An Algorithm for Evaluating the Validity of Singly-Quantified Monadic Predicate Logic Arguments

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An Example

Consider the following argument in the universal set of all beings.

"Every person is a rational being. There exists a being that is irrational and quantified. Therefore, there exists a quantified being that is not a person."

Px = x is a person, Qx = x is quantified , Rx = x is rational

$$(\forall x) (Px \supset Rx)$$

$$\underline{(\exists x) (\neg Rx \wedge Qx)}$$

$$\therefore (\exists x) (Qx \wedge \neg Px)$$

Example/Formal Proof

Steps	**Reasons**	$\vdash \therefore (\exists x)(Qx \wedge \neg Px)$
1. $(\forall x)(Px \supset Rx)$	1. premise	
2. $(\exists x)(\neg Rx \wedge Qx)$	2. premise	
3. $\neg Ry \wedge Qy$	3. 2 E.I.	
4. $Py \supset Ry$	4. 1 U.I.	
5. $\neg Ry$	5. 3 Simp.	
6. Qy	6. 3 Simp.	
7. $\neg Py$	7. 4,5 M.T.	
8. $Qy \wedge \neg Py$	8. 6,7 Conj.	
9. $\therefore (\exists x)(Qx \wedge \neg Px)$	9. 8 E.G.	

$(\forall x)(Px \supset Rx)$
$\underline{(\exists x)(\neg Rx \wedge Qx}$
$\therefore (\exists x)(Qx \wedge \neg Px)$

Q.E.D.

Step 1

Write down the symbolic logic argument underlying each instantiation of the given predicate logic argument.

Consider the example given in Section 1 at the instance when $x = c$, we then have

$$Pc \supset Rc$$
$$\underline{\neg Rc \wedge Qc}$$
$$\therefore Qc \wedge \neg Pc$$

The symbolic logic argument underlying this one would be

$$P \supset R$$
$$\underline{\neg R \wedge Q}$$
$$\therefore Q \wedge \neg P$$

Step 2

Construct the truth table of the symbolic logic argument obtained in Step 1 and put at the topmost part of each column the quantifier of the proposition for that column. Each simple proposition that is not a premise of the argument is considered to be existentially quantified.

(Observe that the truth table has columns for the simple propositions, premises, and conclusion.)

Object Types	∃ P	∃ Q	∃ R	∀ P⊃R	∃ ¬R∧Q	∃ Q∧¬P
1	1	1	1	1	0	0
2	1	1	0	0	1	0
3	1	0	1	1	0	0
4	1	0	0	0	0	0
5	0	1	1	1	0	1
6	0	1	0	1	1	1
7	0	0	1	1	0	0
8	0	0	0	1	0	0

Step 3

Delete the rows of object types that have entries of 0 in a column of a universally quantified premise (i.e., rows with a 0 entry in a premise column).

For this particular example, Rows 2 and 4 are deleted (they are marked with asterisks below).

Object Types	∃	∃	∃	∀	∃	∃
	P	Q	R	P⊃R	¬R∧Q	Q ∧¬ P
1	1	1	1	1	0	0
2	1	1	0	0*	1	0
3	1	0	1	1	0	0
4	1	0	0	0*	0	0
5	0	1	1	1	0	1
6	0	1	0	1	1	1
7	0	0	1	1	0	0
8	0	0	0	1	0	0

Step 4

Delete the rows of object types that have entries of 1 in a column of an existentially quantified conclusion (i.e., rows with a 1 directly below a conclusion column).

For this particular example, Rows 5 and 6 are further deleted.

Object Types	∃	∃	∃	∀	∃	∃
	P	Q	R	P⊃R	¬R∧Q	Q ∧¬ P
1	1	1	1	1	0	0
3	1	0	1	1	0	0
5	0	1	1	1	0	1*
6	0	1	0	1	1	1*
7	0	0	1	1	0	0
8	0	0	0	1	0	0

Step 5

For each ∀ column, conjoin the entries of the remaining rows and indicate the combined truth value for the column at the bottom of the table.

Object Types	∃	∃	∃	∀	∃	∃
	P	Q	R	P⊃R	¬R∧Q	Q ∧¬ P
1	1	1	1	1	0	0
3	1	0	1	1	0	0
7	0	0	1	1	0	0
8	0	0	0	1	0	0
				1		

Step 6

For each ∃ column, disjoin the entries of the remaining rows and indicate the combined truth value for the column at the bottom of the table.

Object Types	∃	∃	∃	∀	∃	∃
	P	Q	R	P⊃R	¬R∧Q	Q ∧¬ P
1	1	1	1	1	0	0
3	1	0	1	1	0	0
7	0	0	1	1	0	0
8	0	0	0	1	0	0
	1	1	1	1	0	0

Step 7

Determine the validity of the argument according to the following rules:

- *If all object types are deleted then **the argument is valid.***
- *If in the universe that contains these, and only these, remaining object types, it is not the case that all the premises are true and the conclusion false, then **the argument is valid.***
- *If in the universe that contains these, and only these, remaining object types, all the existentially quantified simple propositions are true, the premises are true, and the conclusion is false, then **the argument is invalid.***

  *However, if in the universe just described not all of the existentially quantified simple propositions are true but the premises are all true and the conclusion false, then **the argument is valid** in universes in which it has existential import and **invalid** otherwise.*

Object Types	∃ P	∃ Q	∃ R	∀ P⊃R	∃ ¬R∧Q	∃ Q∧¬P
1	1	1	1	1	0	0
3	1	0	1	1	0	0
7	0	0	1	1	0	0
8	0	0	0	1	0	0
	1	1	1	1	0	0

For this particular example, Rule II is applicable and we conclude that the argument is valid (of course, the formal proof presented earlier already shows this).