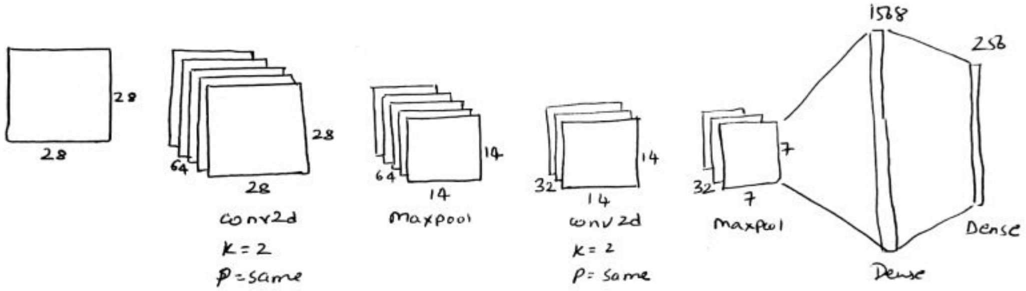
$$O = \frac{W - K + 2P}{S} + 1$$

W = input height or length

K = kernel length or size

P = padding

S = stride



Layer (type)	Output Shape	Param #
conv2d_2 (Conv2D)	(None, 28, 28, 64)	320
max_pooling2d_2 (MaxPooling2	(None, 14, 14, 64)	0
conv2d_3 (Conv2D)	(None, 14, 14, 32)	8224
max_pooling2d_3 (MaxPooling2	(None, 7, 7, 32)	0
flatten_1 (Flatten)	(None, 1568)	0
dense_2 (Dense)	(None, 256)	401664
dense_3 (Dense)	(None, 10)	2570

Total params: 412,778
Trainable params: 412,778
Non-trainable params: 0

Padding=Valid

1	2	3
4	5	6
7	8	9
10	11	12

padding	VALID
filter	2x2
stride	2x2
input	4x3
output	2x1

Padding=Same

1	2	3
4	5	6
7	8	9
10	11	12

padding	SAME
filter	2x2
stride	2x2
input	4x3
output	2x2

Quiz 7

Support Vector Machine (SVM)

SVM (Videos 12.1 to 12.6) Andrew Ng

https://www.youtube.com/watch?v=hCOIMkcsm_g&list=PLLssT5z_DsK-h9vYZkQkYNWcItqhlRJLN&index=70