

# Assignment 15

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Download the latex-tikz codes from

[https://github.com/Bharat437/Matrix\\_Theory/tree/master/Assignment15](https://github.com/Bharat437/Matrix_Theory/tree/master/Assignment15)

## 1 PROBLEM

(UGC,JUNE 2015,68) :

Let  $\mathbf{F} : \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}$  be the function  $\mathbf{F}(\mathbf{x}, \mathbf{y}) = \langle \mathbf{Ax}, \mathbf{y} \rangle$ , where  $\langle \cdot, \cdot \rangle$  is the standard inner product of  $\mathbb{R}^n$  and  $\mathbf{A}$  is a  $n \times n$  real matrix. Here  $D$  denotes the total derivative. Which of the following statements are correct?

- 1)  $(D\mathbf{F}(\mathbf{x}, \mathbf{y}))(\mathbf{u}, \mathbf{v}) = \langle \mathbf{Au}, \mathbf{y} \rangle + \langle \mathbf{Ax}, \mathbf{v} \rangle$ .
- 2)  $(D\mathbf{F}(\mathbf{x}, \mathbf{y}))(0, 0) = 0$ .
- 3)  $D\mathbf{F}(\mathbf{x}, \mathbf{y})$  may not exist for some  $(\mathbf{x}, \mathbf{y}) \in \mathbb{R}^n \times \mathbb{R}^n$ .
- 4)  $D\mathbf{F}(\mathbf{x}, \mathbf{y})$  does not exist at  $(\mathbf{x}, \mathbf{y}) = (0, 0)$ .

## 2 EXPLANATION

<b>Inner product</b>	Inner product between two vectors $\mathbf{x}$ and $\mathbf{y}$ is defined as $\langle \mathbf{x}, \mathbf{y} \rangle = \mathbf{x}^T \mathbf{y} \quad (2.0.1)$ Where $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$
<b>Inner Product Property used</b>	$\langle \mathbf{x}, \mathbf{y} \rangle = \mathbf{x}^T \mathbf{y} = \mathbf{y}^T \mathbf{x} = \langle \mathbf{y}, \mathbf{x} \rangle \quad (2.0.2)$
<b>Total Derivative <math>D</math></b>	Total derivative is a linear transformation. For function $\mathbf{F}(\mathbf{x}, \mathbf{y})$ , the total derivative is given as $D\mathbf{F}(\mathbf{x}, \mathbf{y})$ which says that total derivative of function $\mathbf{F}$ at $(\mathbf{x}, \mathbf{y})$ .

TABLE 1: Definitions and theorem used

## 3 SOLUTION

Statement	Observations
Given	Function $\mathbf{F} : \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}$ , it is given as $\mathbf{F}(\mathbf{x}, \mathbf{y}) = \langle \mathbf{Ax}, \mathbf{y} \rangle = \mathbf{x}^T \mathbf{A}^T \mathbf{y} \quad (3.0.1)$ where $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$ Using property (2.0.2), we can also get $\implies \mathbf{F}(\mathbf{x}, \mathbf{y}) = \langle \mathbf{y}, \mathbf{Ax} \rangle \quad (3.0.2)$ $\implies \mathbf{F}(\mathbf{x}, \mathbf{y}) = \mathbf{y}^T \mathbf{Ax} \quad (3.0.3)$

Total Derivative $D$	<p>Now we will calculate <math>D\mathbf{F}(\mathbf{x}, \mathbf{y})</math></p> $D\mathbf{F}(\mathbf{x}, \mathbf{y}) = \begin{pmatrix} \frac{\partial \mathbf{F}}{\partial \mathbf{x}} & \frac{\partial \mathbf{F}}{\partial \mathbf{y}} \end{pmatrix} \quad (3.0.4)$ <p>From (3.0.1),(3.0.3) we get</p> $\frac{\partial \mathbf{F}}{\partial \mathbf{x}} = \mathbf{y}^T \mathbf{A} \quad (3.0.5)$ $\frac{\partial \mathbf{F}}{\partial \mathbf{y}} = \mathbf{x}^T \mathbf{A}^T \quad (3.0.6)$ <p>Substitute (3.0.5) and (3.0.6) in (3.0.4)</p> $D\mathbf{F}(\mathbf{x}, \mathbf{y}) = \begin{pmatrix} \mathbf{y}^T \mathbf{A} & \mathbf{x}^T \mathbf{A}^T \end{pmatrix}_{1 \times n^2} \quad (3.0.7)$
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TABLE 2: Observations

Option	Solution	True/ False
1	<p>First we calculate <math>(D\mathbf{F}(\mathbf{x}, \mathbf{y}))(\mathbf{u}, \mathbf{v})</math> where <math>\mathbf{u}, \mathbf{v} \in \mathbb{R}^n</math> Using (3.0.7) and block matrix multiplication we get</p> $(D\mathbf{F}(\mathbf{x}, \mathbf{y}))(\mathbf{u}, \mathbf{v}) = \begin{pmatrix} \mathbf{y}^T \mathbf{A} & \mathbf{x}^T \mathbf{A}^T \end{pmatrix} \begin{pmatrix} \mathbf{u} \\ \mathbf{v} \end{pmatrix} \quad (3.0.8)$ $\implies (D\mathbf{F}(\mathbf{x}, \mathbf{y}))(\mathbf{u}, \mathbf{v}) = \mathbf{y}^T \mathbf{A} \mathbf{u} + \mathbf{x}^T \mathbf{A}^T \mathbf{v} \quad (3.0.9)$ $(D\mathbf{F}(\mathbf{x}, \mathbf{y}))(\mathbf{u}, \mathbf{v}) = \langle \mathbf{y}, \mathbf{A} \mathbf{u} \rangle + \langle \mathbf{A} \mathbf{x}, \mathbf{v} \rangle \quad (3.0.10)$ <p>Using property (2.0.2) we get</p> $(D\mathbf{F}(\mathbf{x}, \mathbf{y}))(\mathbf{u}, \mathbf{v}) = \langle \mathbf{A} \mathbf{u}, \mathbf{y} \rangle + \langle \mathbf{A} \mathbf{x}, \mathbf{v} \rangle \quad (3.0.11)$	True
2.	<p>Using (3.0.9), if <math>\mathbf{u} = 0</math> and <math>\mathbf{v} = 0</math> then we get</p> $(D\mathbf{F}(\mathbf{x}, \mathbf{y}))(0, 0) = 0 \quad (3.0.12)$	True
3.	Since from (3.0.7) we can say that $D\mathbf{F}(\mathbf{x}, \mathbf{y})$ will exist for any $(\mathbf{x}, \mathbf{y}) \in \mathbb{R}^n \times \mathbb{R}^n$ .	False
4.	<p>From (3.0.7), if <math>(\mathbf{x}, \mathbf{y}) = (0, 0)</math> we get</p> $D\mathbf{F}(\mathbf{x}, \mathbf{y}) _{(0,0)} = 0 \quad (3.0.13)$ <p>Therefore we can say that <math>D\mathbf{F}(\mathbf{x}, \mathbf{y})</math> will exist at <math>(\mathbf{x}, \mathbf{y}) = (0, 0)</math>.</p>	False

TABLE 3: Solution