UNIT - 2

Concept of Boolean Algebra

Introduction:

→ In 1854 George Boolean introduced a systematic treatment of logic and developed for this purpose an algebraic system now called Boolean Algebra.

→ In 1938 C.E Shanon introduced a two-valued Boolean Algebra called a switching Algebra

Fundamental Postulateds of Boolean Algebra

S.NO	Postulates	Comments
1.	→ Result of each operator is either 0 or 1	$1, 0 \in B$
2.	a) $0+0=0$, $0+1=1+0=1$ b) $1\cdot1=1$; $0\cdot1=1\cdot0=0$	Identity element "0" for "+" and "1" for "·"
3.	a) $(A+B)=(B+A)$ b) $(A\cdot B)=(B\cdot A)$	Commulative law
4.	a) $A\cdot(B+C)=(A\cdot B)+(A\cdot C)$ b) $A+(B\cdot C)=(A+B)\cdot(A+C)$	Distributive law
5.	a) $A+\bar{A}=1$, $0+\bar{0}=1$, $1+\bar{1}=1$ b) $A\cdot\bar{A}=0$, if $\bar{0}\cdot\bar{0}=0\cdot1=0$ $1\cdot\bar{1}=1\cdot0=0$	Complement

$[\bar{0}=1$

$\bar{1}=0]$

Basic theorems and propertys

Duality:

→ The principle of Duality theorem says that starting with a boolean relation, we can derive another boolean relation by

1. Changing each "OR" sign to an "AND" sign
2. Changing each "AND" sign to an "OR" sign
3. Any 0 or 1 operating in the Expression of boolean function.

Ex: Dual of relation $A+\bar{A}=1$ is $A\cdot\bar{A}=0$

$A+\bar{A}=1$ $\quad$ $A\cdot\bar{A}=0$

$A\cdot\bar{A}=0$

principal of Duality theorm says that starting with a

Theorems:

Laws of Boolean algebra:

Boolean Expression	B.A law (or) Rule
⇒ $A+1=1$ $A\cdot 0=0$	Annulment
⇒ $A+0=A$ $A\cdot 1=A$	Identity
⇒ $A+A=A$ $A\cdot A=A$	Idempotent
⇒ $\bar{\bar{A}}=A$ NOT $\bar{A}=A$	Double Nagation

Boolean Expression	B·A law (or) Rule
$\Rightarrow A+\bar{A}=1$ $A\cdot\bar{A}=0$	Complement
$\Rightarrow A+B=B+A$ $A\cdot B=B\cdot A$	Commutative law
*** $\Rightarrow \overline{A+B}=\bar{A}\cdot\bar{B}$ $\overline{AB}=\bar{A}+\bar{B}$	De-morgan

Boolean Algebra functions :

Function	Description	Expression
1.	NULL	0
2.	Identity	1
3.	INput A	A
4.	INput B	B
5.	NOT A	$\bar{A}$
6.	NOT B	$\bar{B}$
7.	A AND B (AND)	$A\cdot B$
8.	A AND NOT B	$A\cdot\bar{B}$
9.	NOT A AND B	$\bar{A}\cdot B$
10.	NOT AND (NAND)	$\overline{A\cdot B}$
11.	A OR B (OR)	$A+B$
12.	A OR NOT B	$A+\bar{B}$

Function	Description	Expression
13.	NOT A OR B	$\bar{A}+B$
14.	NOT OR (NOR)	$\overline{A+B}$
15.	Exclusive OR (X-OR)	$A\bar{B}+\bar{A}B$
16.	Exclusive NOR (X-NOR)	$AB+\bar{A}\bar{B}$

Theorems:

① a) $A+A=A$

b) $A\cdot A=A$

② a) $A+1=1$

b) $A\cdot 0=0$

③ a) $\bar{\bar{A}}=A$

④ a) $A+AB=A$

b) $A(A+B)=A$

⑤ a) $A+\bar{A}B=A+B$

b) $A(\bar{A}+B)=AB$

① ⓐ Proof:

L.H.S

$$A+A=(A+A)\cdot(1)$$
$$=(A+A)(A+\bar{A})$$
$$=AA+A\bar{A}+AA+A\bar{A}$$
$$=AA+A\bar{A}$$
$$=A(A+\bar{A})$$
$$=A \;//$$

$A+A=A$

if $A=0$; $0+0=0$
if $A=1$; $1+1=1$

$[1=A+\bar{A}]$

1 ⓑ Proof:

L.H.S

$$A\cdot A=A\cdot A+(0)$$
$$=A\cdot A+A\cdot\bar{A}$$
$$=A(A+\bar{A})$$
$$=A \;//$$

$A\cdot A=A$

$A=0 \Rightarrow 0\cdot 0=0$
$A=1$; $1\cdot 1=1$

$[A\cdot\bar{A}=0]$

② ⓐ $A+1=1$

Proof: $A+1 = (1)A+1$

$= (A+\bar{A})\cdot(A+1)$

$= A\cdot A + A + \bar{A}\cdot A + \bar{A}$

$= A + A + 0 + \bar{A}$

$= A + A + \bar{A}$

$= A+\bar{A}$

$= 1$

$[A\cdot\bar{A} = 0$

$A+\bar{A} = 1]$

$A+1=1$

$\left\{ \begin{array}{l} \text{if } A=0;\ 0+1=1 \\ A=1;\ 1+1=1 \end{array} \right.$

ⓑ $A\cdot 0 = 0$

Proof: $A\cdot 0 = A\cdot(A\cdot\bar{A})$

$= A(0)$

$= 0$

$A\cdot 0=0$

$A=0;\ 0\cdot 0=0$

$A=1;\ 1\cdot 0=0$

③ $\bar{\bar{A}} = A$

Proof: if $A=0;\ \bar{\bar{0}} = \bar{1} = 0$

$A=1;\ \bar{\bar{1}} = \bar{0} = 1$

$[\bar{0}=1$

$\bar{1}=0]$

④ ⓐ $A+AB=A$

Proof: $A+AB = A(1+B)$

$= A(1)$

$= A$

$[\because 1+B=1]$

ⓑ $A(A+B) = A$

Proof: $A(A+B) = (A+AB)(A+B)$

$= AA + AB + AB\cdot A + AB\cdot B$

$= A + AB + A\cdot AB + AB$

$= A + AB + AB + AB$

$= A(1+B)+AB$

$= A+AB$

$= A(1+B)$

$= A$ //

(5) (a) $A+\bar{A}B = A+B$

proof: $A+\bar{A}B = (A+\bar{A})(A+B)$

$= A+B$ $[\because A+\bar{A}=1]$

(b) $A(\bar{A}+B) = AB$

proof: $A(\bar{A}+B) = A\cdot\bar{A}+AB$ $[A\cdot\bar{A}=0]$

$= 0+AB$

$= AB$ //

***

De-Morgan's theorem:

(i) $\overline{AB} = \bar{A}+\bar{B}$

(ii) $\overline{A+B} = \bar{A}\cdot\bar{B}$

(i) $\overline{AB} = \bar{A}+\bar{B}$

A	B	$\bar{A}$	$\bar{B}$	$\overline{AB}$	$\bar{A}+\bar{B}$
0	0	1	1	1	1
0	1	1	0	1	1
1	0	0	1	1	1
1	1	0	0	0	0

(ii) $\overline{A+B} = \bar{A}\cdot\bar{B}$

A	B	$\bar{A}$	$\bar{B}$	$\bar{A}\cdot\bar{B}$	$\overline{A+B}$
0	0	1	1	1	1
0	1	1	0	0	0
1	0	0	1	0	0
1	1	0	0	0	0

→ Consensus theoram:-

→ In simplification of Boolean Expression, an Expression of the form $AB+\bar{A}C+BC$. The term BC is redented and can be elemin-ated to form the Equvlent expresion $AB+\bar{A}C$.

→ The theoram is used for this simplification is known as Consensus theoram. and it is stated as

$$AB+\bar{A}C+BC = AB+\bar{A}C$$

Proof:- L.H.S $\Rightarrow AB+\bar{A}C+BC = AB+\bar{A}C+BC(1)$ $[\because A+\bar{A}=1]$

$$= AB+\bar{A}C+BC(A+\bar{A})$$

$$= AB+\bar{A}C+BCA+BC\bar{A}$$

$[\because 1+C=1$
$1+B=1]$

$$= AB(1+C)+\bar{A}C(1+B)$$

$$= AB+\bar{A}C$$

* Solve the given Expresion using Consensus theorem

(i) $\bar{A}\bar{B}+AC+B\bar{C}+\bar{B}C+AB$

$\Rightarrow \bar{A}\bar{B}+AC+B\bar{C}+\bar{B}C(1)+AB$ $[A+\bar{A}=1]$

$\Rightarrow \bar{A}\bar{B}+AC+B\bar{C}+\bar{B}C(A+\bar{A})+AB$

$\Rightarrow \bar{A}\bar{B}+AC+B\bar{C}+\bar{B}CA+\bar{B}C\bar{A}+AB$

$\Rightarrow \bar{A}\bar{B}(1+C)+AC(1+\bar{B})+B\bar{C}+AB$ $[1+C=1$
$1+\bar{B}=1]$

$\Rightarrow \bar{A}\bar{B}+AC+B\bar{C}+AB$

$\Rightarrow \bar{A}\bar{B}+AC+B\bar{C}+AB(1)$

$\Rightarrow \bar{A}\bar{B}+AC+B\bar{C}+AB(C+\bar{C})$

$\Rightarrow \bar{A}\bar{B}+AC+B\bar{C}+ABC+AB\bar{C}$ $[1+B=1$
$1+A=1]$

$\Rightarrow \bar{A}\bar{B}+AC(1+B)+B\bar{C}(1+A)$

$\Rightarrow \bar{A}B + AC + B\bar{C}$

Dual of Consensus theorem:

→ The Dual form of consensus theorem is stated as

$(A+B)(\bar{A}+C)(B+C) = (A+B)(\bar{A}+C)$

$(A\bar{A} + AC + \bar{A}B + BC)(B+C) = (A\bar{A} + AC + \bar{A}B + BC)$

$(0 + AC + \bar{A}B + BC)(B+C) = (0 + AC + \bar{A}B + BC)$ $[A\cdot\bar{A} = 0]$

$ABC + \bar{A}B\cdot B + BC\cdot B + AC\cdot C + \bar{A}BC + BC\cdot C = AC + \bar{A}B + BC$

$[BC + BC = BC]$

$\cancel{BC(A+B) + \bar{A}B(B+C}$

$ABC + \bar{A}B + BC + AC + \bar{A}BC + BC = AC + \bar{A}B + BC$

$\bar{A}B(1+C) + BC(1+A) + AC = AC + \bar{A}B + BC$

$$\boxed{\bar{A}B + BC + AC = AC + \bar{A}B + BC}$$

Boolean function (or) switching function:

→ Boolean equations are constructed by connecting the boolean Constants and Variables with the boolean operation.

→ This Boolean Expressions are known as boolean formulas we use boolean Expression to describe boolean functions

boolean Expression to describe

for Example:- If Boolean Expresion $(A+\bar{B})C$ is used to describe the function of f, then boolean function is written as $f(A,B,C) = (A+\bar{B})C$

$f = (A+\bar{B})C$

→ Let us consider the ~~whole~~ four Variable boolean function.

Product-terms:

$f(A,B,C,D) = A + (\bar{B}C) + (A\bar{C}D)$

product terms

Literals

Sum - Terms:-

$$f(A,B,C,D) = (B+\bar{D})\cdot(A+\bar{B}+C)(\bar{A}+C)$$

Sum terms

Literals

→ The literals and terms are arranged in the forms ~~to~~ in two

1. Sum of product (SOP)

2. product of sum (POS)

1. Sum of products:-

→ The sum of product is also called Disjunctional normal form (or) Disjunctional normal formula

→ The words sum & product are discribed from the symbolic representation "OR" and "AND" function (+ and *)

Ex:-1 $f(A,B,C) = AB'C + A\bar{B}\bar{C}$

sum terms

products

Ex:-2 $f(P,Q,R) = \bar{P}Q + QR + RS$

sum terms

Products

2. Product of sum:-

→ The product of sum is also called conjunctional normal form (or) conjunctional normal formula.

→ A product of sum is any group of sum terms ANDed to gether.

Ex:- $f(A,B,C) = (A+B+C)\cdot(A+\bar{B}+\bar{C})$

Product

Sum

Ex: $f(p,q,r) = (p+q)\cdot(r+\bar{p})\cdot q$

Sum product

*Canonical form: (Standard sop and pos forms)

→ Basically canonical forms are two types

1. standard sop (or) minterm canonical form
2. standard pos (or) maxterm canonical form

1. standard sop: (minterm canonical form)

→ The ssop is given by $f(A,B,C) = (A\bar{B}C) + (ABC) + (\bar{A}BC)$

Each product term is consistent all literals in either completed form or uncompleted form

2. standard pos: (maxterm)

$f(A,B,C) = (A+\bar{B}+C)\cdot(A+B+C)\cdot(\bar{A}+B+C)$

Each sum term consists of all literals in the completed form

*Converting Expressions in standard sop (or) pos form

Converting to standard sop form:

Steps to convert sop to standard sop

Steps:

1. find the missing literal in each product term if any
2. "AND" each product term having missing literals which term 1 formed by "OR" ing the literals and its complement
3. Expand the terms by applying Distributive law and recorded the literals in the product terms
4. Reduce the expression omitting (or) remove repeated product terms if any. because $(A+A) = A$

Ex: Convert the given Expression in standard sop form

$f(A,B,C) = AC + AB + BC$

Step: 1 ~~f(A,B,C) =~~ find the missing literal in each product

$f(A,B,C) = AC + AB + BC$

A literal is missing

C literal missing

B literal missing

Step: 2 AND product terms with (missing literal + complement)

$f(A,B,C) = AC(1) + AB(1) + BC(A+\bar{A})$

$= AC(B+\bar{B}) + AB(C+\bar{C}) + BC(A+\bar{A})$

$[A+\bar{A} = 1$
$B+\bar{B} = 1$
$C+\bar{C} = 1]$

Step 3: Expand the term and recorded the terms

Expand:

$f(A,B,C) = ACB + AC\bar{B} + ABC + AB\bar{C} + BCA + BC\bar{A}$

Recorded:

$f(A,B,C) = ABC + A\bar{B}C + ABC + AB\bar{C} + ABC + \bar{A}BC$

Step 4: omitting the repeted product term

$f(A,B,C) = \underline{ABC} + A\bar{B}C + \underline{ABC} + AB\bar{C} + \underline{ABC} + \bar{A}BC$

$f(A,B,C) = ABC + A\bar{B}C + AB\bar{C} + \bar{A}BC$

Ex: Convert the given Expresion in standard sop form

$f(A,B,C) = A + ABC$

Step1: find

$f(A,B,C) = A + ABC$

B literal missing

C literal missing

Step2: AND product term with (missing literal + complement)

$f(A,B,C) = A(1)(1) + ABC$

$= A(B+\bar{B})(C+\bar{C}) + ABC$

Step 3:

$f(A,B,C) = \cancel{AB+A\bar{B}}\,(BC+B\bar{C}+\bar{B}C+\bar{B}\bar{C})+ABC$

$= \underline{ABC} + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + \underline{ABC}$

Step 4: omitting the repeated product term

$f(A,B,C) = ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C}$

Convert pos to standard pos form:

Step 1:

1. find the missing literals in each sum term if any.
2. OR each sum term having missing literal formed by ANDing the literal and its complement
3. Expand the terms by applying distributive law and recorded the literal in the sum terms
4. Reduce the expression by omitting repeated sum term if any

$A \cdot A = A$

Ex: convert the given expression in standard pos form

$f(A,B,C) = (A+B)\cdot(B+C)(A+C)$

$[A\cdot\bar{A}=0$
$B\cdot\bar{B}=0$
$C\cdot\bar{C}=0]$

$[A+(BC) = (A+B)(A+C)]$ distribut

Step 1: find the missing literal in each product

$f(A,B,C) = (A+B)\cdot(B+C)(A+C)$

$(A+C) \to$ B literal missing

$(B+C) \to$ A literal missing

$(A+B) \to$ C literal missing.

Step 2: OR product term with (missing literal + complement)

$f(A,B,C) = [(A+B)+(C\cdot\bar{C})]\cdot[(B+C)+(A\cdot\bar{A})]\cdot[(A+C)+(B\cdot\bar{B})]$

Step 3: Expand the term and recorded the terms

Expand:

$f(A,B,C) = [(A+B+C)\cdot(A+B+\bar{C})]\cdot[(B+C+A)\cdot(B+C+\bar{A})]\cdot[(A+C+B)\cdot(A+C+\bar{B})]$

Recorded:

$f(A,B,C) = [(A+B+C)(A+B+\bar{C})] \cdot [(A+B+C) \cdot (\bar{A}+B+C)] \cdot [(A+B+C)(A+\bar{B}+C)]$

Step 4: omitting the repeated product term

$f(A,B,C) = (A+B+C)(A+B+\bar{C})(A+\bar{B}+C)(\bar{A}+B+C)$

Ex: Convert the given expression in standard pos forms

$f(A,B,C) = (A) \cdot (A+B+C)$

Step 1: find the missing literal in each product

$f(A,B,C) = A \cdot (A+B+C)$

B, C literals missing

Step 2: OR product term with (missing literal + complement)

$f(A,B,C) = [A+(B \cdot \bar{B})+(C \cdot \bar{C})] \cdot (A+B+C)$

Step 3: Expand the term and recorded the terms

Expand: $[A+(B+C) \cdot (B+\bar{C}) \cdot (\bar{B}+C) \cdot (\bar{B}+\bar{C})] \cdot (A+B+C)$

$= (A+B+C) \cdot (A+B+\bar{C}) \cdot (A+\bar{B}+C) \cdot (A+\bar{B}+\bar{C}) \cdot (A+B+C)$

Step 4: omitting the repeated product term

$f(A,B,C) = (A+B+C) \cdot (A+B+\bar{C}) \cdot (A+\bar{B}+C) \cdot (A+\bar{B}+\bar{C})$

M- Notations: (minterms and maxterms)

Decimal NO	Binary Numbers (421)	Minterm (m) (SOP)	Maxterm (M) (POS)
0	000	$\bar{A}\bar{B}\bar{C}$ (m_0)	$A+B+C$ (M_0)
1	001	$\bar{A}\bar{B}C$ (m_1)	$A+B+\bar{C}$ (M_1)
2	010	$\bar{A}B\bar{C}$ (m_2)	$A+\bar{B}+C$ (M_2)
3	011	$\bar{A}BC$ (m_3)	$A+\bar{B}+\bar{C}$ (M_3)
4	100	$A\bar{B}\bar{C}$ (m_4)	$\bar{A}+B+C$ (M_4)
5	101	$A\bar{B}C$ (m_5)	$\bar{A}+B+\bar{C}$ (M_5)
6	110	$AB\bar{C}$ (m_6)	$\bar{A}+\bar{B}+C$ (M_6)
7	111	ABC (m_7)	$\bar{A}+\bar{B}+\bar{C}$ (M_7)

Onlinebd

$\boxed{\text{Minterm} = \text{Complement of maxterm}}$

Example: $f(A,B,C) = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}BC + AB\bar{C}$

$= m_0 + m_1 + m_3 + m_6$

$= \sum m(0,1,3,6)$

Ex: $f(A,B,C) = (A+B+\bar{C}) \cdot (A+\bar{B}+\bar{C}) \cdot (\bar{A}+\bar{B}+C)$

$= M_1 \cdot M_3 \cdot M_6$

$= \pi M(1,3,6)$

Ex: To find sum of product form to the given table

A B C	y
0 0 0	0
0 0 1	0
0 1 0	1
0 1 1	1
1 0 0	0

[ABC is a input variables]

[Sop ki 1's]

A . B C	y
1 0 1	0
1 1 0	1
1 1 1	0

$$f(A,B,C) = \bar{A}B\bar{C} + \bar{A}BC + AB\bar{C}$$

$$= m_2 + m_3 + m_6$$

$$= \Sigma m(2,3,6)$$

To find the product of sum from the given table

[Product of sum ke zero's]

A+B+C	y
0 0 0	1
0 0 1	1
0 1 0	0
0 1 1	1
1 0 0	1
1 0 1	0
1 1 0	1
1 1 1	1

$$f(A,B,C) = (A+\bar{B}+C)\cdot(\bar{A}+B+\bar{C})$$

$$= M_2 \cdot M_5$$

$$= \Pi M(2,5)$$

Algebraic Simplifications:

1. $A \cdot \bar{A}C = 0$
2. $ABCD + ABD = ABD$
3. $A(A+B) = A$
4. $AB + ABC + AB(D+E) = AB$
5. $XY + XYZ + XY\bar{Z} + \bar{X}YZ$
6. $\bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC$
7. $ABC + A\bar{B}C + AB\bar{C}$
8. $A + \bar{A}B + A\bar{B} = A+B$

Ex:- $\overline{\overline{AB} + \bar{A} + AB}$

$\Rightarrow \overline{\bar{A} + \bar{B} + \bar{A} + AB}$

$\Rightarrow \overline{\bar{A} + \bar{B} + AB}$

$\Rightarrow \overline{\bar{A} + (\bar{B}+A)\cdot(\bar{B}+B)}$

$\Rightarrow \overline{\bar{A} + \bar{B} + A\cdot(1)}$

$\Rightarrow \overline{(A+\bar{A}) + \bar{B}}$

$\Rightarrow \overline{1+\bar{B}}$

$\Rightarrow \bar{1}$

$\Rightarrow 0$

$[\overline{AB} = \bar{A}+\bar{B}]$

$[\bar{B} + (AB) = \cancel{\bar{B}A) + (B \cdot \bar{B})}$
$= (\bar{B}+A)\cdot(\bar{B}+B)]$

$[\bar{B}+B = 1]$

$[\bar{1} = 0]$

$\overline{\overline{AB} + \bar{A} + AB}$

$\overline{\bar{A}+\bar{B} + \bar{A} + AB}$

$= \overline{\bar{A}+\bar{B}+AB}$

$= \overline{\bar{A}+A\bar{B} + B\bar{B}}$

$= \overline{\bar{A} + A\bar{B}}$

$= \overline{(\bar{A}+A)\cdot(\bar{A}+\bar{B})}$

$= \overline{1+\bar{B}} = \bar{1} = 0$

(1) $A \cdot \bar{A}C = 0$

$(A\cdot\bar{A})C = 0$

$(0)C = 0$

$A\cdot\bar{A}C = 0$

② $ABCD + ABD = ABD$ $[1 + C = 1]$

$\Rightarrow ABD(1+C)$

$\Rightarrow ABD$

$\Rightarrow ABCD + ABD = ABD$

③ $A(A+B) = A$ $[A \cdot A = 1]$

$\Rightarrow A \cdot A + AB$

$\Rightarrow A + AB$

$\Rightarrow A(1+B)$

$\Rightarrow A$

$A \cdot A + AB = A$

$A + AB = 0$

$\cancel{(A+A)}(A+B)$

$\cancel{(A+A)}(A+B)$

$(A+A+B)$

$A(1+B)$

$= A$

④ $AB + ABC + AB(D+E) = AB$

$\Rightarrow AB + ABC + AB(D+E)$

$\Rightarrow AB(1+C) + ABD + ABE$

$\Rightarrow AB + AB(D+E)$

$\Rightarrow AB(1+D+E)$

$\Rightarrow AB[(1+D)+E]$

$\Rightarrow AB[1+E]$

$\Rightarrow AB(1)$

$\Rightarrow AB$

$xy + xyz + xy\bar{z} + \bar{x}yz = y(x+z)$

$= xy + yz$

$xy + xyz + xy\bar{z} + \bar{x}yz$

$xy + xy(z+\bar{z}) + \bar{x}yz$

$xy + y(x + \bar{x}z)$

$xy + y((x+\bar{x})(x+z))$

$xy + y(x+z)$

$xy + xy + z$

$xy + xy$

$y(x+z)$

5. $xy + xyz + xy\bar{z} + \bar{x}yz = y(x+z)$

$\Rightarrow xy(1) + xyz + xy\bar{z} + \bar{x}yz$

$\Rightarrow xy(z+\bar{z}) + xyz + xy\bar{z} + \bar{x}yz$

$\Rightarrow xyz + xy\bar{z} + xyz + xy\bar{z} + \bar{x}yz$

$\Rightarrow xy(z+\bar{z}) + xy(z+\bar{z}) + \bar{x}yz$

$xy + xyz + xy\bar{z} + \bar{x}yz$

(or) $xy(1+\bar{z}) + yz(x+\bar{x})$

$xy + yz$

$y(x+z)$ //

$\Rightarrow xy + xy + \bar{x}yz$

$[(x) + (\bar{x}z)$
$(x+\bar{x})\cdot(x+z)]$

$\Rightarrow xy + \bar{x}yz$

$\Rightarrow y(x + \bar{x}z) \Rightarrow y((x+\bar{x})\cdot(x+z)) \Rightarrow y(1)\cdot(x+z)$

$\Rightarrow y(x+z)$ //

(6) $\bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC$

$\Rightarrow \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC$

$\Rightarrow \bar{A}\bar{B}\bar{C} + \bar{A}B(\bar{C}+C)$

$\Rightarrow \bar{A}\bar{B}\bar{C} + \bar{A}B$

$\Rightarrow \bar{A}(\bar{B}\bar{C} + B)$

$[(\bar{B}\bar{C}) + B$
$= (\bar{B}+B)\cdot(\bar{C}+B)]$

$\Rightarrow \bar{A}((\bar{B}+B)\cdot(\bar{C}+B))$

$\Rightarrow \bar{A}((1)\cdot(\bar{C}+B))$

$\Rightarrow \bar{A}(\bar{C}+B)$

$\Rightarrow \bar{A}\bar{C} + \bar{A}B$

$\bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC$
$\bar{A}\bar{C}(\bar{B}+B) + \bar{A}BC$
$= \bar{A}\bar{C} + \bar{A}BC$
$= \bar{A}(\bar{C}+BC)$
$= \bar{A}((\bar{C}+B)\cdot(\bar{C}+C))$
$= \bar{A}\bar{C} + \bar{A}B$

(7) $ABC + A\bar{B}C + AB\bar{C}$

$ABC + A\bar{B}C + AB\bar{C}$

$AC(B+\bar{B}) + AB\bar{C}$

$AC + AB\bar{C}$

$A(C + B\bar{C})$

$A((C+B)\cdot(C+\bar{C}))$

$A(C+B)$

$AC + AB$ //

$ABC + A\bar{B}C + AB\bar{C}$
$AC(B+\bar{B}) + AB\bar{C}$
$AC + AB\bar{C}$
$A(C + B\bar{C})$
$A((C+B)\cdot(C+\bar{C}))$
$A(C+B)$
$AC + AB$

⑧ $A+\bar{A}B+A\bar{B} = A+B$

$\Rightarrow A+\bar{A}B+A\bar{B}$

$\Rightarrow A(1+\bar{B})+\bar{A}B$

$\Rightarrow A(1)+\bar{A}B$

$\Rightarrow A+\bar{A}B$

$\Rightarrow (A+\bar{A})\cdot(A+B)$

$\Rightarrow (1)\cdot(A+B)$

$\Rightarrow A+B$ //

$A+\bar{A}B+A\bar{B} = A+B$

$A\cdot(1+\bar{B})+\bar{A}B$

$A+\bar{A}B$

$(A+\bar{A})\cdot(A+B)$

$(1)\cdot(A+B)$

$A+B$

Ex: Simplify the following three variable Expression by using Boolean algebra $Y = \Sigma m(1,3,5,7)$

Given Data $Y = \Sigma m(1,3,5,7)$

$= m_1+m_3+m_5+m_7$

$= \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}C + ABC$

$= \bar{A}C(\bar{B}+B) + AC(\bar{B}+B)$

$= \bar{A}C(1)+AC(1)$

$= C(\bar{A}+A)$

$= C$ //

$Y=\Sigma m(1,3,5,7)$

$= \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}C + ABC$

$= \bar{A}C(B+\bar{B}) + AC(B+\bar{B})$

$= \bar{A}C+AC$

$= C(A+\bar{A})$

$= C(1)$

Ex: Simplify the following three variable Expression by using Boolean algebra $Y = \Pi M(1,3,5,7)$

Given that $Y = \Pi M(1,3,5,7)$

$= M_1\cdot M_3\cdot M_5\cdot M_7$

$= (A+B+\bar{C})(A+\bar{B}+\bar{C})\cdot(\bar{A}+B+\bar{C})\cdot(\bar{A}+\bar{B}+\bar{C})$

$= M_1\cdot M_3\cdot M_5\cdot M_7$

$= (\bar{A}+B+\bar{C})\cdot(A+\bar{B}+C)$

$= [A\cdot A+(A\cdot\bar{B})+(A\cdot\bar{C})+(B\cdot A)+(B\cdot\bar{B})+(B\cdot\bar{C})+(\bar{C}\cdot A)+(\bar{C}\cdot\bar{B})+(\bar{C}\cdot\bar{C})]\cdot$

$[(\bar{A}\cdot\bar{A})+(\bar{A}\cdot B)+(\bar{A}\cdot\bar{C})+(B\cdot\bar{A})+(B\cdot\bar{B})+(B\cdot\bar{C})+(\bar{C}\cdot\bar{A})+(\bar{B}\cdot\bar{C})+(\bar{C}\cdot\bar{C})]$

$= AB\bar{C}+A\bar{B}\bar{C}+A\bar{C}+A\bar{B}\bar{C}+ AB\bar{C}+A\bar{B}\bar{C}+A\bar{C}+ABC+AB\bar{C}+\bar{A}B\bar{C}+\bar{A}B\bar{C}+\bar{A}BC+$

$B\bar{C}+\bar{A}\bar{B}\bar{C}+B\bar{C}+AB\bar{C}+A\bar{B}\bar{C}+A\bar{C}+\bar{A}\bar{B}\bar{C}+\bar{A}\bar{B}\bar{C}+\bar{A}\bar{B}\bar{C}+\bar{A}\bar{B}\bar{C}+\bar{B}\bar{C}$

$+B\bar{C}+\bar{A}\bar{C}+\bar{A}\bar{B}\bar{C}+\bar{A}\bar{C}+\bar{A}B+B\bar{C}+A\bar{C}+\bar{B}\bar{C}+\bar{C}$

$= AB\bar{C}+A\bar{B}\bar{C}+A\bar{C}+\bar{A}B\bar{C}+\bar{A}BC+B\bar{C}+\bar{A}\bar{B}\bar{C}+\bar{B}\bar{C}+\bar{A}C+\bar{A}B+\bar{C}$

$= B\bar{C}(A+\bar{A})+\bar{B}\bar{C}(1+A)+\bar{C}(A+\bar{A})+\bar{A}B(1+C)+\bar{C}(1+B)+\bar{A}\bar{B}\bar{C}$

$= B\bar{C}+\bar{B}\bar{C}+\bar{C}+\bar{A}B+\bar{C}+\bar{A}\bar{B}\bar{C}$

$= \bar{A}B+\bar{A}\bar{B}\bar{C}+\bar{C}$ //

Ex(2) Simplify the following three variable Expresion. Convert the Exprsion into minterm complementary property. ~~and~~ &

$$y = \Pi_M(1,3,5,7)$$

Sol: The given Expresion is maxterm $y = \Pi_M(1,3,5,7)$

To convert given sequences to minterm Complementary form

$\boxed{\text{minterm} = \text{Complementary of Maxterm}}$

The minterm is given by $= \Sigma m(0,2,4,6)$

$y = \Sigma m(0,2,4,6) = \bar{A}\bar{B}\bar{C}+\bar{A}B\bar{C}+A\bar{B}\bar{C}+AB\bar{C}$

$= m_0+m_2+m_4+m_6 \quad = \bar{A}\bar{C}(B+\bar{B})+A\bar{C}(B+\bar{B})$

$= \bar{A}\bar{B}\bar{C}+\bar{A}B\bar{C}+A\bar{B}\bar{C}+AB\bar{C} \quad = \bar{A}\bar{C}+A\bar{C}$

$= \bar{A}\bar{C}(\bar{B}+B)+A\bar{C}(\bar{B}+B) \quad = \bar{C}(A+\bar{A})$

$= \bar{C}$

$= \bar{A}\bar{C}+A\bar{C}$

$= \bar{C}(\bar{A}+A) = \bar{C}$ //

(14m)**

* transform Each of the following Canonical Expresion into its other Canonical form its decimal notations.

(i) $f(x,y,z) = \Sigma m(1,3,5)$

(ii) $f(w,x,y,z) = \Pi M(0,2,5,6,7,8,9,11,12)$

(i) $f(x,y,z) = \Sigma m(1,3,5)$ $\quad f(x,y,z) = \Pi M(0,2,4,6,7)$

$f(x,y,z) = \Pi M(0,2,4,6,7)$

$= M_0 \cdot M_2 \cdot M_4 \cdot M_6 \cdot M_7 \quad (x+y+z)\cdot(x+\bar{y}+z)\cdot(\bar{x}+y+z)\cdot(\bar{x}+\bar{y}+z)\cdot(\bar{x}+\bar{y}+\bar{z})$

$= \cancel{(x+y+z)\cdot(A+\bar{B}+C)\cdot(\bar{A}+B+C)\cdot(\bar{A}+\bar{B}+C)\cdot(\bar{A}+\bar{B}+\bar{C})}$

$= (x+x\bar{y}+xz+xy+yz+xz+\bar{y}z+z)(\bar{x}+\bar{x}\bar{y}+\bar{x}z+\bar{x}y+yz+\bar{x}z+z\bar{y}+z).(\bar{x}+y+\bar{z})$

$= xyz+x\bar{y}z+xz+x\bar{y}z+xyz+x\bar{y}z+xz+xyz+xyz+\bar{x}yz+\bar{x}yz+\bar{x}yz+$
$yz+\bar{x}yz+yz+xyz+x\bar{y}z+xz+\bar{x}\bar{y}z+\bar{x}\bar{y}z+\bar{x}\bar{y}z+\bar{x}\bar{y}z+$
$\bar{y}z+\bar{y}z+\bar{x}z+\bar{x}\bar{y}z+\bar{x}z+\bar{x}yz+yz+\bar{x}z+z\bar{y}+z)$

$= xyz+x\bar{y}z+xz+\bar{x}yz+yz+\bar{x}\bar{y}z+\bar{y}z+\bar{x}z+z$

$= xz(y+\bar{y})+yz(\bar{x}+1)+\bar{y}z(1+\bar{x})+z(x+\bar{x})+z$

$= xz+yz+\bar{y}z+z \Rightarrow x(x+y)+z(1+\bar{y})$

$= xz+xy+z$ //

(ii) $f(w,x,y,z) = \prod M(0,2,5,6,7,8,9,11,12)$

$f(w,x,y,z) = \overline{\prod M(0,2,5,6,7,8,9,11,12)}$

$= \Sigma m(1,3,4,10,14,15)$

$= m_1+m_3+m_4+m_{10}+m_{13}+m_{14}+m_{15}$

$= \bar{w}\bar{x}\bar{y}z+\bar{w}\bar{x}yz+\bar{w}x\bar{y}\bar{z}+w\bar{x}y\bar{z}+wx\bar{y}z+$
$+wxy\bar{z}+wxyz$

$= \bar{w}\bar{x}z(\bar{y}+y)+wxz(y+\bar{y})+wy\bar{z}(x+\bar{x})+\bar{w}x\bar{y}\bar{z}$

$= \bar{w}\bar{x}z+wxz+wy\bar{z}+\bar{w}x\bar{y}\bar{z}$

Decimal NO	Binary number (8421)	minterm (m)	Maxterm (M)
8	1000	$A\overline{B}\overline{C}\overline{D}$	$\overline{A}+B+C+D$
9	1001	$A\overline{B}\overline{C}D$ (m9)	$\overline{A}+B+C+\overline{D}$
10	1010	$A\overline{B}C\overline{D}$ (m	$\overline{A}+B+\overline{C}+D$
11	1011	$A\overline{B}CD$	$\overline{A}+B+\overline{C}+\overline{D}$
12	1100	$ABC\overline{D}$	$\overline{A}+\overline{B}+C+D$
13	1101	$AB\overline{C}D$	$\overline{A}+\overline{B}+C+\overline{D}$
14	1110	$ABC\overline{D}$	$\overline{A}+\overline{B}+\overline{C}+D$
15	1111	$ABCD$	$\overline{A}+\overline{B}+\overline{C}+\overline{D}$

UNIT-III

Gate-Level Minimization

Map:-

→ The map method gives us a systamatic approch for simplyfying a boolean expresion. The map method first proposed by veitch and modified by karnaugh, Hence it is known as the veitch diagram or the karnaugh map (k-map)

One-Variable, two Variable, three Varible and four Variable

Maps:-

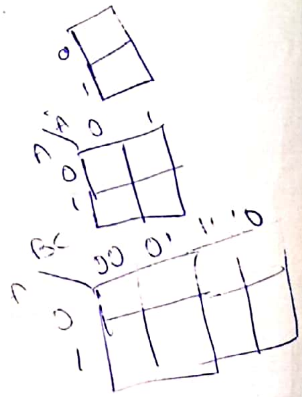
1 Variable $= 2^1 = 2$ cells

2 Variable $= 2^2 = 4$ cells

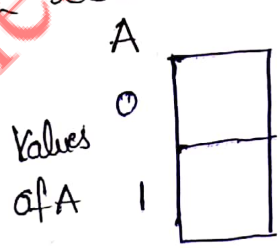
3 Variable $= 2^3 = 8$ cells

4 Variable $= 2^4 = 16$ cells

1 var $= 2^1 = 2$ cells
2 var $= 2^2 = 4$ cells
3 var $= 2^3 = 8$ cells
4 variable $2^4 = 16$ cells

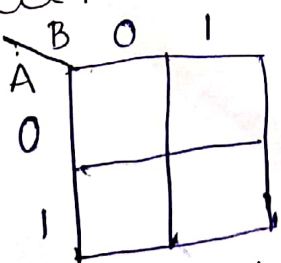


1 Variable K-Map:-



1-Variable Map (2cells)

2-Variable K-Map:-



2-Variable Map (4 cells)

3-Variable K-map:

A \ BC	00	01	11	10
0				
1				

3-Variable K-map (8 cells)

4-Variable K-map:

AB \ CD	00	01	11	10
00				
01				
11				
10				

Values of CD In gray code

4-Variable K-map (16 cells)

1-Variable:

0	$\bar{A}$	$\bar{A}$ 0
1	A	A 1

2-Variable:

A \ B		$\bar{B}$	B
0	$\bar{A}$	$\bar{A}\bar{B}$ 0	$\bar{A}B$ 1
1	A	$A\bar{B}$ 2	AB 3

3-Variable:

A \ BC		$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
0	$\bar{A}$	$\bar{A}\bar{B}\bar{C}$ 0	$\bar{A}\bar{B}C$ 1	$\bar{A}BC$ 3	$\bar{A}B\bar{C}$ 2
1	A	$A\bar{B}\bar{C}$ 4	$A\bar{B}C$ 5	ABC 7	$AB\bar{C}$ 6

4-Variable

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	$\bar{A}\bar{B}\bar{C}\bar{D}$ 0	$\bar{A}\bar{B}\bar{C}D$ 1	$\bar{A}\bar{B}CD$ 3	$\bar{A}\bar{B}C\bar{D}$ 2
$\bar{A}B$	$\bar{A}B\bar{C}\bar{D}$ 4	$\bar{A}B\bar{C}D$ 5	$\bar{A}BCD$ 7	$\bar{A}BC\bar{D}$ 6
AB	$AB\bar{C}\bar{D}$ 12	$AB\bar{C}D$ 13	$ABCD$ 15	$ABC\bar{D}$ 14
$A\bar{B}$	$A\bar{B}\bar{C}\bar{D}$ 8	$A\bar{B}\bar{C}D$ 9	$A\bar{B}CD$ 11	$A\bar{B}C\bar{D}$ 10

Representation of Truth table on k-map

two-Variable k-map

→ The representation of two variable truth table on k-map is given below

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

A \ B	0	1
0	0	1
1	1	0

OR

A \ B	$\bar{B}$	B
$\bar{A}$	0	1
A	1	0

Three-Variable k-map:

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

A \ BC	00	01	11	10
0	0 (0)	0 (1)	0 (3)	1 (2)
1	1 (4)	1 (5)	1 (7)	0 (6)

OR

A \ BC	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	0 (0)	0 (1)	0 (3)	1 (2)
A	1 (4)	1 (5)	1 (7)	0 (6)

four-Variable K-map:

A	B	C	D	y
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1

AB \ CD	00	01	11	10
00	0000 1 (0)	0001 0 (1)	0011 1 (3)	0010 0 (2)
01	0100 1 (4)	0101 1 (5)	0111 1 (7)	0110 0 (6)
11	1100 1 (12)	1101 0 (13)	1111 1 (15)	1110 1 (14)
10	1000 0 (8)	1001 0 (9)	1011 0 (11)	1010 1 (10)

AB \ CD	$\bar C\bar D$	$\bar C D$	CD	$C\bar D$
$\bar A\bar B$	1 (0)	0 (1)	1 (3)	0 (2)
$\bar A B$	1 (4)	1 (5)	1 (7)	0 (6)
AB	1 (12)	0 (13)	1 (15)	1 (14)
$A\bar B$	0 (8)	0 (9)	0 (11)	1 (10)

Ex:- Plot Boolean Expression $Y = AB\bar{C} + ABC + \bar{A}\bar{B}C$ on the k-map

A	B	C	y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

A \ BC	00	01	11	10
0	0 (0)	1 (1)	0 (3)	0 (2)
1	0 (4)	0 (5)	1 (7)	1 (6)

Ex:- plot Boolean expression $y = \bar{A}B\bar{C}\bar{D} + A\bar{B}C\bar{D} + \bar{A}BC\bar{D} + A\bar{B}CD + AB\bar{C}D$ on the k-map.

$$y = \bar{A}B\bar{C}\bar{D} + A\bar{B}C\bar{D} + \bar{A}BC\bar{D} + A\bar{B}CD + AB\bar{C}D$$

$$= 0100 + 1010 + 0110 + 1011 + 1101$$

$$= m_4 + m_{10} + m_6 + m_{11} + m_{13}$$

0100+1010+

AB \ CD	00	01	11	10
00	0000 0	0001 1	0011 3	0010 2
01	0001 1 4	0101 5	0111 7	0110 1 6
11	1100 12	1101 1 13	1111 15	1110 14
10	1000 8	1001 9	1011 1 11	1010 1 10

$\bar{A}BC\bar{D}$

$\bar{A}B\bar{C}\bar{D}$

$AB\bar{C}D$ $\quad A\bar{B}CD$ $\quad A\bar{B}C\bar{D}$

Representation of standard pos on k-map

Three variable k-map:-

A	B	C	Maxterm (M)
0	0	0	$A+B+C$ (M_0)
0	0	1	$A+B+\bar{C}$ (M_1)
0	1	0	$A+\bar{B}+C$ (M_2)
0	1	1	$A+\bar{B}+\bar{C}$ (M_3)
1	0	0	$\bar{A}+B+C$ (M_4)
1	0	1	$\bar{A}+B+\bar{C}$ (M_5)
1	1	0	$\bar{A}+\bar{B}+C$ (M_6)
1	1	1	$\bar{A}+\bar{B}+\bar{C}$ (M_7)

Foure Variable K-map :-

A	B	C	D	Maxterm (M)
0	0	0	0	$A+B+C+D$ (M_0)
0	0	0	1	$A+B+C+\bar{D}$ (M_1)
0	0	1	0	$A+B+\bar{C}+D$ (M_2)
0	0	1	1	$A+B+\bar{C}+\bar{D}$ (M_3)
0	1	0	0	$A+\bar{B}+C+D$ (M_4)
0	1	0	1	$A+\bar{B}+C+\bar{D}$ (M_5)
0	1	1	0	$A+\bar{B}+\bar{C}+D$ (M_6)
0	1	1	1	$A+\bar{B}+\bar{C}+\bar{D}$ (M_7)
1	0	0	0	$\bar{A}+B+C+D$ (M_8)
1	0	0	1	$\bar{A}+B+C+\bar{D}$ (M_9)
1	0	1	0	$\bar{A}+B+\bar{C}+D$ (M_{10})
1	0	1	1	$\bar{A}+B+\bar{C}+\bar{D}$ (M_{11})
1	1	0	0	$\bar{A}+\bar{B}+C+D$ (M_{12})
1	1	0	1	$\bar{A}+\bar{B}+C+\bar{D}$ (M_{13})
1	1	1	0	$\bar{A}+\bar{B}+\bar{C}+D$ (M_{14})
1	1	1	1	$\bar{A}+\bar{B}+\bar{C}+\bar{D}$ (M_{15})

Ex:- plot Boolean Expression $y=(A+\bar{B}+C)(A+\bar{B}+\bar{C})(\bar{A}+\bar{B}+C)(A+B+\bar{C})$ on the K-map.

$y = M_2 \cdot M_3 \cdot M_6 \cdot M_1$

[Minterm ki "1" represent
maxter "0" represent]

A \ BC	00	01	11	10
0	000 0	001 1 0	011 3 0	010 2 0
1	100 4	101 5	111 7	110 6 0

$A+\bar{B}+\bar{C}$

$A+\bar{B}+C$

$\bar{A}+\bar{B}+C$

$A+B+\bar{C}$

Ex-3) plot Boolean Expression $y = (A+B+C+\bar{D})(A+\bar{B}+\bar{C}+D)(A+B+\bar{C}+\bar{D})$
$(\bar{A}+\bar{B}+C+\bar{D})(\bar{A}+\bar{B}+\bar{C}+D)$

$$Y = (A+B+C+\bar{D})\cdot(A+\bar{B}+\bar{C}+D)(A+B+\bar{C}+\bar{D})(\bar{A}+\bar{B}+C+\bar{D})(\bar{A}+\bar{B}+\bar{C}+D)$$

$$Y = M_1 \cdot M_6 \cdot M_3 \cdot M_{13} \cdot M_{14}$$

$(A+B+C+\bar{D})$ $A+B+\bar{C}+\bar{D}$

AB \ CD	00	01	11	10
00	0000 0	0001 1 0	0011 3 0	0010 2
01	0100 4	0101 5	0111 7	0110 6 0
11	1100 12	1101 13 0	1111 15	1110 14 0
10	1000 8	1001 9	1011 11	1010 10

$(A+\bar{B}+\bar{C}+D)$

$(\bar{A}+\bar{B}+\bar{C}+D)$

$\bar{A}+\bar{B}+C+\bar{D}$

Grouping cells for simplification:

Grouping two adjcent ones (pair)

Ex: $y = \bar{A}\bar{B}C + \bar{A}BC$ $[B+\bar{B}=1]$

$= \bar{A}C(B+\bar{B})$

$= \bar{A}C$

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	0	1	1	0
A 1	0	0	0	0

$\bar{A}C$

(or)

A	B	C
0	0	1
0	1	1

$\bar{A}C$

Ex-2 $y = \bar{A}BC + ABC = BC$

$= BC(A+\bar{A})$

$= BC$

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	0	0	1	0
A 1	0	0	1	0

BC

Ex:③ $Y = A\bar{B}\bar{C} + AB\bar{C}$

$= A\bar{C}(\bar{B}+B)$

$= A\bar{C}$

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	0	0	0	0
A 1	1	0	0	1

$A\bar{C}$

Ex:④ $Y = \bar{A}\bar{B}\bar{C}D + A\bar{B}\bar{C}D$

$= \bar{B}\bar{C}D(A+\bar{A})$

$= \bar{B}\bar{C}D$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0	1	0	0
$\bar{A}B$ 01	0	0	0	0
AB 11	0	0	0	0
$A\bar{B}$ 10	0	1	0	0

$\bar{B}\bar{C}D$

Ex:⑤ $Y = \bar{A}\bar{B}C + \bar{A}BC + ABC$

$= \bar{A}\bar{B}C + \bar{A}BC + \bar{A}BC + ABC$

$= \bar{A}C(\bar{B}+B) + BC(\bar{A}+A)$

$= \bar{A}C + BC$

Group A → $\bar{A}C$

A \ BC	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	0	1	1	0
A	0	0	1	0

Group2 → BC

A B C

1

(OR)

Grouping four adjacent ones (Quad)

Ex:①

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	0	0	0	0
A 1	1	1	1	1

A

② $Y = A$

Ex:-2

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0	0	1	0
$\bar{A}B$ 01	0	0	1	0
AB 11	0	0	1	0
$A\bar{B}$ 10	0	0	1	0

CD

(b) $y = CD$

Ex:-③

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$	0	0	0	0
$\bar{A}B$	0	1	1	0
AB	0	1	1	0
$A\bar{B}$	0	0	0	0

BD

③ $y = BD$

Ex:-④

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0	0	0	0
$\bar{A}B$ 01	0	0	0	0
AB 11	1	0	0	1
$A\bar{B}$ 10	1	0	0	1

$A\bar{D}$

$y = A\bar{D}$

Ex:- 5

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1	0	0	1
$\bar{A}B$ 01	0	0	0	0
AB 11	0	0	0	0
$A\bar{B}$ 10	1	0	0	1

$\bar{B}\bar{D}$

A B C D
0 0 0 0
1 0 0 0
$\bar{B}\bar{C}\bar{D}$

A B C D
0 0 1 0
1 0 1 0
$\bar{B}C\bar{D}$

$= \bar{B}\bar{C}\bar{D} + \bar{B}C\bar{D}$

$= \bar{B}\bar{D}(\bar{C}+C)$

Ans:- $Y = \bar{B}\bar{D}$

Ex:-6

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0	0	0	0
$\bar{A}B$ 01	0	0	0	0
AB 11	1	1	1	1
$A\bar{B}$ 10	0	1	1	1

Group1 → AB

Group 3 → AC

Group 2 ⇒ AD

$\therefore Y = AB + AD + AC$

Grouping Eight Adjacent ones (Octet)

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0	0	0	0
$\bar{A}B$ 01	1	1	1	1
AB 11	1	1	1	1
$A\bar{B}$ 10	0	0	0	0

B

(a) $Y = B$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0	1	1	0
$\bar{A}B$ 01	0	1	1	0
AB 11	0	1	1	0
$A\bar{B}$ 10	0	1	1	0

D

(b) $Y = D$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1	1	1	1
$\bar{A}B$ 01	0	0	0	0
AB 11	0	0	0	0
$A\bar{B}$ 10	1	1	1	1

$\bar{B}$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1	0	0	1
$\bar{A}B$ 01	1	0	0	1
AB 11	1	0	0	1
$A\bar{B}$ 10	1	0	0	1

$\bar{D}$

A	B	C	D
0	0	0	0
0	0	0	1
0	0	1	1
0	0	1	0
1	0	0	0
1	0	0	1
1	0	1	1
1	0	1	0

$\bar{B}$

A	B	C	D
0	0	0	0
0	1	0	0
1	1	0	0
1	0	0	0
0	0	1	0
0	1	1	0
1	1	1	0
1	0	1	0

$\bar{D}$

Simplifications of SOP Expressions

→ from the above discussion we can outline generalized procedure to simplify Boolean Expressions as follows:

1. plot the k-map and place 1s in those cells corresponding to the 1s in the truth table or sum of product expression. place 0s in other cells.

2. check the k-map for adjacent 1s and encircle those 1's which are not adjacent to any other 1s. these are called isolated 1s.

3. Check for those 1's which are adjacent to only one other 1 and encircle such pairs.

4. Check for quads and octets of adjacent 1s even if it contains some 1s that have already bee encircled. While doing this make sure that there are minimum no. of groups.

5. Combaine any pairs necessary to include any 1s that have not yet been grouped.

6. From the simplified expresion by summing product terms of all groups.

Ex: Minimize the expresion $Y = A\bar{B}C + \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C}$

Solution:

$Y = 101 + 001 + 011 + 100 + 000$

$= m_5 + m_1 + m_3 + m_4 + m_0$

$= \Sigma m(5, 3, 4, 0)$

Step 1: The k-map for three variables and it is ploted according to the given expression.

AB \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
0	000 1 (0)	001 1 (1)	011 1 (3)	010 (2)
1	100 1 (4)	101 1 (5)	111 (7)	110 (6)

Step 2: There are no isolated 1s

Step 3: 1 in the cell 3 adjacent only to 1 in the cell 1. This pair is combained and refered to as group 1

Step 4: There is no octet, but there is a quad cells 0, 1, 4 and 5 from a quard. This quad is combained and reffered to as a group.

AB \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
0	1	1	1	0
1	1	1	0	0

Step 5: All 1's have alredy been grouped

Step 6: Each group generates a term in expression for y. In group 1 B variable is eliminated and in group 2 variables A and C are eliminated and we get

$A \backslash BC$	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	1	1	1	0
A 1	1	1	0	0

$\bar{A}C$

$\bar{B}$

$$y = \bar{A}C + \bar{B}$$

Ex: minimize the expression

$$y = \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}D + AB\bar{C}\bar{D} + AB\bar{C}D + A\bar{B}\bar{C}D + \bar{A}\bar{B}C\bar{D}$$

$$= 0100 + 0101 + 1100 + 1101 + 1001 + 0010$$

$$= m_4 + m_5 + m_{12} + m_{13} + m_9 + m_2$$

$$= \Sigma m(4,5,12,13,9,2)$$

$AB \backslash CD$	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0	1	3	2 (1)
$\bar{A}B$ 01	4 (1)	5 (1)	7	6
AB 11	12 (1)	13 (1)	15	14
$A\bar{B}$ 10	8	9 (1)	11	10

Group 3: $\bar{A}\bar{B}C\bar{D}$

Group 1: $B\bar{C}$

Group 2: $A\bar{C}D$

G1

A	B	C	D
0	1	0	0
0	1	0	1
1	1	0	0
1	1	0	1

$B\bar{C}$

G2

A	B	C	D
1	1	0	1
1	0	0	1

$A\,\bar{C}\,D$

G3

ABCD
0010

$\bar{A}\bar{B}C\bar{D}$

$$Y = \bar{A}\bar{B}C\bar{D} + A\bar{C}D + B\bar{C}$$

* Simplify the logic function specified by the truth table by using the Kanaugh map method. Y is the output variable and A, B and C are the input variables

A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

$Y = \bar{A}\bar{B}\bar{C} + \bar{A}BC + A\bar{B}\bar{C} + ABC$

$= m_0 + m_3 + m_4 + m_7$

$= \Sigma m(0,3,4,7)$

A \ BC	00	01	11	10
0	0: 1	1	3: 1	2
1	4: 1	5	7: 1	6

Group 1 $\bar{B}\bar{C}$ Group 2 BC

Group 1

A	B	C
0	0	0
1	0	0

$\bar{B}\bar{C}$

Group 2

A	B	C
0	1	1
1	1	1

BC

$\therefore Y = \bar{B}\bar{C} + BC$

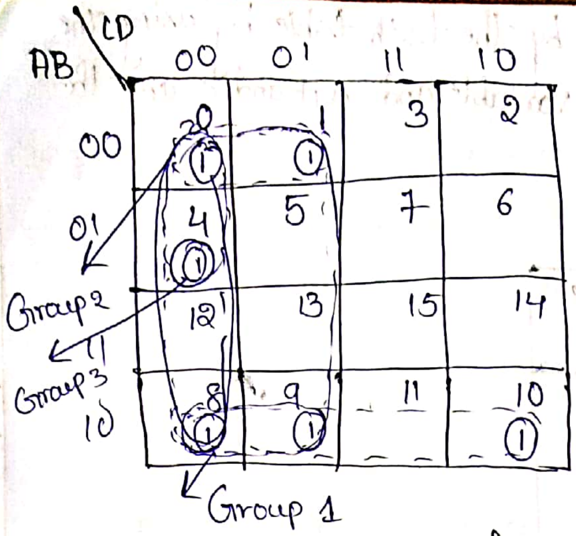
Ex: Reduce the following function using K-map technique.

$\bar{A}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{D} + \bar{C}$

$f(A,B,C,D) = \Sigma m(0,1,4,8,9,10)$

$= m_0 + m_1 + m_4 + m_8 + m_9 + m_{10}$

$= \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}B\bar{C}\bar{D} + A\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}D + A\bar{B}C\bar{D}$



Group 1

A	B	C	D
1	0	0	0
1	0	1	0

$A\bar{B}\bar{D}$

Group 2

A	B	C	D
0	0	0	0
0	0	0	1
1	0	0	0
1	0	0	1

$\bar{B}\bar{C}$

Group 3

A	B	C	D
0	1	0	0

$\bar{A}B\bar{C}\bar{D}$

$$Y = A\bar{B}\bar{D} + \bar{B}\bar{C} + \bar{A}B\bar{C}\bar{D}$$

Product of Sum simplification:

1. plot the k-map and place 0s in those cells corresponding to the 0s in the truth table or maxterms in the product of sum expression
2. check the k-map for adjacent 0s and Encircle those 0s which are not adjacent to any other 0s. These are called isolated 0s.
3. Check for those 0s which are adjacent to only one other 0 and Encircle such pairs.
4. check for quads and octets of adjacent 0s even if it contains some 0s that have already been Encircled. while doing this make sure that there are minimum no. of groups.
5. Combine any pairs necessary to include any 0s that have not yet been grouped.
6. From the simplified pos Expression for F by taking product of sum terms of all the groups

To get familiar with these steps we will solve some Examples.

Ex:- minimize the Expression

$$Y = (A+B+\bar{C})(A+\bar{B}+\bar{C})(\bar{A}+\bar{B}+\bar{C})(\bar{A}+B+C)(A+B+C)$$

$= \cancel{(001)(011)}$

$= (A+B+\bar{C}) = M_1,\ (A+\bar{B}+\bar{C}) = M_3,\ (\bar{A}+\bar{B}+\bar{C}) = M_7$

$(\bar{A}+B+C) = M_4,\ (A+B+C) = M_0$

Step 1: (a) shows the k-map for three variable and it is plotted according to given maxterms

A \ BC	00	01	11	10
0	0 (0)	0 (1)	0 (3)	(2)
1	0 (4)	(5)	0 (7)	(6)

Step 2: three are no isolated 0s

Step 3: 0 in the cell 4 is adjacent only to 0 in the cell 0 and 0 in cell 7 is adjacent only to 0 in the cell 3. These two pairs are combined and refered to as group 1 and group 2 respectively.

A \ BC	00	01	11	10
0	0 (0)	0 (1)	0 (3)	(2)
1	0 (4)	(5)	0 (7)	(6)

Step 4: There are no quads and octets

Step 5: The 0 in the cell 1 can be combained with 0 in the cell 3 to be from a pair. This pair is referred to as group 3

A \ BC	00	01	11	10
0	0 (0)	0 (1)	0 (3)	(2)
1	0 (4)	(5)	0 (7)	(6)

Step 6: In group 1 and in group 2, A is eliminated, where in group 3 variable B is eliminated and we get

Group 1

A	B	C
0	0	0
1	0	0
	$\bar{B}\bar{C}$	

Group 2

A	B	C
0	1	1
1	1	1
	BC	

Group 3

A	B	C
0	0	1
0	1	1
	$\bar{A}C$	

$$\bar{Y} = B\bar{C} + BC + \bar{A}C$$

$$\bar{\bar{Y}} = Y = \overline{\bar{B}\bar{C} + BC + \bar{A}C}$$ [According to Demorgans]

$$Y = \overline{\bar{B}\bar{C} + BC + \bar{A}C}$$

$$= (\overline{\bar{B}\bar{C}}) \cdot (\overline{BC}) \cdot (\overline{\bar{A}C})$$

$$= (\bar{\bar{B}} + \bar{\bar{C}}) \cdot (\bar{B} + \bar{C}) \cdot (\bar{\bar{A}} + \bar{C})$$

$$Y = (B+C)(\bar{B}+\bar{C})(A+\bar{C})$$

Ex: Minimize the following Expression in the pos form

$$X = (\bar{A}+\bar{B}+C+D)(\bar{A}+\bar{B}+\bar{C}+D)(\bar{A}+\bar{B}+\bar{C}+\bar{D})(\bar{A}+B+C+D)(A+\bar{B}+\bar{C}+D)$$
$$(A+\bar{B}+\bar{C}+\bar{D})(A+B+C+D)(\bar{A}+\bar{B}+C+\bar{D})$$

Sol: $(\bar{A}+\bar{B}+C+D) = M_{12}$, $(\bar{A}+\bar{B}+\bar{C}+D) = M_{14}$, $(\bar{A}+\bar{B}+\bar{C}+\bar{D}) = M_{15}$

$(\bar{A}+B+C+D) = M_8$, $(A+\bar{B}+\bar{C}+D) = M_6$, $(A+\bar{B}+\bar{C}+\bar{D}) = M_7$

$(A+B+C+D) = M_0$ and $(\bar{A}+\bar{B}+C+\bar{D}) = M_{13}$

AB \ CD	$C+D$ 00	$C+\bar{D}$ 01	$\bar{C}+\bar{D}$ 11	$\bar{C}+D$ 10
$A+B$ 00	0: 0	1	3	2
$A+\bar{B}$ 01	4	5	7: 0	6: 0
$\bar{A}+\bar{B}$ 11	12: 0	13: 0	15: 0	14: 0
$\bar{A}+B$ 10	8: 0	9	11	10

Step 1: shows the k-map for four Variable and it plotted according to given max terms.

Step 2: There are no isolated 1's

Step 3: 0 in the cell 0 is adjacent Only to in the cell this pair is combined and refferd to as group 1

Step 4: There are two quads cells 12, 13, 14 and 15 form a quad 1 and cells 6, 7, 14, 15 forms a quad 2. These two quads are reffered to as group 2 and group 3, respectively

Step 5: All 0s have already been grouped

Step 6: In group 1, Variable A is eliminated in group 2, Variable C and D are eliminated and 01 in group 3 variable A and D are Eliminated.

∴ we get simplified pos Expression

Group 1

AB \ CD	00	01	11	10
00	0: 0	1	3	2
01	4	5	7: 0	6: 0
11	12: 0	13: 0	15: 0	14: 0
10	8: 0	9	11	10

Group 2 Group 3

Group 1

A	B	C	D
0	0	0	0
1	0	0	0

$\bar{B}\,\bar{C}\,\bar{D}$

Group 2

A	B	C	D
1	1	0	0
1	1	0	1
1	1	1	1
1	1	1	0

A B

Group 3

A	B	C	D
0	1	1	1
0	1	1	0
1	1	1	1
1	1	1	0

B C

$$\overline{Y} = \overline{B}\,\overline{C}\,\overline{D} + AB + BC$$

$$Y = \overline{\overline{Y}} = \overline{\overline{B}\,\overline{C}\,\overline{D} + AB + BC}$$

$$Y = (B+C+D)\cdot(\overline{A}+\overline{B})\cdot(\overline{B}+\overline{C})$$

Incompletely specified functions (Don't Care terms (or) conditions)

Ex:

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	X
1	1	1	X

Sol:

A \ BC	00	01	11	10
0	(0)	(1) 1	(3) 1	(2)
1	(4)	(5) 1	(7) X	(6) X

Don't Care Conditions

A \ BC	00	01	11	10
0	(0)	(1) 1	(3) 1	(2)
1	(4)	(5) 1	(7) 1	(6) 0

Group 1

Group 1

A	B	C
0	0	1
0	1	1
1	0	1
1	1	1
		C

$$\boxed{\therefore Y = C}$$

Describing Incomplete Boolean function

Ex: (i) $f(A,B,C) = \Sigma m(0,2,4) + d(1,5)$

(ii) $f(A,B,C) = \Pi M(2,5,7) + d(1,3)$

(i) $f(A,B,C) = \Sigma m(0,2,4) + d(1,5)$

A \ BC	00	01	11	10
0	(0) 1	(1) X	(3)	(2) 1
1	(4) 1	(5) X	(7)	(6)

A \ BC	00 ($\bar{B}\bar{C}$)	01 ($\bar{B}C$)	11 (BC)	10 ($B\bar{C}$)
$\bar{A}$ 0	1	1		1
A 1	1	1		

Group 1, ~~Group 2~~, Group 3

G_1

A B C
0 0 0
1 0 0

$\bar{B}\ \bar{C}$

G_2

A B C
0 0 0
0 0 1

$\bar{A}\ \bar{B}$

$y = \cancel{\bar{B}\bar{C} + \bar{A}\bar{B} + B\bar{C}}$ //

Group 1

A B C
0 0 0
0 0 1
1 0 0
1 0 1

$\bar{B}$

$y = \bar{B} + B\bar{C}$

(ii) $f(A,B,C) = \Pi M(2,5,7) + d(1,3)$

A \ BC	00	01	11	10
0	(0)	(1) X	(3) X	(2) 0
1	(4)	(5) 0	(7) 0	(6)

A \ BC	00	01	11	10
0	(0)	(1) 0	(3) 0	(2) 0
1	(4)	(5) 0	(7) 0	(6)

Group 1, Group 2, Group 3

G_1

A B C
0 0 1
1 0 1
1 0 1

BC

G_2

A B C
1 0 1
1 1

AC

G_3

A B C
0 1 0

$\bar{A}B\bar{C}$

G_1

A B C
0 0 1
0 1 1
1 0 1
1 1 1

C

$y = \bar{A}B\bar{C} + C$ //

Don't care conditions in logic Design

	A	B	C	D	P
0	0	0	0	0	0
1	0	0	0	1	1
2	0	0	1	0	1
3	0	0	1	1	0
4	0	1	0	0	1
5	0	1	0	1	0
6	0	1	1	0	0
7	0	1	1	1	1
8	1	0	0	0	1
9	1	0	0	1	0
10	1	0	1	0	–
11	1	0	1	1	–
12	1	1	0	0	–
13	1	1	0	1	–
14	1	1	1	0	–
15	1	1	1	1	–

$P = \Sigma m(1,2,4,7,8) + d(10,11,12,13,14,15)$

Ex:- find the reduced sop form of the following function.

$f(A,B,C,D) = \Sigma m(1,3,7,11,15) + \Sigma d(0,2,4)$

$f(A,B,C,D) = \Sigma m(1,3,7,11,15) + \Sigma d(0,2,4)$

AB \ CD	00	01	11	10
00	X (0)	1 (1)	1 (3)	X (2)
01	X (4)	(5)	1 (7)	(6)
11	(12)	(13)	1 (15)	(14)
10	(8)	(9)	1 (11)	(10)

AB \ CD	00	01	11	10
00	1	1	1	1
01	0		1	
11			1	
10			1	

Group2

Group1

G_1

A	B	C	D
0	0	1	1
0	1	1	1
1	1	1	1
1	0	1	1

CD

G_2

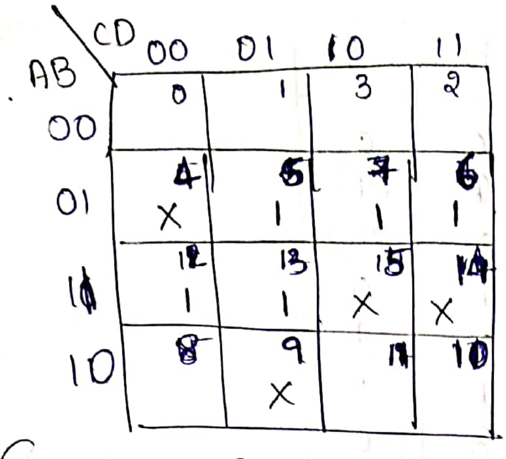
A	B	C	D
0	0	0	0
0	0	0	1
0	0	1	1
0	0	1	0

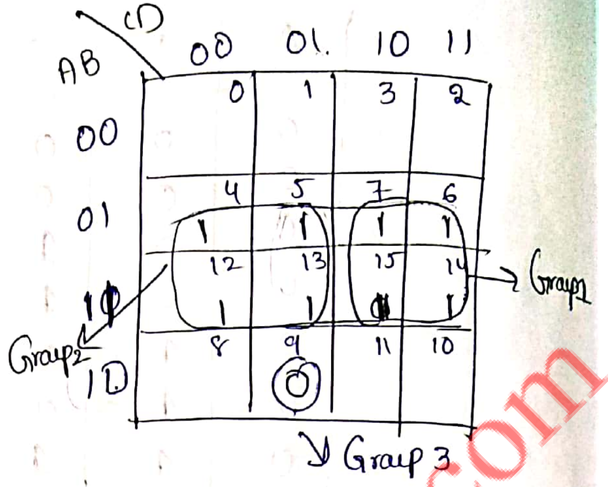
$\bar{A}\bar{B}$

$\therefore Y = \bar{A}\bar{B} + CD$

Ex: Reduce the following function using K-map technique

$f(A,B,C,D) = \Sigma m(5,6,7,12,13) + \Sigma d(4,9,14,15)$





G_1

A	B	C	D
0	1	0	0
0	1	0	1
1	1	0	0
1	1	0	1

$B\bar{C}$

G_2

A	B	C	D
0	1	1	0
0	1	1	1
1	1	1	0
1	1	1	1

BC

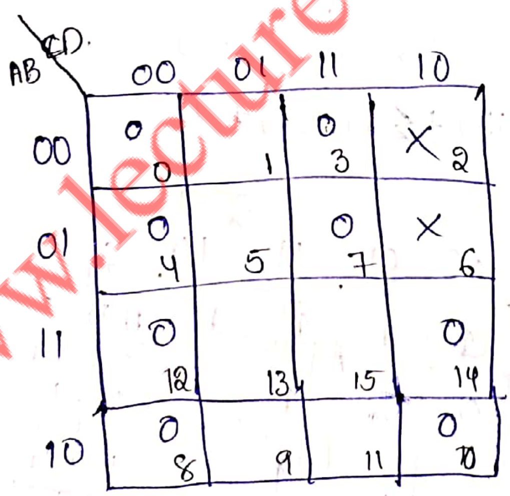
A	B	C	D
1	0	0	1

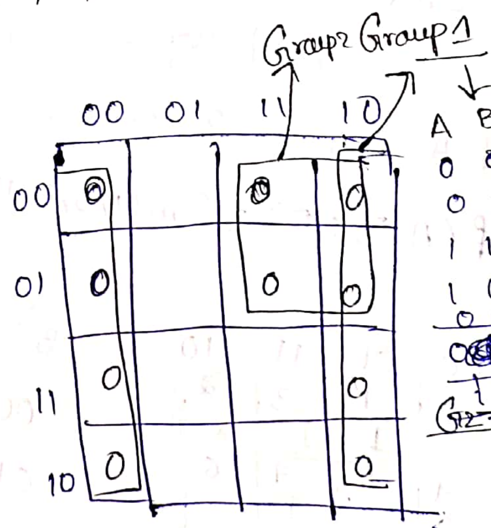
$A\bar{B}\bar{C}D$

$$Y = A\bar{B}\bar{C}D + B\bar{C} + BC$$

Ex: Reduce the following function by using K-map technique

$f(A,B,C,D) = \Pi M(0,3,4,7,8,10,12,14) + d(2,6)$





A	B	C	D
0	0	1	0
0	1	1	0
1	1	1	0
1	0	1	0
0	0	0	0
0	1	0	0
1	0	0	0

$\bar{D}$

G_2

A	B	C	D
0	0	1	1
0	0	1	0
0	1	1	1
0	1	1	0

$\bar{A}C$

$$\bar{Y} = \bar{D} + \bar{A}C$$

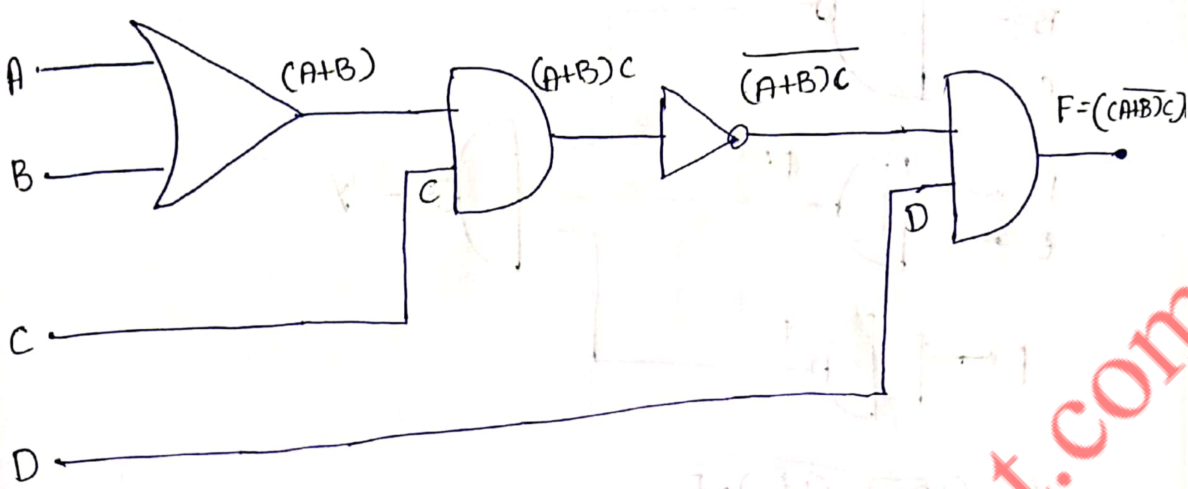
$$Y = \bar{\bar{Y}} = \overline{\bar{D} + \bar{A}C}$$

$$= (\bar{\bar{D}}) \cdot (\overline{\bar{A}C})$$

$$= D \cdot (A + \bar{C})$$

* ~~f(A+B, C+D) =~~

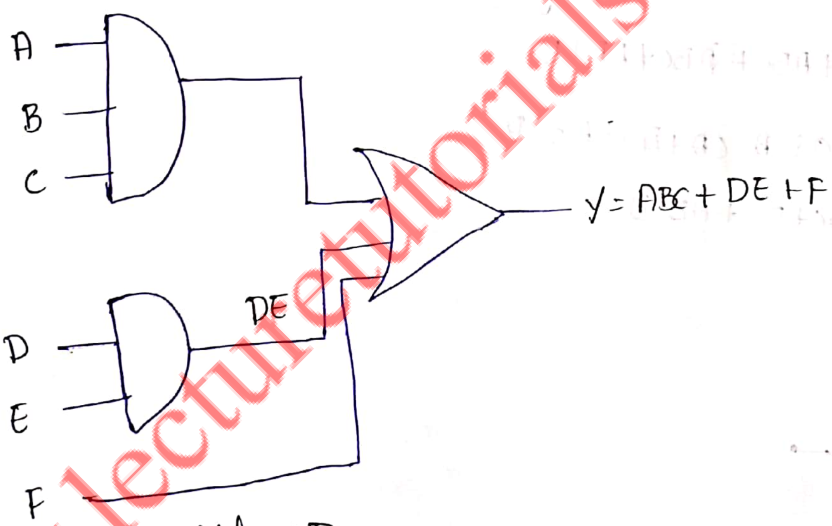
Ex:- $F = (\overline{(A+B)C})D$ to Implement logic Design?



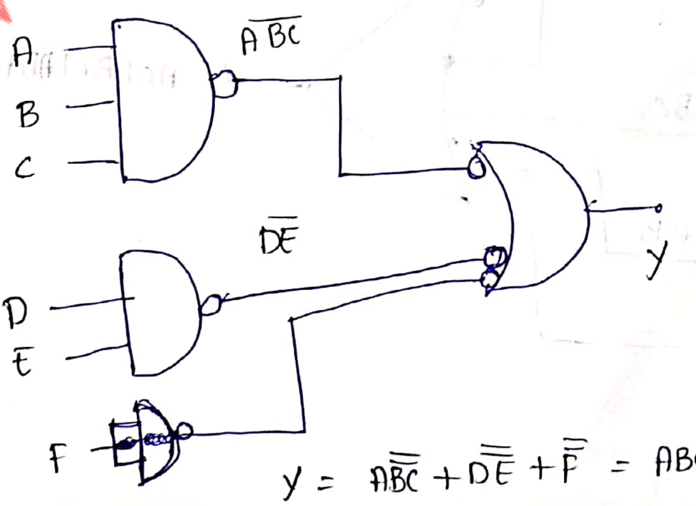
NAND to NAND Implementation:-

$y = ABC + DE + F$

(i) AND - OR

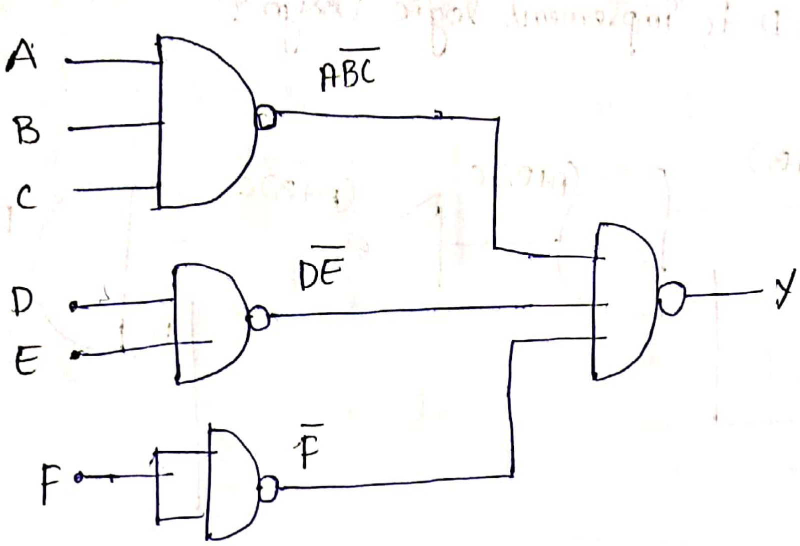


(ii) NAND - Bubble OR



$y = \overline{\overline{ABC}} + \overline{\overline{DE}} + \overline{\overline{F}} = ABC + DE + F$

(iii) NAND – NAND



$$Y = \overline{(\overline{ABC}) \cdot (\overline{DE}) \cdot \overline{F}}$$

$$= (\overline{\overline{ABC}}) + (\overline{\overline{DE}}) + \overline{\overline{F}}$$

$$= ABC + DE + F$$

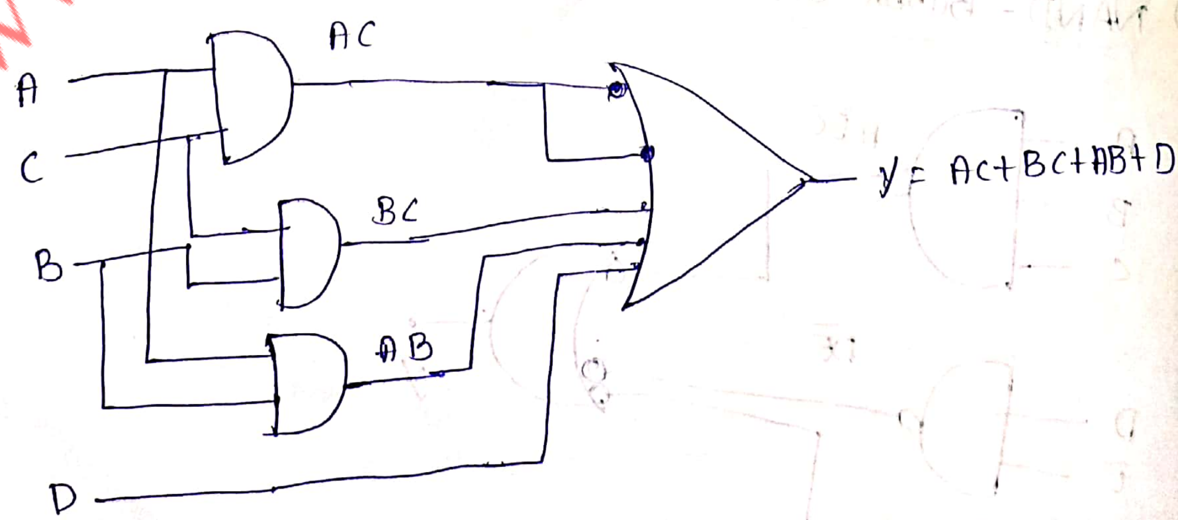
Ex: $Y = AC + ABC + \bar{A}BC + AB + D$ Implement the following Boolean function with NANA to NAND logic.

$$Y = AC + ABC + \bar{A}BC + AB + D$$

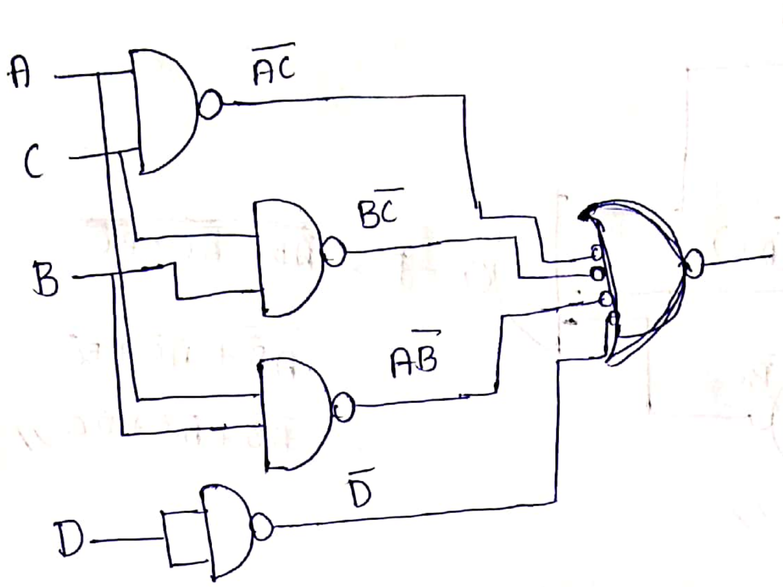
$$= AC + BC(A + \bar{A}) + AB + D$$

$$Y = AC + BC + AB + D$$

(i) AND – OR



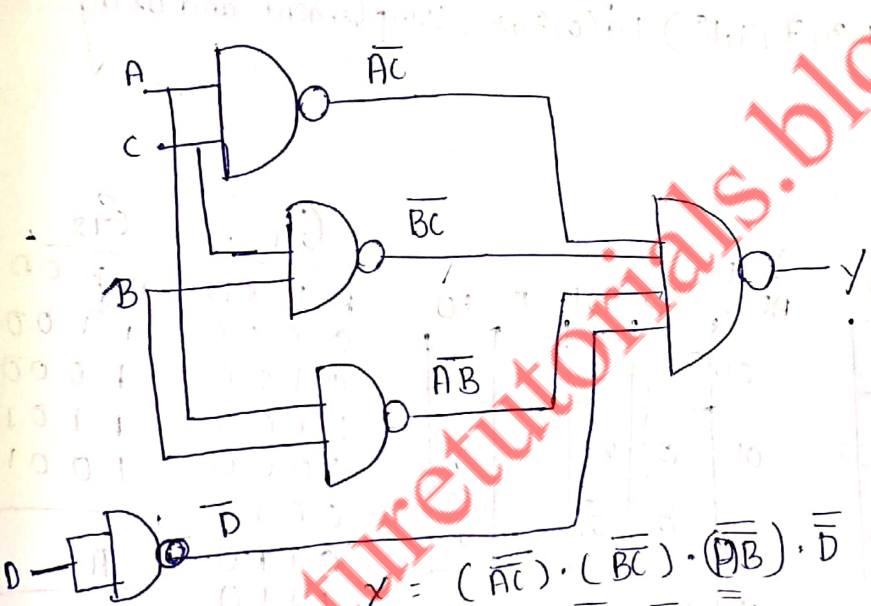
(ii) NAND - Bubble OR



$Y = \overline{\overline{AC}} + \overline{\overline{BC}} + \overline{\overline{AB}} + \overline{\overline{D}}$

$= AC + BC + AB + D$

(iii) NAND - NAND



$Y = \overline{(\overline{AC}) \cdot (\overline{BC}) \cdot (\overline{AB}) \cdot \overline{D}}$

$= \overline{\overline{AC}} + \overline{\overline{BC}} + \overline{\overline{AB}} + \overline{\overline{D}}$

$= AC + BC + AB + D$

Ex: Implementation the following Boolean Expresion with NAND-NAND logic.

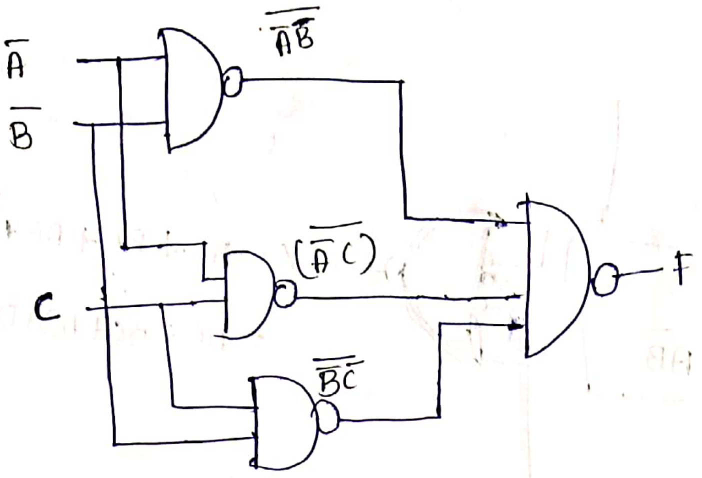
$F(A,B,C) = \Sigma m(0,1,3,5)$

A \ BC	00	01	11	10
0	1 (0)	1 (1)	1 (3)	(2)
1	(4)	1 (5)	(7)	(6)

G1, G2, G3

$F(A,B,C) = \bar{A}\bar{B} + \bar{A}C + \bar{B}C$

NAND - NAND



$F = \overline{\overline{\bar{A}\bar{B}} \cdot \overline{\bar{A}C} \cdot \overline{\bar{B}C}}$

$= \overline{\overline{\bar{A}\bar{B}}} + \overline{\overline{\bar{A}C}} + \overline{\overline{\bar{B}C}}$

$= \bar{A}\bar{B} + \bar{A}C + \bar{B}C$ //

$F = \bar{A}\bar{B} + \bar{A}C + \bar{B}C$ //

Ex:① Find the reduced pos form of the following Equation $F(A,B,C,D) = \Sigma m(1,3,7,11,15) + d(0,2,5)$ Implement and using NAND logic?

AB \ CD	00	01	11	10
00	X $_0$	1 $_1$	1 $_3$	X $_2$
01	0 $_4$	X $_5$	1 $_7$	0 $_6$
11	0 $_{12}$	0 $_{13}$	1 $_{15}$	0 $_{14}$
10	0 $_8$	0 $_9$	1 $_{11}$	0 $_{10}$

AB \ CD	00	01	11	10
00	0			0
01	0	X		0
11	0	0		0
10	0	0		0

Group2 Group1

G_1

A	B	C	D
0	0	0	0
0	1	0	0
1	1	0	0
1	0	0	0
0	0	1	0
0	1	1	0
1	1	1	0
1	0	1	0

$\bar{D}$

G_2

A	B	C	D
1	1	0	0
1	0	0	0
1	1	0	1
1	0	0	1

$A\bar{C}$

$\bar{Y} = \bar{D} + A\bar{C}$

$Y = \bar{\bar{Y}} = (\bar{\bar{D}})(\overline{A\bar{C}})$

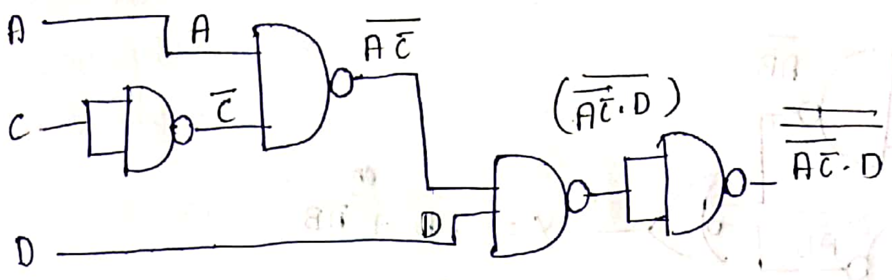
$\therefore Y = D \cdot (\bar{A} + C)$

Ex:② using ~~k-map, Determine the minimal sop Expression~~ and ~~resize the simplified~~ ~~Expression using~~ NAND logic?

~~f(w,x,y,z) = ΠM(0,2,3,7,8,9,10)~~

Sol: ~~f(w,x,y,z) = Σm(1,4,5,6,11,12,13,14,15)~~

$\therefore Y = D\cdot(\bar{A}+C)$



$\overline{AB+\bar{A}B}$

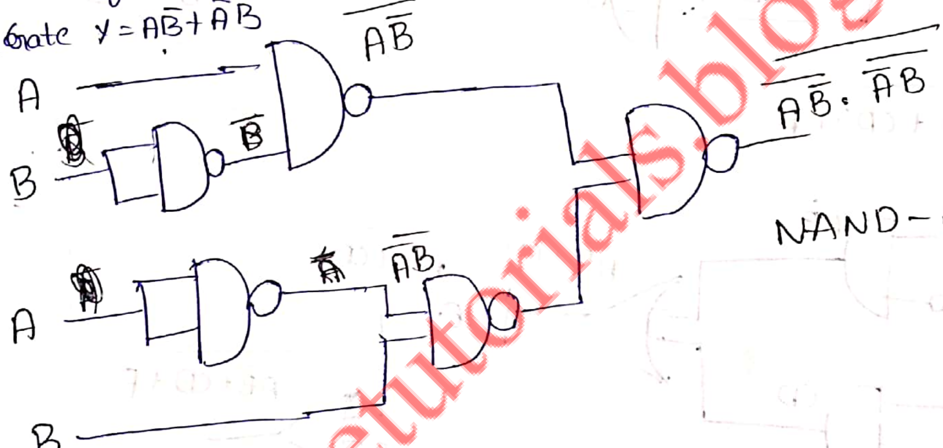
$$Y = (\overline{A\bar{C}}\cdot D)$$
$$= (\bar{A}+\bar{\bar{C}})\cdot D$$
$$Y = (\bar{A}+C)\cdot D \;//$$

Ex:- To Implement X-OR gate by using NAND-NAND logic gate

$$Y = A\bar{B}+\bar{A}B$$

The X-OR gate function is two Inputs and one output. The i/p variables are A and B. The o/p variable Y

X-OR Gate $Y = A\bar{B}+\bar{A}B$

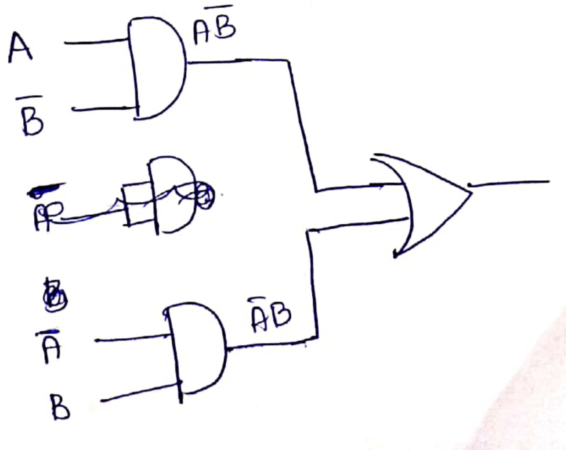


NAND-NAND

$$Y = \overline{\overline{A\bar{B}+\bar{A}B}}$$

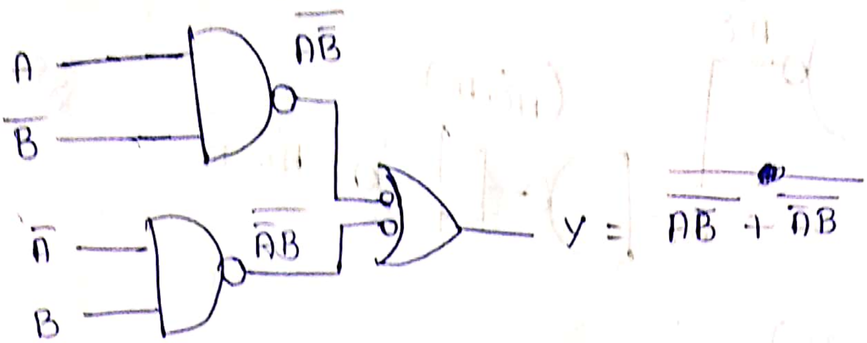
$$Y = A\bar{B}+\bar{A}B \;//$$

(9) AND-OR



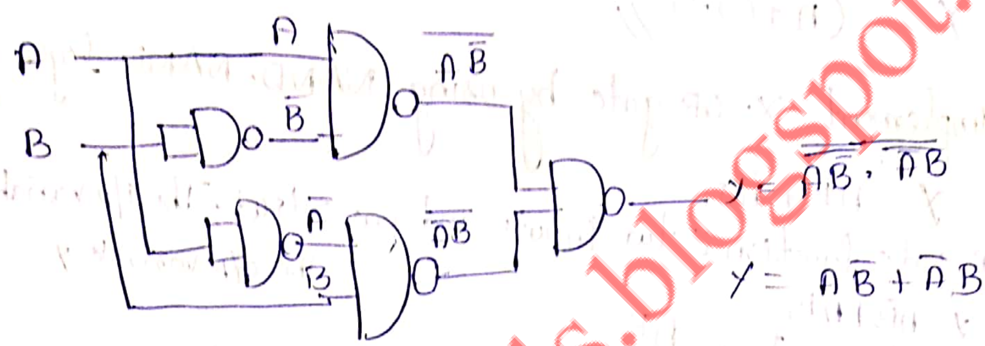
(ii) NAND - Bubbled OR

$Y = A\bar{B} + \bar{A}B$

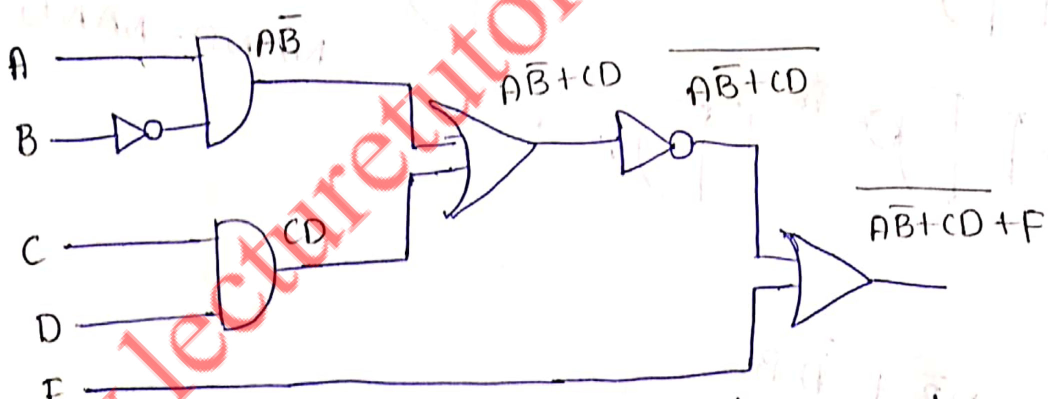


$Y = A\bar{B} + \bar{A}B$

(iii) NAND - NAND



Ex: $\overline{(A\bar{B} + CD)} + F$



Ex: Implement the following function with NAND to NAND logic

$f(A, B, C) = \Sigma m(0, 1, 3, 5)$

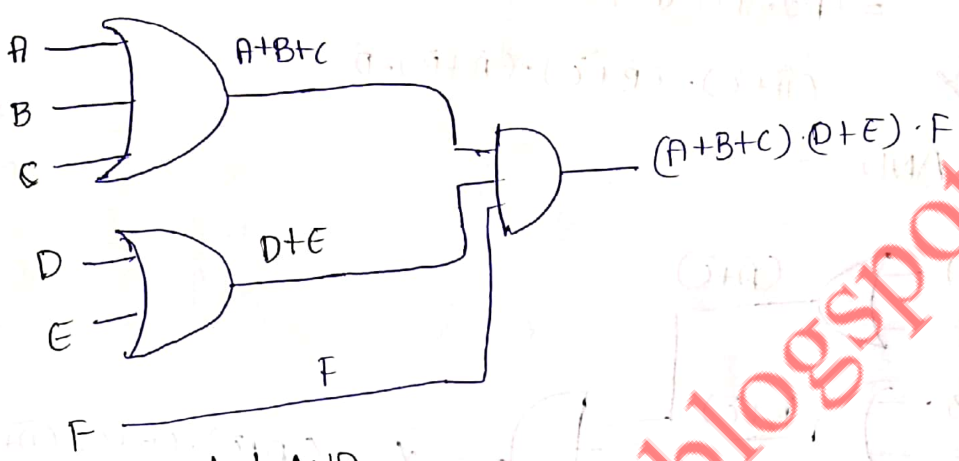
$Y = AC + BC + AB + D$

NOR- NOR Implementation:-

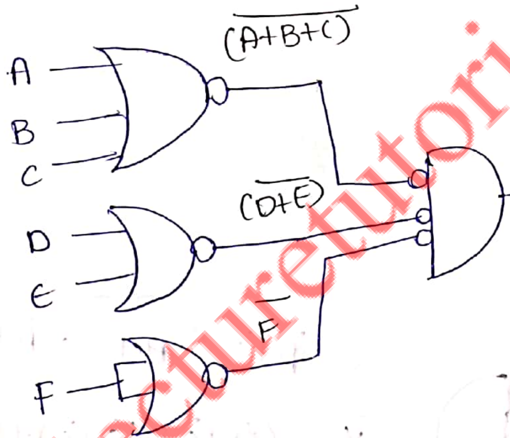
(i) OR – AND
(ii) NOR- Bubbled AND
(iii) NOR- NOR

Ex:- $Y = (A+B+C) \cdot (D+E) \cdot F$

(i) OR- AND



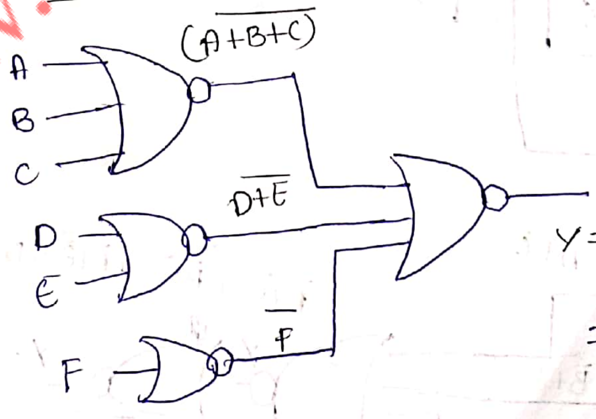
(ii) NOR- Bubbled AND



$Y = \overline{\overline{(A+B+C)}} \cdot \overline{\overline{(D+E)}} \cdot \overline{\overline{F}}$

$= (A+B+C)\,(D+E) \cdot F$

(iii) NOR- NOR



$Y = \overline{\overline{(A+B+C)} + \overline{(D+E)} + \overline{F}}$

$= (A+B+C) \cdot (D+E) \cdot F$

$[\because \overline{A+B} = \overline{A} \cdot \overline{B}]$

$\overline{\overline{A}} = A]$

Ex:- Implement the following Boolean function with NOR-NOR logic

$Y = AC + BC + AB + D$

The given Boolean Expression

$Y = AC + BC + AB + D$

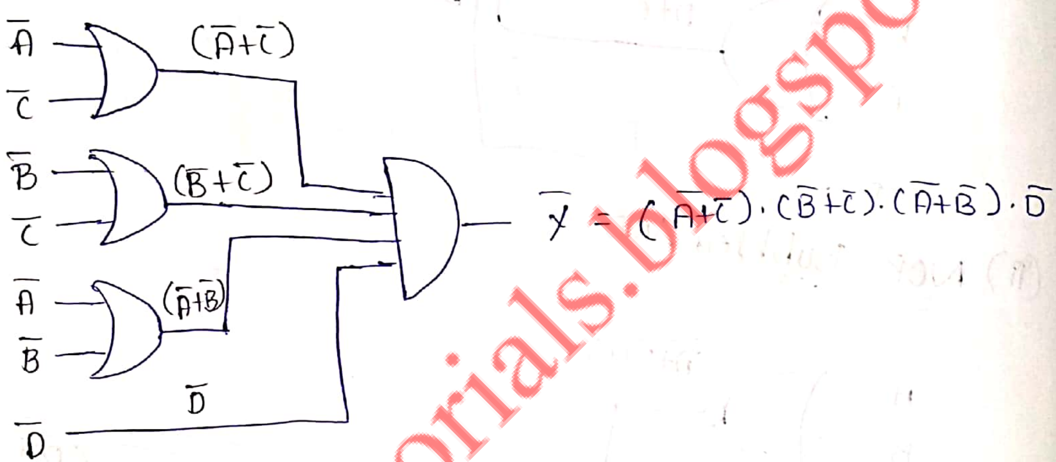
Assumption. By using Duality theorem

$\bar{Y} = \overline{AC + BC + AB + D}$

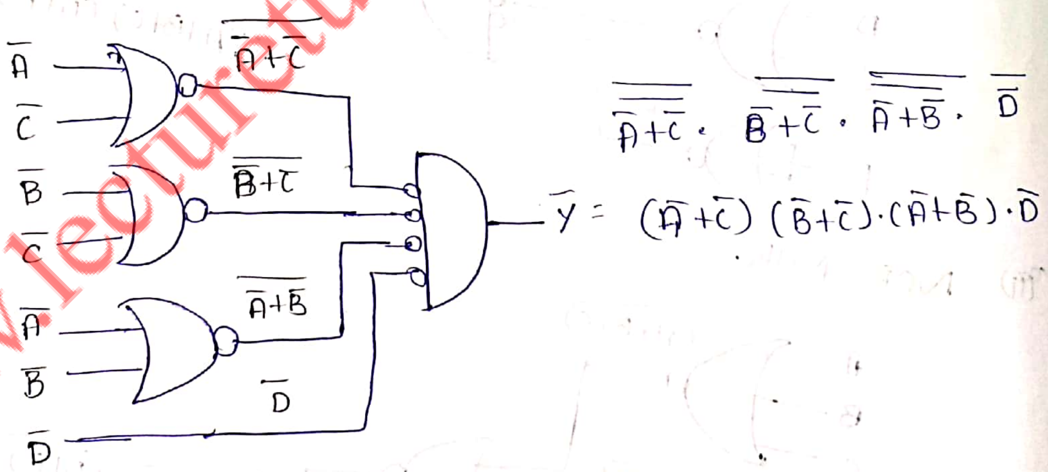
$= (\overline{AC}) \cdot (\overline{BC}) \cdot (\overline{AB}) \bar{D}$

$\bar{Y} = (\bar{A} + \bar{C}) \cdot (\bar{B} + \bar{C}) \cdot (\bar{A} + \bar{B}) \cdot \bar{D}$

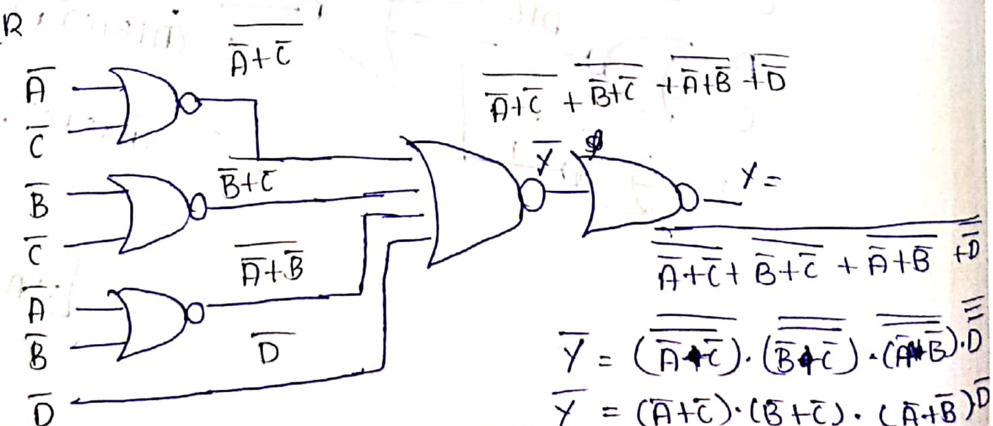
(i) OR-AND



(ii) NOR- Bubbled-AND



(iii) NOR-NOR



$$\overline{\overline{Y}} = \overline{(\bar{A}+\bar{C})}\cdot\overline{(\bar{B}+\bar{C})}\cdot\overline{(\bar{A}+\bar{B})}\cdot\bar{\bar{D}} \qquad \left[\because\ \overline{\bar{A}+\bar{B}} = \bar{\bar{A}}\cdot\bar{\bar{B}} = AB\right]$$

$$Y = AC+BC+AB+D$$

Ex:- Implement the following Boolean function with NOR-NOR logic

$$f(A,B,C) = \Pi_M(0,2,4,5,6)$$

Ex:- Implement the X-NOR gate by using only NOR gate $(AB+\bar{A}\bar{B})$

Ex:-① $f(A,B,C) = \Pi_M(0,2,4,5,6)$

A \ BC	00	01	11	10
0	0 (0)	(1)	(3)	0 (2)
1	0 (4)	0 (5)	(7)	0 (6)

G1, G2, G3

G1

A	B	C
0	0	0
1	0	0

$\bar{B}\bar{C}$

G2

A	B	C
1	0	0
1	0	1

$A\bar{B}$

G3

A	B	C
0	1	0
1	1	0

$B\bar{C}$

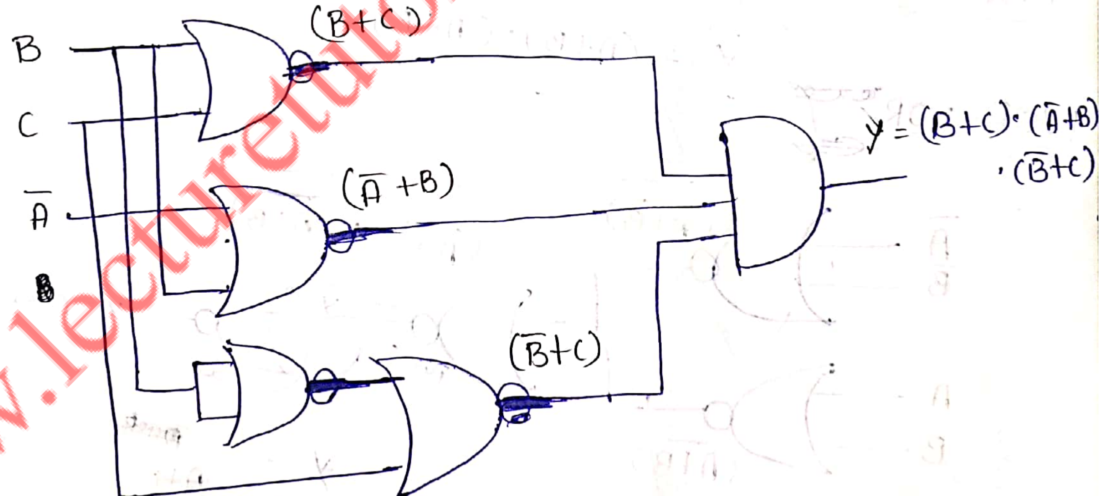
$$\bar{Y} = \bar{B}\bar{C} + A\bar{B} + B\bar{C}$$

$$\bar{\bar{Y}} = \overline{\bar{B}\bar{C}+A\bar{B}+B\bar{C}}$$

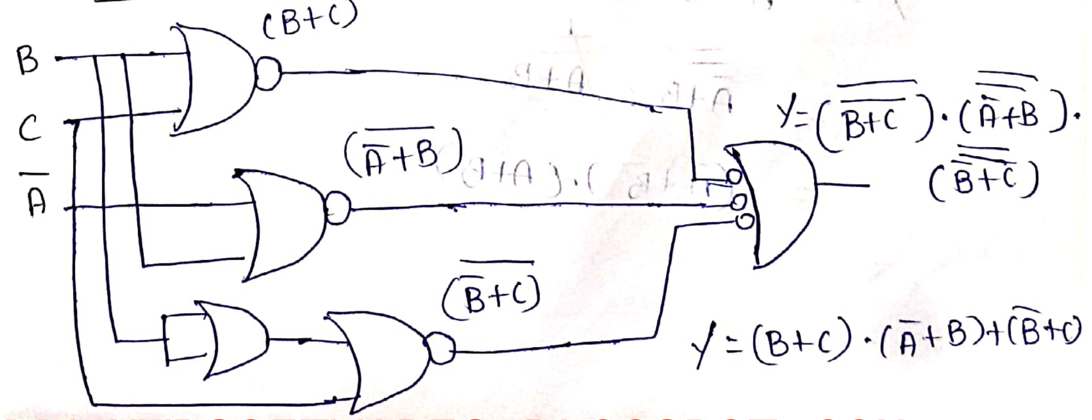
$$= (\bar{\bar{B}}+\bar{\bar{C}})\cdot(\bar{A}+\bar{\bar{B}})\cdot(\bar{B}+\bar{\bar{C}})$$

$$Y = (B+C)\cdot(\bar{A}+B)\cdot(\bar{B}+C)$$

(i) OR – AND

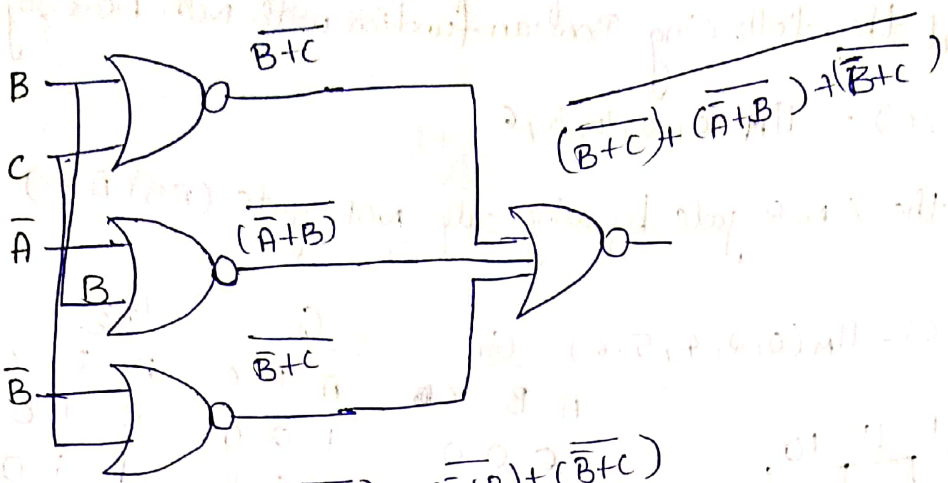


(ii) NOR – Bubbled AND



(iii) NOR-NOR

$Y = (B+C) \cdot (\bar{A}+B) \cdot (\bar{B}+C)$



$Y = \overline{(\overline{B+C}) + (\overline{\bar{A}+B}) + (\overline{\bar{B}+C})}$

$= (\overline{\overline{B+C}}) \cdot (\overline{\overline{\bar{A}+B}}) \cdot (\overline{\overline{\bar{B}+C}})$

$Y = (B+C) \cdot (\bar{A}+B) \cdot (\bar{B}+C)$

(2) Implement the X-NOR gate by using only NOR gate. The X-OR gate function is two inputs and one output. The input Variables are A and B and the output Variable is Y

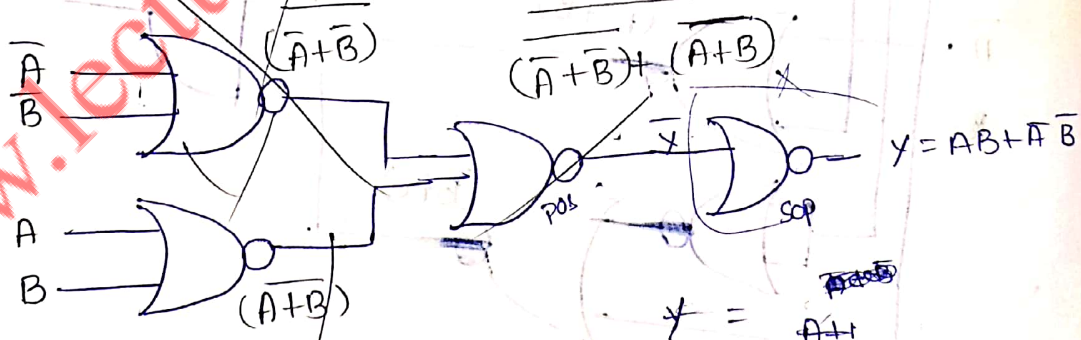
$Y = AB + \bar{A}\bar{B}$

~~NOR-NOR~~

$\bar{Y} = \overline{AB + \bar{A}\bar{B}}$

$\bar{Y} = (\bar{A}+\bar{B}) \cdot (A+B)$

NOR-NOR

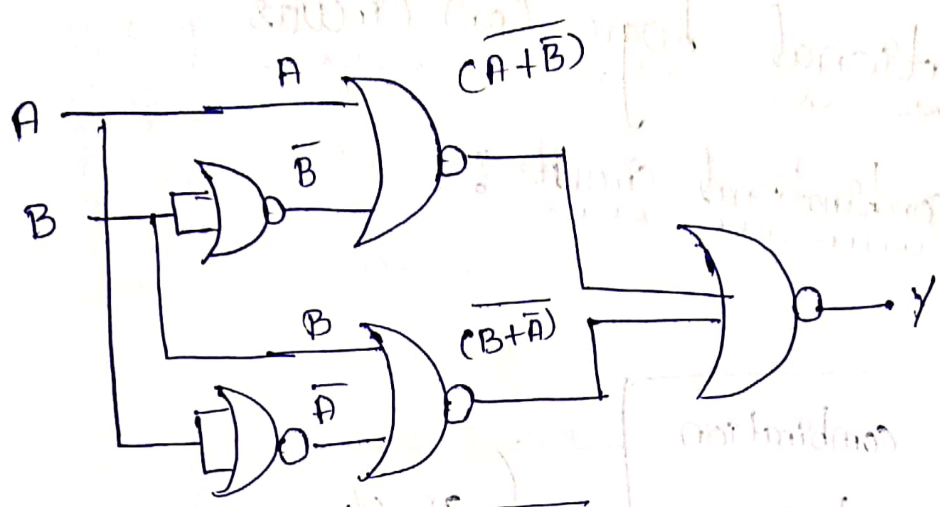


$\bar{Y} = \overline{\overline{\bar{A}+\bar{B}} + (\overline{A+B})}$

$\overline{\overline{\bar{A}+\bar{B}}} \cdot \overline{\overline{A+B}}$

$= (\bar{A}+\bar{B}) \cdot (A+B)$

Q: Implement the x-



$$y = \overline{\overline{(A+\bar{B})} + \overline{(B+\bar{A})}}$$

$$y = \overline{\overline{(A+\bar{B})}} \cdot \overline{\overline{(B+\bar{A})}}$$

$$y = (A+\bar{B})(B+\bar{A})$$

$$= AB + \bar{B}B + A\bar{A} + \bar{A}\bar{B}$$

$$= AB + \bar{A}\bar{B}$$

$\left[\because B\bar{B} = 0,\ A\bar{A} = 0\right]$