

Assignment 4

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Geometry

Abstract—This documnet contains the solution to prove angles of a equilateral triangles are 60 degrees through Linear Algebra .

Download all python codes from

https://github.com/shivangi-975/EE5609-Matrix_Theory/tree/master/Assignment4/Codes

Download latex-tikz codes from

https://github.com/shivangi-975/EE5609-Matrix_Theory/blob/master/Assignment4/Assignment4.tex

1 PROBLEM

To prove angles of equilateral triangles are 60° each.

2 SOLUTION

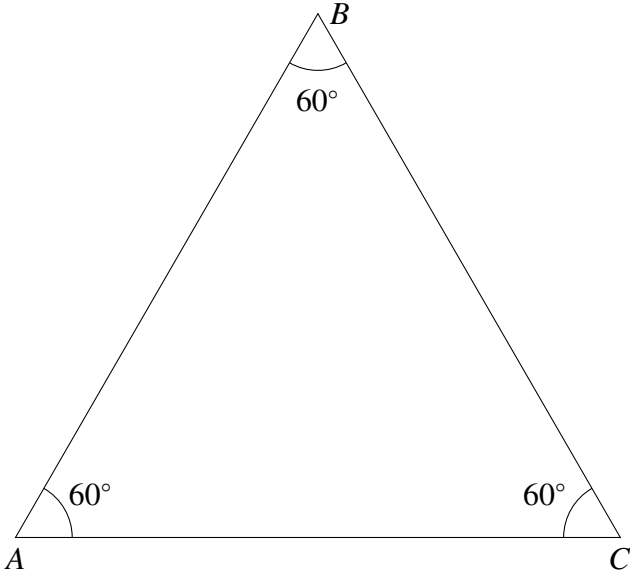


Fig. 1: Equilateral $\triangle ABC$ with A,B and C as vertices

Considering A,B and C as the vertices of triangle:

$$A = \begin{pmatrix} 0 \\ 0 \end{pmatrix} B = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} C = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$$

In equilateral triangle we have:

$$\|\mathbf{A} - \mathbf{B}\| = \|\mathbf{B} - \mathbf{C}\| = \|\mathbf{A} - \mathbf{C}\| \quad (2.0.1)$$

Taking square of Norm forms we have

$$\|\mathbf{A} - \mathbf{B}\|^2 = x_1^2 + y_1^2 \quad (2.0.2)$$

$$\|\mathbf{A} - \mathbf{C}\|^2 = x_2^2 + y_2^2 \quad (2.0.3)$$

$$\|\mathbf{B} - \mathbf{C}\|^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 \quad (2.0.4)$$

Taking inner products of side \mathbf{AB} and \mathbf{BC} we get $\angle BAC$. Hence,

$$\begin{aligned} \angle BAC &= \|\mathbf{A} - \mathbf{B}\|^T \|\mathbf{B} - \mathbf{C}\| \\ \angle BAC &= (x_1^2 + y_1^2) \cos \theta \end{aligned} \quad (2.0.5)$$

Taking inner products of side \mathbf{BC} and \mathbf{AC} we get $\angle BCA$.Hence,

$$\begin{aligned} \angle BCA &= \|\mathbf{A} - \mathbf{B}\|^T \|\mathbf{B} - \mathbf{C}\| \\ \angle BCA &= (x_1^2 + y_1^2) \cos \theta \end{aligned} \quad (2.0.6)$$

Taking inner products of side \mathbf{AB} and \mathbf{BC} we get $\angle ABC$. Hence,

$$\begin{aligned} \angle ABC &= \|\mathbf{A} - \mathbf{B}\|^T \|\mathbf{B} - \mathbf{C}\| \\ \angle ABC &= (x_1^2 + y_1^2) \cos \theta \end{aligned} \quad (2.0.7)$$

Hence from 2.0.5 ,2.0.6 and 2.0.7 we have for equilateral triangle $\angle BAC = \angle BCA = \angle ABC$.