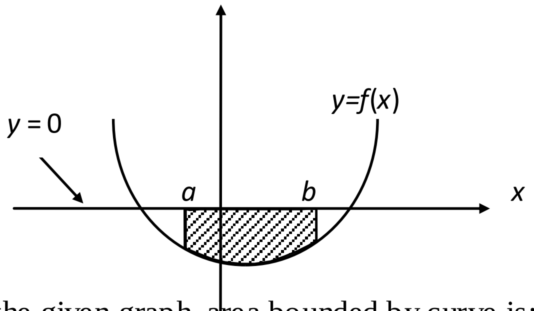
Question Bank: AML5101

Objective	Type of questions	Approximate time expected to be spent by a student on answering every single question
To test memory	MCQ	1 minute

1. $$Area = \int_a^b [\ldots\ldots\ldots\ldots\ldots\ldots]dx$$
   a) Top Function – Bottom Function
   b) Top Function
   c) Bottom Function
   d) Top Function + Bottom Function

2. 

In the given graph, area bounded by curve is:

a) $\int_a^b f(x)dx$

b) $-\int_a^b f(a)dx$

c) $-\int_a^b f(b)dx$

d) $-\int_a^b f(x)dx$

3. If A is a square matrix, then A^{-1} exist iff
   a) $|A| = 0$
   b) $|A| \neq 0$
   c) $|A| > 0$
   d) $|A| < 0$

4. If A and B are two equivalent matrices, then
   a) Rank A = Rank B
   b) Rank A ≠ Rank B
   c) Rank A > Rank B
   d) None of these.

5. Rank of matrix $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is
   a) 1
   b) 2
   c) 3
   d) 0

6. For which of the following type of matrix, rank may not be equal to its order?
   a) Square
   b) Scalar
   c) Identity
   d) Non singular

7. Which of the following is not normal form?
   a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
   b) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$
   c) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$
   d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

8. If $A^2 + A - I = 0$, then $A^{-1} =$
   a) $A + A^2$
   b) $A + I$
   c) A
   d) I

9. Using normal notations, the system of non-homogeneous system of equations have unique solution if:
   a) Rank of (A:B) = rank of (A) = number of variables.
   b) Rank of (A:B) $\neq$ rank of (A).
   c) Rank of (A:B) = rank of (A) < number of variables.
   d) Rank of (A:B) = rank of (A) > number of variables.

10. The system of homogeneous system of equations have Trivial solution if:
    a) Rank of (A) < number of variables.
    b) $|A| = 1$.
    c) Rank of (A) = number of variables.
    d) Rank of (A) > number of variables.

11. In system of non-homogeneous linear equations rank of augmented matrix is equal to the:
    a) Number of zero rows in its Row Echelon Form.
    b) Number of non- zero rows in its Row Echelon Form.
    c) Number of zero in its Row Echelon Form.
    d) Number of non-zero elements in its Row Echelon Form.

12. Which of the following is the solution of $4x - 3y = 2$ and $3x - 4y = 5$:
    a) (1, 2)
    b) (-1, 2)
    c) (1, -2)
    d) (-1, -2)

13. $2x + 3y = 9$ and $2x + \lambda y = \mu$ has infinite solutions, if:
    a) $\lambda = -3$ and $\mu = 9$
    b) $\lambda = 3$ and $\mu = -9$
    c) $\lambda = 3$ and $\mu = 9$
    d) $\lambda = -3$ and $\mu = -9$

14. The given system of equations $x - 5/8y = 0$ and $8/5x - y = 0$ has:
    a) Infinite solutions.
    b) No solution.
    c) Unique solution.
    d) Trivial solution.

15. The given system of equations $x + 2y + 3z = 0$, $3x + 4y + 4z = 0$ and $7x + 10y + 12z = 0$ has:

a) Infinite solutions.
b) No solution.
c) Can't be determined.
d) Trivial solution.

16. For some scalar λ, if matrix A is such that $(A-\lambda I)$ is singular, then

a) λ is a characteristic root of A.
b) λ is not a characteristic root of A.
c) λ is zero.
d) λ is eigen vector.

17. If $\lambda_1, \lambda_2, \lambda_3$ are the eigen values of matrix A then A^3 has the eigen values

a) $\lambda_1^2, \lambda_2^2, \lambda_3^2$
b) $\lambda_1, \lambda_2, \lambda_3$
c) $\lambda_1^3, \lambda_2^3, \lambda_3^3$
d) $\lambda_1 = \lambda_2 = \lambda_3$

18. The eigen values for matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$ are given as

a) 1,2,3
b) 1,-2,3
c) 3,2,-2
d) 2,3,0

19. Cayley Hamilton theorem is ………………satisfied for $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 1 \end{bmatrix}$

a) Always
b) Never
c) Sometimes
d) Rarely

20. Product of eigen values of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 0 & 5 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ is

a) 5
b) 7
c) 9
d) 0

21. Characteristic equation for a matrix $\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$ is given by

a) $\lambda^2 - 2\lambda - 5 = 0$
b) $\lambda^2 - 2\lambda + 5 = 0$
c) $\lambda^2 + 2\lambda - 5 = 0$
d) $\lambda^2 + 2\lambda + 5 = 0$

22. Which of the following matrix has same eigen values as of matrix $A = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$

a) $\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$
b) $\begin{bmatrix} 2 & 6 \\ 8 & 4 \end{bmatrix}$

c) $\begin{bmatrix} 13 & 9 \\ 12 & 16 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

23. If A is a square matrix and $A^2 = A$, then A is
    a) Idempotent Matrix.
    b) Symmetric Matrix.
    c) Involuntary Matrix. → $A^2 = I$
    d) Orthogonal Matrix.

24. Rank of the matrix, whose every element is unity, is
    a) Greater than one.
    b) Equal to one.
    c) Zero.
    d) Negative

25. Rank of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$ is
    a) 0
    b) 1
    c) 2
    d) 3

26. By applying elementary transformation to a matrix, its rank
    a) Decreases.
    b) Increases.
    c) Do not change.
    d) Cannot say anything.

27. If the rank of matrix A is 3, then rank of $3A^T$ is
    a) 1
    b) 2
    c) 3
    d) 9

28. Which of the following statements is not true?
    a) Rank of A and A^T is same.
    b) Rank of null matrix is not defined.
    c) For a n-square matrix, if rank = n then $|A| \neq 0$.
    d) For a n-square matrix, if rank < n then $|A| = 0$.

29. Which of the following is not associated with system of non-homogeneous equations?
    a) Rank of coefficient Matrix.
    b) Augmented Matrix.
    c) Infinite number of solutions.
    d) Trivial solution.

30. The system of homogeneous system of equations have infinite solutions if:
    a) Rank of (A) < number of variables.
    b) $|A| \neq 0$.
    c) Rank of (A) = number of variables.
    d) Rank of (A) > number of variables.

31. The system of homogeneous linear equations is:
a) Always inconsistent.
b) Always consistent.
c) Always have trivial solution.
d) Consistency depends upon the given problem to be solved.

32. For the system, $2x + 3y = 10$ and $x/3 + y/2 = 5/3$ the values of x and y respectively are:
a) (1,2)
b) (1,1)
c) (2,2)
d) (0,0)

33. The given system of equations $(8-\lambda)x - 4y = 0$ and $2x + (2 -\lambda)y = 0$ has infinite number of solutions if λ is equal to:
a) 4
b) -4
c) 1
d) 0

34. The system has $x + 3y + 2z = 0$, $2x - y + 3z = 0$ and $3x - 5y + 4z = 0$ has:
a) Unique solution.
b) No solution.
c) Infinite solutions.
d) Can't be determined.

35. The rank of the coefficient matrix of $x - y + 2z = 3$, $x + 2y + 3z = 5$ and $3x - 4y - 5z = -13$ is:
a) 0
b) 1
c) 2
d) 3

36. If scalar λ is a characteristic root of the matrix A, then $(A-\lambda I)$ is
a) Singular matrix.
b) Non –singular matrix.
c) Diagonal matrix.
d) Triangular matrix.

37. The product of Eigen values of a matrix A is equal to
a) A
b) A^T
c) Determinant of matrix A
d) Rank(A)

38. The characteristic root for the matrix $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ are
a) 5, 1, 1
b) 5, 2, 2
c) 0, 0, 0
d) 15, 14, 15

39. Every square matrix satisfies its own characteristic equation is known as
a) Taylor's theorem
b) Maclaurin's theorem
c) Cayley Hamilton theorem
d) Euler's theorem

40. If the product of two Eigen values of $\begin{bmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ is -4 then the third eigen value is

a) -2
b) 3
c) 1
d) 2

41. Characteristic equation for a matrix $\begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}$ is given by

a) $\lambda^2-4\lambda-5=0$
b) $\lambda^2-4\lambda+5=0$
c) $\lambda^2+4\lambda-5=0$
d) $\lambda^2+4\lambda+5=0$

42. A square matrix A is nilpotent if

a) $A^m = I$, where m is any positive integer.
b) $A^m = 0$, where m is any positive integer.
c) $A^m = A$, where m is any positive integer.
d) None of these.

43. If A is a square matrix of order n, then A^{-1} exists if

a) Rank A = 0
b) Rank A = n
c) Rank A < n
d) Rank A > n

44. Find rank of $A=\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$.

a) 0
b) 1
c) 2
d) Not possible

45. Rank of the matrix is the number of

a) Non-zero elements in Row Echelon form of the matrix.
b) Non-zero rows in Row Echelon form of the matrix.
c) Non-zero elements in the matrix.
d) Non-zero rows in the matrix.

46. If $A= [I_2 \quad 0]_{2X3}$ rank of A is

a) 1
b) 2
c) 0
d) Not possible

47. If A is any n-order square matrix and k is any scalar then

a) $|kA|=k^n |A|$
b) $|kA|=k |A|$
c) $|kA|= |A|$
d) None of these

48. Using normal notations, the system of non-homogeneous system of equations have infinite solutions if:

a) Rank of (A:B) = rank of (A) = number of variables.
b) Rank of (A:B) $\neq$ rank of (A).
c) Rank of (A:B) = rank of (A) < number of variables.
d) Rank of (A:B) = rank of (A) > number of variables.

49. The system of linear equations is said to be consistent if it has:

a) Only unique solution.
b) Only infinite solutions.
c) Unique as well as infinite solutions.
d) No solution.

50. Let 'A' be the coefficient matrix of a homogeneous system of equations having infinite number of solutions, then determinant of A must be equal to:

a) -1.
b) 1
c) 0
d) Can't be determined.

51. Solution of the system of equations x + y + z =3 , x + 2y + 3z = 4 and x + 4y + 9z = 6 is (x, y, z):

a) (2, 1, 0)
b) (3, 1, 0)
c) (2, 0, 0)
d) (0, 1, 0)

52. For which value/s of 'k' the system has kx + y = 1 and 6x +(k+1)y = 3 has infinite number of solutions:

a) 2
b) -3
c) 2 and -3
d) -2 and -3

53. The given system of equations 8x - 4y = 0 and 2x + 2y = 0 has:

a) Infinite solutions.
b) No solution.
c) Unique solution.
d) Trivial solution.

54. Maximum possible rank of $[A]_{5X4}$ is

a) 4
b) 5
c) 0
d) 1

55. Characteristic equation of a square matrix is given by

a) (A-λI)
b) |A-λI|
c) |A-λI| = 0
d) |A-λI|≠ 0

56. The sum of eigen values of a matrix is equal to

a) Determinant of A
b) A^T
c) Sum of principal diagonal elements
d) Rank(A)

57. The Eigen values of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ are

a) 2,3,6
b) 2,6,7
c) -2,3,6
d) 1,1,1

58. For Cayley Hamilton Theorem matrix A must be:
a) Rectangular matrix
b) Row matrix
c) Column matrix
d) Square matrix

59. The sum of the squares of eigen values of $\begin{bmatrix} 2 & 3 & 4 \\ 0 & 3 & 6 \\ 0 & 0 & 6 \end{bmatrix}$ is
a) 0
b) 11
c) 49
d) 121

60. Characteristic equation for a matrix $\begin{bmatrix} 3 & 1 \\ -1 & 3 \end{bmatrix}$ is given by
a) $\lambda^2 - 6\lambda - 10 = 0$
b) $\lambda^2 - 6\lambda + 10 = 0$
c) $\lambda^2 + 6\lambda - 10 = 0$
d) $\lambda^2 + 6\lambda + 10 = 0$

61. A square matrix is orthogonal if
a) $AA^T \neq I$
b) $AA^T = A^TA = I$
c) $AA^T = 0$
d) None of these.

62. If two rows of a matrix are identical then
a) $|A|=0$
b) $|A|= 1$
c) $|A|= 2$
d) $|A| \neq 0$

63. Rank of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$ is
a) 0
b) 1
c) 2
d) 3

64. Maximum possible rank of $[A]_{5X4}$ is
a) 4
b) 5
c) 0
d) 1

65. Normal form of a matrix is used to find
a) Characteristic equation of that matrix.
b) Characteristic roots of that matrix.
c) Determinant of that matrix.
d) Rank of that matrix.

66. If Gauss Jordan method of finding inverse of a matrix, we can apply
   a) Elementary row operations.
   b) Elementary column operations.
   c) Both (a) and (b).
   d) None of these.

67. Using normal notations, the system of non-homogeneous system of equations have no solution if:
   a) Rank of (A:B) = rank of (A) = number of variables.
   b) Rank of (A:B) $\neq$ rank of (A).
   c) Rank of (A:B) = rank of (A) < number of variables.
   d) Rank of (A:B) = rank of (A) > number of variables.

68. If a linear system of equations have Trivial Solution, then:
   a) All the variables have zero value.
   b) All the variables have non-zero value.
   c) All the variables have infinite values.
   d) All the variables do not have any value.

69. In homogeneous system of equations, which of following is not possible:
   a) Unique solution.
   b) Infinite solutions.
   c) No solution.
   d) Trivial solution

70. $x + y + 4z = 1$, $x + 2y - 2z = 1$ and $\lambda x + y + z = 1$ has unique solution if the value of λ is:
   a) $=7/10$
   b) $\neq 7/10$
   c) $= -7/10$
   d) $\neq -7/10$

71. Solution of system of equations, $3x + y + 2z = 3$, $2x - 3y - z = -3$, $x + 2y + z = 4$ is given by
   a) $x=1, y=2, z=1$
   b) $x=1, y=2, z=-1$
   c) $x=1, y=-2, z=1$
   d) $x=-1, y=2, z=1$

72. The given system of equations $2x - 3y = 5$ and $3x + y = 13$ is:
   a) Consistent
   b) Inconsistent
   c) Homogeneous system
   d) Can't be determined

73. If A be a square matrix and k be any scalar the Characteristic Polynomial is:
   a) Det (A-kI)
   b) Det (A-kI)=0
   c) (A-kI)
   d) A-k

74. If $1/\lambda_1, 1/\lambda_2$ are the eigen values of matrix A^{-1}, then eigen values of matrix A are given by
   a) $1/\lambda_1$, $1/\lambda_2$
   b) λ_1^2, λ_2^2
   c) λ_1, λ_2
   d) 1,1

75. Cayley Hamilton theorem states
   a) Every square matrix satisfies its own characteristic equation.
   b) Every matrix satisfies its own characteristic equation.

c) Every rectangular matrix satisfies its own characteristic equation.
d) Every square matrix satisfies its own characteristic polynomial.

Objective	Type of questions	Approximate time expected to be spent by a student on answering every single question
To test comprehension	MCQ or True/False or Fill in the blanks	2 minutes

1. Find the area of the region that is enclosed by $y=-x^2+1$, $y=2x+4$, x = -1 and x = 2.
   a) 15
   b) 16
   c) 17
   d) 18

2. Area of the region that is enclosed by the x-axis, the graph $y=-x^2+4x-8$ over the interval [-1, 4].
   a) 31.67
   b) 31.38
   c) 30.08
   d) 32.11

3. Area of the region that is enclosed by the graphs of the functions:
$f(x)=2x-1$ and $g(x)=x^2-4$
   a) 32/3
   b) 3/32
   c) 23/3
   d) 3/23

4. Area of the region that is enclosed by $y=x^2-4x+3$ and the x- axis.
   a) 1.25
   b) 1.55
   c) 1.33
   d) 1.56

5. The value of $\iint r^3 dr d\theta$ over the region included between the circles $r=2\sin\theta$ and $r=4\sin\theta$ is
   a) 21π
   b) 21.5π
   c) 22π
   d) 22.5π

6. $\int_0^1 \int_{x^2}^{2-x} xydydx$ is equal to
   a) $\frac{1}{2}$
   b) $\frac{3}{8}$
   c) $\frac{3}{7}$
   d) $\frac{5}{2}$

7. $\int_{-a}^{a}\int_{0}^{\sqrt{a^2-y^2}} f(x,y)dxdy$ is equal to

a) $\int_{-a}^{a}\int_{0}^{\sqrt{a^2-x^2}} f(x,y)dxdy$

b) $\int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} f(x,y)dxdy$

c) $\int_{0}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} f(x,y)dxdy$

d) $\int_{-a}^{0}\int_{-\sqrt{a^2-x^2}}^{0} f(x,y)dxdy$

8. The value of the integral $\int_0^a \int_{x/a}^{\sqrt{x/a}} (x^2+y^2)dxdy$ is

a) $\frac{a^2}{28}+\frac{a^3}{20}$

b) $\frac{a^2}{28}+\frac{a}{20}$

c) $\frac{a^3}{28}+\frac{a}{20}$

d) $\frac{a^2}{20}+\frac{a^3}{28}$

9. $\iint_R \frac{r}{\sqrt{a^2+r^2}} drd\theta$, where R is one loop of the lemniscates $r^2 = a^2\cos^2 2\theta$, is equal to

a) $a\left(1-\frac{\pi}{4}\right)$

b) $a\left(1-\frac{\pi}{8}\right)$

c) $2a\left(1-\frac{\pi}{4}\right)$

d) $2a\left(1-\frac{\pi}{8}\right)$

10. $\iint_R xydxdy$, where R is the positive quadrant of the circle $x^2+y^2=a^2$, is equal to

a) $\frac{a}{8}$

b) $\frac{a^2}{8}$

c) $\frac{a^3}{8}$

d) $\frac{a^4}{8}$

11. If $f(x)$ is an odd function of x, then $\int_{-a}^{a} f(x)dx$ is equal to

a) 0

b) $2\int_{0}^{a} f(x)dx$

c) $-2\int_{0}^{a} f(x)dx$

d) 1

12. $\int_{0}^{b}\int_{0}^{a\sqrt{1-\frac{y^2}{b^2}}} dxdy$ is equal to

a) $\frac{\pi ab}{4}$

b) $\frac{\pi ab}{3}$

c) $\frac{\pi ab}{2}$

d) πqb

13. Minimum distance of the point $(3,4,12)$ from the sphere $x^2+y^2+z^2=1$ is equal to

a) **8**

b) 10

c) 12

d) 14

14. The value of the integral $\int_{0}^{\infty}\int_{0}^{\infty} e^{-(x^2+y^2)}dxdy$ is

a) π

b) $\frac{\pi}{2}$

c) $\frac{\pi}{4}$

d) $\frac{\pi}{8}$

15. The value of the integral $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x^2+y^2+z^2)dxdydz$ is

a) 0

b) $abc(a^2+b^2+c^2)$

c) 2

d) $\frac{8}{3}abc(a^2+b^2+c^2)$

16. The condition for the saddle point is:-

a) $f_{xx}f_{yy}-f_{xy}^2=0$

b) $f_{xx}f_{yy}-f_{xy}^2<0$

c) $f_{xx}f_{yy}-f_{xy}^2>0$

d) $f_{xx}f_{yy}-f_{xy}<0$

17. $\int\limits_R f(x,y)\,dxdy = \int\limits_{R_1} f(x,y)\,dxdy + \int\limits_{R_2} f(x,y)\,dxdy$

a) where R is the union of two overlapping regions R_1 and R_2
b) where R is the union of two non-overlapping regions R_1 and R_2
c) R_1 and R_2 may be overlapping or non-overlapping
d) none of the above

18. $\int\limits_{-a}^{a} \int\limits_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} \int\limits_{-\sqrt{a^2-x^2-y^2}}^{\sqrt{a^2-x^2-y^2}} dxdydz$ is equal to

a) $\frac{4}{3}\pi a^3$
b) $\frac{2}{3}\pi a^3$
c) $2\pi a^3$
d) 0

19. Changing to cylindrical coordinates π, θ and z by substituting x = $\pi\cos\theta$, y = $\pi\sin\theta$, z = z. The element dxdydz becomes

a) $\pi\cos\theta d\pi d\theta dz$
b) $\pi\sin\theta d\pi d\theta dz$
c) $\pi d\pi d\theta dz$
d) $\pi^2 d\pi d\theta dz$

20. A given quantity of metal is to be cast into a half-cylinder with a rectangular base and semicircular ends. For total surface area to be minimum, the ratio of the length of the cylinder to the diameter of its semi-circular ends is

a) $\pi : \pi + 3$
b) $\pi : \pi + 2$
c) $\pi : \pi + 4$
d) 0

21. $\int_0^1 \int_0^{1-x} \int_0^{1-x^2-y^2} dzdydx =$:

a) $\frac{1}{3}$
b) $\frac{8xyz}{3}$
c) $\frac{8}{3}$
d) $\frac{xy}{3}$

22. $\int\limits_0^1 \int\limits_0^2 \int\limits_0^{1-z} dydxdz$ is equal to :-

a) 0
b) 1
c) -1
d) 3

23. $\int\limits_0^{\pi} \int\limits_0^1 e^{r^2} r dr d\theta$ is equal to :-

a) $\frac{\pi}{2}$
b) $\frac{\pi}{2}(e)$
c) $\frac{\pi}{2}(e+1)$
d) $\frac{\pi}{2}(e-1)$

24. Find the value of 'a' if $\int_0^1\int_0^a\int_0^3 dzdydx = 9$

a) 3
b) 9
c) -3
d) 1

25. Given elliptic paraboloid $z = \frac{2x^2 + y^2}{3}$ in cylindrical polar coordinates (r, θ, z) will be as:-

a) $z = \frac{r^2}{3}(1 + \cos^2\theta)$

b) $z = \frac{r^2}{3}(1 - \cos^2\theta)$

c) $z = \frac{1}{3}(r^2 + \cos^2\theta)$

d) $z = \frac{1}{3}(1 + r^2\cos^2\theta)$

26. Critical points in $F(x, y, z) = f(x, y, z) + \lambda\phi(x, y, z)$ are find by solving

a) $f_x = 0, f_y = 0, f_z = 0$
b) $F_x = 0, F_y = 0, F_z = 0$
c) Both (a) and (b)
d) None of these

27. The value of the integral given below is: $\int_0^1\int_0^1\int_0^1(x + y + z)dxdydz$

a) 1
b) 1/3
c) 2/3
d) 3/2

28. Determine the value of the triple integral given below: $\int_0^3\int_0^2\int_0^1(x^2y^2z^2)dxdydz$

a) 0
b) 6
c) 8
d) 9

29. The value of the integral given below is: $\int_0^1\int_0^1\int_0^1(3xyz)dxdydz$

a) 1
b) 1/3
c) 2/3
d) 3/8

30. The solution of the integral given below is: $\int_0^a\int_0^b\int_0^c(xyz)dxdydz$

a) abc/4
b) $a^2b^2c^2/4$
c) $a^2b^2c^2/8$
d) abc/8

31. Evaluate the integral $\iiint (x^2 + y^2 + z^2)\,dx\,dy\,dz$, by transforming into cylindrical polar coordinates taken over the region $0 \le z \le x^2 + y^2 \le 1$. (Use equation of circle $x^2 + y^2 = 1$, where r varies from 0 to 1 and θ from 0 to 2)
   a) $5\pi/6$
   b) $2\pi/3$
   c) $7\pi/9$
   d) $\pi/3$

32. The value of $\int_0^1 \int_0^z \int_0^y (xy^2z^3)\,dx\,dy\,dz$
   a) 1/90
   b) 1/50
   c) 1/45
   d) 1/10

33. The value of the integral $\int_0^2 \int_0^3 (1+8xy)\,dx\,dy$ is:
   a) **57**
   b) 75
   c) 12
   d) 13

78

34. Evaluate $\int_0^{\pi/2} \int_0^1 r^3 \sin\theta \cos\theta \, dr\, d\theta$
   a) 1/4
   b) $-1/8$
   c) $1/8$
   d) -1/4

35. Evaluate $\int_0^3 \int_0^1 \frac{u^2}{3}\, du\, dv$
   a) 1/3
   b) $1/9$
   c) $-1/9$
   d) -1/3

Objective	Type of questions	Approximate time expected to be spent by a student on answering every single question
To test application of knowledge	Short answer types/Numerical based using application	20 minutes

1. Evaluate $\int_0^{\infty} x^{\frac{1}{4}} e^{-\sqrt{x}}\, dx$.
2. Evaluate $\int_0^{\infty} x^{-\frac{5}{2}} (1 - e^{-x})\, dx$.
3. Convert the integral into polar coordinates $\int_0^a \left(\int_0^{\sqrt{a^2-y^2}} (x^2 + y^2)\, dx \right) dy$
4. For the function $\phi = x^2 y^3 z^4$, calculate the maximum rate of change and the corresponding direction at the point $2\vec{i} + 3\vec{j} - \vec{k}$.
5. Find the curl of vector $F = (x^2 + yz)\vec{i} + (y^2 + zx)\vec{j} + (z^2 + xy)\vec{k}$.
6. Use cylindrical coordinates to compute the integral $\iiint_D \frac{z}{\sqrt{x^2+y^2}}\, dV$ where D is the solid bounded above by the plane $z = 2$ and below by the surface $x^2 + y^2 - 2z = 0$.
7. Compute the volume of the tetrahedron T bounded by coordinate planes and $x + 2y + z = 2$.

8. Find the area common to the circles $r = a$ and $r = \sqrt{2}\,a\cos\theta$.
9. Calculate the area bounded by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
10. Find the area bounded by $y^2 = x^3$ and $y^3 = x^2$.
11. Determine the critical points and locate any relative minima, maxima and saddle points of function f defined by
12. Determine the critical points and locate any relative minima, maxima and saddle points of function f defined by
13. Determine the critical points of the function below and find out whether each point corresponds to a relative minimum, maximum, saddle point or no conclusion can be made.

$f(x, y) = x^3 - 12x + y^3 + 3y^2 - 9y$

14. Change the order of integration in $I = \int_0^1 \int_{x^2}^{2-x} xy\,dx\,dy$, and hence evaluate.

15. Evaluate the integral by changing the order of integration $\int_0^a \int_{\sqrt{ax}}^a \frac{y^2 dydx}{\sqrt{y^4 - a^2x^2}}$.

16. Evaluate the integral by changing the order of integration $\int_0^\infty \left(\int_x^\infty \frac{e^{-y}}{y} dy \right) dx$.

17. Evaluate $\iint x(e^{\frac{-x^2}{y}})dy\,dx$ where $0 \le x \le \infty$ and $0 \le y \le x$ by change of order of integration
18. Evaluate the integral $\iint dx\,dy$ over the region bounded by $y^2 + x^2 = 1$, $y = 0$ and $5y = 3$
19. Evaluate the integral $\iint xy^5 dx\,dy$ over the region bounded by $y = 0, y - x = 0$ and $x + y = 1$
20. Evaluate the integral $\iint (x^2 + y^2)dx\,dy$ over the region bounded by y=x, y=2x and x=1 in the first quadrant
21. Evaluate $\int_0^\infty 3^{-4x^2} dx$.
22. Evaluate $\int_0^1 (x \log x)^3 dx$.
23. If $F = (x + y + 1)\vec{i} + \vec{j} - (x + y)\vec{k}$, Prove that $F.\,curl\,F = 0$.
24. Find the equations of the tangent plane and normal to the surface $xyz = 4$ at the point (2, 1, 2).
25. If $\emptyset = xyz$, $A = 2yz\vec{i} - x^2y\vec{j} + xz^2\vec{k}$ and $B = x^2\vec{i} + yz\vec{j} - xy\vec{k}$ find the value of $(A.\nabla)B$.

26. Integrate $f(x, y, z) = x^2 + y^2 - z$ over the tetrahedron with vertices (0, 0, 0), (1, 1, 0), (0, 1, 0), (0, 0, 3).
27. Find the volume of the body obtained by intersecting the solid cylinders $x^2 + z^2 \le 1$ and $y^2 + z^2 \le 1$.
28. Evaluate $\int_0^1 \sqrt{\frac{1-x}{x}}\, dx$.
29. Evaluate $\int_0^2 (4 - x^2)^{\frac{8}{2}} dx$.
30. Find the equations of the tangent plane and normal to the surface $xyz = 4$ at the point (2, 1, 2).
31. solve $\iint (x^2 + y\ \ )dx\,dy$ where $-\frac{1}{2} \le x \le 1$ and $-x \le y \le 1 + x$
36. solve $\iint dx\,dy$ where $x = 0, y = 0, x^2 + y^2 = 1, y = 1/2$

37. Evaluate $\iint \left[\frac{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}}{1 + \frac{x^2}{a^2} + \frac{y^2}{b^2}} \right]^{\frac{1}{2}} dx\,dy$ over the positive quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ by the transformation x = au & y = bv . (8)
38. Evaluate the line integral $\int_c y^2 dx - 2x^2 dy$ along the parabola y= x^2 from (0, 0) to (2, 4)
39. Find the value of integral $\frac{1}{x^2 + y^2 + 1}$ where $0 \le y \le \sqrt{1 + x^2}$ & $0 \le x \le 1$
40. Evaluate $\iint e^{2x+3y}$ over the triangle bounded by x = 0, y = 0, & x + y = 1.
41. Evaluate $\int_0^1 \int_0^2 \int_0^2 x^2yz\,dxdydz$
42. By changing the order of integration solve the integral $\int_0^\infty \int_0^x xe^{-x^2/y} dydx$
43. Find the length and breadth of rectangle having 50 feet of fencing so that area enclosed by it is maximum

44. Find the maximum and minimum value of $\mathbf{x}^2 + \mathbf{y}^2$ with the constraint $\mathbf{x}^4 + \mathbf{y}^4 = 1$.

45. Using triple integral to find the volume of solid bounded by coordinate planes and the surface $4x^2 + 9y^2 + z^4 = 1$. (Ans. $\frac{\pi}{45}$)

46. Using spherical coordinates, find $\iiint_R z^2 dV$ for the solid obtained by intersecting $\{1 \leq x^2 + y^2 + z^2 \leq 4\}$ with the double cone $\{z^2 \geq x^2 + y^2\}$.

47. Evaluate $\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} dx \times \int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta$.

48. Prove that $\nabla \times (r \times F) = (\nabla . F)r - (r.\nabla)F - 2F$.

49. If $'a'$ is a constant vector, show that $curl\left(\frac{a \times r}{r^3}\right) = \frac{-a}{r^3} + 3\frac{r}{r^5}(a.r)$.

50. Evaluate the integral $\int_1^2 \int_1^3 \cos(x) \sin(y)\, dx\, dy$.