Assignment2

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Q 40

A digital communication system uses a rep-etition code for channel encoding/decoding.During transmission, each bit is repeated threetimes(0 is transmitted as 000, and 1 is trans-mitted as 111). It is assumed that the sourceputs out symbols independently and with equalprobability. The decoder operates as follows: Ina block of three received bits, if the number ofzeros exceeds the number of ones, the decoderdecides in favour of a 0, and if the number ofones exceeds the number of zeros, the decoderdecides in favour of a 1. Assuming a binarysymmetric channel with crossover probability p = 0.1, the average probability of error is

solution

case i : The sender has sent 000 bit which has a probability of $\left(\frac{1}{2}\right)$
Let X be the number of 1's recieved due to crossover and p = 0.1 be the crossover probability

$$P(X = i) = \binom{n}{i} \times p^i \times (1-p)^{n-i} \tag{1}$$

$$P(X = 0) = \binom{3}{0} \times p^0 \times (1-p)^3$$

$$P(X = 1) = \binom{3}{1} \times p^1 \times (1-p)^2$$

$$P(X = 2) = \binom{3}{2} \times p^2 \times (1-p)^1$$

$$P(X = 3) = \binom{3}{3} \times p^3 \times (1-p)^0$$

When $X \geq 2$ the reciever interprets it as 1, which is an error. And by Total Probability theorem we have

$$P_1 = \frac{P(X = 2) + P(X = 3)}{\sum_{i=0}^{3} P(X = i)} \tag{2}$$

where P_1 is the probability of error when the sender has sent 0
case ii : The sender has sent 111 bit which has a probability of $\left(\frac{1}{2}\right)$
Let X be the number of 1's recieved and p = 0.1 be the crossover probability

$$P(X = i) = \binom{n}{i} \times p^{n-i} \times (1-p)^i \tag{3}$$

$$P(X = 0) = \binom{3}{0} \times p^3 \times (1-p)^0$$

$$P(X = 1) = \binom{3}{1} \times p^2 \times (1-p)^1$$

$$P(X = 2) = \binom{3}{2} \times p^1 \times (1-p)^2$$

$$P(X = 3) = \binom{3}{3} \times p^0 \times (1-p)^3$$

When $X \leq 1$ the reciever interprets it as 0, which is an error. And by Total Probability theorem we have

$$P_2 = \frac{P(X = 0) + P(X = 1)}{\sum_{i=0}^{3} P(X = i)} \tag{4}$$

where P_2 is the probability of error when the sender has sent 1

$$\sum_{i=0}^{3} P(X=i) = 1 \times 0.9^3 + 3 \times 0.1 \times 0.9^2$$
$$+ 3 \times 0.1^2 \times 0.9 + 1 \times 0.1^3 = 1$$

Let Y be the number sent by the sender

$$P_1 = 0.028$$
$$P_2 = 0.028$$

The average probability is

$$P_{avg} = P(Y=0) \times P_1 + P(Y=1) \times P_2$$
$$= 0.028 \quad (5)$$

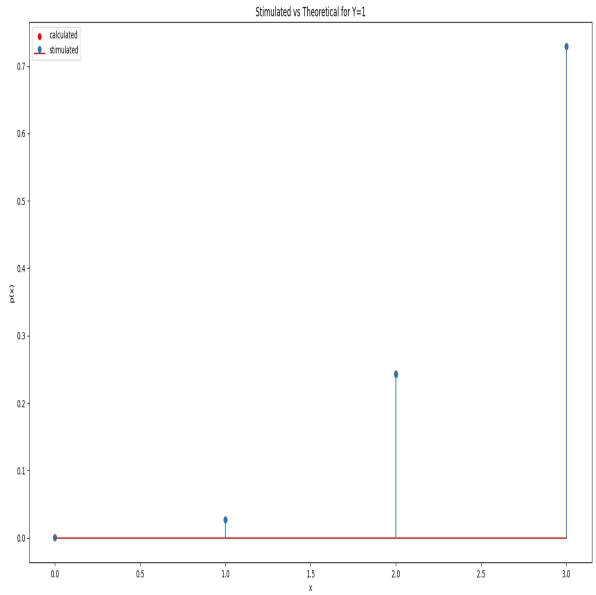

Figure 2: graph for Y = 1

	X	0	1	2	3
Y=0	P(X)	0.729	0.243	0.027	0.001
Y=1	P(X)	0.001	0.027	0.243	0.729

Table 1: Probability of number of 1's recieved

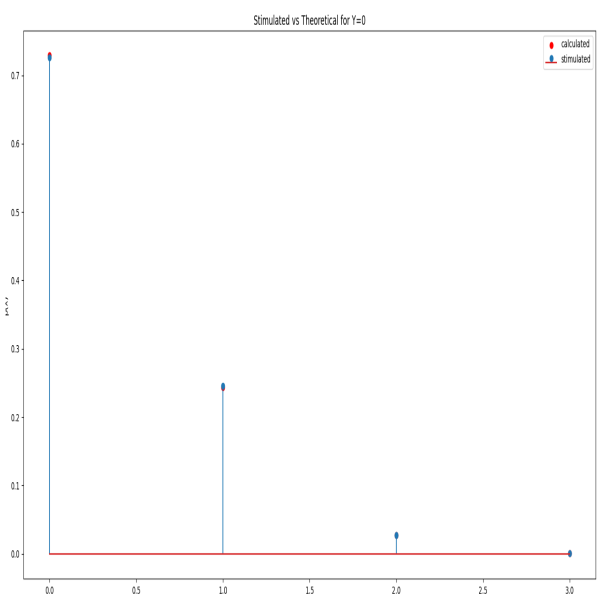

Figure 1: graph for Y = 0