Assignment 13

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Download the latex-tikz codes from

https://github.com/Bharat437/Matrix_Theory/tree/master/Assignment13

1 Problem

(UGC,Dec 2018,77) :
Define a real values function $\mathbf{B}$ on $\mathbb{R}^2 \times \mathbb{R}^2$ as follows. If $v = (x_1, x_2)$, $w = (y_1, y_2)$ belong to $\mathbb{R}^2$ define $\mathbf{B}(v, w) = x_1y_1 - x_1y_2 - x_2y_1 + 4x_2y_2$. Let $v_0 = (1, 0)$ and let $\mathbf{W} = \left\{v \in \mathbb{R}^2 : \mathbf{B}(v_0, v) = 0\right\}$. Then $\mathbf{W}$

1) is not a subspace of $\mathbb{R}^2$
2) equals $\{(0, 0)\}$
3) is the y axis
4) is the line passing through (0,0) and (1,1)

2 Explanation

Subspace	A non-empty subset $\mathbf{W}$ of $\mathbf{V}$ is a subspace of $\mathbf{V}$ if and only if for each pair of vectors α, β in $\mathbf{W}$ and each scalar c in $\mathbf{F}$ the vector $c\alpha + \beta$ is again in $\mathbf{W}$.

TABLE 1: Definitions and theorem used

3 Solution

Statement	**Observations**
Given	$$\mathbf{W} = \left\{\mathbf{v} \in \mathbb{R}^2 : \mathbf{B}(\mathbf{v_0}, \mathbf{v}) = 0\right\} \quad (3.0.1)$$ $$\mathbf{v} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad (3.0.2)$$ $$\mathbf{w} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \quad (3.0.3)$$ $$\mathbf{v_0} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (3.0.4)$$ $$\mathbf{B}(\mathbf{v}, \mathbf{w}) = x_1y_1 - x_1y_2 - x_2y_1 + 4x_2y_2 \quad (3.0.5)$$ we will express (3.0.5) in quadratic form. $$\mathbf{B}(\mathbf{v}, \mathbf{w}) = \mathbf{v}^T \begin{pmatrix} 1 & -1 \\ -1 & 4 \end{pmatrix} \mathbf{w} \quad (3.0.6)$$ From (3.0.2), (3.0.4), (3.0.6) we will calculate $\mathbf{B}(\mathbf{v_0}, \mathbf{v})$

	$\implies \mathbf{B}(\mathbf{v_0}, \mathbf{v}) = \mathbf{v_0}^T \begin{pmatrix} 1 & -1 \\ -1 & 4 \end{pmatrix} \mathbf{v}$ (3.0.7) $\implies \mathbf{B}(\mathbf{v_0}, \mathbf{v}) = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ (3.0.8) $\implies \mathbf{B}(\mathbf{v_0}, \mathbf{v}) = x_1 - x_2$ (3.0.9) $\mathbf{B}(\mathbf{v_0}, \mathbf{v}) = 0$ if and only if $x_1 = x_2$ Therefore, **W** consists points which have same x and y coordinates.

TABLE 2: Observations

Option	Solution	True/False
1.	Now we will see whether **W** is a subspace or not. Let $\alpha = \begin{pmatrix} m \\ m \end{pmatrix}$ and $\beta = \begin{pmatrix} n \\ n \end{pmatrix}$ be two pair of vectors in **W** where $\alpha, \beta \in \mathbb{R}^2$ and c be a scalar value in $\mathbb{R}$. Now we will see whether the vector $c\alpha + \beta$ is in **W** or not. Here $c\alpha + \beta = \begin{pmatrix} cm + n \\ cm + n \end{pmatrix}$ (3.0.10) Now we will calculate $\mathbf{B}(\mathbf{v_0}, c\alpha + \beta)$ using (3.0.6) $\implies \mathbf{B}(\mathbf{v_0}, c\alpha + \beta) = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} cm + n \\ cm + n \end{pmatrix}$ (3.0.11) $= (cm + n) - (cm + n)$ (3.0.12) $\implies \mathbf{B}(\mathbf{v_0}, c\alpha + \beta) = 0$ (3.0.13) From (3.0.13), we can say that vector $c\alpha + \beta \in \mathbf{W}$. Therefore, **W** is a subspace of $\mathbb{R}$	False
2.	From Table 2, we got **W** consists points which have same x and y coordinates. For example vector $\mathbf{u} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \in \mathbb{R}^2$, we will calculate $\mathbf{B}(\mathbf{v_0}, \mathbf{u})$ $\implies \mathbf{B}(\mathbf{v_0}, \mathbf{u}) = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 1 - 1 = 0$ (3.0.14) From (3.0.14), we can say that vector $\mathbf{u} \in \mathbf{W}$. Therefore, $\mathbf{W} \neq \{(0, 0)\}$	False
3.	Let us consider a vector on y-axis, $\mathbf{p} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$ we will calculate $\mathbf{B}(\mathbf{v_0}, \mathbf{p})$ $\implies \mathbf{B}(\mathbf{v_0}, \mathbf{p}) = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} 3 \\ 0 \end{pmatrix} = 3 - 0 = 3$ (3.0.15) $\implies \mathbf{B}(\mathbf{v_0}, \mathbf{p}) \neq 0$ (3.0.16) From (3.0.16), we can say that vector $\mathbf{p} \notin \mathbf{W}$.	False

	Therefore, all vectors in $\mathbf{W}$ are not on y-axis.	
4.	The direction vector $\mathbf{m}$ and normal vector $\mathbf{n}$ of the line through $\mathbf{M} = (0, 0)$ and $\mathbf{N} = (1, 1)$ is $$\mathbf{m} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{3.0.17}$$ $$\mathbf{n} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \mathbf{m} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{3.0.18}$$ $$\implies \mathbf{n} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \tag{3.0.19}$$ The equation of line can be obtained as $$\mathbf{n}^T (\mathbf{x} - \mathbf{M}) = 0 \tag{3.0.20}$$ $$\implies \begin{pmatrix} -1 & 1 \end{pmatrix} \left(\mathbf{x} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right) = 0 \tag{3.0.21}$$ $$\implies \begin{pmatrix} -1 & 1 \end{pmatrix} \mathbf{x} = 0 \tag{3.0.22}$$ $$\implies \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ -1 & 4 \end{pmatrix} \mathbf{x} = 0 \tag{3.0.23}$$ $$\implies \mathbf{v_0}^T \begin{pmatrix} 1 & -1 \\ -1 & 4 \end{pmatrix} \mathbf{x} = 0 \tag{3.0.24}$$ $$\implies \mathbf{B}(\mathbf{v_0}, \mathbf{x}) = 0 \tag{3.0.25}$$ (3.0.22) is the equation of line and From (3.0.25) we can say that $\mathbf{x} \in \mathbf{W}$. Therefore, the vectors in $\mathbf{W}$ are on the line passing through (0,0) and (1,1).	True

TABLE 3: Solution