Assignment 4

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Geometry

***Abstract*—This documnet contains the solution to prove angles of a equilateral triangles are 60 degrees through Linear Algebra .**

Download all python codes from

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https://github.com/shivangi−975/EE5609−Matrix_Theory/tree/master/Assignment4/Codes
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Download latex-tikz codes from

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https://github.com/shivangi−975/EE5609−Matrix_Theory/blob/master/Assignment4/Assignment4.tex
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1 Problem

To prove angles of equilateral triangles are 60° each.

2 Solution

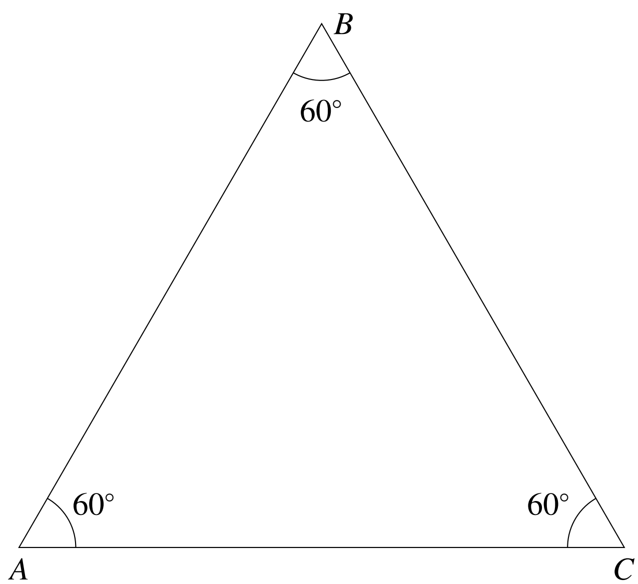

Fig. 1: Equilateral $\triangle ABC$ with A,B and C as vertices

Considering A,B and C as the vertices of triangle:

$$A = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} B = \begin{pmatrix} 0 \\ 0 \end{pmatrix} C = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

In equilateral triangle we have:

$$\|\mathbf{A}-\mathbf{B}\| = \|\mathbf{B}-\mathbf{C}\| = \|\mathbf{A}-\mathbf{C}\| = k \tag{2.0.1}$$

Taking the inner product of sides AB,BC and sides CA,BC and sides and BA,AC .

$$(\mathbf{A}-\mathbf{B})^T(\mathbf{B}-\mathbf{C}) = \|\mathbf{A}-\mathbf{B}\|\,\|\mathbf{B}-\mathbf{C}\|\cos ABC \tag{2.0.2}$$

$$(\mathbf{A}-\mathbf{C})^T(\mathbf{B}-\mathbf{C}) = \|\mathbf{A}-\mathbf{C}\|\,\|\mathbf{B}-\mathbf{C}\|\cos ACB \tag{2.0.3}$$

$$(\mathbf{B}-\mathbf{A})^T(\mathbf{A}-\mathbf{C}) = \|\mathbf{B}-\mathbf{A}\|\,\|\mathbf{A}-\mathbf{C}\|\cos BAC \tag{2.0.4}$$

The angles from the both the above equations are,

$$\implies \cos ABC = \frac{(\mathbf{A}-\mathbf{B})^T(\mathbf{B}-\mathbf{C})}{\|\mathbf{A}-\mathbf{B}\|\,\|\mathbf{B}-\mathbf{C}\|} \tag{2.0.5}$$

$$\implies \cos ACB = \frac{(\mathbf{A}-\mathbf{C})^T(\mathbf{B}-\mathbf{C})}{\|\mathbf{A}-\mathbf{C}\|\,\|\mathbf{B}-\mathbf{C}\|} \tag{2.0.6}$$

$$\implies \cos BAC = \frac{(\mathbf{B}-\mathbf{A})^T(\mathbf{A}-\mathbf{C})}{\|\mathbf{B}-\mathbf{A}\|\,\|\mathbf{A}-\mathbf{C}\|} \tag{2.0.7}$$

From 2.0.1 since triangle is equilateral:

$$\cos ABC = \frac{(\mathbf{A}-\mathbf{B})^T(\mathbf{B}-\mathbf{C})}{k^2} \tag{2.0.8}$$

$$\cos ACB = \frac{(\mathbf{A}-\mathbf{C})^T(\mathbf{B}-\mathbf{C})}{k^2} \tag{2.0.9}$$

$$\cos BAC = \frac{(\mathbf{A}-\mathbf{B})^T(\mathbf{B}-\mathbf{C})}{k^2} \tag{2.0.10}$$

Angles of a equilateral triangles are equal:

$$\frac{(\mathbf{A}-\mathbf{B})^T(\mathbf{B}-\mathbf{C})}{k^2} = \frac{(\mathbf{A}-\mathbf{C})^T(\mathbf{B}-\mathbf{C})}{k^2} = \frac{(\mathbf{A}-\mathbf{B})^T(\mathbf{B}-\mathbf{C})}{k^2} \tag{2.0.11}$$

Simplifying we have:

$$(\mathbf{A}-\mathbf{B})^T = (\mathbf{A}-\mathbf{C})^T = (\mathbf{A}-\mathbf{B})^T \tag{2.0.12}$$