AI1103: Assignment 2

Damaragidda Bharadwaja Rao - CS20BTECH11012

Download all python codes from

https://github.com/Bharadwaja−rao−D/AI1103/blob/main/assignment2/assignment2.py

and latex-tikz codes from

https://github.com/Bharadwaja−rao−D/AI1103/blob/main/assignment2/assignment2.tex

PROBLEM GATE-EC-Q40:

A digital communication system uses a repetition code for channel encoding/decoding. During transmission, each bit is repeated three times(0 is transmitted as 000, and 1 is transmitted as 111). It is assumed that the source puts out symbols independently and with equal probability. The decoder operates as follows: In a block of three received bits, if the number of zeros exceeds the number of ones, the decoder decides in favour of a 0, and if the number of ones exceeds the number of zeros, the decoder decides in favour of a 1. Assuming a binary symmetric channel with crossover probability p = 0.1, the average probability of error is

SOLUTION:

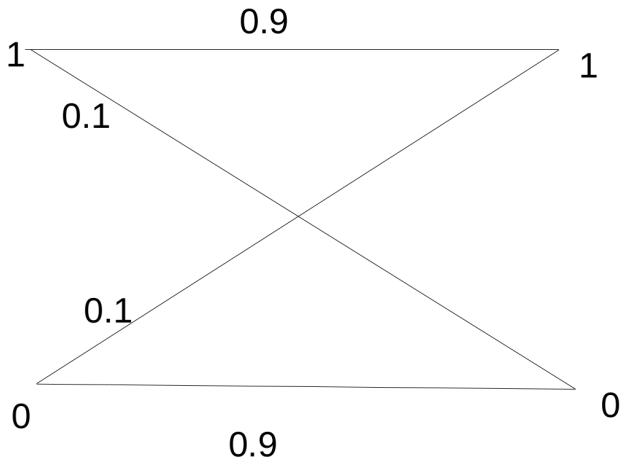

Fig. 0: symmetric binary channel with crossover probability 0.1

Let Y be the bit sent by the sender and X be the number of 1's received by the receiver and p = 0.1 is the crossover probability

Case 1: Y = 0

$$\Pr(X = i) = \binom{n}{i} \times p^i \times (1-p)^{n-i} \tag{0.0.1}$$

When $X \geq 2$ the receiver interprets it as 1, which is an error. And by Total Probability theorem we have

$$P_1 = \frac{P(X=2) + P(X=3)}{\sum_{i=0}^{3} P(X=i)} \tag{0.0.2}$$

where P_1 is the probability of error when Y = 0

Case 2: Y = 1

$$\Pr(X = i) = \binom{n}{i} \times p^{n-i} \times (1-p)^i \tag{0.0.3}$$

When $X \leq 1$ the receiver interprets it as 0, which is an error. And by Total Probability theorem we have

$$P_2 = \frac{\Pr(X=0) + \Pr(X=1)}{\sum_{i=0}^{3} \Pr(X=i)} \tag{0.0.4}$$

where P_2 is the probability of error when Y = 1

$$\sum_{i=0}^{3} \Pr(X=i) = 1 \times 0.9^3 + 3 \times 0.1 \times 0.9^2 + 3 \times 0.1^2 \times 0.9 + 1 \times 0.1^3 = 1 \tag{0.0.5}$$

$$P_1 = 0.028 \tag{0.0.6}$$

$$P_2 = 0.028 \tag{0.0.7}$$

The average probability is

$$P_{avg} = \Pr(Y=0) \times P_1 + \Pr(Y=1) \times P_2 = 0.028 \tag{0.0.8}$$

	X	0	1	2	3
Y=0	Pr(X)	0.729	0.243	0.027	0.001
Y=1	Pr(X)	0.001	0.027	0.243	0.729

TABLE 0: Probability of number of 1's recieved

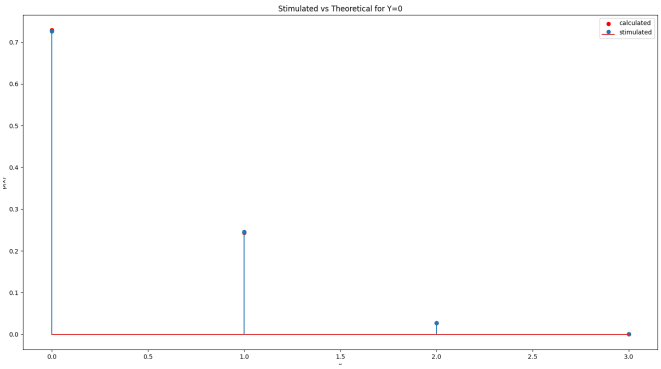

Fig. 0: Plot when Y = 0

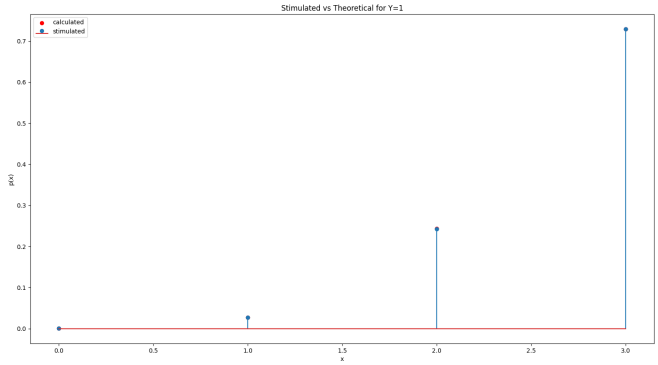

Fig. 0: Plot when Y = 0