Challenging Problem1

Shivangi Parashar AI20MTECH14012

Lines and Planes

Abstract—This documnet contains the solution to find the points on the lines that are closest to each other.Given Lines are skew

Download latex-tikz codes from

https://github.com/shivangi–975/Challenge_1/blob/master/Challenge_1.tex

1 Problem 79

Find the points on the skew lines that are closest to eachother in 3-Dimensions? skew line 1 passing through the point $A(1, 1, 0)$ with directional vector $S_1(2, -1, 1)$ and skew line 2 passing through the point $B(2, 1, -1)$ with directional vector $S_2(3, -5, 2)$

$$L_1 : x = \begin{pmatrix}1\\1\\0\end{pmatrix} + \lambda\begin{pmatrix}2\\-1\\1\end{pmatrix} \tag{1.0.1}$$

and

$$L_2 : x = \begin{pmatrix}2\\1\\-1\end{pmatrix} + \omega\begin{pmatrix}3\\-5\\2\end{pmatrix} \tag{1.0.2}$$

2 Solution

Let the closest points be $P(p_1, p_2, p_3)$ on skew line1 and $Q(q_1, q_2, q_3)$ on skew line2,

$$\mathbf{P} = \begin{pmatrix}1\\1\\0\end{pmatrix} + \lambda\begin{pmatrix}2\\-1\\1\end{pmatrix} \tag{2.0.1}$$

$$\mathbf{Q} = \begin{pmatrix}2\\1\\-1\end{pmatrix} + \omega\begin{pmatrix}3\\5\\-2\end{pmatrix} \tag{2.0.2}$$

$$\mathbf{Q} - \mathbf{P} = \begin{pmatrix}2\\1\\-1\end{pmatrix} - \begin{pmatrix}1\\1\\0\end{pmatrix} + \omega\begin{pmatrix}3\\-5\\2\end{pmatrix} - \lambda\begin{pmatrix}2\\-1\\1\end{pmatrix} \tag{2.0.3}$$

$$= \begin{pmatrix}1\\0\\-1\end{pmatrix} + \omega\begin{pmatrix}3\\-5\\2\end{pmatrix} + \lambda\begin{pmatrix}-2\\1\\-1\end{pmatrix} \tag{2.0.4}$$

$$= \begin{pmatrix}1\\0\\-1\end{pmatrix} + \begin{pmatrix}3 & -2\\-5 & 1\\2 & -1\end{pmatrix}\begin{pmatrix}\omega\\\lambda\end{pmatrix} \tag{2.0.5}$$

points P and Q are closest points,Q-P will be perpendicular to both the skew lines, Therefore,

$$\mathbf{S}_1^T(\mathbf{Q} - \mathbf{P}) = 0 \tag{2.0.6}$$

$$\mathbf{S}_2^T(\mathbf{Q} - \mathbf{P}) = 0 \tag{2.0.7}$$

From 2.0.6 and 2.0.7 we have:

$$\begin{pmatrix}2\\-1\\1\end{pmatrix}^T\left(\begin{pmatrix}1\\0\\-1\end{pmatrix} + \begin{pmatrix}3 & -2\\-5 & 1\\2 & -1\end{pmatrix}\begin{pmatrix}\omega\\\lambda\end{pmatrix}\right) = 0 \tag{2.0.8}$$

$$\begin{pmatrix}3\\-5\\2\end{pmatrix}^T\left(\begin{pmatrix}1\\0\\-1\end{pmatrix} + \begin{pmatrix}3 & -2\\-5 & 1\\2 & -1\end{pmatrix}\begin{pmatrix}\omega\\\lambda\end{pmatrix}\right) = 0 \tag{2.0.9}$$

Solving 2.0.8 and 2.0.9 we have:

$$13\omega - 6\lambda = -1 \tag{2.0.10}$$

$$38\omega - 13\lambda = -1 \tag{2.0.11}$$

Solving 2.0.10 and 2.0.11, we have λ=25/59 and ω=7/59

Substituting λ=25/59 and ω=7/59 coordinates of points would be.

$$P = \begin{pmatrix}109/59\\34/59\\23/59\end{pmatrix} \tag{2.0.12}$$

$$Q = \begin{pmatrix}139/59\\24/59\\-45/59\end{pmatrix} \tag{2.0.13}$$