

Today's Content:

- Workers allocation
- Aggressive Cows
- ∞ Array[]

Q1: Given N tasks, k workers & time taken for each task, find min time in which we can complete all tasks → no sorting

Notes: A single worker can only do continuous set of tasks
 All workers start their assigned tasks at same time
 A task can only be assigned to 1 worker

Ex1: 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14

$N = 15$: 3 5 1 7 8 2 5 3 10 1 4 7 5 4 6

$k = 3$: $W_1 = 9$ $W_2 = 17$ $W_3 = 45$

$k = 3$: $W_1 = 24$ $W_2 = 21$ $W_3 = 26$

$k = 3$: $W_1 = 24$ $W_2 = 25$ $W_3 = 22$

Time Taken

45

26

25

Ex2:
 $arr[6] = 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 0 \ 1$
 $k = 2$ $W_1 = 5$ $W_2 = 10 \ 1$

Time taken
 101
Idea:
 → avg time
 → N/k length subarrays

Idea: In $N-1$ Empty slots, we need to keep $k-1$ sticks

#ways = $N-1 C_{k-1} \times N$ → Time taken to get amount of work assigned to each perm

To distribute tasks to workers

opt: TC: $N + N-1 C_{k-1} \times k$

Idea2: Binary Search:

a) : Target \rightarrow Min time to finish tasks

b) Search Space: $\left[\begin{array}{cc} \text{low} & \text{high} \\ \text{min of time} & \text{sum of all times} \end{array} \right]$

Ex1: $ar[4] = \{10, 2, 9, 8\}$ Ex2: $ar[4] = \{10, 15, 11, 13\}$
 $k=1$, Time taken = 29 w_1 w_2 w_3 w_4

$k=4$ = Time taken = 15

c) discard: 

Case-I: If we can do task in mid time:
 $ans = mid$ goto left



Case-II: If we cannot do task in mid time:
 goto right

$N=15$:

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|----|---|----|----|----|----|----|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 3 | 5 | 1 | 7 | 8 | 2 | 5 | 3 | 10 | 1 | 4 | 7 | 5 | 4 | 6 |

$k=4$ $w_1=26$ $w_2=30$ $w_3=15$

$w_4 = \{ \}$

$k=4$ $w_1=9$ 7 $w_3=10$ $w_4=8$ Tasks are left out

Can it finished in 10 min?

..... 8 9 10
 F F F F F

Can it finished in 30 min?

30 31 32 33 ... -
 T T T T ...

Trace & Pseudocode

→ // Task → // worker → Time for each task

int minTime(int N, int k, int time[]) { TC: $\log_2(\frac{\text{ele}}{2}) \times N$

l = min of arr[] h = sum of arr[] ans = ____

while (l <= h) {

 m = (l+h)/2

 // Can we finish N tasks in m time

 if (check(N, k, time[], m) == True) {

 ans = m; // goto left

 h = m-1

 else { // not possible goto right

 l = m+1

return ans

#ele on which we
apply BS = h-l+1

ele ≈ [sum of arr[] ele]
 - min of arr[]

bool check(int N, int k, int time[], int T) { TC: O(N) SC: O(1)

 c = 0, s = 0

 for (i = 0; i < N; i++) {

 s = s + arr[i] // add task time

 if (s > T) { // re-assign arr[i] task to another worker

 c = c+1, s = arr[i]

 if (c == k) { // utilised k worker but task left?

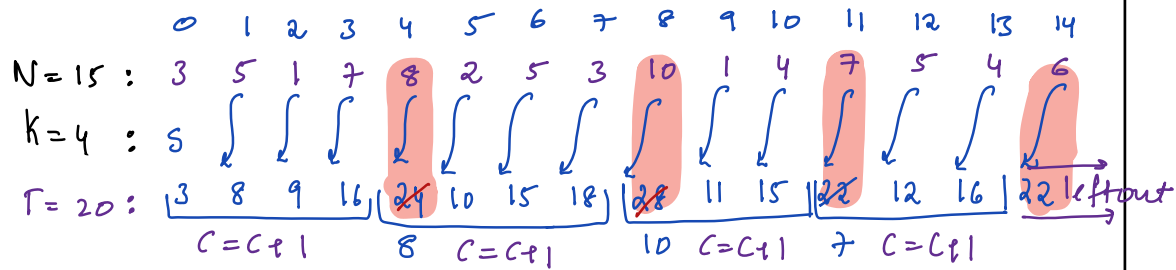
 return false

 s ≠ 0, some work

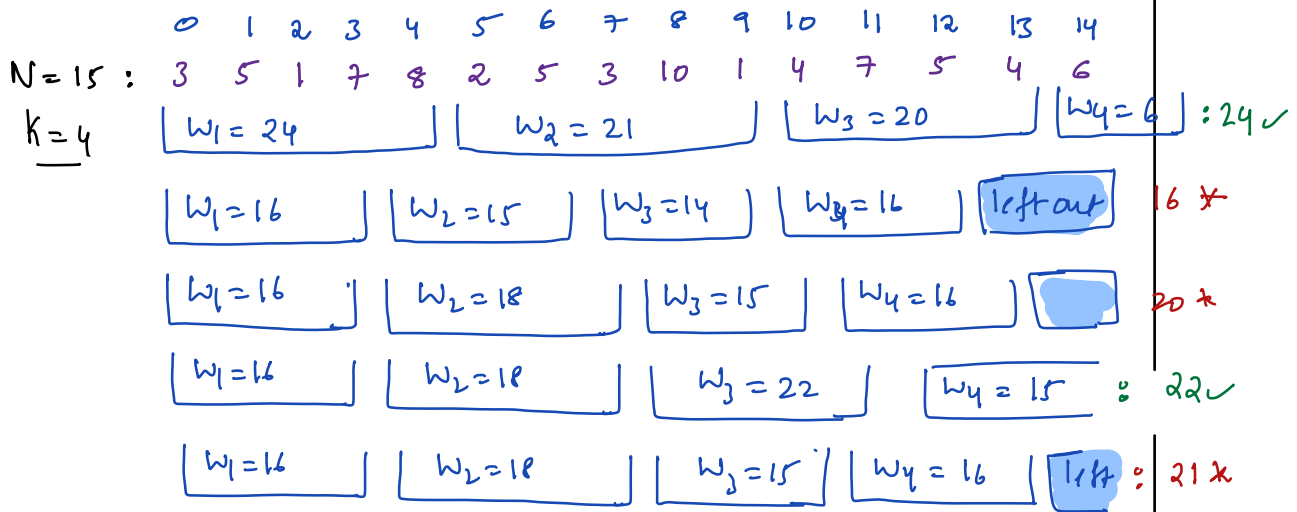
 is to be done

return true

Check func Idea:



obs: Even after utilizing 4 people tasks are left out when false



l h m, Can we finish tasks in m time ans = _____

10 71 40 ✓ ans=40, goto left h=m-1

10 39 24 ✓ ans=24, goto left h=m-1

10 23 16 ✗ goto right l=m+1

17 23 20 ✗ goto right l=m+1

21 23 22 ✓ ans=22, goto left h=m-1

21 21 21 ✗ goto right l=m+1

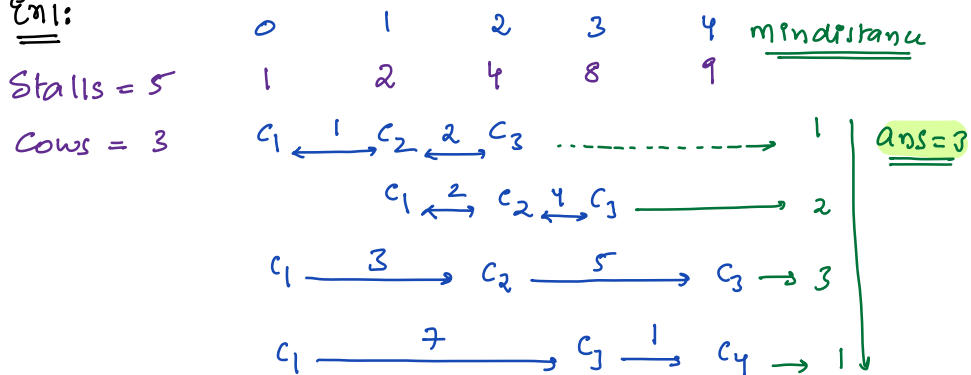
22 21 if break return ans = 21

Ques) Given k cows & N stalls, all stalls are on a line at different locations, Place all k cows such a way min distance between any 2 cows is maximized, maximize min dist

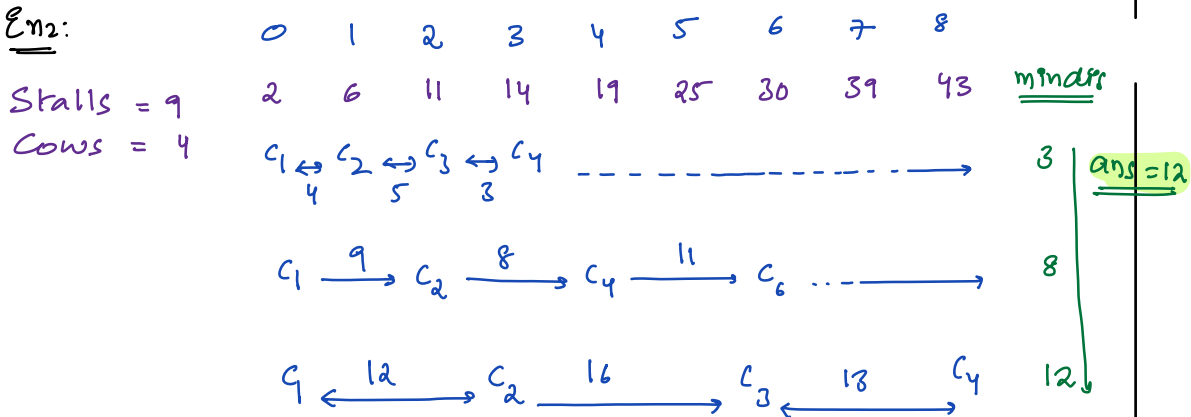
Note1: In a stall only 1 cow can be present

Note2: All cows have to be placed, $N \geq k$, & stall pos are sorted
 ↳ if not please sort

Ex1:



Ex2:



Idea1: Given N stalls, ways to choose k stalls for k cows

$N C_k \times k$
 $\swarrow \quad \searrow$
 $\# \text{ways} \quad \hookrightarrow \text{Total min distance for a selection}$

Idea2: Binary Search


a) Target: max min distance between 2 cows

b) Search space: $\begin{bmatrix} \text{low} & \text{high} \\ \text{min adj diff} & \text{stall}[N-1] - \text{stall}[0] \end{bmatrix}$

Ex1:

stalls[4] = { 3 8 14 20 } stalls[4] = { 3 8 14 20 }
k=2 $\begin{matrix} \uparrow & & \uparrow \\ c_1 & \longleftrightarrow & c_2 \end{matrix}$ k=4 $\begin{matrix} \uparrow & \uparrow & \uparrow & \uparrow \\ c_1 & \longleftrightarrow & c_2 & \longleftrightarrow & c_3 & \longleftrightarrow & c_4 \end{matrix}$
Cows = 17 Cows = 5

c) discard:

Case-I: 

Say we can place all cows atleast at mid dist apart
ans = mid, goto right

Case-II: 

Say cannot place all cows atleast at mid dist apart
goto left

Trace:

Stalls = 9 0 1 2 3 4 5 6 7 8
 2 6 11 14 19 25 30 39 43

Cows = 4 $c_1 \xrightarrow{23} c_2 \xrightarrow{\quad} c_3 \xrightarrow{\quad} c_4$: 20*

$c_1 \xrightarrow{9} c_2 \xrightarrow{8} c_3 \xrightarrow{6} c_4$: We can place atleast 5 dist apart

Can we place cows atleast at 5 dist apart?

Can we place cows atleast at 20 dist apart?

1 2 3 4 5 20 21 22 23 24...
... T T T F F F F F

\rightarrow // stalls \rightarrow // cows \rightarrow // position of all stalls
 int mandis(int N, int k, int stalls[]) { TC: $\log_2(\text{ele}) \times N$ }

\rightarrow // no. of ele on which we apply BS
 $l = \text{min adj diff in stalls[]}, h = \text{stalls[N-1] - stalls[0]}, \text{ans} = _$

while (l <= h) {

#ele = h - l + 1

m = (l + h) / 2

// if we can place k cows atleast at mid distance apart

if (check(N, k, stalls[], m) == True) { TODO

ans = mid; // goto right

l = m + 1

}

TC: O(N)

else { // cannot place cows at mid distance apart

h = m - 1

}

}

return ans;

}

bool check() { TODO

|

}

final obs in BS

check function:



T T T T T . . T (T) F F F . . . F

if we last in True
ans = mid
goto right

if we last in False
goto left

check function:

F F F . . . F (T) T T . . . T

if we last in False
goto right

if we last in True
ans = mid
goto left

Q8: Search in a sorted array, search $k = 32$

arr = { 0, 1, 2, 3, 4, 5, 6, 7, 8 }
 2, 4, 10, 13, 19, 24, 27, 30, 33

// Say target is at some index p position

Idea1: linear search in arr : p steps

Idea2:

l h

1 2 : k is $> \text{arr}[h]$ not present in range

↓ ↓

2 4 : k is $> \text{arr}[h]$ not present in range

↓ ↓

4 8 : k is $\leq \text{arr}[h]$ possible apply Binary Search

After every step we double l & h : $\log l + \log h \approx \log p$

→ it will $\log p$ steps or elements to be range

→ Again apply BS in given range, to check if p is there or not $\approx \log p$