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B+

1.

Tutorial - sheet

Q. What is the complexity of the following piece of code:-

1. What is the time, space complexity of following code:-

Ans 1. (3.) O(N+M) time, O(1) space.

Ans. 2. O(n) time, O(1) space

Ans. 3. O(log n) time, O(1) space

Ans. 4. O($\sqrt{n}$) time, O(1) space

Ans. 5. O($\sqrt{n}$) time, O(1) space.

Ans. 6.
```
void recursion(int n)        T(n)
{
    if(n == 1) return;
    recursion(n-1);
    print(n);
    recursion(n-1);
}
```

$$T(n) = \begin{cases} \text{———} & n = 1 \\ 2T(n-1)+1 & n > 1 \end{cases} \quad -(1)$$

$T(n-1) = 2T(n-2) + 1$ — (2).

putting in (1):-

$T(n) = 2(2T(n-2)) + 1 + 1$

$T(n) = 4T(n-2) + 2 + 1$

$T(n-2) = 2T(n-3) + 1$ — (3)

$T(n) = 4(2T(n-3) + 1) + 2 + 1$

$T(n) = 8T(n-3) + 4 + 2 + 1$ — (4)

$T(n) = 2^3 T(n-3) +$

$T(n) = 2^k T(n-k) + 2^{k-2} + 2^{k-3} + 2^1 + 2^0$

$T(k) = 2^k T(n-k) + (1 + 2 + 4 + \ldots$ k times).

$T(1) = T(n-k)$.

$n = k + 1$.

$\boxed{k = n-1}$

$T(n) = 2^{n-1} T(1) + (1 + 2 + 4 + 8 \ldots$ (n-1) times)

$T(n) = 2^{n-1}(T(1)) + \frac{2^{n-1}}{2}$

Dt.
Pg. B+ 3.

$T(n) = 2^{n-1} + 2^{n-1}$

$T(n) = O(2^n)$ time

$O(2^n)$ time, $O(1)$ space.

Ans. 7 $T(n) = T\left(\frac{n}{2}\right) + 1$

as $T\left(\frac{n}{2}\right) = T\left(\frac{n}{4}\right) + 1$

$T(n) = T\left(\frac{n}{4}\right) + 2$

$T\left(\frac{n}{4}\right) = T\left(\frac{n}{8}\right) + 1$.

$T(n) = T\left(\frac{n}{8}\right) + 3$

$T(n) = T\left(\frac{n}{2^3}\right) + 3$.

$T(n) = T\left(\frac{n}{2^k}\right) + k$.

$\frac{n}{2^k} = 1$.

$n = 2^k$

Taking log on both sides.

$\log_2 n = k \log_2 2$

Dt. Pg. B+ 4.

$$\log_2 n = k$$

$$k = \log_2 n.$$

$$T(n) = T(1) + k.$$

$$T(n) = 1 + \log_2 n.$$

$$T(n) = \log_2 n.$$

$$T(n) = O(\log_2 n).$$

Ans.

Dt.
Pg. B+ 5-

Ans. 8 (1.) $T(1) = 1$

$T(n) = T(n-1) + 1$ — (1).

$T(n-1) = T(n-2) + 1$ — (2).

putting (2) in (1) :-

$T(n) = T(n-2) + 1 + 1$

$T(n) = T(n-2) + 2$ — (3)

$T(n-2) = T(n-3) + 1$ — (4).

putting (4) in (3) :-

$T(n) = T(n-3) + 1 + 2$.

$T(n) = T(n-3) + 3$ — (5).

$T(n) = T(n-k) + k$

$n - k = 1$

$k = n - 1$

$T(n) = T(1) + (n-1)$.

$T(n) = 1 + (n-1)$

$T(n) = n$.

$O(n)$ time, ~~$O(1)$~~

8.(2.) ⇒ $T(n) = T(n-1) + n$ — (1)

$T(n-1) = T(n-2) + (n-1)$ — (2)

putting (2) in (1) :-

$T(n) = T(n-2) + (n-1) + n$ — (3)

$T(n-2) = T(n-3) + (n-2)$ — (4)

putting (4) in (3) :-

$T(n) = T(n-3) + (n-2) + (n-1) + n$ — (5)

$T(n-3) = T(n-4) + (n-3)$ — (6)

putting (6) in (5) :-

$T(n) = T(n-4) + (n-3) + (n-2) + (n-1) + n$

$T(n) = T(n-k) + (n-(k-1)) + (n-(k-2)) + (n-(k-3)) + \cdots n$

$T(1) = T(n-k)$

$n-k = 1$, $k = n-1$

$n = k+1$,

$T(n) = T(1) + (n-(n-1-1)) + (n-(n-1-2)) + (n-(n-1-3)) + \cdots n$

$T(n) = 1 + 2 + 3 + 4 + \cdots n$

$T(n) = n * (n+1)/2$

$T(n) = O(n^2)$ time

Dt.
Pg. B+ 7.

8. (3.) $T(n) = T(n/2) + 1$

Using Master's Equation :-

$T(n) = aT(n/b) + f(n)$.

$a = 1,\ b = 2,\ f(n) = 1$

$c = \log_b a \Rightarrow \log_2 1 = 0$

$c = 0$

$n^c = n^0 = 1,\ f(n) = 1$

$n^c = f(n)$
$1 = 1$

$\cancel{T(n) = \theta(\log n)}$ $\boxed{T(n) = \theta(\log_2 n)}$
Ans

8. (4.) $T(n) = 2T(n/2) + 1$

Using Master's Equation :-

$T(n) = aT(n/b) + f(n)$

$a = 2,\ b = 2,\ f(n) = 1$.

$c = \log_b a \Rightarrow \log_2 2 = 1$

$c = 1$
$n^c = n^1 = n,\ f(n) = 1$
$n^c > f(n)$
$n > 1$

Dt
Pg. B+

$\boxed{\cancel{T(n) = \Theta(n\log_2 n)}}$ $\boxed{T(n) = \Theta(n)}$ Ans.

(5.) $T(n) = 2T(n-1) + 1$

as $T(n) = 2T(n-1) + 1$ — (1)

$T(n-1) = 2T(n-2) + 1$ — (2)

putting (2) in (1) :-

$T(n) = 2(2T(n-2) + 1) + 1$

$T(n) = 4T(n-2) + 2 + 1$ — (3)

$T(n-2) = 2T(n-3) + 1$ — (4).

putting (4) in (3) :-

$T(n) = 4(2T(n-3) + 1) + 2 + 1$

$T(n) = 8T(n-3) + 4 + 2 + 1$.

$T(n-3) = 2T(n-4) + 1$

$T(n) = 16T(n-4) + 8 + 4 + 2 + 1$.

$T(n) = 2^k T(n-k) + 2^{k-1} + 2^{k-2} + 2^{k-3} + 2^0$

$(n-k) = 1$

$k = n - 1$

$T(n) = 2^{n-1} T(1) + 2^{(n-1-1)} + 2^{(n-1-2)} + 2^{(n-1-3)} + 2^2 + 2^1 + 2^0.$

$= T(n) = 2^{n-1} + 2^{n-2} + 2^{n-3} + 2^{n-4} + 2^{n-n}$.

Dt
Pg
B+ 9

$$T(n) = \frac{2^n}{2^1} + \frac{2^n}{2^2} + \frac{2^n}{2^3} + \frac{2^n}{2^4} + \frac{2^n}{2^n}.$$

$$T(n) = 2^n \left(\underbrace{\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{2^n}}_{G.P.}\right).$$

$$T(n) = 2^n \left(\cancel{2^n} \left(1 - \frac{1}{2^n}\right)\right).$$

$$T(n) = (2^n)$$

$$T(n) = O(2^n) \text{ time.}$$

Ans.

8.(6.) $T(n) = 3T(n-1)$, $T(0) = 1$,

$T(n) = 3T(n-1)$ — (1)

$T(n-1) = 3T(n-2)$ — (2)

$T(n) = 3(3T(n-2))$

$T(n) = 9T(n-2)$

$T(n-2) = 3T(n-3)$

$T(n) = 27T(n-3)$

$T(n) = 3^3 T(n-3)$

$T(n) = 3^k T(n-k)$

$(n-k) = 0$

$n = k$.

$T(n) = 3^n T(0)$

$T(n) = 3^n$, $\boxed{T(n) = O(3^n)}$ time

8.(7) $T(n) = T(\sqrt{n}) + 1$.

$T(n^{\frac{1}{2}}) = T(n^{\frac{1}{4}}) + 1$.

$T(n) = T(n^{\frac{1}{4}}) + 1 + 1$

$T(n^{\frac{1}{4}}) = T(n^{\frac{1}{8}}) + 1$

Dt____ Pg____ B+ 17-

$$T(n) = T(n^{\frac{1}{8}}) + 1 + 1 + 1$$

$$T(n) = T\left(n^{\frac{1}{2^3}}\right) + 3(1)$$

$$T(n) = T\left(n^{\frac{1}{2^k}}\right) + k.$$

$$\boxed{n^{\frac{1}{2^k}} = 2}. \qquad T(2) = 1.$$

let $x = 2^k$.

$$\frac{1}{x} \log n = 1$$

$$\log n = x.$$

$$\log n = 2^k$$

$$\log_2(\log_2 n) = k \log_2 2$$

$$\log_2(\log_2 n) = k$$

$$T(n) = T(1) + k$$

$$T(n) = k$$

$$T(n) = \log(\log n)$$

$$\boxed{T(n) = O(\log(\log n))}$$ Ans.

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8 (8.) $T(n) = T(\sqrt{n}) + n$, $T(2) = 1$.

$T(n^{\frac{1}{2}}) = T(n^{\frac{1}{4}}) + n^{\frac{1}{2}}$

$T(n) = T(n^{\frac{1}{4}}) + n^{\frac{1}{2}} + n$

$T(n^{\frac{1}{4}}) = T(n^{\frac{1}{8}}) + n^{\frac{1}{4}}$

$T(n) = T(n^{\frac{1}{8}}) + n^{\frac{1}{4}} + n^{\frac{1}{2}} + n$

$T(n^{\frac{1}{8}}) = T(n^{\frac{1}{16}}) + n^{\frac{1}{8}} + n^{\frac{1}{4}} + n^{\frac{1}{2}} + n$.

$T(n) = T\left(n^{\left(\frac{1}{2^4}\right)}\right) + n^{\frac{1}{2^3}} + n^{\frac{1}{2^2}} + n^{\frac{1}{2^1}} + n^{\frac{1}{2^0}}$

$T(n) = T\left(n^{\frac{1}{2^k}}\right) + n^{\frac{1}{2^{k-1}}} + n^{\frac{1}{2^{k-2}}} + n^{\frac{1}{2^0}}$

$n^{\frac{1}{2^k}} = 2$.

$\frac{1}{2^k} \log n = 1$.

$\log n = 2^k$.

$\log \log n = k$.

$T(n) = 1 + n \log \log(n)$

$\boxed{T(n) = O(n \log(\log(n)))}$.

Ans

Ans. 9. $O(n)$ time, $O(1)$ space.

Ans. 10. (4) $O(n^2)$ time,

Ans. 11. (2) $O(n \log_2 n)$ time

Ans. 12. (2) X will always be a better choice for large input.

Ans. 13. (4) $O(\log n)$

Ans. 14. $T(n) = 7\,T\left(\frac{n}{2}\right) + 3n^2 + 2$

Using Master's Equation:-

$T(n) = a\,T\left(\frac{n}{b}\right) + (3n^2 + 2)$

$a = 7,\ b = 2,\ f(n) = 3n^2 + 2$

$c = \log_b a = \log_2 7 = 2.8,\ c = 2.8$

$n^c = n^{2.8}$

$n^c > f(n)$

$n^{2.8} > 3n^2 + 2$

(C) $\boxed{T(n) = \Theta(n^{2.8})}$ Ans.

Ans. 15. (a) $f_2 > f_4 > f_1 > f_3$

Ans. 16. (a). $O(2^n)$

Ans. 17. $T(n) = 2T\left(\frac{n}{2}\right) + n^2$.

By using master's Equation:—

$T(n) = aT\left(\frac{n}{b}\right) + f(n)$

$a = 2, \; b = 2, \; f(n) = n^2$

$c = \log_b a \Rightarrow \log_2 2 = 1$

$c = 1,$

$n^c = n^1 = n$

$f(n) = n^2$

$f(n) > n^c$

$n^2 > n$

$T(n) = \Theta(n^2),$

(a) $T(n) = O(n^2)$.

Ans. 18. $O(\log n)$

Ans. 19. $T(n) = O(n^2)$