Relativistic effects in quantum computing using qiskit

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Abstract

In this study, we performed a set of quantum simulations and attempted to understand the effect of relativity on spin and entanglement using Qiskit. First, we simulated the Wigner rotation for a spin-$\frac{1}{2}$ particle that undergoes a Lorentz boost in the z-direction and examined how the spin state transforms across six distinct momentum directions and velocities ranging from 0.1c to 0.99c. Second, inspired by the work of Jafarizadeh and Mahdian, we construct a quantum circuit to demonstrate how the spin of a particle entangles with momentum under the Lorentz boost and how Wigner rotation acts as a controlled rotation gate and, under special conditions, as a CNOT gate. Third, based on the work of Gingrich and Adani, we construct a quantum circuit of a 2-particle system, initially prepared in a Bell state, and explore how spin-spin entanglement degrades under relativistic transformations. We quantify this measure using von Neumann entropy and concurrence as functions of the boost parameters.

1 Introduction

Quantum mechanics and special relativity have been studied extensively within their own domains; however, their intersection is a subtle and profound phenomenon [1], such as the transformation of spin states under Lorentz boosts, which leads to effects such as Wigner rotation, spin-momentum entanglement, and entanglement degradation. Understanding the behavior of spin, momentum, and entanglement under relativistic transformations is essential for their application in quantum information, relativistic quantum communication, and potential space-based quantum technologies [2].

When a spin-$\frac{1}{2}$ particle with momentum $\vec{p}$ undergoes a Lorentz transformationΛ, the spin state, in addition to being boosted, is rotated by an amount that depends on the momentum [1]. The transformed state can be written as

$$|\psi\rangle \to D(W(\Lambda, p))|\psi\rangle,$$

where $D(W(\Lambda, p))$ is the unitary representation of the Wigner rotation corresponding to the boost Λ and momentum p. The Wigner rotation itself is defined by:

$$W(\Lambda, p) = L^{-1}(\Lambda p)\Lambda L(p),$$

where $L(p)$ is the standard boost that transforms the rest momentum to p, and Λp is the boosted momentum Wigner. This dependence of the spin on the momentum introduces a coupling between the two degrees of freedom. A conceptual representation of this process is shown in Figure 1, where the spin transformation D depends on both the particle's momentum and the applied Lorentz boost.

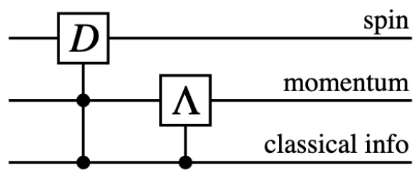

Figure 1: Conceptual quantum circuit adapted from [1], illustrating how spin transformations under Lorentz boosts (D) depend on both momentum state and classical frame informationΛ.

This entanglement between the two degrees of freedom was explored in [3], where it was shown that in a single-particle system, spin-momentum entanglement can be induced by a Lorentz boost. If the state is initially $|\Psi\rangle = |\vec{p}\rangle \otimes |\chi\rangle$, after the Lorentz boost, it becomes

$$|\Psi'\rangle = |\Lambda\vec{p}\rangle \otimes D(W(\Lambda, \vec{p}))|\chi\rangle,$$

resulting in a non-separable state. The axis and angle of the Wigner rotation depend on the momentum vector$\vec{p}$. The spin state evolves differently depending on the momentum state occupied by the particle. This is how the Wigner rotation acts as a controlled rotation gate, where the momentum serves as a control and the spin as a target. For specific values of the boost parameters, the Wigner rotation closely resembles a CNOT gate [3].

Earlier studies have shown that entanglement is observer-dependent in relativistic settings [4], motivating further investigations into how such transformations affect information-theoretic quantities. [5] investigated how spin-spin entanglement behaves under relativistic transformations. They considered a 2-particle system initially prepared in a Bell state and boosted along a common direction. Because the Wigner rotation acts locally on each particle, the reduced spin density matrix becomes mixed from the perspective of a moving observer. This degradation was quantified using von Neumann entropy

$$S(\rho) = -\mathrm{Tr}(\rho \log_2 \rho),$$

and concurrence:

$$C(\rho) = \max\left(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\right),$$

where λ_i is the square root of the eigenvalue $\rho\tilde{\rho}$, and $\tilde{\rho}$ is the spin-flipped matrix of ρ.

While the theoretical foundations of relativistic quantum information are now well established and studied, the simulation of such phenomena using gate-based quantum frameworks remains underexplored. In this study, we implemented circuit simulations of relativistic quantum effects, namely Wigner rotation, spin-momentum entanglement, and spin-spin entanglement degradation. While some of these studies and theoretical works rely on continuous momentum distributions or 4-momentum, our approach, for simplicity, uses 3-dimensional discrete momentum eigenstates and fixed Lorentz boosts along the z-direction. This strategy allows for optimal quantum circuit implementation while retaining the behavior predicted by theoretical frameworks, making it suitable for study and analysis.

2 Simulation of Wigner Rotation

2.1 Simulation Objective

The goal of this section is to simulate the relativistic spin transformation of a spin-$\frac{1}{2}$ particle under a Lorentz boost, as described in the foundational works on relativistic quantum information [1]. We implement the Wigner rotation, which acts as a unitary operator on the spin degree of freedom.

2.2 Methodology

In our simulation, we fixed the Lorentz boost in the z-direction and considered six discrete momentum directions:

$$\vec{p} \in \{\pm\hat{x}, \pm\hat{y}, \pm\hat{z}\}.$$

We varied the boost velocity v from $0.01c$ to $0.99c$, computing the corresponding rapidity as

$$\eta = \tanh^{-1}(v).$$

For each momentum–velocity pair, we computed the Wigner rotation angle Ω using

$$\Omega = 2\tan^{-1}\left(\frac{\sin\alpha\sinh\eta}{\cosh\eta + \cos\alpha\sinh\eta}\right),$$

where α is the angle between the boost direction and momentum vector.

The rotation axis is given by the normalized cross product of the boost and momentum directions as follows:

$$\vec{n} = \frac{\vec{v}\times\vec{p}}{|\vec{v}\times\vec{p}|}.$$

This axis is used to construct the Hermitian generator:

$$G = n_x\sigma_x + n_y\sigma_y + n_z\sigma_z,$$

The Wigner rotation unitary is implemented as

$$U = \exp\left(-i\frac{\Omega}{2}G\right).$$

In Qiskit [6], this is realized by applying a single-qubit unitary rotation to the spin qubit, as shown in Figure 2, with each momentum direction being mapped to a unique simulation run. We prepare the spin in state$|0\rangle$, apply the Wigner rotation, and measure the probability of obtaining the outcome $|1\rangle$ as a function of the velocity and momentum direction.

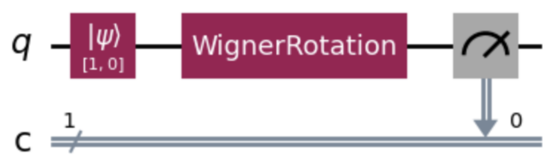

Figure 2: Quantum circuit implemented in Qiskit to simulate Wigner rotation on a spin-$\frac{1}{2}$ particle under Lorentz boost. The unitary rotation, derived from the Wigner angle, is applied to the spin qubit based on the particle's momentum configuration.

2.3 Results and Circuit Implementation

The simulation was implemented in Qiskit using a parameterized single-qubit rotation gate. For each direction of momentum and boost velocity, we computed the Wigner angle Ω and the axis $\vec{n}$, and constructed the Hermitian generator $G = \vec{n} \cdot \vec{\sigma}$ to apply the unitary $\exp(-i\Omega G/2)$ to the spin qubit. Each configuration was run independently.

We analyze the results by plotting the probability of measuring the spin in state$|1\rangle$, denoted$P(1)$, as a function of the boost velocity$v$, across six momentum directions, as shown in figure 3.

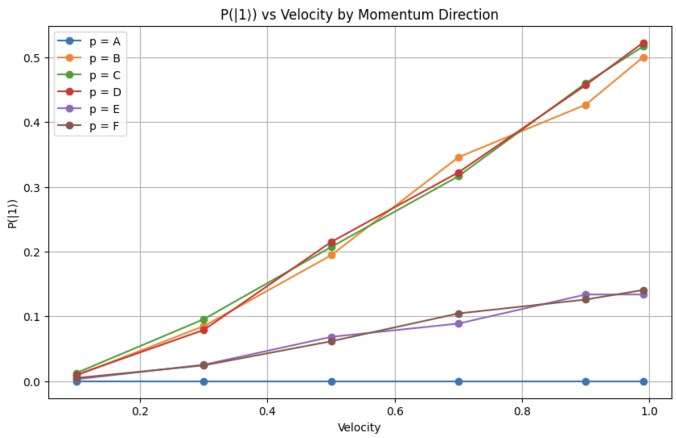

Figure 3: Probability of measuring the spin in state$|1\rangle$, denoted $P(1)$, as a function of boost velocity v for six different momentum directions. When the momentum is aligned with the boost direction ($\vec{p} \parallel \vec{v}$), $P(1) \approx 0$, indicating no rotation. For transverse momenta ($\vec{p} \perp \vec{v}$), $P(1)$ increases significantly with velocity, indicating stronger Wigner rotations.

Key observations: - When $\vec{p} \parallel \vec{v}$ (i.e., along the boost direction), the Wigner angle is zero, and the spin remains unchanged, resulting in $P(1) \approx 0$. - When $\vec{p} \perp \vec{v}$ (e.g., $\hat{x}, \hat{y}$), the Wigner angle is maximal, and $P(1)$ increases sharply with velocity, exceeding 0.5 for $v \approx 0.99$. Intermediate behavior is observed for momentum vectors partially aligned with the boost.

Additionally, plotting the Wigner angle vs $P(1)$ as shown in Figure 4, reveals that the velocity alone does not fully determine the spin transformation; the momentum direction plays a crucial role in determining the final state.

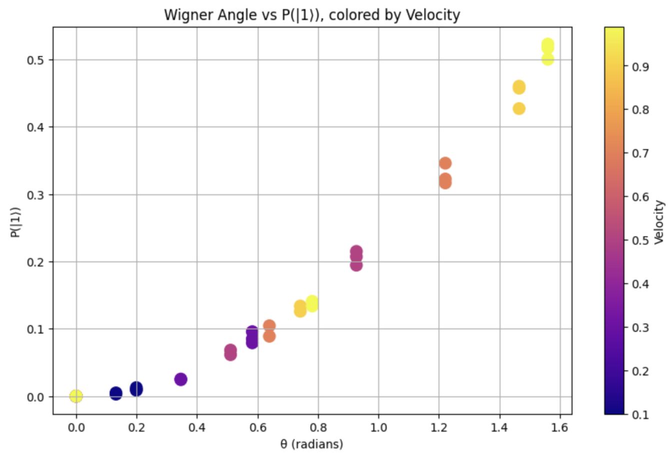

Figure 4: Plot of Wigner angle θ versus the probability $P(1)$ of measuring the spin in state $|1\rangle$, with data points colored according to the boost velocity v. This plot illustrates that while the Wigner angle plays a central role in determining the spin transformation, it does not uniquely determine the measurement outcome. Identical rotation angles can result in different probabilities, depending on the underlying momentum configuration and spin orientation. This confirms that the spin dynamics under Lorentz boosts depend on both the boost magnitude and momentum direction.

3 Simulation of Spin-Momentum Entanglement

3.1 Objective and Theoretical Framework

This section simulates how Lorentz boosts lead to spin-momentum entanglement in a single-particle system, as described by [3]. The Wigner rotation introduces momentum-dependent spin transformations, leading to entanglement, even when the initial state is a product state. We modeled this behavior in Qiskit using a controlled rotation gate acting on the spin qubit, conditioned on the momentum state.

The total unitary evolution is represented as

$$U = |0\rangle\langle 0|_{\mathrm{p}} \otimes R_y(\omega_1) + |1\rangle\langle 1|_{\mathrm{p}} \otimes R_y(\omega_2),$$

where ω_1 and ω_2 are the Wigner rotation angles for the momentum states $|0\rangle$ and $|1\rangle$, respectively. The full state after Hadamard on the momentum qubit is

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left(|0\rangle_{\mathrm{p}} \otimes R_y(\omega_1)|0\rangle_{\mathrm{s}} + |1\rangle_{\mathrm{p}} \otimes R_y(\omega_2)|0\rangle_{\mathrm{s}}\right).$$

3.2 Simulation Methodology

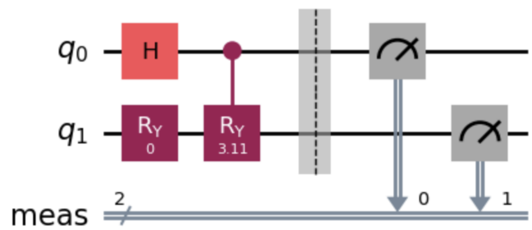

Figure 5: Quantum circuit implemented in Qiskit to simulate spin-momentum entanglement under Lorentz boost, based on the framework by [3]. The momentum qubit is prepared in superposition, and the spin qubit undergoes a base rotation $R_y(\omega_1)$, followed by a controlled rotation $R_y(\omega_2 - \omega_1)$ conditioned on the momentum qubit. This circuit models how the Wigner rotation acts as a controlled gate that can replicate CNOT-like entanglement under certain boost parameters. The rotation angles shown correspond to a specific boost velocity configuration. In the full simulation, ω_1 and ω_2 were varied programmatically across a range of values to study the entanglement dynamics under Lorentz boosts.

We construct a two-qubit quantum circuit, as shown in Figure 5.

- The momentum qubit is initialised$|0\rangle$, followed by a Hadamard gate to prepare$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$.
- The spin qubit is initialized in $|0\rangle$.
- A base rotation $R_y(\omega_1)$ is applied unconditionally.
- A controlled rotation $R_y(\omega_2 - \omega_1)$ is applied, controlled by the momentum qubit.

Two main simulation cases were explored.

1. **Case 1:** $\omega_1 = 0$, $\omega_2 = \pi v$, where $v \in [0, 1]$
2. **Case 2:** $\omega_1 = \frac{\pi}{2}$, $\omega_2 = v \cdot \frac{\pi}{2}$

3.3 Results and Interpretation

To measure entanglement, we perform a partial trace over the momentum qubit to obtain the reduced spin density matrixρ_{spin}. From this, we calculate the von Neumann entropy:

$$S(\rho) = -\text{Tr}(\rho \log_2 \rho).$$

Case 1: $\omega_1 = 0$, $\omega_2 = \pi v$

Here, the base rotation on the spin qubit is the identity, that is, no rotation is applied to the spin for the $|0\rangle$ momentum state. The spin qubit is initialized in $|0\rangle$. The controlled Wigner rotation acts on the spin only when the momentum is $|1\rangle$, applying a rotation $R_y(\omega_2 - \omega_1) = R_y(\pi v)$.

The resulting two-qubit state is:

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left(|0\rangle_{\mathrm{p}} \otimes |0\rangle_{\mathrm{s}} + |1\rangle_{\mathrm{p}} \otimes R_y(\pi v)|0\rangle_{\mathrm{s}} \right).$$

As $v \to 1$, the controlled rotation approaches $R_y(\pi)$, which flips the spin state from $|0\rangle$ to $|1\rangle$. The final state is expressed as

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left(|0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle \right),$$

which is a Bell state $|\Phi^+\rangle$. This is a maximally entangled spin-momentum state that is generated.

As a result, the von Neumann entropy of the reduced spin state increases with v and reaches its maximum at $v = 1$, as shown in figure 6, confirming maximal entanglement in the boosted frame. At low velocities, the controlled rotation is weak ($R_y(0) = I$), and the system remains nearly separable, with an entropy close to zero.

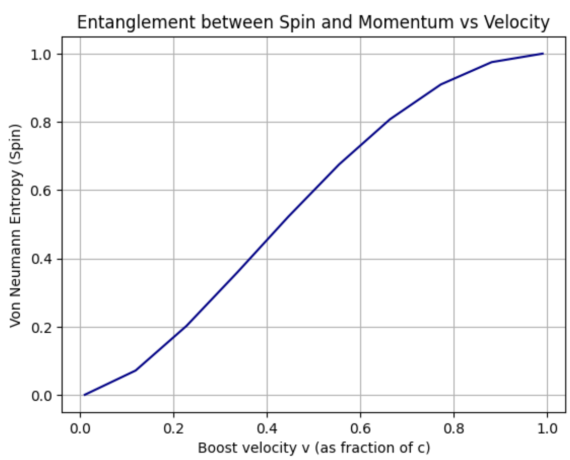

Figure 6: Von Neumann entropy of the reduced spin state for Case 1 ($\omega_1 = 0, \omega_2 = \pi v$). As the velocity increases, the controlled rotation approaches $R_y(\pi)$, leading to maximal spin-momentum entanglement and maximal entropy at $v = 1$. This matches the expected Bell state outcome $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$.

In Case 2: $\omega_1 = \frac{\pi}{2}$, $\omega_2 = v \cdot \frac{\pi}{2}$

In this case, a base rotation of $R_y(\pi/2)$ was applied to the spin qubit before the controlled rotation. This transforms the spin state from $|0\rangle$ to $|+\rangle$. The controlled gate then applies $R_y(\omega_2 - \omega_1)$, which becomes $R_y(-\pi/2)$ when $v \to 0$, effectively undoing the base rotation on the $|1\rangle$ momentum branch. The resulting state is:

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left(|0\rangle_{\mathrm{p}} \otimes |+\rangle_{\mathrm{s}} + |1\rangle_{\mathrm{p}} \otimes |0\rangle_{\mathrm{s}} \right),$$

which is a nonseparable state, that is, the spin and momentum are entangled.

As $v \to 1$, the controlled rotation becomes $R_y(0)$, the identity, and the spin state remains $|+\rangle$ in both the branches. The system becomes separable, and the entanglement disappears. This behavior is reflected in the entropy plot in figure 7: the von Neumann entropy starts at a non-zero value at $v = 0$ and decreases toward zero as $v \to 1$.

Although Bloch sphere representations are not included here, we observe that at $v = 0.5$ for case 1, the spin state begins to deviate significantly between branches. At $v = 1.0$, the state becomes maximally entangled, with orthogonal spin states for $|0\rangle_{\mathrm{p}}$ and $|1\rangle_{\mathrm{p}}$.

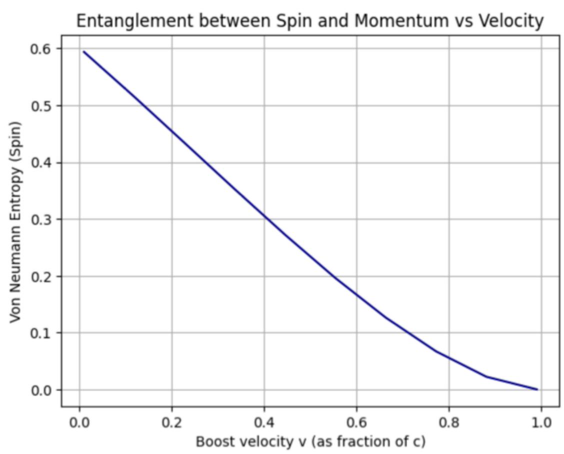

Figure 7: Von Neumann entropy of the reduced spin state for Case 2 ($\omega_1 = \frac{\pi}{2}, \omega_2 = v \cdot \frac{\pi}{2}$). At low velocities, the system is entangled owing to destructive interference between the base and controlled rotation. As the velocity increases, the rotation becomes trivial, and the system transitions to a separable product state, reducing the entropy.

3.4 Conclusion

This simulation confirmed that relativistic effects can induce entanglement between the spin and momentum in single-particle systems. For specific boost parameters, the circuit behaves identically to a CNOT gate [3], producing maximally-entangled Bell states and enabling control over entanglement. The use of entropy as an entanglement measure and the divergence of the spin state under Lorentz boosts provide both quantitative and qualitative insights into this phenomenon.

4 Simulation of Spin-Spin Entanglement Degradation

4.1 Objective and Theoretical Framework

We now study the effect of relativistic transformations on spin-spin entanglement in a two-particle system, following the framework introduced by [5]. The system is initialized in a maximally entangled Bell state as follows:

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle),$$

where each qubit represents the spin state of the particle. Under a Lorentz boost, each spin undergoes a local unitary transformation (Wigner rotation), which depends on the momentum direction and boost velocity. The transformed state is

$$|\Psi'\rangle = (U_1 \otimes U_2)|\Phi^+\rangle,$$

where U_1 and U_2 are local unitaries that encode the Wigner rotations.

Under Lorentz boosts, the spin of each particle undergoes a Wigner rotation depending on the momentum and magnitude of the Lorentz boost. This induces spin-momentum entanglement, causing the pure spin-spin state to become mixed. As a result, the reduced spin density matrix became mixed, reflecting the loss of spin-spin entanglement.

We use two standard entanglement measures: 1. Von Neumann entropy, which quantifies mixedness of the reduced state:

$$S(\rho) = -\mathrm{Tr}(\rho \log_2 \rho),$$

2. Concurrence quantifies two-qubit entanglement as follows:

$$C(\rho) = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4),$$

where λ_i are the square roots of the eigenvalues of $\rho\tilde{\rho}$, and $\tilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$ is the spin-flipped state.

Note: In the original paper, the behaviour of concurrence with respect to rapidity is shown considering Gaussian wave packets (e.g., $\sigma_r/m = 1$ or $\sigma_r/m = 4$). Our simulation simplifies the setup by using discrete momentum eigenstates similar to the previous section and based on [3] and not continuous distributions, thus better resembling the $\sigma_r/m = 1$ case.

4.2 Simulation Methodology

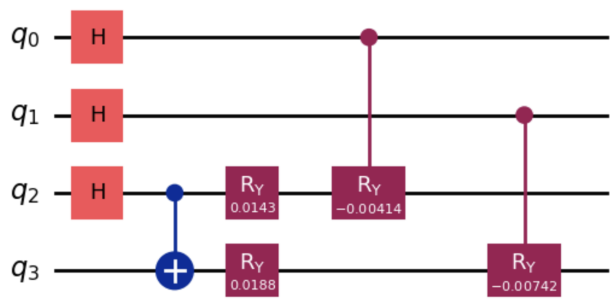

Figure 8: Quantum circuit used to simulate the degradation of spin-spin entanglement under Lorentz boosts, based on the framework of [5]. A Bell state is prepared between the two spin qubits. Local Wigner rotations, modeled as $R_y(\omega_A)$ and $R_y(\omega_B)$, are applied to each spin qubit, depending on the associated momentum. The rotation angles shown correspond to a specific boost velocity configuration. In the full simulation, ω_1 and ω_2 are varied programmatically across a range of values to study entanglement dynamics under Lorentz boosts.

We simulated a 4-qubit system in Qiskit, as shown in figure 8. - Qubits q_0 and q_1 represent the momentum eigenstates of particles A and B, respectively. - Qubits q_2 and q_3 represent their spins, initialized in a Bell state$|\Phi^+\rangle$.

Each momentum qubit is prepared in superposition using Hadamard gates. The momentum directions are chosen to produce distinct Wigner rotations under a fixed Lorentz boost along the z-axis.

Spin qubits undergo: - A base Wigner rotation based on their momentum (using $R(\Omega_A)$), - An additional controlled rotation to simulate boost-induced differences $R(\Omega_B - \Omega_A)$

After the boost simulation, the momentum qubits are traced out to obtain the reduced spin state ρ_{spin}. From this, we computed the entropy and concurrence as functions of the velocity.

4.3 Results and Analysis

Reduced Density Matrix: At a low velocity ($v = 0.01$), the spin state remains nearly pure and maximally entangled. The reduced density matrix is given by

$$\rho_{v=0.01} \approx \begin{bmatrix} 0.4999 & -0.0007 & 0.0007 & 0.4999 \\ -0.0007 & 0.000003 & -0.000003 & -0.0007 \\ 0.0007 & -0.000003 & 0.000003 & 0.0007 \\ 0.4999 & -0.0007 & 0.0007 & 0.4999 \end{bmatrix}$$

At a high velocity ($v = 0.99$), the entanglement degrades.

$$\rho_{v=0.99} \approx \begin{bmatrix} 0.4791 & -0.0574 & 0.0574 & 0.4791 \\ -0.0574 & 0.0209 & -0.0209 & -0.0574 \\ 0.0574 & -0.0209 & 0.0209 & 0.0574 \\ 0.4791 & -0.0574 & 0.0574 & 0.4791 \end{bmatrix}$$

Entropy and Concurrence Behaviour: As velocity increases: The von Neumann entropy increases, as shown in figure 9, indicating that the spin subsystem becomes more mixed owing to Wigner rotation-induced decoherence.

As rapidity increases: - The concurrence decreases, as shown in figure 10, approaching zero at high rapidities. This confirms the loss of spin-spin entanglement from the perspective of an observer under relativistic motion.

This behavior matches theoretical expectations, that is, Lorentz boosts induce momentum-dependent spin rotations. These local transformations create entanglement between the spin and momentum, reducing the entanglement between spins when the momentum is traced out. Spin-spin entanglement is redistributed into spin-momentum entanglement, which aligns with the results in [5].

Although not shown here, Bloch sphere simulations reveal that at low velocities, the spin qubits remain pure and maximally entangled. As the velocity increases, the spin states diverge due to the differing Wigner angles, contributing to the observed entropy increase.

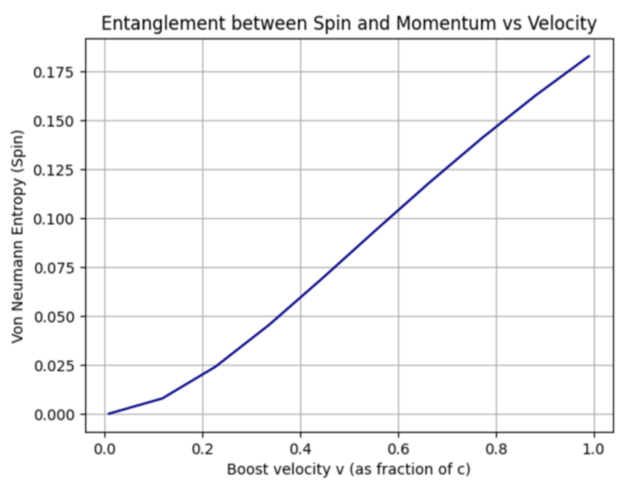

Figure 9: Von Neumann entropy of the reduced spin state as a function of boost velocity. As the velocity increases, the local Wigner rotations applied to each particle's spin qubit induce decoherence, resulting in a mixed-spin state. This confirms the degradation of entanglement from the perspective of boosted observers.

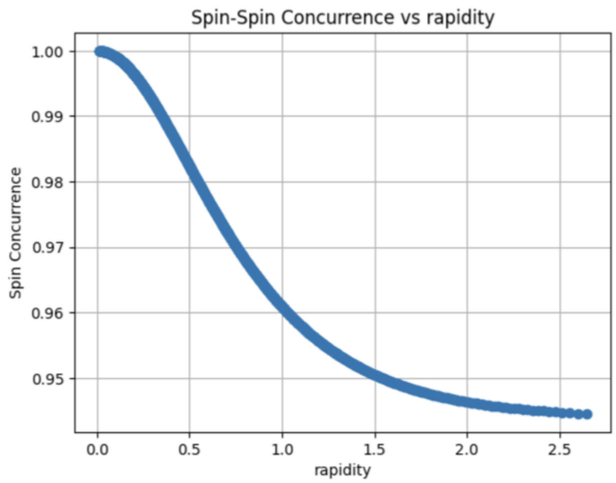

Figure 10: Concurrence as a function of rapidity ξ. As the rapidity increases (i.e., as the relativistic boost becomes stronger), the concurrence decreases, approaching zero. This quantitatively confirms the loss of spin-spin entanglement due to momentum-induced Wigner rotations, in agreement with the findings of [5].

4.4 Conclusion

This simulation confirms that Lorentz boosts degrade spin entanglement in a two-particle Bell state by inducing spin-momentum entanglement. Using entropy and concurrence, we demonstrate that the relativistic motion acts as a source of decoherence. Although we used discrete momentum eigenstates rather than Gaussian wave packets, our output and results are consistent with the behavior described in [5], closely matching the case ($\sigma_r/m = 1$). This establishes a practical framework for modeling the relativistic effects on entanglement using quantum circuits.

5 Discussion and Future Work

This study demonstrated how relativistic effects, particularly Lorentz boosts, can be modeled using circuit-level simulations in Qiskit. Across three simulations, we observed how boosts impact spin through Wigner rotations, induce spin-momentum entanglement, and how local transformations degrade spin-spin entanglement in bipartite systems. The results are in line with the theoretical predictions from the relativistic quantum information literature and show that the use of quantum circuits is a viable path for studying these effects.

An important insight is that entanglement is not lost under Lorentz transformations but is redistributed between the internal and external degrees of freedom. Our simulations show that even with simplified ideas, such as discrete momentum eigenstates and fixed boost direction, the essential effects of relativistic quantum transformations can be captured using quantum gates.

Future work could extend this framework in several ways. One path is to use variational quantum algorithms, such as the Variational Quantum Eigensolver (VQE), to optimize the Wigner rotation parameters. By defining a cost function that quantifies the deviation from a desired entangled target state, classical optimizers can be used for dynamic Wigner angle adjustments to preserve entanglement under specific relativistic transformations. This approach offers practical tools for mitigating relativistic decoherence in space-based quantum communication.

Other intuitive extensions include moving from discrete momentum eigenstates to Gaussian wave packets, exploring arbitrary boost directions, and simulating multipartite entangled systems that undergo relativistic transformations.

6 Conclusion

We presented a set of quantum simulations that explored the relativistic effects on spin and entanglement using Wigner rotations. By simulating three key phenomena — Wigner rotation, spin-momentum entanglement, and entanglement degradation in Bell states — we demonstrated how Lorentz boosts affect quantum states. Our results offer a concrete, circuit-level framework for studying relativistic quantum information using present-day quantum computing tools such as Qiskit

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