AI1103: Assignment 2

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Download all python codes from

https://github.com/Bharadwaja−rao−D/AI1103/blob/main/assignment2/assignment2.py

and latex-tikz codes from

https://github.com/Bharadwaja−rao−D/AI1103/blob/main/assignment2/assignment2.tex

Problem GATE-EC-Q40:

A digital communication system uses a repetition code for channel encoding/decoding. During transmission, each bit is repeated three times(0 is transmitted as 000, and 1 is transmitted as 111). It is assumed that the source puts out symbols independently and with equal probability. The decoder operates as follows: In a block of three received bits, if the number of zeros exceeds the number of ones, the decoder decides in favour of a 0, and if the number of ones exceeds the number of zeros, the decoder decides in favour of a 1. Assuming a binary symmetric channel with crossover probability p = 0.1, the average probability of error is

Solution:

Let Y be the bit sent by the sender and X be the number of 1's received by the receiver and p = 0.1 is the crossover probability

Case 1: Y = 0

$$\Pr(X = i) = \binom{n}{i} \times p^i \times (1-p)^{n-i} \tag{0.0.1}$$

$$\Pr(X = 0) = \binom{3}{0} \times p^0 \times (1-p)^3 \tag{0.0.2}$$

$$\Pr(X = 1) = \binom{3}{1} \times p^1 \times (1-p)^2 \tag{0.0.3}$$

$$\Pr(X = 2) = \binom{3}{2} \times p^2 \times (1-p)^1 \tag{0.0.4}$$

$$\Pr(X = 3) = \binom{3}{3} \times p^3 \times (1-p)^0 \tag{0.0.5}$$

When $X \geq 2$ the receiver interprets it as 1, which is an error. And by Total Probability theorem we have

$$P_1 = \frac{P(X=2) + P(X=3)}{\sum_{i=0}^{3} P(X=i)} \tag{0.0.6}$$

where $\Pr_1$ is the probability of error when Y = 0

Case 2: Y = 1

$$\Pr(X = i) = \binom{n}{i} \times p^{n-i} \times (1-p)^{i} \tag{0.0.7}$$

$$\Pr(X = 0) = \binom{3}{0} \times p^3 \times (1-p)^0 \tag{0.0.8}$$

$$\Pr(X = 1) = \binom{3}{1} \times p^2 \times (1-p)^1 \tag{0.0.9}$$

$$\Pr(X = 2) = \binom{3}{2} \times p^1 \times (1-p)^2 \tag{0.0.10}$$

$$\Pr(X = 3) = \binom{3}{3} \times p^0 \times (1-p)^3 \tag{0.0.11}$$

When $X \leq 1$ the receiver interprets it as 0, which is an error. And by Total Probability theorem we have

$$P_2 = \frac{\Pr(X=0) + \Pr(X=1)}{\sum_{i=0}^{3} \Pr(X=i)} \tag{0.0.12}$$

where $\Pr_2$ is the probability of error when Y = 1

$$\begin{aligned}\sum_{i=0}^{3} \Pr(X = i) &= 1 \times 0.9^3 + 3 \times 0.1 \times 0.9^2 \\ &\quad + 3 \times 0.1^2 \times 0.9 + 1 \times 0.1^3 = 1\end{aligned} \tag{0.0.13}$$

$$P_1 = 0.028 \tag{0.0.14}$$

$$P_2 = 0.028 \tag{0.0.15}$$

The average probability is

$$\begin{aligned}P_{avg} &= \Pr(Y=0) \times P_1 + \Pr(Y=1) \times P_2 \\ &= 0.028\end{aligned} \tag{0.0.16}$$

	X	0	1	2	3
Y=0	Pr(X)	0.729	0.243	0.027	0.001
Y=1	Pr(X)	0.001	0.027	0.243	0.729

TABLE 0: Probability of number of 1's recieved

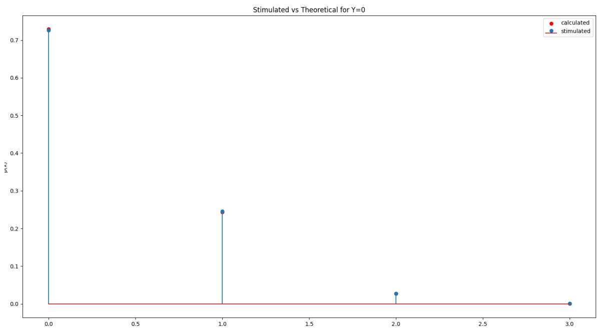

Fig. 0: Plot when Y = 0

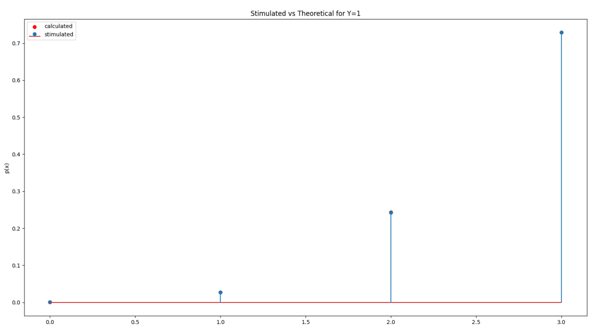

Fig. 0: Plot when Y = 0