Assignment 1

AVVARU BHARAT

Download all python codes from

https://github.com/Bharat437/Matrix_Theory/tree/master/Assignment1/Codes

and latex-tikz codes from

https://github.com/Bharat437/Matrix_Theory/tree/master/Assignment1

1 Question No. 41

Find the equation of the right bisector of the line segment joining the points $\begin{pmatrix}3\\4\end{pmatrix}$ and $\begin{pmatrix}-1\\2\end{pmatrix}$

2 Explanation

The right bisector of the line segment joining two points passes through mid-point between two points and it is perpendicular to the line segment.

Let $\mathbf{M}$ be the midpoint of two points $\mathbf{A}$=$\begin{pmatrix}3\\4\end{pmatrix}$ and $\mathbf{B}$=$\begin{pmatrix}-1\\2\end{pmatrix}$.

$$\mathbf{M} = \frac{\mathbf{A}+\mathbf{B}}{2} = \frac{1}{2}\begin{pmatrix}2\\6\end{pmatrix} \tag{2.0.1}$$

$$\implies \mathbf{M} = \begin{pmatrix}1\\3\end{pmatrix}$$

The direction vector of AB is

$$\begin{pmatrix}-1\\2\end{pmatrix} - \begin{pmatrix}3\\4\end{pmatrix} = \begin{pmatrix}-4\\-2\end{pmatrix} = -\frac{1}{4}\begin{pmatrix}1\\\frac{1}{2}\end{pmatrix} \tag{2.0.2}$$

$$\implies \text{slope of line AB} = m = \frac{1}{2} \tag{2.0.3}$$

Let the slope of right bisector be m_p. We know that product of slopes of two perpendicular lines is -1. Then

$$mm_p = -1 \implies m_p = -\frac{1}{m} \implies m_p = -2 \tag{2.0.4}$$

The direction vector of right bisector is $\mathbf{d} = \begin{pmatrix}1\\-2\end{pmatrix}$

Hence, the normal vector

$$\mathbf{n} = \begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}\mathbf{d} \tag{2.0.5}$$

$$\implies \mathbf{n} = \begin{pmatrix}2\\1\end{pmatrix} \tag{2.0.6}$$

The equation of line in terms of normal vector is then obtained as

$$\mathbf{n}^T(\mathbf{x}-\mathbf{M}) = 0 \tag{2.0.7}$$

$$\implies \begin{pmatrix}2 & 1\end{pmatrix}\left(\mathbf{x} - \begin{pmatrix}1\\3\end{pmatrix}\right) = 0 \tag{2.0.8}$$

$$\implies \begin{pmatrix}2 & 1\end{pmatrix}\mathbf{x} = 5 \tag{2.0.9}$$

We got equation of the right bisector of line segment joining points $\mathbf{A}$ and $\mathbf{B}$. The line also passes through point $\mathbf{M}$ with slope -2 .

Plot of Line segment and Right bisector: