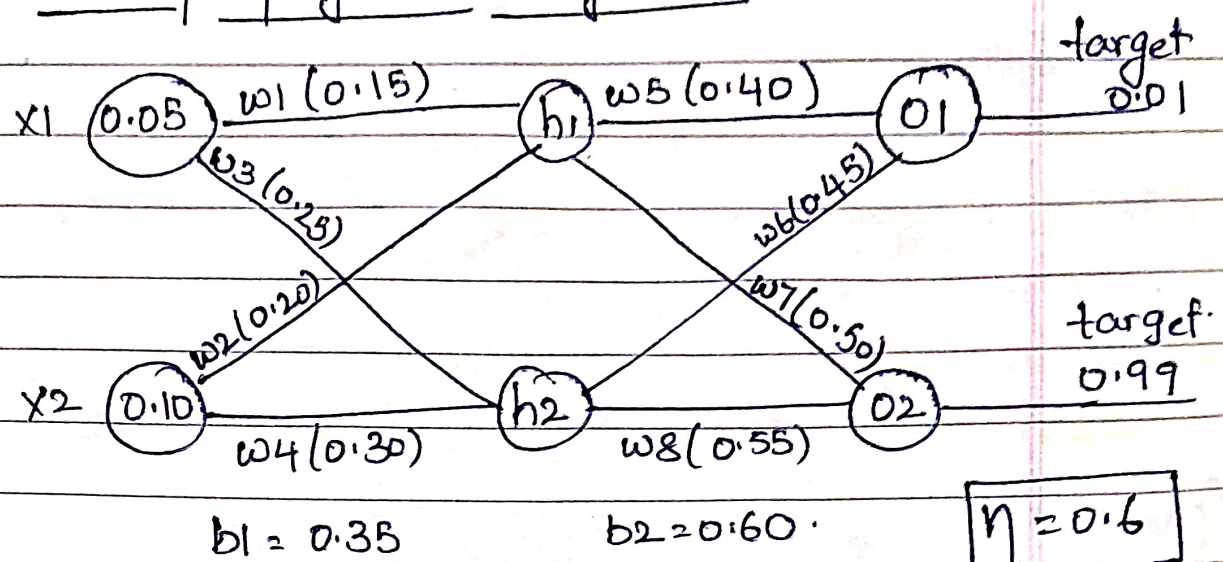


# Back Propagation algorithm:



I calculate forward propagation error

$$h_1(in) = w_1x_1 + w_2x_2 + b_1$$

$$= (0.15 \times 0.05) + (0.2 \times 0.10) + 0.35$$

$$= 0.377$$

$$h_1(out) = \frac{1}{1 + e^{-h_1(in)}} = \frac{1}{1 + e^{-0.377}} = 0.5932$$

$$h_2(in) = w_3x_1 + w_4x_2 + b_1$$

$$= (0.25 \times 0.05) + (0.30 \times 0.10) + 0.35$$

$$= 0.3925$$

$$h_2(out) = \frac{1}{1 + e^{-h_2(in)}} = \frac{1}{1 + e^{-0.3925}} = 0.5968$$

$$o_1(in) = w_5h_1(out) + w_6h_2(out) + b_2$$

$$= (0.4 \times 0.593) + (0.45 \times 0.596) + 0.6$$

$$= 1.105$$

$$o_1(out) = \frac{1}{1 + e^{-o_1(in)}} = \frac{1}{1 + e^{-1.105}} = 0.7513$$

$$\begin{aligned} O_2(\text{in}) &= w_7 h_1(\text{out}) + w_8 h_2(\text{out}) + b_2 \\ &= 0.50 \times 0.5932 + 0.55 \times 0.5968 + 0.6 \\ &= 1.22484 \end{aligned}$$

$$O_2(\text{out}) = \frac{1}{1 + e^{-O_2(\text{in})}} = \frac{1}{1 + e^{-1.22484}} = 0.7729$$

$$E_{\text{total}} = \sum \frac{1}{2} (\text{target} - \text{output})^2$$

$$= \frac{1}{2} (0.01 - 0.7513)^2 + \frac{1}{2} (0.99 - 0.7729)^2$$

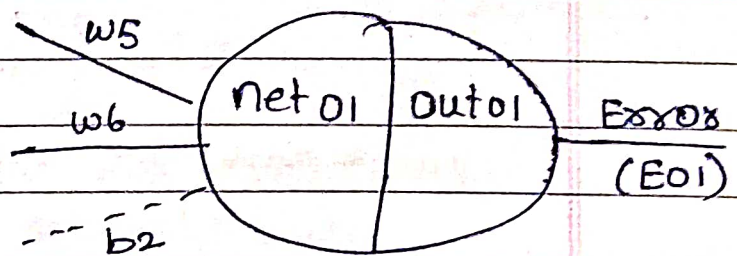
$$= 0.2983$$

$$\begin{aligned} E_{O_1} &= \frac{1}{2} (0.01 - 0.7513)^2 \\ &= 0.274 \end{aligned}$$

$$\begin{aligned} E_{O_2} &= \frac{1}{2} (0.99 - 0.7729)^2 \\ &= 0.0235 \end{aligned}$$

## II Calculating backward propagation error output to hidden layer

$w_5, w_6, w_7, w_8$



$$\frac{\partial E_{\text{total}}}{\partial w_5} = \frac{\partial E_{\text{total}}}{\partial \text{out } o1} \times \frac{\partial \text{out } o1}{\partial \text{net } o1} \times \frac{\partial \text{net } o1}{\partial w_5}$$

$$\frac{\partial E_{\text{total}}}{\partial \text{out } o1} = (\text{out } o1 - \text{target } o1)$$

$$= 0.7513 - 0.01 = 0.7413$$



$$\frac{\partial \text{out}_1}{\partial \text{net}_1} = \text{out}_1 (1 - \text{out}_1)$$

$$= 0.751365 (1 - 0.751365)$$

$$= 0.186815$$

$$\frac{\partial \text{net}_1}{\partial w_5} = \text{out}_{h1} = 0.59326$$

$$\frac{\partial E_{\text{total}}}{\partial w_5} = 0.08216$$

$$w_5^* = w_5 - \eta * \frac{\partial E_{\text{total}}}{\partial w_5}$$

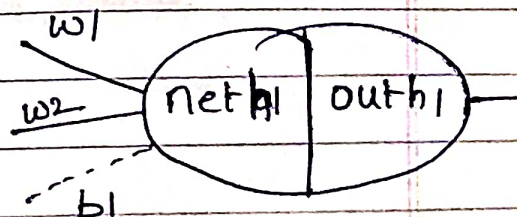
$$= 0.4 - 0.6 * 0.08216$$

$$= 0.3506$$

do the same process for  $w_6, w_7, w_8$

### III Calculating backpropagation error hidden to input layer

$w_1, w_2, w_3, w_4$



$$\frac{\partial E_{\text{total}}}{\partial w_1} = \frac{\partial E_{\text{total}}}{\partial \text{out}_1} * \frac{\partial \text{out}_1}{\partial \text{net}_1} * \frac{\partial \text{net}_1}{\partial w_1}$$

$$\frac{\partial E_{total}}{\partial out_1} = \frac{\partial E_{01}}{\partial out_1} + \frac{\partial E_{02}}{\partial out_1}$$

$$\frac{\partial E_{01}}{\partial net_{01}} * \frac{\partial net_{01}}{\partial out_1} + \frac{\partial E_{02}}{\partial net_{02}} * \frac{\partial net_{02}}{\partial out_1}$$

$$\left[ \frac{\partial E_{01}}{\partial out_{01}} * \frac{\partial out_{01}}{\partial net_{01}} \right] * \frac{\partial net_{01}}{\partial out_1} + \left[ \frac{\partial E_{02}}{\partial out_{02}} * \frac{\partial out_{02}}{\partial net_{02}} \right] * \frac{\partial net_{02}}{\partial out_1}$$

$$\frac{\partial E_{01}}{\partial out_{01}} = (out_{01} - target_{01})$$

$$= 0.7513 - 0.01$$

$$= 0.7413$$

$$\frac{\partial E_{02}}{\partial out_{02}} = (out_{02} - target_{02})$$

$$= 0.7729 - 0.99$$

$$= -0.2171$$

$$\frac{\partial out_{01}}{\partial net_{01}} = out_{01}(1 - out_{01})$$

$$= 0.7513(1 - 0.7513)$$

$$= 0.1868$$

$$\frac{\partial out_{02}}{\partial net_{02}} = out_{02}(1 - out_{02})$$

$$= 0.7729(1 - 0.7729)$$

$$= 0.1755$$

$$\frac{\partial net_{01}}{\partial out_1} = w_5 = 0.4$$

$$\frac{\partial net_{02}}{\partial out_1} = w_7 = 0.5$$

$$\frac{\partial E_{01}}{\partial out_1} = 0.7413 * 0.1868 * 0.4 = 0.055389$$

$$\frac{\partial E_{02}}{\partial out_1} = -0.2171 * 0.1755 * 0.5 = -0.019050$$



$$\frac{\partial E_{total}}{\partial out_1} = 0.055389 - 0.019050$$

$$= 0.036339$$

$$\frac{\partial out_1}{\partial net_1} = out_1 (1 - out_1)$$

$$= 0.5932 (1 - 0.5932)$$

$$= 0.2413$$

$$\frac{\partial net_1}{\partial w_1} = \frac{\partial}{\partial w_1} (w_1 x_1 + w_2 x_2 + b_1)$$

$$= x_1 = 0.05$$

$$\frac{\partial E_{total}}{\partial w_1} = 0.036339 * 0.2413 * 0.05$$

$$= 0.000438$$

$$w_1^* = w_1 - \eta \frac{\partial E_{total}}{\partial w_1}$$

$$= 0.15 - (0.6 * 0.000438)$$

$$= 0.1497$$

do the same process for  $w_2, w_3, w_4$