Towards Meaningful Visual Abstraction of Mathematical Notation

Volker Sorge

MathJax Consortium
joint work with Davide Cervone and Peter Krautzberger

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Introduction

- Generation of responsive equations in MathJax
- MathJax is a JavaScript library for rendering Mathematics in all browsers
- Can take LaTeX, AsciiMath, and MathML as input
- Generates browser output, e.g. HTML/CSS, SVG
- Standard Maths rendering solution for: stackexchange, wordpress blogs, mediawiki, etc.
- Internal format is MathML

Introduction (ctd.)

- Content has to be displayed well, regardless of the form factor
- Responsiveness is ubiquitous
- Reading/browsing is more and more dominated by mobile devices
- There is a need to work on responsiveness for advanced content
- Mathematics is particularly challenging
- Authoring is still mainly geared towards static print
- Most content is created from LaTeX or AsciiMath which is not "web friendly"

Responsive Web Content

- Responsive design enhances a core feature of HTML: reflow
- Originally focused on re-arranging and optimising content
- New tools transform the content itself
    - cropping images
    - abstracting icons
    - modifying tables

Reflow in Mathematics

- Combines the properties of text, tables, and graphics into a single problem
- Good line-breaking algorithms exist for print
- They are often counter-productive on the web
- Damage legibility of larger equations beyond repair
- Content is usually created with print in mind
  - Manual line breaks
  - arrangements across tabular layout
  - Tweaks of spacing, etc.
- Sensible reflow is much harder to accomplish.

Responsive Equations

- Automatic reflow adapting to form factor of display
- Intelligent linebreaking
  - Don't break in the middle of an expression
- Chunking: Abstracting over large elements
  - Find meaningful subexpressions
- Adapt tabular layout to different screen sizes

Example

$$I_\nu(\nu^{-1},1) = \underbrace{\frac{\pi^2}{4}\ln\left(\frac{(1+\nu)^{1+\nu}}{\nu^\nu}\right) - \frac{7\zeta(3)}{8}\nu}_{\text{Let this be C}} + 2\int_1^{\frac{1-\nu}{1+\nu}} \frac{\chi_3(v)}{(1+v)^2}\mathrm{d}v$$

$$= C - \frac{2\chi_3(v)}{1+v}\bigg|_1^{\frac{1-\nu}{1+\nu}} + 2\int_1^{\frac{1-\nu}{1+\nu}} \frac{\chi_2(v)}{v(1+v)}\mathrm{d}v$$

$$= C + (1-\nu)\chi_3\left(\frac{1-\nu}{1+\nu}\right) - \frac{7\zeta(3)}{8} - \quad 2\chi_2(v)\ln(1+v)\bigg|_1^{\frac{1-\nu}{1+\nu}} + \int_1^{\frac{1-\nu}{1+\nu}} \frac{\ln(1+v)\ln}{v}$$

$$= C + (1-\nu)\chi_3\left(\frac{1-\nu}{1+\nu}\right) - \frac{7\zeta(3)}{8} + 2\chi_2\left(\frac{1-\nu}{1+\nu}\right)\ln\left(\frac{1+\nu}{2}\right) + \frac{\pi^2}{4}\ln 2$$

$$+ \frac{1}{2}\int_1^{\frac{1-\nu}{1+\nu}} \frac{\ln^2(1+v) - \ln^2(1-v) + \ln^2\left(\frac{1-v}{1+v}\right)}{v}\mathrm{d}v$$

Example of mathematics “in the wild” taken from math.stackexchange.com.

Responsive Equations in MathJax

- reduce the size of equations by abstracting well-defined parts of formulas without obscuring the overall structure of an expression
- embedding a semantic structure into the MathML representation underlying the rendering process
- collapsing mathematically meaningful sub-expressions
- exploit information also for linebreaking

Semantic Enrichment

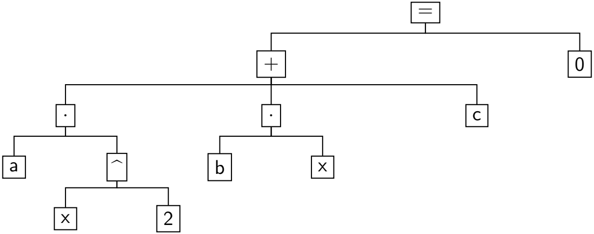
- Basis is semantic tree transformation of MathML elements
- Uses a rule based implementation implemeneted in a speech rule engine (SRE)
- SRE drives math interpretation in ChromeVox (Google) and MathML-Cloud (Benetech)
- Mainly based on interpreting meaning of symbols.
- Rewrite syntax tree into a term tree.

Semantic Tree Example

$$ax^2 + bx + c = 0$$

is rewritten from its Presentation MathML representation into its semantic interpretation:

```
<math>
  <mi>a</mi>
  <msup>
    <mi>x</mi>
    <mn>2</mn>
  </msup>
  <mo>+</mo>
  <mi>b</mi>
  <mi>x</mi>
  <mo>+</mo>
  <mi>c</mi>
  <mo>=</mo>
  <mn>0</mn>
</math>
```



Main Heuristics

- Determine potential function applications,
- break up symbol sequences into elided products respecting spaced element,
- combine bracketed expressions as much as possible,
- recognise scope and nesting of big operators (e.g., sums, integrals),
- distinguish tables into matrices, vectors, and case statements,
- combine punctuated expressions and determine the meaning of ellipses.

“Type of Semantic”

- A shallow interpretation rather than a full blown semantic markup language
- “Semantics of display”
- Aims to stay faithful to a given notation, without fixing too much semantics
- Originally developed to deal mainly with K-12 Mathematics
- Well honed on some quite horrible MathML input
- But it’s heuristic, hence with limitations
- Useful intermediary step towards further semantic enrichment

Combining Semantic and MathML

- MathML is internal representation in MathJax
- Embed the semantic interpretation directly using HTML5 data attributes
- Alternative view on the MathML element, by providing an orthogonal tree structure
- No exposure to the outside yet
- Some issues with embedding:
  - Adds some `mrow` elements, invisible elements.
  - Collapsed elements have to be noted explicitly.
  - Special cases for `mfenced`, `mmultiscripts`

Rendering Enhancements

- Initial enrichment can already lead to enhanced rendering
- E.g. breaking up single `mrows` with stretchy characters:
  Before

$$\left|\tau_0\right| = \left|\sum_m \left(a_b + b_m\right)\right|$$

  After

$$|\tau_0| = \left|\sum_m (a_b + b_m)\right|$$

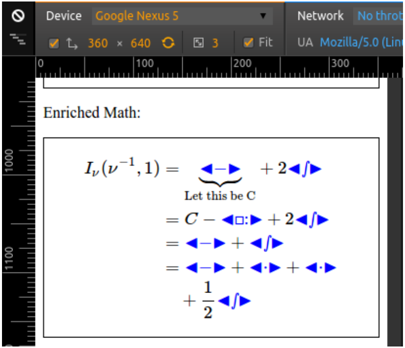
- Similar effects for line breaking

Responsive Equations

- Targets “casual reading”, i.e. reader browses without the need for full details
- Collapse and re-arrange sub-expressions on small screens to provide the reader with a meaningful overview of the expression
- Realise an interface for exploration of collapsed equations.

Exploring Equations

- Subexpressions over a certain complexity are hidden
- Complexity measure mainly aims at shortening an equation
- Collapsed parts are represented by a simple meaning Unicode construction, $\blacktriangleleft X \blacktriangleright$. E.g., $\blacktriangleleft () \blacktriangleright, \blacktriangleleft f() \blacktriangleright, \blacktriangleleft + \blacktriangleright, \blacktriangleleft \sqrt{\ } \blacktriangleright$


Device Google Nexus 5 ▾ Network No throt
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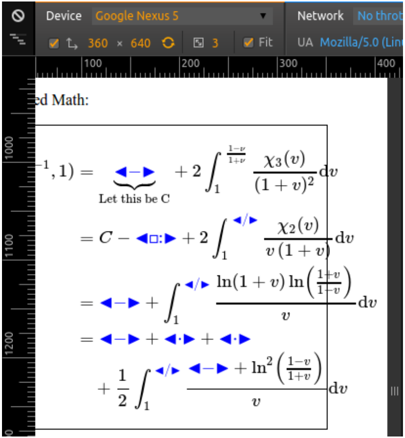
Enriched Math:

$$
\begin{aligned}
I_\nu(\nu^{-1},1) &= \underbrace{\blacktriangleleft - \blacktriangleright}_{\text{Let this be C}} + 2 \blacktriangleleft \int \blacktriangleright \\
&= C - \blacktriangleleft \square : \blacktriangleright + 2 \blacktriangleleft \int \blacktriangleright \\
&= \blacktriangleleft - \blacktriangleright + \blacktriangleleft \int \blacktriangleright \\
&= \blacktriangleleft - \blacktriangleright + \blacktriangleleft \cdot \blacktriangleright + \blacktriangleleft \cdot \blacktriangleright \\
&\quad + \frac{1}{2} \blacktriangleleft \int \blacktriangleright
\end{aligned}
$$

Exploring Equations (ctd.)

- standard `maction` elements on collapsible parts of equation
- `maction` toggles to show or hide those parts
- exploration via expanding and collapsing nested layers of the equation



Device Google Nexus 5

Network No throt

☑ 360 × 640 3 ☑ Fit

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ed Math:

$$
{}^{-1},1) = \underbrace{\blacktriangleleft\!-\!\blacktriangleright}_{\text{Let this be C}} + 2\int_1^{\frac{1-v}{1+v}} \frac{\chi_3(v)}{(1+v)^2}\mathrm{d}v
$$

$$
= C - \blacktriangleleft\square{:}\blacktriangleright + 2\int_1^{\blacktriangleleft/\blacktriangleright} \frac{\chi_2(v)}{v\,(1+v)}\mathrm{d}v
$$

$$
= \blacktriangleleft\!-\!\blacktriangleright + \int_1^{\blacktriangleleft/\blacktriangleright} \frac{\ln(1+v)\ln\left(\frac{1+v}{1-v}\right)}{v}\mathrm{d}v
$$

$$
= \blacktriangleleft\!-\!\blacktriangleright + \blacktriangleleft\cdot\blacktriangleright + \blacktriangleleft\cdot\blacktriangleright
$$

$$
+ \frac{1}{2}\int_1^{\blacktriangleleft/\blacktriangleright} \frac{\blacktriangleleft\!-\!\blacktriangleright + \ln^2\left(\frac{1-v}{1+v}\right)}{v}\mathrm{d}v
$$

Discussion

- UX is not ideal:
  - collapse can be accidentally triggered
  - expansion via single steps only
- Complexity measure mainly tries to determine visual layout size. Other measures could capture a notion of interestingness, etc.
- Display vs exploration
  - Sometimes collapsed parts do not save any more space.
  - Stepwise exploration might still be interesting.

$$\frac{1}{\blacktriangleleft\cdot\blacktriangleright} = 1 + \blacktriangleleft/\blacktriangleright$$

$$\frac{1}{\left(\blacktriangleleft\sqrt{}\blacktriangleright - \phi\right)e^{\frac{2}{5}\pi}} = 1 + \frac{e^{-2\pi}}{1 + \blacktriangleleft/\blacktriangleright}$$

$$\frac{1}{\left(\blacktriangleleft\sqrt{}\blacktriangleright - \phi\right)e^{\frac{2}{5}\pi}} = 1 + \frac{e^{-2\pi}}{1 + \frac{e^{-4\pi}}{1+\blacktriangleleft/\blacktriangleright}}$$

$$\frac{1}{\left(\sqrt{\phi\sqrt{5}} - \phi\right)e^{\frac{2}{5}\pi}} = 1 + \frac{e^{-2\pi}}{1 + \frac{e^{-4\pi}}{1+\blacktriangleleft/\blacktriangleright}}$$

$$\frac{1}{\left(\sqrt{\phi\sqrt{5}} - \phi\right)e^{\frac{2}{5}\pi}} = 1 + \frac{e^{-2\pi}}{1 + \frac{e^{-4\pi}}{1+\frac{e^{-6\pi}}{1+\blacktriangleleft/\blacktriangleright}}}$$

$$\frac{1}{\left(\sqrt{\phi\sqrt{5}} - \phi\right)e^{\frac{2}{5}\pi}} = 1 + \frac{e^{-2\pi}}{1 + \frac{e^{-4\pi}}{1+\frac{e^{-6\pi}}{1+\frac{e^{-8\pi}}{1+\dots}}}}$$

Accessiblity Unit

- Exploit semantic information directly for voicing of equations
- Use collapsed equations for intelligent summarisation
- Interactive exploration with respect to current maction units for improved chunking
- Further assistive technology support
  - Make information available for other AT systems
  - Push data attributes into the actual DOM content as micro data that can be exploited by some AT and Accessibility APIs

Conclusion

- Responsive equations with collapsing and enhanced line breaking
- Semantics is shallow but effective
- Good results with mathematics in the wild

- Work on breaking up tables and tabular alignment.
- Intelligent accessibility via summarisation etc.
- Support of assistive technology

Demo this evening.

Web References

- Demo:
  - http://mathjax.github.io/MathJax-RespEq/Semantics-Lab/Struik.html
  - http://mathjax.github.io/MathJax-RespEq/Semantics-Lab/Semantics-Lab-TeX.html
  - http://mathjax.github.io/MathJax-RespEq/Semantics-Lab/Semantics-Lab-TeX-linebreaking.html
- Systems:
  - https://github.com/mathjax/MathJax/
  - https://github.com/mathjax/MathJax-RespEq/
  - https://github.com/zorkow/speech-rule-engine/
  - https://github.com/mathjax/MathJax-node/