

Assignment 4

Shivangi Parashar

Geometry

Abstract—This documnet contains the solution to prove angles of a equilateral triangles are 60 degrees through Linear Algebra .

Download all python codes from

https://github.com/shivangi-975/EE5609-Matrix_Theory/tree/master/Assignment4/Codes

Download latex-tikz codes from

https://github.com/shivangi-975/EE5609-Matrix_Theory/blob/master/Assignment4/Assignment4.tex

1 PROBLEM

To prove angles of equilateral triangles are 60° each.

2 SOLUTION

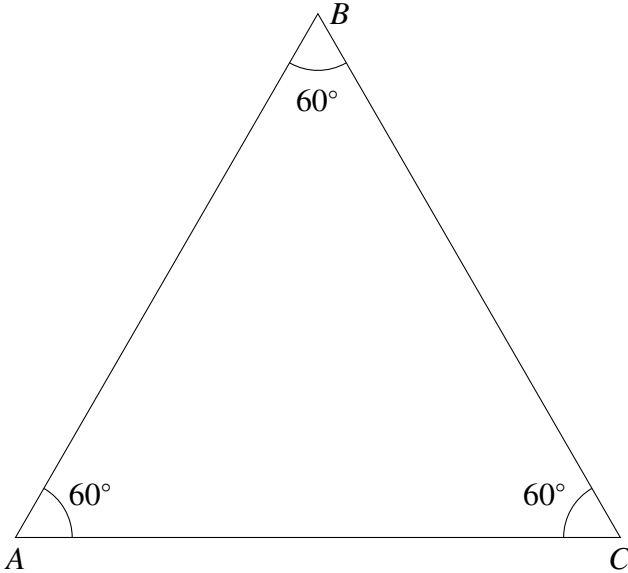


Fig. 1: Equilateral $\triangle ABC$ with A,B and C as vertices

Considering A,B and C as the vertices of triangle:

$$A = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} B = \begin{pmatrix} 0 \\ 0 \end{pmatrix} C = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

In equilateral triangle we have:

$$\|A - B\| = \|B - C\| = \|A - C\| = k \quad (2.0.1)$$

Taking the inner product of sides AB, BC and sides CA, BC and sides and BA, AC .

$$(A - B)^T (B - C) = \|A - B\| \|B - C\| \cos ABC \quad (2.0.2)$$

$$(A - C)^T (B - C) = \|A - C\| \|B - C\| \cos ACB \quad (2.0.3)$$

$$(B - A)^T (A - C) = \|B - A\| \|A - C\| \cos BAC \quad (2.0.4)$$

The angles from the both the above equations are,

$$\Rightarrow \cos ABC = \frac{(A - B)^T (B - C)}{\|A - B\| \|B - C\|} \quad (2.0.5)$$

$$\Rightarrow \cos ACB = \frac{(A - C)^T (B - C)}{\|A - C\| \|B - C\|} \quad (2.0.6)$$

$$\Rightarrow \cos BAC = \frac{(B - A)^T (A - C)}{\|B - A\| \|A - C\|} \quad (2.0.7)$$

From 2.0.1 since triangle is equilateral we have:

$$\cos ABC = \frac{(A - B)^T (B - C)}{k^2} \quad (2.0.8)$$

$$\cos ACB = \frac{(A - C)^T (B - C)}{k^2} \quad (2.0.9)$$

$$\cos BAC = \frac{(A - B)^T (B - C)}{k^2} \quad (2.0.10)$$

Angles of a equilateral triangles are equal .Hence we have

$$\frac{(A - B)^T (B - C)}{k^2} = \frac{(A - C)^T (B - C)}{k^2} = \frac{(A - B)^T (B - C)}{k^2} \quad (2.0.11)$$

Simplifying we have

$$(A - B)^T = (A - C)^T = (A - B)^T \quad (2.0.12)$$