

Today's Content:

- Max submatrix sum
- Find k in 2D matrix
- Beggars outside temple
- Merge Intervals
 - a) Merge all overlapping intervals
 - b) Insert a interval into non-overlapping interval

1Q) Given row wise column wise sorted matrix, find max submatrix sum

Ex1:

	0	1	2	3
0	-20	-16	-4	8
1	-10	-8	12	14
2	-1	6	21	30
3	5	7	28	42

Sum = 152

ideas:

1) For every submatrix, get the sum, & find overall max

TC: $O(N^2M^2) * O(N*M) = O(N^3M^3)$

// Can be optimized using pfm [][]

TC: $O(N*M) + O(N^2M^2) * O(1)$

SC: $O(N*M)$

// All submatrices

// Calculating sum of each submatrix using pfm [][]

Ex2:

	0	1	2	3
0	-20	-16	-4	-1
1	-10	-8	-2	5
2	-4	2	4	8

Sum = 15

Ex3:

	0	1	2
0	-50	-40	-30
1	-35	-20	-15
2	-19	-14	-3

Sum = -3

Obs: Our ans submatrix bottom right = [n-1, m-1] cell

↳ Take every cell as TL

Create pfm [][]

TC: $O(N^2M + N*M * O(1))$

SC: $O(N*M)$

i = 0; i < N; i++

j = 0; j < M; j++

TL = (i, j) BR = (n-1, m-1)

Using pfm [][] get submatrix sum & overall max

3

28) Given row wise column wise sorted matrix, find k?

return True / False

k=12

TL TR

	0	1	2	3	4	5	
0	-10	-5	-2	2	4	7	12
1	-7	-4	-1	3	6	9	12
2	-2	3	5	7	10	14	12
3	3	6	8	11	14	17	
4	7	11	12	15	19	20	
5	13	14	18	20	24	29	
	BL						BR

Idea:

1) Iterate on entire matrix

TC: $O(N \times M)$ SC: $O(1)$

2) Iterate on only those rows, which can contain k

TC: $O(N \times M)$ SC: $O(1)$

Pseudocode

i=0, j=M-1

while (i < n && j >= 0)

if (mat[i][j] == k)

return true

}

if (mat[i][j] < k)

// skip row

i++

}

else // skip column

j--

}

return False

k=16

	0	1	2	3	4	5	
0	-10	-5	-2	2	4	7	16
1	-7	-4	-1	3	6	9	16
2	-2	3	5	7	10	14	16
3	3	6	8	11	14	17	16
4	7	11	12	15	19	20	
5	13	16	18	20	24	29	

TODO: TL, BL, BR, TR ✓

row wise column
Both inf helped us

TC: $O(N+M)$: At every step, we either skip a row or column

↳ Total Rows we can skip: N

↳ Total Cols we can skip: M

Total skips at max = N+M

TC: $O(N+M)$ SC: $O(1)$

3Q) Given an array $ar[N]$, s.t $ar[0] = 0$, initially return the final array after performing all the queries

Queries $\rightarrow (i, n) = \text{Add } n \text{ to all the numbers from } \overrightarrow{A[i] \text{ to } A[N-1]}$

Ex1:

	0	1	2	3	4	5	6
$A =$	0	0	0	0	0	0	0
		3	3	3	3	3	3
				-2	-2	-2	
				1	1	1	1
<hr/>							
	0	3	3	4	2	2	2
<hr/>							

$(i, n) : 3 \text{ Queries}$

$(1, 3)$

$(4, -2)$

$(3, 1)$

Idea1: For every query
iterate from $[i, N-1]$
and add n

TC: $O(Q \cdot N)$ SC: $O(1)$

Ex2:

	0	1	2	3	4	5	6
$A =$	0	0	0	0	0	0	0
$mod\ A =$	0	3	0	1	-2	0	0
$Pf\ A[i]$	0	3	3	4	2	2	2

$(i, n) : 3 \text{ Queries}$

$(1, 3)$

$(4, -2)$

$(3, 1)$

Idea2: TC: $O(Q + N)$

1) For every query (i, n)
 $ar[i] += n$

2) After all queries apply
 $Pf[i]$ in $ar[i]$

Queries $\rightarrow (i, j, n)$ = Add n to all numbers from $A[i] \rightarrow A[j]$

$A[] = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$
 $i \ j \ n : 3$
Ideal: For every query iterate & add n
 $TC: O(Q \times N)$

$\begin{matrix} & & 2 & 2 & 2 \\ & & & 3 & 3 & 3 & 3 \\ & & & & -1 & -1 & -1 \end{matrix}$
 $\begin{matrix} (1 \ 3 \ 2) \\ (2 \ 5 \ 3) \\ (2 \ 4 \ -1) \end{matrix}$

$A[]: \underline{0 \ 2 \ 4 \ 4 \ 2 \ 3 \ 0}$

0 1 2 ... i i+1 ... j j+1 j+2 ... N-1

$(i \ j \ n) :$ $n \ n \dots n \ n \ n \dots n : O(i, n)$

$\rightarrow (i, n) + (j+1, -n)$ $-n \ -n \dots -n : O(j+1, -n)$

$arr[i] += n$ $arr[j+1] -= n$

$A[] = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$
 $i \ j \ n : 3$

$mod \ 4 \ A[] \ 0 \ 2 \ 2 \ 0 \ -2 \ 1 \ -3$
 $\begin{matrix} (1 \ 3 \ 2) \\ (2 \ 5 \ 3) \\ (2 \ 4 \ -1) \end{matrix}$

$PfA[] : 0 \ 2 \ 4 \ 4 \ 2 \ 3 \ 0$

Pseudocode : $\rightarrow TC: O(Q + N)$

1) For all Queries

$Q: (i, j, n) :$

$arr[i] += n$

$if (j+1 < n) \downarrow$

$arr[j+1] -= n$ // if $j = n-1$, $j+1 = n$, out of bounds

2) Apply $Pf[]$ on $arr[]$

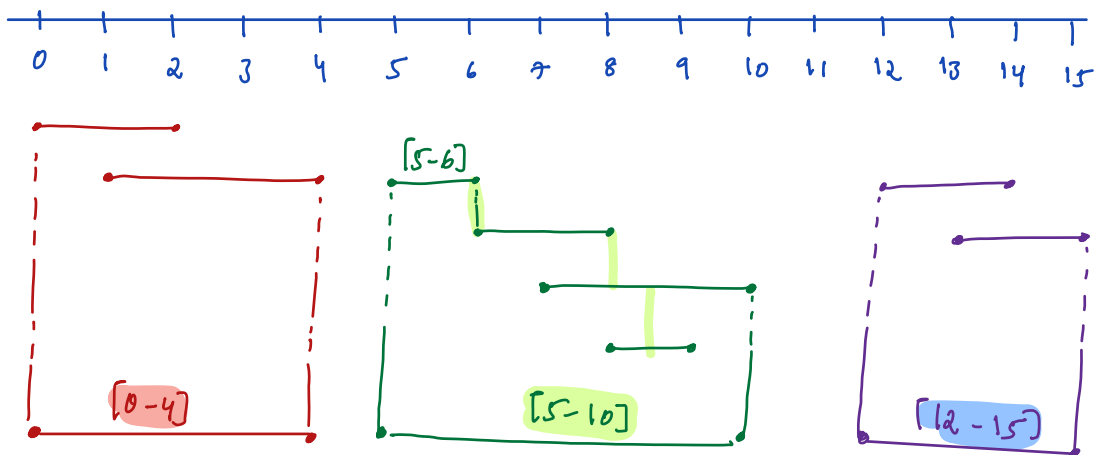
$8: arr \rightarrow 8: 37arr$

2m no Ques

48) Given a sorted list of overlapping intervals, sorted based on start time, merge all overlapping intervals & return a sorted list of non-overlapping intervals : $[start, end]$

Intervals:

	0	1	2	3	4	5	6	7
S	[0	1	5	6	7	8	12	13]
E	[2	4	6	8	10	9	14	15]



Final Intervals: [Non Overlapping Intervals, Sorted on start time]

$$\begin{bmatrix} [0, 5] \\ [5, 10] \\ [12, 15] \end{bmatrix}$$

s e \longrightarrow n y

[0 2] ✓ [0 2]

[1 4] ✓ [0 4]

[5 6] $\xrightarrow[\substack{\text{update} \\ n, y}]{}$ [5, 6]

[6 8] ✓ [5, 8]

[7 10] ✓ [5, 10]

[8 9] ✓ [5, 10]

[12 14] $\xrightarrow[\substack{\text{update} \\ n, y}]{}$ [12, 14]

[13 15] [12, 15]

[Take care of last interval]

Final ans

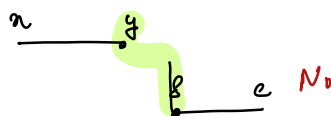
[0, 4]

[5, 10]

[12, 15]

} non-overlapping
intervals sorted
based on start time
TC: O(N)

Merged Int = [n, max(y, e)]



void merge(int s[], int e[]) { TC: O(N)

int n = s[0], y = e[0]

i = 1; i < n; i++ {

// ith interval is overlapping n — y

if (y >= s[i]) { // merged interval

y = max(y, e[i])

} else { // non-overlapping

print(n, y)

n = s[i], y = e[i]

}
print(n, y)

5Q) Given N non-overlapping intervals, sorted based on start time,

Insert new interval & return non-overlapping intervals

[s e]

N=8

new Interval

[1 3]

[4 7]

[10 14] → [12 22] : {10, 22}

[16 19] → {10, 22} : {10, 22}

[21 24] → {10, 22} : {10, 24}

[27 30] * {10, 24}

[32 35]

[36 40]

[1 3]

[4 7]

[10, 24]

[27 30]

[32 35]

[36 40]

} Intervals before
new interval

} Intervals merging
new interval

} Intervals coming
after new interval

N=5:

new Interval

[1 5]

[8 10]

[11 14] → [12 22] : {11, 22}

[15 20] → {11, 22} : {11, 22}

[20 24] → {11, 22} : {11, 24}

[1 5]

[8 10]

[11 24]

TODO: TC: O(N)

1) [8 e] coming before [n y]

2) [8 e] coming after [n y]

3) [8 e] merging [n y]