SEMANTIC ENRICHMENT OF MATHEMATICS FOR ACCESSIBILITY AND DISPLAY ON THE WEB

VOLKER SORGE, DAVIDE CERVONE, PETER KRAUTZBERGER

We propose to develop and integrate heuristics for semantic enrichment of MathML into MathJax, the open-source JavaScript display engine for mathematics. The enrichment will be applied to solve two major problems for mathematics on the web: ubiquitous math accessibility, and responsive rendering of equations. The proposed work is based on our existing and reliable open-source technology, which has been developed for over 5 years and is deployed across the scientific, educational, and industrial ecosystem on the web.

1. Mathematics and the Open Web Platform

Mathematical notation is a core component of educational, scientific, and industrial communication. It provides a universal method to communicate complex ideas in a precise form, balancing the flexibility to develop new forms of expression with a rigor to remain accessible. The World Wide Web Consortium's (W3C) Open Web Platform is the most important communication platform today and it is rapidly becoming the dominating publishing platform. Despite the many advances in browser and web-authoring technologies, however, scientific content on the web frequently still is forced into static images.

Mathematics plays a vital role in emerging web standards as the only field with a W3C specification – MathML. Today, MathML is the primary format for mathematical content, in particular in publishing workflows. While authors favor a variety of options such as equation editors and LaTeX, conversion to MathML is commonplace.

Semantic enrichment. Despite its success, MathML currently is used more for output than exchange. One might compare this to the situation with graphics formats, where JPG and PNG are good for output, but SVG is a better format for exchange of graphical data.

MathML comes in two flavors: Presentation MathML and Content MathML. Presentation MathML is geared towards visual display, playing a role similar to Microsoft Office XML and LaTeX for print. It provides common layout elements (sub-/superscripts, fractions, roots, etc.), styles, and attributes. While it is sufficient to lay out mathematics, expressions usually do not exhibit enough structure to infer meaning.

In contrast, Content MathML aims to serve as a semantic exchange format for mathematical software systems. It provides elements with relatively precise mathematical semantics (divide, interval, matrix, etc.) leading to representations inspired by mathematical logic (e.g., lambda calculus). Thus, while Content MathML is suitable for conveying semantics between systems, it does not lend itself to a human-oriented style, which often mixes semantic with presentational elements. This limitation also shows in the lack of authoring tools and existing content repositories for Content MathML.

The dominance of Presentation MathML combined with the shortcomings of Content MathML severely limit the use and re-use of MathML content, as it adds significant complexity for any technology that digests MathML, in particular for accessibility (e.g., voicing, navigation), search, and scientific computing.

Real-world mathematics mixes semantics and presentation to enhance both. A successful exchange of MathML, therefore, depends on pragmatic solutions within the existing ecosystem. This means building on Presentation MathML instead of overly abstract Content MathML, with openly documented and extensible APIs to enrich expressions.

We propose to develop an open-source JavaScript solution as part of the MathJax project to help solve this problem. It will provide an implementation of heuristics and APIs to create semantically enriched MathML from TeX, Asciimath, and MathML. We will use this to build solutions for two major problems for mathematical content on the web today: ubiquitous accessibility, and rendering on mobile devices.

Accessibility. Ensuring accessibility to scientific material has always been a challenging task and a major obstacle for full inclusiveness in education in the traditional STEM subjects (science, technology, engineering, and mathematics), in particular at the late secondary and tertiary stages. As teaching moves towards the web, we are faced with rapidly changing content, employing media ranging from traditional articles containing mathematical formulas and scientific diagrams to highly interactive web pages with dynamic diagrams or simulations. This makes traditional approaches to accessibility all but obsolete.

In addition, accessibility goes beyond the traditional ideas of aural rendering and magnification that make content accessible for users who are visually impaired or deaf. It also includes support for those with cognitive impairments or learning disabilities, who often need solutions such as highlighting, changes in contrast, or spatial re-representation.

The Open Web Platform holds the promise of greatly enhancing accessibility. Still, the reliance on expensive, specialist software creates an even higher obstacle for inclusive education. To make matters worse, users and publishers currently are left without a reliable solution for mathematics on the open web.

We will leverage the semantically enriched MathML to generate speech text (e.g. "f of x equals x squared"), enable exploration of mathematical expressions, and provide APIs for accessibility tools like screenreaders. Incorporating accessibility directly into the MathJax core will provide a ubiquitous accessibility solution on all browsers and platforms.

Responsive equations. Rendering equations on mobile devices is extremely challenging: they often fail to reflow or become illegible through iterated line-breaks. We will use the semantic enrichment to develop a novel rendering for small screens by re-arranging and collapsing subexpressions. This will present an "outline" to the reader that fits faithfully on the screen while providing the means to navigate and explore and expand the expression further.

Future continuation. We hope to significanly advance this work in all three aspects in a second development year as described in the appendix.

2. Prior Work

MathML authoring and conversion. A key problem of MathML is the lack of good authoring solutions. In fact, MathML is rarely used as a direct authoring format. Instead, authors use tools with custom formats such as graphical equation editors, TEX/LATEX, or computing software. This diversity is desirable so that authors can find solutions that fit their needs.

While some tools export to MathML, others (notably LATEX) do not. This leads to an ecosystem of conversion tools of varying output quality. In either case, the resulting markup is usually limited to Presentation MathML. Similarly, enriching MathML will happen during import into an internal format.

Accessibility. Until 2013, the only MathML accessibility solution was MathPlayer, available as a plugin for Internet Explorer. Unfortunately, Microsoft abandoned support for plug-ins like MathPlayer starting with Internet Explorer 10. In 2013, Google's screen reader, ChromeVox, gained MathML support. In late 2013, Apple VoiceOver incorporated rudimentary MathML support.

MathPlayer and ChromeVox apply heuristics to generate a semantic structure from Presentation MathML and use this to voice and navigate equations. While MathPlayer is the gold standard, it is proprietary, undocumented, and now effectively unavailable.

MathJax and ChromeVox. Our proposal's main focus is the open-source MathJax project, a joint venture of the American Mathematical Society and the Society for Industrial and Applied Mathematics. Founded in 2008, MathJax solves the chicken-and-egg problem of MathML: no browser support $\Rightarrow$ no published MathML $\Rightarrow$ no browser support $\Rightarrow \ldots$

MathJax is centered around its eponymous JavaScript library for rendering mathematics. It is a highly modular, input- and output-agnostic display engine that "just works". Today, MathJax is the default solution for math on the web thanks to universal browser support, a large test suite, and a rich API. It is also used in desktop and mobile applications and the free MathJax CDN service has grown to 100 million unique visitors per month.

While MathJax is known for ease-of-use and visual quality, it was designed with accessibility in mind. This includes a user interface for magnification and copy&paste, as well as smooth interaction with MathPlayer and ChromeVox. In 2013, the MathJax and ChromeVox teams collaborated to enhance ChromeVox's voicing and navigation of MathJax output. This collaboration will find a continuation in a joint project with Benetech in August 2014 to provide server-side, accessible SVG image rendering for non-JavaScript environments.

In the project proposed here, we will combine our expertise with mathematical rendering and accessibility to extend MathJax to ubiquitous enrichment, accessibility, and responsive rendering of mathematics.

3. Research Approach

We will develop JavaScript applications to generate semantically enriched MathML as well as two rendering tools to leverage this enrichment. To summarize we propose to build

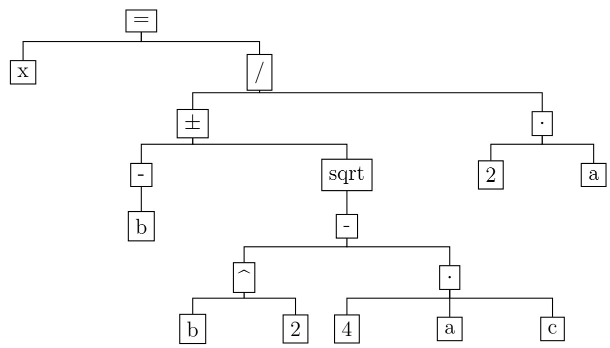
(1) an application implementing heuristic algorithms to process MathML and embed semantic information in the original MathML,
(2) an application that creates and embeds speech-strings in MathML for voicing, navigation, and synchronized highlighting by screenreaders, and,
(3) an application for "responsive equations", i.e., meaningful rendering on mobile devices by re-arranging and collapsing content as well as a user interface for exploration.

3.1. **Semantic enrichment.** For semantic enrichment we decided against Content MathML, as it is impractical for in human-oriented presentation. We therefore propose to adapt and improve an approach used by the ChromeVox screen reader. It is based on the heuristic interpretation of Presentation MathML into a custom semantic representation that is modeled on K-16 mathematics.

The main problem for semantic enrichment of Presentation MathML is to transform its flat structure into one that correctly determines the scope of operators, relations, etc. Our approach aims to represent a formula in a semantic tree structure akin to a term tree. The semantic tree is assembled bottom-up, where we first classify the single components of an expression, giving each an immutable type and a mutable role. The former aims to capture the basic nature of the symbol, while the latter is used to describe the role of a symbol in the context of the formula. For example, f, which has the type "Latin letter" can have the role of an identifier in $f + g$ but that of a function in $f(x)$.

A central heuristic then builds term trees from flat structures by promoting relations and defining operator precedence orders as well as determining properly delimited structures. As an example of this heuristic we observe how the quadratic formula is rewritten from its Presentation MathML representation into its semantic interpretation below:

```
<math>
  <mi>x</mi>
  <mo>=</mo>
  <mfrac>
    <mrow>
      <mo>&#x2212;</mo>
      <mi>b</mi>
      <mo>&#x00B1;</mo>
      <msqrt>
        <msup>
          <mi>b</mi>
          <mn>2</mn>
        </msup>
        <mo>&#x2212;</mo>
        <mn>4</mn>
        <mi>a</mi>
        <mi>c</mi>
      </msqrt>
    </mrow>
    <mrow>
      <mn>2</mn>
      <mi>a</mi>
    </mrow>
  </mfrac>
</math>
```



In addition, our procedure contains heuristics to (1) determine potential function applications, (2) break up symbol sequences into elided products, (3) recognize scope and nesting of big operators (e.g., sums, integrals), (4) distinguish tables into matrices, vectors, and case statements, (5) combine punctuated expressions and determine the meaning of ellipses.

Since it is not useful to replace the existing MathML structures by purely semantic structures, part of our work will revolve around embedding the semantic information into the original MathML. While the connection between the semantic tree and the original MathML may appear straightforward, a naive approach can complicate the exploitation of the semantic information. We will therefore investigate several strategies including custom data-attributes, micro-data, and RDFa.

Within the scope of this project we will experiment with these strategies by testing their effectiveness directly in the context of accessibility and responsive equations. This will, in particular, provide answers to the question how best to represent the different relationships of single constituents of an expression to allow for the exploitation of semantic information not only for an entire expression but also for sub-expressions. In addition to experimenting within the context of our own work, we will also involve collaborators to obtain feedback on the proposed approaches in terms of their usefulness for application like search or computing.

3.2. **Accessibility.** As in our work on semantic enrichment, we can build on our work for providing accessibility in the ChromeVox screen reader. Our goals are to (a) generate high-quality speech text and (b) enable semantically meaningful formula browsing.

Semantic Speech Annotations. In an initial step, we will exploit the semantic enrichment of formulas given in MathML to generate appropriate annotations for aural rendering of mathematical formulas. Consequently this task will serve as a first application of different embeddings of the semantic content as described in the previous section. Our main focus here will be on meaningful rendering and embedding of sub-expressions in the context of the overall expression as well as APIs for access by screenreaders.

Semantic Formula Browsing. Formula exploration is the main tool to allow reader engagement with content and provide synchronized highlighting to assist users with learning disabilities. In current implementations, formula exploration only works on the syntactic representation, i.e., the MathML tree, as the semantic tree representation is not a straight forward DOM representation that can be meaningfully browsed recursively. Hence we will design a method to traverse the semantic tree so as to only expose meaningful substructures in the correct order, exploiting the semantic annotations provided.

Since our semantic markup yields a term tree structure, we can exploit well understood techniques from symbolic computation and logic, such as anti-unification, in order to compute sub-term abstractions. This task is closely related to our efforts on responsive equations, as detailed in the next section, so we can expect a synergy effect here. The examples in the next section also serve as a visual representation of how formula browsing and abstraction would appear to non-sighted users.

In a later continuation of this project we want to develop these abstractions further to provide similarity-based, structural, and elliptical abstraction; see the appendix for more information.

3.3. **Responsive Equations.** The second rendering tool we propose is related to responsive web design, which emerged from the challenges of the mobile web. Responsive design enhances a core feature of HTML: reflow. Originally focusing on re-arranging and optimizing content, new tools transform the content itself, e.g., cropping images, abstracting icons, and modifying tables.

Reflowing mathematics poses a great challenge as it combines the properties of text, tables, and graphics into a singular problem. While good line-breaking algorithms exist for print, they are often counter-productive on the web, damaging legibility of larger equations beyond repair. The problem is exacerbated by the fact that content is created with print in mind, manually fitting it to page dimensions – manual line breaks, arrangements across tabular layout, and other "tweaks" prevent a sensible reflow.

We want to leverage the semantic enrichment to create "responsive equations", a completely novel way of dynamically presenting math on small screens. Our tool will (a) collapse and re-arrange subexpressions on small screens to provide the reader with a meaningful overview of the expression and (b) implement an interface for exploration of collapsed equations.

As a simple example, below is a typical number-theoretical identity. On the left hand side, typeset normally and in two collapsed versions, on the right hand with line-breaking in a 10em container. Similarly, arrays of equations can be re-arranged as a single column before further collapsing.

$$1+\frac{q^2}{(1-q)}+\frac{q^6}{(1-q)(1-q^2)}+\cdots=\prod_{j=0}^{\infty}\frac{1}{(1-q^{5j+2})(1-q^{5j+3})},\qquad \text{for } |q|<1.$$

$$1+\frac{q^2}{\bullet}+\frac{q^6}{\bullet\bullet}+\cdots=\prod_{j=0}^{\infty}\frac{1}{\bullet\bullet},\ \text{for } |q|<1.$$

$$\sum\bullet=\prod_{j=0}^{\infty}\bullet,\ \text{for } |q|<1.$$

$$\begin{array}{l} 1+\dfrac{q^2}{(1-q)} \\ +\dfrac{q^6}{(1-q)(1-q^2)} \\ +\cdots= \\ \displaystyle\prod_{j=0}^{\infty}\frac{1}{(1-q^{5j+2})(1\qquad} , \\ \quad -\,q^{5j+3}) \\ \qquad \text{for } |q|<1. \end{array}$$

An interface will allow the user to navigate these responsive equations, expanding and contracting them as fits their needs and screen real estate. A simple demonstration, including the example above, is available at codepen.io/mathjax/full/dgJHx .

4. Team background

4.1. **The team.** Volker Sorge, Senior Lecturer (Associate Professor) at the University of Birmingham, UK, has been working on semantic enrichment and accessibility of mathematical documents with his research group for 10 years. He integrated accessibility solutions in the context of the European Digital Mathematics Library and developed and implemented the mathematics component for ChromeVox while at Google, Inc., Mountain View, CA.

The MathJax team consists of Davide Cervone, Christian Perfect, and Peter Krautzberger. Davide Cervone, professor at Union College, NY, is MathJax's creator and lead developer. He has pioneered the use of JavaScript for mathematical layout since 2004. He also works on WeBWorK, an open-source online homework system for math and sciences courses.

Peter Krautzberger is an independent consultant in Bonn, Germany, managing the MathJax project. He joined MathJax in 2012 after completing a DFG-funded postdoc at the University of Michigan, Ann Arbor. He is an invited expert at the W3C Digital Publishing Working Group where he leads the STEM task force. Christian Perfect joined MathJax in 2014. He is also the lead developer of the open-source assessment system NUMBAS and will not be directly involved in this project.

In short, our team has extensive expertise in the fields of math display, processing, and accessibility and a proven track record of producing production-ready results combined with an ease of use that enables the widest possible audience.

4.2. **Partners and Collaborators.** MathJax's growing network of collaborators in education, research, and industry will provide us with feedback during the project. For example, (a) we work closely with the MathJax sponsors, (b) we have worked with the Readium Foundation on MathML support in their epub3 SDK, and (c) we have good relations with the teams at Hypothes.is, IPython, and Sage, all of whom are interested in better semantics.

5. Deliverables

5.1. **Semantic enrichment.** We will develop a JavaScript application that will process MathML, TEX/LATEX, and Asciimath to generate MathML with embedded semantic information. This information will be generated from heuristics that analyze the MathML, identifying its structure, subexpressions, and subject area.

5.2. **Accessibility tool.** Using this enrichment, we will develop an application to generate speech strings as well as navigational information and embed these into MathJax output. This will allow accessibility tools to access and navigate the expressions for voicing, synchronized highlighting, and other modern accessibility methods.

5.3. **Responsive equations.** Using the enrichment, we will develop an application to provide a novel way of rendering mathematics on small screens by re-arranging and collapsing equations meaningfully with a user interface for exploration.

6. Budget justification

Our proposal requests funding for a small team that has an excellent track record in leading the MathJax project and making math accessible on the web. The proposed funds will support these established researchers/developers to focus on the complex problems in this proposal.

7. Other sources

The MathJax Consortium and its sponsors support the regular MathJax maintenance and development. These resources are not available to support the work outlined in this proposal. However, the MathJax Consortium will support travel to appropriate conferences and workshops to disseminate the results of the research.