Todays Content:

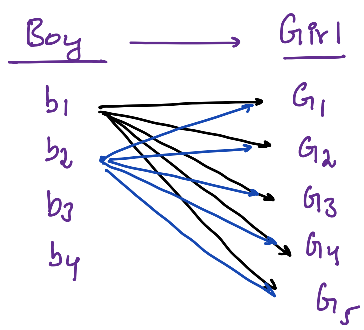
- Addition & multiplication rule
- Permutation Basics
- Combination
- Properties

Q) 3 T/F, how many ways we can answer them?

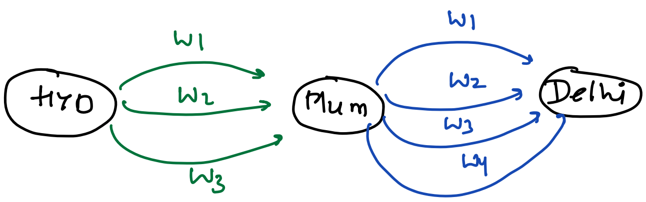
$\underline{2} * \underline{2} * \underline{2} = 8$ ways

$$\begin{bmatrix} T & T & T \\ T & T & F \\ T & F & T \\ T & F & F \\ F & T & T \\ F & T & F \\ F & F & T \\ F & F & F \end{bmatrix}$$

Q) 5 girls & 4 boys, how many pair: $\underset{\text{Boy}}{\underline{4}} \underset{\text{and}}{*} \underset{\text{Gir}}{\underline{5}} = 20$



Q):



ways HYD → Delhi via Mum :

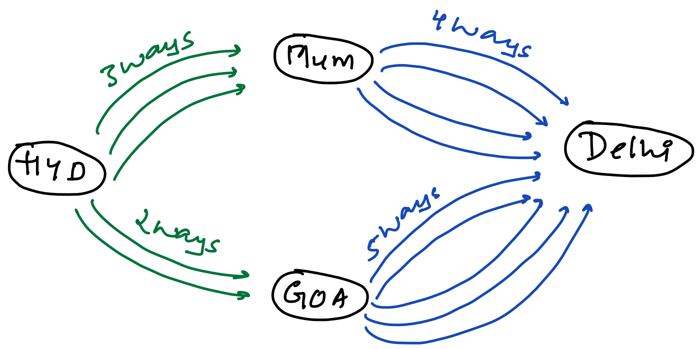
$\underbrace{\text{HYD} \longrightarrow \text{Mum}}_{\#3\text{ways}}$ && $\underbrace{\text{Mum} \longrightarrow \text{Delhi}}_{\#4\text{ways}}$ = 12ways

4Q) In Class 9 Girls & 8 Boys, number a we can select either a boy or a girl

$\underbrace{\text{9 Girls}}_{\text{9 ways}}$ or $\underbrace{\text{8 Boys}}_{\text{8 ways}}$

9 ways + 8 ways = 17 ways

5Q)



#ways Hyd → Delhi :

Hyd → Delhi via Mum or Hyd → Delhi via GOA

Hyd → Mum && Mum → Del or Hyd → GOA && GOA → Delhi

3 * 4 + 2 * 5

12 ways + 10 ways = 22 ways

Permutations : # ways to arrange {order matters}

Pair (i, j) (j, i) : Both are different

ways to arrange P_1, P_2, P_3 = 6 ways

P_1	P_2	P_3
P_1	P_3	P_2
P_2	P_1	P_3
P_2	P_3	P_1
P_3	P_1	P_2
P_3	P_2	P_1

$\underline{3}$ & $\underline{2}$ & $\underline{1}$ = $3!$

$P_1 \to P_2$ --- P_3
$P_1 \to P_3$ ---- P_2

$P_2 \to P_1$ --- P_2
$P_2 \to P_3$ ---- P_1

$P_3 \to P_1$ ---- P_2
$P_3 \to P_2$ ---- P_1

ways to arrange 4, P_1 P_2 P_3 P_4

$\underline{4} * \underline{3} * \underline{2} * \underline{1} = 4! = 24$

$P_1 * \{P_2, P_3, P_4\}$

$P_2 : \{P_1, P_3, P_4\}$

$P_3 : \{P_1, P_2, P_4\}$

$P_4 : \{P_1, P_2, P_3\}$

obs: # ways to arrange N

↳ $\underline{N} \times \underline{N-1} \times \underline{N-2} \times \underline{N-3} \times \cdots \underline{1} = N!$

ways to arrange 2, from 4 people {P1 P2 P3 P4}

$\underline{4} * \underline{3} = 12$

P1 P2	P2 P1	P3 P1	P4 P1
P1 P3	P2 P3	P3 P2	P4 P2
P1 P4	P2 P4	P3 P4	P4 P3

ways to arrange 3, from 5 people = {P1 P2 P3 P4 P5}

$\underline{5} * \underline{4} * \underline{3} = 60$

ways to arrange r, from N people = = $N!/(N-r)! = {}^{N}P_{r}$

$\underset{1}{\underline{N}} * \underset{2}{\underline{N-1}} * \underset{3}{\underline{N-2}} * \underset{4}{\underline{N-3}} * \ldots\ldots \underset{r}{\underline{N-r+1}}$

ways =

$$\frac{N * (N-1) * (N-2) \ldots\ldots (N-r+1) * (N-r) * (N-r-1) * (N-r-2) \ldots 1}{(N-r) * (N-r-1) * (N-r-2) \ldots 1}$$

$= N!/(N-r)!$

$$\boxed{\# \text{ arrange } r \text{ from } N \text{ items} = {}^{N}P_{r} = \frac{N!}{(N-r)!}}$$

Permutate with Repeation

Permutate 4 items = $\underline{\underline{P_1 \; P_2 \; P_3 \; P_4}}$

$P_1 P_2 P_3 P_4$	$P_2 P_1 P_3 P_4$	$P_3 P_1 P_2 P_4$	$P_4 P_1 P_2 P_3$
$P_1 P_2 P_4 P_3$	$P_2 P_1 P_4 P_3$	$P_3 P_1 P_4 P_2$	$P_4 P_1 P_3 P_2$
$P_1 P_3 P_2 P_4$	$P_2 P_3 P_1 P_4$	$P_3 P_2 P_1 P_4$	$P_4 P_2 P_1 P_3$
$P_1 P_3 P_4 P_2$	$P_2 P_3 P_4 P_1$	$P_3 P_2 P_4 P_1$	$P_4 P_2 P_3 P_1$
$P_1 P_4 P_2 P_3$	$P_2 P_4 P_1 P_3$	$P_3 P_4 P_1 P_2$	$P_4 P_3 P_1 P_2$
$P_1 P_4 P_3 P_2$	$P_2 P_4 P_3 P_1$	$P_3 P_4 P_2 P_1$	$P_4 P_3 P_2 P_1$

Permutate 4 items = $P_1 \; P_2 \; P_3 \; P_4$, but $P_1 = P_2 = P$ $= 12$

$P \; P \; P_3 \; P_4$ —	$P \; P \; P_3 \; P_4$	$P_3 \; P \; P \; P_4$	$P_4 \; P \; P \; P_3$
$P \; P \; P_4 \; P_3$ —	$P \; P \; P_4 \; P_3$	$P_3 \; P \; P_4 \; P$	$P_4 \; P \; P_3 \; P$
$P \; P_3 \; P \; P_4$ —	$P \; P_3 \; P \; P_4$	$P_3 \; P \; P \; P_4$	$P_4 \; P \; P \; P_3$
$P \; P_3 \; P_4 \; P$ —	$P \; P_3 \; P_4 \; P$	$P_3 \; P \; P_4 \; P$	$P_4 \; P \; P_3 \; P$
$P \; P_4 \; P \; P_3$ —	$P \; P_4 \; P \; P_3$	$P_3 \; P_4 \; P \; P$	$P_4 \; P_3 \; P \; P$
$P \; P_4 \; P_3 \; P$ ↔	$P \; P_4 \; P_3 \; P$	$P_3 \; P_4 \; P \; P$	$P_4 \; P_3 \; P \; P$

Permutate 4 items = P_1 P_2 P_3 P_4, but $P_1 = P_2 = P_3 = 4$

$P\ P\ P\ P_4$	$P\ P\ P\ P_4$	$P\ P\ P\ P_4$	$P_4\ P\ P\ P$
$P\ P\ P_4\ P$	$P\ P\ P_4\ P$	$P\ P\ P_4\ P$	$P_4\ P\ P\ P$
$P\ P\ P\ P_4$	$P\ P\ P\ P_4$	$P\ P\ P\ P_4$	$P_4\ P\ P\ P$
$P\ P\ P_4\ P$	$P\ P\ P_4\ P$	$P\ P\ P_4\ P$	$P_4\ P\ P\ P$
$P\ P_4\ P\ P$	$P\ P_4\ P\ P$	$P\ P_4\ P\ P$	$P_4\ P\ P\ P$
$P\ P_4\ P\ P$	$P\ P_4\ P\ P$	$P\ P_4\ P\ P$	$P_4\ P\ P\ P$

$$\boxed{P_1\ P_2\ P_3}\ P_4 = \frac{4!}{3!} = 4$$

Permutate 4 items = P_1 P_2 P_3 P_4, but $\overbrace{P_1 = P_2}^{P_1}$ $\overbrace{P_3 = P_4}^{P_3}$

$P_1\ P_1\ P_3\ P_3$	$P_1\ P_1\ P_3\ P_3$	$P_3\ P_1\ P_1\ P_3$	$P_3\ P_1\ P_1\ P_3$
$P_1\ P_1\ P_3\ P_3$	$P_1\ P_1\ P_3\ P_3$	$P_3\ P_1\ P_3\ P_1$	$P_3\ P_1\ P_3\ P_1$
$P_1\ P_3\ P_1\ P_3$	$P_1\ P_3\ P_1\ P_3$	$P_3\ P_1\ P_1\ P_3$	$P_3\ P_1\ P_1\ P_3$
$P_1\ P_3\ P_3\ P_1$	$P_1\ P_3\ P_3\ P_1$	$P_3\ P_1\ P_3\ P_1$	$P_3\ P_1\ P_3\ P_1$
$P_1\ P_3\ P_1\ P_3$	$P_1\ P_3\ P_1\ P_3$	$P_3\ P_3\ P_1\ P_1$	$P_3\ P_3\ P_1\ P_1$
$P_1\ P_3\ P_3\ P_1$	$P_1\ P_3\ P_3\ P_1$	$P_3\ P_3\ P_1\ P_1$	$P_3\ P_3\ P_1\ P_1$

$$P_1\ P_2\ P_3\ P_4 = \frac{4!}{2! * 2!} = \frac{24}{4} = 6$$

// Say ways to arrange N items, but in items

$$\frac{N!}{a! * b! * c!}$$

a items repeat =

b items repeat =

c items repeat

Selection: #ways to select {order won't matter}

$\{ (\underbrace{i, j) \ (j, i}_{\text{Same items}}) \}$ Both are same

Say 4 people, how many ways we can select 2 people?

$P_1 \ P_2 \ P_3 \ P_4$: = 6

$P_1 \ P_2$	$P_2 \ P_1$	$P_3 \ P_1$	$P_4 \ P_1$
$P_1 \ P_3$	$P_2 \ P_3$	$P_3 \ P_2$	$P_4 \ P_2$
$P_1 \ P_4$	$P_2 \ P_4$	$P_3 \ P_4$	$P_4 \ P_3$

Say 4 people, how may ways we can select 3 people

$P_1 \ P_2 \ P_3 \ P_4$:

Arrange 3 people : → Selections → 4 diff ways

$P_2 \ P_3 \ P_4$	$P_1 \ P_3 \ P_4$	$P_3 \ P_1 \ P_2$	$P_4 \ P_1 \ P_3$
$P_2 \ P_4 \ P_3$	$P_1 \ P_4 \ P_3$	$P_3 \ P_2 \ P_1$	$P_4 \ P_3 \ P_1$
$P_2 \ P_1 \ P_3$	$P_1 \ P_2 \ P_4$	$P_3 \ P_2 \ P_4$	$P_4 \ P_2 \ P_3$
$P_2 \ P_3 \ P_1$	$P_1 \ P_4 \ P_2$	$P_3 \ P_4 \ P_2$	$P_4 \ P_3 \ P_2$
$P_2 \ P_1 \ P_4$	$P_1 \ P_2 \ P_3$	$P_3 \ P_1 \ P_4$	$P_4 \ P_1 \ P_2$
$P_2 \ P_4 \ P_1$	$P_1 \ P_3 \ P_2$	$P_3 \ P_4 \ P_1$	$P_4 \ P_2 \ P_1$

$\boxed{P_1 \; P_2 \; P_3}$ → arrange → select items

	arrange	select items
$P_1 \; P_2 \; P_3$	$3!$	1
Total	24	x

$x * 3! = 24$

$x = \frac{24}{3!} = 4$

ways to select r from N items $= N! / r! * [N-r]! = \boxed{{}^{N}C_r}$

	arrange	select
$\boxed{r \text{ items}}$	$r!$	1
# ways to arrange r from N	$\frac{N!}{(N-r)!}$	x

$$x * r! = \frac{N!}{(N-r)!}$$

$$x = \frac{N!}{r!\,(N-r)!}$$

${}^{N}P_r$: # arrange r from N

Say ways to arrange N items, but in items

a items repeat =

b items repeat =

c items repeat

$$\left[\frac{N!}{a! * b! * c!} \right.$$

${}^{N}C_r$: # select r from N $=$ ${}^{N}C_r = \frac{N!}{r!\,(N-r)!} = \frac{{}^{N}P_r}{r!}$

$$\boxed{{}^{N}C_r = \frac{{}^{N}P_r}{r!}}$$

Properties:

$$^{N}C_{0} = 1 = \frac{N!}{(0!)(N!)} = 1$$

$$^{N}C_{N} = 1$$

$\boxed{0! = 1}$

{Extra/Doubts

Q) 5 people, #ways to select 2 people

P_1 P_2 P_3 P_4 P_5

$\underbrace{P_1 \quad P_2}$ → { P_3 P_4 P_5 }
P_1 P_3 → { P_2 P_4 P_5 }
P_1 P_4 → { P_2 P_3 P_5 }
P_1 P_5 → { P_2 P_3 P_4 }
P_2 P_3 → { P_1 P_4 P_5 }
P_2 P_4 → { P_1 P_3 P_5 }
P_2 P_5 → { P_1 P_3 P_4 }
P_3 P_4 → { P_1 P_2 P_5 }
P_3 P_5 → { P_1 P_2 P_4 }
P_4 P_5 → { P_1 P_2 P_3 }

$$\boxed{^{N}C_{R} = {}^{N}C_{N-R}}$$

#ways to select R items

#ways to select N-R items

// Say # ways to select r items from N: ${}^{N}C_{R}$

items: $a_1 \quad a_2 \quad a_3 \; .. \; a_{N-1}$ $\quad a_N$

pick it: N-1, r-1 items : ${}^{N-1}C_{r-1}$

+

leave it: N-1, r items ${}^{N-1}C_{r}$

way1 { Select r-1 from N-1 $\quad a_N$ } taking

way2 { Select r from N-1 } leave it

$$\boxed{{}^{N}C_{R} = {}^{N-1}C_{R} + {}^{N-1}C_{R-1}}$$

expand formula TODO

$R! = R * (R-1)!$

$(N-R)! = N-R * (N-R-1)!$

$$\frac{N!}{(N-R)!\,R!} = \frac{(N-1)!}{R!\,(N-R-1)!} + \frac{(N-1)!}{(R-1)!\,(N-R)!}$$

$$= \frac{(N-1)!}{R*(R-1)!\,(N-R-1)!} + \frac{(N-1)!}{(N-R)(N-R-1)!(R-1)!}$$

$$= \frac{(N-1)!}{(N-R-1)!\,(R-1)!}\left[\frac{1}{R} + \frac{1}{N-R}\right]$$

LCM

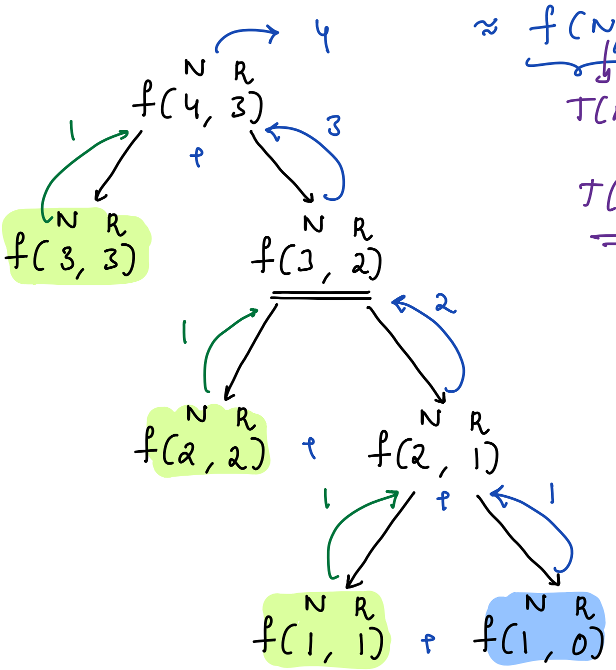
$$= \frac{(N-1)!}{(N-R-1)!\,(R-1)!}\left[\frac{N-\cancel{R}+\cancel{R}}{R * N-R}\right]$$

$$= \frac{N!}{(N-R)!\,R!} = {}^{N}C_{R}$$

$$^{N}C_{R} = {}^{N-1}C_{R} + {}^{N-1}C_{R-1}$$

$$\boxed{f(N, R) = f(N-1, R) + f(N-1, R-1)}$$

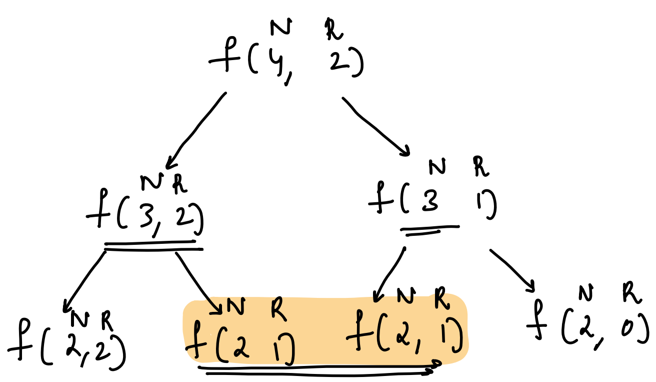
```
int fun(N, R){   ≈ O(2N)
    if(r==0 || N==R) return 1
    return fun(N-1, R) + fun(N-1, R-1)
}
```



$$\approx f(N, R) = f(N-1, R) + f(N-1, R)$$

$$T(N) = T(N-1) + T(N-1) + 1$$

$$T(N) = 2T(N-1)+1 : O(2^N)$$



Using recursion, same subproblem is repeating, calculate it once store it & use it again

// $f(\overset{N}{4}, \overset{R}{3})$:

mat[5, 4] :

	0	1	2	3
0	0	0	0	0
1	1	1	0	0
2	1	2	1	0
3	1	3	3	1
4	1	4	6	4

final ans

$${}^{N}C_{R} = {}^{N-1}C_{R} + {}^{N-1}C_{R-1}$$

$${}^{2}C_{1} = {}^{1}C_{1} + {}^{1}C_{0}$$

$${}^{3}C_{1} = {}^{2}C_{1} + {}^{2}C_{0}$$

$${}^{3}C_{2} = {}^{2}C_{2} + {}^{2}C_{1}$$

$${}^{4}C_{1} = {}^{3}C_{1} + {}^{3}C_{0}$$

$${}^{4}C_{2} = {}^{3}C_{2} + {}^{3}C_{1}$$

$${}^{4}C_{3} = {}^{3}C_{3} + {}^{3}C_{2}$$

TC: O(N*R) SC: O(N*R)

```
__ cal ( N, R ){
    int mat[N+1][R+1] = 0
    i= 1; i<= N; i++){
        mat[i][0] = 1
        j=1; j<= R; j++){
            if( j>i ){ mat[i][j] = 0}
            else{ mat[i][j] = mat[i-1][j] + mat[i-1][j-1]
        }
    }
    return mat[N][R]
}
```