Matrix theory Assignment 1

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***Abstract*—This documnet contains the solution to a variable *a,b* in a set of linear equations for infinite solution**

Download all python codes from

https://github.com/shivangi−975/−EE5609−Matrix−Theory−/tree/master/codes

and latex-tikz codes from

https://github.com/shivangi−975/−EE5609−Matrix−Theory−/blob/master/Assignment1.tex

1 Problem

For which value of *a,b* will the following pair of linear equations have infinite solution

$$\begin{pmatrix}2 & 3\end{pmatrix}\mathbf{x} = 7$$
$$\begin{pmatrix}a\text{-b} & a\text{+b}\end{pmatrix}\mathbf{x} = 3a\text{+b-2}$$

2 Solution

Constructing the augmented matrix

$$\begin{pmatrix}2 & 3 & 7\\ a-b & a+b & 3a+b-2\end{pmatrix}$$

Transforming the matrix into row-echelon form

$$\begin{pmatrix}2 & 3 & 7\\ a-b & a+b & 3a+b-2\end{pmatrix} \xleftrightarrow{R2\leftarrow R1\times\frac{a-b}{2}-R2} \begin{pmatrix}2 & 3 & 7\\ 0 & \frac{3a-b}{2}-(a-b) & \frac{7a-b}{2}-(3a+b-2)\end{pmatrix} \tag{2.0.1}$$

For the linear equations to have infinite solution,
Rank(Coefficient matrix) =Rank(Augmented matrix)
and both $\neq$ Rank(Full matrix)

$$\begin{pmatrix}1 & -5 & 0\\ 1 & -9 & -4\end{pmatrix} \xleftrightarrow{R2\leftarrow R2-R1} \begin{pmatrix}1 & -5 & 0\\ 0 & -4 & -4\end{pmatrix} \tag{2.0.2}$$

$$\begin{pmatrix}1 & -5 & 0\\ 0 & -4 & -4\end{pmatrix} \xleftrightarrow{R1\leftarrow 4\times R1} \begin{pmatrix}4 & -20 & 0\\ 0 & -4 & -4\end{pmatrix} \tag{2.0.3}$$

$$\begin{pmatrix}4 & -20 & 0\\ 0 & -4 & -4\end{pmatrix} \xleftrightarrow{R2\leftarrow 5\times R2} \begin{pmatrix}4 & -20 & 0\\ 0 & -20 & -20\end{pmatrix} \tag{2.0.4}$$

$$\begin{pmatrix}4 & -20 & 0\\ 0 & -20 & -20\end{pmatrix} \xleftrightarrow{R1\leftarrow R2-R1} \begin{pmatrix}-4 & 0 & -20\\ 0 & -20 & -20\end{pmatrix} \tag{2.0.5}$$

$$\begin{pmatrix}-4 & 0 & -20\\ 0 & -20 & -20\end{pmatrix} \xleftrightarrow{R1\leftarrow R1\div -4} \begin{pmatrix}1 & 0 & 5\\ 0 & -20 & -20\end{pmatrix} \tag{2.0.6}$$

$$\begin{pmatrix}1 & 0 & 5\\ 0 & -20 & -20\end{pmatrix} \xleftrightarrow{R2\leftarrow R2\div -20} \begin{pmatrix}1 & 0 & 5\\ 0 & 1 & 1\end{pmatrix} \tag{2.0.7}$$

Now writing matrix in the form AX=B to obtain solution we have

$$\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}a\\ b\end{pmatrix} = \begin{pmatrix}5\\ 1\end{pmatrix} \tag{2.0.8}$$

Solving the above equation

$$\implies a = 5 \quad \& \quad b = 1 \tag{2.0.9}$$

From equation (2.0.9),
For a=5 and b=1
the given set of linear equations will have infinite solution.