EE3900-assign1

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1 4.2 solution

$$x(n) = \{1, 2, 3, 4, 2, 1\}$$

$$X(z) = Z\{x(n)\} = \sum_{0}^{5} x(n) * (z^{-n}))$$

$$X(z) = 1 * z^0 + 2 * Z^{-1} + 3 * z^{-2} + 4 * z^{-3} + 2 * z^{-4} + 1 * z^{-5}$$

2 4.5 solution

$$y = a^n u(n)$$

$$u(n) = \begin{cases} 1 & \text{if } n \geq 0 \\ 0 & \text{if } n < 0 \end{cases}$$

$$a^n u(n) = \begin{cases} a^n & \text{if } n \geq 0 \\ 0 & \text{if } n < 0 \end{cases}$$

$$Z\{a^n u(n)\} = \sum_{-\infty}^{\infty} a^n u(n) z^{-n}$$

$$Z\{a^n u(n)\} = \sum_{0}^{\infty} a^n z^{-n}$$

$$Z\{a^n u(n)\} = \sum_{0}^{\infty} \frac{a}{z}^n$$

$$if \quad |z| > |a| \quad then \quad sum \quad of \quad G.P$$

$$Z\{a^n u(n)\} = \frac{1}{1 - az^{-1}}$$

3 5.1 solution

$$h(n), n < 5 \tag{1}$$

$$H(z) = \frac{1 + z^{-2}}{1 + \frac{1}{2}z^{-1}} \tag{2}$$

$$\begin{array}{r}
1 + \frac{1}{2}z^{-1})\overline{1 + z^{-2}}(2z^{-1} \\
\underline{2z^{-1} + z^{-2}} \\
1 + \frac{1}{2}z^{-1})\overline{1 - 2z^{-1}}(- 4 \\
\underline{-4 - 2z^{-1}} \\
5 + 0z^{-1}
\end{array}$$

Hence by long division will be

$$H(z) = 2z^{-1} - 4 + \frac{5}{1 + \frac{1}{2}z^{-1}} \tag{3}$$

4 5.3 solution

From the graph we can say that maximim value of h(n) is 1.25. The minimum value of graph is around -0.6.

$$-0.6 > h(n) \geq .25$$

The function is bounded between 1.25 and -0.6.

5 5.4 solution

$$h(n) = \left(-\frac{1}{2} \right)^n u(n) + \left(-\frac{1}{2} \right)^{n-2} u(n-2)$$

Ratio test

$$L = \lim_{n \to \infty} \left| \frac{h(n+1)}{h(n)} \right|$$

$$h(n+1) = \left(-\frac{1}{2} \right)^{n+1} u(n+1) + \left(-\frac{1}{2} \right)^{n-1} u(n-1)$$

$$h(n) = \left(-\frac{1}{2} \right)^n u(n) + \left(-\frac{1}{2} \right)^{n-2} u(n-2)$$

$$as \quad n \to \infty \qquad u(n) = u(n-1) = u(n+1) = u(n-2) = 1$$

$$h(n+1) = \{-\frac{1}{2}\}h(n)$$

$$\left| \frac{h(n+1)}{h(n)} \right| = \left| -\frac{1}{2} \right| = \frac{1}{2}$$

$$L = \lim_{n \to \infty} \left| \frac{h(n+1)}{h(n)} \right| = \frac{1}{2}$$

L is less than 1 h(n) is convergent

6 5.5 solution

$$h(n) = \left(-\frac{1}{2}\right)^n u(n) + \left(-\frac{1}{2}\right)^{n-2} u(n-2)$$

$$\sum_{n=-\infty}^{\infty} h(n) = \sum_{n=-\infty}^{\infty} \left(-\frac{1}{2}\right)^n u(n) + \sum_{n=-\infty}^{\infty} \left(-\frac{1}{2}\right)^{n-2} u(n-2)$$

$$\sum_{n=-\infty}^{\infty} h(n) = \sum_{n=0}^{\infty} \left(-\frac{1}{2}\right)^n u(n) + \sum_{n=2}^{\infty} \left(-\frac{1}{2}\right)^{n-2} u(n-2)$$

$$\sum_{n=-\infty}^{\infty} h(n) = \sum_{n=0}^{\infty} \left(-\frac{1}{2}\right)^n + \sum_{n=2}^{\infty} \left(-\frac{1}{2}\right)^{n-2}$$

$$\sum_{n=-\infty}^{\infty} h(n) = \frac{4}{3}$$

7 5.6

8 5.9 solution

9 5.10 solution

from 5.13 we know that

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

$$t = n - k$$

$$y(n) = \sum_{n-t=-\infty}^{\infty} x(n-t)h(t)$$

since n is finite

$$-\infty < \infty$$

the above equation is equivalet to

$$y(n) = \sum_{n=-\infty}^{\infty} x(n-t)h(t)$$

hence proved

10 6.1 solution