

Challenging Problem1

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Lines and Planes

Abstract—This documnet contains the solution to find the points on the lines that are closest to each other. Given Lines are skew

Download latex-tikz codes from

https://github.com/shivangi-975/Challenge_1/blob/master/Challenge_1.tex

1 PROBLEM

Find the points on the skew lines that are closest to each other in 3-Dimensions? skew line 1 passing through the point $A(1, 1, 0)$ with directional vector $v_1(2, -1, 1)$ and skew line 2 passing through the point $B(2, 1, -1)$ with directional vector $v_2(3, -5, 2)$

$$L_1 : x = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \quad (1.0.1)$$

and

$$L_2 : x = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} + \omega \begin{pmatrix} 3 \\ -5 \\ 2 \end{pmatrix} \quad (1.0.2)$$

2 SOLUTION

Let the closest points be $P(p_1, p_2, p_3)$ on skew line1 and $Q(q_1, q_2, q_3)$ on skew line2, Let p, q be two points on the lines L_1, L_2

$$p = x_1 + \lambda_1 v_1 \quad (2.0.1)$$

$$q = x_2 + \lambda_2 v_2 \quad (2.0.2)$$

$$pq = x_2 + \lambda_2 v_2 - x_1 - \lambda_1 v_1$$

$$pq = (x_2 \ v_2) \begin{pmatrix} 1 \\ \lambda_2 \end{pmatrix} - (x_1 \ v_1) \begin{pmatrix} 1 \\ \lambda_1 \end{pmatrix} \quad (2.0.3)$$

points P and Q are closest points, p-q will be perpendicular to both the skew lines, Therefore,

$$v_1^T(q - p) = 0 \quad (2.0.4)$$

$$v_2^T(q - p) = 0 \quad (2.0.5)$$

From 2.0.4 and 2.0.5 we have: The dot product of v_1 with the line pq is

$$v_1^T (x_2 \ v_2) \begin{pmatrix} 1 \\ \lambda_2 \end{pmatrix} - v_1^T (x_1 \ v_1) \begin{pmatrix} 1 \\ \lambda_1 \end{pmatrix} = 0 \quad (2.0.6)$$

The dot product of v_2 with the line pq is

$$v_2^T (x_2 \ v_2) \begin{pmatrix} 1 \\ \lambda_2 \end{pmatrix} - v_2^T (x_1 \ v_1) \begin{pmatrix} 1 \\ \lambda_1 \end{pmatrix} = 0 \quad (2.0.7)$$

solving 2.0.6 and 2.0.7 we get

$$\begin{pmatrix} v_1^T x_2 & v_1^T v_2 & -v_1^T x_1 & -v_1^T v_1 \\ v_2^T x_2 & v_2^T v_2 & -v_2^T x_1 & -v_2^T v_1 \end{pmatrix} \begin{pmatrix} 1 \\ \lambda_1 \\ 1 \\ \lambda_2 \end{pmatrix} = 0 \quad (2.0.8)$$

simplifying it further

$$\begin{pmatrix} v_1^T v_1 & -v_1^T v_2 \\ v_2^T v_1 & -v_2^T v_2 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} v_1^T (x_2 - x_1) \\ v_2^T (x_2 - x_1) \end{pmatrix} \quad (2.0.9)$$

Substituting values we have $\lambda_1 = 25/59$ and $\lambda_2 = 7/59$ and coordinates of points would be.

$$P = \begin{pmatrix} 109/59 \\ 34/59 \\ 23/59 \end{pmatrix} \quad (2.0.10)$$