

# Assignment 4

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[https://github.com/Bharat437/Matrix\\_Theory/tree/master/Assignment4](https://github.com/Bharat437/Matrix_Theory/tree/master/Assignment4)

## 1 QUESTION

(Geometry,1.10) Q. Using cosine formula in an equilateral triangle, show that  $\cos 60^\circ = \frac{1}{2}$ .

## 2 EXPLANATION

Consider an equilateral  $\triangle ABC$  as shown in below figure:

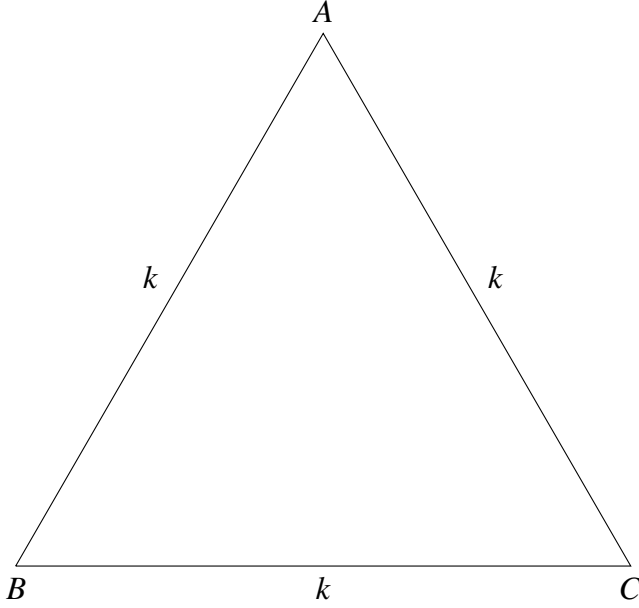


Fig. 1: Equilateral  $\triangle ABC$

In equilateral triangle all sides have equal length

$$\Rightarrow \|\mathbf{A} - \mathbf{B}\| = \|\mathbf{B} - \mathbf{C}\| = \|\mathbf{A} - \mathbf{C}\| = k \quad (2.0.1)$$

Let  $\mathbf{B} = 0$ . Then substituting in (2.0.1) will give

$$\|\mathbf{A}\| = \|\mathbf{C}\| \quad (2.0.2)$$

$$\|\mathbf{A}\| = \|\mathbf{A} - \mathbf{C}\| \quad (2.0.3)$$

Taking square on both sides in (2.0.3).

$$\Rightarrow \|\mathbf{A}\|^2 = \|\mathbf{A} - \mathbf{C}\|^2 \quad (2.0.4)$$

$$\Rightarrow \|\mathbf{A}\|^2 = \|\mathbf{A}\|^2 + \|\mathbf{C}\|^2 - 2\mathbf{A}^T \mathbf{C} \quad (2.0.5)$$

$$\Rightarrow \|\mathbf{A}\|^2 = \|\mathbf{A}\|^2 + \|\mathbf{A}\|^2 - 2\mathbf{A}^T \mathbf{C} \quad (2.0.6)$$

$$\Rightarrow 0 = \|\mathbf{A}\|^2 - 2\mathbf{A}^T \mathbf{C} \quad (2.0.7)$$

$$\Rightarrow 2\mathbf{A}^T \mathbf{C} = \|\mathbf{A}\|^2 \quad (2.0.8)$$

$$\Rightarrow \mathbf{A}^T \mathbf{C} = \frac{\|\mathbf{A}\|^2}{2} \quad (2.0.9)$$

let  $\theta = \angle ABC$ .

Taking the inner product of sides AB and BC.

$$(\mathbf{A} - \mathbf{B})^T (\mathbf{B} - \mathbf{C}) = \|\mathbf{A} - \mathbf{B}\| \|\mathbf{B} - \mathbf{C}\| \cos \theta \quad (2.0.10)$$

$$\Rightarrow \cos \theta = \frac{(\mathbf{A} - \mathbf{B})^T (\mathbf{B} - \mathbf{C})}{\|\mathbf{A} - \mathbf{B}\| \|\mathbf{B} - \mathbf{C}\|} \quad (2.0.11)$$

Substitute  $\mathbf{B} = 0$  in (2.0.11)

$$\Rightarrow \cos \theta = \frac{\mathbf{A}^T \mathbf{C}}{\|\mathbf{A}\| \|\mathbf{C}\|} \quad (2.0.12)$$

Substitute (2.0.2),(2.0.9) in (2.0.12)

$$\Rightarrow \cos \theta = \frac{\frac{\|\mathbf{A}\|^2}{2}}{\|\mathbf{A}\|^2} \quad (2.0.13)$$

$$\Rightarrow \cos \theta = \frac{1}{2} \quad (2.0.14)$$

In equilateral triangle,  $\angle ABC = 60^\circ$

$$\Rightarrow \cos 60^\circ = \frac{1}{2} \quad (2.0.15)$$

Hence proved.