Assignment 11

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Download the latex-tikz codes from

https://github.com/Bharat437/Matrix_Theory/tree/master/Assignment11

1 Problem

(UGC-dec2016,73) :

Let $\mathbf{A} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$ and let α_n and β_n denote the two eigenvalues of $\mathbf{A}^n$ such that $|\alpha_n| \geq |\beta_n|$.

Then

1) $\alpha_n \to \infty$ as $n \to \infty$
2) $\beta_n \to 0$ as $n \to \infty$
3) β_n is positive if n is even.
4) β_n is negative if n is odd.

2 Solution

Options	Solutions	True/False
1.	Given $$\mathbf{A} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$ Now lets find the eigen values of matrix $\mathbf{A}$ $$\left|\mathbf{A} - \lambda\mathbf{I}\right| = 0$$ $$\implies \begin{vmatrix} 1-\lambda & 1 \\ 1 & -\lambda \end{vmatrix} = 0$$ $$\implies \lambda^2 - \lambda - 1 = 0$$ On solving we get 2 eigen values $$\alpha_1 = \tfrac{1+\sqrt{5}}{2} \quad \beta_1 = \tfrac{1-\sqrt{5}}{2}$$ We know that if eigenvalue of $\mathbf{A}$ is λ then eigenvalue of $\mathbf{A}^n$ is λ^n. In this problem we can say that the eigenvalues α_n and β_n of $\mathbf{A}^n$ are $$\alpha_n = \alpha_1^n \quad \beta_n = \beta_1^n$$ Since $\alpha_1 > 1$ we can say that $\alpha_n \to \infty$ as $n \to \infty$.	True
2.	We got $\beta_1 = \frac{1-\sqrt{5}}{2}$ and $\beta_n = \beta_1^n$. Since $-1 < \beta_1 < 0$,we can say that $\beta_n \to 0$ as $n \to \infty$.	True
3.	We got $\beta_1 = \frac{1-\sqrt{5}}{2}$ and $\beta_n = \beta_1^n$. Since β_1 is negative because $-1 < \beta_1 < 0$, if n is even then β_n is positive.	True
4.	We got $\beta_1 = \frac{1-\sqrt{5}}{2}$ and $\beta_n = \beta_1^n$. Since β_1 is negative, if n is odd then β_n is negative.	True