AI1103: Assignment 5

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Download all latex-tikz codes from

https://github.com/Bharadwaja−rao−D/AI1103/blob/main/assignment5/assignment5.tex

Problem UGC-MATH 2019 Q 105:

Consider a simple symmetric random walk on integers, where from every state i you move to i-1 and i+1 with the probability half each. Then which of the following are true?

1) The random walk is aperiodic
2) The random walk is irreducible
3) The random walk is null recurrent
4) The random walk is positive recurrent

Solution:

We say a random walk or Markov chain is irreducible if all pairs of state space communicate, that is $(i \to j) \wedge (j \to i)\ \forall (i, j) \in$ state space.
In case of integers, without loss of generality let us assume that the random walk starts at i and We can say that j is accessible from i if

$$\mathbb{P}_i \left\{ \bigcup_{n=0}^{\infty} \{X_n = j\} \right\} = \mathbb{P}_i(A) > 0 \tag{0.0.1}$$

$$\mathbb{P}_i(A) = \Pr\{A|X_0 = i\} \tag{0.0.2}$$

$$\mathbb{P}_i(A = j) = \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^{|j-i|+2k} \tag{0.0.3}$$

$$= \left(\frac{1}{2}\right)^{|j-i|} \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^{2k} \tag{0.0.4}$$

$$= \left(\frac{1}{2}\right)^{|j-i|} \frac{4}{3} \tag{0.0.5}$$

$$\therefore \mathbb{P}_i(A = j) > 0 \tag{0.0.6}$$

similarly we can show that

$$\mathbb{P}_j(A = i) > 0 \tag{0.0.7}$$

$\therefore \forall (i, j) \in$ sample space i and j are connected
Hence The random walk on integers is irreducible.