# Initial Empirical Evaluation of Anytime Lifted Belief Propagation

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### Abstract

Lifted first-order probabilistic inference, which manipulates first-order representations of graphical models directly, has been receiving increasing attention. Most lifted inference methods to date need to process the entire given model before they can provide information on a query's answer, even if most of it is determined by a relatively small, local portion of the model. Anytime Lifted Belief Propagation (ALBP) performs Lifted Belief Propagation but, instead of first building a supernode network based on the entire model, incrementally processes the model on an as-needed basis, keeping a guaranteed bound on the query's answer the entire time. This allows a user to either detect when the answer has been already determined, before actually processing the entire model, or to choose to stop when the bound is narrow enough for the application at hand. Moreover, the bounds can be made to converge to the exact solution when inference has processed the entire model. This paper shows some preliminary results of an implementation of ALBP, illustrating how bounds can sometimes be narrowed a lot sooner than it would take to get the exact answer.

#### 1 Introduction

Recently, there has been a surge in interest in the area of Statistical Relational Learning (SRL) [1] and several different representations have been proposed. The central problem in all these models is that of efficient inference. Earlier inference methods were based on sampling methods or methods that completely instantiate all the objects in the domain and hence were prohibitively expensive. Lifted inference [2, 3], inference that manipulates and keeps the first-order structure,

avoiding extensive propositionalization, has been receiving increasing attention recently. To date, all lifted inference methods require a model to be *shattered* against itself and evidence, before inference starts. Shattering means dividing the random variables of the model into clusters of exactly symmetric variables. Evidence is often provided at the level of random variables on specific individuals, typically causing all random variables involving them to form singleton clusters. For many problems this is very close to propositionalization, and the gains from lifted inference are greatly decreased.

The reason shattering is needed in advance is because the algorithms that have been lifted (belief propagation and variable elimination) do require the entire model in order to compute a query's belief. So in general the entire model needs to be used, requiring it to be entirely shattered. However recent work on box propagation [4] shows how to derive bounds on beliefs from using only a portion of a model. This allows us to gradually shatter the model while obtaining useful bounds on the query. Interestingly, this also corresponds to the intuition that reasoning only considers sub- or individual cases in an as-needed basis, as it is done in theorem proving where unification and resolution are gradually used.

The box propagation algorithm was later extended to the SRL case as Anytime Lifted BP (ALBP) [5]. The key idea in ALBP is to perform incremental shattering with the propagation of bounds. In this work, we extend the previous paper by presenting the pseudocode for the ALBP algorithm and by empirically evaluating the algorithm on a small domain. This is a work-in-progress and the results that we present are initial observations. Nonetheless, some of these results clearly show the advantage of box propagation when the bounds can be narrowed much sooner than converging to a point estimate. A similar approach has been taken in [6] where the messages are grouped after every few iterations without constructing the full

lifted network.

# 2 Algorithmic Details

Algorithm 1 Pseudocode explaining the iterative process of Anytime Lifted Belief Propagation.

```
1: for all Query q in queryList do
        rn \leftarrow \text{createSupernode}(\text{lift}(q));
 3:
        cn \leftarrow rn;

⊳ Start at query

 4:
        addSupernode(fq, rn);
 5:
        stillSplittingAndShattering \leftarrow true;
 6:
        ▶ Use rules and evidence to shatter and split
 7:
        while stillSplittingAndShattering do
 8:
            for all Rule r in ruleList do
 9:
                if (r \text{ contains } cn.predicate) and
                   (not splitBy(cn, r)) then
10:
                    performSplit(cn, r, fg);
                                                       ▶ Extends
11:
                end if
12:
            end for
13:
            for all Evidence e in evidenceList do
14:
                if (e contains cn.predicate) and
                   \overline{(\mathbf{not} \text{ shatteredBy}(cn, e))} and
                   (isSubset(e.constriants, cn.groundings))
15:
                    performSplit(cn, e, fg);
                                                       ▶ Extends
16:
                    performShatter(cn, e, fq);
                                                        ▶ Breaks
17:
                end if
18:
            end for
19:
            ▶ Ground the query on the first iteration
20:
            if 1<sup>st</sup> iteration then
21:
                performShatter(rn, q, fg);
                                                        ▶ Breaks
22:
            end if
23:
            ▶ Box propagation gets marginal bound
24:
            interval \leftarrow runBoxPropagation(rn, fg);
25:
            if interval interval < \delta then
26:
                break;
27:
            end if
28:
             ▷ Select the next node to split and shatter
29:
            cn \leftarrow \text{getNextNode}(fg, \text{METHOD});
30:
            if \underline{cn} == \mathbf{null} then
31:
                stillSplittingAndShattering \leftarrow \mathbf{false};
32:
            end if
33:
        end while
34: end for
```

Lifted belief propagation [7] is based on the idea that symmetric variables (that is, variables with exactly the same set of dependencies) will receive and generate the same belief messages. It determines these sets (called *supernodes*) by shattering as a preprocessing step, and performs message passing between them. The main problem with this and the other lifted inference methods is that the nodes that are not even part of the evidence relevant to the query are shattered in advance. For example, consider the following two parfactors:  $\forall Y \phi_1(funny(Y))$  and  $\forall X, Y \phi_1(funny(Y), likes(X, Y))$ . Now if the query is P(funny(fred) | likes(Tom, Fred)), and if it is observed that Tom is a friend of Fred, then by shattering, likes(tom, Fred),  $\langle likes(X, Fred), X \neq tom \rangle$  and  $\langle likes(X,Y)|X \neq tom|Y \neq Fred \rangle$  will form the three clusters. Note that though there are no other evidence present, we still have to split likes(X,Y) into two more clusters, one for Y=Fred and one for  $Y\neq Fred$ . If we had another evidence, say that Mary is a friend of Fred, then we will have more clusters. As the amount of evidence increases, the number of shatterings also increases as the clusters may need to be shattered even if they are not part of the evidence.

ALBP on the other hand, does not consider shattering the model completely against the evidence and shatters only if required. Instead, ALBP extends box propagation to the relational setting. ALBP works by using only a subset of the model for box propagation, but with supernodes, as in Lifted BP. Shattering is performed only as needed for accommodating the parfactors considered at each step, thus minimizing the number of shatterings. Also, the propagation of the bounds can possibly allow for decision making even if the marginal has not converged.

Algorithm 1 presents the outer loop of the algorithm<sup>1</sup>. The outer loop explains the overall process of each iteration of the ALBP algorithm. Beginning with a single supernode that represents the query predicate with no constraints (root node rn), the lifted factor graph (fq) is further extended and broken on each iteration until the marginal probability bounds are satisfactory or fgis completely shattered. In each iteration, a chosen node (cn) in fg is extended by all rules that involve cn's predicate; this mimics the unraveling process of theorem proving. Any evidence on a subset of the constraints of cn also results in breaking fg in order to separate the constraints with respect to the difference in observation. Since the query likely applies to a subset of rn's constraints, we also break at rn in the first iteration with respect to the query's constraints.

## 3 Initial Experiments

We have run our implementation of Anytime Lifted Belief Propagation on a variation of the popular smokers example with the following MLN [8]:

```
2.3
         \neg cancer(x)
1.4
         \neg smokes(x)
2.0
         \neg friends(x,y)
-0.5
        hospital(x)
2.5
        party(x)
1.5
        smokes(x) \rightarrow cancer(x)
1.1
         friends(x, y) \rightarrow (smokes(x) \leftrightarrow smokes(y))
1.4
        party(x) \land smokes(x)
0.5
        cancer(x) \rightarrow hospital(x)
        party(x) \rightarrow hospital(x)
-1.0
```

Our domain contained five persons, and we queried how likely each person had cancer given various observations. For each person, up to one randomly cho-

<sup>&</sup>lt;sup>1</sup>An example and additional pseudocode can be found at http://tsi.wfubmc.edu/labs/strait/ALBP.pdf

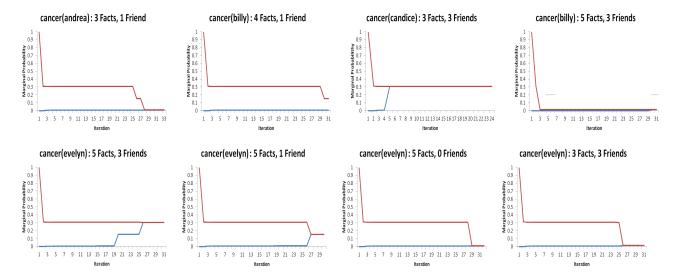


Figure 1: Plots of the marginal probability bounds during ALBP for various queries on the dataset. The red line represents the upper bound and the blue line represents the lower bound.

sen attribute was observed. Either zero, one or three friends relation were observed for each individual. The effects the varying observations had on the bounds of a single query (cancer(X)) are presented in Figure 1.

We observe three effects – (1) If there is immediate evidence about the query then ALBP converges quickly where the bound becomes zero rapidly (right two cases in the top figure). (2) If there is evidence close to the query but is not very strong, the bound becomes narrow quickly but convergence happens later. (3) Finally, if multiple evidences appear as the network is expanded, the bounds progressively decrease to zero. Currently, the choice of the next parfactor to be shattered is made at random. A better heuristic can lead to faster convergence of the bound in many cases. Our initial results are still promising as in most cases, the bounds become quite narrow before ALBP converges.

#### 4 Conclusion

We presented the ALBP algorithm that performs lifted inference without shattering the entire model in advance. This allows for faster useful inference when bounds are enough (such as for decision-making). Importantly, it allows us to treat classes of random variables as a group even when they are not exactly symmetric; in other words, it allows us to reason in general about objects that are only approximately symmetric, where the notion of approximation becomes more restricted the more precise an answer is required. Our initial experiments are promising as the bounds become narrower before the algorithm converges.

An interesting extension in that direction would be making the algorithm consider the cost of obtaining evidence or sharper bounds on certain variables. Similarly, exploring the different methods of identifying the next parfactor to shatter would be an important future work. Finally, the algorithm should be evaluated on large real domains.

Acknowledgements The authors gratefully acknowledge the support of Defense Advanced Research Projects Agency (DARPA) Machine Reading Program under Air Force Research Laboratory (AFRL) prime contract no. FA8750-09-C-0181. Any opinions, findings, and conclusion or recommendations expressed in this material are those of the authors and do not necessarily reflect the view of the DARPA, AFRL, or the US government.

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