Assignment-1

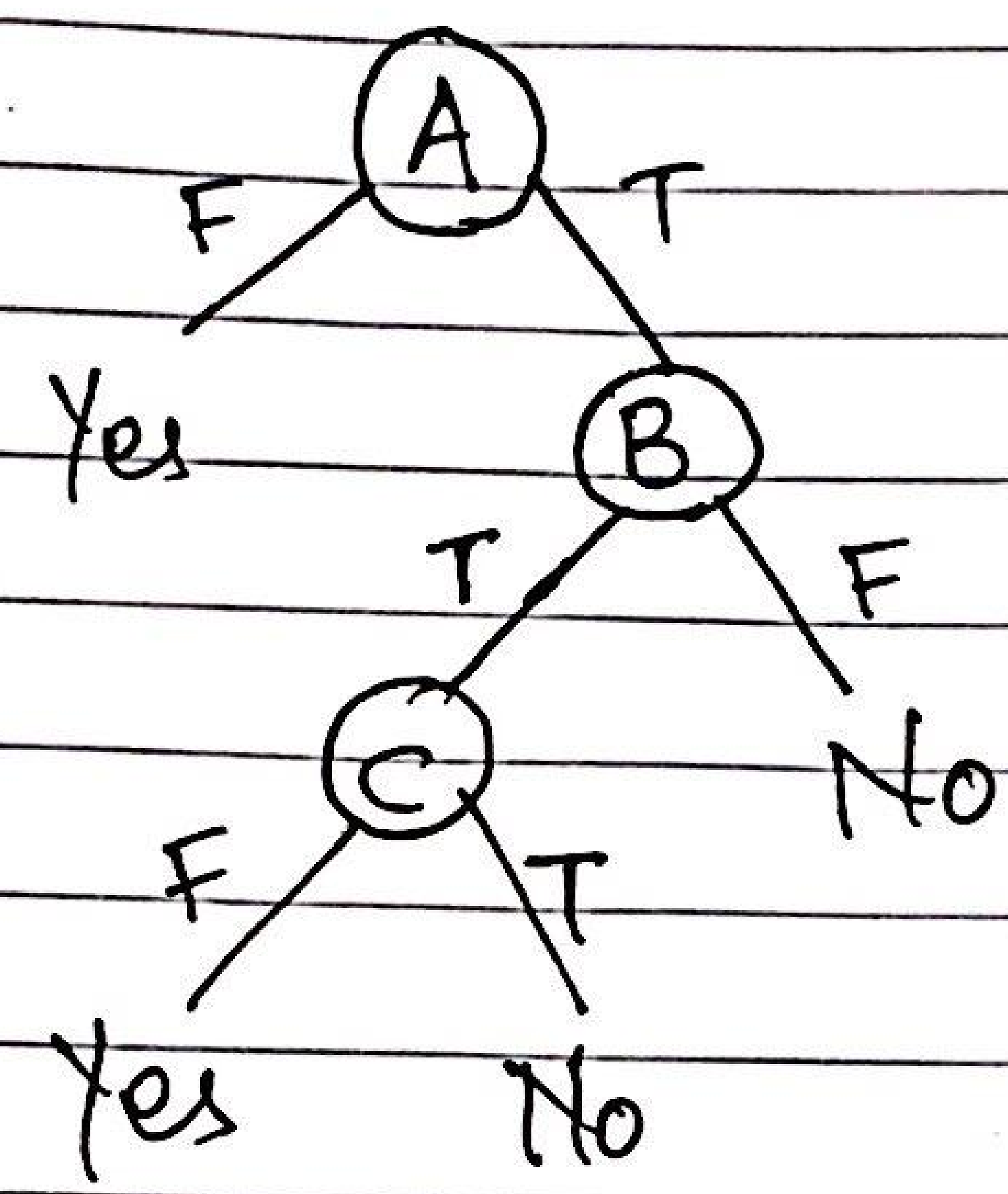
⇒ Part I: Written Problems

1. Representing Boolean Functions

(a.) $(\neg A \vee B) \wedge \neg(C \wedge A)$

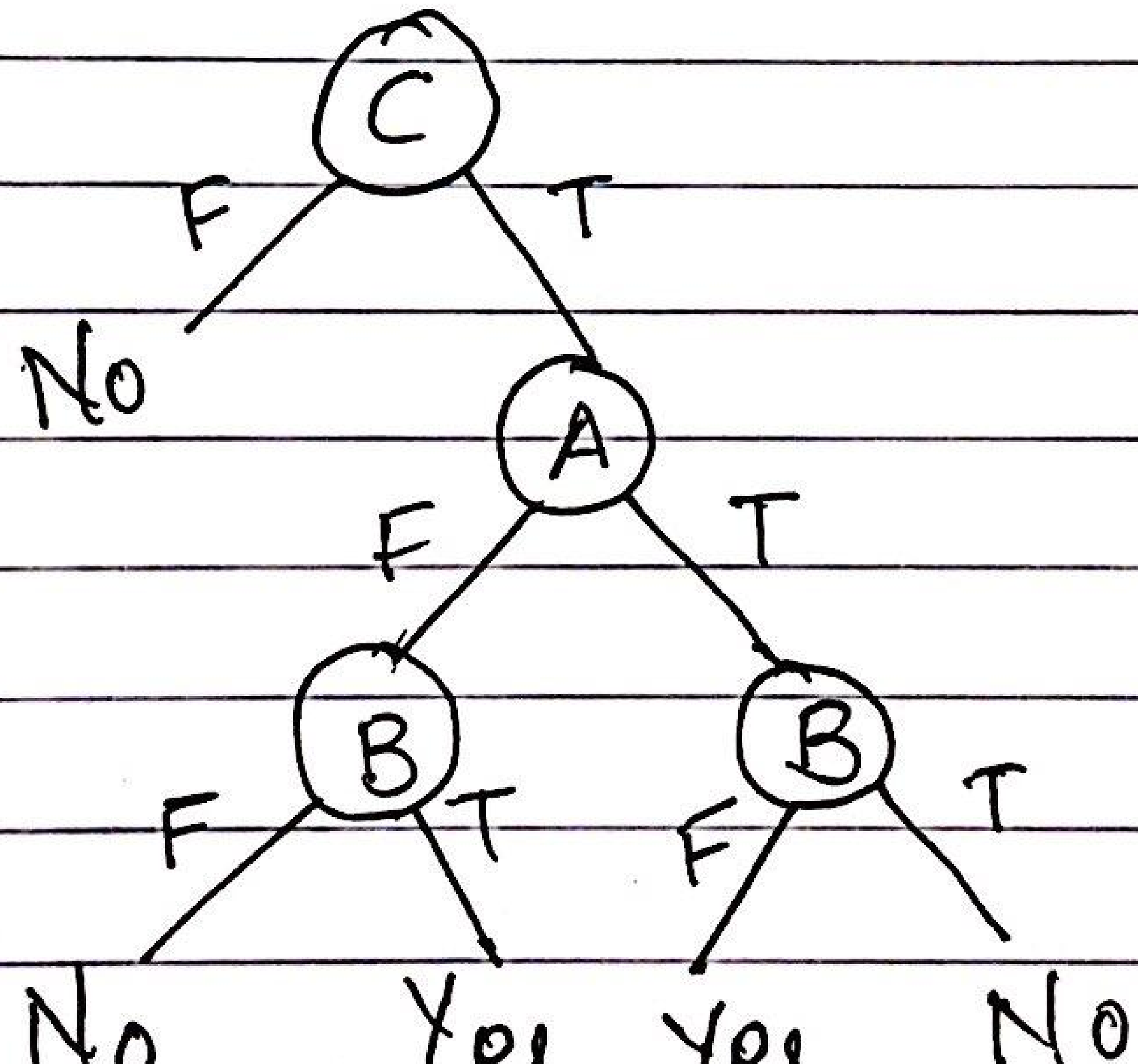
$\therefore (A' \vee B) \wedge (C' \vee A')$

$\therefore A' \vee (B \wedge C')$



(b.) $(A \oplus B) \wedge C = ([A \wedge B'] \vee [A' \wedge B]) \wedge C$

$= (A \wedge B' \wedge C) \vee (A' \wedge B \wedge C)$



2. Decision Trees

$\hookrightarrow$ $P(Class = 1) = \frac{5}{10}$

$P(Class = 0) = \frac{5}{10}$

$$\therefore H(Y) = -\sum_{i=Y}^{N} P(Y=y_i) \log_2 P(Y=y_i)$$

$$= -0.5 \log_2 0.5 - 0.5 \log_2 0.5$$

$$= -0.5(-1) - 0.5(-1)$$

$$= 0.5 + 0.5$$

$$\therefore H(Y) = 1$$

$$H(Class \mid X_1 = 0) = \frac{-4}{1+4} \log_2 \frac{4}{1+4} - \frac{1}{5} \log_2 \frac{1}{5}$$

$$= \frac{-4}{5} \log_2 \frac{4}{5} - \frac{1}{5} \log_2 \frac{1}{5}$$

$$= -\frac{4}{5}(-0.321) - \frac{1}{5}(-2.32)$$

$$= 0.257 + 0.464$$

$$= 0.721$$

$$\therefore H(Class \mid X_1 = 1) = -\frac{1}{5} \log_2 \frac{1}{5} - \frac{4}{5} \log_2 \frac{4}{5}$$

$$= -0.2 \log_2 0.2 - 0.8 \log_2 0.8$$

$$= -(0.2)(-2.32) - (0.8)(-0.321)$$

$$= 0.464 + 0.257$$

$$= 0.721$$

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$\therefore H(\text{Class}|X_1) = \frac{5}{10}(0.721) + \frac{5}{10}(0.721)$

$= 0.36 + 0.36$

$\therefore H(\text{Class}|X_1) = 0.721$

$H(\text{Class}|X_2=0) = -\frac{4}{7}\log_2\frac{4}{7} - \frac{3}{7}\log_2\frac{3}{7}$

$= 0.462 + 0.523$

$= 0.985$

$H(\text{Class}|X_2=1) = -\frac{1}{3}\log_2\frac{1}{3} - \frac{2}{3}\log_2\frac{2}{3}$

$= 0.528 + 0.389$

$= 0.917$

$\therefore H(\text{Class}|X_2) = \frac{7}{10}(0.985) + \frac{3}{10}(0.917)$

$= 0.6895 + 0.2751$

$\therefore H(\text{Class}|X_2) = 0.9646$

$H(\text{Class}|X_3=1) = -\frac{2}{2}\log_2\frac{2}{2} - \frac{0}{2}\log_2\frac{0}{2}$

$= -1\log 1 - 0$

$= 0$

$H(\text{Class}|X_3=0) = -\frac{3}{8}\log_2\frac{3}{8} - \frac{5}{8}\log_2\frac{5}{8}$

$= 0.530 + 0.423$

$= 0.954$

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$$H(\text{Class} \mid X_3) = \frac{8}{10}(0.954) + \frac{2}{10}(0)$$

$$= 0.7632$$

$$\therefore H(\text{Class} \mid X_1) = 0.721$$

$$H(\text{Class} \mid X_2) = 0.965$$

$$H(\text{Class} \mid X_3) = 0.763$$

$$I(Y; X_1) = H(Y) - H(Y \mid X_1)$$

$$= 1 - 0.721$$

$$= 0.279$$

$$I(Y; X_2) = H(Y) - H(Y \mid X_2)$$

$$= 1 - 0.965$$

$$= 0.035$$

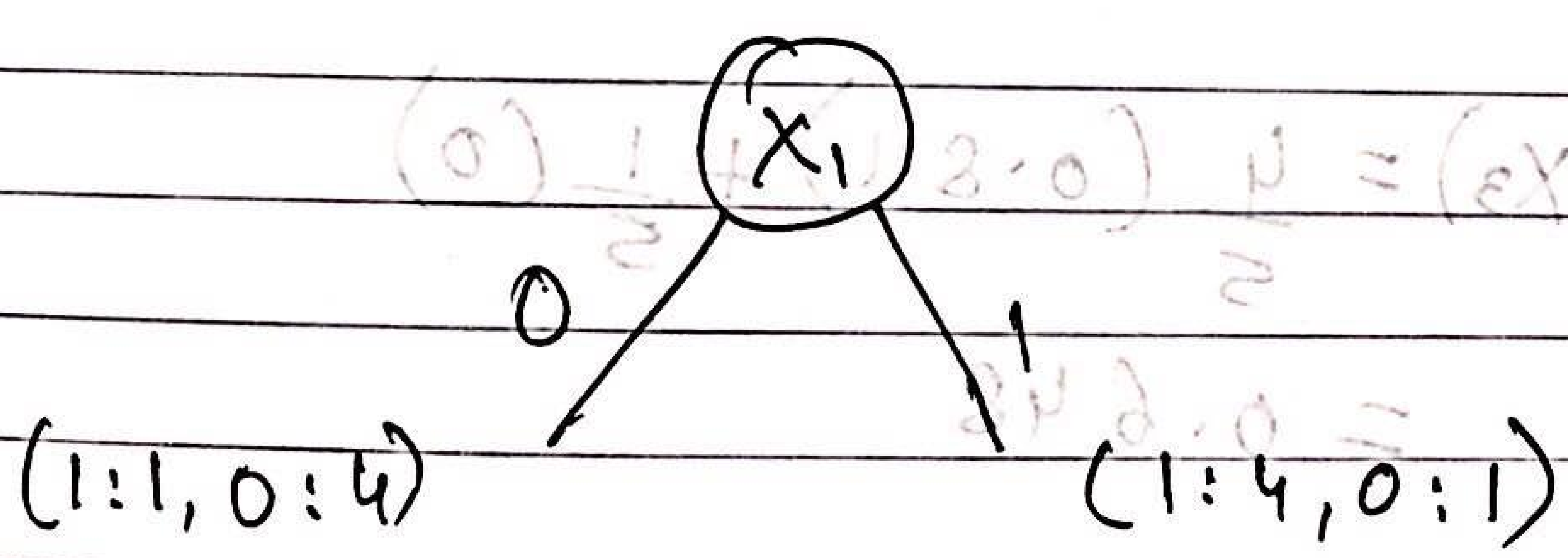
$$I(Y; X_3) = H(Y) - H(Y \mid X_3)$$

$$= 1 - 0.763$$

$$= 0.237$$

$\therefore$ X_1 has the maximum information gain.

$\therefore$ X_1 is the root node.



$$\therefore H(Y) = -\frac{1}{5}\log_2\frac{1}{5} - \frac{4}{5}\log_2\frac{4}{5}$$

$$= 0.464 + 0.257$$

$$\therefore H(Y) = 0.721$$

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$$H(Class \mid X_2=0) = -\frac{3}{3}\log_2\frac{3}{3} - \frac{0}{3}\log_2\frac{0}{3}$$
$$= -1\log_2 1 - 0$$
$$= 0$$

$$H(Class \mid X_2=1) = -\frac{1}{2}\log_2\frac{1}{2} - \frac{1}{2}\log_2\frac{1}{2}$$
$$= 0.5 + 0.5$$
$$= 1$$

$$\therefore H(Class \mid X_2) = \frac{3}{5}(0) + \frac{2}{5}(1)$$
$$= 0 + 0.4$$
$$= 0.4$$

$$H(Class \mid X_3=0) = -\frac{3}{4}\log_2\frac{3}{4} - \frac{1}{4}\log_2\frac{1}{4}$$
$$= 0.311 + 0.5$$
$$= 0.811$$

$$H(Class \mid X_3=1) = -\frac{1}{1}\log\frac{1}{1} - \frac{0}{1}\log\frac{0}{1}$$
$$= 0$$

$$\therefore H(Class \mid X_3) = \frac{4}{5}(0.811) + \frac{1}{5}(0)$$
$$= 0.648$$

$$\therefore I(Y;X_2) = 0.721 - 0.4 = 0.321$$

$$\therefore I(Y;X_3) = 0.721 - 0.648 = 0.073$$

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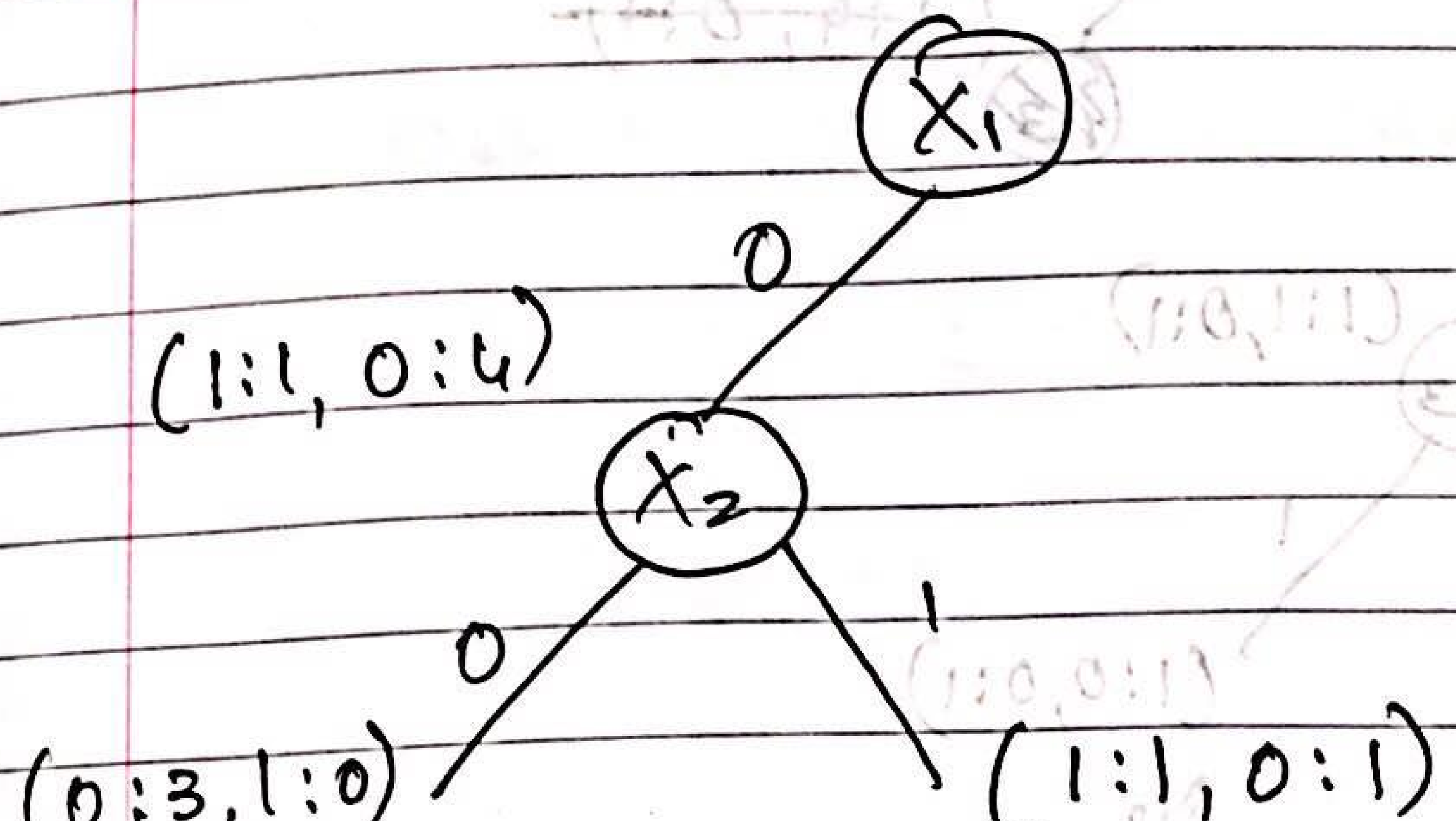
$\therefore$ Here X_2 has maximum information gain

$\therefore$ X_2 is the next node.



$$\therefore H(Y) = \frac{-1}{2}\log_2\frac{1}{2} - \frac{1}{2}\log_2\frac{1}{2}$$

$$= 0.5 + 0.5$$

$$= 1$$

$$H(\text{Class} \mid X_3 = 0) = \frac{-1}{1}\log_2\frac{1}{1} - \frac{1}{1}\log_2\frac{1}{1}$$

$$= 0$$

$$H(\text{Class} \mid X_3 = 1) = \frac{-1}{1}\log_2\frac{1}{1} - \frac{1}{1}\log_2\frac{1}{1}$$

$$= 0$$

$$H(\text{Class} \mid X_3) = \frac{1}{2}(0) + \frac{1}{2}(0)$$

$$= 0$$

$$\therefore I(Y; X_3) = 1 - 0 = 1$$

$\therefore$ X_3 is the next node for $X_2 = 1$.

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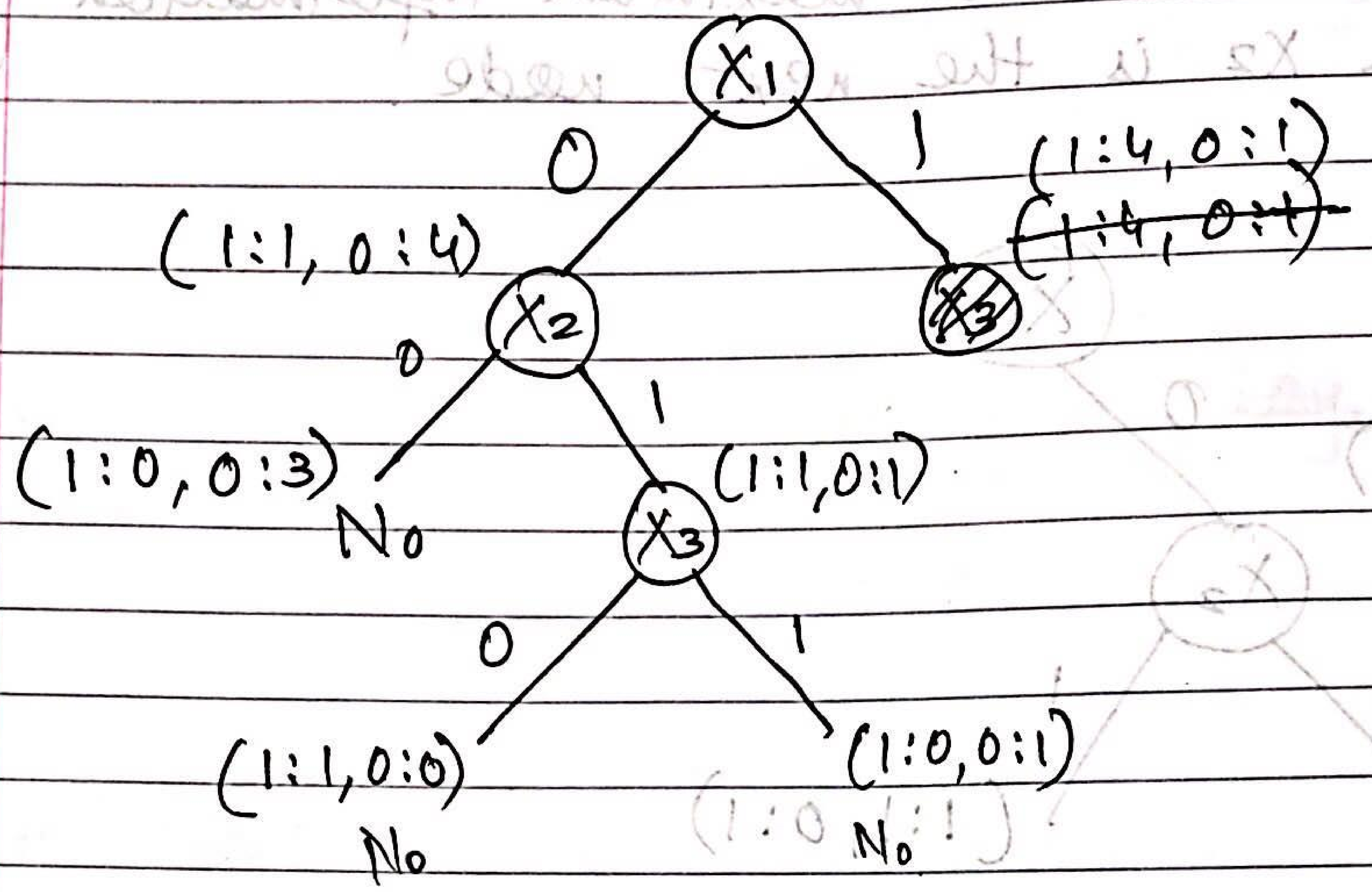

$$\therefore H(Y) = -\frac{4}{5}\log_2\frac{4}{5} - \frac{1}{5}\log_2\frac{1}{5}$$

$$= 0.257 + 0.464$$

$$= 0.721$$

$$H(\text{Class} \mid X_2 = 0) = -\frac{1}{4}\log_2\frac{1}{4} - \frac{3}{4}\log_2\frac{3}{4}$$

$$= 0.5 + 0.311$$

$$= 0.811$$

$$H(\text{Class} \mid X_2 = 1) = -\frac{1}{1}\log_2\frac{1}{1} - \frac{0}{1}\log_2\frac{0}{1}$$

$$= 0$$

$$\therefore H(\text{Class} \mid X_2) = \frac{4}{5}(0.811) + 0$$

$$= 0.648$$

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$$H(\text{Class} \mid X_3=0) = -\frac{0}{4}\log_2\frac{0}{4} - \frac{4}{4}\log_2\frac{4}{4}$$
$$= 0$$

$$H(\text{Class} \mid X_3=1) = -\frac{1}{1}\log_2\frac{1}{1} - \frac{0}{1}\log_2\frac{0}{1}$$
$$= 0$$

$$\therefore H(\text{Class} \mid X_3) = 0$$

$$\therefore I(Y; X_2) = 0.721 - 0.648 = 0.073$$

$$\therefore I(Y; X_3) = 0.721 - 0 = 0.721$$

$\therefore$ Here X_3 has maximum information gain.
$\therefore$ X_3 is the next node for $X_1 = 1$.