Assignment 9

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Download latex-tikz codes from

https://github.com/Bharat437/Matrix_Theory/tree/master/Assignment9

1 Problem

Let

$$\mathbf{A} = \begin{pmatrix} 3 & -1 & 2 \\ 2 & 1 & 1 \\ 1 & -3 & 0 \end{pmatrix} \tag{1.0.1}$$

For which triples (y_1, y_2, y_3) does the system $\mathbf{AX} = \mathbf{Y}$ have a solution ?

2 Solution

Given ,

$$\mathbf{AX} = \mathbf{Y} \tag{2.0.1}$$

$$\begin{pmatrix} 3 & -1 & 2 \\ 2 & 1 & 1 \\ 1 & -3 & 0 \end{pmatrix} \mathbf{X} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \tag{2.0.2}$$

Now we try to find the matrix $\mathbf{B}$ such that $\mathbf{BA}$ gives the row echelon form of matrix $\mathbf{A}$.

Here, $\mathbf{B}$ is given by ,

$$\mathbf{B} = \begin{pmatrix} 1 & 0 & 0 \\ -\frac{2}{3} & 1 & 0 \\ -\frac{7}{5} & \frac{8}{5} & 1 \end{pmatrix} \tag{2.0.3}$$

$$\implies \mathbf{BA} = \begin{pmatrix} 3 & -1 & 2 \\ 0 & \frac{5}{3} & -\frac{1}{3} \\ 0 & 0 & -\frac{6}{5} \end{pmatrix} \tag{2.0.4}$$

Therefore, from (2.0.4) rank of matrix $\mathbf{A}$ is 3 and it is a full rank matrix.

Hence the columns of $\mathbf{A}$ are linearly independent.

Therefore, the triples (y_1, y_2, y_3) are linear combination of columns of matrix $\mathbf{A}$.

$$\implies \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = a \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} + b \begin{pmatrix} -1 \\ 1 \\ -3 \end{pmatrix} + c \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \tag{2.0.5}$$

where a,b,c can be any real value.