ABSOLUTE CONVERGENCE:

DEFINITION:

A series $\sum_{n=1}^{\infty} a_n$ is called absolutely convergent if the series of absolute values $\sum_{n=1}^{\infty} |a_n|$ is convergent.

NOTE:

If $\sum_{n=1}^{\infty} a_n$ is a series with positive terms then $|a_n| = a_n$ and so absolute convergence is same as the convergence in this case.

THEOREM:

If the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent, then it is convergent.

DEFINITION:

A series $\sum_{n=1}^{\infty} a_n$ is called conditionally convergent if it is convergent but not absolutely convergent.

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PROBLEMS:

1) Determine whether the series is absolutely convergent, conditionally convergent or divergent.

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$

Soln:

Let $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$

Now, $\sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} \frac{1}{n^2}$ which is a p-series with

$p = 2 > 1$. Hence, it is convergent.

Since $\sum_{n=1}^{\infty} |a_n|$ is convergent, the given series $\sum_{n=1}^{\infty} a_n$

is absolutely convergent.

(b) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$

Soln:

Let $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$

By Leibnitz test, $\sum_{n=1}^{\infty} a_n$ is convergent.

But, $\sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} \frac{1}{n}$ which is a p-series with $p = 1$. Hence, it is divergent.

∴ The given series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ is conditionally convergent.