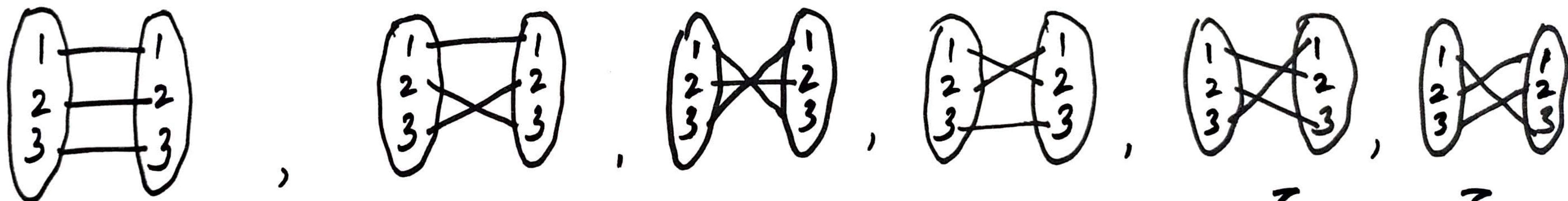
Permutation Groups

$G = \{ f : S \rightarrow S \mid f \text{ is bijective } fn \text{ on set } S \}$

under the operation 'composition'

Eg:- $S = \{1, 2, 3\}$ or $\{a, b, c\}$



e, σ_1, σ_2, σ_3, τ_1, τ_2

$G = S_3 = \{e, \sigma_1, \sigma_2, \sigma_3, \tau_1, \tau_2\}$

$|S_3| = 3! = 6$

In general, $|S_n| = n!$

Representation of S_n

$S_3 \Rightarrow$

$$\underset{e}{\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}}, \underset{\sigma_1}{\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}}, \underset{\sigma_2}{\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}}, \underset{\sigma_3}{\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}}$$

$$\underset{\tau_1}{\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}} \quad \underset{\tau_2}{\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}}$$

σ_2 fixes '2' interchanges 1 and 3

τ_1 does not fix any entity,
does not interchanges any entity
τ_1 maps 1 to 2, 2 to 3, 3 to 1

CYCLIC Representation

$e = (1)(2)(3)$

$\sigma_1 = (1)(23) = (23)$

$\sigma_2 = (13)(2) = (13)$

$\sigma_3 = (12)(3) = (12)$

$\tau_1 = (123)$

$\tau_2 = (132)$

$S_3 = \{e, (12), (13), (23), (123), (132)\}$

$(12), (13), (23)$ are of cycles of length 2

$(123), (132)$ are of cycles of length 3

Composition of elements

Eg :- $\tau_1 = (1\,2\,3)$

$$\tau_1^2 = \underbrace{(1\,2\,3)}_{x}\underbrace{(1\,2\,3)}_{y}$$

* Start from right. Start from 1

In y, 1 goes to 2, STOP [Don't see where 2 goes to]

In x, 2 goes to 3, STOP [Don't see where 3 goes to]

check if any other cycle is there on left.

If not, 1 goes to 3 → (1 3

* Now start with 3,

$$(1\,2\,3)(1\,2\,3)$$

In y, 3 goes to 1,

In x, 1 goes to 2. Finally 3 goes to 2

⇒ (1 3 2

Similarly check for 2. 2 will go to 1. ⇒ (1 3 2)

Order of elements

* If there is only one cycle, then cycle length is its order

  Eg :- Order of (12) is 2
  (123) is 3

* If there are more cycles,

  a) If they are disjoint, then LCM of those cycles length is its order

  b) If they are not disjoint, compose them & make disjoint, then take LCM of cycle lengths

JAM-2019

Q. 41) Let x be the 100-cycle $(1\ 2\ 3 \ldots 100)$ and let y be the transposition $(49\ 50)$ in the permutation group S_{100}. and let Then the order of xy is 99 ✓

$(1\ 2\ 3 \ldots\ 100)(49\ 50)$

$(1\ 2\ 3 \ldots 48\ 49\ 50\ 51 \ldots 99\ 100)(49\ 50)$

$\leftarrow \underbrace{(1\ 2\ 3\ 4 \ldots 48\ 49\ 51\ 52 \ldots 100)}\ (50)$

$\underline{99}$

$\boxed{\text{Ans :- } 99}$

JAM-2018

Q. 33) Suppose f, g, h are permutations of the set $\{\alpha, \beta, \gamma, \delta\}$ where 'f interchanges α & β but fixes γ & δ'

'g interchanges β and γ, but fixes α & δ'

'h interchanges γ and δ, but fixes α & β'

Which of the following interchanges α and δ but fixes β and γ?

✓ A) $f \cdot g \cdot h \cdot g \cdot f$

B) $g \cdot h \cdot f \cdot h \cdot g$

C) $g \cdot f \cdot h \cdot f \cdot g$

✓ D) $h \cdot g \cdot f \cdot g \cdot h$

Soln :- $\{\alpha, \beta, \gamma, \delta\} = \{1, 2, 3, 4\}$

$f = (12)(3)(4) = (12)$, $g = (23)(1)(4) = 23$

$h = (34)(1)(2)$.

Which of the following gives $(14)(2)(3) = (14)$??

A) $f \cdot g \cdot h \cdot g \cdot f = (12)(23)(34)(23)(12) = (14)(2)(3) = (14)$

B) $g \cdot h \cdot f \cdot h \cdot g = (23)(34)(12)(34)(23) = (13)(2)(4) = (13)$ X

C) $g \cdot f \cdot h \cdot f \cdot g = (23)(12)(34)(12)(23) = (1)(24)(3) = (24)$ X

D) $h \cdot g \cdot f \cdot g \cdot h = (34)(23)(12)(23)(34) = (14)(2)(3) = (14)$

$\boxed{\text{Ans :- A, D}}$