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Assignment 4

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Download latex-tikz codes from

https://github.com/Bharat437/Matrix_Theory/tree/master/Assignment4

1 Question

(Geometry,1.10) Q. Using cosine formula in an equilateral triangle, show that $\cos 60^{\circ} = \frac{1}{2}$.

2 EXPLANATION

Consider an equilateral $\triangle ABC$ as shown in below figure:

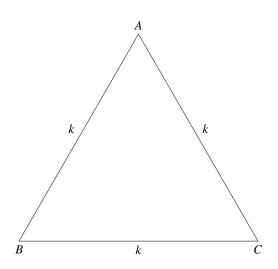


Fig. 1: Equilateral $\triangle ABC$

In equilateral triangle all sides have equal length

$$\implies \|\mathbf{A} - \mathbf{B}\| = \|\mathbf{B} - \mathbf{C}\| = \|\mathbf{A} - \mathbf{C}\| = k \quad (2.0.1)$$

Let $\mathbf{B} = 0$. Then substituting in (2.0.1) will give

$$||\mathbf{A}|| = ||\mathbf{C}|| \tag{2.0.2}$$

$$||\mathbf{A}|| = ||\mathbf{A} - \mathbf{C}|| \tag{2.0.3}$$

Taking square on both sides in (2.0.3).

$$\implies \|\mathbf{A}\|^2 = \|\mathbf{A} - \mathbf{C}\|^2 \qquad (2.0.4)$$

$$\implies \|\mathbf{A}\|^2 = \|\mathbf{A}\|^2 + \|\mathbf{C}\|^2 - 2\mathbf{A}^T\mathbf{C}$$
 (2.0.5)

$$\implies \|\mathbf{A}\|^2 = \|\mathbf{A}\|^2 + \|\mathbf{A}\|^2 - 2\mathbf{A}^T\mathbf{C}$$
 (2.0.6)

$$\implies 0 = ||\mathbf{A}||^2 - 2\mathbf{A}^T \mathbf{C} \qquad (2.0.7)$$

$$\implies 2\mathbf{A}^T\mathbf{C} = ||\mathbf{A}||^2 \qquad (2.0.8)$$

$$\implies \mathbf{A}^T \mathbf{C} = \frac{\|\mathbf{A}\|^2}{2} \qquad (2.0.9)$$

let $\theta = \angle ABC$.

Taking the inner product of sides AB and BC.

$$(\mathbf{A} - \mathbf{B})^{T}(\mathbf{B} - \mathbf{C}) = \|\mathbf{A} - \mathbf{B}\| \|\mathbf{B} - \mathbf{C}\| \cos \theta$$
 (2.0.10)

$$\implies \cos \theta = \frac{(\mathbf{A} - \mathbf{B})^T (\mathbf{B} - \mathbf{C})}{\|\mathbf{A} - \mathbf{B}\| \|\mathbf{B} - \mathbf{C}\|} (2.0.11)$$

Substitute $\mathbf{B} = 0$ in (2.0.11)

$$\implies \cos \theta = \frac{\mathbf{A}^T \mathbf{C}}{\|\mathbf{A}\| \|\mathbf{C}\|}$$
 (2.0.12)

Substitute (2.0.2),(2.0.9) in (2.0.12)

$$\implies \cos \theta = \frac{\frac{\|\mathbf{A}\|^2}{2}}{\|\mathbf{A}\|^2} \tag{2.0.13}$$

$$\implies \cos \theta = \frac{1}{2} \tag{2.0.14}$$

In equilateral triangle, $\angle ABC = 60^{\circ}$

$$\implies \cos 60^\circ = \frac{1}{2} \tag{2.0.15}$$

Hence proved.