$$y = f(w^T x + w_0)$$

$$P(x|C_k) = \frac{1}{(2\pi)^{d/2} |\Sigma|^{1/2}} \exp\left\{-\frac{1}{2}(x-\mu_k)^T \Sigma^{-1} (x-\mu_k)\right\}$$

$$P(C_1|x) = \frac{P(x|C_1)P(C_1)}{P(x|C_1)P(C_1) + P(x|C_2)P(C_2)}$$

$$= \frac{1}{1 + \frac{P(x|C_2)P(C_2)}{P(x|C_1)P(C_1)}}$$

$$= \frac{1}{1 + e^{-\ln \frac{P(x|C_1)P(C_1)}{P(x|C_2)P(C_2)}}}$$

$$= \frac{1}{1 + e^{-z}}$$

$$z = \ln \frac{P(x|C_1)P(C_1)}{P(x|C_2)P(C_2)} = w^T x + w_0$$

Logistic Regression – Method 1

STEP ① Read data file (pandas)

STEP ② Process data file

(i) Drop column id

(ii) map label column to 0 and 1

STEP ③ Split data frame into train, val & Test

STEP ④ Normalize the dataset

STEP ⑤ Initialize weights and biases & learning Rate

STEP ⑥ for epoch in Range (10000)

$$z = \theta^T X + b$$

$$a = \sigma(z)$$

$$L = \frac{-(y \log a + (1-y)\log(1-a))}{m}$$

$$= -\frac{1}{m}(y \log a + (1-y)\log(1-a))$$

$$= -\frac{1}{m}\left(y \log(\sigma(z)) + (1-y)\log(1-\sigma(z))\right)$$

~~$W = W - \eta \Delta W$~~

$$\theta = \theta - \eta \Delta\theta$$

$$b = b - \eta \Delta b$$

$$L = \frac{-1}{m}\left\{ y \log a + (1-y)\log(1-a) \right\}$$

$$= \frac{-1}{m}\left\{ y \log \sigma(z) + (1-y)\log(1-\sigma(z)) \right\}$$

$$\Delta\theta_1 = \frac{\partial L}{\partial \theta_1} = \frac{-1}{m}\frac{\partial}{\partial \theta_1}\left\{ y \log \sigma(z) + (1-y)\log(1-\sigma(z)) \right\}$$

$$= \frac{-1}{m}\left\{ y \cdot \frac{1}{\sigma(z)} \cdot \frac{\partial}{\partial \theta_1}\sigma(z) + (1-y) \cdot \frac{1}{1-\sigma(z)} \frac{\partial}{\partial \theta_1}(1-\sigma(z)) \right\}$$

$$= \frac{-1}{m}\left\{ y \cdot \frac{1}{\sigma(z)} \sigma(z)(1-\sigma(z)) \frac{\partial}{\partial \theta_1} z \; + \right.$$

$$\left. (1-y) \cdot \frac{1}{1-\sigma(z)} \sigma(z)(1-\sigma(z)) \frac{\partial}{\partial \theta_1}(-z) \right.$$

$$= \frac{-1}{m}\left\{ y(1-\sigma(z))x_1 + (1-y)\sigma(z)(-x_1) \right\}$$

$$= \frac{-1}{m}\left\{ y - y\sigma(z) - \sigma(z) + y\sigma(z) \right\} x_1$$

$$\Delta\theta_1 = \frac{-1}{m}\left\{ y - \sigma(z) \right\} x_1$$

For updating Bias,

$$\Delta b = \frac{-1}{m} \left\{ y - \sigma(z) \right\} . 1$$