ALTERNATING SERIES:

An alternating series is a series whose terms are alternatively positive and negative.

ALTERNATING SERIES TEST / LEIBNITZ' TEST:

If the alternating series

$$\sum_{n=1}^{\infty} (-1)^{n-1} b_n = b_1 - b_2 + b_3 - b_4 + \dots \quad , b_n > 0$$

satisfies

(i) $b_{n+1} < b_n$ for all n

(ii) $\lim_{n \to \infty} b_n = 0$

then the series is convergent.

PROBLEMS:

1) Test for the convergence of the series

(a) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$

Soln:

Let $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} (-1)^{n-1} b_n = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$

$\Rightarrow b_n = \frac{1}{n}$

Clearly, $n < n+1 \quad \forall n \geq 1$

$\Rightarrow \quad \frac{1}{n+1} < \frac{1}{n} \, , \; \forall n \geq 1$

$\Rightarrow \quad b_{n+1} < b_n \, , \; \forall n \geq 1$

Also, $\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{1}{n} = 0$

Hence, by the alternating series test/ Leibnitz test, the given series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$ is convergent.

(b) $\sum_{n=1}^{\infty} (-1)^{n-1} e^{2/n}$

Soln:
Let $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} (-1)^{n-1} b_n = \sum_{n=1}^{\infty} (-1)^{n-1} e^{2/n}$

Clearly, the given series is an alternating series.

w.k.t $\quad n < n+1 \, , \; \forall n \geq 1$

$\Rightarrow \frac{1}{n+1} < \frac{1}{n} \, , \; \forall n \geq 1$

$\Rightarrow \frac{2}{n+1} < \frac{2}{n} \, , \; \forall n \geq 1$

$\Rightarrow e^{2/n+1} < e^{2/n}$ [$\because$ exponential function is an increasing function]

$\Rightarrow b_{n+1} < b_n \, , \; \forall n \geq 1$

Now,
$\lim_{n\to\infty} b_n = \lim_{n\to\infty} e^{1/n} = e^0 = 1 \neq 0$

Hence, by the alternating series test | Leibnitz test, the given series is divergent.