Prefix Sums
Difference Arrays
Circular Arrays

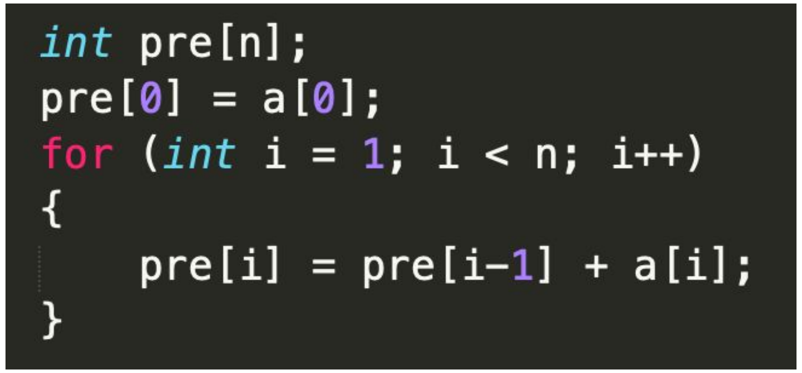
- Shivansh

Goals:

- Learn about 2D prefix sums.
- Learn about difference arrays.
- Learn about various use cases of difference arrays.
- Learn about circular arrays and solve related problems.

Recap on Prefix Sums

Prefix sum is an array consisting of the sum of the prefixes of the original array.



```
int pre[n];
pre[0] = a[0];
for (int i = 1; i < n; i++)
{
    pre[i] = pre[i-1] + a[i];
}
```

Sum of a range [L, R] can found with $P_R - P_{L-1}$

2D Prefix Sums – Computation

2D prefix sums are similar to 1D prefix sums, but extended to two dimensions arrays/grids.

The index of (i, j) in the prefix sum will store the sum of the sub-grid [0…i][0…j] in the original grid.

Prefix sum of index (i, j) can be computed as

$$\left(P_{i-1,j} + P_{i,j-1} - P_{i-1,j-1}\right) + A_{i,j}$$

2D Prefix Sums – Range Sums

To calculate the sum of the subarray [a…c][b…d]:

$$P_{c,d} - P_{c,b-1} - P_{a-1,d} + P_{a-1,b-1}$$

With $O(NM)$ precomputation, we can compute the sum of any submatrix in $O(1)$.

This can be used to answer range queries in $O(1)$ in two dimensional grids/arrays.

Difference Arrays

You are given Q queries of the form $[L, R], K$ meaning you add the value K to the range $[L, R]$. The array is initially filled with zeros. What is the final array?

The brute-force solution would be the update the array for each query, and output the array at the end.

The time complexity of this approach is $O(QN)$.

Difference Arrays

Difference arrays can update the array in $O(1)$, but the changes won't be visible until we perform a specific function on it.

We initially have an array D of size $N + 1$ filled with zeros.

To add K to range $[L, R]$, we just take $D_L \mathrel{+}= K$ and $D_{R+1} \mathrel{-}= K$.

At the end of the queries, we can take the prefix sum of D for the final array.

If our array had some values originally, it has to be added later.

Problems to Solve:

- https://cses.fi/problemset/task/1652
- https://www.codechef.com/ZCOPRAC/problems/ZCO22001
- Range product updates in $O(1)$ where $K \leq 1e9$ (will not overflow)
- https://www.hackerrank.com/challenges/crush/problem
- https://www.codechef.com/BYTR20B/problems/AGCY

Circular Arrays

Circular arrays are arrays where the index after $N - 1$ is the index 0.

Say we traverse through this circular array X times. Then if we take $\mathrm{X} \equiv R \ (mod\ N)$, then R is the index we will end up at.

Traversing through a circular array would mean that we loop infinitely.

In a way, rotating a circular array would mean that the array does not change at all, only the indices change.

output: standard output

Monocarp has just learned a new card trick, and can't wait to present it to you. He shows you the entire deck of n cards. You see that the values of cards from the topmost to the bottommost are integers $a_1, a_2, \dots, a_n$, and all values are different.

Then he asks you to shuffle the deck m times. With the j-th shuffle, you should take b_j topmost cards and move them under the remaining $(n - b_j)$ cards without changing the order.

And then, using some magic, Monocarp tells you the topmost card of the deck. However, you are not really buying that magic. You tell him that you know the topmost card yourself. Can you surprise Monocarp and tell him the topmost card before he shows it?

Circular Arrays

A trick to solve problems related to circular arrays is to double the array's length.

Example problem:

https://leetcode.com/problems/next-greater-element-ii/

https://codeforces.com/contest/1681/problem/B

Resources

- https://usaco.guide/silver/more-prefix-sums?lang=cpp
- https://codeforces.com/blog/entry/78762 (basic)
- https://codeforces.com/blog/entry/86420 (advanced)
- https://en.wikipedia.org/wiki/Modulo_operation (properties of mod)