

Todays Content:

- Quick sort
- Countsort
- Rabin Sort :

Q) Given  $\text{arr}[N]$  elements, re-arrange the array such that

- $\text{arr}[0]$  should go to correct sorted position } Expected SC:  $O(1)$
- All elements  $\leq \text{arr}[0]$  leftside of  $\text{arr}[0]$
- All elements  $> \text{arr}[0]$  rightside of  $\text{arr}[0]$

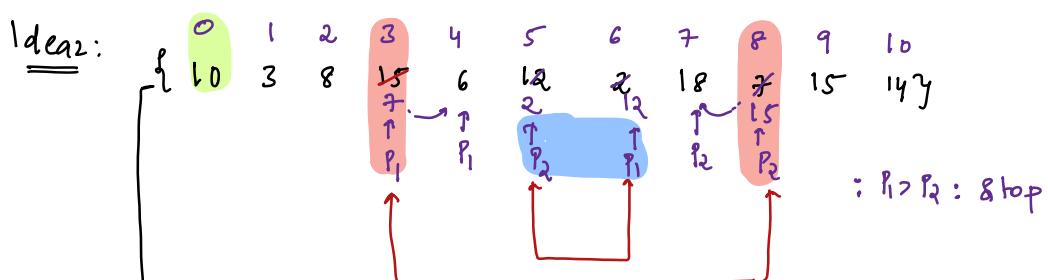
$$\text{arr}[11] = \{ 0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \\ 10 \ 3 \ 8 \ 15 \ 6 \ 12 \ 2 \ 18 \ 7 \ 15 \ 14 \}$$

re-arrange =  $\boxed{\leq 10} \quad 10 \quad \boxed{> 10}$

Idea1: Sort  $\text{arr}[]$ ,  $T.C: O(N \log N)$

$$\text{Sort arr}[] = \{ 0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \\ 2 \ 3 \ 6 \ 7 \ 8 \ 10 \ 12 \ 14 \ 15 \ 15 \ 18 \}$$

$\leq 10 \checkmark \quad > 10 \checkmark$



$$\{ 0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \\ 10 \ 3 \ 8 \ 7 \ 6 \ 2 \ 12 \ 18 \ 15 \ 15 \ 14 \}$$

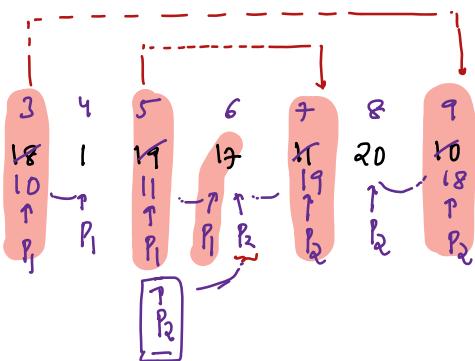
$P_1$        $P_2$

Swap  $\text{arr}[P_1]$  &  $\text{arr}[P_2]$

$$\{ 2 \ 3 \ 8 \ 7 \ 6 \ 10 \ 12 \ 18 \ 15 \ 15 \ 14 \\ \leq 10 \quad > 10$$

E<sub>n2</sub>:

$$ar[12] = \{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 14, 6, 8, 18, 10, 1, 19, 11, 12, 17, 19, 20, 18, 33, 29 \}$$



$$ar[12] = \{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 14, 6, 8, 10, 1, 11, 12, 17, 19, 20, 18, 33, 29 \}$$

swap ar[0] → ar[P<sub>2</sub>] =

$$ar[12] = \{ 11, 6, 8, 10, 1, 14, 12, 19, 20, 18, 33, 29 \}$$

E<sub>n3</sub>: ar[4]: { 0, 1, 2, 3, 4 } no element, P<sub>1</sub>=4, P<sub>2</sub>=3: break

Swap ar[0] → ar[P<sub>2</sub>] =

$$ar[4]: \{ 7, 4, 1, 8 \}$$

```
void re-arrange(int arr[N]) { TC: O(N) SC: O(1)
```

```
//reference = arr[0]
```

```
P1 = 1, P2 = n - 1
```

```
while (P1 <= P2) {
```

```
    if (arr[P1] <= arr[0]) { P1++ }
```

```
    else if (arr[P2] > arr[0]) { P2-- }
```

```
    else {
```

```
        swap(arr[P1], arr[P2])
```

```
        P1++, P2--
```

```
} swap(arr[0] & arr[P2])
```

```
}
```

```
3
```

```
3
```

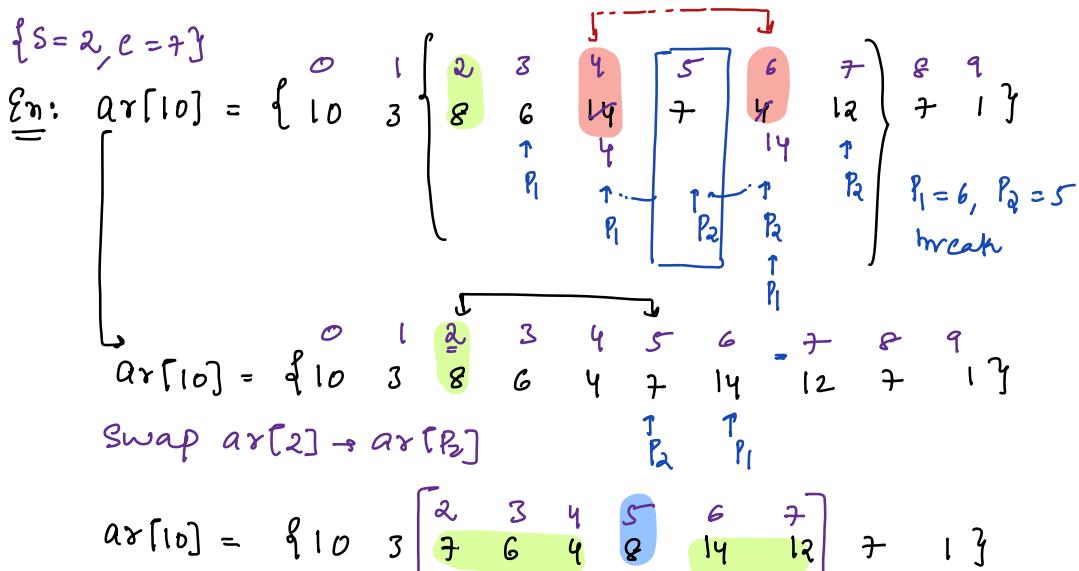
```
}
```

Q8) Given  $\text{arr}[N]$  & subarray  $[s, e]$  Re-arrange sub  $[s - e]$  such that

→  $\text{arr}[s]$  should come to correct pos

→ all elements  $x = \text{arr}[s]$  should left

→ all elements  $> \text{arr}[s]$  should right



PT re-arrange( $\text{int arr}[N], \text{int } s, \text{int } e\}$ ) TC:  $O(N)$  SC:  $O(1)$

//reference =  $\text{arr}[s]$

$P_1 = s+1, P_2 = e$

while ( $P_1 <= P_2$ ) {

    if ( $\text{arr}[P_1] <= \text{arr}[s]$ ) {  $P_1++$  }

    else if ( $\text{arr}[P_2] > \text{arr}[s]$ ) {  $P_2--$  }

    else

        swap( $\text{arr}[P_1], \text{arr}[P_2]$ )

$P_1++, P_2--$

}

swap( $\text{arr}[s] \& \text{arr}[P_2]$ )

return  $P_2$ ; // returning index at which we keep  $\text{arr}[s]$

### 3Q] Sort entire array:

Ass: Sort entire subarray from [s e]

```
void Quicksort(int ar[], int s, int e){
```

```
    if(s >= e) { return; } // Doubt
```

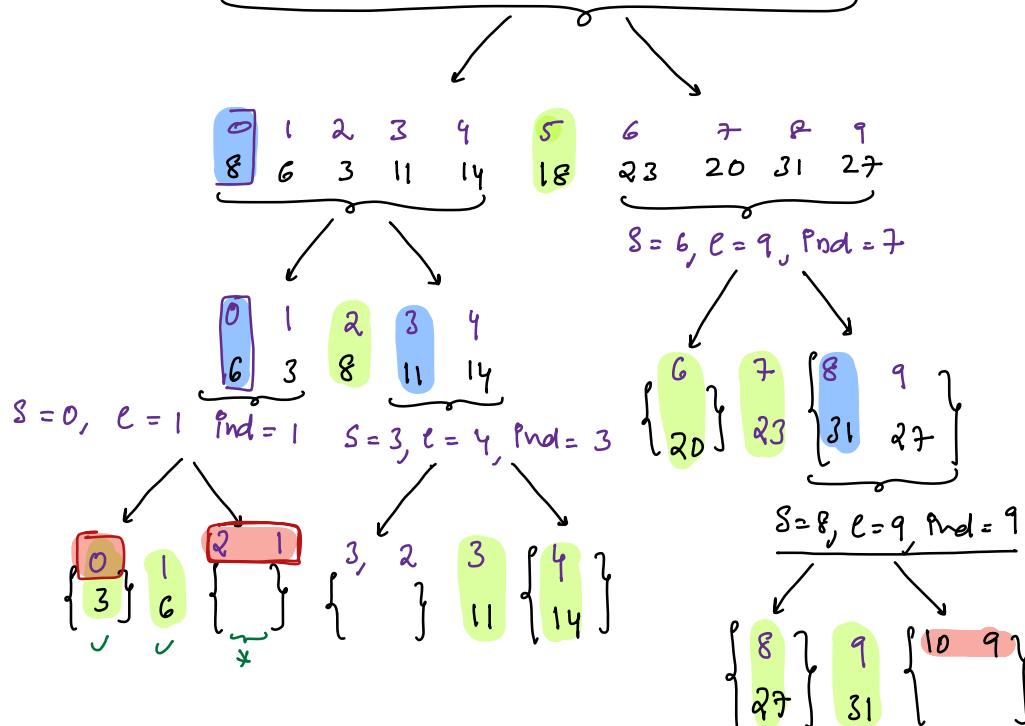
```
    int ind = rearrange(ar, s, e); // keep ar[s] in correct pos
```

```
    sort(ar, s, ind-1)
```

```
} sort(ar, ind+1, e)
```



Dryrun:  $ar[10]: \underline{18} \quad 8 \quad 6 \quad 3 \quad 11 \quad 14 \quad 23 \quad 20 \quad 31 \quad 27$



Final data:

$\rightarrow \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 6 & 8 & 11 & 14 & 18 & 20 & 23 & 27 & 31 \end{matrix}$

8:20 → 8:30 am

```
void Quicksort(int ar[], int s, int e) {

```

```
    if(s >= e) { return; } // Doubt
    int ind = rearrange(ar, s, e)
    sort(ar, s, ind-1)
    sort(ar, ind+1, e)
}
```

Recursive Relations:

$$\underline{\text{Best}}: T(N) = N + 2T(N/2)$$

$$\downarrow : T(N) = N \log N$$

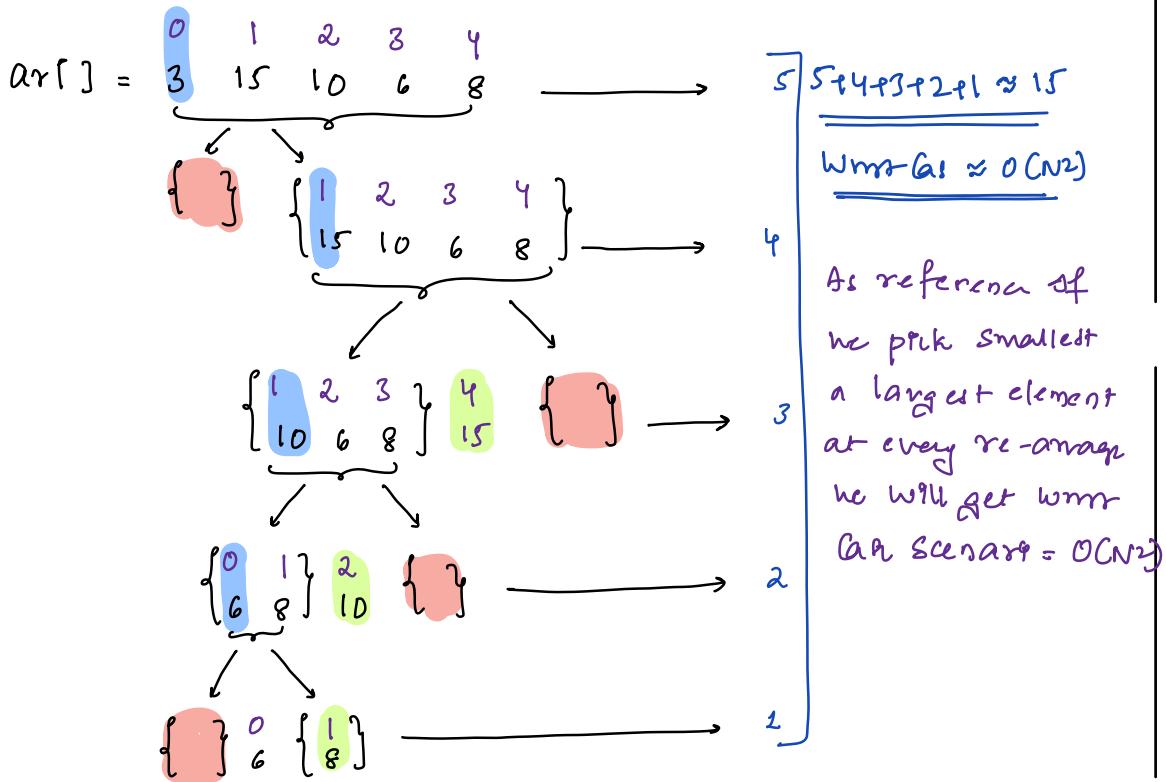
$$\underline{\text{Worst}}: T(N) = N + T(N-1)$$

all elements goes to one side

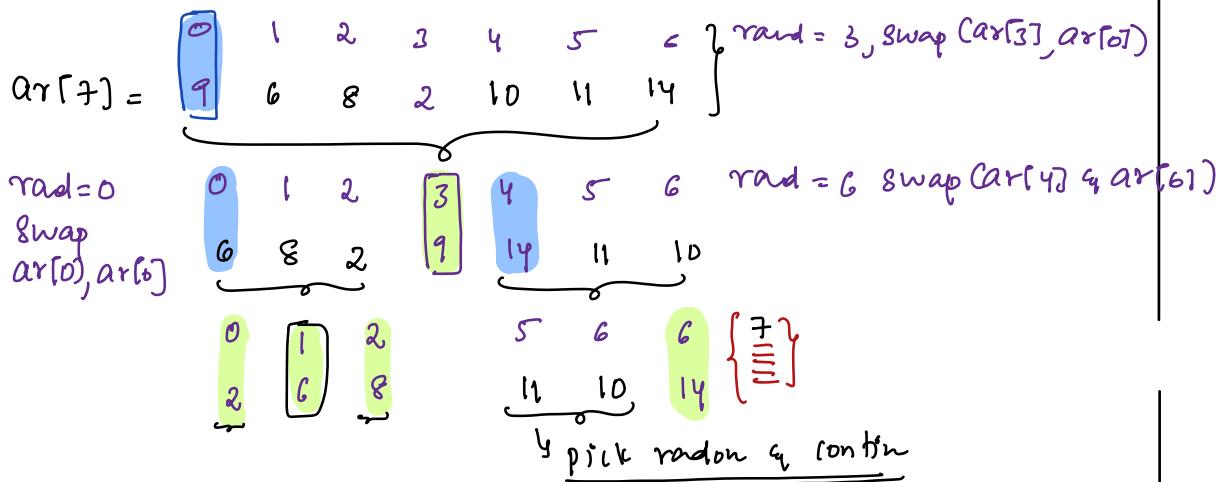
$$\Rightarrow T(N) = O(N^2)$$

Worst Case:

re-arrange iterations



Optimized: pick any random index ele in given range as ref



### Final Quick Sort

```

int re-arrange(int ar[], int s, int e) {
    TC: O(N log N)
    //random index from [s - e]
    r = rand(s, e) //return random number
    swap(ar[s], ar[r])
    p1 = s+1, p2 = e
    while (p1 <= p2) {
        if (ar[p1] <= ar[s]) { p1++ }
        else if (ar[p1] > ar[s]) { p2-- }
        else {
            swap(ar[p1], ar[p2])
            p1++, p2--
        }
    }
    swap(ar[s] & ar[p2])
    return p2; //returning indices at which we keep ar[s]
}

```

## Inbuilt :

Java → Tim Sort : mergeSort + QuickSort of a reference/pivot

C++ → IntrSort : QuickSort + Heap Sort  
↳ of future

python → Tim Sort : Insertion, Merge

→ pick k min : Selection

→ pick man in stable : Bubble

→ Insert in sortedT : Insertion

→ Sort entire data : Inbuilt sort

### CountSort:

// Given arr[N], all elements in range [1-4] sort arr[]

$$\text{arr}[] = \{ \begin{smallmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 1 & 4 & 4 & 2 & 1 & 3 & 3 & 2 & 1 \end{smallmatrix} \}$$

Idea1: Sort arr[]

TC:  $N \log N$

Idea2: Store freq of all elements [1-4] in hashmap

ele : freq

1 : 3

2 : 2

3 : 3

4 : 2

1 1 1 7

2 2

3 3 3

4 4

1 1 1 2 2 3 3 3 4 4

→ Pseudo Code:

Step1: Store all elements with freq in hashmap

Step2:  $k=0$

$i = 1; i \leq 4; i++ \{$

// how many time i occurs = hm[i]

$j = 0; j \leq hm[i]; j++ \{$

$\quad \quad \quad \text{arr}[k] = i, k++;$

// arr[] is sorted

SC = Data in range  $[s \ e]$  ?  $\underline{TC} = O(N)$   $\underline{SC} = O(N)$

Step 1: Store all elements with freq in hashmap  $\Rightarrow O(N)$

Step 2:  $k=0$   $\xrightarrow{\text{data in range } [s \ e]}$

```
i = s; i <= e; i++ {  
    // how many time i occurs = hm[i]  
    j = 0; j <= hm[i]; j++ {  
        ar[k] = i, k++;  
    }  
}
```

// ar[] is sorted

$\underline{TC} = O(N \times R)$

$$R = e - s$$

$\rightarrow O(N + R)$   
 $\downarrow$   
R: depends on data

Ex: ar[10] all elements in range  $[1 \ 100]$

$\hookrightarrow \{ 1 \ 2 \ 2 \ 6 \ 100 \ 2 \ 2 \ 1 \ 9 \ 80 \ 80 \ 1 \ 9 \ 9 \}$

$i = s \Rightarrow 1$

j: freq of  $[i]$

1	$\text{freq}[1]$
2	$\text{freq}[2]$
3	2
4	1
5	1
6	$\text{freq}[6]$
7	1
8	$\text{freq}[9]$
9	$\text{freq}[8]$
:	
50	$\text{freq}[50]$
:	
100	$\text{freq}[100]$

Total:  $\begin{cases} \text{sum of all freq}() = N \\ \text{sum of all } i \approx e-s \end{cases}$

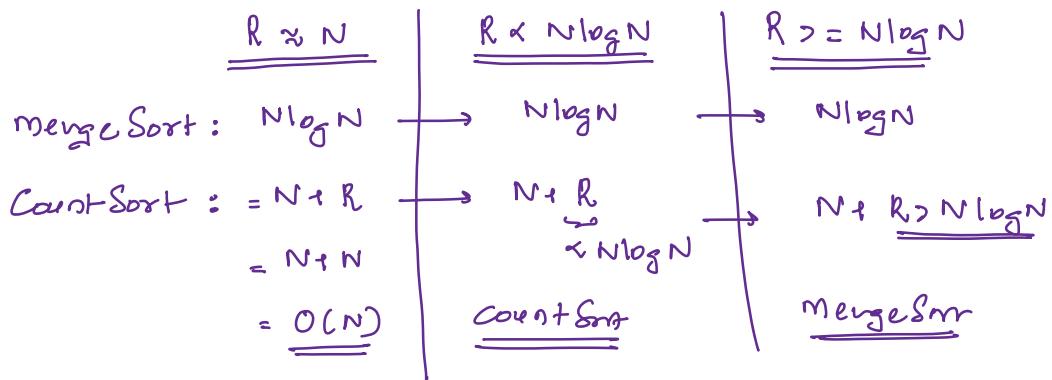
TC:  $O(N + e - s)$

In general  $e-s$  is called Range R

TC:  $O(N + R)$

Sorting data with freq = CountSort

//  $arr[N]$  : Range R = max - min of array

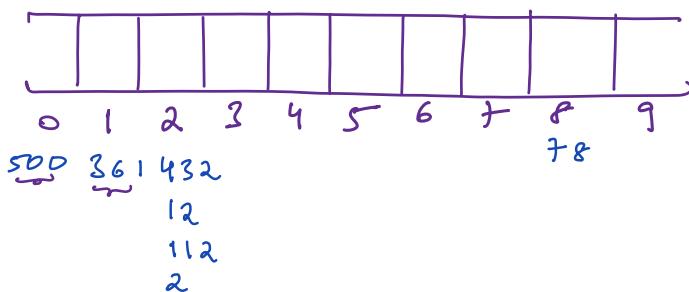


if Range R  $\approx N$  : Good to go with CountSort

RadixSort: Sort data digit by digit from one place  $\rightarrow \dots$

$arr[] = 361 \quad 432 \quad 12 \quad 78 \quad 500 \quad 112 \quad 2$

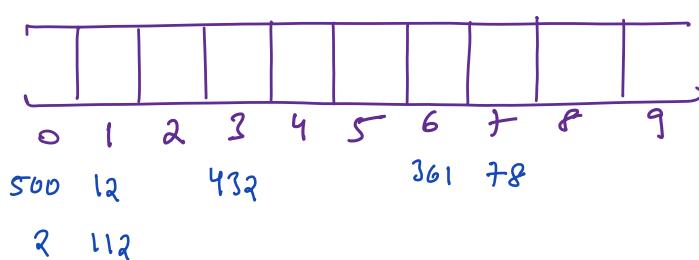
Stable Sort data based on 1's place



$Tc: N$

$arr[]: 500 \quad 361 \quad 432 \quad 12 \quad 112 \quad 2 \quad 78$

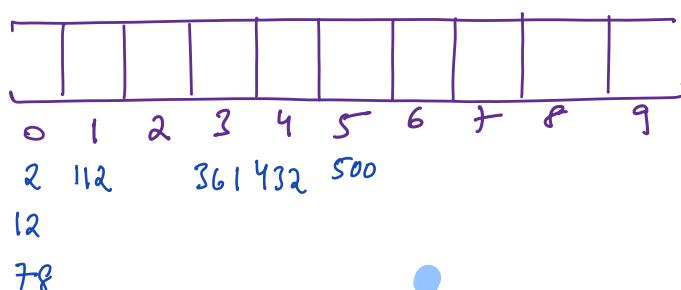
Stable Sort data based on 10's place



$\frac{N}{10}$

$arr[]: 500 \quad 2 \quad 12 \quad 112 \quad 432 \quad 361 \quad 78$

Stable Sort data based on 100's place



$\frac{N}{100}$

$(arr[]): 2 \quad 12 \quad 78 \quad 112 \quad 361 \quad 432 \quad 500$

If given integers  $\approx 10^9 \rightarrow$  digits  $\approx 10 \rightarrow$  radix sort  $= N \times 10 \Rightarrow O(N)$

$Sc = O(N)$

## TODO: Radix Sort real time applications TODO

→ BS + 2P + Hashing + Strings: 9

(41) : —

→ Linked / Stacks : 7

$$\begin{array}{r} 41 \ 11 \ 22 \ 33 \\ \hline 48 \ 3 \\ = (18) \Rightarrow \end{array}$$

→ Trees → 6

→ Heaps / Trees / Greedy = 5

Collections:

→ Backtracking = 2

Sep: 27



→ DP → 7

Out: 4



11



18



25

Nov: 1



8



15



22



29

Dec: 6



13



20



1.

