

Assignment 4

AVVARU BHARAT

Download latex-tikz codes from

https://github.com/Bharat437/Matrix_Theory/tree/master/Assignment4

1 QUESTION

(Geometry,1.10) Q. Using cosine formula in an equilateral triangle, show that $\cos 60^\circ = \frac{1}{2}$.

2 EXPLANATION

Consider an equilateral $\triangle ABC$ as shown in below figure:

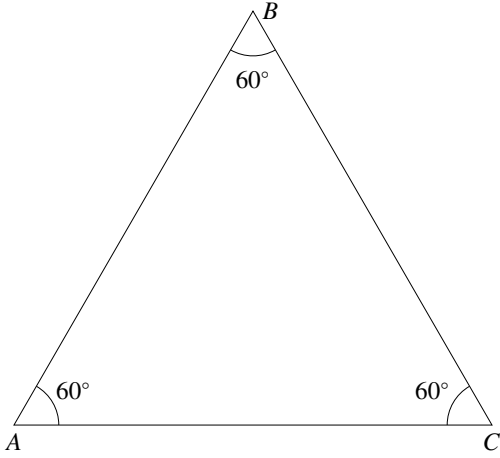


Fig. 1: Equilateral $\triangle ABC$

In equilateral triangle, all sides have equal length. Lets consider coordinates of points as $A(0,0)$, $B\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, $C(1,0)$ such that each side has length equal to 1 and forms an equilateral triangle.

Direction vector of AC and AB is given as below

$$\mathbf{m}_{AC} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.1)$$

$$\mathbf{m}_{AB} = \begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix} \quad (2.0.2)$$

We can say that line AB is obtained by rotating line AC by $+60^\circ$. Using rotation matrix, we can obtain direction vector of AB as below.

$$\begin{pmatrix} \cos 60^\circ & -\sin 60^\circ \\ \sin 60^\circ & \cos 60^\circ \end{pmatrix} \mathbf{m}_{AC} = \mathbf{m}_{AB} \quad (2.0.3)$$

$$\Rightarrow \begin{pmatrix} \cos 60^\circ & -\sin 60^\circ \\ \sin 60^\circ & \cos 60^\circ \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix} \quad (2.0.4)$$

$$\Rightarrow \begin{pmatrix} \cos 60^\circ \\ \sin 60^\circ \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix} \quad (2.0.5)$$

$$\cos 60^\circ = \frac{1}{2} \quad (2.0.6)$$

Hence proved.