

Todays Content:

- Recursion?
- How to write a Recursive Code / Tracing
- TC/SC of Recursive Codes → Thursday Session

Why Recursim?

- Merge Sort / Quick Sort
- Binary Tree / BST / BBST / Segment Tree / Trees
- Dynamic Programming
- Backtracking
- Graphs

Recursion: → function calling itself → Recursion
 ↓
 → Solving problem, Using Smaller Instance of Same Problem
 Sub Problem

$$\text{sum}(N) = \underbrace{1 + 2 + 3 + \dots}_{\circ} N - 1 + N$$

$$\boxed{\text{sum}(N) = \text{sum}(N-1) + N} \rightarrow \text{sum}(4) = \text{sum}(3) + 4$$

→ Smaller Instance of Same Problem / Sub Problem

How to write Recursive Code?

Assumption: Decide what your function does

Main logic: Solve Assumption using Sub Problems
 Smaller Instance of Same Problem

Base Condition: Inputs for which we need to stop

```

int sum(N) {
    Ass: Given N, calculate & return sum of
          N Natural Numbers
    if (N == 1) { return 1 } // Base Condition at Start?
    return (sum(N-1) + N)
}

$$\text{Sum of } \underbrace{1 \text{ to } N-1 \text{ Natural Numbers}}_{\text{Sum of } N-1 \text{ Natural Numbers}} + N = \text{Sum of } N \text{ Natural}$$


```

$$\text{fact}(3) = 3 * 2 * 1 = 6, \quad \text{fact}(4) = 4 * 3 * 2 * 1 = 24$$

```

int fact(N) {
    Ass: Given N calculate & return N!
    if (N == 1) { return 1 }
    return [ fact(N-1) * N ]
}

```

Function Call Tracing:

```

int add(n, m) {
    return (n + m)
}

int mul(n, y) {
    return (n * y)
}

int sub(n, y) {
    return (n - y)
}

main() {
    n = 10, y = 20
    print(sub(mul(add(n, y), 30), 75))
}

print(sub(mul(add(n, y), 30), 75)) → 825
    ↓
    sub(mul(add(n, y), 30), 75) → return 825
        ↓
        mul(add(n, y), 30) → return 900
            ↓
            add(n, y) { return n + y → 30
                ↑
                n = 10, y = 20
}

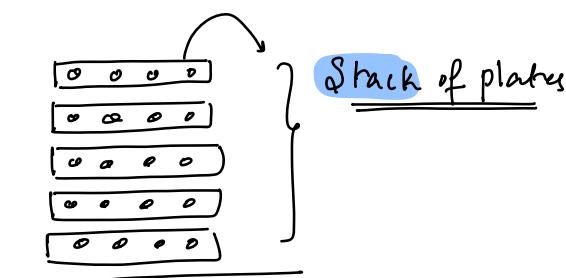
```

= Data Structure :

- {
 - obs1: When ever function call, Insert function call top
 - obs2: When we return function come out
 - obs3: Insert at top & We delete at top \Rightarrow Stack

```
10 20  
add(x,y) : return 30 // Once returned it will come out  
          )  
mul(add(x,y),30) : return 900 // Once returned it will come out  
          )  
sub(mul(add(x,y),30),75) : return 825 // It will come out
```

Insert at top & We delete at top



// Sum Tracing: ID

```
int sum(N=4)
if (N==1) {return 1}
return (sum(N-1)+N)
          ↓
          6 + 4
```

```
int sum(N=3)
if (N==1) {return 1}
return (sum(N-1)+N)
          ↓
          3 + 3
```

```
int sum(N=2)
if (N==1) {return 1}
return (sum(N-1)+N)
          ↓
          1 + 2
```

```
int sum(N=1)
if (N==1) {return 1}
return (sum(N-1)+N)
```

Stack Tra:

```
Sum(1): return 1
          ↓
Sum(2): return (sum(1)+2)
          ↓
Sum(3): return (sum(2)+3)
          ↓
Sum(4): return (sum(3)+4)
          ↓
          10 =
```

Without Base Condition: Function Calls won't stop

Infinite loop

TLE

+ we get below error
Insert function calls in Stack

Stack overflow / Memory limit Exceeded

Note: In Recursion, If your code gives Memory limit exceed,
that means, code is not properly stopped, verify base conditions

$N > 0$

#Input	0	1	2	3	4	5	6	7	8	9	10
<u>Fib():</u>	0	1	1	2	3	5	8	13	21	34	55

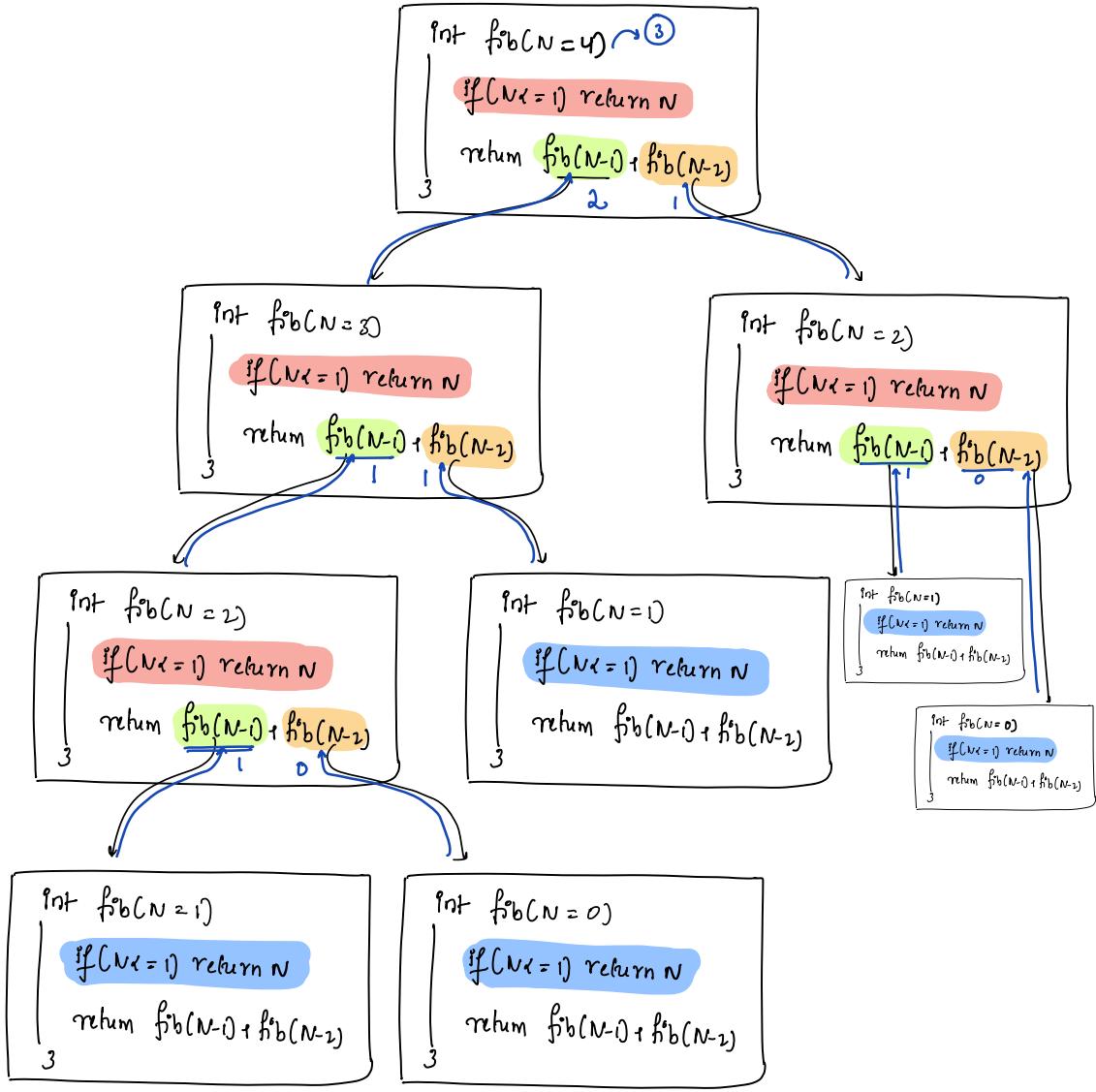
int fib(N) { Ass: Given N, Calculate & return N^{th} fib number

If $N = 1$ return N

return $\left[\underbrace{\text{fib}(N-1)}_{\downarrow} + \underbrace{\text{fib}(N-2)}_{\downarrow} \right]$

$N-1^{\text{th}}$ fib number $N-2^{\text{nd}}$ fib number

$$\begin{cases} \text{fib}(0) = \text{fib}(-1) + \text{fib}(-2) \\ \text{fib}(1) = \text{fib}(0) + \text{fib}(-1) \\ \text{fib}(2) = \text{fib}(1) + \text{fib}(0) \end{cases}$$



→ TODO for Stack Trace

// given N, print all numbers from $\underline{1-N}$ in increasing order

void $\text{Inc}(N)$ { Ass: Given N, print all numbers from 1 - N }

If ($N == 1$) { $\text{print}(1)$ return; }

$\text{Inc}(N-1)$ } // 1, 2, 3, N-1
 $\text{print}(N)$ } // N

output

$\text{Inc}(3) \rightarrow 1 \ 2 \ 3$

$\text{Inc}(4) \rightarrow 1 \ 2 \ 3 \ 4$

$\text{Inc}(N) = \underbrace{1 \ 2 \ 3 \ 4 \ 5 \ \dots \ N-1 \ N}_{\text{...}}$

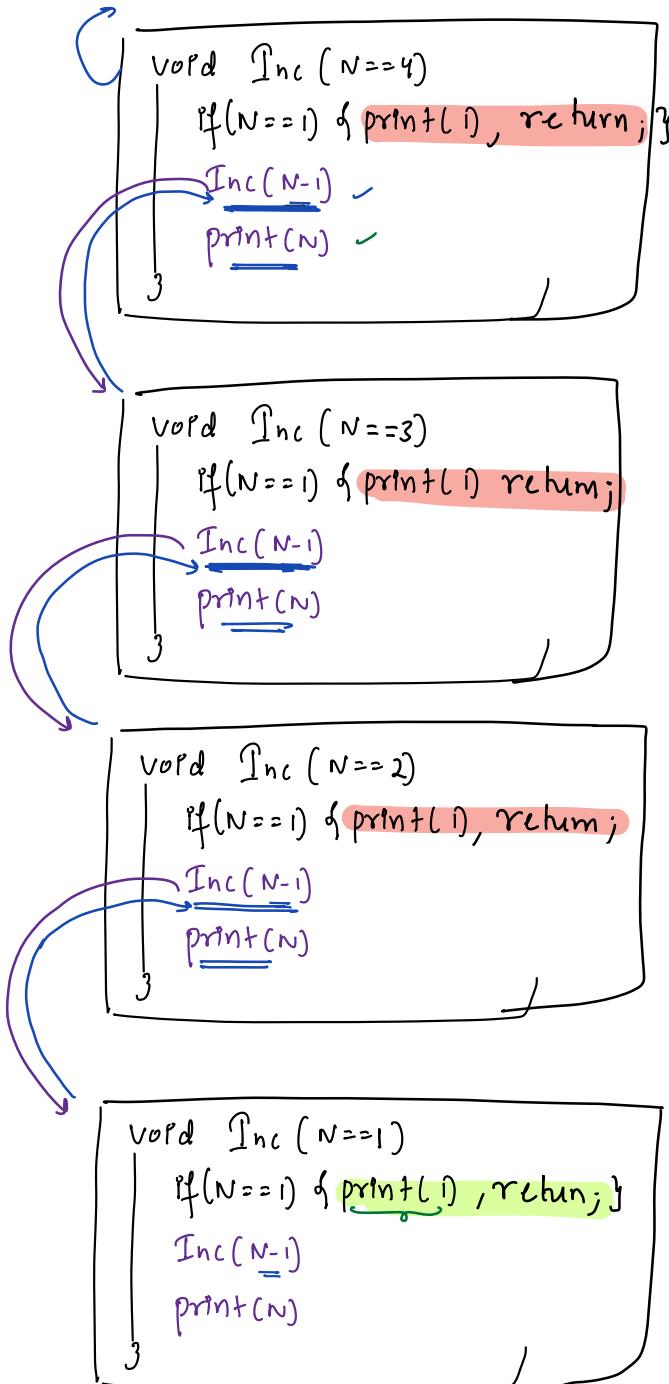
$$\text{Inc}(N) = \begin{cases} \text{Inc}(N-1) \\ \text{print}(N) \end{cases}$$

$$\text{Inc}(1) = \begin{cases} \text{Inc}(0) \\ \text{print}(1) \end{cases}$$

3

TODO:

void $\text{dec}(N)$: Ass: Given N print all number from $N \rightarrow 1$



Output

print(1)
print(2)
print(3)
print(4)

Note: When function is completely enclosed
it will automatically return to function
which calls

Q8) Given a Substring Check if it's palindrome or Not?

Ex1:  $s=4, e=6 \rightarrow$ return True

Ex2:  $s=2, e=5 \rightarrow$ return False

→ Ass: Return If given substring [s e] is palindrome or Not

bool isPal(char ch[], int s, int e)

if ($s > e$) { return true }

if ($ch[s] == ch[e]$ &&

 isPal(ch, s+1, e-1)) {

 return True

 }

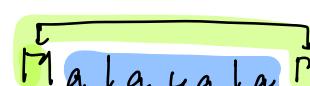
$ch[]: \underline{s} \underline{s+1} \dots \underline{e-1} \underline{e}$

To check, $ch[s-e]$ is palind

$ch[s] == ch[e]$ &&

Substring from $[s+1, e-1]$ should
also be a palindrome

Ex1: 

Ex2: 

0 1 2 3 4 5

Input: m a d d a m , s=0, e=5

bool ispal(ch[], int s=0, int e=5){

 if(s > e) { return true; }

 if(ch[s] == ch[e] && ispal(ch, s+1, e-1)) {
 return true;
 }

 return false;

True

bool ispal(ch[], int s=1, int e=4){

 if(s > e) { return true; }

 if(ch[s] == ch[e] && ispal(ch, s+1, e-1)) {
 return true;
 }

 return false;

True

bool ispal(ch[], int s=2, int e=3){

 if(s > e) { return true; }

 if(ch[s] == ch[e] && ispal(ch, s+1, e-1)) {
 return true;
 }

 return false;

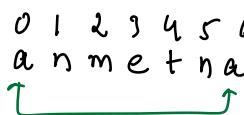
True

bool ispal(ch[], int s=3, int e=2){

 if(s > e) { return true; }

 if(ch[s] == ch[e] && ispal(ch, s+1, e-1)) {
 return true;
 }

 return false;

Input:  $s=0, e=6$

```
bool ispal(ch[], int s=0, int e=6) {
    if(s > e) { return true; }
    if(ch[s] == ch[e]) { ispal(ch, s+1, e-1); }
    return false;
}
```

```
if(s > e) { return true; }
```

↓
return True

} return False

```
bool ispal(ch[], int s=1, int e=5) {
```

```
if(s > e) { return true; }
```

```
if(ch[s] == ch[e]) { ispal(ch, s+1, e-1); }
```

↓
return True

} return False

```
bool ispal(ch[], int s=2, int e=4) {
```

```
if(s > e) { return true; }
```

```
if(ch[s] == ch[e]) { ispal(ch, s+1, e-1); }
```

↓
return True

} return False

// Increment:

postinc preinc

Note: Don't use post increment
operators, while writing
recursion, Use pre increment

```
int n = 10
int y = n++
```

printf(y)

10

11

```
int n = 10
int y = ++n
```

printf(y)