Challenge Problem

Shivangi Parashar

Curve Fitting

***Abstract*—This document contains the solution to interpolate the curve**

Download all python codes from Download latex-tikz codes from

https://github.com/shivangi−975/EE5609−Matrix_Theory/blob/master/Challenge/Challenge.tex

1 Problem

Suppose that we are given n distinct pairs of points $(x_1, y_1), (x_2, y_2).....(x_n, y_n)$.How do you check whether all these points lie on a polynomial of degree at most m? Given n distinct pairs of points, is there always a polynomial of degree at most n1 which passes through all these points?

2 Construction

Given a set of n points with distinct x values, there is a unique interpolating polynomial of degree n or less that interpolates the points. Given n points, we construct n equations in n unknowns. Let a_k be the coefficient for x^kin the unknown polynomial, and let (x_k, y_k)be the data point (given in the problem). The k th equation is given

$$(a^n)(x_K^n) + ...(a_1)(x_K) + a_0 = y_K \tag{2.0.1}$$

In matrix-vector form the equation looks like:

$$\begin{pmatrix} x_1^n & x_1^n - 1 & \cdots & 1 \\ x_2^n & x_2^n - 1 & \cdots & 1 \\ \vdots & \vdots & \vdots & \vdots \\ x_n^n & x_n^n - 1 & \cdots & 1 \end{pmatrix} \begin{pmatrix} a_n \\ a_n - 1 \\ \vdots \\ a0 \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} \tag{2.0.2}$$

If the points x_i are distinct, then the matrix is invertible,and that will always gives us the coefficients of our polynomial and hence the equation of curve. combination

3 Solution

Let us consider $P(x) = a_0 + a_1x + a_2x^2$. Taking $P(2) = 17$,$P(3) = 11$ and $P(7) = 2$
From above we will get equations like

$$a_0 + 2(a_1) + 4(a_2) = 17 \tag{3.0.1}$$

$$a_0 + 3(a_1) + 9(a_2) = 11 \tag{3.0.2}$$

$$a_0 + 7(a_1) + 49(a_2) = 2 \tag{3.0.3}$$

Now converting above equations in matrix form we have:

$$\begin{pmatrix} 1 & 2 & 4 \\ 1 & 3 & 4 \\ 1 & 7 & 49 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 17 \\ 11 \\ 12 \end{pmatrix} \tag{3.0.4}$$

Now writing equation in form $X = A^-1B$ we have

$$\begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = A^-1 \begin{pmatrix} 17 \\ 11 \\ 12 \end{pmatrix} \tag{3.0.5}$$

On solving 3.0.5 we have :

$$\begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 67/2 \\ -39/4 \\ 3/4 \end{pmatrix} \tag{3.0.6}$$

Hence equation of the curve is
$67/2 + -39/4x + 3/4x^2$