21/1/19

UNIT - 4

DYNAMIC PROGRAMMING

* All pairs shortest path problem(or) Floyd Warshall
* Single source shortest paths general weights (Dijkstra's)
* String Edition.
* zero by one 0/1 Knapsack problem
* Reliability Design

* Dynamic programming is applicable for only optimization problems.
* For a given problem we may get any no.of solutions from all those solutions we seek for the optimal solutions. and such an optimal solution becomes the solution ~~of~~ to the given problem.

*2M

Principle of optimality:-

* The dynamic programming algorithm obtains the solutions using principle optimality.
* The principle of optimality states that in an optimal sequences of decisions (or) choices.
* Each sub sequences must be optimal.
* When it is not possible to apply the principle of optimality it is almost impossible to obtain the solution using the dynamic programming approach.
* For example finding of shortest path in a given graph uses the principle of optimality.

All pairs shortest path Problem:-

* Suppose the given graph is weighted and connected graph
* The objective of all pairs shortest path problem is to find shortest path between each and every pair of nodes present in the given graph
* To find shortest path between every pair of nodes using dynamic programming we need to find $A^{k}[i,j]$

where $k = 0$ to n

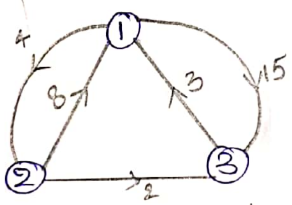
$A^{k}_{[i,j]}$ represents path between i to j via k.

* If $k=0$ we can write A^0 from given graph i.e $A^0 = W[i,j]$

If $k \geq 1$ we can compute the following

$$\boxed{A^{k}[i,j] = \min\{A^{k-1}[i,j],\ A^{k-1}[i,k] + A^{k-1}[k,j]\}}$$

Solve Minimum distances (or) All pair shortest path problem using Dynamic programming.



Now convert the given graph into matrix form.

$$\begin{bmatrix} 0 & 4 & 15 \\ 8 & 0 & 2 \\ 3 & \infty & 0 \end{bmatrix} = A^0$$

Now find out A^1 Matrix

$A^1_{11} = 0$

$A^1_{12} = \min\{A^{1-1}[1,2],\ A^{1-1}[1,1] + A^{1-1}[1,2]\}$

$= \min\{A^0[1,2], A^0[1,1] + A^0[1,2]\}$

$= \min\{4, 0+4\}$

$= 4$

$A^1_{13} = \min\{A^{1-1}_{[1,3]},\ A^{1-1}_{[1,1]} + A^{1-1}_{[1,3]}\}$

$= \min\{15,\ 0+15\}$

$= 15$

$A^1_{21} = \min\{A^{1-1}_{[2,1]},\ A^{1-1}_{[2,1]} + A^{1-1}_{[1,1]}\}$

$= \min\{8,\ 8+0\}$

$= 8$

$A^1_{22} = 0$

$A^1_{23} = \min\{A^{1-1}_{[2,3]},\ A^{1-1}_{[2,1]} + A^{1-1}_{[1,3]}\}$

$= \min\{2,\ 8+15\}$

$= \min\{2,\ 23\}$

$= 2$

$A^1_{31} = \min\{A^{1-1}_{[3,1]},\ A^{1-1}_{[3,1]} + A^{1-1}_{[1,1]}\}$

$= \min\{3,\ 3+0\}$

$= \cancel{0}\ 3$

$A^1_{32} = \min\{A^{1-1}_{[3,2]},\ A^{1-1}_{[3,1]} + A^{1-1}_{[1,2]}\}$

$= \min\{\infty,\ 3+4\}$

$= \min\{\infty,\ 7\}$

$= 7$

$A^1_{33} = 0$

A^2_{11} = ~~min~~ $\{$ $A^1 = \begin{bmatrix} 0 & 4 & 15 \\ 8 & 0 & 2 \\ 3 & 7 & 0 \end{bmatrix}$

$A^2_{11} = \min\{A^{2-1}_{[1,1]},\ A^{2-1}_{[1,2]} + A^{2-1}_{[2,1]}\}$

$= \min\{0,\ 4+8\}$

$= 0$

$A^2_{12} = \min\{A^{2-1}_{[1,2]},\ A^{2-1}_{[1,2]} + A^{2-1}_{[2,2]}\}$

$= \min\{4,\ 4+0\}$

$= 4$

$A^2_{13} = \min\{A^{2-1}[1,3],\ A^{2-1}[1,2] + A^{2-1}[2,3]\}$

$= \min\{15, 4+2\}$

$= 6$

$A^2_{21} = \min\{A^{2-1}[2,1],\ A^{2-1}[2,2] + A^{2-1}[2,1]\}$

$= \min\{8, 0+8\}$

$= 8$

$A^2_{22} = 0$

$A^2_{23} = \min\{A^{2-1}[2,3],\ A^{2-1}[2,2] + A^{2-1}[2,3]\}$

$= \min\{2, 0+2\}$

$= 2$

$A^2_{31} = \min\{A^{2-1}[3,1],\ A^{2-1}[3,2] + A^{2-1}[2,1]\}$

$= \min\{3, 7+8\} = \min\{3, 15\}$

$= 3$

$A^2_{32} = \min\{A^{2-1}[3,2],\ A^{2-1}[3,2] + A^{2-1}[2,2]\}$

$= \min\{7, 7+0\}$

$= 7$

$A^2_{33} = 0$

$$A^2 = \begin{bmatrix} 0 & 4 & 6 \\ 8 & 0 & 2 \\ 3 & 7 & 0 \end{bmatrix}$$

Now findout A^3 matrix, here $k=3$

$A^3_{11} = 0$

$A^3_{12} = \min\{A^2[1,2],\ A^2[1,3] + A^2[3,2]\}$

$= \min\{4, 6+7\}$

$= 4$

$A^3_{13} = \min\{A^2[1,3],\ A^2[1,3] + A^2[3,3]\}$

$= \min\{6, 6+0\}$

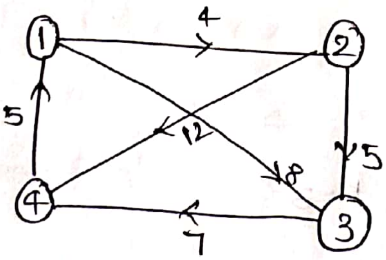
$= 6$

$$A^3_{21} = \min\{A^2_{[2,1]}, A^2_{[2,3]} + A^2_{[3,1]}\}$$
$$= \min\{8, 2+3\}$$
$$= 5$$
$$A^3_{22} = 0$$
$$A^3_{23} = \min\{A^2_{[2,3]}, A^2_{[2,3]} + A^2_{[3,3]}\}$$
$$= \min\{2, 2+0\}$$
$$= 2$$
$$A^3_{31} = \min\{A^2_{[3,1]}, A^2_{[3,3]} + A^2_{[3,1]}\}$$
$$= \min\{3, 0+3\}$$
$$= 3$$
$$A^3_{32} = \min\{A^2_{[3,2]}, A^2_{[3,3]} + A^2_{[3,2]}\}$$
$$= \min\{7, 0+7\}$$
$$= 7$$
$$A^3_{33} = 0$$

$$A^3 = \begin{bmatrix} 0 & 4 & 6 \\ 5 & 0 & 2 \\ 3 & 7 & 0 \end{bmatrix}$$

Hence the distances between all pairs obtain

ex:



Now convert the given graph into matrix form

$$\begin{bmatrix} 0 & 4 & 8 & \alpha \\ \alpha & 0 & 5 & 12 \\ \alpha & \alpha & 0 & 7 \\ 5 & \alpha & \alpha & 0 \end{bmatrix}$$

Now find out A' Matrix

$$A'_{11} = 0$$

$$A'_{12} = \min\{A^{1-1}_{[1,2]}, A^{1-1}_{[1,1]} + A^{1-1}_{[1,2]}\}$$
$$= \min\{4, 0+4\}$$
$$= \cancel{\alpha}\ 4$$
$$A'_{13} = \min\{A^{1-1}_{[1,3]}, A^{1-1}_{[1,1]} + A^{1-1}_{[1,3]}\}$$
$$= \min\{8, 0+8\}$$
$$= 8$$
$$A'_{14} = \min\{A^{1-1}_{[1,4]}, A^{1-1}_{[1,1]} + A^{1-1}_{[1,4]}\}$$
$$= \min\{\alpha, 0+\alpha\}$$
$$= \alpha$$
$$A'_{21} = \min\{A^{1-1}_{[2,1]}, A^{1-1}_{[2,1]} + A^{1-1}_{[1,1]}\}$$
$$= \min\{\alpha, \alpha+0\}$$
$$= \alpha$$
$$A'_{22} = 0$$
$$A'_{23} = \min\{A^{1-1}_{[2,3]}, A^{1-1}_{[2,1]} + A^{1-1}_{[1,3]}\}$$
$$= \min\{5, \alpha+8\}$$
$$= 5$$
$$A'_{24} = \min\{A^{1-1}_{[2,4]}, A^{1-1}_{[2,1]} + A^{1-1}_{[1,4]}\}$$
$$= \min\{12, \alpha+\alpha\}$$
$$= 12$$
$$A'_{31} = \min\{A^{0}_{[3,1]}, A^{0}_{[3,1]} + A^{0}_{[1,1]}\}$$
$$= \min\{\alpha, \alpha+0\}$$
$$= \alpha$$
$$A'_{32} = \min\{A^{0}_{[3,2]}, A^{0}_{[3,1]} + A^{0}_{[1,2]}\}$$
$$= \min\{\alpha, \alpha+4\}$$
$$= \cancel{4}\ \alpha$$
$$A'_{33} = 0$$
$$A'_{34} = \min\{A^{0}_{[3,4]}, A^{0}_{[3,1]} + A^{0}_{[1,4]}\}$$
$$= \min\{7, \alpha+\alpha\}$$
$$= 7$$

$A^1_{41} = \min\{A^0_{[4,1]}, A^0_{[4,1]} + A^0_{[1,1]}\}$

$= \min\{5, 5+0\}$

$= 5$

$A^1_{42} = \min\{A^0_{[4,2]}, A^0_{[4,1]} + A^0_{[1,2]}\}$

$= \min\{\alpha, 5+4\}$

$= 9$

$A^1_{43} = \min\{A^0_{[4,3]}, A^0_{[4,1]} + A^0_{[1,3]}\}$

$= \min\{\alpha, 5+8\}$

$= 13$

$A^1_{44} = 0$

$$A^1 = \begin{bmatrix} 0 & 4 & 8 & \alpha \\ \alpha & 0 & 5 & 12 \\ \alpha & \alpha & 0 & 7 \\ 5 & 9 & 13 & 0 \end{bmatrix}$$

Now find A^2 matrix, here $k=2$

$A^2_{11} = 0$

$A^2_{12} = \min\{A^1_{[1,2]}, A^1_{[1,2]} + A^1_{[2,2]}\}$

$= \min\{4, 4+0\}$

$= 4$

~~$A^2_{22} = 0$~~

$A^2_{13} = \min\{A^1_{[1,3]}, A^1_{[1,2]} + A^1_{[2,3]}\}$

$= \min\{8, 4+5\}$

$= 8$

$A^2_{14} = \min\{A^1_{[1,4]}, A^1_{[1,2]} + A^1_{[2,4]}\}$

$= \min\{\alpha, 4+12\}$

$= 16$

$A^2_{21} = \min\{A'_{[2,1]}, A'_{[2,2]} + A'_{[2,1]}\}$
$= \min\{\alpha, 0+\alpha\}$
$= \alpha$

$A^2_{22} = 0$

$A^2_{23} = \min\{A'_{[2,3]}, A'_{[2,2]} + A'_{[2,3]}\}$
$= \min\{\cancel{8}^5, 0+5\}$
$= 5$

$A^2_{24} = \min\{A'_{[2,4]}, A'_{[2,2]} + A'_{[2,4]}\}$
$= \min\{12, 0+12\}$
$= 12$

$A^2_{31} = \min\{A'_{[3,1]}, A'_{[3,2]} + A'_{[2,1]}\}$
$= \min\{\alpha, \alpha+\alpha\}$
$= \alpha$

$A^2_{32} = \min\{A'_{[3,2]}, A'_{[3,2]} + A'_{[2,2]}\}$
$= \min\{\alpha, \alpha+0\}$
$= \alpha$

$A^2_{33} = 0$

$A^2_{34} = \min\{A'_{[3,4]}, A'_{[3,2]} + A'_{[2,4]}\}$
$= \min\{7, \alpha+12\}$
$= 7$

$A^2_{41} = \min\{A'_{[4,1]}, A'_{[4,2]} + A'_{[2,1]}\}$
$= \min\{5, 9+\alpha\}$
$= 5$

$A^2_{42} = \min\{A'_{[4,2]}, A'_{[4,2]} + A'_{[2,2]}\}$
$= \min\{9, 9+0\}$
$= 9$

$A^2_{43} = \min\{A'_{[4,3]}, A'_{[4,2]} + A'_{[2,3]}\}$
$= \min\{13, 9+5\}$
$= 13$

$A^2_{44} = 0$

$$A^2 = \begin{bmatrix} 0 & 4 & 8 & 16 \\ \alpha & 0 & 5 & 12 \\ \alpha & \alpha & 0 & 7 \\ 5 & 9 & 13 & 0 \end{bmatrix}$$

$A^3_{11} = 0$

$A^3_{12} = \min\{A^2_{[1,2]}, A^2_{[1,3]} + A^2_{[3,2]}\}$

$= \min\{4, 8+\alpha\}$

$= 4$

$\cancel{A^3_{21} = \min\{A^2_{[1,2]}}$

$\cancel{A^3_{24} = }$

$A^3_{13} = \min\{A^2_{[1,3]}, A^2_{[1,3]} + A^2_{[3,3]}\}$

$= \min\{8, 8+0\}$

$= 8$

$A^3_{14} = \min\{A^2_{[1,4]}, A^2_{[1,3]} + A^2_{[3,4]}\}$

$= \min\{16, 8+7\}$

$= 15$

$A^3_{21} = \min\{A^2_{[2,1]}, A^2_{[2,3]} + A^2_{[3,1]}\}$

$= \min\{\alpha, 5+\alpha\}$

$= \alpha$

$A^3_{22} = 0$

$A^3_{23} = \min\{A^2_{[2,3]}, A^2_{[2,3]} + A^2_{[3,3]}\}$

$= \min\{5, 5+0\}$

$= 5$

$A^3_{24} = \min\{A^2_{[2,4]}, A^2_{[2,3]} + A^2_{[3,4]}\}$

$= \min\{12, 5+7\}$

$= 12$

$A^3_{31} = \min\{A^2_{[3,1]}, A^2_{[3,3]} + A^2_{[3,1]}\}$

$= \min\{\alpha, 0+\alpha\}$

$= \alpha$

$A^3_{32} = \min\{A^2_{[3,2]}, A^2_{[3,3]} + A^2_{[3,2]}\}$

$= \min\{\alpha, 0+\alpha\}$

$= \alpha$

$A^3_{33} = 0$

$A^3_{34} = \min\{A^2_{[3,4]}, A^2_{[3,3]} + A^2_{[3,4]}\}$

$= \min\{7, 0+7\}$

$= 7$

$A^3_{41} = \min\{A^2_{[4,1]}, A^2_{[4,3]} + A^2_{[3,1]}\}$

$= \min\{5, 13+\alpha\}$

$= 5$

$A^3_{42} = \min\{A^2_{[4,2]}, A^2_{[4,3]} + A^2_{[3,2]}\}$

$= \min\{9, 13+\alpha\}$

$= 9$

$A^3_{43} = \min\{A^2_{[4,3]}, A^2_{[4,3]} + A^2_{[3,3]}\}$

$= \min\{13, 13+0\}$

$= 13$

$A^3_{44} = 0$

$$A^3 = \begin{bmatrix} 0 & 4 & 8 & 15 \\ \alpha & 0 & 5 & 12 \\ \alpha & \alpha & 0 & 7 \\ 5 & 9 & 13 & 0 \end{bmatrix}$$

$A^4_{11} = 0$

$A^4_{12} = \min\{A^3_{[1,2]}, A^3_{[1,4]} + A^3_{[4,2]}\}$

$= \min\{4, \cancel{8}\ 15+9\}$

$= 4$

$A^4_{13} = \min\{A^3_{[1,3]}, A^3_{[1,4]} + A^3_{[4,3]}\}$

$= \min\{8, 15+13\}$

$= 8$

$A^4_{14} = \min\{A^3_{[1,4]}, A^3_{[1,4]} + A^3_{[4,4]}\}$

$= \min\{15, 15+0\}$

$= 15$

$A^4_{21} = \min\{A^3_{[2,1]}, A^3_{[2,4]} + A^3_{[4,1]}\}$

$= \min\{\alpha, 12+5\}$

$= 17$

$A^4_{22} = 0$

$A^4_{23} = \min\{A^3_{[2,3]}, A^3_{[2,4]} + A^3_{[4,3]}\}$

$= \min\{5, 12+13\}$

$= 5$

$A^4_{24} = \min\{A^3_{[2,4]}, A^3_{[2,4]} + A^3_{[4,4]}\}$

$= \min\{12, 12+0\}$

$= 12$

$A^4_{31} = \min\{A^3_{[3,1]}, A^3_{[3,4]} + A^3_{[4,1]}\}$

$= \min\{\alpha, 7+5\}$

$= 12$

$A^4_{32} = \min\{A^3_{[3,2]}, A^3_{[3,4]} + A^3_{[4,2]}\}$

$= \min\{\alpha, 7+9\}$

$= 16$

$A^4_{33} = 0$

$A^4_{34} = \min\{A^3_{[3,4]}, A^3_{[3,4]} + A^3_{[4,4]}\}$

$= \min\{7, 7+0\}$

$= 7$

$A^4_{41} = \min\{A^3_{[4,1]}, A^3_{[4,4]} + A^3_{[4,1]}\}$

$= \min\{5, 0+5\}$

$= 5$

$A^4_{42} = \min\{A^3_{[4,2]}, A^3_{[4,4]} + A^3_{[4,2]}\}$

$= \min\{9, 0+9\}$

$= 9$

$A^4_{43} = \min\{A^3_{[4,3]}, A^3_{[4,4]} + A^3_{[4,3]}\}$

$= \min\{13, 0+13\}$

$= 13$

$A^4_{44} = 0$

$$A^4 = \begin{bmatrix} 0 & 4 & 8 & 15 \\ 17 & 0 & 5 & 12 \\ 12 & 16 & 0 & 7 \\ 5 & 9 & 13 & 0 \end{bmatrix}$$

1/1/19 All pairs shortest path Algorithm:-

```
Algorithm Allpaths(cost, A, n)
{
  for i:=1 to n do
     for j:=1 to n do
          A[i, j] := cost[i, j];

  for k:=1 to n do
    for i:=1 to n do
       for j:=1 to n do
           A[i,j] := min (A[i,j], A[i,k] + A[k,j]);
}
```

Note:-

Time complexity all pairs shortest path problem is $O(n^3)$

0/1 Knapsack problem:-

* 0/1 Knapsack problem represents 0 means not consider particular object total weight, 1 means consider total weight. M(or)W represents total weight of the knapsack.

* Maximum profit is $\sum_{i=1}^{n} p_i x_i$

where x_i is exact 0 (or) 1

Total weight is $\sum_{i=1}^{n} w_i x_i$ then Maximum

$$\boxed{\text{Maximized } \sum_{i=1}^{n} p_i x_i \text{ subject to } \sum_{i=1}^{n} w_i x_i \le m}$$

PURGING RULE (or) Dominance Rule:-

* If S^{i+1} contains (P_j, W_j) & (P_k, W_k) is two pairs such that $P_j \le P_k$ & $W_j \ge W_k$ then (P_j, W_j) can be eliminated

* In Dominance Rule remove the pair with less profit and more weight

eg: Obtain the solutions for the following n=3 & m=6 profits $(P_1, P_2, P_3) = (1,2,4)$ corresponding weights $(W_1, W_2, W_3) = (2,3,3)$

Consider $S^0 = \{(0,0)\}$

Now $S_1^0 = \{$select first pair (P_1, W_1) and add it with $S^0\}$

$= \{S^0 + (1,2)\}$

$= \{(0,0) + (1,2)\}$

$S_1^0 = \{(1,2)\}$

$S^1 = \{$merge S^0 and $S_1^0\}$

$S^1 = \{(0,0), (1,2)\}$

$S_1^2 = \{$Select next pair (P_2, W_2) and add it with $S^1\}$

$= \{S^1 + (2,3)\}$

$S_1^2 = \{(2,3), (3,5)\}$

$S^2 = \{$merge S^1 and $S_1^2\}$

$S^2 = \{(0,0), (1,2), (2,3), (3,5)\}$

$S_1^3 = \{$select next pair $(4,3)$ and add it with $S^2\}$

$= \{S^2 + (4,3)\}$

$= \{(4,3), (5,5), (6,6), (7,8)\}$

$S^3 = \{$merge S^2 and $S_1^3\}$

$= \{(0,0), (1,2), (2,3), (3,5), (4,3), (5,5), (6,6), (7,8)\}$

Now apply PURGING RULE on $(3,5)$ and $(4,3)$

$3 \leq 4$ (T) and $5 \geq 3$ (T)

Hence Remove $(3,5)$

apply purging rule $(2,3)$ and $(4,3)$

$2 \leq 4$ (T) and $3 \geq 3$ (T)

Hence Remove $(2,3)$

$S^3 = \{(0,0), (1,2), (4,3), (5,5), (6,6), (7,8)\}$

S^3 contain pair (7,8) but our bag capacity is only 6.
Hence remove (7,8) pair from S^3.

$S^3 = \{(0,0), (1,2), (4,3), (5,5), (6,6)\}$

From S^3 consider (6,6) pair

(6,6) $\in S^3$ select third object pair (4,3)

$\{(6-4), (6-3)\} = (2,3)$

now set x_3 $x_3 = 1$

Now $(2,3) \in S^2$ consider second object pair (2,3)

$\{(2-2), (3-3)\} = \{(0,0)\}$

now set $x_2 = 1$

now $\sum_{i=1}^{n} w_i x_i = w_1 x_1 + w_2 x_2 + w_3 x_3$
$= 2\times 0 + 3\times 1 + 3\times 1$
$= 0 + 3 + 3$
$= 6.$

The maximum profit is $\sum_{i=1}^{n} p_i x_i = p_1 x_1 + p_2 x_2 + p_3 x_3$
$= 1\times 0 + 2\times 1 + 4\times 1$
$= 0 + 2 + 4$
$= 6.$

eg:- Consider $n=7$, $(w_1, w_2, w_3, w_4, w_5, w_6, w_7) = (2,3,5,7,1,4,1)$
$(P_1, P_2, P_3, P_4, P_5, P_6, P_7) = (10,5,15,7,6,18,3)$ $M=15$ solve 0/1 knapsack problem using dynamic programming

~~consider~~ $S^0 = \{(0,0)\}$.

Now $S_1^0 = \{$select first pair (P_1, W_1) and add it with $S^0\}$
$= \{S^0 + (10,2)\}$
$= \{(0,0) + (10,2)\}$

$S_1^0 = \{(10,2)\}$

$S^1 = \{$merge S^0 and $S_1^0\}$

$S^1 = \{(0,0), (10,2)\}$

$S_1^2 = \{$select next ~~pair (P_2, W_2)~~ and add it with $S^1\}$

Q: Consider $n=7$ $(w_1, w_2, w_3, w_4, w_5, w_6, w_7) = (2,3,5,7,1,4,1)$
$(P_1, P_2, P_3, P_4, P_5, P_6, P_7) = (10,5,15,7,6,18,3)$ $M=15$ solve
0/1 knapsack problem using dynamic programming.

Consider $S^0 = \{(0,0)\}$

Now $S_1^0 = \{$select first pair (P_1, W_1) and add it with $S^0\}$

$= \{S^0 + (10,2)\}$

$= \{(0,0) + (10,2)\}$ $\qquad P_j \le P_k$, $W_j \ge W_k$

$S_1^0 = \{(10,2)\}$

$S^1 = \{$merge S^0 and $S_1^0\}$

$S^1 = \{(0,0), (10,2)\}$

Now apply purging rule on

$0 \le 10$ T

$0 \ge 2$ F

$S_1^2 = \{$select next pair (P_2, W_2) and add it with $S^1\}$

$= \{S^1 + (5,3)\}$

$S_1^2 = \{(5,3), (15,5)\}$

$S^2 = \{$merge S^1 and $S_1^2\}$

$S^2 = \{(0,0), (5,3), (10,2), (15,5)\}$

apply purging rule $(5,3)$ and $(10,2)$

$5 \le 10$ (T) and $3 \ge 2$ (T)

Hence remove $(5,3)$

$S^2 = \{(0,0), (10,2), (15,5)\}$

$S_1^3 = \{$select next pair (P_3, W_3) and add it with $S^2\}$

$= \{S^2 + (15,5)\}$

$S_1^3 = \{(15,5), (25,7), (30,10)\}$

$S^3 = \{$merge S^2 and $S_1^3\}$

$(15,5)$

$S^3 = \{(0,0), (10,2), (15,5), (25,7), (30,10)\}$

purging rule $(15,5)$ and $(15,5)$

$15 \le 15$ (T) and $5 \ge 5$ (T)

Hence remove (15,5)

$S^3 = \{(0,0), (10,2), (15,5), (25,7), (30,10)\}$

$S_1^4 = \{$select next pair (7,7) and add it with $S^3\}$

$= \{S^3 + (7,7)\}$ $X_2 = 1$

$= \{(7,7), (17,9), (22,12), (32,14), (37,17)\}$ 15 5

$S^4 = \{$merge S^3 and $S_1^4\}$ (10,2)

$S^4 = \{(0,0), \cancel{(7,7)}, (10,2), (15,5), \cancel{(17,9)}, \cancel{(22,12)}, (25,7), (30,10),$
$(32,14), \cancel{(37,17)}\}$

apply purging rule on (7,7) and (10,2)

$7 \le 10$ (T) and $7 \ge 2$ (T)

Now remove (7,7)

apply purging rule on (22,12) and (25,7)

$22 \le 25$ (T) and $12 \ge 7$ (T)

Hence remove (22,12)

apply purging rule on (17,9) and (25,7)

$17 \le 25$ (T) and $9 \ge 7$ (T)

Hence remove (17,9)

$S^4 = \{(0,0), (10,2), (15,5), (25,7), (30,10), (32,14)\}$

$S_1^5 = \{$select next pair (6,1) and add it with $S^4\}$

$= \{S^4 + (6,1)\}$

$= \{(6,1), (16,3), (21,6), (31,8), (36,11), (38,15)\}$

$S^5 = \{$merge S^4 and $S_1^5\}$

$S^5 = \{(0,0), (6,1), (10,2), \cancel{(15,5)}, (16,3), (21,6), (25,7), \cancel{(30,10)},$
$(31,8), \cancel{(32,14)}, (36,11), (38,15)\}$

apply purging rule (15,5) and (16,3) $X_1 = 1$ X_1

$15 \le 16$ (T) and $(5 \ge 3)$ (T) $\cancel{(10,2)}$ 0

Hence remove (15,5) $\cancel{10,2}$

apply purging rule (30,10) and (31,8) $X_4=0$

$30 \le 31$ (T) and $10 \ge 8$ (T) $X_5=1$

Hence remove (30,10) $X_3=1$

apply purging rule (32,14) and (36,11) (36,11)

$32 \le 36$ (T) and $14 \ge 11$ (T) 6,1

Hence remove (32,14) (30,10)

$S^5 = \{(0,0), (6,1), (10,2), (16,3), (21,6), (25,7), (31,8), (36,11), (38,15)\}$

$S_1^6 = \{$Select next pair (18,4) and add it with $S^5\}$

$= \{S^5 + (18,4)\}$

$= \{(18,4), (24,5), (28,6), (34,7), (39,10), (43,11), (49,12), (54,15), (56,19)\}$

$S^6 = \{$merge S^5 and $S_1^6\}$

$= \{(0,0), (6,1), (10,2), (16,3), (18,4), (21,6), (24,5), (25,7), (28,6), (31,8), (34,7), (36,11), (38,15), (39,10), (43,11), (49,12), (54,15), (56,19)\}$

apply purging rule (21,6) and (24,5) (30,10)

$21 \le 24$ (T) and $6 \ge 5$ (T) 15,5

Hence remove (21,6) (15,5)

apply purging rule (25,7) and (28,6)

$25 \le 28$ (T) and $7 \ge 6$ (T)

Hence remove (25,7)

apply purging rule (31,8) and (34,7)

$31 \le 34$ (T) and $8 \ge 7$ (T)

Hence remove (31,8)

apply purging rule (38,15) and (39,10)

$38 \le 39$ (T) and $15 \ge 10$ (T)

Hence remove (38,15)

$S^6 = \{(0,0), (6,1), (10,2), (16,3), (18,4), (24,5), (28,6), (34,7), (36,11), (39,10), (43,11), (49,12), (54,15)\}$

$S_1^7 = \{$Select next pair (3,1) and add it with $S^6\}$

$= \{S^6 + (3,1)\}$

$= \{(3,1), (9,2), (13,3), (19,4), (21,5), (27,6), (31,7), (37,8), (39,12),$

(42,11), (46,12), (52,13), (57,16)}

S^7 = {merge S^6 and S_1^7}

= {(0,0), ~~(3,1)~~, (6,1), ~~(9,2)~~, (10,2), ~~(13,3)~~, (16,3), ~~(18,4)~~, (19,4), ~~(21,5)~~, (24,5), ~~(27,6)~~, (28,6), ~~(31,7)~~, (34,7), ~~(36,11)~~, (37,8), (39,10), ~~(39,12)~~, ~~(42,11)~~, (43,11), ~~(46,12)~~, (49,12), ~~(52,13)~~, (54,15), ~~(57,16)~~}

apply purging rule (3,1) and (6,1)

$3 \leq 6$ (T) and $(1 \geq 1)$ T

remove (3,1)

apply purging rule (9,2) and (10,2)

$9 \leq 10$ (T) and $2 \geq 2$ (T)

remove (9,2)

apply purging rule (13,3) and (16,3)

$13 \leq 16$ (T) and $3 \geq 3$ (T)

remove (13,3)

apply purging rule (18,4) and (19,4)

$18 \leq 19$ (T) and $4 \geq 4$ (T)

remove (18,4)

apply purging rule (21,5) and (24,5)

$21 \leq 24$ (T) and $5 \geq 5$ (T)

remove (21,5)

apply purging rule (27,6) and (28,6)

$27 \leq 28$ (T) and $(6 \geq 6)$ T

remove (27,6)

apply purging rule (31,7) and (34,7)

$31 \leq 34$ (T) and $7 \geq 7$ (T)

remove (31,7)

apply purging rule (36,11) and (37,8)

$36 \leq 37$ (T) and $11 \geq 8$ (T)

remove (36,11)

apply purging rule (39,12) and (42,11)

$39 \leq 42$ (T) and $12 \geq 11$ (T)

remove (39,12)

apply purging rule (42,11) and (43,11)

$42 \le 43$ (T) and $11 \ge 11$ (T)

remove (42, 11)

apply purging rule (46,12) and (49,12)

$46 \le 49$ (T) and $12 \ge 12$ (T)

remove (46, 12)

$S^7 = \{(0,0), (6,1), (10,2), (16,3), (19,4), (24,5), (28,6), (34,7), (37,8), (39,10), (43,11), (49,12), (52,13), (54,15)\}$

from S^7 consider (54, 15) pair

$(54,15) \notin S^7$ select seventh object pair

$x_7 = 0$

we take $x_6 = 1$ $(54-18, 15-4) = (36,11) \in S^5$

$x_5 = 1$ $(36-6, 11-1) = (30,10) \notin S^4$

$x_4 = 0$ $(30,10) \in S^3$

$x_3 = 1$ $(30-15, 10-5) = (15,5) \in S^2$

$x_2 = 1$, $(15-5, 5-3) = (10,2) \in S^1$

$x_1 = 1$, $(10-10, 2-2) = 0$

$x_7 = 0$
$x_6 =$

$S_6 =$
18, 4
(54-18)
(15-4)
= (36,11)

eg:- consider $n = 4$, $m = 8$ $(P_1, P_2, P_3, P_4) = (1,2,5,6)$, and weights $(w_1, w_2, w_3, w_4) = (2,3,4,5)$.

Consider $S^0 = \{(0,0)\}$

Now $S_1^0 = \{$select first pair (P_1, W_1) and add it with $S^0\}$

$= \{S^0 + (1,2)\}$

$S_1^0 = \{(1,2)\}$

$S^1 = \{$merge S^0 and $S_1^0\}$

$S^1 = \{(0,0), (1,2)\}$

Now apply purging rule on

$0 \le 1$ (T) and $0 \ge 2$ (F)

$S_1^2 = \{$select next pair (P_2, W_2) and add it with $S^1\}$

$= \{S^1 + (2,3)\}$

$S_1^2 = \{(2,3), (3,5)\}$

$S^2 = \{$merge S^1 and $S_1^2\}$

$S^2 = \{(0,0), (1,2), (2,3), (3,5)\}$

$S_1^3 = \{$select next pair $(5,4)$ and add it with $S^2\}$

$= \{S^2 + (5,4)\}$

$= \{(5,4), (6,6), (7,7), (8,9)\}$

$S^3 = \{$merge S^2 and $S_1^3\}$

$= \{(0,0), (1,2), (2,3), (3,5), (5,4), (6,6), (7,7), (8,9)\}$

apply purging rule $(3,5)$ and $(5,4)$

$3 \leq 5$ (T) and $(5 \geq 4)$ T

Hence remove $(3,5)$

$S^3 = \{(0,0), (1,2), (2,3), (5,4), (6,6), (7,7), (8,9)\}$

$S_1^4 = \{$Select next pair $(6,5)$ and add it with $S^3\}$

$= \{S^3 + (6,5)\}$

$= \{(6,5), (7,7), (8,8), (9,10), (11,9), (12,11), (13,11), (13,12), (14,14)\}$

$S^4 = \{$merge S^3 and $S_1^4\}$

$= \{(0,0), (1,2), (2,3), (5,4), \cancel{(6,6)}, (6,5), \cancel{(7,7)}, (7,7), \cancel{(8,9)}, (8,8), (9,10), (11,9), \cancel{(12,11)}, (13,11), (13,12), (14,14)\}$

apply purging rule on $(6,6)$ and $(6,5)$

$6 \leq 6$ (T) and $6 \geq 5$ (T)

Hence remove $(6,6)$

apply purging rule on $(7,7)$ and $(7,7)$

$7 \leq 7$ (T) and $7 \geq 7$ (T)

remove $(7,7)$

apply purging rule on $(8,9)$ and $(8,8)$

$8 \leq 8$ (T) and $9 \geq 8$ (T)

Hence remove $(8,9)$

apply purging rule on $(12,11)$ and $(13,11)$

$12 \leq 13$ (T) and $11 \geq 11$ (T)

Hence remove $(12,11)$

apply purging rule on (9,10) and (11,9)

$9 \leq 11$ (T) and $10 \geq 9$ (T)

Hence remove (9,10)

$S^4 = \{(0,0), (1,2), (2,3), (5,4), (6,5), (7,7), (8,8)\}$

from S^4 consider (8,8) pair

$(8,8) \in S^4$ select fourth object pair

$S^4 = \phi$

$(8-6, 8-5) = (2,3)$

$(2-2, 3-3) = (0,0)$ $S^2 = 1$

$(8,8) \in S^4$

$x_4 = 1$

$x_2 = 1$

$x_1 = 0$

$x_3 = 0$

{2, 3, 4}

(2,3)

(2,3)

(0,0)

$\sum w_i x_i \leq M$

$8 \leq 8$

$2 \times 0 + 3 \times 1 + 4 \times 0 + 5 \times 1 = 8$

0/1 KNAPSACK ALGORITHM:-

Algorithm DKP(p, w, n, m)
{
  $S^0 := \{(0,0)\}$;
  for i := 1 to n-1 do
  {
    $S_1^{i-1} := \{(P,W) \mid (P-p_i, W-w_i) \in S^{i-1}$ and $W \le m\}$;
    $S^i :=$ MergePurge (S^{i-1}, S_1^{i-1});
  }
  $(PX, WX) :=$ last pair in S^{n-1};
  $(PY, WY) := (P' + p_n, W' + w_n)$ where W' is the largest W in any pair in S^{n-1} such that $W + w_n \le m$;
  if $(PX > PY)$ then $x_n := 0$;
  else $x_n := 1$;
  TraceBackFor $(x_{n-1}, \ldots x_1)$;
}

12) SINGLE SOURCE SHORTEST PATH PROBLEM

* Single source shortest path problem in dynamic programming is solved by using BellMan & Ford algorithm

* Greedy single source shortest path problem does not allow negative cost. But where as Bellman Ford algorithm allows negative

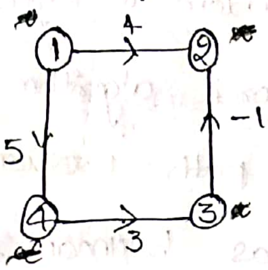
* To solve single source shortest path problem in dynamic programming, BellMan & Ford introduced Relaxiation Rule that is

$$\boxed{\begin{array}{l}\text{if } d(u)+\text{cost}(u,v) < d(v) \text{ then} \\ \text{change as } d(v) = d(u)+\text{cost}(u,v)\end{array}}$$

* The differences between single source shortest path in Greedy Method and single source shortest path in Dynamic programming

SSSP in GM	SSSP in DP
* Greedy single source shortest path problem is solved by using Dijkstra's algorithm.	* Dynamic programming single source shortest path problem is solved by using BellMan & Ford algorithm
* It is connected and weighted graph	* It is also a connected and weighted graph.
* In greedy SSSP we can obtain an optimal solution from possible no. of feasible solutions.	* In Dynamic programming SSSP we can satisfy all choices (or) decisions.
* In greedy SSSP we can't solve -ve edge cost in the given graph.	* In Dynamic programming we can solve the problem even the edge cost may be -ve
* In greedy SSSP, finally we can written shortest path and minimum distance.	* In dynamic SSSP we can written individual minimum distances.

eg:- Solve SSSP using Dynamic programming.



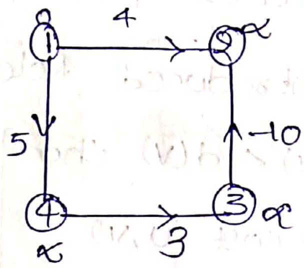
Sol:- Initially, the distance of source vertex is "0" and remaining vertex distances is α

$d(1) = 0$

$d(2) = \alpha$

$d(3) = \alpha$

$d(4) = \alpha$

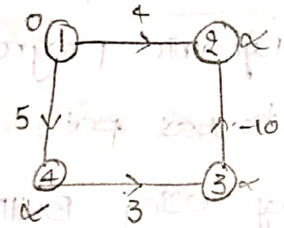


Now form edge list is $(1,2), (1,4), (3,2), (4,3)$

Now apply relaxation Rule

~~the~~ i.e $d(u) + cost(u,v) < d(v)$ then change as

$$d(v) = d(u) + cost(u,v)$$



$0+4 < \alpha$ (T) then

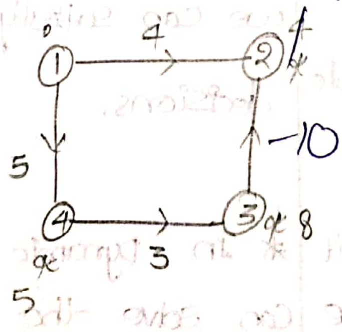
$d(v) = 0+4 = 4$

$0+5 < \alpha$ (T)

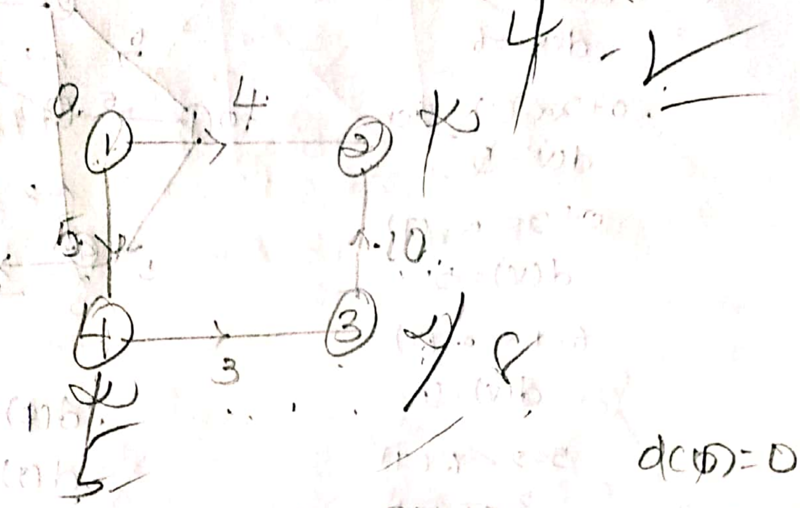
$d(v) = 0+5 = 5$

$5+3 < \alpha$ (T)

$8 < \alpha$ $\quad d(v) = 5+3 = 8$



1. $0+4 < 4$ (F)



d(1)=0

(1,2) (1,4), (3,2), (4,3)

d(2)=∞

d(3)=∞

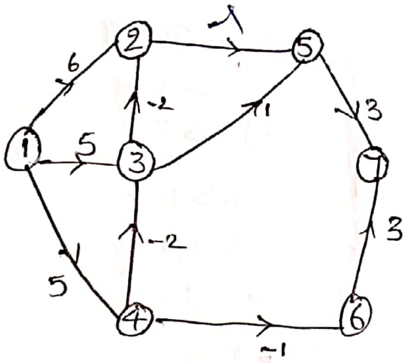
① $0+4 < \infty$ True

d(4)=∞

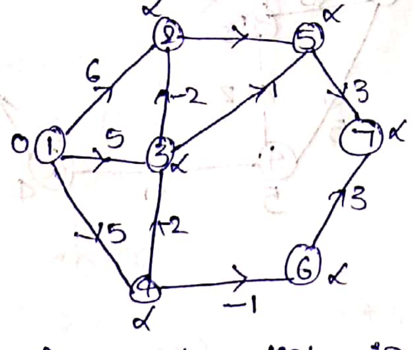
$d(2) = 0+4 = 4$

d(v)

Solve



Initially the distance of source vertex is "0" and remaining vertex distances is '∞'



d(1)=0
d(2)=∞
d(3)=∞
d(4)=∞
d(5)=∞
d(6)=∞
d(7)=∞

Now form edge list is (1,2), (1,3), (1,4), (2,5), (3,2), (3,5), (4,3), (4,6), (5,7), (6,7).

i.e $d(u) + cost(u,v) < d(v)$ then change as.

$d(v) = d(u) + cost(u,v)$.

① $0+6 < \alpha$ (T) then

$d(v)=6$

$0+5<\alpha$ (T) then

$d(v)=5$

$0+5 < \alpha$ (T)

$d(v)=5$

$6-1 < \alpha$ (T)

$d(v)=5$

$5-2 < 6$ (T)

$3 < 6$ (T)

$d(v)=3$

$5+1 < 5$

$6 < 5$ (F)

$d(v)=4$

$5-2 < 5$

$3 < 5$ (T)

$d(v)=3$

$5-1 < \alpha$

$4 < \alpha$ (T)

$d(v)=4$

$5+3 < \alpha$

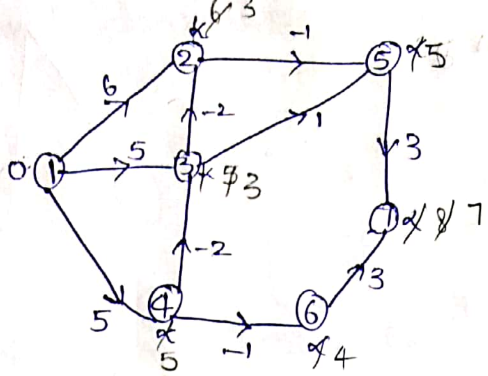
$8 < \alpha$

$d(v)=8$

$4+3 < 8$

$7 < 8$ (T)

$d(v)=7$



$d(1)=0$

$d(2)=3$

$d(3)=3$

$d(4)=5$

$d(5)=5$

$d(6)=4$

$d(7)=7$

② $0+6 < 3$ (F)

$0+5 < 3$ (F)

$0+5 < 5$ (F)

$3-1 < 5$

$2 < 5$ (T)

$d(v)=2$

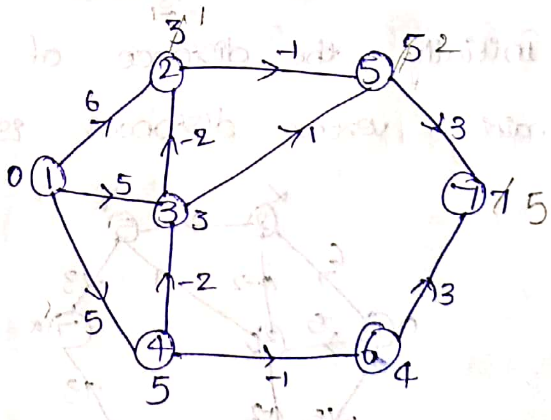
$3-2 < 3$

$1 < 3$ (T)

$d(v)=1$

$3-1 < 2$

$2 < 2$ (F)



5 − 2 < 3

3 < 3 (F)

5 − 1 < 4

4 < 4 (F)

2 + 3 < 7

5 < 7 (T)

d(V) = 5

4 + 3 < 5

7 < 5 (F)

d(1) = 0

d(2) = 1

d(3) = 3

d(4) = 5

d(5) = 2

d(6) = 4

d(7) = 5

(3)

0 + 6 < 1 (F)

0 + 5 < 3 (F)

0 + 5 < 5 (F)

1 − 1 < 2

0 < 2 (T)

d(V) = 0

3 − 2 < 1

1 < 1 (F)

3 − 1 < 0

2 < 0 (F)

5 − 2 < 3

3 < 3 (F)

5 − 1 < 4

4 < 4 (F)

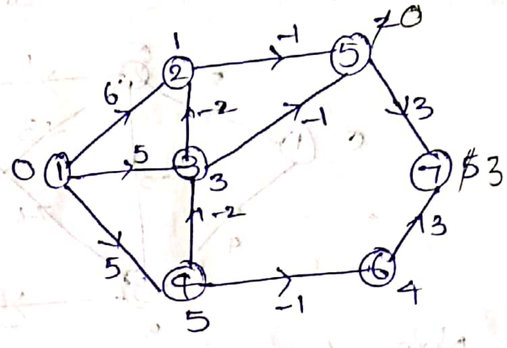
0 + 3 < 5

3 < 5 (T)

d(V) = 3

4 + 3 < 3

7 < 3 (F)



d(1) = 0

d(2) = 1

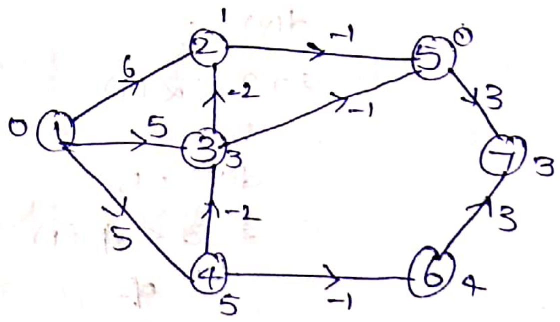
d(3) = 3

d(4) = 5

d(5) = 0

d(6) = 4

d(7) = 3



(4) 0 + 6 < 1 (F)

0 + 5 < 3 (F)

0 + 5 < 5 (F)

1 − 1 < 0

0 < 0 (F)

3 − 2 < 1

1 < 1 (F)

3 − 1 < 0 (F)

5 − 2 < 3

3 < 3 (F)

5 − 1 < 4

4 < 4 (F)

0 + 3 < 3 (F)

4 + 3 < 3 (F)

d(1) = 0

d(2) = 1

d(3) = 3

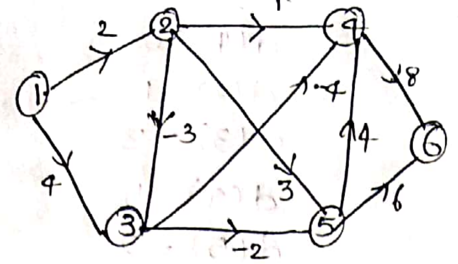
d(4) = 5

d(5) = 0

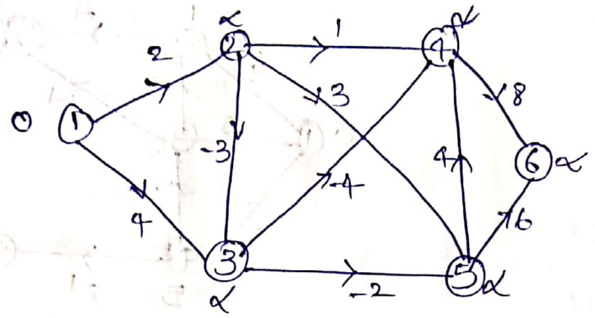
d(6) = 4

d(7) = 3

eg:



Initially the vertex distance of source vertex is "0" and remaining vertex distances is "α"



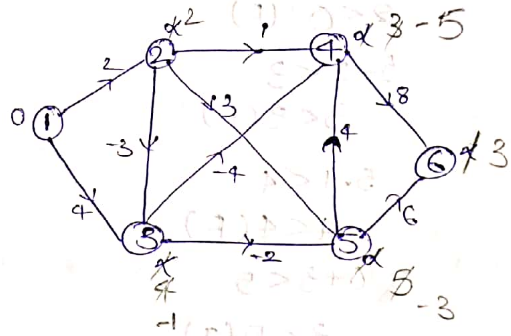
d(1) = 0
d(2) = α
d(3) = α
d(4) = α
d(5) = α
d(6) = α

Now form edge list is (1,2), (1,3), (2,4), (2,3), (2,5), (3,5), (3,4), (4,6), (5,4), (5,6).

d(u) + cost(u,v) < d(v) then change as

d(v) = d(u) + cost(u,v)

① 0+2 < α (T) then
d(v) = 2



0 + 4 < α (T) then
d(v) = 4

2+1 < α (T)
3 < α
d(v) = 3

2+3 < α (T)
5 < α
d(v) = 5

2 − 3 < 4 (T)
−1 < 4
d(v) = −1

−1 − 2 < 5
−3 < 5 (T)
d(v) = −3

−1 − 4 < 3
−5 < 3 (T)
d(v) = −5

−5 + 8 < α
3 < α (T)
d(v) = 3

−3 + 6 < 3
3 < 3 (F)

−3 + 4 < −5
1 < −5 (F)

d(1) = 0
d(2) = 2
d(3) = −1
d(4) = −5
d(5) = −3
d(6) = 3

② $0+2<2$ (F)

$0+4<-1$ (F)

$2+1<-5$ (F)

$2+3<-3$ (F)

$2-3<-1$

$-1<-1$ (F)

$-1-4<-5$

$-5<-5$ (F)

$-1-2<-3$

$-3<-3$ (F)

$-5+8<3$

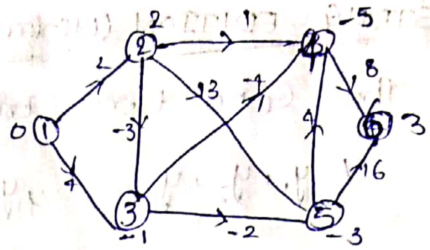
$3<3$ (F)

$-3+4<-5$

$1<-5$ (F)

$-3+6<3$

$3<3$ (F)



d(1) = 0

d(2) = 2

d(3) = -1

d(4) = -5

d(5) = -3

d(6) = 3.

Algorithm for SINGLE SOURCE SHORTEST PATH :-

```
Algorithm Bellmanford (v, cost, dist, n)
{
  for i:=1 to n do
    dist [i]:=cost (v,i);
  for k:=2 to n-1 do
    for each u such that u≠v and u has
     at least one incoming edge do
    for each (i,u) in the graph do
       if dist [u] > dist [i] + cost [i,u] then
          dist [u] := dist [i] + cost [i,u];
}
```

Time complexity :-

Time complexity for SSSP General weight problem is $O(n^3)$

STRING EDITION (Minimum Edit Distance):-

* The Given two strings $X = x_1, x_2, x_3 \ldots x_n$ and $Y = y_1, y_2, y_3, \ldots, y_m$ where x_i is $1 \le i \le n$

y_j is $1 \le j \le m$

are members of a finite set of symbols

* We want to transform X Y using a sequence of edit operations on X.

* The permissible edit operations are insert, Delete (or) Remove and Replace (or) change.

* The cost of a sequence of operations is the sum of the costs of individual operations in the sequence.

* The problem of string editing is to identify a minimum cost sequence of editing operations that will transform X into Y.

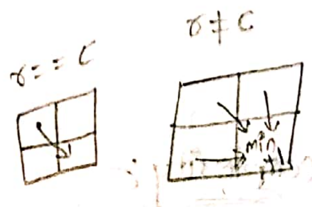
* let $D(x_i)$ be the cost of Deleting the symbol x_i from X.

* let $I(y_j)$ be the cost of inserting the symbol y_j into X

* let $C(x_i, y_j)$ be the cost of changing the symbol x_i of X into y_j of Y

* To solve string edition problem we can use the formula

$$\boxed{cost(i,j) = \min \left\{ \begin{array}{l} cost(i-1, j) + D(x_i), \\ cost(i-1, j-1) + C(x_i, y_j), \\ cost(i, j-1) + I(y_j) \end{array} \right\}}$$



Consider String Edit problem of $X = aabaababaa$, $Y = babaabab$.

consider string $X = aabaababaa$

string $Y = babaabab$

Now Generate cost Table.

i \ j	0	b	a	b	a	a	b	a	b
0	0	1	2	3	4	5	6	7	8
a	1	1	1	2	3	4	5	6	7
a	2	2	1	2	2	3	4	5	6
b	3	2	2	1	2	3	3	4	5
a	4	3	2	2	1	2	3	3	4
a	5	4	3	3	2	1	2	3	4
b	6	5	4	3	3	2	1	2	3
a	7	6	5	4	3	3	2	1	2
b	8	7	6	5	4	4	3	2	1
a	9	8	7	6	5	4	4	3	2
a	10	9	8	7	6	5	5	4	3

Now $X = a\ a\ b\ a\ a\ b\ a\ b\ \cancel{a}\ \cancel{a}$

$Y = b\ a\ b\ a\ a\ b\ a\ b$

If $a \neq b$ then apply remove operation and add with '1'

Hence the string a is remove from 'X'

If $a \neq b$ then apply remove operation and add with '1'

Hence the string a is removed from 'X'.

If $b = b$ then apply replace operation without adding of '1"

hence we can not perform any operation on x & y

likewise perform the replace operations on x & y

Untill we get r=c.

Now if $a \neq b$ then apply replace operation from minimum cost add with 1

Finally we can perform Three operations on X transform to Y.

i.e One replace operation a→b

Two remove operations a, a

Hence final minimum cost is 3.

2) Consider the string Editing problem X = a a b a b, Y = b a bb.

i \ j	0	b	a	b	b
0	0 →	1 →	2 →	3 →	4
a	1	1	1 →	2 →	3
a	2	2	1 →	2 →	3
b	3	2	2	1	2
a	4	3	2	2	2
b	5	4	3	2	2

Now x = a a b a b

Y = b a b b

→ If b = b then apply replace operation without adding 1, hence we can not perform any operation on x & y

→ If a ≠ b then apply remove operation and add with 1 hence the string a is removed from 'X'.

→ If b = b then apply replace operation without adding 1, hence we can not perform any operation on x & y.

→ If a = a then apply replace operation without adding 1, hence the can not perform any operation on x & y.

→ If a ≠ b then apply replace operation from minimum cost add with 1

Finally we can perform two operations on x transform to y.

i.e one remove operation a

one replace operation a→b

Hence final minimum cost is 2

13/2/19

RELIABLE DESIGN:-

* let m_i no.of devices in D_i are connected in parallel with reliability r_i
* $(1-r_i)$ is the probability that one copy of the device will malfunction $\to (1-r_i)$
* The probability that all the devices of this type malfunction at the same time is $(1-r_i)^{m_i}$
* Hence the reliability of device d_i D_i can be expressed as

$$\boxed{\phi_i(m_i) = (1-(1-r_i)^{m_i})}$$

* To find no.of devices in a particular machine we can use the formula as u_i

$$\boxed{U_i = \left[\left(C + C_i - \sum_{j=1}^{n} C_j \right) \Big/ C_i \right]}$$

where C = Total cost.

C_i = Individual costs C_1, C_2, C_3

$C_j = C_1 + C_2 + C_3$

Dominance Rule:-

(f_1, x_1) dominates (f_2, x_2) if $f_1 \geq f_2$ for $x_1 \leq x_2$ which means that if we can achieve more reliability by spending less, discard all tuples which spend more for less reliability.

Ex:- construct m_1, m_2, m_3 with the help of $C_1 = 30$, $C_2 = 15$ & $C_3 = 20$ and its corresponding reliabilities are $r_1 = 0.9$, $r_2 = 0.8$ and $r_3 = 0.5$. Total cost $C = 105$

Now $U_i = \left[\left[C + C_i - \sum_{j=1}^{n} C_j \right] \Big/ C_i \right]$

$U_1 = C + C_1 - (C_1 + C_2 + C_3) / C_1$

$= \left[(105 + 30 - (30 + 15 + 20)) / 30 \right]$

$= \lfloor (135-65)/30 \rfloor = \lfloor 2.333 \rfloor = 2$

$U_2 = C + C_2 - (C_1+C_2+C_3)/C_2$

$= \lfloor (105 + 15 - (30+15+20))/15 \rfloor$

$= \lfloor (120-65)/15 \rfloor = \lfloor 3.66 \rfloor = 3$

$U_3 = \lfloor (105+20-(30+15+20))/20 \rfloor$

$= \lfloor (125-65)/20 \rfloor = \lfloor 3 \rfloor = 3$

Hence $U_1 = 2$ $U_2 = 3$, $U_3 = 3$

The reliability, the cost pair is (r, x) that is $(1,0)$

Hence $S^0 = \{(1,0)\}$

Now

In S^1_1, $i=1$, $j=1$ i.e $m_1 = 1$

Now $\phi_i m_1 = (1-(1-r_i)^{m_i})$

$= (1-(1-0.9)^1)$

$= (1-(0.1)^1)$

$= 0.9$

cost $C_1 = 1 \times C_1$ i.e $1 \times 30 = 30$

$S^1_1 = \{(0.9, 30)\}$

In S^1_2, $i=1$, $j=2$ i.e $m_1 = 2$

Now $\phi_i m_1 = (1-(1-0.9)^2)$

$= 1-(0.01)$

$= 0.99$

cost $C_1 = 2 \times C_1$ i.e $2 \times 30 = 60$.

S^1 can be obtained by merging the sets S^1_1, S^1_2

$S^1 = \{(0.9, 30), (0.99, 60)\}$

calculate S^2_1

here $i=2$, $j=1$ i.e $m_2 = 1$

Now $\phi_2(m_2) = 1-(1-r_2)^{m_2}$

$= 1-(1-0.8)^1 = 1-(0.02) = 0.8$

Since $j=1$, $1 \times C_2 = 1 \times 15 = 15$ is added to S' pair

$S_1^2 = \{(0.9 \times 0.8, 30+15), (0.99 \times 0.8, 60+15)\}$

$= \{(0.72, 45), (0.792, 75)\}$

Now S_2^2 $j=2$, $i=2$, $m_2=2$

$\phi_2(m_2) = 1-(1-r_2)^{m_2} = 1-(1-0.8)^2 = 1-0.02$

$= 0.96$

Since $j=2$, $2 \times C_2 = 2 \times 15 = 30$ is added to S' pairs

$S_2^2 = \{(0.9 \times 0.96, 30+30), (0.99 \times 0.96, 60+30)\}$

$= \{(0.864, 60), (0.9504, 90)\}$

Now find S_3^2

Here $i=2$, $j=3$, $m_2=3$

$\phi_2(m_2) = 1-(1-r_2)^{m_2} = 1-(0.02)^3$

$= 0.992$

Since $j=3$, $3 \times C_2 = 3 \times 15 = 45$

$S_3^2 = \{(0.9 \times 0.992, 30+45), (0.99 \times 0.992, 60+45)\}$

$= \{(0.8928, 75), (0.98208, 105)\}$

S^2 can be obtained by merging set S_1^2, S_2^2, S_3^2

$S^2 = \{(0.72, 45), (0.792, 75), (0.864, 60), (0.9504, 90),$

$(0.8928, 75), (0.98208, 105)\}$

$0.792 \leq 0.864$ (T) $75 \geq 60$ (T)

Remove $(0.792, 75)$

$S^2 = \{(0.72, 45), (0.864, 60), (0.8928, 75), (0.98208, 105)\}$

Calculate S_1^3

Here $i=3$, $j=1$, $m_3=1$

$\phi_3(m_3) = 1-(1-r_3)^{m_3} = 1-(1-0.5)^1 = 0.5$

Since $j=1$, $C_3 = 20$

$S_1^3 = \{(0.72 \times 0.5, 45+20), (0.864 \times 0.5, 60+20), (0.8928 \times 0.5,$
$75+20), (0.98208 \times 0.5, 105+20)\}$

last tuple is removed from S_1^3, since cost 125 exceeding the given cost 105.

$S_1^3 = \{(0.36, 65), (0.432, 80), (0.4464, 95)\}$

In S_2^3

Here $i=3$; $j=2$, i.e $m_3=2$

since $j=2$, $2 \times C_3 = 2 \times 20 = 40$

$\phi_3(m_3) = 1-(1-r_3)^{m_3} = 1-(1-0.5)^2 = 1-0.25$
$= 0.75$.

$S_2^3 = \{(0.72 \times 0.75, 45+40), (0.864 \times 0.75, 60+40)\}$

$= \{(0.54, 85), (0.648, 100)\}$

In S_3^3

Here $i=3$, $j=3$, $m_3=3$

since $j=3$, $3C_3 = 3 \times 20 = 60$

$\phi_3(m_3) = 1-(1-r_3)^{m_3}$

$= 1-(1-0.5)$

$= 1-0.125 = 0.875$

$S_3^3 = \{(0.72 \times 0.875, 45+60)\}$

S^3 can be obtained by merging the set S_1^3, S_2^3, S_3^3

$S^3 = \{(0.36, 65), (0.432, 80), (0.4464, 95), (0.54, 85),$
$(0.648, 100), (0.63, 105)\}$

Apply purging rule

$S^3 = \{(0.36, 65), (0.432, 80), (0.54, 85), (0.648, 100)\}$

⇒ The best design has reliability of 0.648 and cost 100

(0.648, 100) pair present in S_2^3, $j=2$ so $m_3=2$

⇒ The (0.648, 100) pair obtained from (0.864, 60), which is present in S_2^2, $j=2$ $m_2=2$

⇒ The (0.864, 60) pair obtained from (0.9, 30)

S_1^1, $j=1$ so $m_1=1$

$m_1=1$ $m_2=2$ $m_3=2$

1 device of D_1 | 2 device of D_2 | 2 device of D_3

construct m_1, m_2 & m_3 $c_1 = 25$ $c_2 = 18$, $c_3 = 25$ Total cost is 120, $r_1 = 0.5$, $r_2 = 0.8$, $r_3 = 0.7$.

Now $u_i = \left\lfloor \left[c + c_i - \sum_{j=1}^{n} c_j\right] / c_i \right\rfloor$

$U_1 = c + c_1 - (c_1 + c_2 + c_3) / c_1$

$= \lfloor 120 + 25 - (25 + 18 + 25) / 25 \rfloor$

$= (145 - 68) / 25$

$= 3$

$$\begin{array}{r} 145 \\ 68 \\ \hline 77 \end{array} \qquad 25)\,77\,(3 \quad \begin{array}{r} 75 \\ \hline 20 \end{array}$$

$U_2 = c + c_2 - (c_1 + c_2 + c_3) / c_2$

$= 120 + 18 - (25 + 18 + 25) / 18$

$= (138 - 68) / 18$

$= 3$

$$\begin{array}{r} 138 \\ 68 \\ \hline 70 \end{array} \qquad 18)\,70\,(3 \quad \begin{array}{r} 54 \\ \hline 16 \end{array}$$

$U_3 = c + c_3 - (c_1 + c_2 + c_3) / c_3$

$= 120 + 25 - (25 + 18 + 25) / 25$

$= (145 - 68) / 25$

$= 3$

Hence $U_1 = 3$, $U_2 = 3$ and $U_3 = 3$

The reliability, the cost pair is (r, x) that is $(1, 0)$

Hence, $S^0 = \{(1, 0)\}$

now S^1_1 $i = 1$, $j = 1$ $m_1 = 1$

$\phi_i m_1 = (1 - (1 - r_i)^{m_i})$

$= (1 - (1 - 0.5)^1)$

$= (1 - 0.5)$

$= 0.5$

$C_1 = 1 \times C_1$ i.e $1 \times 25 = 25$ is added to S^0 pair

$S^1_1 = \{(0.5, 25)\}$

In S^1_2 $i = 1$, $j = 2$ $m_1 = 2$

$\phi_i m_1 = (1 - (1 - 0.5)^2)$

$= (1 - (0.5)^2)$

$= (1 - 0.25) = 0.75$

$C_1 = 2 \times C_1$ i.e $2 \times 25 = 50$ is added to S^0 pair.

$S_2^1 = \{(0.75, 50)\}$

now S_3^1 $i=1$, $j=3$, $m1=3$

$\phi_1 m_1 = (1-(1-0.5)^3)$

$= (1-(0.125))$

$= 0.875$

$$\begin{array}{r} 0.75 \\ \times 0.5 \\ \hline 375 \\ 000\times \\ \hline 0.375 \end{array}$$

$C_1 = 3 \times C_1$ i.e $3 \times 25 = 75$ is added to S^0 pair

$S_3^1 = \{(0.875, 75)\}$

S^1 can be merged S_1^1, S_2^1, S_3^1

$S^1 = \{(0.5, 25), (0.75, 50), (0.875, 75)\}$

Calculate S_1^2

here $i=2$, $j=1$ $m2=1$

$\phi_2 m_2 = (1-(1-0.8)^{m2})$

$= (1-(1-0.8)^1)$

$= (1-0.2)$

$= 0.8$

$1 \times C_2 = 1 \times 18 = 18$ is added to S^1 pair

$S_1^2 = \{(0.5 \times 0.8, 25+18), (0.75 \times 0.8, 50+18), (0.875 \times 0.8, 75+18)\}$

$= \{(0.4, 43), (0.6, 68), (0.7, 93)\}$

$$\begin{array}{r} 1.00 \\ 0.04 \\ \hline 0.96 \end{array} \qquad \begin{array}{r} 0.2 \\ \times 0.2 \\ \hline 04 \\ 00\times \\ \hline 0.04 \end{array}$$

Now S_2^2 $i=2$, $j=2$ $m2=2$

$\phi_2 m_2 = (1-(1-0.8)^2) = 1-(0.2)^2 = 1-0.04 = 0.96$

~~$= (1-(1-0.64)) = 1-0.36 = 0.64$~~

$2 \times C2 = 2 \times 18 = 36$ is added to S^1 pair

$S_2^2 = \{(0.5 \times \cancel{0.64}\ 0.96, 25+36), (0.75 \times \cancel{0.64}\ 0.96, 50+36), (0.875 \times \cancel{0.64}\ 0.96, 75+36)\}$

$= \{(0.32, 61), (0.48, 86), (0.56, 111)\}$

Now find S_3^2 $i=2$, $j=3$, $m2=3$

$\phi_2 m_2 = (1-(1-0.8)^3)$

$= (1-(1-0.512))$

$S_2^2 = \{(0.5\times0.96, 25+36), (0.75\times0.96, 50+36), (0.875\times0.96, 75+36)\}$

$\{(0.480, 61), (0.7200, 86), (0.84000, 111)\}$

Now S_3^2 $i=2$, $j=3$, $m2=3$

$\phi_2 m_2 = (1-(1-0.8)^3)$

$= (1-(0.2)^3) = 1-0.008$

$= 0.992$

$3\times c2 = 3\times18 = 54$ is added to s^1 pair

S_3^2 $\{(0.5\times0.992, 25+54), (0.75\times0.992, 50+54), (0.875\times0.992, 75+54)\}$

$= \{(0.4960, 79), (0.744, 104), (0.868, 129)\}$

S^2 can be merged S_1^2, S_2^2, S_3^2

$S^2 = \{(0.4, 43), (0.6, \cancel{68}), (\cancel{0.7, 93}), (0.480, 61), (0.7\cancel{200}, 86), (\cancel{0.84000, 111}),$

$(0.4960, 79), (0.744, 104), (\cancel{0.868, 129})\}$

$S^2 = \{(0.4, 43), (0.480, 61), (0.4960, 79), (0.744, 104)\}$

calculate S_1^3 $i=3$, $j=1$, $m3=1$

$\phi_3 m_3 = 1-(1-0.7)^1 = 1-0.3 = 0.7$

$1\times c3 = 1\times25 = 25$ is added to S^2 pair

$S_1^3 = \{(0.4\times0.7, 43+25), (0.48\times0.7, 61+25), (0.496\times0.7, 79+25),$

$(0.744\times0.7, 104+25)\}$

$= \{(0.28, 68), (0.336, 86), (0.3472, 104), (0.52\cancel{08}, 129)\}$

last tuple is removed from S_1^3, since cost 129 exceed the given cost 120.

Now S_2^3 $i=3$, $j=2$, $m3=2$

$\phi_3 m_3 = 1-(1-0.7)^2 = 1-(0.3)^2 = 1-0.09 = 0.91$

$2\times c3 = 2\times25 = 50$ is added to S^2 pair

$S_2^3 = \{(0.4\times0.91, 43+50), (0.48\times0.91, 61+50), (0.496\times0.91, 79+50),$

$(0.744\times0.91, 104+50)\}$

$= \{(0.364, 93), (0.4368, 111), (0.45136, \cancel{129}), (0.67\cancel{704}, 154)\}$

last two tuples are removed from S_2^3 since cost 129, 154 exceeding the given cost 120

$S_2^3 = \{(0.364, 93), (0.4368, 111)\}$

Now S_3^3 $i=3$, $j=3$, $m_3=3$

$$\phi_3 m_3 = 1-(1-0.7)^3 = 1-(0.3)^3 = 1-0.027$$
$$= 0.973$$

$3 \times C_3 = 3 \times 25 = 75$ is added to S^2 pair

$S_3^3 = \{(0.4 \times 0.973, 43+75), (0.48 \times 0.973, 61+75), (0.496 \times 0.973, 79+75)$

$(0.744 \times 0.973, 104+75)\}$

$= \{(0.3892, 118), (0.46704, 136), (0.482608, 154), (0.723912, 179)\}$

Last three tuples are removed from S_3^3 since cost is 136, 154, 179 exceeding the given cost is 120.

$S_3^3 = \{(0.3892, 118)\}$

S^3 can be merged S_1^3, S_2^3, S_3^3

$S^3 = \{(0.28, 68), (0.336, 86), \cancel{(0.3472, 104)}, (0.364, 93),$

$(0.4368, 111), \cancel{(0.3892, 118)}\}$