Assignment 4

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Download latex-tikz codes from

https://github.com/Bharat437/Matrix_Theory/tree/master/Assignment4

1 Question

(Geometry,1.10) Q. Using cosine formula in an equilateral triangle, show that $\cos 60^\circ = \frac{1}{2}$.

2 Explanation

Consider an equilateral $\triangle ABC$ as shown in below figure:

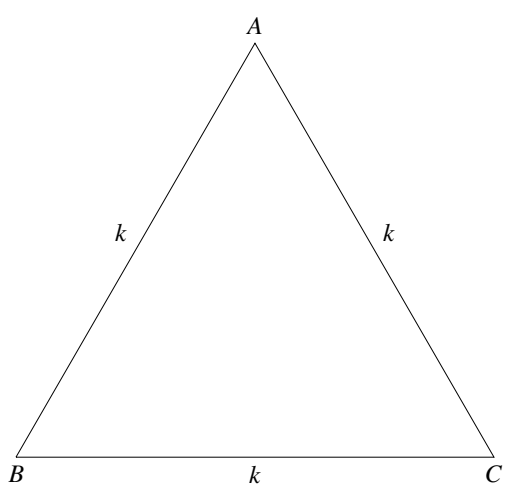

Fig. 1: Equilateral $\triangle ABC$

In equilateral triangle all sides have equal length

$$\implies \|\mathbf{A}-\mathbf{B}\| = \|\mathbf{B}-\mathbf{C}\| = \|\mathbf{A}-\mathbf{C}\| = k \tag{2.0.1}$$

Let $\mathbf{B} = 0$. Then substituting in (2.0.1) will give

$$\|\mathbf{A}\| = \|\mathbf{C}\| \tag{2.0.2}$$

$$\|\mathbf{A}\| = \|\mathbf{A}-\mathbf{C}\| \tag{2.0.3}$$

Taking square on both sides in (2.0.3).

$$\implies \|\mathbf{A}\|^2 = \|\mathbf{A}-\mathbf{C}\|^2 \tag{2.0.4}$$

$$\implies \|\mathbf{A}\|^2 = \|\mathbf{A}\|^2 + \|\mathbf{C}\|^2 - 2\mathbf{A}^T\mathbf{C} \tag{2.0.5}$$

$$\implies \|\mathbf{A}\|^2 = \|\mathbf{A}\|^2 + \|\mathbf{A}\|^2 - 2\mathbf{A}^T\mathbf{C} \tag{2.0.6}$$

$$\implies 0 = \|\mathbf{A}\|^2 - 2\mathbf{A}^T\mathbf{C} \tag{2.0.7}$$

$$\implies 2\mathbf{A}^T\mathbf{C} = \|\mathbf{A}\|^2 \tag{2.0.8}$$

$$\implies \mathbf{A}^T\mathbf{C} = \frac{\|\mathbf{A}\|^2}{2} \tag{2.0.9}$$

let $\theta = \angle ABC$.

Taking the inner product of sides AB and BC.

$$(\mathbf{A}-\mathbf{B})^T(\mathbf{B}-\mathbf{C}) = \|\mathbf{A}-\mathbf{B}\|\,\|\mathbf{B}-\mathbf{C}\|\cos\theta \tag{2.0.10}$$

$$\implies \cos\theta = \frac{(\mathbf{A}-\mathbf{B})^T(\mathbf{B}-\mathbf{C})}{\|\mathbf{A}-\mathbf{B}\|\,\|\mathbf{B}-\mathbf{C}\|} \tag{2.0.11}$$

Substitute $\mathbf{B} = 0$ in (2.0.11)

$$\implies \cos\theta = \frac{\mathbf{A}^T\mathbf{C}}{\|\mathbf{A}\|\,\|\mathbf{C}\|} \tag{2.0.12}$$

Substitute (2.0.2),(2.0.9) in (2.0.12)

$$\implies \cos\theta = \frac{\frac{\|\mathbf{A}\|^2}{2}}{\|\mathbf{A}\|^2} \tag{2.0.13}$$

$$\implies \cos\theta = \frac{1}{2} \tag{2.0.14}$$

In equilateral triangle, $\angle ABC = 60^\circ$

$$\implies \cos 60^\circ = \frac{1}{2} \tag{2.0.15}$$

Hence proved.