

Assignment 13

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Download the latex-tikz codes from

https://github.com/Bharat437/Matrix_Theory/tree/master/Assignment13

1 PROBLEM

(UGC, Dec 2018, 77) :

Define a real values function \mathbf{B} on $\mathbb{R}^2 \times \mathbb{R}^2$ as follows. If $v = (x_1, x_2)$, $w = (y_1, y_2)$ belong to \mathbb{R}^2 define $\mathbf{B}(v, w) = x_1y_1 - x_1y_2 - x_2y_1 + 4x_2y_2$. Let $v_0 = (1, 0)$ and let $\mathbf{W} = \{v \in \mathbb{R}^2 : \mathbf{B}(v_0, v) = 0\}$. Then \mathbf{W}

- 1) is not a subspace of \mathbb{R}^2
- 2) equals $\{(0, 0)\}$
- 3) is the y axis
- 4) is the line passing through $(0, 0)$ and $(1, 1)$

2 EXPLANATION

Subspace	A non-empty subset \mathbf{W} of \mathbf{V} is a subspace of \mathbf{V} if and only if for each pair of vectors α, β in \mathbf{W} and each scalar c in \mathbf{F} the vector $c\alpha + \beta$ is again in \mathbf{W} .
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TABLE 1: Definitions and theorem used

3 SOLUTION

Statement	Observations
Given	$\mathbf{W} = \{v \in \mathbb{R}^2 : \mathbf{B}(v_0, v) = 0\} \quad (3.0.1)$
	$v = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad (3.0.2)$
	$w = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \quad (3.0.3)$
	$v_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (3.0.4)$
	$\mathbf{B}(v, w) = x_1y_1 - x_1y_2 - x_2y_1 + 4x_2y_2 \quad (3.0.5)$
	we will express (3.0.5) in quadratic form.
	$\mathbf{B}(v, w) = v^T \begin{pmatrix} 1 & -1 \\ -1 & 4 \end{pmatrix} w \quad (3.0.6)$
	From (3.0.2), (3.0.4), (3.0.6) we will calculate $\mathbf{B}(v_0, v)$

	$\Rightarrow \mathbf{B}(\mathbf{v}_0, \mathbf{v}) = \mathbf{v}_0^T \begin{pmatrix} 1 & -1 \\ -1 & 4 \end{pmatrix} \mathbf{v} \quad (3.0.7)$
	$\Rightarrow \mathbf{B}(\mathbf{v}_0, \mathbf{v}) = \begin{pmatrix} 1 & 0 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad (3.0.8)$
	$\Rightarrow \mathbf{B}(\mathbf{v}_0, \mathbf{v}) = \begin{pmatrix} 1 & -1 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad (3.0.9)$
	<p>Now we find the basis vector for \mathbf{W}, which is the basis vector of null space of $\mathbf{B}(\mathbf{v}_0, \mathbf{v})$.</p>
	$\Rightarrow \mathbf{B}(\mathbf{v}_0, \mathbf{v}) = 0 \quad (3.0.10)$
	$\Rightarrow \begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0 \quad (3.0.11)$
	$\Rightarrow \begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0 \quad (3.0.12)$
	$\Rightarrow x_1 = x_2 \quad (3.0.13)$
	<p>Therefore, the basis vector for \mathbf{W} is</p>
	$\mathbf{b} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (3.0.14)$
	<p>Therefore</p>
	$\mathbf{W} = \{k\mathbf{b} : \forall k \in \mathbb{R}\} \quad (3.0.15)$

TABLE 2: Observations

Option	Solution	True/False
1.	<p>Now we will see whether \mathbf{W} is a subspace or not. Let α, β be two pair of vectors in \mathbf{W} where</p> $\alpha = m\mathbf{b} \quad (3.0.16)$ $\beta = n\mathbf{b} \quad (3.0.17)$ <p>Here $m, n \in \mathbb{R}$ and now we will see whether the vector $c\alpha + \beta$ is in \mathbf{W} or not where c is a scalar value in \mathbb{R}. Here</p> $c\alpha + \beta = \begin{pmatrix} cm + n \\ cm + n \end{pmatrix} = (cm + n) \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (3.0.18)$ $\Rightarrow c\alpha + \beta = (cm + n)\mathbf{b} \quad (3.0.19)$ <p>From (3.0.19), $(cm + n) \in \mathbb{R}$ and we can say that the vector $c\alpha + \beta \in \mathbf{W}$. Therefore, \mathbf{W} is a subspace of \mathbb{R}^2</p>	
2.	<p>From Table 2, we got \mathbf{W} contains the vectors which are all linear combination of basis vector \mathbf{b} as shown in (3.0.15) (3.0.14). Therefore,</p>	False

	$\mathbf{W} \neq \{(0, 0)\}$ (3.0.20)	
3.	<p>Let us consider a vector on y-axis</p> $\mathbf{p} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \quad (3.0.21)$ <p>Here</p> $\mathbf{p} \neq k\mathbf{b} \quad (3.0.22)$ <p>for any $k \in \mathbb{R}$ The vector \mathbf{p} can not be written in terms of the basis vector \mathbf{b}. Then $\mathbf{p} \notin \mathbf{W}$. Therefore, the vectors in \mathbf{W} is not y-axis.</p>	False
4.	<p>There is only one basis vector \mathbf{b} for \mathbf{W}. Therefore the vectors in \mathbf{W} forms a straight line in vector space \mathbb{R}^2. Since,</p> $\begin{pmatrix} 0 \\ 0 \end{pmatrix} = 0\mathbf{b} \quad (3.0.23)$ $\begin{pmatrix} 1 \\ 1 \end{pmatrix} = 1\mathbf{b} \quad (3.0.24)$ <p>Therefore, the line passes through (0,0) and (1,1).</p>	True

TABLE 3: Solution