Assignment 16

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Download the latex-tikz codes from

https://github.com/Bharat437/Matrix_Theory/tree/master/Assignment16

1 Problem

(UGC,JUNE 2014,75) :
Let $\mathbf{A}$ be 5×5 matrix and let $\mathbf{B}$ be obtained by changing one element of $\mathbf{A}$. Let r and s be the ranks of $\mathbf{A}$ and $\mathbf{B}$ respectively. Which of the following statements is/are correct?

1) $s \leq r + 1$
2) $r - 1 \leq s$
3) $s = r - 1$
4) $s \neq r$

2 Explanation

Theorem	If $\mathbf{M}$ and $\mathbf{N}$ are two matrices whose ranks are $rank(\mathbf{M})$ and $rank(\mathbf{N})$ respectively. Then $$rank(\mathbf{M}+\mathbf{N}) \leq rank(\mathbf{M}) + rank(\mathbf{N}) \quad (2.0.1)$$

TABLE 1: Definitions and theorem used

3 Solution

Option	Solution	True/ False
1.	Given matrix $\mathbf{A}$ has rank r and $\mathbf{B}$ has rank s. Also given matrix $\mathbf{B}$ is obtained by changing only one element of $\mathbf{A}$. Lets assume another matrix $\mathbf{P}$ whose addition to matrix $\mathbf{A}$ results to matrix $\mathbf{B}$ as below. $$\mathbf{A} + \mathbf{P} = \mathbf{B} \quad (3.0.1)$$ Since matrix $\mathbf{P}$ consists only single element we can say that $rank(P) = 1$ From (2.0.1), (3.0.1), we get $$rank(\mathbf{A}+\mathbf{P}) \leq rank(\mathbf{A}) + rank(\mathbf{P}) \quad (3.0.2)$$ $$\implies rank(\mathbf{B}) \leq rank(\mathbf{A}) + rank(\mathbf{P}) \quad (3.0.3)$$ $$\implies s \leq r + 1 \quad (3.0.4)$$	True
2.	From (3.0.1), If $\mathbf{P} = -\mathbf{Q}$ then we can get as below	

	$\mathbf{A} - \mathbf{Q} = \mathbf{B} \qquad (3.0.5)$ $\implies \mathbf{B} + \mathbf{Q} = \mathbf{A} \qquad (3.0.6)$ Since matrix **Q** also consists only single element we can say that $rank(Q) = 1$ From (2.0.1), (3.0.6), we get $rank(\mathbf{B} + \mathbf{Q}) \leq rank(\mathbf{B}) + rank(\mathbf{Q}) \qquad (3.0.7)$ $\implies rank(\mathbf{A}) \leq rank(\mathbf{B}) + rank(\mathbf{Q}) \qquad (3.0.8)$ $\implies r \leq s + 1 \qquad (3.0.9)$ $\implies r - 1 \leq s \qquad (3.0.10)$	True
3.	Let matrix **A** be identity matrix then $rank(\mathbf{A})$ is 5 and matrix **B** can be $\mathbf{A} = \mathbf{I}_{5\times5} \qquad (3.0.11)$ $\mathbf{B} = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad (3.0.12)$ Then $rank(\mathbf{B})$ is also 5.Therefore $s = r - 1$ is always not true.	False
4.	Similarly from (3.0.11),(3.0.12) we can say that $s \neq r$ is not true always.	False

TABLE 2: Solution