

1) a) $1 - 0.3 = 0.7$

b) $0.6 + 0.5 - 0.7 = 0.4$

c) $0.6 - 0.4 = 0.2$

d) $0.7 - 0.4 = 0.3$

2) $S \rightarrow$ $\begin{pmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & & & & & \\ (3,1) & & & & & \\ (4,1) & & & & & \\ (5,1) & & & & & \\ (6,1) & & & & & \end{pmatrix} \quad (x_1, x_2)$

$P(A) = \frac{5}{36} \quad ((2,6) (6,2) (4,4) (3,5) (5,3))$

3) $\frac{1}{2^n - 1} \quad (\because P = \frac{\frac{1}{2^n}}{\frac{1}{2}})$ (independent).

4) 1. $F_X(x) = p F_d(x) + (1-p) F_c(x)$

2. Couldn't understand how PDF defined for discrete

3. $E[X] = p E[X_d] + (1-p) E[X_c]$

4. Probably $p^2 \text{var}(X_d) + (1-p)^2 \text{var}(X_c) + 2p^2(1-p)^2 \text{cov}(X_d, X_c)$. (Need clarity in this & topic further)

5) $\text{Cov}(Z, W) = E[ZW] - E[Z]E[W]$
 $= 3 - 1 = 2$

$E[Z] = 1 + E[X] + E[X]E[Y^2]$
 $= 1$

$E[W] = 1$

$E[ZW] = 1 + E[X^2]E[Y^2]$

$E[X^2] = E[Y^2] = 1 + E[X^2]$
 $= 3$

6) No

7) $\mu = 100, \sigma^2 = 90 \quad n = 1000 \Rightarrow$ Assuming Normal distrib

$P \approx 0.01355$ (from CDF table)

8) $\approx 64 + 2\sigma \approx 64 + 2\sqrt{32} \approx 75.3$ (But answer slightly less because we have to be 95% sure on only one side of dist from mean (no shortage \rightarrow))

9) 1. $E[X] = E[Y] = 0$

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 2. $\text{Var}(X) = \text{Var}(Y) = 1$
 3. $\text{Cov}(X, Y) = \rho$

10) $X = 1 + K(Y-2) + \dots$ (Gaussian $\sim N(0, 4 - \frac{1}{3})$)

$$= 1 + \left(\frac{1}{3}\right)(Y-2) + \dots \sim N\left(0, \frac{11}{3}\right)$$

$$X|Y=y = 1 + \frac{1}{3}(y-2) + \dots \sim N\left(0, \frac{11}{3}\right)$$

$$\Rightarrow X|Y=y \sim N\left(1 + \frac{1}{3}(y-2), \frac{11}{3}\right)$$

11) $E[Z] = 0$

$$\text{Var}(Z) = \text{Var}(3X - 2Y) = 9\text{Var}(X) + 4\text{Var}(Y)$$

$$\text{Var}(Z) = E[9X^2 + 4Y^2 - 12XY] = 0$$

$$= 9 + 4 \times 4 - 12 = 13$$

$$\text{Cov}(Z, X) = E[ZX] - 0 = E[3X^2 - 2XY] = 3 - 2 = 1$$

12) $E[X|Y=y, Z=z] = \mu_X + \frac{1}{\sigma_Y^2 - \sigma_Z^2} \left((\sigma_{XY}\sigma_Z^2 - \sigma_{XZ}\sigma_{YZ})(y - \mu_Y) + (\sigma_{XZ}\sigma_Y^2 - \sigma_{XY}\sigma_{YZ})(z - \mu_Z) \right)$

$$= \frac{1}{14} (11y + 5z)$$

\rightarrow I simplified matrices, don't know why

$$\sigma_{\cdot}^2 = \sigma_X^2 - \frac{1}{\sigma_Y^2 - \sigma_Z^2} \left(\sigma_{XY}^2 \sigma_Z^2 - 2\sigma_{XY}\sigma_{XZ}\sigma_{YZ} + \sigma_{XZ}^2 \sigma_Y^2 \right)$$

$$= 4 - \frac{1}{14} (27) = \frac{29}{14}$$