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Assignment 4

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Geometry

Abstract—This documnet contains the solution to prove angles of a equilateral triangles are 60 degrees through Linear Algebra .

Download all python codes from

https://github.com/shivangi-975/EE5609-Matrix_Theory/tree/master/Assignment4/ Codes

Download latex-tikz codes from

https://github.com/shivangi-975/EE5609-Matrix_Theory/blob/master/Assignment4/ Assignment4.tex

1 Problem

To prove angles of equilateral triangles are 60° each.

2 Solution

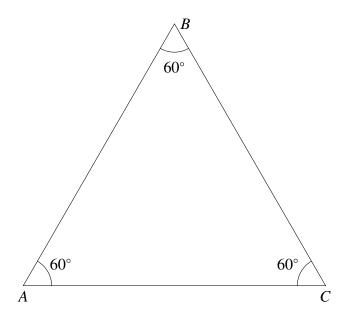


Fig. 1: Equilateral $\triangle ABC$ with A,B and C as vertices

Considering A,B and C as the vertices of triangle:

$$A = \begin{pmatrix} 0 \\ 0 \end{pmatrix} B = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} C = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$$

In equilateral triangle we have:

$$\|\mathbf{A} - \mathbf{B}\| = \|\mathbf{B} - \mathbf{C}\| = \|\mathbf{A} - \mathbf{C}\|$$
 (2.0.1)

Taking square of Norm forms we have

$$\|\mathbf{A} - \mathbf{B}\|^2 = x_1^2 + y_1^2$$
 (2.0.2)

$$\|\mathbf{A} - \mathbf{C}\|^2 = x_2^2 + y_2^2$$
 (2.0.3)

$$\|\mathbf{B} - \mathbf{C}\|^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$
 (2.0.4)

Taking inner products of side **AB** and **BC** we get $\angle BAC.Hence$,

$$\angle BAC = \|\mathbf{A} - \mathbf{B}\|^T \|\mathbf{B} - \mathbf{C}\|$$

$$\angle BAC = (x_1^2 + y_1^2)\cos\theta \qquad (2.0.5)$$

Taking inner products of side **BC** and **AC** we get $\angle BCA$. Hence,

$$\angle BCA = \|\mathbf{A} - \mathbf{B}\|^T \|\mathbf{B} - \mathbf{C}\|$$

$$\angle BCA = (x_1^2 + y_1^2)\cos\theta \qquad (2.0.6)$$

Taking inner products of side **AB** and **BC** we get $\angle ABC$. Hence,

$$\angle ABC = \|\mathbf{A} - \mathbf{B}\|^T \|\mathbf{B} - \mathbf{C}\|$$

$$\angle ABC = (x_1^2 + y_1^2) \cos \theta$$
 (2.0.7)

Hence from 2.0.5 ,2.0.6 and 2.0.7 we have for equilateral triangle $\angle BAC = \angle BCA = \angle ABC$.