Assignment 7

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Download latex-tikz codes from

https://github.com/Bharat437/Matrix_Theory/tree/master/Assignment7

1 Question

Q. Perform QR decomposition on matrix $\mathbf{A}$

$$\mathbf{A} = \begin{pmatrix} 1 & 4 \\ 3 & -5 \end{pmatrix} \tag{1.0.1}$$

2 Explanation

The columns of matrix $\mathbf{A}$ can be represented in α and β as

$$\implies \alpha = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \tag{2.0.1}$$

$$\beta = \begin{pmatrix} 4 \\ -5 \end{pmatrix} \tag{2.0.2}$$

For QR decomposition, matrix $\mathbf{A}$ can be expressed as

$$\mathbf{A} = \mathbf{QR} \tag{2.0.3}$$

where, $\mathbf{Q}$ and $\mathbf{R}$ are expressed as

$$\mathbf{Q} = \begin{pmatrix} \mathbf{u_1} & \mathbf{u_2} \end{pmatrix} \tag{2.0.4}$$

$$\mathbf{R} = \begin{pmatrix} k_1 & r_1 \\ 0 & k_2 \end{pmatrix} \tag{2.0.5}$$

Note that $\mathbf{R}$ is an upper triangular matrix.
Now,we calculate

$$k_1 = \|\alpha\| = \sqrt{10} \tag{2.0.6}$$

$$\mathbf{u_1} = \frac{\alpha}{k_1} = \frac{1}{\sqrt{10}}\begin{pmatrix} 1 \\ 3 \end{pmatrix} \tag{2.0.7}$$

$$r_1 = \frac{\mathbf{u_1}^T\beta}{\|\mathbf{u_1}\|^2} = \frac{1}{\sqrt{10}}\begin{pmatrix} 1 & 3 \end{pmatrix}\begin{pmatrix} 4 \\ -5 \end{pmatrix} \tag{2.0.8}$$

$$\implies r_1 = -\frac{11}{\sqrt{10}} \tag{2.0.9}$$

$$\mathbf{u_2} = \frac{\beta - r_1\mathbf{u_1}}{\|\beta - r_1\mathbf{u_1}\|} \tag{2.0.10}$$

Consider

$$\beta - r_1\mathbf{u_1} = \begin{pmatrix} 4 \\ -5 \end{pmatrix} + \frac{11}{\sqrt{10}}\frac{1}{\sqrt{10}}\begin{pmatrix} 1 \\ 3 \end{pmatrix} \tag{2.0.11}$$

$$\implies \beta - r_1\mathbf{u_1} = \begin{pmatrix} \frac{51}{10} \\ -\frac{17}{10} \end{pmatrix} \tag{2.0.12}$$

$$\|\beta - r_1\mathbf{u_1}\| = \frac{17}{\sqrt{10}} \tag{2.0.13}$$

Substitute (2.0.12),(2.0.13) in (2.0.10), we get

$$\mathbf{u_2} = \begin{pmatrix} \frac{3}{\sqrt{10}} \\ -\frac{1}{\sqrt{10}} \end{pmatrix} \tag{2.0.14}$$

$$k_2 = \mathbf{u_2}^T\beta = \begin{pmatrix} \frac{3}{\sqrt{10}} & -\frac{1}{\sqrt{10}} \end{pmatrix}\begin{pmatrix} 4 \\ -5 \end{pmatrix} \tag{2.0.15}$$

$$\implies k_2 = \frac{17}{\sqrt{10}} \tag{2.0.16}$$

Therefore, from (2.0.4) and (2.0.5)

$$\mathbf{Q} = \begin{pmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{3}{\sqrt{10}} & -\frac{1}{\sqrt{10}} \end{pmatrix} \tag{2.0.17}$$

$$\mathbf{R} = \begin{pmatrix} \sqrt{10} & -\frac{11}{\sqrt{10}} \\ 0 & \frac{17}{\sqrt{10}} \end{pmatrix} \tag{2.0.18}$$

Note that,

$$\mathbf{Q}^T\mathbf{Q} = \begin{pmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{3}{\sqrt{10}} & -\frac{1}{\sqrt{10}} \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{3}{\sqrt{10}} & -\frac{1}{\sqrt{10}} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \mathbf{I} \tag{2.0.19}$$

Now matrix $\mathbf{A}$ can be written as (2.0.3)

$$\begin{pmatrix} 1 & 4 \\ 3 & -5 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{3}{\sqrt{10}} & -\frac{1}{\sqrt{10}} \end{pmatrix}\begin{pmatrix} \sqrt{10} & -\frac{11}{\sqrt{10}} \\ 0 & \frac{17}{\sqrt{10}} \end{pmatrix} \tag{2.0.20}$$