Assignment 13

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Download the latex-tikz codes from

https://github.com/Bharat437/Matrix_Theory/tree/master/Assignment13

1 Problem

(UGC,Dec 2018,77) :
Define a real values function **B** on $\mathbb{R}^2 \times \mathbb{R}^2$ as follows. If $v = (x_1, x_2)$, $w = (y_1, y_2)$ belong to $\mathbb{R}^2$ define $\mathbf{B}(v, w) = x_1y_1 - x_1y_2 - x_2y_1 + 4x_2y_2$. Let $v_0 = (1, 0)$ and let $\mathbf{W} = \left\{v \in \mathbb{R}^2 : \mathbf{B}(v_0, v) = 0\right\}$. Then **W**

1) is not a subspace of $\mathbb{R}^2$
2) equals $\{(0, 0)\}$
3) is the y axis
4) is the line passing through (0,0) and (1,1)

2 Explanation

Subspace	A non-empty subset **W** of **V** is a subspace of **V** if and only if for each pair of vectors α, β in **W** and each scalar c in **F** the vector $c\alpha + \beta$ is again in **W**.

TABLE 1: Definitions and theorem used

3 Solution

Statement	Observations
Given	$$\mathbf{W} = \left\{\mathbf{v} \in \mathbb{R}^2 : \mathbf{B}(\mathbf{v_0}, \mathbf{v}) = 0\right\} \quad (3.0.1)$$ $$\mathbf{v} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad (3.0.2)$$ $$\mathbf{w} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \quad (3.0.3)$$ $$\mathbf{v_0} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (3.0.4)$$ $$\mathbf{B}(\mathbf{v}, \mathbf{w}) = x_1y_1 - x_1y_2 - x_2y_1 + 4x_2y_2 \quad (3.0.5)$$ we will express (3.0.5) in quadratic form. $$\mathbf{B}(\mathbf{v}, \mathbf{w}) = \mathbf{v}^T \begin{pmatrix} 1 & -1 \\ -1 & 4 \end{pmatrix} \mathbf{w} \quad (3.0.6)$$ From (3.0.2), (3.0.4), (3.0.6) we will calculate $\mathbf{B}(\mathbf{v_0}, \mathbf{v})$

	$$\implies \mathbf{B}(\mathbf{v_0}, \mathbf{v}) = \mathbf{v_0}^T \begin{pmatrix} 1 & -1 \\ -1 & 4 \end{pmatrix} \mathbf{v} \tag{3.0.7}$$ $$\implies \mathbf{B}(\mathbf{v_0}, \mathbf{v}) = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \tag{3.0.8}$$ $$\implies \mathbf{B}(\mathbf{v_0}, \mathbf{v}) = \begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \tag{3.0.9}$$ Now we find the basis vector for **W**, which is the basis vector of null space of $\mathbf{B}(\mathbf{v_0}, \mathbf{v})$. $$\implies \mathbf{B}(\mathbf{v_0}, \mathbf{v}) = 0 \tag{3.0.10}$$ $$\implies \begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0 \tag{3.0.11}$$ $$\implies \begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0 \tag{3.0.12}$$ $$\implies x_1 = x_2 \tag{3.0.13}$$ Therefore, the basis vector for **W** is $$\mathbf{b} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{3.0.14}$$

TABLE 2: Observations

Option	Solution	True/False
1.	Now we will see whether **W** is a subspace or not. Let $\alpha = \begin{pmatrix} m \\ m \end{pmatrix}$ and $\beta = \begin{pmatrix} n \\ n \end{pmatrix}$ be two pair of vectors in **W** where $\alpha, \beta \in \mathbb{R}^2$ and c be a scalar value in $\mathbb{R}$. Now we will see whether the vector $c\alpha + \beta$ is in **W** or not. Here $$c\alpha + \beta = \begin{pmatrix} cm + n \\ cm + n \end{pmatrix} \tag{3.0.15}$$ Now we will calculate $\mathbf{B}(\mathbf{v_0}, c\alpha + \beta)$ using (3.0.6) $$\implies \mathbf{B}(\mathbf{v_0}, c\alpha + \beta) = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} cm + n \\ cm + n \end{pmatrix} \tag{3.0.16}$$ $$= (cm + n) - (cm + n) \tag{3.0.17}$$ $$\implies \mathbf{B}(\mathbf{v_0}, c\alpha + \beta) = 0 \tag{3.0.18}$$ From (3.0.18), we can say that vector $c\alpha + \beta \in \mathbf{W}$. Therefore, **W** is a subspace of $\mathbb{R}^2$	False
2.	From Table 2, we got **W** contains the vectors which are all linear combination of basis vector **b** in (3.0.14). Therefore, $$\mathbf{W} \neq \{(0, 0)\} \tag{3.0.19}$$	False

<table>
<tr>
<td>3.</td>
<td>Let us consider a vector on y-axis
$$\mathbf{p} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \tag{3.0.20}$$
Here
$$\mathbf{p} \neq k\mathbf{b} \tag{3.0.21}$$
for any $k \in \mathbb{R}$
The vector $\mathbf{p}$ can not be written in terms of the basis vector $\mathbf{b}$. Then $\mathbf{p} \notin \mathbf{W}$.
Therefore, the vectors in $\mathbf{W}$ is not y-axis.</td>
<td>False</td>
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<tr>
<td>4.</td>
<td>There is only one basis vector $\mathbf{b}$ for $\mathbf{W}$. Therefore the vectors in $\mathbf{W}$ forms a straight line in vector space $\mathbb{R}^2$.
Since,
$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} = 0\mathbf{b} \tag{3.0.22}$$
$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} = 1\mathbf{b} \tag{3.0.23}$$
Therefore, the line passes through (0,0) and (1,1).</td>
<td>True</td>
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TABLE 3: Solution