

Todays Content:

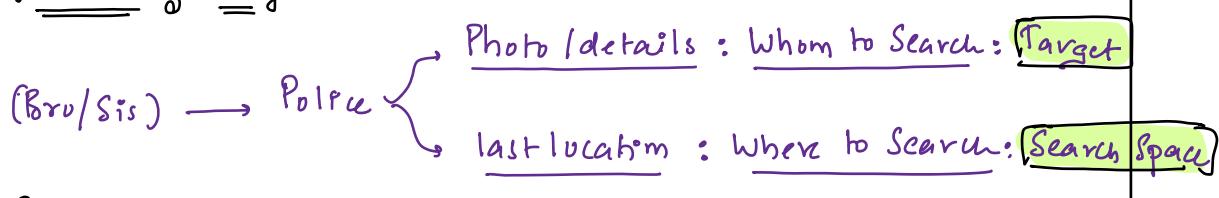
Oct 4th holiday: Navami ✓

- Searching Basics
- Why mid at half?

Problems:

- a) Search in sorted arr[]
- b) finding floor in a sorted arr[]
- c) finding 1st occurrence in sorted arr[]
- d) finding local minima

Searching Story:



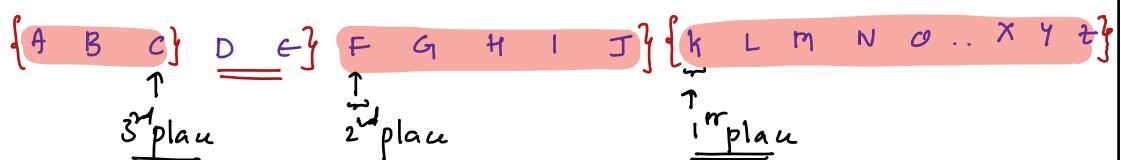
Example:

Word → { Dict / Books / News paper }

phonenumber → { Contacts / phone books }

If search space is ordered / searching become easier

Search Dog in Dictionary:

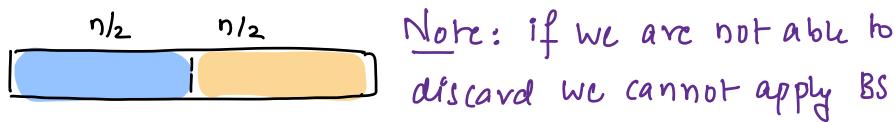


Why land at mid?



When to apply BS:

After dividing search space into 2 halves
we should be able to discard 1 half of search space
using some conditions.



Q) Given a sorted $ar[N]$ search if k is present or not?

$$ar[10] = \{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 \\ | 3, 6, 9, 12, 14, 19, 20, 23, 25, 27 \} \quad k=12$$

Idea1: Linear Search $T(: O(N)) \quad SC: O(1)$

Idea2: Check, if we discard a half



Case-I: $ar[mid] == k$ return True



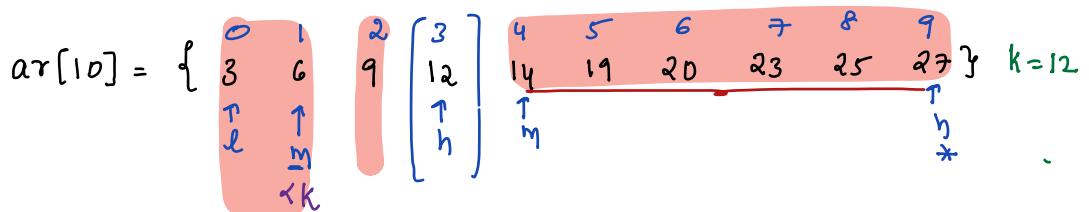
Case-II: $ar[mid] > k$:

- discard right
- search left



Case-III: $ar[mid] < k$

- discard left
- search right



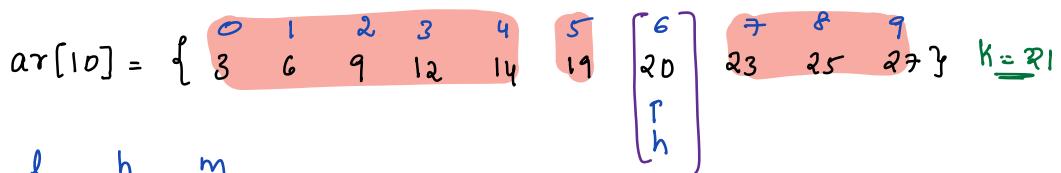
$$\frac{l \quad h}{m} = \left[\frac{l+h}{2} \right] \quad k=12$$

0 9 4 $\rightarrow \text{ar}[m] > k$: search left, $h=m-1$

0 3 1 $\rightarrow \text{ar}[m] < k$: search right, $l=m+1$

2 3 2 $\rightarrow \text{ar}[m] < k$: search right, $l=m+1$

3 3 3 $\rightarrow \text{ar}[m] == k$: return True



0 9 4 : $\text{ar}[4] < k$: search right = $l=m+1$

5 9 7 : $\text{ar}[7] > k$: search left = $h=m-1$

5 6 5 : $\text{ar}[5] < k$: search right = $l=m+1$

6 6 6 : $\text{ar}[6] < k$: search right = $l=m+1$

$\underbrace{7}_{6}$: $l > h$ Stop return False

```
bool search(int arr[N], int k)
```

```
l = 0, h = N-1
```

```
while(l <= h) {
```

$$m = \frac{l+h}{2}$$

```
if (arr[m] == k) { return True }
```

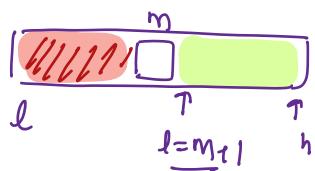
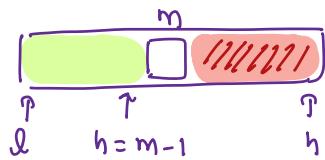
```
if (arr[m] > k) {
```

```
    // Search left  
    h = m-1
```

```
} else {
```

```
    // Search right  
    l = m+1
```

```
}
```

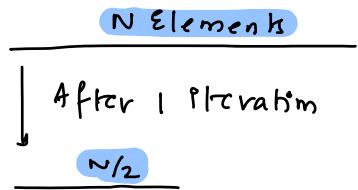


```
}
```

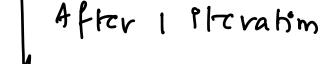
```
return false
```

```
}
```

TC:



$$\begin{aligned} N &\rightarrow N/2 \rightarrow N/4 \rightarrow N/8 \rightarrow \dots L \\ &= \log_2 N \text{ iterations} \end{aligned}$$



:

- :-)

Q8) Given a sorted arr[] find floor of given num k
 greatest ele $\leq k$ in arr[]

$$arr[] = \{ \overset{'}{0}, \overset{'}{1}, \overset{'}{2}, \overset{'}{3}, \overset{'}{4}, \overset{'}{5}, \overset{'}{6}, \overset{'}{7}, \overset{'}{8} \\ -5, 2, 3, 6, 9, 10, 11, 14, 18 \}$$

$k=5 : 3$

$k=4 : 3$

$k=10 : 10$

$k=-7 : \text{nothing}$

↳ INT_MIN

$k=24 : 18$

$k=12$ ans = INT_MIN Ideal: Iteration from

i : arr[i]

0 : $-5 : \text{ans} = -5$

1 : $2 : \text{ans} = 2$

2 : $3 : \text{ans} = 3$

3 : $6 : \text{ans} = 6$

4 : $9 : \text{ans} = 9$

5 : $10 : \text{ans} = 10$

6 : $11 : \text{ans} = 11$

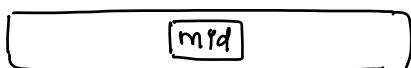
7 : $14 : 14 \text{ cannot be ans, break return ans}$

left \rightarrow right

update ans

TC: O(N)

Ideas:



Case-I: $arr[mid] == k$: return k



Case-II: $arr[mid] < k$: ans = arr[mid] go to right



Case-III $arr[mid] > k$: go to left

Tracing:

$ar[9] = \{ -5, 2, 3, 6, 9, 10, 11, 14, 18 \} \quad k=5$

$\begin{matrix} 0 & 1 & 2 & 3 \\ -5 & 2 & 3 & 6 \\ \uparrow & \uparrow & \uparrow & \uparrow \\ l & h & m \end{matrix}$

| l | h | m | $ans = INT_MIN$ |
|-----|-----|-----|---|
| 0 | 8 | 4 | $ar[m] > k$: goto left, $h=m-1$: $ans = INT_MIN$ |
| 0 | 3 | 1 | $ar[m] < k$: goto right, $l=m+1$: $ans = ar[m] = 2$ |
| 2 | 3 | 2 | $ar[m] < k$: goto right, $l=m+1$: $ans = ar[m] = 3$ |
| 3 | 3 | 3 | $ar[m] > k$: goto left, $h=m-1$: $ans = 3$ |
| 3 | 2 | | 2 : break return ans |

int floor(int ar[N], int k) { TC: $O(\log N)$ SC: $O(1)$

$l=0, h=N-1, ans = INT_MIN$
while ($l <= h$) {
 $m = (l+h)/2$
 if ($ar[m] == k$) { return k }
 if ($ar[m] < k$) {
 $ans = ar[m]$, goto right
 $l = m+1$
 }
 else // $ar[m] > k$
 goto left
 $h = m-1$
 }
}
return ans;

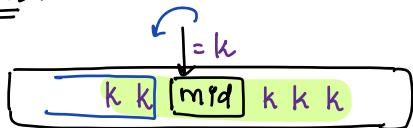
Q8) Given a sorted arr] find first occurrence index of given element?

$\text{arr}[] = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18\}$
 $\{ -5, -5, -3, 0, 0, 1, 1, 5, 5, 5, 5, 5, 5, 5, 8, 10, 10, 15, 15 \}$

k: target
5 7
-5 0
20 -1 → no element

Idea: Iterate on array from [0, n-1]
to get 1st occurrence index of k
 $Tc: O(N)$ $Sc: O(1)$

Idea:



Case-I: $\text{arr}[\text{mid}] == k$: $\text{ans} = \text{mid}, \text{goto left}$



Case-II: $\text{arr}[\text{mid}] < k$: goto right



Case-III $\text{arr}[\text{mid}] > k$: goto left

$\text{arr[]} = \{ -5, -5, -3, 0, 0, 1, 1, 5, 5, 5, 5, 5, 5, 8, 10, 10, 15, 15 \}$ $k=5$

$$l \quad h \quad m \quad \text{ans} = -1$$

0 18 9 $\text{arr[m]} == k$, goto left $h = m - 1$ $\text{ans} = 9$

0 8 4 arr[m] < k, goto right l = m + 1

5 8 6 ar[m] < k, goto right l = m+1

$\neq \& \neq \& \text{ax}[m] == k, \text{ goto left } h = m - 1 \quad \text{ans} = ?$

7 G : break return ans

int linear(int arr[], int k) { TC: O(N) SC: O(1)

$$l=0, h=N-1, ans = -1$$

```
while ( l <= h) {
```

$$m = (l+n)/1$$

if ($\text{arr}[m] == k$) {

ans = mid, goto left

$$h = m - 1$$

else if (arr[m] < k) {

| go to right

l = m + 1

3 100% =

else // arr[m] > k
| - - - - -

go to left

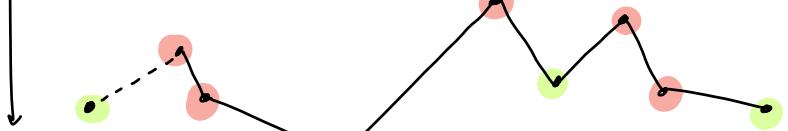
$$j = M - 1$$

return ans;

$$8:32 \rightarrow 8:40 \text{ hr:min}$$

48) Given unsorted arr[N] distinct elements return ^{any} local minima

An element is said to be local minima, if its less than than its adjacent elements



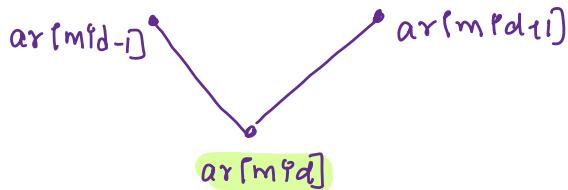
$$\begin{array}{c} \left[\begin{array}{ccc} \frac{n-1}{n} & \downarrow & \frac{n+1}{n} \\ & 0 & 1 \\ \hline n-2 & & n-1 \end{array} \right] \end{array}$$

$$\text{E8: } ar[8] = \left\{ \begin{array}{cccccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 9 & 8 & 7 & 3 & 6 & 4 & 1 & 5 & 2 \end{array} \right\} \quad \begin{array}{l} \text{return any local} \\ \text{ans} = \{3, 1, 2\} \end{array}$$

Ideal: Iterate on arr[], check if a particular element is local minima or not? Tc: O(N) Sc: O(1)

Idea 2:

Cases-I:

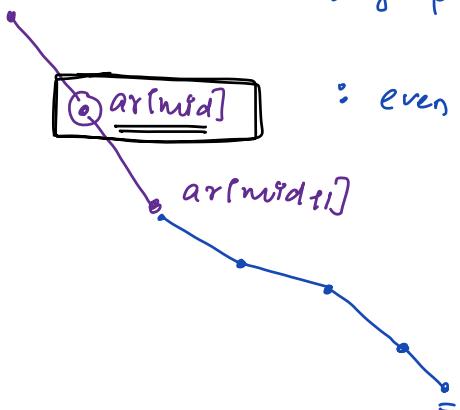


Case - II: goto right side: discard left side

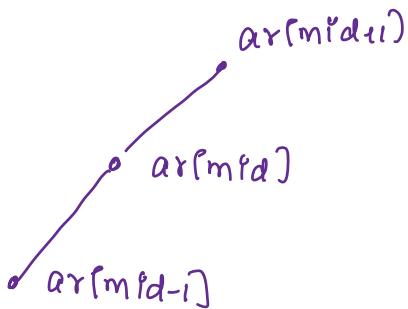
$\arctan(-x)$: graphs always decrease,

last element is local minima

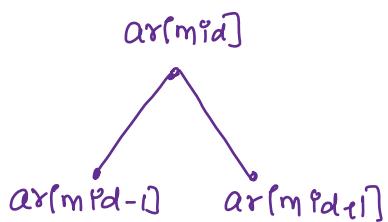
: even if there is an increase, that element is local min.



Case - I: goto left side, discard right side



Case - II: goto any side, we will find local minima



$$\underline{\text{Ex}}: \text{ar}[8] = \{ 0 \ 1 \ 2 \ \boxed{3} \ 4 \ 5 \ 6 \ 7 \ 8 \}$$

The array elements are 0, 1, 2, 3, 4, 5, 6, 7, 8. The element 3 is highlighted with a green box and has an upward arrow pointing to it from the label 'm'.

$l \ h \ m$

0 8 4 : if ($\text{ar}[m \text{id}] > \text{ar}[m \text{id}-1]$) goto left $h = m-1$

0 3 1 : if ($\text{ar}[m \text{id}] > \text{ar}[m \text{id}+1]$) goto right $l = m+1$

2 3 2 : if ($\text{ar}[m \text{id}] > \text{ar}[m \text{id}+1]$) goto right $l = m+1$

3 3 3 : $\text{ar}[m \text{id}]$ is local min. return $\text{ar}[m \text{id}] = 3$

int local_min (int $\underbrace{ar[N]}$) { Tc: O(log N) Sc: O(1)

$$\hookrightarrow \underline{N \geq 2}$$

To Do: local minima

Edge Case:

$\left[\begin{array}{l} \text{if } (ar[0] < ar[1]) \text{ return } ar[0] \\ \text{if } (ar[n-2] > ar[n-1]) \text{ return } ar[n-1] \end{array} \right]$

$l = 1, h = n - 2$

while ($l <= h$) {

$$m = (l+h)/2$$

if ($ar[m-1] > ar[m]$ & $ar[m] < ar[m+1]$) {

 return $ar[m]$

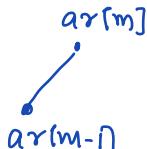
 if ($ar[m-1] < ar[m]$) {

 // goto left

$$h = m - 1$$

\nearrow if $m = n-1$ edge case

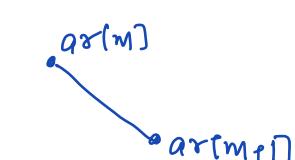
\nearrow if $m = 0$ edge case



 else // $ar[m] > ar[m+1]$

 // goto right

$$l = m + 1$$



}