



I forward pass (Compute  $y_3, y_4, y_5, y_6$ )

Summation term  $a_j = \sum_i (w_{ij} * x_i)$

Sigmoid / activation function  $y_j = f(a_j)$

$$= \frac{1}{1 + e^{-a_j}}$$

$$a_3 = (w_{13} x_1 + w_{23} x_2) + b_1$$

$$= (0.15 \times 0.05) + (0.2 \times 0.10) + 0.35$$

$$= 0.377$$

$$y_3 = \frac{1}{1 + e^{-a_3}} = \frac{1}{1 + e^{-0.377}} = 0.5932$$

$$a_4 = (w_{14} x_1 + w_{24} x_2) + b_1$$

$$= (0.25 \times 0.05) + (0.30 \times 0.10) + 0.35$$

$$= 0.3925$$

$$y_4 = \frac{1}{1 + e^{-0.3925}}$$

$$= 0.5968$$

$$a_5 = w_{35}y_3 + w_{45}y_4 + b_2$$

$$= (0.4 \times 0.593) + (0.45 \times 0.596) + 0.6$$

$$= 1.105$$

$$y_5 = \frac{1}{1 + e^{-a_5}} = \frac{1}{1 + e^{-1.105}} = 0.7513$$

$$a_6 = w_{36}y_3 + w_{46}y_4 + b_2$$

$$= (0.50 \times 0.5932) + (0.55 \times 0.5968) + 0.6$$

$$= 1.22484$$

$$y_6 = \frac{1}{1 + e^{-a_6}} = \frac{1}{1 + e^{-1.22484}} = 0.7729$$

$$E_{\text{total}} = \sum_{K \in \text{outputs}} \frac{1}{2} (\text{target}_K - o/p_K)^2$$

$$= \frac{1}{2} (0.01 - 0.7513)^2 + \frac{1}{2} (0.99 - 0.7729)^2$$

$$= 0.2983$$

$$E_{05} = \frac{1}{2} (0.01 - 0.7513)^2$$

$$= 0.274$$

$$E_{06} = \frac{1}{2} (0.99 - 0.7729)^2$$

$$= 0.0235$$



II Propagate the errors backward through the network.

for each network output unit  $K$ ,  
 Calculate error term  $\delta_K$   

$$\delta_K \leftarrow O_K (1 - O_K) (t_K - O_K)$$

for each hidden unit  $h$ ,  
 Calculate error term  $\delta_h$

$$\delta_h \leftarrow O_h (1 - O_h) \sum_{K \in \text{outputs}} w_{Kh} \delta_K$$

for output units, Calculate  $\delta_5, \delta_6$

$$\delta_5 \leftarrow Y_5 (1 - Y_5) (\text{target}_5 - Y_5)$$

$$= 0.7513 (1 - 0.7513) (0.01 - 0.7513)$$

$$= 0.7513 \times 0.2487 \times -0.7413$$

$$= -0.1385$$

$$\delta_6 \leftarrow Y_6 (1 - Y_6) (\text{target}_6 - Y_6)$$

$$= 0.7729 (1 - 0.7729) (0.99 - 0.7729)$$

$$= 0.7729 \times 0.2271 \times 0.2171$$

$$= 0.0381$$

## Output to hidden layer

let us adjust weights  $w_{53}, w_{63}, w_{54}, w_{64}$

$$w_{ji} \leftarrow w_{ji} + \Delta w_{ji}$$

$$\text{i.e., } w_{ji} \leftarrow w_{ji} + \eta \delta_j x_{ji}$$

$$w_{53} = w_{53} + \eta * \delta_5 * y_3$$

$$= 0.40 + [0.6 * -0.1385 * 0.5932]$$
$$= 0.3507$$

$$w_{63} = w_{63} + \eta * \delta_6 * y_3$$

$$= 0.50 + [0.6 * 0.0381 * 0.5932]$$
$$= 0.5135$$

$$w_{54} = w_{54} + \eta * \delta_5 * y_4$$

$$= 0.45 + [0.6 * -0.1385 * 0.5968]$$
$$= 0.4005$$

$$w_{64} = w_{64} + \eta * \delta_6 * y_4$$

$$= 0.55 + [0.6 * 0.0381 * 0.5968]$$
$$= 0.5636$$



hidden to input layer

Adjust weights  ~~$w_{13}, w_{14}, w_{23}, w_{24}$~~   
 $w_{31}, w_{41}, w_{32}, w_{42}$

$$\delta_3 = y_3 (1 - y_3) [w_{53} \delta_5 + w_{63} \delta_6]$$

$$= 0.5932 (1 - 0.5932) [0.3507 * -0.1385 + 0.5135 * 0.0381]$$

$$= 0.5932 (0.4068) [-0.0485 + 0.0195]$$

$$= -0.00699.$$

$$\delta_4 = y_4 (1 - y_4) [w_{54} \delta_5 + w_{64} \delta_6]$$

$$= 0.5968 (1 - 0.5968) [0.4005 * -0.1385 + 0.5636 * 0.0381]$$

$$= -0.0081$$

$$w_{31} \leftarrow w_{31} * \eta \delta_3 * x_1$$

$$= 0.15 * [0.6 * -0.00699 * 0.05]$$

$$= 0.1497$$

$$w_{41} \leftarrow w_{41} + \eta * \delta_4 * x_1$$

$$= 0.25 + 0.6 * -0.0081 * 0.05$$

$$= 0.2497$$

$$w_{32} \leftarrow w_{32} + \eta * \delta_3 * x_2$$

$$= 0.20 + 0.6 * -0.00699 * 0.10$$

$$= 0.1995$$

$$w_{42} \leftarrow w_{42} + \eta * \delta_4 * x_2$$

$$= 0.30 + 0.6 * -0.0081 * 0.10$$

$$= 0.2995$$