

AI1103: Assignment 2

Damaragidda Bharadwaja Rao - CS20BTECH11012

Download all python codes from

<https://github.com/Bharadwaja-rao-D/AI1103/blob/main/assignment2/assignment2.py>

and latex-tikz codes from

<https://github.com/Bharadwaja-rao-D/AI1103/blob/main/assignment2/assignment2.tex>

PROBLEM GATE-EC-Q40:

A digital communication system uses a repetition code for channel encoding/decoding. During transmission, each bit is repeated three times (0 is transmitted as 000, and 1 is transmitted as 111). It is assumed that the source puts out symbols independently and with equal probability. The decoder operates as follows: In a block of three received bits, if the number of zeros exceeds the number of ones, the decoder decides in favour of a 0, and if the number of ones exceeds the number of zeros, the decoder decides in favour of a 1. Assuming a binary symmetric channel with crossover probability $p = 0.1$, the average probability of error is

SOLUTION:

Let Y be the bit sent by the sender and X be the number of 1's received by the receiver and $p = 0.1$ is the crossover probability

Case 1: $Y = 0$

$$\Pr(X = i) = \binom{n}{i} \times p^i \times (1 - p)^{n-i} \quad (0.0.1)$$

$$\Pr(X = 0) = \binom{3}{0} \times p^0 \times (1 - p)^3 \quad (0.0.2)$$

$$\Pr(X = 1) = \binom{3}{1} \times p^1 \times (1 - p)^2 \quad (0.0.3)$$

$$\Pr(X = 2) = \binom{3}{2} \times p^2 \times (1 - p)^1 \quad (0.0.4)$$

$$\Pr(X = 3) = \binom{3}{3} \times p^3 \times (1 - p)^0 \quad (0.0.5)$$

When $X \geq 2$ the receiver interprets it as 1, which is an error. And by Total Probability theorem we have

$$P_1 = \frac{P(X = 2) + P(X = 3)}{\sum_{i=0}^3 P(X = i)} \quad (0.0.6)$$

where P_1 is the probability of error when $Y = 0$

Case 2: $Y = 1$

$$\Pr(X = i) = \binom{n}{i} \times p^{n-i} \times (1 - p)^i \quad (0.0.7)$$

$$\Pr(X = 0) = \binom{3}{0} \times p^3 \times (1 - p)^0 \quad (0.0.8)$$

$$\Pr(X = 1) = \binom{3}{1} \times p^2 \times (1 - p)^1 \quad (0.0.9)$$

$$\Pr(X = 2) = \binom{3}{2} \times p^1 \times (1 - p)^2 \quad (0.0.10)$$

$$\Pr(X = 3) = \binom{3}{3} \times p^0 \times (1 - p)^3 \quad (0.0.11)$$

When $X \leq 1$ the receiver interprets it as 0, which is an error. And by Total Probability theorem we have

$$P_2 = \frac{\Pr(X = 0) + \Pr(X = 1)}{\sum_{i=0}^3 \Pr(X = i)} \quad (0.0.12)$$

where P_2 is the probability of error when $Y = 1$

$$\begin{aligned} \sum_{i=0}^3 \Pr(X = i) &= 1 \times 0.9^3 + 3 \times 0.1 \times 0.9^2 \\ &+ 3 \times 0.1^2 \times 0.9 + 1 \times 0.1^3 = 1 \end{aligned} \quad (0.0.13)$$

$$P_1 = 0.028 \quad (0.0.14)$$

$$P_2 = 0.028 \quad (0.0.15)$$

The average probability is

$$\begin{aligned} P_{avg} &= \Pr(Y = 0) \times P_1 + \Pr(Y = 1) \times P_2 \\ &= 0.028 \end{aligned} \quad (0.0.16)$$

	X	0	1	2	3
Y=0	Pr(X)	0.729	0.243	0.027	0.001
Y=1	Pr(X)	0.001	0.027	0.243	0.729

TABLE 0: Probability of number of 1's recieved

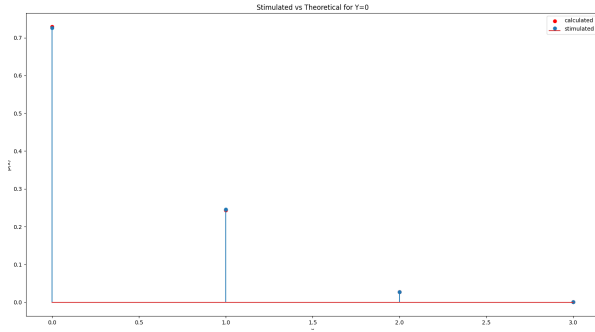


Fig. 0: Plot when $Y = 0$

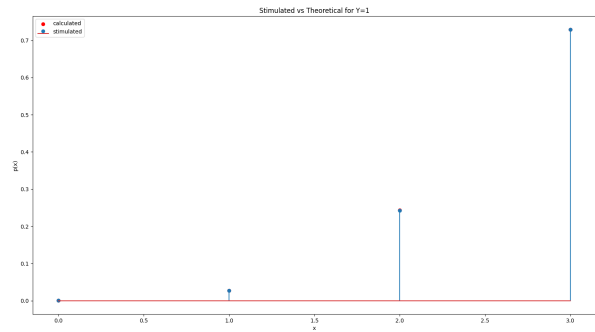


Fig. 0: Plot when $Y = 0$