$$\max_{c_1, c_2} u = \ln(c_1) + \beta \ln(c_2)$$

Subject to: $c_1 + s = w_1$ + $s(1+r) + w_2 = c_2$

where: $w_2 = 0$ + $s = w_1 - c_1$

Constraint: $c_2 = (w_1 - c_1)(1+r)$

$$\mathcal{L} = \ln(c_1) + \beta \ln(c_2) + \lambda((w_1 - c_1)(1+r) - c_2)$$

$$\frac{d\mathcal{L}}{dc_1} = 1/c_1 - \lambda(1+r) = 0 \Rightarrow \frac{1}{(1+r)c_1} = \lambda \quad ①$$

$$\frac{d\mathcal{L}}{dc_2} = \beta/c_2 - \lambda = 0 \Rightarrow \beta/c_2 = \lambda \quad ②$$

$$\frac{d\mathcal{L}}{d\lambda} = (w_1 - c_1)(1+r) - c_2 = 0 \Rightarrow c_2 = (w_1 - c_1)(1+r) \quad ③$$

From ① + ②: $\frac{1}{(1+r)c_1} = \frac{\beta}{c_2} \Rightarrow c_2 = \beta(1+r)c_1$ ④

From ④ → ③ $\beta(1+r)c_1 = (w_1 - c_1)(1+r)$

$$\beta(1+r)c_1 = w_1(1+r) - c_1(1+r)$$

$$\beta(1+r)c_1 + (1+r)c_1 = w_1(1+r)$$

$$\frac{(1+r)(\beta+1)c_1}{(1+r)(\beta+1)} = \frac{w_1(1+r)}{(1+r)(\beta+1)}$$

⑤ $\boxed{c_1 = \frac{w_1}{\beta+1}}$

Contd. →

Utilizing 4 w/ 5

$$c_2 = \beta(1+r)\left(w_1/\beta_{+1}\right)$$

$$\boxed{c_2 = \frac{\beta(1+r)(w_1)}{\beta+1}} \quad 6$$