

Today's Content:

→ Adhoc Problems

→ 22nd → 26th vacation

20th Thursday : _____

21 : Friday

22 : Sat

23 : Sun

24 : Mond

25 : Tue

26 : Wed

27 : Thursday : Session

1Q) Give unsorted arr[] of N distinct elements,

find k^{th} index pos in its sorted form {arr}.

Note: We cannot modify arr[] / We cannot use extra space

Ex1: arr[5] = {
0 1 2 3 4
2 8 3 11 14 } $k=2$ ans=8
0 2 1

Ex2: arr[9] = {
0 1 2 3 4 5 6 7 8
11 24 18 3 5 27 34 9 40 } $k=9$
3 4 0 1 2 ans=18

Idea: find k^{th} index element in sorted arr[]

0 1 2 ... k-1 k
 $[a_0, a_1, a_2, \dots, a_{k-1}] < a_k$

Obs: no. of elements less than that == k

arr[9] = {
0 1 2 3 4 5 6 7 8
11 24 18 3 5 27 34 9 40 } $k=6$
elem arr[i] = 3 5 4 0 1 6
* * * * *

Idea: for every arr[i], iterate & calculate no. of elem < arr[i] == k

Pseudo Code:

int k^{th} smallest(int arr[N], int k) { TC: $O(N^2)$ SC: $O(1)$

```
{  
    i = 0; j = N; i < j {  
        if (class(arr[j], arr[i]) == k) {  
            return arr[i];  
        }  
    }  
}
```

Implement class on your own

Idea 2: Binary Search

1) Target: k^{th} smallest element

2) Search space: $\text{arr}[]$

3) We are not able to discard $\text{arr}[]$ *

→ Change Search space: $[\min \text{ of } \text{arr}[] \quad \max \text{ of } \text{arr}[]]$

Ex:

$\text{arr}[5] = \{ \overset{0}{4} \quad \overset{1}{1} \quad \overset{2}{5} \quad \overset{3}{15} \quad \overset{4}{6} \quad \overset{5}{2} \} \quad k=3$

<u>l</u>	<u>h</u>	<u>m</u>	<u>#count of $\text{arr}[] < m$</u>	
1	15	8	$5 > 3$	8 9 10 ... goto left F F F $h = m-1$
1	7	4	$2 < 3$.. 2 3 4 goto right F F F $l = m+1$
5	7	6	$4 > 3$	goto left.. $h = m-1$
5	5	5	$3 == 3$	return 5

Idea2: Pseudo Code :

int th k smallest (int arr[N], int k) { TC: $\left[\log_2^{max-min+1} \right] \times N$

l = min of arr[], h = max of arr[] ans = — SC: O(1)

while (l <= h) {

 m = (l + h) / 2

 // m is kth index elem in sorted arr[]?

 // calculate no. of elements < m

 int c = countLess(arr[], m)

 if (c == k) {

 ans = m;

 l = m + 1

 }

 if (c < k) { // goto right

 l = m + 1

 }

 else { // goto left

 h = m - 1

 }

}

return ans;

Note: If data repeats above logic fails.

Edge Case:

Ans: { 11 24 20 3 5 27 34 9 40 } $k=4$

l	h	m	#count of arr $< m$
3	40	21	$5 > 4$ 21 22 23 ... 40 goto left F F F F $h = m-1$
3	20	11	$3 < 4$... 9 10 11 goto right $l = m+1$ F F F
12	20	16	$4 == 4$ 16 is ans, but it's not even in arr

{ 11 24 20 3 5 27 34 9 40 }

SearchSpace: { 3 40 }

3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22 ... 40
0	1	1	2	2	2	2	3	3	4	4	4	4	4	4	4	4	4	5	5 ... 8
$class < k$ goto right									$class = k$ ans = m; goto right;									$class > k$: goto left	

Q) Element can repeat:

$arr[8] = \{ 15 \ 4 \ 15 \ 10 \ 16 \ 19 \ 10 \ 15 \}$ $k=4$

l	h	m	#count of $arr[i] < m$
-----	-----	-----	------------------------

4	19	11	$3 < 4$: $ans = 11$, goto right $l = m+1$
---	----	----	---

12	19	15	$3 < 4$: $ans = 15$, goto right $l = m+1$
----	----	----	---

16	19	17	$7 > 4$: goto left
----	----	----	---------------------

16	16	16	$5 > 4$: goto left
----	----	----	---------------------

16	15		} Break
----	----	--	---------

searchSpace [4 19] $k=4$

4	5	6	7	8	9	10	11	12	13	14	15
0	1	1	1	1	1	1	3	3	3	3	3

$ans = mid$
goto right

16	17	18	19
6	7	7	7

$c > k$:
goto left

Repeated call:

int k smallest (int arr[N], int k) { TC: $\left[\log_2^{man - min + 1} \right] \cdot N$

l = min of arr[], h = max of arr[] ans = — SC: $O(1)$

while (l <= h) {

 m = (l + h) / 2

 // m is kth index elem in sorted arr[]?

 // calculate no. of elements < m

 int c = countLess(arr[], m)

 if (c > k) {

 // goto left

 h = m - 1

 }

 else {

 ans = m

 l = m + 1

 }

}

return ans;

}

Note: Above logic works for all cases

8:35 $\xrightarrow{\text{10m}}$ 8:45

28) Given 2 sorted arrays[], find k^{th} pos in overall sorted data

$A[8] = \{ \overset{0}{3}, \overset{1}{3}, \overset{2}{6}, \overset{3}{7}, \overset{4}{7}, \overset{5}{11}, \overset{6}{14}, \overset{7}{17} \}$
 $B[7] = \{ \overset{0}{2}, \overset{1}{2}, \overset{2}{10}, \overset{3}{10}, \overset{4}{13}, \overset{5}{20}, \overset{6}{20} \}$

} $k = 8$

Idea1: Merge 2 sorted arr[] $TC: O(N+m)$

Idea2: Using Binary Search:

Target : k^{th} pos element

Search Space : min of $(A[], B[])$ max of $(A[], B[])$

Discard : Yes

<u>l</u>	<u>h</u>	<u>m</u>	<u>#count of $\leq m$ in (A, B)</u>	<u>$k=8$</u>
2	20	11	9 > 8	$h = m - 1$
2	10	6	4 < 8	$ans = 6, l = m + 1$
7	10	8	7 < 8	$ans = 8, l = m + 1$
9	10	9	7 < 8	$ans = 9, l = m + 1$
10	10	10	7 < 8	$ans = 10, l = m + 1$
11	10	{break}		


```
int countlessSorted (int ar[N], int k){
```

```
// given sorted arr(), calculate no: of elements < k  
using binary search TC: O(log N) SC: O(1)
```

```
}
```

```
int kthSmallest (int ar[N], int br[M], k)
```

```
l = min(ar[0], br[0]) h = max(ar[N-1], br[M-1]) ans = —
```

```
while (l <= h) {
```

TC: $\log_2^{(hi-lo+1)} * [\log_2 N + \log_2 M]$

```
    m = (l+h)/2
```

```
    // m is kth index elem in sorted arr()?
```

```
    // calculate no: of elements < m
```

```
    int c = countlessSorted (ar[], m) }  $\log N$ 
```

```
    c = c + countlessSorted (br[], m) }  $\log M$ 
```

↳ TC: $\log_2^{[n+m]}$ TODO
↳ median of 2 sorted arrays

```
    if (c > k) { // goto left
```

```
        h = m-1
```

```
    }
```

```
    else {
```

```
        ans = m
```

```
        l = m+1
```

```
    }
```

```
return ans;
```

```
}
```

// Given $\text{mat}[N][M]$ every row sorted, find k^{th} pos in overall sorted data

$\text{mat}[3][4] = \begin{bmatrix} 0 & 3 & 6 & 8 \\ -2 & 1 & 4 & 11 \\ 2 & 4 & 3 & 6 \end{bmatrix}$

min of 0^{th} col

int k^{th} smallest (int $\text{mat}[N][M]$, int k) {

$l = \text{min of } 0^{\text{th}} \text{ col}$ $h = \text{max of } M-1^{\text{th}} \text{ col}$

while ($l \neq h$) {

TC: $\log_2^{h-l+1} * (N * \log m)$ SC: $O(1)$

$m = (l+h)/2$

// m is k^{th} index elem in sorted arr?

// calculate no. of elements $\leq m$

$c = 0$

$i = 0; i < N; i++$ {

$c = c + \text{countLessSorted}(\text{mat}[i], m)$

if ($c > k$) { // go left

$h = m-1$

}

else {

$\text{ans} = m$

$l = m$

}

return ans;

// passing i^{th} row as parameter

// Each row contains m elements

}

// Give $\text{mat}[N][M]$, every row is sorted, find median? Interview

// Say $N \times M = 21 \text{ elem} = 0 \ 1 \ 2 \ \dots \ 10 \ 11 \ \dots \ 20$
 // ele at index 10 is median, $k=10$
 // Say $N \times M = 10 \text{ ele} = 0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9$
 // avg of ele at index 4, 5, $k=4, k=5$

if $(N \times M) \% 2 == 1$ {

// only 1 median
 $\text{th } k \text{ smallest}(\text{mat}[N][M], \frac{N \times M}{2})$

else {

// 2 median find = $\frac{N \times M}{2}$

$$\left[\begin{array}{c} \text{th } k \text{ smallest}(\text{mat}[N][M], \text{mid}) \\ + \\ \text{th } k \text{ smallest}(\text{mat}[N][M], \text{mid}-1) \end{array} \right] / 2$$

$\Rightarrow \left\lceil l + \frac{(h-l)}{2} \right\rceil$ to avoid overflows
 $\Rightarrow \left\lfloor \frac{l+h}{2} \right\rfloor$ at times it can overflow