Assignment 11

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Download latex-tikz codes from

https://github.com/Bharat437/Matrix_Theory/tree/master/Assignment11

1 Problem

Let $\mathbf{A} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$ and let α_n and β_n denote the two eigenvalues of $\mathbf{A}^n$ such that $|\alpha_n| \geq |\beta_n|$.
Then

1) $\alpha_n \to \infty$ as $n \to \infty$
2) $\beta_n \to 0$ as $n \to \infty$
3) β_n is positive if n is even.
4) β_n is negative if n is odd.

2 Solution

Given

$$\mathbf{A} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \tag{2.0.1}$$

Now lets find the eigenvalues of matrix $\mathbf{A}$.

$$\left|\mathbf{A} - \lambda\mathbf{I}\right| = 0 \tag{2.0.2}$$

$$\implies \begin{vmatrix} 1-\lambda & 1 \\ 1 & -\lambda \end{vmatrix} = 0 \tag{2.0.3}$$

$$\implies \lambda^2 - \lambda - 1 = 0 \tag{2.0.4}$$

On solving (2.0.4), we get 2 eigenvalues α_1 and β_1 such that we satisfy the given condition $|\alpha_1| \geq |\beta_1|$.

$$\implies \alpha_1 = \frac{1+\sqrt{5}}{2} \quad \beta_1 = \frac{1-\sqrt{5}}{2} \tag{2.0.5}$$

From (2.0.5), we can say that $\alpha_1 > 1$ and $-1 < \beta_1 < 0$.
We know that if eigenvalue of $\mathbf{A}$ is λ then eigenvalue of $\mathbf{A}^n$ is λ^n.
Therefore in this problem we can say that the eigenvalues α_n and β_n of $\mathbf{A}^n$ are

$$\alpha_n = \alpha_1^n \tag{2.0.6}$$

$$\beta_n = \beta_1^n \tag{2.0.7}$$

Now we can conclude that

1) Since $\alpha_1 > 1$, from (2.0.6) we can say that $\alpha_n \to \infty$ as $n \to \infty$.
2) Since $|\beta_1| < 1$, from (2.0.7) we can say that $\beta_n \to 0$ as $n \to \infty$.
3) Since β_1 is negative because $-1 < \beta_1 < 0$, from (2.0.7) if n is even then β_n is positive. and also
4) If n is odd then β_n is negative.