Challenging Problem1

Shivangi Parashar AI20MTECH14012

Lines and Planes

***Abstract*—This documnet contains the solution to find the points on the lines that are closest to each other.Given Lines are skew**

Download latex-tikz codes from

https://github.com/shivangi−975/Challenge_1/blob/master/Challenge_1.tex

1 Problem 79

Find the points on the skew lines that are closest to eachother in 3-Dimensions? skew line 1 passing through the point $A(1,1,0)$ with directional vector $S_1(2,-1,1)$ and skew line 2 passing through the point $B(2,1,-1)$ with directional vector $S_2(3,-5,2)$

$$L_1 := \begin{pmatrix}1\\1\\0\end{pmatrix} + \lambda\begin{pmatrix}2\\-1\\1\end{pmatrix} \tag{1.0.1}$$

and

$$L_2 := \begin{pmatrix}2\\1\\-1\end{pmatrix} + \omega\begin{pmatrix}3\\-5\\2\end{pmatrix} \tag{1.0.2}$$

2 Solution

Let the closest points be $P(p_1,p_2,p_3)$ on skew line1 and $Q(q_1,q_2,q_3)$ on skew line2,

$$P = \begin{pmatrix}1+2\lambda\\1-\lambda\\\lambda\end{pmatrix} Q = \begin{pmatrix}2+3\omega\\1-5\omega\\-1+2\omega\end{pmatrix} \tag{2.0.1}$$

$$\begin{aligned}\mathbf{PQ} &= \mathbf{Q}-\mathbf{P}\\ &= \begin{pmatrix}3\omega+1-2\lambda\\\lambda-5\omega\\-1+2\omega-\lambda\end{pmatrix}\end{aligned} \tag{2.0.2}$$

points P and Q are closest points,Q-P will be perpendicular to both the skew lines,
Therefore,

$$(\mathbf{Q}-\mathbf{P})^T\mathbf{S_1} = 0 \tag{2.0.3}$$

$$(\mathbf{Q}-\mathbf{P})^T\mathbf{S_2} = 0 \tag{2.0.4}$$

Solving above equation we have:

$$13\lambda_2 - 6\lambda_1 = -1 \tag{2.0.5}$$

$$38\lambda_2 - 13\lambda_1 = -1 \tag{2.0.6}$$

Solving the above equations 2.0.5 and 2.0.6, we have λ=25/59 and ω=7/59
Substituting λ=25/59 and ω=7/59 coordinates of points would be.

$$P = \begin{pmatrix}109/59\\34/59\\23/59\end{pmatrix} \tag{2.0.7}$$

$$Q = \begin{pmatrix}139/59\\24/59\\-45/59\end{pmatrix} \tag{2.0.8}$$