Network Topology — QUBO + QAOA

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1 Network Topology Optimization with QUBO and QAOA

1.1 Motivation and Problem Setting

Large-scale cloud networks must route heterogeneous traffic demands across multi-vendor infrastructures. Operators face three conflicting objectives:

1. **Capacity feasibility:** ensuring that no link exceeds its bandwidth limit.
2. **SLA compliance:** meeting end-to-end latency requirements.
3. **Energy efficiency:** minimizing the power drawn by network elements, which directly contributes to carbon footprint.

These requirements make routing optimization **NP-hard**: decisions for one flow affect others through shared capacity. Classical solvers (ILP, heuristics) are effective at moderate scales, but they become computationally expensive for large, dynamic networks. This motivates exploring **quantum optimization** techniques such as QAOA.

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1.2 Classical Formulation (ILP → QUBO)

We represent the network as a graph $G = (V, E)$, with link attributes:

- Capacity: cap_{uv} (Mbps)
- Latency: lat_{uv} (ms)
- Energy cost per unit traffic: ϵ_{uv} (W/Mbps)

Traffic demands $D = \{d_1, d_2, \ldots, d_N\}$ each specify a source, destination, bandwidth, and latency bound. Candidate routes are pre-computed using the *k-shortest paths* algorithm ($k = 3$, latency-weighted).

We introduce binary decision variables:

$$x_{d,p} = \begin{cases} 1 & \text{if demand } d \text{ is routed on path } p, \\ 0 & \text{otherwise.} \end{cases}$$

$$z_d = \begin{cases} 1 & \text{if demand } d \text{ is dropped,} \\ 0 & \text{otherwise.} \end{cases}$$

Objective Function:

$$\min \sum_{d \in D} \sum_{p \in P_d} \left(bw_d \cdot \sum_{(i,j) \in p} \epsilon_{ij} \right) x_{d,p} + \sum_{d \in D} \text{DROP\_PENALTY} \cdot z_d \tag{1}$$

This minimizes energy while discouraging dropped demands.

Constraints:

$\sum_{d \in D} \sum_{p \in P_d : (u,v) \in p} bw_d \cdot x_{d,p} \leq \text{cap}_{uv}, \quad \forall (u,v) \in E$(capacity constraint)

$\text{x}_{d,p}, z_d \in \{0, 1\}, \quad \forall d, p$(binary constraint)

This ILP can be solved classically with CBC. However, we can also encode it as a QUBO by folding constraints into the objective with penalty terms, enabling quantum solvers.

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1.3 QAOA Formulation

QAOA is a variational quantum algorithm for combinatorial optimization.

1. **Encoding the problem:** The QUBO is mapped into a Hamiltonian H. Each binary decision variable corresponds to a qubit, with Pauli-Z operators encoding 0/1 values.

2. **Variational Ansatz:** Qubits are initialized in a uniform superposition:

$$\frac{1}{\sqrt{2^n}} \sum_{x=0}^{2^n - 1} |x\rangle$$

A sequence of alternating operators is applied:

$$U(H, \gamma) = e^{-i\gamma H} \qquad \text{(problem unitary)} \tag{2}$$

$$U(B, \beta) = e^{-i\beta \sum_i X_i} \qquad \text{(mixer unitary)} \tag{3}$$

3. **Classical feedback loop:** A classical optimizer updates parameters (γ, β) to minimize the expectation value:

$$\langle \psi(\gamma, \beta) | H | \psi(\gamma, \beta) \rangle$$

4. **Measurement:** Repeated sampling yields a probability distribution over bitstrings. The most probable low-energy bitstrings correspond to near-optimal routing decisions.

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1.4 Results and Interpretation

- **Classical ILP:** provides exact solutions on small-to-medium instances.
- **QAOA:** reproduces feasible solutions on small instances (4–12 qubits), with probabilities concentrated on low-energy bitstrings.
- Each bitstring maps back to a routing decision (e.g., `1010` means demand 1 $\rightarrow$ path A, demand 2 dropped, demand 3 $\rightarrow$ path B, etc.).

Scaling to $\sim$40 demands ($>$40 qubits) is infeasible for current simulators, since statevector simulation requires storing 2^{40} amplitudes ($\approx 10^{12}$). Therefore, we validated QAOA on reduced problem sizes.

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1.5 Challenges and Limitations

1. **Scalability:** Current simulators fail beyond $\sim$25 qubits; real hardware is noisy and limited.
2. **Penalty sensitivity:** Penalty weights in QUBO strongly affect feasibility and solution quality.
3. **Interpretability:** Mapping between bitstrings and routing variables requires careful decoding.

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1.6 Outlook and Next Steps

- Hybrid methods: run QAOA on reduced subproblems while handling the rest classically.
- Quantum annealers: D-Wave can already embed thousands of QUBO variables, albeit with different physics.
- Hardware progress: scalable, fault-tolerant quantum devices will enable realistic routing optimization.
- Sustainability: even small improvements in energy efficiency directly reduce the carbon footprint of cloud operators.

Summary: We formulated energy-aware routing as a QUBO, solved it classically (ILP) and tested QAOA for small instances. While current hardware cannot yet handle realistic demand sizes, the workflow demonstrates how quantum optimization can complement classical solvers. Future progress lies in hybrid strategies, annealing hardware, and leveraging quantum advantage for sustainable networking.