Todays Content :

→ Follow Slack

- → % operator
- → Modular Arthmetic
- → 1 Hard Problem

Range :

int x : $-2*10^9$, $2*10^9$

long y : $-8*10^{18}$, $-8*10^{18}$

% Basics

n%a = It is remainder when n is divided by a

Rem = Div − (Quo * div)

10%4 = 2 ⇒ 2 = 10 − (8)

13%5 = 3 ⇒ 3 = 13 − 10

Dividend = Quotient * Divisr + Remainder

$$\boxed{\text{Remainder} = \text{Divident} - (\underbrace{\text{Quotient} * \text{Divisor}}_{\text{largest multiple of divird = divident}})}$$

Quizes:

$150\%11 = 150 - (\underbrace{\text{greatest mul of } 11 <= 150}_{143}) = 7$

$100\%7 = 100 - (\underbrace{\text{greatest mul of } 7 <= 100}_{98}) = 2$

$-40\%7 = -40 - (\underbrace{\text{greatest mul of } 7 <= (-40)}_{-42})$

$= -40 - (-42) = -40 + 42 = 2$

$-60\%9 = -60 - (\underbrace{\text{greatest mul of } 9 <= -60}_{-63})$

$= -60 - (-63) = 3$

$-40\%9 = -40 - (\underbrace{\text{greatest mul of } 9 <= -40}_{-45})$

$= -40 - (-45) = 5$

	Python:		C/C++/ Java/ JS /C#
-40%7	2	$\xleftarrow{+7}$	-5
-60%9	3	$\xleftarrow{+9}$	-6
-40%9	5	$\xleftarrow{+9}$	-4

Why? [Extra Content]

In above language

$a\%p \xrightarrow{a<0}$

Conceptually $\rightarrow$

Correct data

$\{a\%p + p\}$

↓

divisor

Why %. → To limit our data to requried Range
- ↳ Hash function → DSA
- ↳ Consistent Hashing for load balancing } → HLD/LLD

$$\left.\begin{matrix}-\infty \\ \\ +\infty\end{matrix}\right] \%10 \longrightarrow [0 \quad 9]$$

output range

Input

$$\left.\begin{matrix}-\infty \\ \\ \infty\end{matrix}\right] \%p \longrightarrow [0, \quad p-1]$$

output range

Modular Arthimetic:

$$(a+b)\%p = (a\%p + b\%p)\%p$$

a	b	p			
6	8	10	4	=	(6+8) = 14% 10 = 4
7	9	5	1	=	(2+4) = 6%5 = 1

$$(a*b)\%p = (a\%p * b\%p)\%p$$

$(a-b)\%p = (a\%p - b\%p)\%p$ ✗ → TODO

$(a/b)\%p = (a\%p / b\%p)\%p$ ✗ Only in Advanced Content

→ Inverse Modulo

1) $(\underbrace{a\%p}) \% p = a\%p$

$\hookrightarrow (\underline{a\%p}) \longrightarrow [\underbrace{0, p}-1] \% p = a\%p$

2) $\boxed{(\underbrace{a\%p}_{x} * \underbrace{b}_{y}) \% p = (a * b) \% p}$

$x \longrightarrow y$

$\hookrightarrow (x \times y) \% p = (x\%p * y\%p) \% p$

$\longrightarrow = ((\underbrace{(a\%p)\%p}) * b\%p) \% p$

$= (\underline{a\%p * b\%p}) \% p$

$= (a * b) \% p$

8:25 → 8:35am

Divisibily Rules

%3 → Sum of digits should be divisibly by 3
%9 → Sum of digits should be divisibly by 9
%4 → {last 2 digits should be divisble by 4}
%8 → {last 3 digits should be divisble by 8}

Proof: → %3

$$(2472)\%3 = \{2*10^3 + 4*10^2 + 7*10^1 + 2*10^0\}\%3$$

$$\{(2*10^3)\%3 + (4*10^2)\%3 + (7*10^1)\%3 + (2*10^0)\%3\}\%3$$

$$\underline{\{2+4+7+2\}\%3} = \{\text{sum of digits}\}\%3$$

//obs: ⟶ obs2

$10^0\%3 = 1$	$10^0\%9 = 1$
$10^1\%3 = 1$	$10^1\%9 = 1$
$10^2\%3 = 1$	$10^2\%9 = 1$
$10^3\%3 = 1$	$10^3\%9 = 1$
$10^n\%3 = 1$	$10^n\%9 = 1$

Proof2 → %4

$$(2392)\%4 \to \{2300 + 92\}\%4$$

$$= \{\underset{0}{2300\%4} + 92\%4\}\%4$$

$$(92\%4)\%4$$

$$\Rightarrow (92)\%4$$

Obs3:

$$10^2\%4 = 0$$

Proof %8: ⟶ $(34262)\%8$

$$10^3\%8 = 0$$

↳ $(34000 + 262)\%8$

↳ $((\underset{0}{34000)\%8} + (262)\%8)\%8$

↳ $(262)\%8$

1) Given a, n, p calulate $a^n \% p$, without inbuilt functions

Constraints $1 <= a <= 10^9$ $2 <= p <= 10^9$ $1 <= n <= 10^5$

Eg1: $a = 3,\ n = 4,\ p = 7 \rightarrow (3^4) \% p \Rightarrow 81 \% 7 \Rightarrow 4$

```
—— powmod(int a, int n, int p){   TC→O(N)  SC: O(1)
      long ans=1;
      for(int i=1; i<=n; i++){
          ans = (ans * a)%p  → 1
          ans = (ans%p * a%p)%p  → 2
      }
      return ans%p
}
```

$\underbrace{ans\%p}_{[0\ p-1]} * \underbrace{a\%p}_{[0\ p-1]} \approx p^2$; $p = 10^9 \rightarrow 10^{18} \rightarrow$ long

$\%p \rightarrow\ \approx [0\ p-1]$

return ans%p → not needed

Issues:→ {overflow}

a	n	p	ans
2	30	41	2^{30}
2	40	41	2^{40}
2	100	65	2^{100}

// ans = (ans * a) %p $\overset{\text{Same}}{\longleftrightarrow}$ ans = (ans%p * a%p)%p

Dry Run: // Given a, p, n=5

ans :	i	ans = (ans*a)%p
1	1	$= a\%p \Rightarrow$
$a\%p$	2	$= (a\%p * a)\%p \Rightarrow (a*a)\%p = a^2\%p$ $((a\%p)\%p * a\%p)\%p \Rightarrow (a\%p * a\%p)\%p$
$a^2\%p$	3	$= (a^2\%p * a)\%p = (a^2 * a)\%p = a^3\%p$
$a^3\%p$	4	$= (a^3\%p * a)\%p = (a^3 * a)\%p = a^4\%p$
$a^4\%p$	5	$= (a^4\%p * a)\%p = (a^4 * a)\%p = a^5\%p$

$a^n \% p$
- → Iterative : $O(N)$
- → Bit manipulation : $O(\log N)$ *
- → Recursion : $O(\log N)$ ✓ → { Incoming Recursion Sessions }

2Q) Given 1 number in ar[] format Calulate ar[]%p

↳ Each ar[i] represents a single digit of number

Constraints :

$1 <= N <= 10^5$

$0 <= ar[i] <= 9$

$2 <= p <= 10^9$

Ex 1:

N			p
		0 1 2 3 4	
5	ar[5] =	[3 | 2 | 6 | 4 | 9]	24
		0 1 2 3 4 5 6	
7	ar[7] :	[2 | 1 | 4 | 3 | 2 | 0 | 6]	35

→ (32649) % 24 = 9

→ (2143206) % 35 = 16

Obs:

		max
99	:	$10^2 - 1$
999	:	$10^3 - 1$
9999	:	$10^4 - 1$

// for N digit number max = $10^N - 1$

// Max N according Constraints : 10^5

↳ for 10^5 dig number max = $10^{10^5} - 1$

Idea :

↳ Getting divisibility rules for all p's is not possible

Hint :

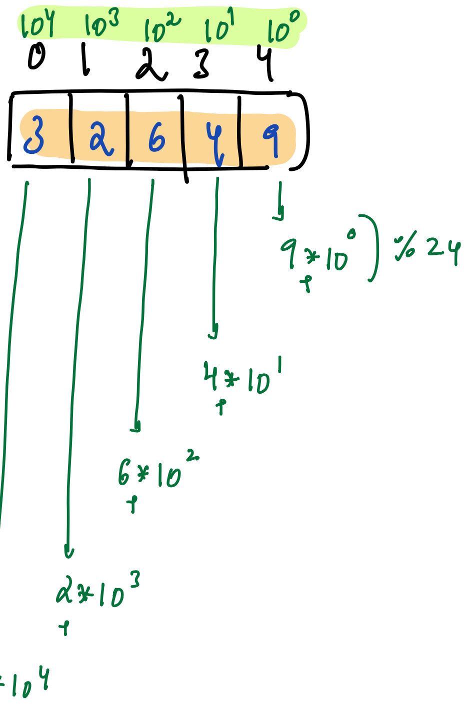
↳ split the number

Hint:

N P

5 24

ar[5] =



$$\begin{array}{ccccccc} & & 0 & 1 & 2 & 3 & 4 \\ (3\,2\,6\,4\,9)\%p & = & (3*10^4 + & 2*10^3 + & 6*10^2 + & 4*10^1 + & 9*10^0)\%p \end{array}$$

$$= \{ (3*10^4)\%p + (2*10^3)\%p + (6*10^2)\%p + (4*10^1)\%p + (9*10^0)\%p \}\%p$$

$t = 10^0\%p * ar[4]$

$t = t*10\%p$

$t = 10^1\%p * ar[3]$

$t = (t*10)\%p$

$t = 10^2\%p * ar[2]$

$t = (t*10)\%p$

$t = 10^3\%p * ar[1]$

$t = t*10\%p$

$t = 10^4\%p * ar[0]$

🟦 → Add all blue colour boxes

```
armod( int ar[], int p){   TC → O(N)   SC: O(1)
    int n = ar.length;
    long t = 1;
    long sum = 0;

    for( i = N-1 ; i >= 0 ; i--){
        sum = (sum + ar[i] * t) %p
        (t = t*10) %p
    }
} return sum
```

max p-1

→ sum = ≈ 10^9

→ t = ≈ 10^9

→ ar[i] = ≈ 9

≈ p-1 = $\underline{\underline{10^9 + 9 * 10^9}} \approx 10^{10}$ max → int ×

↳ p-1 ≈ $10^9 * 10 \approx 10^{10}$ → int ×

// Why % different , / → In Java/C/C++/C# → integer divisio

a%b

$$\text{Remainder} = \text{Divident} - (\overbrace{\text{Quotient}}^{a/b} * \overbrace{\text{Divisor}}^{\text{Divisor}})$$

a b

$100 \% 7 = 100 - (\overset{100/7}{14} * 7) = 100 - 98 = 2$

$-40 \% 7 = -40 - (\overset{-40/7}{-5} * 7) = -40 + 35 = -5$

$-60 \% 9 = -60 - (\overset{-60/9}{-6} * 9) = -60 + 54 = -6$

In Python /, $a/b \Rightarrow$ is not integer division

$$\left.\begin{array}{l} 4/3 \Rightarrow \lfloor 1 \rfloor .333 \\ 10/4 \Rightarrow \lfloor 2 \rfloor .555 \end{array}\right\} \quad \underline{\text{Quotient}} : \text{floor}(a/b)$$

$$: \text{floor}(1.333) = 1$$

$$: \text{floor}(2.55) = 2$$

$$\text{Remainder} = \text{Divident} - (\overset{\overset{\text{floor}(a/b)}{\uparrow}}{\underbrace{\text{Quotient}}} * \text{Divisor})$$

a b

$$\text{floor}\left(\frac{100}{7}\right) = \text{floor}(14.3..)$$

$$100 \% 7 = 100 - (14 * 7) = 100 - 98 = 2$$

$$\text{floor}(-40/7) = \text{floor}(-5.633) = -6$$

$$-40 \% 7 = -40 - (-6 * 7) = -40 + 42 = 2$$

$$\text{floor}(-60/9) = \text{floor}(-6.666) = -7$$

$$-60 \% 9 = -60 - (-7 * 9) = -60 + 63 = 3$$