

Assignment 6

AVVARU BHARAT

Download latex-tikz codes from

https://github.com/Bharat437/Matrix_Theory/tree/master/Assignment6

1 QUESTION

(Ramsey 3.4.4)Q. What does the equation

$$\mathbf{x}^T \begin{pmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix} \mathbf{x} = 2a^2 \quad (1.0.1)$$

become when the axes are turned through 30°

2 EXPLANATION

The general second degree equation is expressed as follows,

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (2.0.1)$$

Comparing (1.0.1) and (2.0.1), we get

$$\mathbf{V} = \begin{pmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix} \quad (2.0.2)$$

$$\mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.0.3)$$

$$f = -2a^2 \quad (2.0.4)$$

Now we find

$$|\mathbf{V}| = \begin{vmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{vmatrix} \quad (2.0.5)$$

$$\Rightarrow |\mathbf{V}| = -4 \quad (2.0.6)$$

$$\Rightarrow |\mathbf{V}| < 0 \quad (2.0.7)$$

Therefore the given equation (1.0.1) represents hyperbola.

Now from affine transformations,

$$\mathbf{x} = \mathbf{P} \mathbf{y} + \mathbf{c} \quad (2.0.8)$$

We have to rotate the axes by $\theta = 30^\circ$, Then using rotation matrix.

$$\mathbf{P} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \quad (2.0.9)$$

$$\Rightarrow \mathbf{P} = \begin{pmatrix} \cos 30^\circ & -\sin 30^\circ \\ \sin 30^\circ & \cos 30^\circ \end{pmatrix} \quad (2.0.10)$$

$$\Rightarrow \mathbf{P} = \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \quad (2.0.11)$$

From eigenvalue decomposition,

$$\mathbf{P}^T \mathbf{V} \mathbf{P} = \mathbf{D} \quad (2.0.12)$$

$$\Rightarrow \mathbf{D} = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix} \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \quad (2.0.13)$$

$$\Rightarrow \mathbf{D} = \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \quad (2.0.14)$$

$$\mathbf{P} = (\mathbf{p}_1 \quad \mathbf{p}_2) \quad (2.0.15)$$

$$\mathbf{P}^T = \mathbf{P}^{-1} \quad (2.0.16)$$

The equation (2.0.1) becomes as below due to affine transformation

$$\mathbf{y}^T \mathbf{D} \mathbf{y} = \mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f \quad (2.0.17)$$

with

$$\mathbf{c} = -\mathbf{V}^{-1} \mathbf{u} \quad (2.0.18)$$

Substitute (2.0.2),(2.0.3),(2.0.4),(2.0.14) in (2.0.17) and (2.0.18), we get

$$\mathbf{y}^T \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix} \mathbf{y} = 2a^2 \quad (2.0.19)$$

with centre,

$$\mathbf{c} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.0.20)$$

Therefore the given equation (1.0.1) becomes (2.0.19) when the axes are turned through 30° .