Assignment 13

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Download the latex-tikz codes from

https://github.com/Bharat437/Matrix_Theory/tree/master/Assignment13

1 Problem

(UGC,Dec 2018,77) :
Define a real values function $\mathbf{B}$ on $\mathbb{R}^2 \times \mathbb{R}^2$ as follows. If $v = (x_1, x_2)$, $w = (y_1, y_2)$ belong to $\mathbb{R}^2$ define $\mathbf{B}(v, w) = x_1y_1 - x_1y_2 - x_2y_1 + 4x_2y_2$. Let $v_0 = (1, 0)$ and let $\mathbf{W} = \left\{v \in \mathbb{R}^2 : \mathbf{B}(v_0, v) = 0\right\}$. Then $\mathbf{W}$

1) is not a subspace of $\mathbb{R}^2$
2) equals $\{(0, 0)\}$
3) is the y axis
4) is the line passing through (0,0) and (1,1)

2 Explanation

Subspace	A non-empty subset $\mathbf{W}$ of $\mathbf{V}$ is a subspace of $\mathbf{V}$ if and only if for each pair of vectors α, β in $\mathbf{W}$ and each scalar c in $\mathbf{F}$ the vector $c\alpha + \beta$ is again in $\mathbf{W}$.

TABLE 1: Definitions and theorem used

3 Solution

Statement	Observations
Given	$$\mathbf{W} = \left\{\mathbf{v} \in \mathbb{R}^2 : \mathbf{B}(\mathbf{v_0}, \mathbf{v}) = 0\right\} \quad (3.0.1)$$ $$\mathbf{v} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad (3.0.2)$$ $$\mathbf{w} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \quad (3.0.3)$$ $$\mathbf{v_0} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (3.0.4)$$ $$\mathbf{B}(\mathbf{v}, \mathbf{w}) = x_1y_1 - x_1y_2 - x_2y_1 + 4x_2y_2 \quad (3.0.5)$$ we will express (3.0.5) in quadratic form. $$\mathbf{B}(\mathbf{v}, \mathbf{w}) = \mathbf{v}^T \begin{pmatrix} 1 & -1 \\ -1 & 4 \end{pmatrix} \mathbf{w} \quad (3.0.6)$$ From (3.0.2), (3.0.4), (3.0.6) we will calculate $\mathbf{B}(\mathbf{v_0}, \mathbf{v})$

	$$\implies \mathbf{B}(\mathbf{v_0}, \mathbf{v}) = \mathbf{v_0}^T \begin{pmatrix} 1 & -1 \\ -1 & 4 \end{pmatrix} \mathbf{v} \quad (3.0.7)$$ $$\implies \mathbf{B}(\mathbf{v_0}, \mathbf{v}) = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad (3.0.8)$$ $$\implies \mathbf{B}(\mathbf{v_0}, \mathbf{v}) = \begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad (3.0.9)$$ Now we find the basis vector for $\mathbf{W}$, which is the basis vector of null space of $\mathbf{B}(\mathbf{v_0}, \mathbf{v})$. $$\implies \mathbf{B}(\mathbf{v_0}, \mathbf{v}) = 0 \quad (3.0.10)$$ $$\implies \begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0 \quad (3.0.11)$$ $$\implies \begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0 \quad (3.0.12)$$ $$\implies x_1 = x_2 \quad (3.0.13)$$ Therefore, the basis vector for $\mathbf{W}$ is $$\mathbf{b} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (3.0.14)$$ Therefore $$\mathbf{W} = \{k\mathbf{b} : \forall k \in \mathbb{R}\} \quad (3.0.15)$$

TABLE 2: Observations

Option	Solution	True/False
1.	Now we will see whether $\mathbf{W}$ is a subspace or not. Let $\alpha = \begin{pmatrix} m \\ m \end{pmatrix}$ and $\beta = \begin{pmatrix} n \\ n \end{pmatrix}$ be two pair of vectors in $\mathbf{W}$ where $\alpha, \beta \in \mathbb{R}^2$ and c be a scalar value in $\mathbb{R}$. Now we will see whether the vector $c\alpha + \beta$ is in $\mathbf{W}$ or not. Here $$c\alpha + \beta = \begin{pmatrix} cm + n \\ cm + n \end{pmatrix} = (cm + n) \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (3.0.16)$$ $$\implies c\alpha + \beta = (cm + n)\mathbf{b} \quad (3.0.17)$$ From (3.0.17), $(cm + n) \in \mathbb{R}$ and we can say that the vector $c\alpha + \beta \in \mathbf{W}$. Therefore, $\mathbf{W}$ is a subspace of $\mathbb{R}^2$	
2.	From Table 2, we got $\mathbf{W}$ contains the vectors which are all linear combination of basis vector $\mathbf{b}$ as shown in (3.0.15) (3.0.14). Therefore, $$\mathbf{W} \neq \{(0, 0)\} \quad (3.0.18)$$	False
3.	Let us consider a vector on y-axis	

	$$\mathbf{p} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \tag{3.0.19}$$ Here $$\mathbf{p} \neq k\mathbf{b} \tag{3.0.20}$$ for any $k \in \mathbb{R}$ The vector **p** can not be written in terms of the basis vector **b**. Then $\mathbf{p} \notin \mathbf{W}$. Therefore, the vectors in **W** is not y-axis.	False
4.	There is only one basis vector **b** for **W**. Therefore the vectors in **W** forms a straight line in vector space $\mathbb{R}^2$. Since, $$\begin{pmatrix} 0 \\ 0 \end{pmatrix} = 0\mathbf{b} \tag{3.0.21}$$ $$\begin{pmatrix} 1 \\ 1 \end{pmatrix} = 1\mathbf{b} \tag{3.0.22}$$ Therefore, the line passes through (0,0) and (1,1).	True

TABLE 3: Solution