Assignment 5

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Download latex-tikz codes from

https://github.com/Bharat437/Matrix_Theory/tree/master/Assignment5

1 Question

(loney 13.8) Q. Find the value of k so that the following equation may represent pair of straight lines: $12x^2 + kxy + 2y^2 + 11x - 5y + 2 = 0$.

2 Explanation

Comparing the given equation with the general equation of second degree given as below:

$$ax^2 + 2bxy + cy^2 + +2dx + 2ey + f = 0 \quad (2.0.1)$$

we will get $a = 12$, $b = \frac{k}{2}$, $c = 2$, $d = \frac{11}{2}$, $e = -\frac{5}{2}$, $f = 2$.

The general equation can be expressed as:

$$\mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \quad (2.0.2)$$

where

$$\mathbf{V} = \mathbf{V}^T = \begin{pmatrix} a & b \\ b & c \end{pmatrix} = \begin{pmatrix} 12 & \frac{k}{2} \\ \frac{k}{2} & 2 \end{pmatrix} \quad (2.0.3)$$

$$\mathbf{u} = \begin{pmatrix} d \\ e \end{pmatrix} = \begin{pmatrix} \frac{11}{2} \\ -\frac{5}{2} \end{pmatrix} \quad (2.0.4)$$

The given equation represents pair of straight lines if

$$\begin{vmatrix} \mathbf{V} & \mathbf{u} \\ \mathbf{u}^T & f \end{vmatrix} = 0 \quad (2.0.5)$$

$$\implies \begin{vmatrix} 12 & \frac{k}{2} & \frac{11}{2} \\ \frac{k}{2} & 2 & -\frac{5}{2} \\ \frac{11}{2} & -\frac{5}{2} & 2 \end{vmatrix} = 0 \quad (2.0.6)$$

The matrix in (2.0.6) must be singular matrix and in echelon form of the matrix should consist a row with all zeros.

$$\implies \begin{pmatrix} 12 & \frac{k}{2} & \frac{11}{2} \\ \frac{k}{2} & 2 & -\frac{5}{2} \\ \frac{11}{2} & -\frac{5}{2} & 2 \end{pmatrix} \quad (2.0.7)$$

$$\implies \begin{pmatrix} 24 & k & 11 \\ k & 4 & -5 \\ 11 & -5 & 4 \end{pmatrix} \quad (2.0.8)$$

$$\xleftrightarrow{R_2 \leftarrow 24R_2 - kR1} \begin{pmatrix} 24 & k & 11 \\ 0 & 96 - k^2 & -120 - 11k \\ 11 & -5 & 4 \end{pmatrix} \quad (2.0.9)$$

$$\xleftrightarrow{R_3 \leftarrow 24R_3 - 11R1} \begin{pmatrix} 24 & k & 11 \\ 0 & 96 - k^2 & -120 - 11k \\ 0 & -120 - 11k & -25 \end{pmatrix} \quad (2.0.10)$$

$$\xleftrightarrow{R_3 \leftarrow (96-k^2)R_3 - (-120-11k)R2} \begin{pmatrix} 24 & k & 11 \\ 0 & 96 - k^2 & -120 - 11k \\ 0 & 0 & -96k^2 - 2640k - 16800 \end{pmatrix} \quad (2.0.11)$$

In (2.0.11), the elements in last row must consist all zeros. For this to happen we should find k value.

$$\implies -96k^2 - 2640k - 16800 = 0 \quad (2.0.12)$$

$$\implies 2k^2 + 55k + 350 = 0 \quad (2.0.13)$$

$$\implies (10 + k)(2k + 35) = 0 \quad (2.0.14)$$

$$\implies k = -10 \text{ and } k = -\frac{35}{2} \quad (2.0.15)$$

Therefore, for $k = -10$ and $k = -\frac{35}{2}$ the given equation represents pair of straight lines.