Assignment 4

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Download latex-tikz codes from

https://github.com/Bharat437/Matrix_Theory/tree/master/Assignment4

1 Question

(Geometry,1.10) Q. Using cosine formula in an equilateral triangle, show that $\cos 60^\circ = \frac{1}{2}$.

2 Explanation

Consider an equilateral $\triangle ABC$ as shown in below figure:

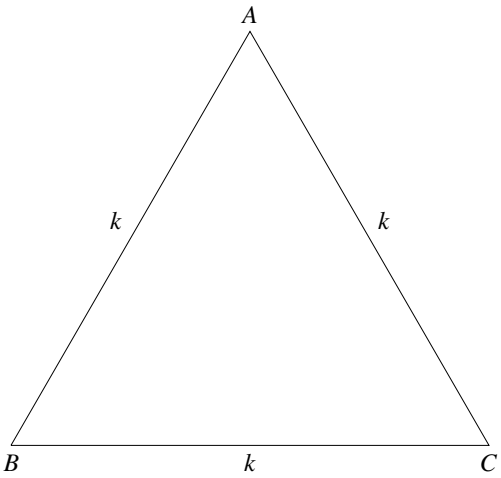

Fig. 1: Equilateral $\triangle ABC$

Length of three sides are given as below:

$$\mathbf{AB} = \|\mathbf{A}-\mathbf{B}\| \tag{2.0.1}$$

$$\mathbf{BC} = \|\mathbf{B}-\mathbf{C}\| \tag{2.0.2}$$

$$\mathbf{AC} = \|\mathbf{A}-\mathbf{C}\| \tag{2.0.3}$$

In equilateral triangle all sides have equal length

$$\implies \mathbf{AB} = \mathbf{BC} = \mathbf{AC} = k \tag{2.0.4}$$

Let $\mathbf{B}$ be origin. Then $\mathbf{A} = \begin{pmatrix} \frac{k}{2} \\ \frac{\sqrt{3}k}{2} \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} k \\ 0 \end{pmatrix}$.

let $\theta = \angle ABC$.

Taking the inner product of sides AB and BC.

$$(\mathbf{A}-\mathbf{B})^T(\mathbf{B}-\mathbf{C}) = \|\mathbf{A}-\mathbf{B}\|\,\|\mathbf{B}-\mathbf{C}\|\cos\theta \tag{2.0.5}$$

$$\implies \cos\theta = \frac{(\mathbf{A}-\mathbf{B})^T(\mathbf{B}-\mathbf{C})}{\|\mathbf{A}-\mathbf{B}\|\,\|\mathbf{B}-\mathbf{C}\|} \tag{2.0.6}$$

$$\implies \cos\theta = \frac{\begin{pmatrix} \frac{k}{2} \\ \frac{\sqrt{3}k}{2} \end{pmatrix}^T \begin{pmatrix} k \\ 0 \end{pmatrix}}{k^2} \tag{2.0.7}$$

$$\implies \cos\theta = \frac{\frac{k^2}{2}}{k^2} \tag{2.0.8}$$

$$\implies \cos\theta = \frac{1}{2} \tag{2.0.9}$$

In equilateral triangle, $\angle ABC = 60^\circ$

$$\implies \cos 60^\circ = \frac{1}{2} \tag{2.0.10}$$

Hence proved.