Assignment 13

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Download the latex-tikz codes from

https://github.com/Bharat437/Matrix_Theory/tree/master/Assignment13

1 Problem

(UGC,Dec 2018,77) :
Define a real values function $\mathbf{B}$ on $\mathbb{R}^2 \times \mathbb{R}^2$ as follows. If $v = (x_1, x_2)$, $w = (y_1, y_2)$ belong to $\mathbb{R}^2$ define $\mathbf{B}(u, w) = x_1y_1 - x_1y_2 - x_2y_1 + 4x_2y_2$. Let $v_0 = (1, 0)$ and let $\mathbf{W} = \left\{v \in \mathbb{R}^2 : \mathbf{B}(v_0, v) = 0\right\}$. Then $\mathbf{W}$

1) is not a subspace of $\mathbb{R}^2$
2) equals $\{(0, 0)\}$
3) is the y axis
4) is the line passing through (0,0) and (1,1)

2 Explanation

Subspace	A non-empty subset $\mathbf{W}$ of $\mathbf{V}$ is a subspace of $\mathbf{V}$ if and only if for each pair of vectors α, β in $\mathbf{W}$ and each scalar c in $\mathbf{F}$ the vector $c\alpha + \beta$ is again in $\mathbf{W}$.

TABLE 1: Definitions and theorem used

3 Solution

Statement	**Observations**
Given	$$\mathbf{W} = \left\{v \in \mathbb{R}^2 : \mathbf{B}(v_0, v) = 0\right\} \tag{3.0.1}$$ $$v_0 = (1, 0) \tag{3.0.2}$$ $$\mathbf{B}(u, w) = x_1y_1 - x_1y_2 - x_2y_1 + 4x_2y_2 \tag{3.0.3}$$ From (3.0.2) and (3.0.3), we will calculate $\mathbf{B}(v_0, v)$ $$\mathbf{B}(v_0, v) = y_1 - y_2 \tag{3.0.4}$$ $\mathbf{B}(v_0, v) = 0$ if and only if $y_1 = y_2$ Therefore, $\mathbf{W}$ consists points which have same x and y coordinates.

TABLE 2: Observations

Option	**Solution**	**True/False**
1.	Now we will see whether $\mathbf{W}$ is a subspace or not. Let $\alpha = (m, m)$ and $\beta = (n, n)$ be two pair of vectors in $\mathbf{W}$ where $\alpha, \beta \in \mathbb{R}^2$ and c be a scalar value in $\mathbb{R}$. Now we will see whether the vector $c\alpha + \beta$ is in $\mathbf{W}$ or not.	

<table>
<tr>
<td></td>
<td>Here
$$c\alpha+\beta=(cm+n,cm+n) \tag{3.0.5}$$
Now we will calculate $\mathbf{B}(v_0,c\alpha+\beta)$ using (3.0.4)
$$\implies \mathbf{B}(v_0,c\alpha+\beta)=(cm+n)-(cm+n) \tag{3.0.6}$$
$$\implies \mathbf{B}(v_0,c\alpha+\beta)=0 \tag{3.0.7}$$
From (3.0.7), we can say that vector $c\alpha+\beta\in\mathbf{W}$.
Therefore, $\mathbf{W}$ is a subspace of $\mathbb{R}$</td>
<td>False</td>
</tr>
<tr>
<td>2.</td>
<td>From Table 2, we got $\mathbf{W}$ consists points which have same x and y coordinates.
For example vector $\mathbf{u}=(1,1)\in\mathbb{R}^2$, we will calculate $\mathbf{B}(v_0,u)$
$$\implies \mathbf{B}(v_0,u)=1-1=0 \tag{3.0.8}$$
From (3.0.8), we can say that vector $\mathbf{u}\in\mathbf{W}$.
Therefore, $\mathbf{W}\neq\{(0,0)\}$</td>
<td>False</td>
</tr>
<tr>
<td>3.</td>
<td>Let us consider a point on y-axis, $p=(3,0)$ we will calculate $\mathbf{B}(v_0,p)$
$$\implies \mathbf{B}(v_0,p)=3-0=3 \tag{3.0.9}$$
$$\implies \mathbf{B}(v_0,p)\neq 0 \tag{3.0.10}$$
From (3.0.10), we can say that vector $\mathbf{p}\notin\mathbf{W}$.
Therefore, all points in $\mathbf{W}$ are not on y-axis.</td>
<td>False</td>
</tr>
<tr>
<td>4.</td>
<td>The direction vector $\mathbf{m}$ and normal vector $\mathbf{n}$ of the line through $\mathbf{M}=(0,0)$ and $\mathbf{N}=(1,1)$ is
$$\mathbf{m}=\begin{pmatrix}1\\1\end{pmatrix}-\begin{pmatrix}0\\0\end{pmatrix}=\begin{pmatrix}1\\1\end{pmatrix} \tag{3.0.11}$$
$$\mathbf{n}=\begin{pmatrix}0&-1\\1&0\end{pmatrix}\mathbf{m}=\begin{pmatrix}0&-1\\1&0\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix} \tag{3.0.12}$$
$$\implies \mathbf{n}=\begin{pmatrix}-1\\1\end{pmatrix} \tag{3.0.13}$$
The equation of line can be obtained as
$$\mathbf{n}^T(\mathbf{x}-\mathbf{M})=0 \tag{3.0.14}$$
$$\implies \begin{pmatrix}-1&1\end{pmatrix}\left(\mathbf{x}-\begin{pmatrix}0\\0\end{pmatrix}\right)=0 \tag{3.0.15}$$
$$\implies \begin{pmatrix}-1&1\end{pmatrix}\mathbf{x}=0 \tag{3.0.16}$$
(3.0.16) is the equation of line. Therefore, We can say that the line passes through the points which is having same x and y coordinates.
Therefore From Table 2, all points in $\mathbf{W}$ are on the line passing through $\mathbf{M}$ and $\mathbf{N}$</td>
<td>True</td>
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TABLE 3: Solution