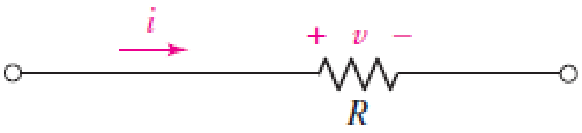
Basic laws and Theorems

Basic laws and Theorems

- Ohms law
- Kirchoff’s Laws
- Series, Parallel circuits and Delta- Wye Conversion
- Mesh Analysis & Nodal Analysis

Ohms law



Ohm's law states that the voltage across conducting materials is directly proportional to the current flowing through the material

$$V = RI$$

where the constant of proportionality *R is called the resistance. The unit of* resistance is the '**ohm (Ω)**'.

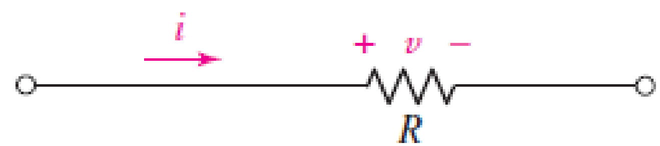
Power Absorption

The absorbed power appears physically as heat and/or light and is always positive; a (positive) resistor is a passive element that cannot deliver power or store energy.

$$P = vi = i^2 R = \frac{v^2}{R}$$

Numericals

The 560Ω resistor shown in Fig. is connected to a circuit which causes a current of 42.4mA to flow through it. Calculate the voltage across the resistor and the power it is dissipating.



Ans: Voltage=23.7V, Power = 1.005 W

Numericals

Compute the following

a) R if $i = -2\ \mu A$ *and* $v = -44\ V$*.*

Ans: 22 MΩ

b) The power absorbed by the resistor if $v = 1\ V$ *and* $R = 2\ k\Omega$.

Ans: 500 μW;

c) The power absorbed by the resistor if $i = 3\ nA$ *and* $R = 4.7\ M\Omega$.

Ans: *42.3 pW*

Numericals

An electric vehicle is driven by a single motor rated at 40 hp. If the motor is run continuously for 3 h at maximum output, calculate the electrical energy consumed.

Ans:89.4 kwh

Real resistors can only be manufactured to a specific tolerance, so that in effect the value of the resistance is uncertain. For example, a 1 Ω resistor specified as 5% tolerance could in practice be found to have a value anywhere in the range of **0.95Ω to 1.05Ω .**

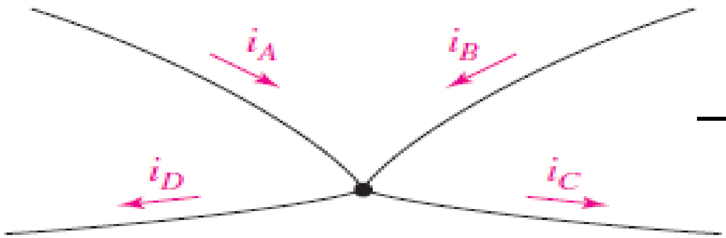
Calculate the voltage across a 2.2 kΩ, 10% tolerance resistor if the current flowing through the element is (a) 1 mA; (b) 4 sin 44t mA.

Ans: 2.42V, 1.98V, 9.68sin(44t)V, 7.92sin(44t)V.

Kirchoff’s Current Law

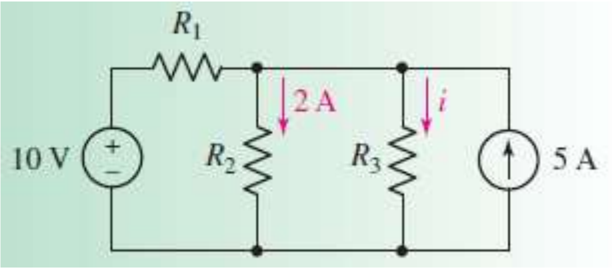
The algebraic sum of the currents entering any node is zero.

Note: ***A node is not a circuit element and it* cannot store, destroy, or generate charge**



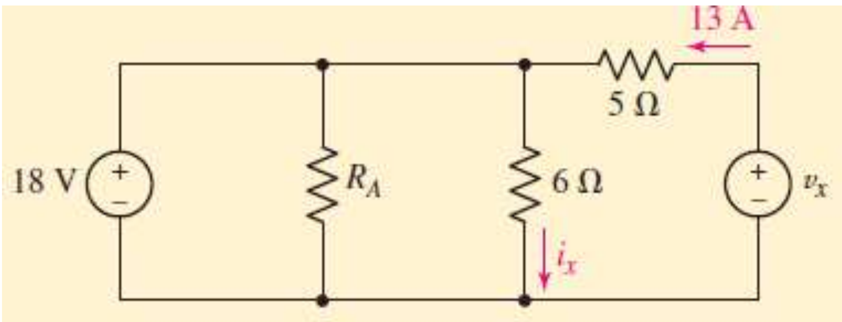
$$-i_A - i_B + i_C + i_D = 0$$

Compute the current through resistor R_3 if it is known that the voltage source supplies a current of 3 A.



Ans:6A

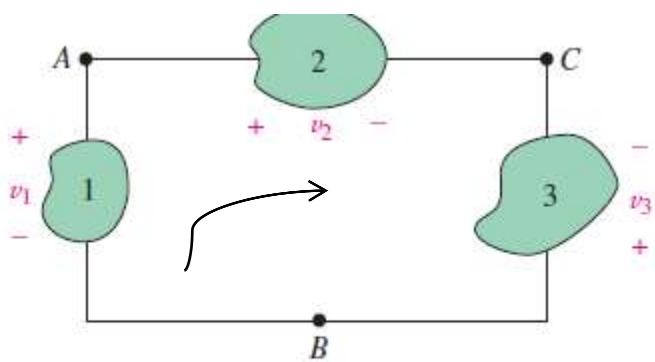
Count the number of branches and nodes in the circuit in Fig. If $i_x = 3$ A and the 18 V source delivers 8 A of current, what is the value of R_A?



Ans:5,3,1Ω

Kirchoff’s Voltage Law

The algebraic sum of the voltages around any closed path is zero.



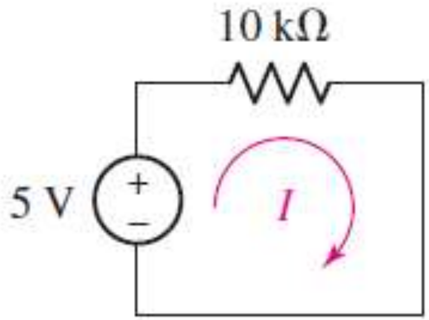
$$-V_1 + V_2 - V_3 = 0$$

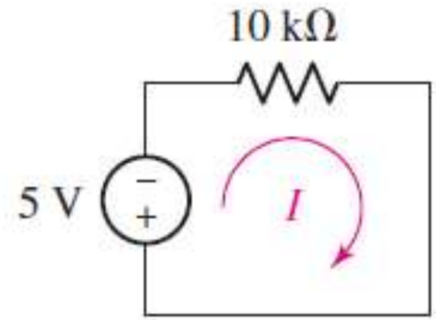
“Move around the closed path in a clockwise direction and writing down directly the voltage of each element whose (+) terminal is entered, and writing down the negative of every voltage first met at the (−) sign”.

Numericals

find the current I and compute the power absorbed by the resistor.

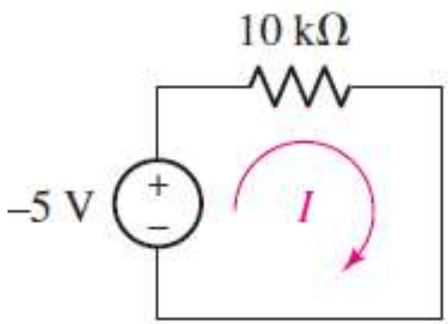
Ans:0.5mA, 2.5mW

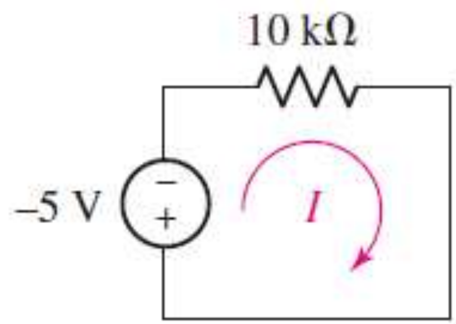




Ans:-0.5mA, 2.5mW

Ans:-0.5mA, 2.5mW



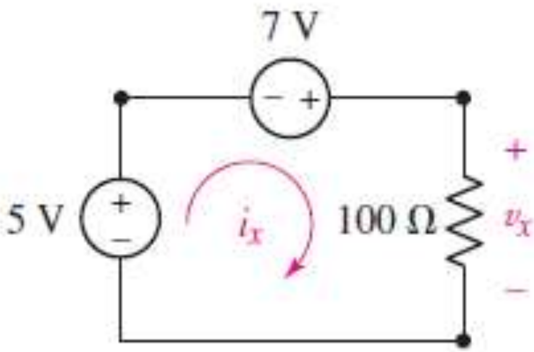


Ans:0.5mA, 2.5mW

Batteries come in a wide variety of types and sizes. Two of the most common are called “AA” and “AAA.” A single battery of either type is rated to produce a terminal voltage of 1.5 V when fully charged. So what are the differences between the two, other than size?

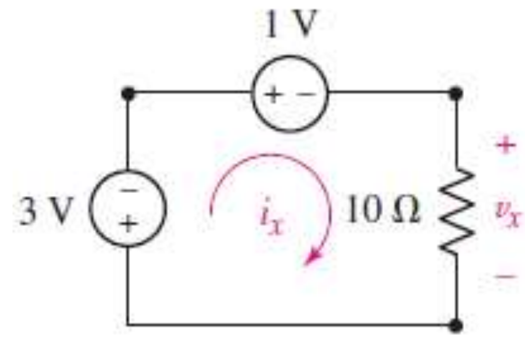
Ans: ‘AA’ Battery **AH** rating is more than the ‘AAA’ Battery.

Find v_x *and* i_x.



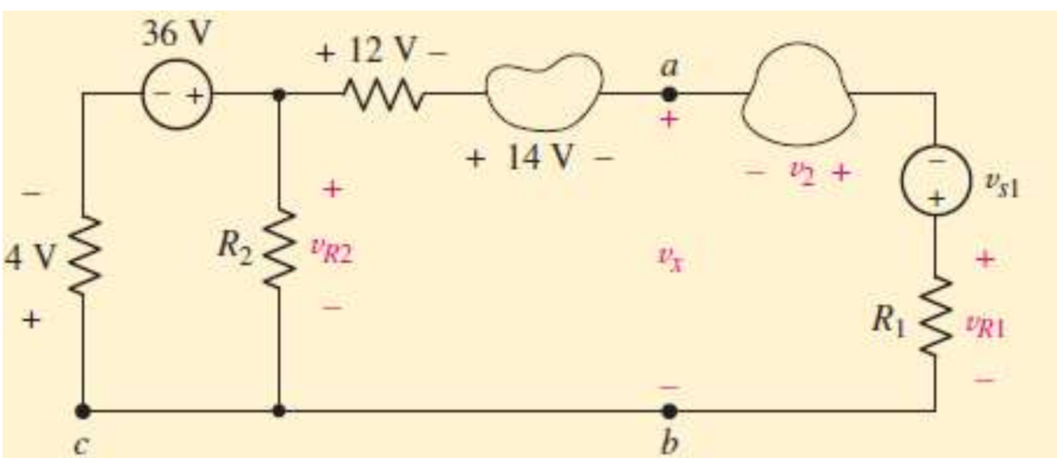
Ans:12V,0.12A

Determine i_x and V_x in the circuit of Fig.



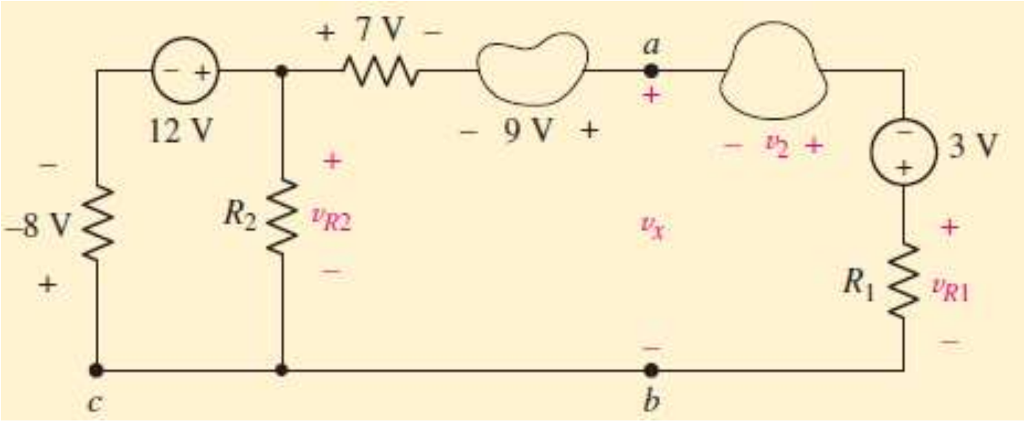
Ans:-4V,-0.4A

Find V_{R2} (the voltage across R2) and the voltage labeled v_x



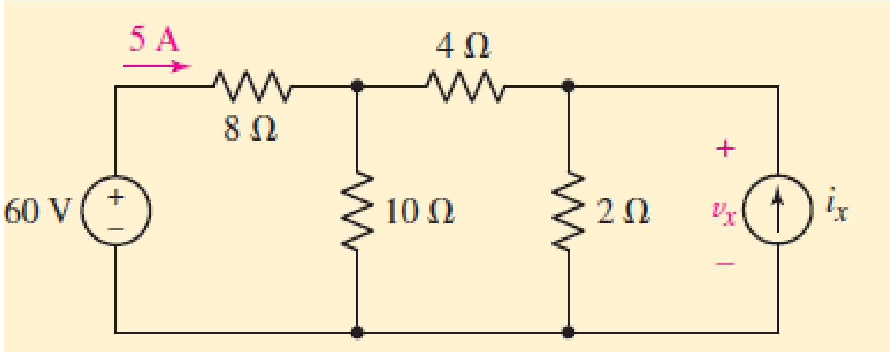
Ans:32V,6V

Determine (a) V_{R2} and (b) V_2, if V_{R1} = 1 V.



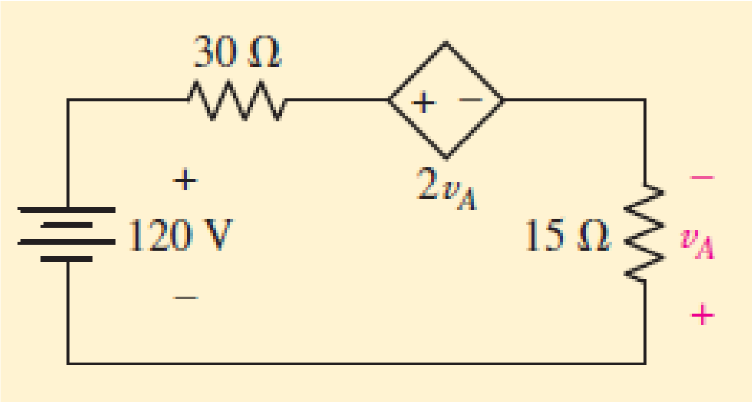
Ans:20V,-24V

Determine V_x in the circuit of Fig.



Ans:8V

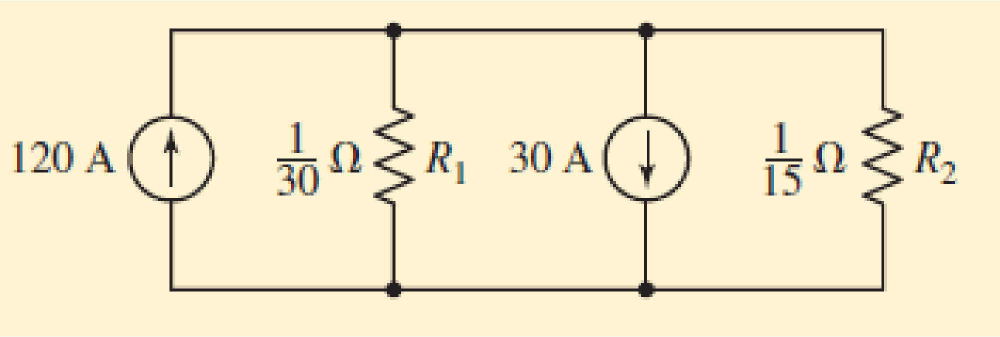
Compute the power absorbed in each element for the circuit shown in Fig.



Ans:-960W,1920W,-1920W,960W

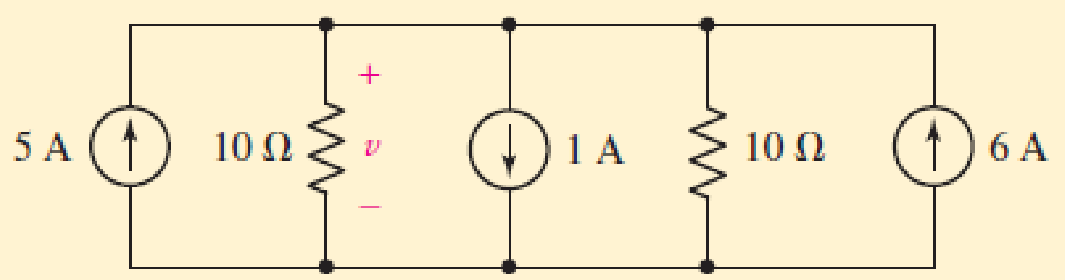
Find the voltage, current, and power associated with each element in the circuit of Fig.

Ans:2V, i_{R1}= 60A, i_{R2}= 30A,
P_{R1}= 120W, P_{R2}= 60W,
P_{120A} = -240W, P_{30A} =60W

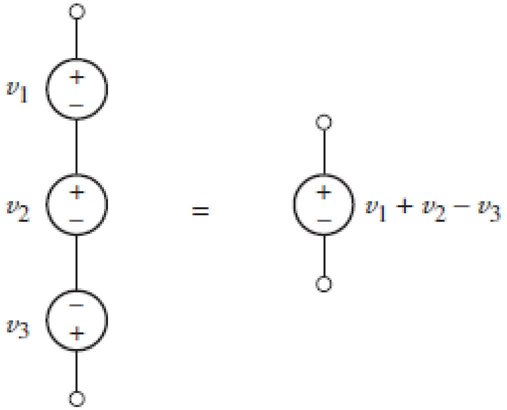


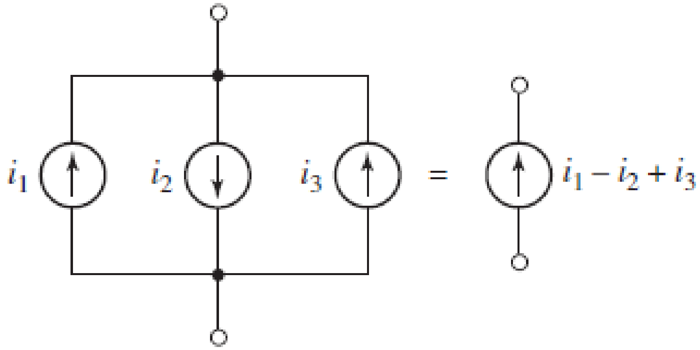
Determine *v* in the circuit of Fig.

Ans:50V

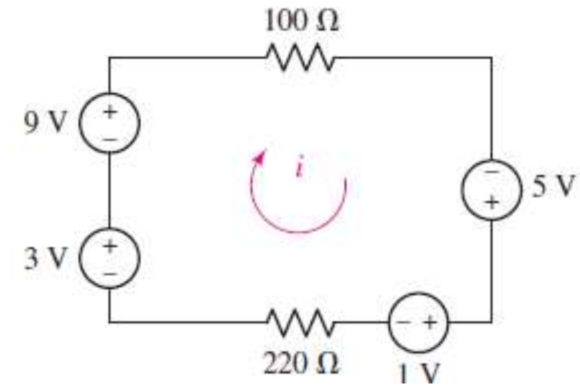


Series and Parallel Connected Sources



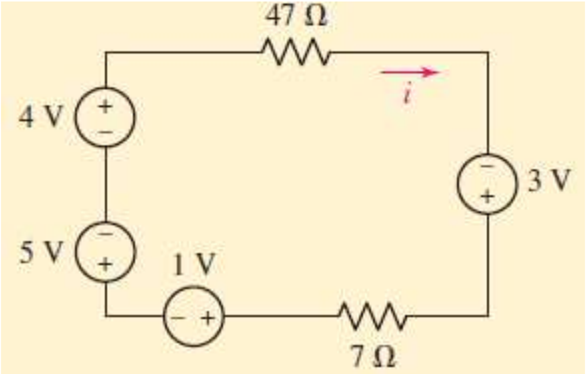


Determine the current i in the circuit of Fig.



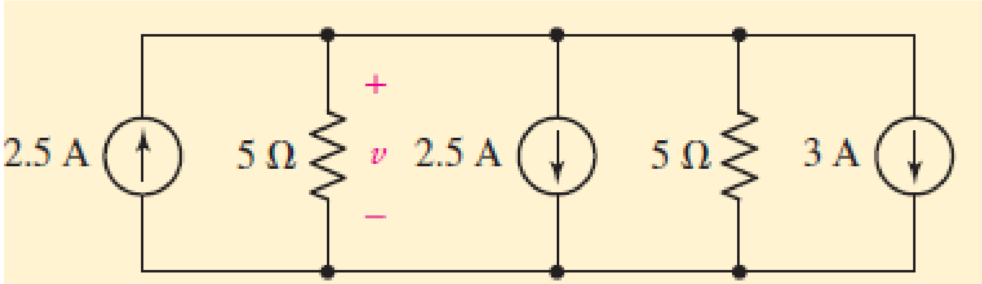
Ans:50mA

Determine the current i in the circuit of Fig.



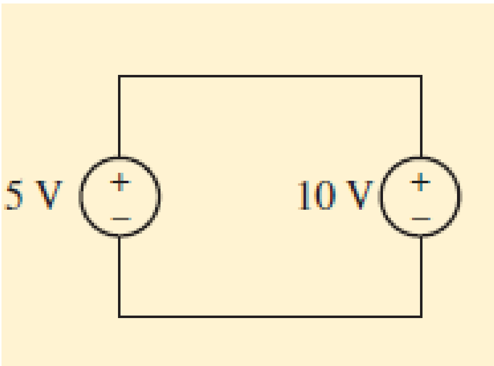
Ans:1/54A

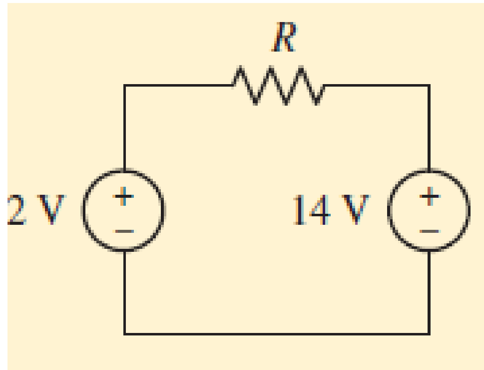
Determine the voltage v in the circuit of Fig.

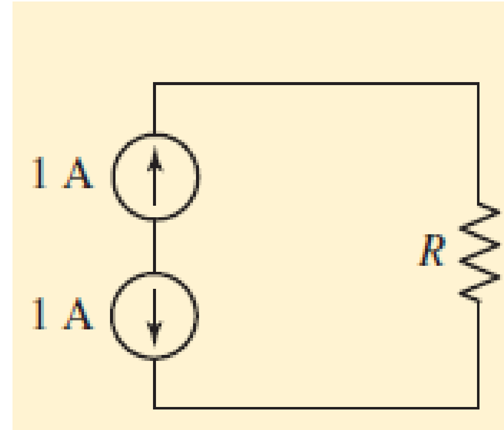


Ans:-7.5V

Determine which of the circuits of Fig. are valid.







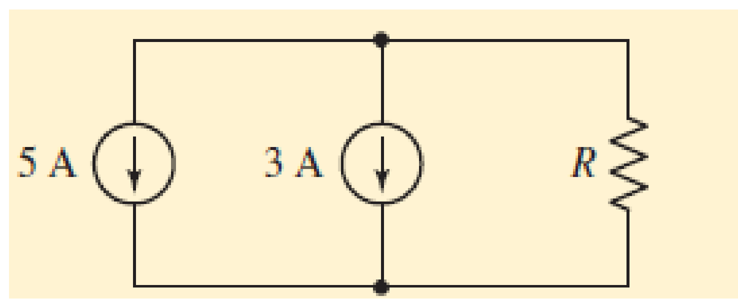
Not valid.

Valid.

Not valid.

Determine whether the circuit of Fig. violates either of Kirchhoff's laws.

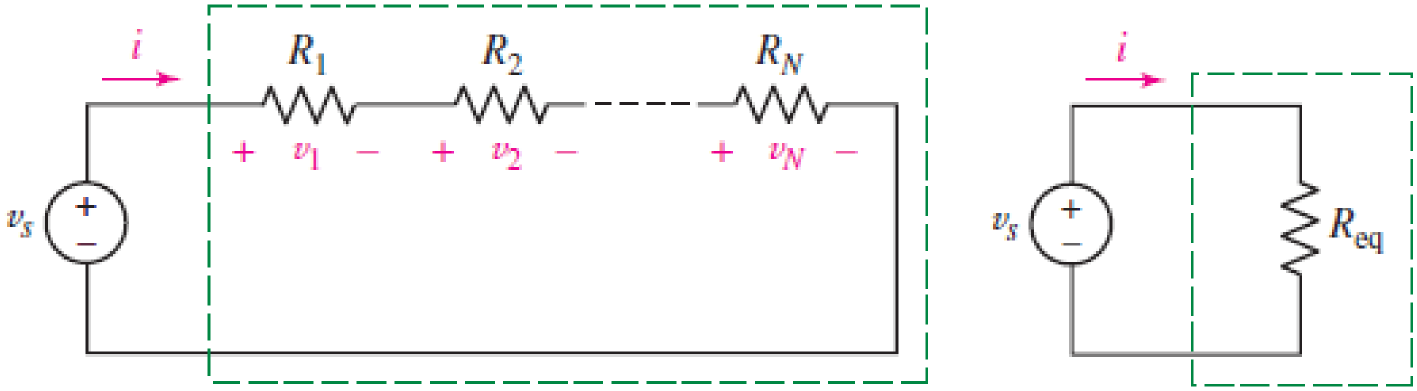
No.



Resistors in Series

It is often possible to replace relatively complicated resistor combinations with a single equivalent resistor.

All the current, voltage, and power relationships in the remainder of the circuit will be unchanged.



$$v_s = v_1 + v_2 + \cdots + v_N$$

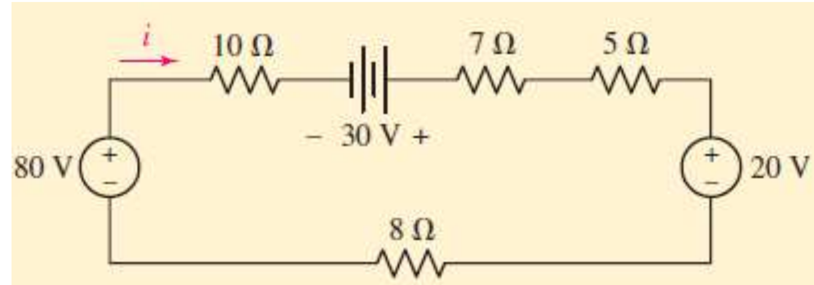
$$v_s = R_1 i + R_2 i + \cdots + R_N i = (R_1 + R_2 + \cdots + R_N)i$$

$$v_s = R_{eq} i$$

$$\boxed{R_{eq} = R_1 + R_2 + \cdots + R_N}$$

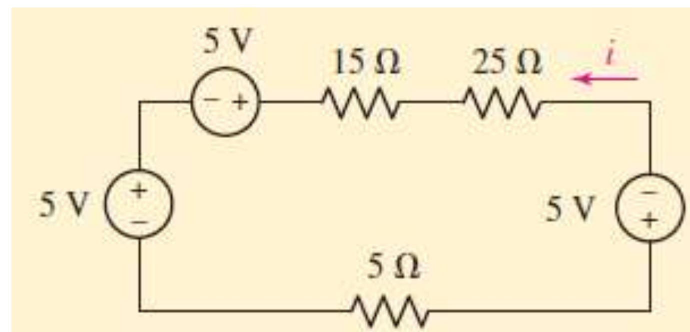
Use resistance and source combinations to determine the current I in Fig. and the power delivered by the 80 V source.

Ans:3A, 240W



Determine i in the circuit of Fig.

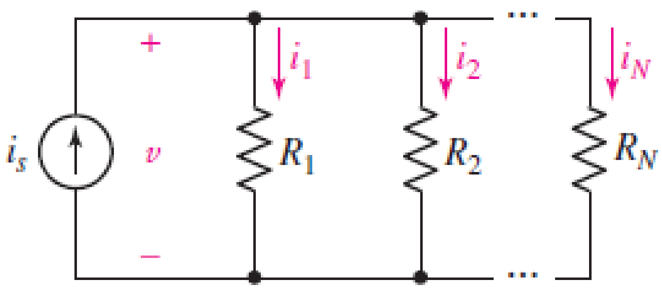
Ans:-333mA



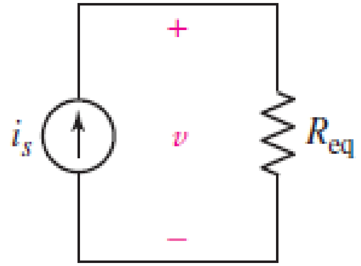
Resistors in Parallel

All the current, voltage, and power relationships in the remainder of the circuit will be unchanged.

$$i_s = i_1 + i_2 + \cdots + i_N$$



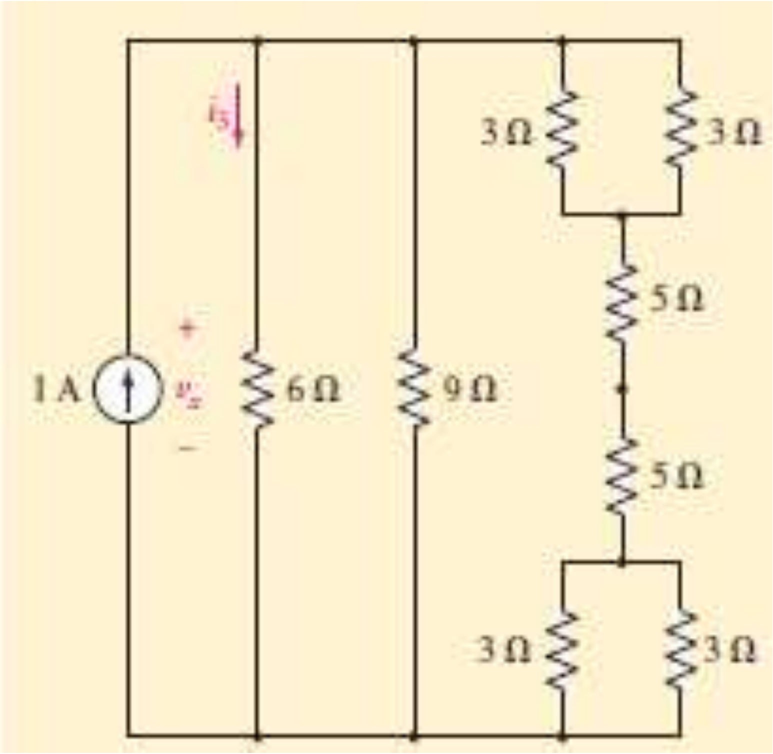
$$i_s = \frac{v}{R_1} + \frac{v}{R_2} + \cdots + \frac{v}{R_N}$$
$$= \frac{v}{R_{eq}}$$



$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}}$$

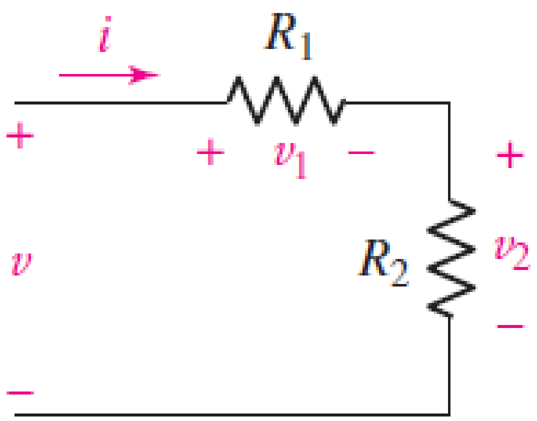
$$R_{eq} = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}}$$

Calculate the voltage V_x for the circuit of Fig.



Ans: 2.819V

Voltage Division



$$v = v_1 + v_2 = iR_1 + iR_2 = i(R_1 + R_2)$$

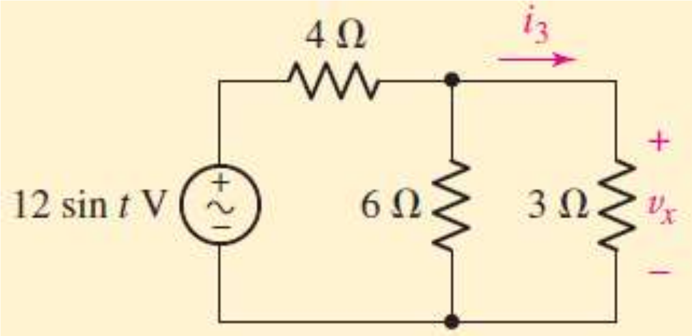
$$i = \frac{v}{R_1 + R_2}$$

$$v_2 = \frac{R_2}{R_1 + R_2} v \qquad v_1 = \frac{R_1}{R_1 + R_2} v$$

If the network of is generalized series combination of R_1, R_2, R_3, . . , R_N then the general result for voltage division across a string of N series resistors

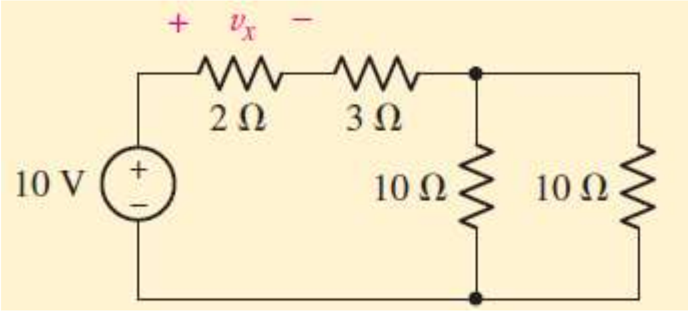
$$\boxed{v_k = \frac{R_k}{R_1 + R_2 + \cdots + R_N} v}$$

Use voltage division determine Vx in the circuit of Fig.



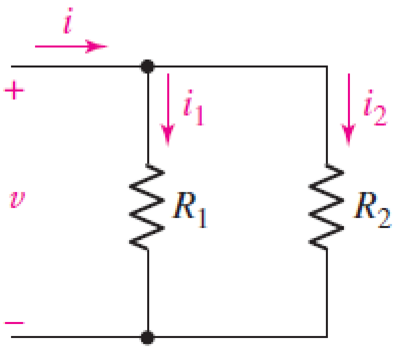
Ans: 4sint V

Determine V_x in the circuit of Fig.



Ans: 2V

Current Division



$$i_2 = \frac{v}{R_2} = \frac{i(R_1 \| R_2)}{R_2} = \frac{i}{R_2}\frac{R_1 R_2}{R_1 + R_2}$$

$$\boxed{i_2 = i\frac{R_1}{R_1 + R_2}}$$

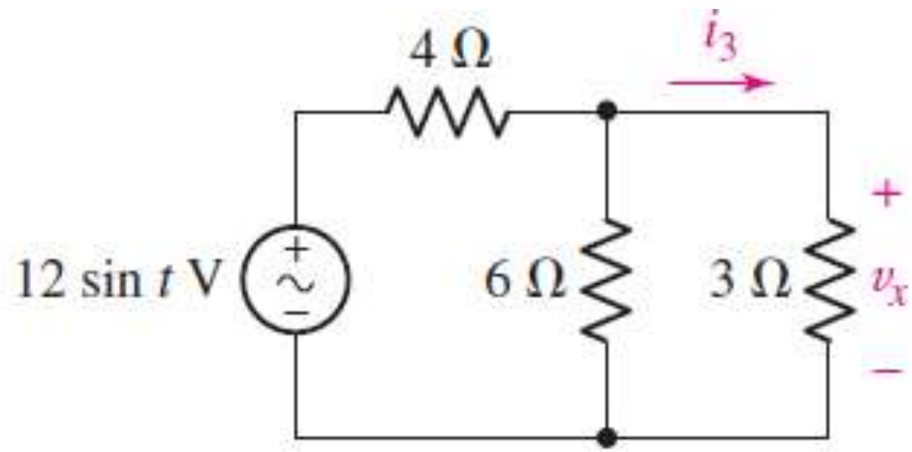
$$\boxed{i_1 = i\frac{R_2}{R_1 + R_2}}$$

For a parallel combination of N resistors, the current through resistor R_k is

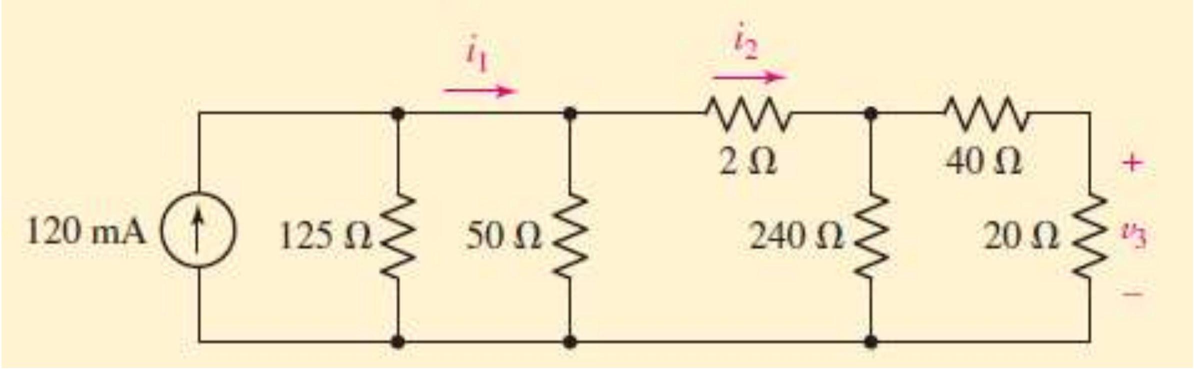
$$\boxed{i_k = i\frac{\dfrac{1}{R_k}}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \cdots + \dfrac{1}{R_N}}}$$

Write an expression for the current through the 3Ω resistor in the circuit of Fig.

Ans:4sint/3

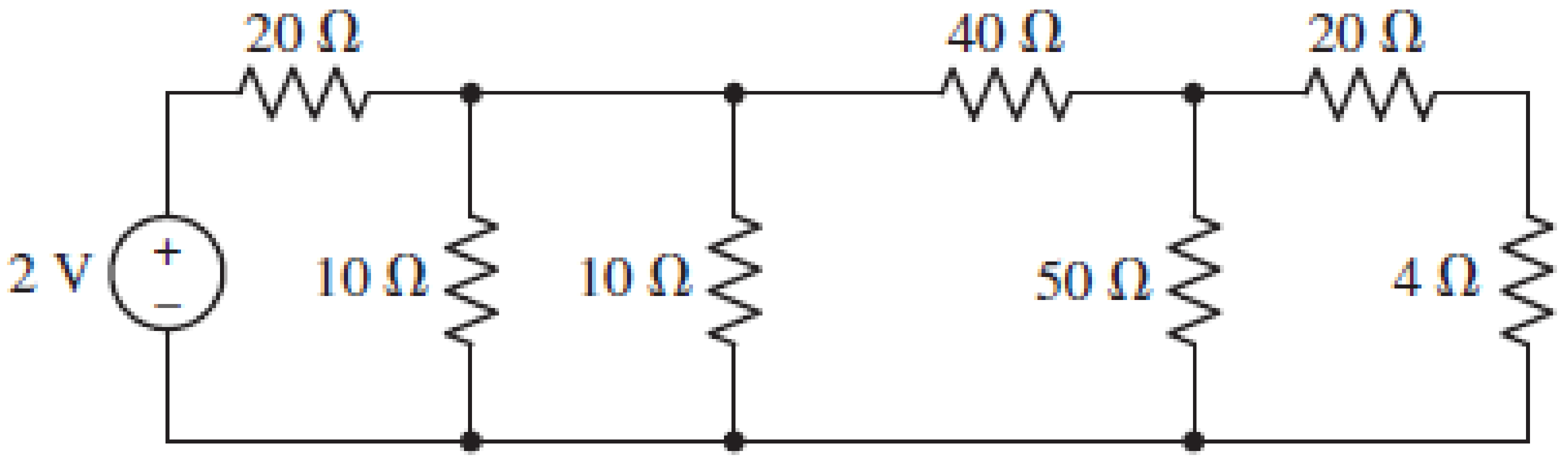


In the circuit of Fig., use resistance combination methods and current division to find i_1, i_2, and V_3.

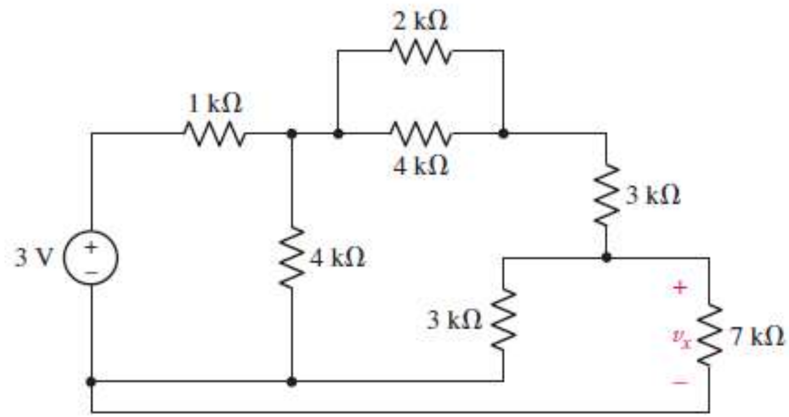


Ans:100 mA; 50 mA; 0.8 V.

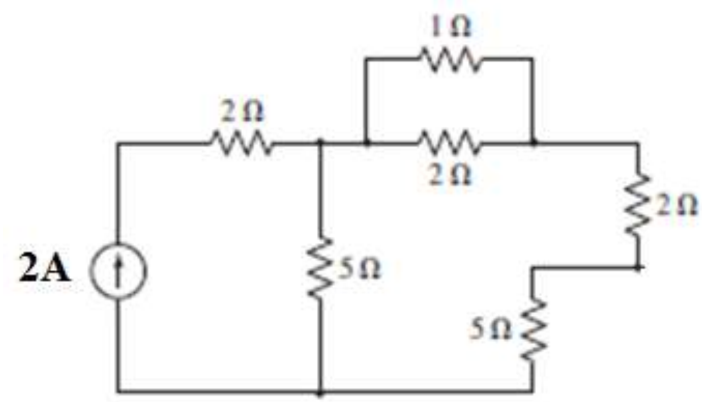
With regard to the circuit shown in Fig. Compute
(a) the voltage across the two 10 resistors
(b) the power dissipated by the 4 resistor.



1. Simplify the circuit using appropriate resistor combinations and iteratively employ voltage division to determine V_x.

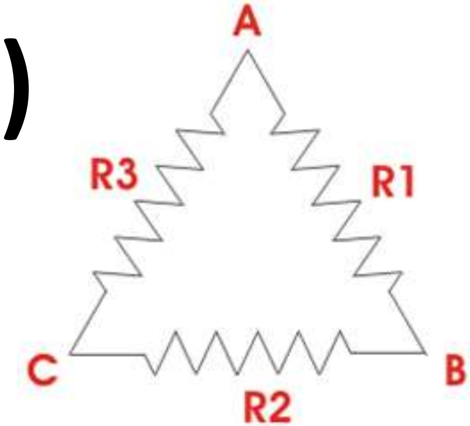


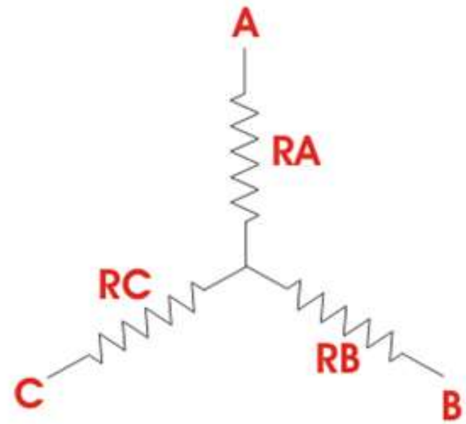
2. (a) Calculate the current flowing through each resistor. (b)Determine the voltage across the current source, assuming the top terminal is the positive reference terminal.



3. A network is constructed from a series connection of five resistors having values 1Ω, 3Ω, 5Ω, 7Ω, and 9Ω . If 9 V is connected across the terminals of the network, employ voltage division to calculate the voltage across the 3Ω resistor, and the voltage across the 7 Ω resistor.

Delta(Δ)

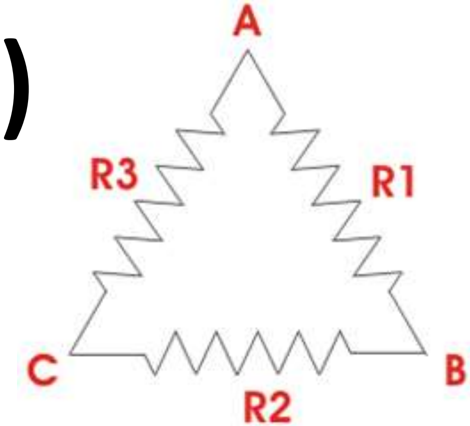




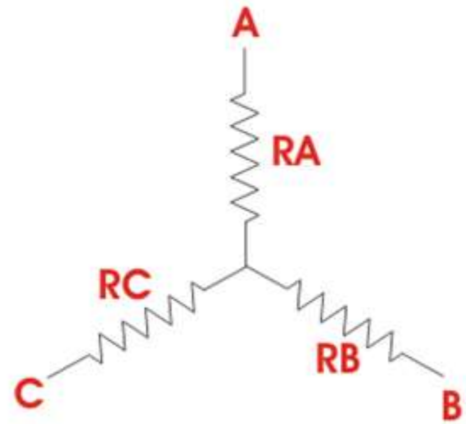
Star(Y)

Delta	Star
Consider a delta system that's three corner points are A, B and C as shown in the figure. The resistance between the points A and B will be $R_{AB} = \frac{R_1(R_2+R_3)}{R_1+R_2+R_3}$ *Similarly* $R_{BC} = \frac{R_2(R_1+R_3)}{R_1+R_2+R_3}$ $R_{CA} = \frac{R_3(R_1+R_2)}{R_1+R_2+R_3}$	Consider a Star system that's three corner points are A, B and C as shown in the figure. The resistance between the points A and B will be $R_{AB} = R_A + R_B$ *Similarly* $R_{BC} = R_B + R_C$ $R_{CA} = R_C + R_A$

Delta(Δ)



Star(Y)



Since the two systems are identical, resistance measured between terminals A and B in both systems must be equal.

$$R_A + R_B = \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3} ----(I)$$

Similarly

$$R_B + R_C = \frac{R_2(R_1 + R_3)}{R_1 + R_2 + R_3} ----(II)$$

$$R_C + R_A = \frac{R_3(R_1 + R_2)}{R_1 + R_2 + R_3} ----(III)$$

Adding equations (I), (II) and (III) we get

$$R_A + R_B + R_C = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1 + R_2 + R_3} ---(IV)$$

Subtractin g equations (I), (II) and (III) from equation (IV) we get

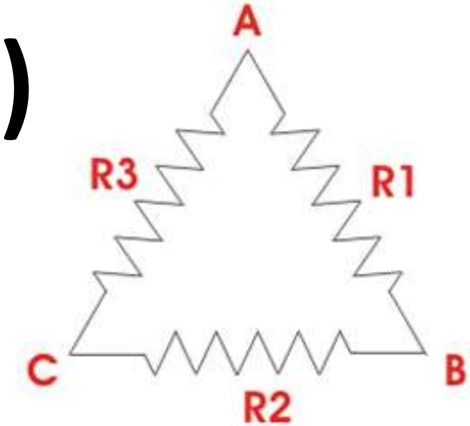
Delta to Star Conversion

$$\mathrm{R_A} = \frac{R_3R_1}{R_1 + R_2 + R_3} ----(V)$$

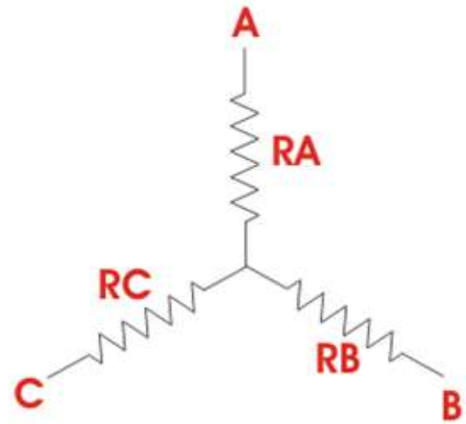
$$\mathrm{R_B} = \frac{R_1R_2}{R_1 + R_2 + R_3} ----(VI)$$

$$\mathrm{R_C} = \frac{R_2R_3}{R_1 + R_2 + R_3} ----(VII)$$

Delta(Δ)



Star(Y)



Star to Delta Conversion

we just multiply equations (v), (VI) and (VI), (VII) and (VII), (V)

and sum of products

$$(v) \times (VI) + (VI) \times (VII) + (VII) \times (V)$$

$$R_A R_B + R_B R_C + R_C R_A = \frac{R_1 R_2^2 R_3 + R_1 R_2 R_3^2 + R_1^2 R_2 R_3}{(R_1 + R_2 + R_3)^2} ----(VIII)$$

$$R_A R_B + R_B R_C + R_C R_A = \frac{R_1 R_2 R_3}{(R_1 + R_2 + R_3)}$$

Now dividing equation (VIII)

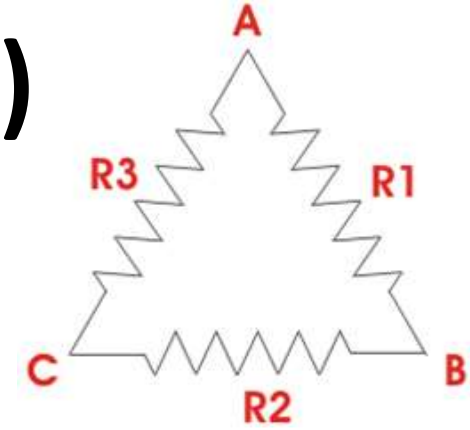
by equations (V), (VI) and equations (VII) separately,

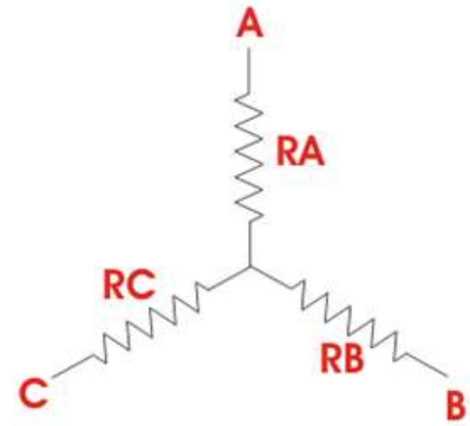
$$R_1 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_C}$$

$$R_2 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_A}$$

$$R_3 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_B}$$

Delta(Δ)





Star(Y)

Star to Delta Conversion

$$R_1 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_C}$$

$$R_2 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_A}$$

$$R_3 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_B}$$

Delta to Star Conversion

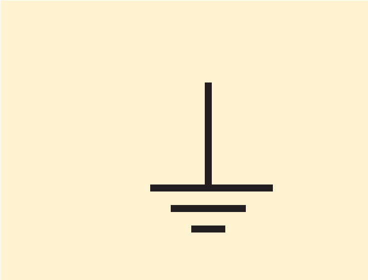
$$R_A = \frac{R_3 R_1}{R_1 + R_2 + R_3}$$

$$R_B = \frac{R_1 R_2}{R_1 + R_2 + R_3}$$

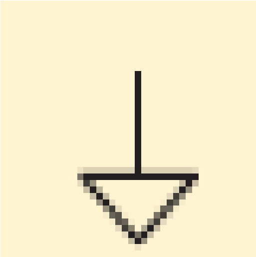
$$R_C = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$

Earth Ground

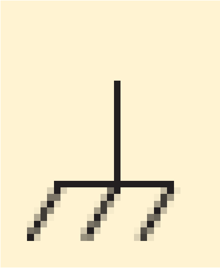
Group Discussion



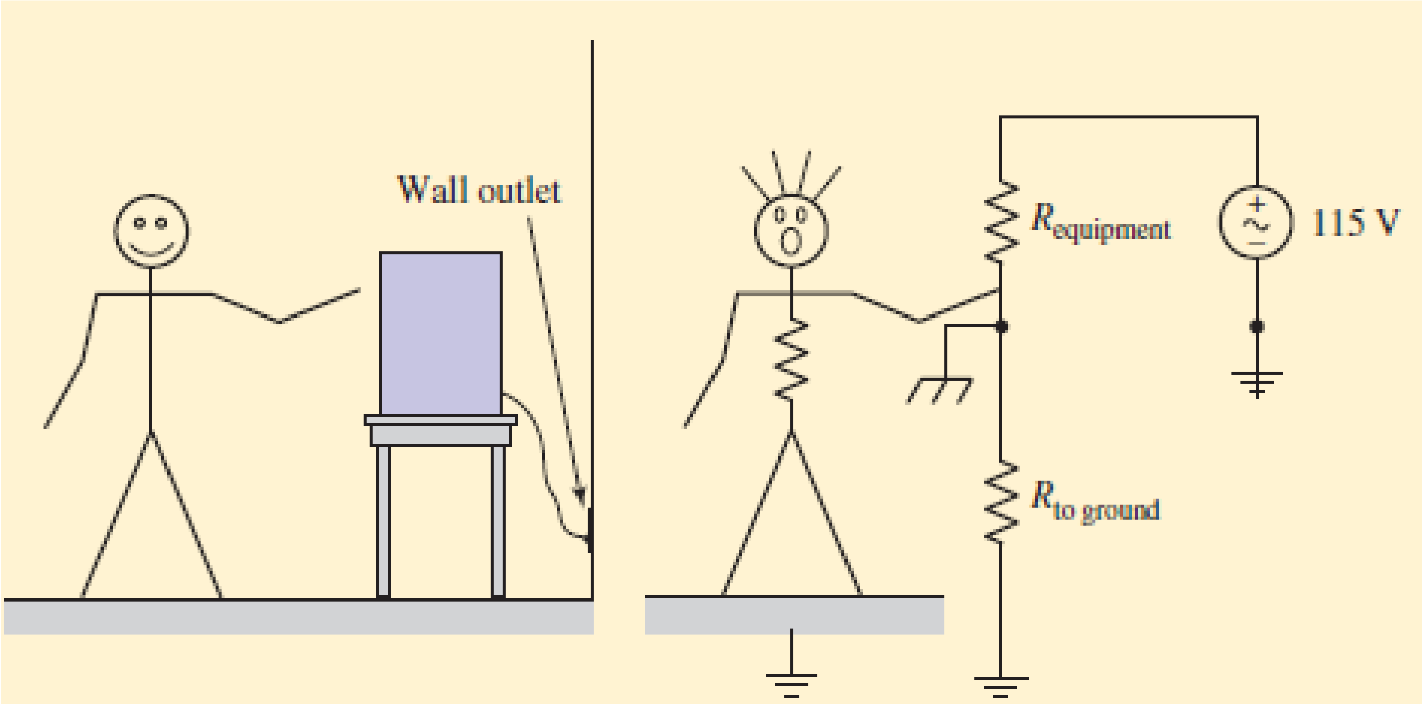
Earth Ground

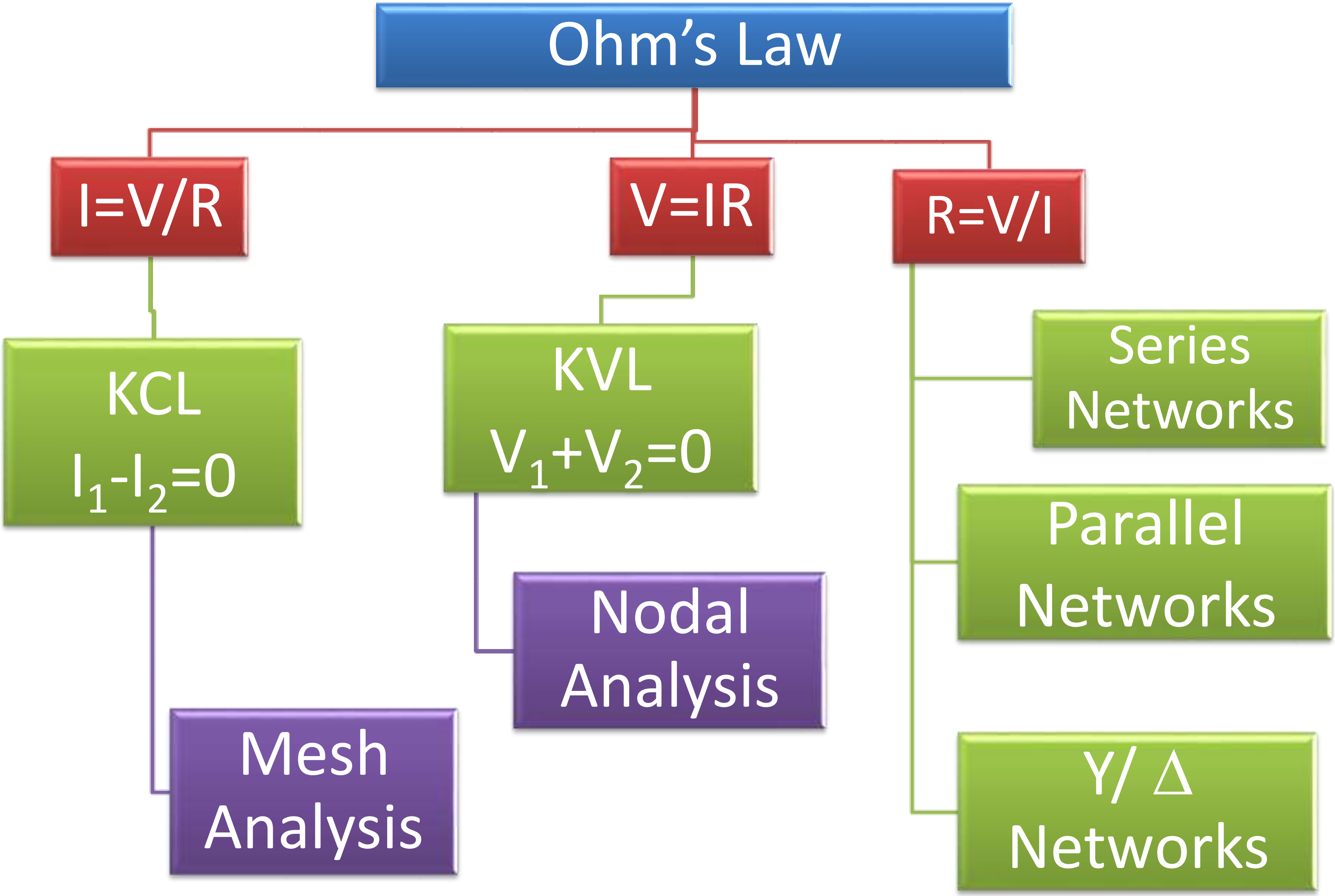


Signal Ground



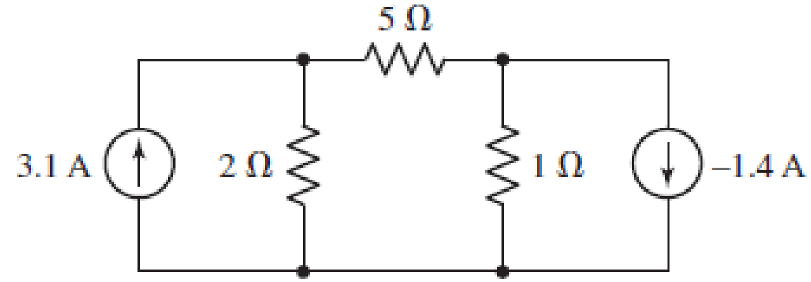
Chassis Ground



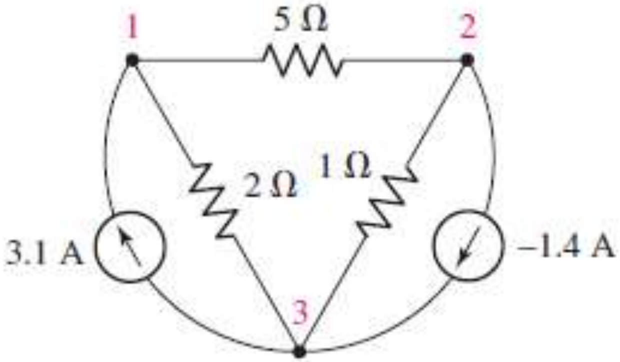

Ohm's Law
I=V/R
V=IR
R=V/I
KCL
$I_1 - I_2 = 0$
KVL
$V_1 + V_2 = 0$
Series Networks
Parallel Networks
Y/ Δ Networks
Mesh Analysis
Nodal Analysis

Nodal Analysis

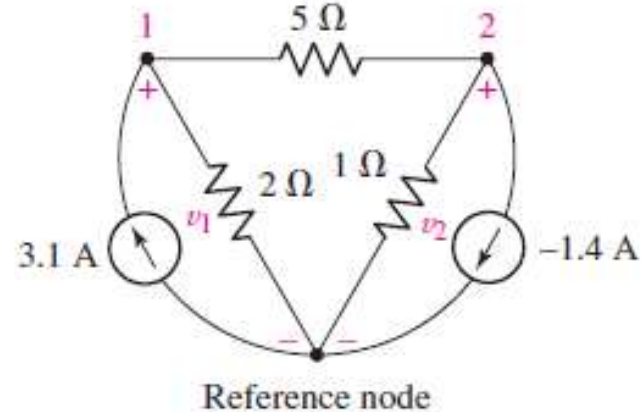
Find the Voltage across 5Ω resistor of Fig.



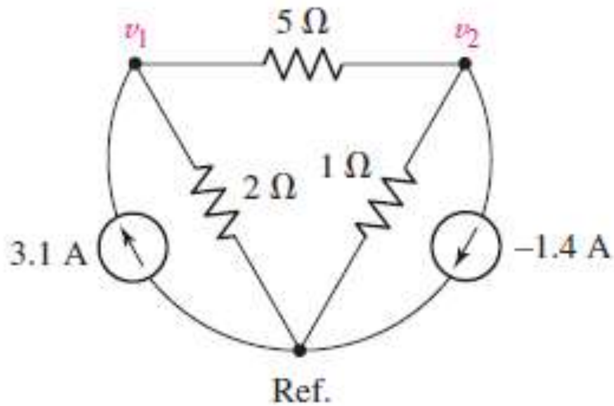
Step-1



Step-2



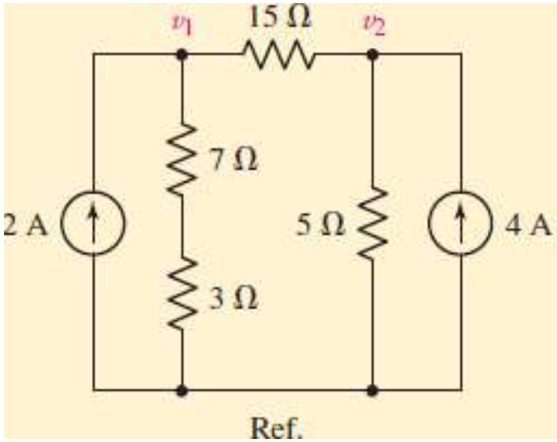
Step-3



Ans:3V

Note: The reference node in a schematic is implicitly defined as zero volts. However, it is important to remember that any terminal can be designated as the reference terminal. Thus, the reference node is at zero volts with respect to the other defined nodal voltages, and not necessarily with respect to earth ground.

Determine the current flowing left to right through the 15 Ω resistor of Fig.

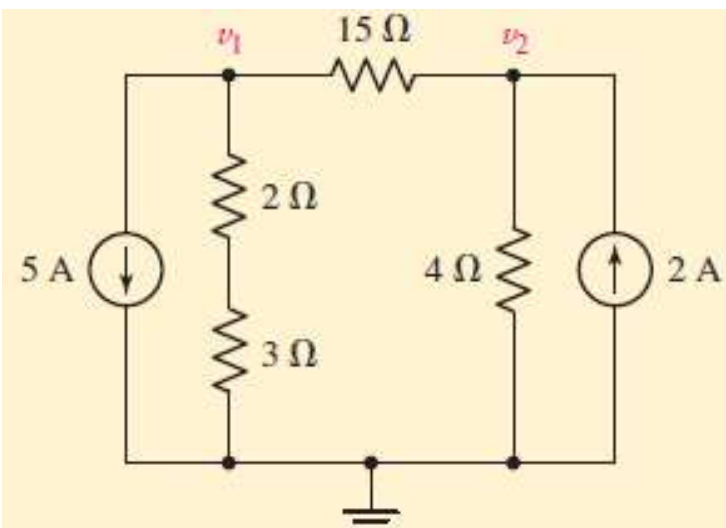


$$-2+\frac{V_1-0}{7+3}+\frac{V_1-V_2}{15}=0---(1)$$

$$-4+\frac{V_2-0}{5}+\frac{V_2-V_1}{15}=0---(2)$$

Ans:0A

Determine the nodal voltages V_1 and V_2

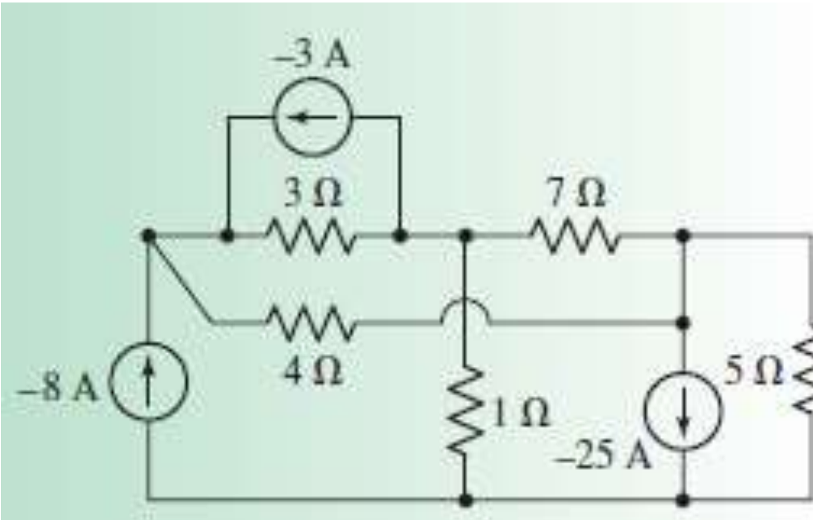


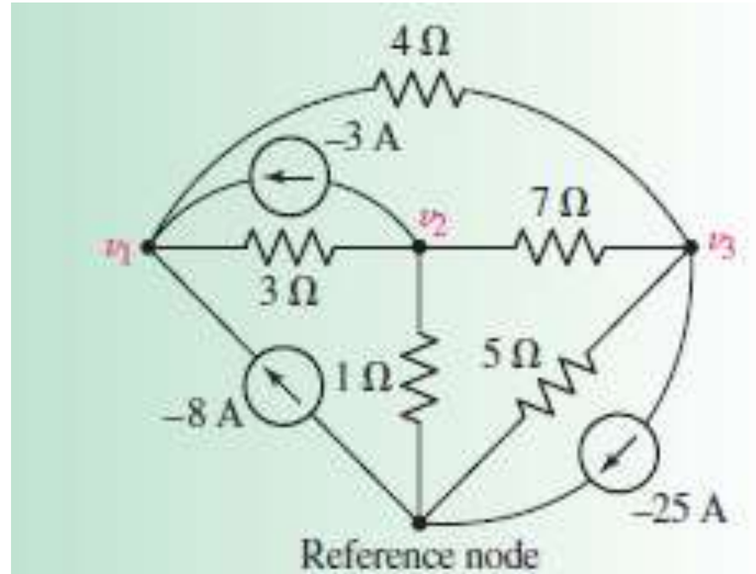
$$5+\frac{V_1-0}{2+3}+\frac{V_1-V_2}{15}=0---(1)$$

$$-2+\frac{V_2-0}{4}+\frac{V_2-V_1}{15}=0---(2)$$

Ans: V_1 = -145/8V & V_2 =5/2V

Determine the Nodal voltages for the circuit of Fig.





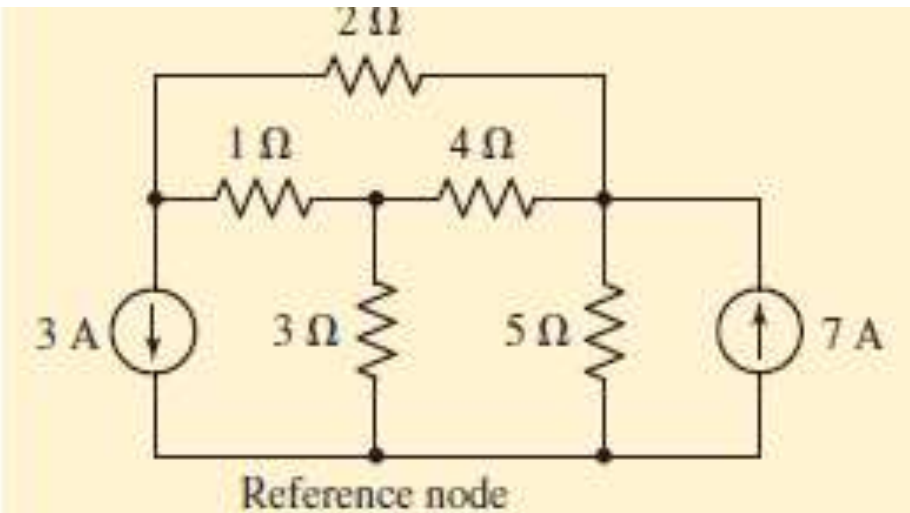
$$-(-3)-(-8)+\frac{V_1-V_2}{3}+\frac{V_1-V_3}{4}=0--(1)$$

$$-3+\frac{V_2-0}{1}+\frac{V_2-V_1}{3}+\frac{V_2-V_3}{7}=0--(2)$$

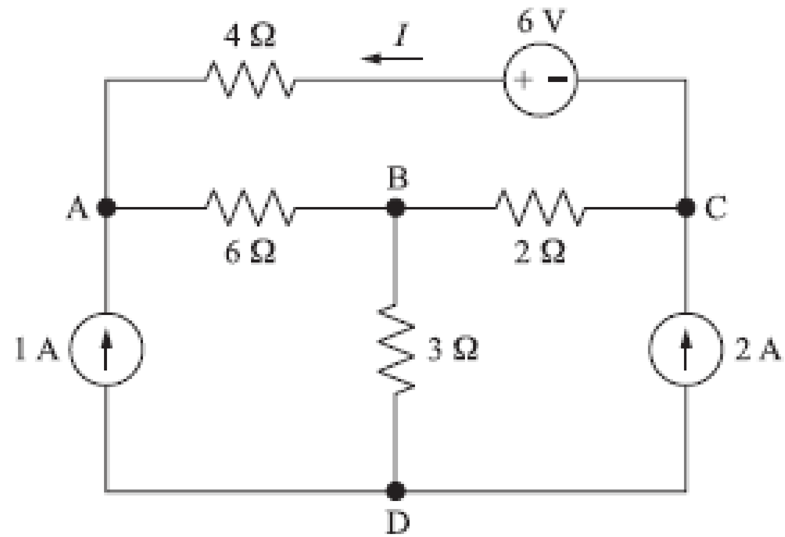
$$-25+\frac{V_3-0}{5}+\frac{V_3-V_1}{4}+\frac{V_3-V_2}{7}=0--(3)$$

Ans: V_1**=5.412V,**
V_2**=7.736V,**
V_3**=46.32V.**

I) Compute the voltage across each current source. **Ans: V_{3A}= 5.235V, V_{7A}=11.47V.**

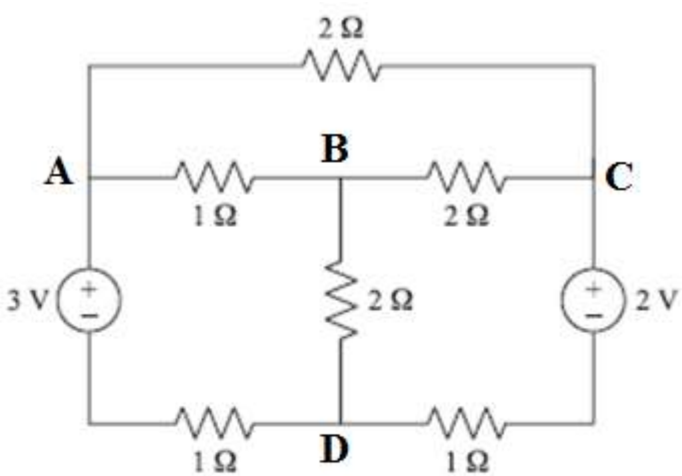


II) Compute the current 'I' in the circuit.



Ans: I=0.3325A

III) Compute the Current between A&B in the circuit.



Ans: I=0.627A

Summary of Basic Nodal Analysis Procedure

1. Count the number of nodes (N).

2. Designate a reference node. The number of terms in your nodal equations can be minimized by selecting the node with the greatest number of branches connected to it.

3. Label the nodal voltages (there are $N - 1$ of them).

4. Write a KCL equation for each of the nonreference nodes. Sum the currents flowing into a node from sources on one side of the equation. On the other side, sum the currents flowing out of the node through resistors. Pay close attention to "-" signs.

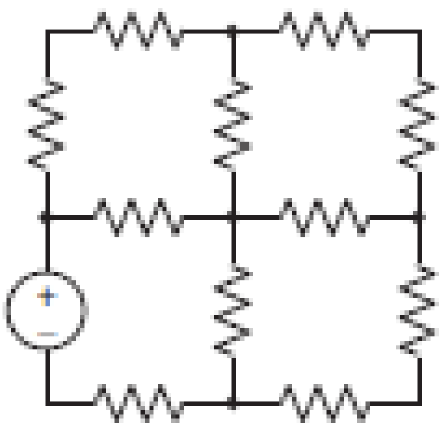
5. Express any additional unknowns such as currents or voltages other than nodal voltages in terms of appropriate nodal voltages. This situation can occur if voltage sources or dependent sources appear in our circuit.

6. Organize the equations. Group terms according to nodal voltages.

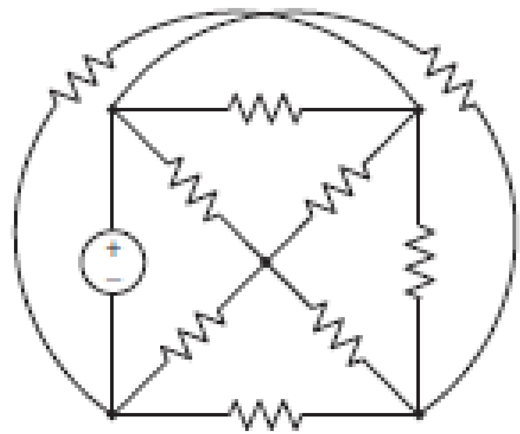
7. Solve the system of equations for the nodal voltages (there will be $N - 1$ of them).

Mesh Analysis

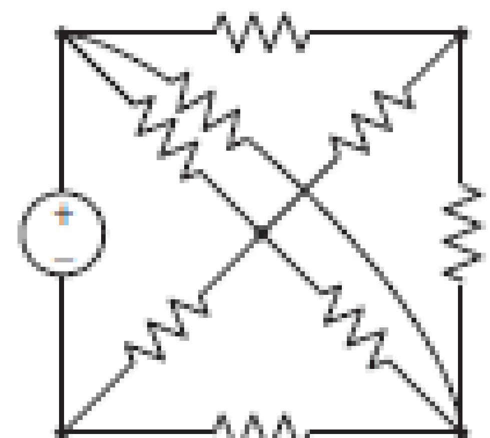
If it is possible to draw the diagram of a circuit on a plane surface in such a way that no branch passes over or under any other branch, then that circuit is said to be a **planar circuit.**



Planar Circuit



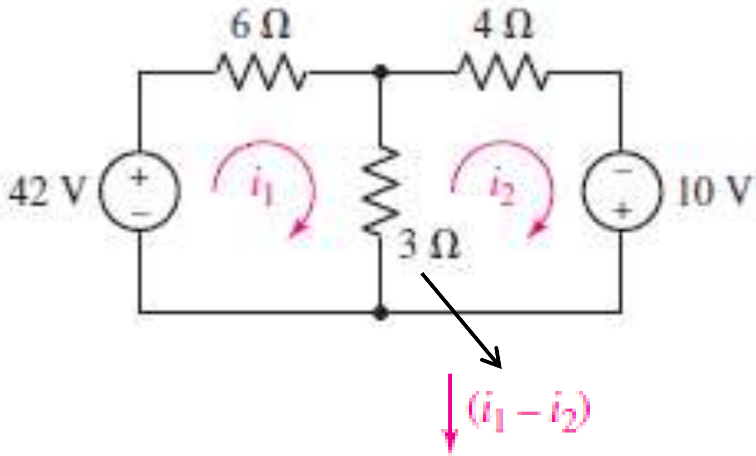
Non Planar Circuit

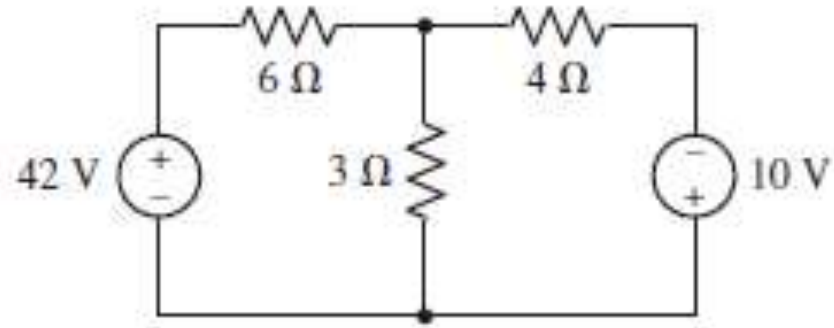


Planar Circuit

A '**Mesh'** as a loop that does not contain any other loops within it.
If a network is planar, mesh analysis can be used to accomplish the analysis.
This technique involves the concept of a **Mesh Current.**
Mesh current as a current that flows only around the perimeter of a mesh.

Find the Current through 3Ω using Mesh Analysis.





Ans: 2A

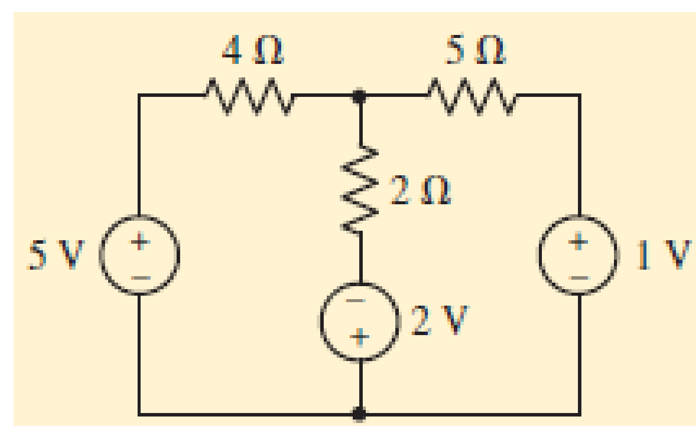
$$-42+6i_1+3(i_1-i_2)=0---(1);-10+3(i_2-i_1)+4i_2=0---(2)$$

Note: If circuit contains M meshes, then we expect to have M mesh currents and therefore will be required to write M independent equations.

Determine the power supplied by the 2 V source of Fig.

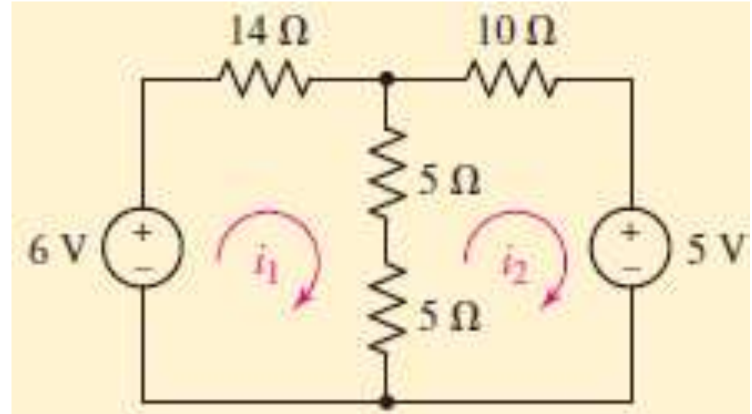
$$-5+4i_1+2(i_1-i_2)-2=0---(1)$$

$$2+2(i_2-i_1)+5i_2+1=0---(2)$$



Ans:i_1=1.13A,i_2= -0.105A ,2.474W

Determine i_1 and i_2 in the circuit in Fig. using Mesh Analysis.



$$-6+14i_1+5(i_1-i_2)+5(i_1-i_2)=0---(1)$$

$$5(i_2-i_1)+5(i_2-i_1)+10i_2+5=0---(2)$$

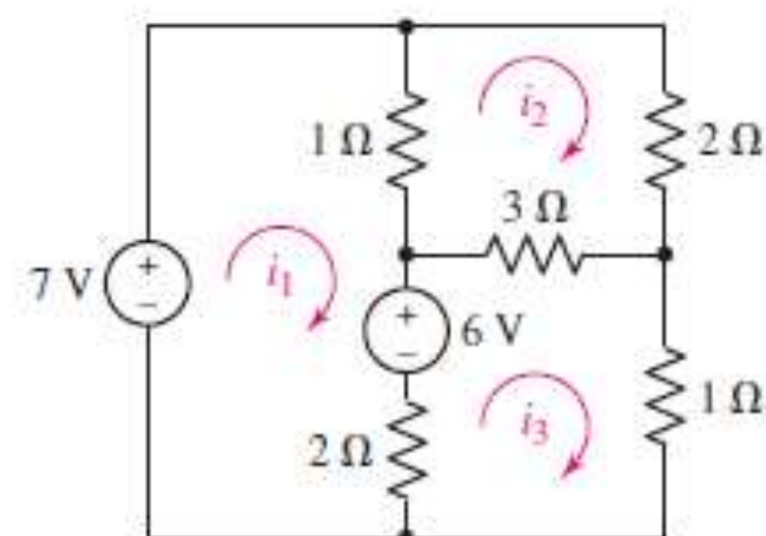
Ans: +184.2 mA, −157.9 mA.

Use mesh analysis to determine the three mesh currents in the circuit of Fig.

$$-7+1(i_1-i_2)+6+2(i_1-i_3)=0---(1)$$

$$1(i_2-i_1)+2i_2+3(i_2-i_3)=0---(2)$$

$$2(i_3-i_1)-6+3(i_3-i_2)+1i_3=0---(3)$$



Ans: 3A, 2A, 3A.