GROUP WORK PROJECT # ___
GROUP NUMBER: _5657__________

MScFE 622: Stochastic Modeling

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Statement of integrity: By typing the names of all group members in the text boxes below, you confirm that the assignment submitted is original work produced by the group (excluding any non-contributing members identified with an "X" above).

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Note: You may be required to provide proof of your outreach to non-contributing members upon request.

SM Group Work Project 1

This Notebook is created by Bharat Swami,Wycliffe Kipkoech Cheruiyot and Christopher Enny Ofikwu for Stochatic Modeling Group Work Project 1.

Group Number : 5657

```
import numpy as np
import pandas as pd
from scipy.integrate import quad
from scipy.optimize import brute, fmin
import matplotlib.pyplot as plt
from numpy.fft import fft
from scipy.interpolate import splev, splrep

import warnings
warnings.filterwarnings("ignore")
```

Step1 MemberA Heston Characteristic Function

```python
def H93_char_func(u, T, r, kappa_v, theta_v, sigma_v, rho, v0):
    c1 = kappa_v * theta_v
    c2 = -np.sqrt(
        (rho * sigma_v * u * 1j - kappa_v) ** 2 - sigma_v**2 * (-u * 1j - u**2)
    )
    c3 = (kappa_v - rho * sigma_v * u * 1j + c2) / (
        kappa_v - rho * sigma_v * u * 1j - c2
    )
    H1 = r * u * 1j * T + (c1 / sigma_v**2) * (
        (kappa_v - rho * sigma_v * u * 1j + c2) * T
        - 2 * np.log((1 - c3 * np.exp(c2 * T)) / (1 - c3))
    )
    H2 = (
        (kappa_v - rho * sigma_v * u * 1j + c2)
        / sigma_v**2
        * ((1 - np.exp(c2 * T)) / (1 - c3 * np.exp(c2 * T)))
    )

    char_func_value = np.exp(H1 + H2 * v0)

    return char_func_value
```

Integral Value in Lewis (2001)

```python
def H93_int_func(u, S0, K, T, r, kappa_v, theta_v, sigma_v, rho, v0):
    char_func_value = H93_char_func(
        u - 1j / 2, T, r, kappa_v, theta_v, sigma_v, rho, v0
    )
    int_func_value = (
        1 / (u**2 + 0.25) * (np.exp(1j * u * np.log(S0 / K)) * char_func_value).real
    )
    return int_func_value

def H93_put_value(S0, K, T, r, kappa_v, theta_v, sigma_v, rho, v0):
    int_value = quad(
        lambda u: H93_int_func(u, S0, K, T, r, kappa_v, theta_v, sigma_v, rho, v0),
        0,
        np.inf,
        limit=250,
    )[0]
    call_value = max(0, S0 - np.exp(-r * T) * np.sqrt(S0 * K) / np.pi * int_value)
    put_value = call_value + K * np.exp(- r * T) - S0 # Put-Call parity
    return put_value
```

Heston Calibration

```python
S0 = 232.90
r0 = 1.5/100

data = pd.read_csv("MScFE 622_Stochastic Modeling_GWP1_Option data.xlsx - 1.csv")
data["r"] = r0
data["T"] = data["Days to maturity"] / 250 # 250days / year

options = data[(data["Days to maturity"] == 15) & (data["Type"] == "P")] # Put Option with DTM is 15
options
```

	Days to maturity	Strike	Price	Type	r	T
15	15	227.5	4.32	P	0.015	0.06
16	15	230.0	5.20	P	0.015	0.06
17	15	232.5	6.45	P	0.015	0.06
18	15	235.0	7.56	P	0.015	0.06
19	15	237.5	8.78	P	0.015	0.06

```
def H93_error_function(p0):
    global i, min_MSE
    kappa_v, theta_v, sigma_v, rho, v0 = p0
    if kappa_v < 0.0 or theta_v < 0.005 or sigma_v < 0.0 or rho < -1.0 or rho > 1.0:
        return 500.0
    if 2 * kappa_v * theta_v < sigma_v**2:
        return 500.0
    se = []
    for row, option in options.iterrows():
        model_value = H93_put_value(
            S0,
            option["Strike"],
            option["T"],
            option["r"],
            kappa_v,
            theta_v,
            sigma_v,
            rho,
            v0,
        )

        se.append((model_value - option["Price"]) ** 2)

    MSE = sum(se) / len(se)
    min_MSE = min(min_MSE, MSE)
    if i % 25 == 0:
        print("%4d |" % i, np.array(p0).round(2), "| %7.3f | %7.3f" % (MSE, min_MSE))
    i += 1
    return MSE

def H93_calibration_full():
    p0 = brute(
      H93_error_function,
      (
          (1.5, 6.5, 5.0),
          (0.1, 0.4, 0.1),
          (0.01, 0.03, 0.01),
          (-0.5, 0.25, 0.25),
          (0.04, 0.09, 0.01),
      ),
      finish=None,
  )
    opt = fmin(
      H93_error_function, p0, xtol=0.00001, ftol=0.00001, maxiter=750, maxfun=900
```

```
  )
    return opt
```

```
i = 0
min_MSE = 500

params_H93 = H93_calibration_full()
```

```
      0 | [ 1.5   0.1   0.01 -0.5   0.04] |   3.488 |   3.488
     25 | [1.5  0.1  0.02 0.   0.04] |   3.490 |   0.055
     50 | [ 1.5   0.2   0.02 -0.25  0.04] |   2.704 |   0.034
     75 | [ 1.5   0.3   0.02 -0.5   0.04] |   2.046 |   0.034
    100 | [1.5  0.4  0.01 0.   0.04] |   1.503 |   0.034
    125 | [ 1.5   0.4   0.02 -0.5   0.07] |   0.048 |   0.034
    150 | [ 1.48  0.4   0.02 -0.52  0.07] |   0.034 |   0.034
    175 | [ 1.28  0.43  0.03 -0.68  0.07] |   0.033 |   0.033
    200 | [ 0.95  0.49  0.05 -1.    0.07] |   0.031 |   0.031
    225 | [ 0.96  0.37  0.07 -0.98  0.08] |   0.029 |   0.029
    250 | [ 1.07  0.02  0.13 -0.96  0.09] |   0.026 |   0.026
    275 | [ 0.98  0.01  0.14 -0.98  0.09] |   0.024 |   0.024
    300 | [ 1.08  0.01  0.14 -0.99  0.09] |   0.023 |   0.023
    325 | [ 1.09  0.01  0.14 -1.    0.09] |   0.023 |   0.023
    350 | [ 1.08  0.01  0.14 -1.    0.09] |   0.023 |   0.023
    375 | [ 1.08  0.01  0.14 -1.    0.09] |   0.023 |   0.023
    400 | [ 1.08  0.01  0.14 -1.    0.09] |   0.023 |   0.023
    425 | [ 1.08  0.01  0.14 -1.    0.09] |   0.023 |   0.023
    450 | [ 1.08  0.01  0.14 -1.    0.09] |   0.023 |   0.023
     Optimization terminated successfully.
              Current function value: 0.023166
              Iterations: 287
              Function evaluations: 508
```

```
params_H93.round(4)
```

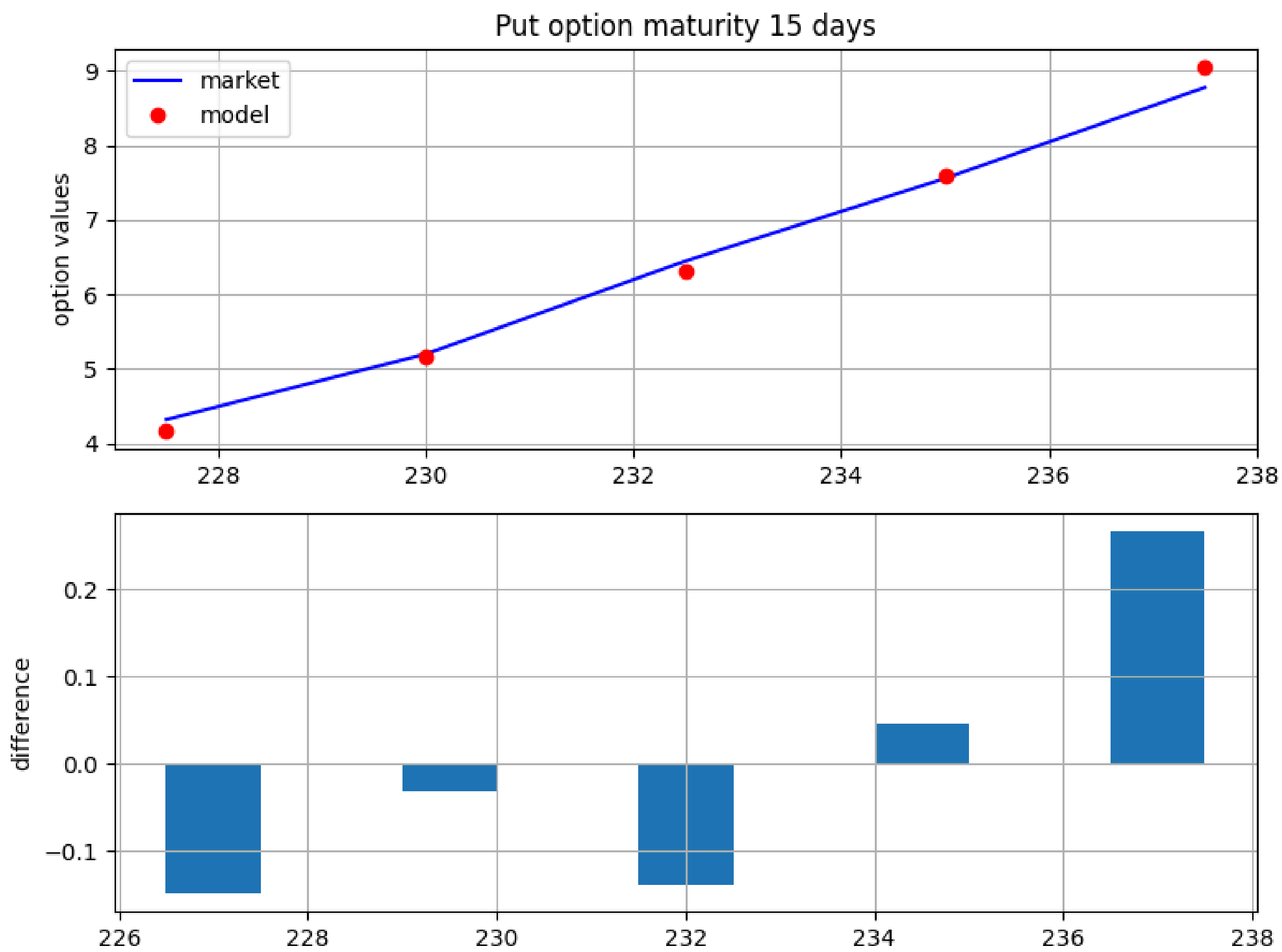
```
     array([ 1.0825,  0.0091,  0.14  , -1.    ,  0.0872])
```

Given MSE around 0.023 The graph after calibration as shown below:

```
def plot_calibration_results(p0):
    kappa_v, theta_v, sigma_v, rho, v0 = p0
    options["Model"] = 0.0
    for row, option in options.iterrows():
        options.loc[row, "Model"] = H93_put_value(
            S0, option["Strike"], option["T"], option["r"], kappa_v, theta_v, sigma_v, rho, v0
        )

    plt.figure(figsize=(8, 6))
    plt.subplot(211)
    plt.grid()
    plt.title("Put option maturity %s days" % str(options["Days to maturity"].iloc[0])[:10])
    plt.ylabel("option values")
    plt.plot(options.Strike, options.Price, "b", label="market")
    plt.plot(options.Strike, options.Model, "ro", label="model")
    plt.legend(loc=0)
    plt.subplot(212)
    plt.grid()
    wi = 1.0
    diffs = options.Model.values - options.Price.values
    plt.bar(options.Strike.values - wi / 2, diffs, width=wi)
    plt.ylabel("difference")
    plt.tight_layout();

plot_calibration_results(params_H93)
```


Put option maturity 15 days
market
model
option values
difference
9
8
7
6
5
4
228
230
232
234
236
238
0.2
0.1
0.0
−0.1
226

Member B

```python
def H93_char_func(u, T, r, kappa_v, theta_v, sigma_v, rho, v0):
    c1 = kappa_v * theta_v
    c2 = -np.sqrt(
        (rho * sigma_v * u * 1j - kappa_v) ** 2 - sigma_v**2 * (-u * 1j - u**2)
    )
    c3 = (kappa_v - rho * sigma_v * u * 1j + c2) / (
        kappa_v - rho * sigma_v * u * 1j - c2
    )
    H1 = r * u * 1j * T + (c1 / sigma_v**2) * (
        (kappa_v - rho * sigma_v * u * 1j + c2) * T
        - 2 * np.log((1 - c3 * np.exp(c2 * T)) / (1 - c3))
    )
    H2 = (
        (kappa_v - rho * sigma_v * u * 1j + c2)
        / sigma_v**2
        * ((1 - np.exp(c2 * T)) / (1 - c3 * np.exp(c2 * T)))
    )

    char_func_value = np.exp(H1 + H2 * v0)
    return char_func_value

def M76J_char_func(u, T, lamb, mu, delta):
    omega = -lamb * (np.exp(mu + 0.5 * delta**2) - 1)
    char_func_value = np.exp(
        (1j * u * omega + lamb * (np.exp(1j * u * mu - u**2 * delta**2 * 0.5) - 1))
        * T
    )
    return char_func_value

def B96_char_func(u, T, r, kappa_v, theta_v, sigma_v, rho, v0, lamb, mu, delta):
    H93 = H93_char_func(u, T, r, kappa_v, theta_v, sigma_v, rho, v0)
    M76J = M76J_char_func(u, T, lamb, mu, delta)

    return H93 * M76J
def  B96_put_FFT(S0, K, T, r, kappa_v, theta_v, sigma_v, rho, v0, lamb, mu, delta):
    """
    Put option price in Bates (1996) under FFT
    """

    k = np.log(K / S0)
    g = 1  # Factor to increase accuracy
    N = g * 4096
    eps = (g * 150) ** -1
    eta = 2 * np.pi / (N * eps)
```

```python
b = 0.5 * N * eps - k
u = np.arange(1, N + 1, 1)
vo = eta * (u - 1)

# Modifications to ensure integrability
if S0 >= 0.95 * K:  # ITM Case
    alpha = 1.5
    v = vo - (alpha + 1) * 1j
    modcharFunc = np.exp(-r * T) * (
        B96_char_func(v, T, r, kappa_v, theta_v, sigma_v, rho, v0, lamb, mu, delta)
        / (alpha**2 + alpha - vo**2 + 1j * (2 * alpha + 1) * vo)
    )

else:
    alpha = 1.1
    v = (vo - 1j * alpha) - 1j
    modcharFunc1 = np.exp(-r * T) * (
        1 / (1 + 1j * (vo - 1j * alpha))
        - np.exp(r * T) / (1j * (vo - 1j * alpha))
        - B96_char_func(
            v, T, r, kappa_v, theta_v, sigma_v, rho, v0, lamb, mu, delta
        )
        / ((vo - 1j * alpha) ** 2 - 1j * (vo - 1j * alpha))
    )

    v = (vo + 1j * alpha) - 1j

    modcharFunc2 = np.exp(-r * T) * (
        1 / (1 + 1j * (vo + 1j * alpha))
        - np.exp(r * T) / (1j * (vo + 1j * alpha))
        - B96_char_func(
            v, T, r, kappa_v, theta_v, sigma_v, rho, v0, lamb, mu, delta
        )
        / ((vo + 1j * alpha) ** 2 - 1j * (vo + 1j * alpha))
    )

# Numerical FFT Routine
delt = np.zeros(N)
delt[0] = 1
j = np.arange(1, N + 1, 1)
SimpsonW = (3 + (-1) ** j - delt) / 3
if S0 >= 0.95 * K:
    FFTFunc = np.exp(1j * b * vo) * modcharFunc * eta * SimpsonW
    payoff = (np.fft.fft(FFTFunc)).real
```

```
        CallValueM = np.exp(-alpha * k) / np.pi * payoff
    else:
        FFTFunc = (
            np.exp(1j * b * vo) * (modcharFunc1 - modcharFunc2) * 0.5 * eta * SimpsonW
        )
        payoff = (np.fft.fft(FFTFunc)).real
        CallValueM = payoff / (np.sinh(alpha * k) * np.pi)

    pos = int((k + b) / eps)
    CallValue = CallValueM[pos] * S0
    PutValue = CallValue + K * np.exp(-r * T) - S0 # Put-Call parity

    return PutValue


# upload the option data
data = pd.read_csv("MScFE 622_Stochastic Modeling_GWP1_Option data.xlsx - 1.csv")
S0 = 232.90
r0 = 1.5 / 100
data["r"] = r0
data["T"] = data["Days to maturity"] / 250


# Select put options with a maturity of 15 days for calibration
options = data[(data["Days to maturity"] == 15) & (data["Type"] == "P")]

# Initial parameters for calibration
p0 = [1.0825, 0.0091, 0.14, -1., 0.0872]
```

```
# Calibration using brute force and fmin
def H93_error_function(p0):
    global i, min_MSE
    kappa_v, theta_v, sigma_v, rho, v0 = p0
    if kappa_v < 0.0 or theta_v < 0.005 or sigma_v < 0.0 or rho < -1.0 or rho > 1.0:
        return 500.0
    if 2 * kappa_v * theta_v < sigma_v**2:
        return 500.0
    se = []
    for row, option in options.iterrows():
        model_value = H93_put_value(
            S0,
            option["Strike"],
            option["T"],
            option["r"],
            kappa_v,
            theta_v,
            sigma_v,
            rho,
            v0,
        )

        se.append((model_value - option["Price"]) ** 2)

    MSE = sum(se) / len(se)
    min_MSE = min(min_MSE, MSE)
    if i % 25 == 0:
        print("%4d |" % i, np.array(p0).round(2), "| %7.3f | %7.3f" % (MSE, min_MSE))
    i += 1
    return MSE


# Brute force optimization
p0_brute = (
    (1.5, 10.6, 5.0),
    (0.01, 0.041, 0.01),
    (0.0, 0.251, 0.1),
    (-0.75, 0.01, 0.25),
    (0.01, 0.031, 0.01),
)
params_B96_brute = brute(H93_error_function, p0_brute, finish=None)
```

```
 475 | [ 1.5   0.01  0.   -0.5   0.03] |  53.761 |   0.023
 500 | [ 1.5   0.02  0.   -0.25  0.01] |  53.761 |   0.023
```

```
      525 | [ 1.5   0.02  0.2  -0.25  0.02] |  10.612 |   0.023
      550 | [ 1.5   0.03  0.1  -0.25  0.03] |   6.671 |   0.023
      575 | [1.5  0.04 0.   0.   0.01] |  53.761 |   0.023
      600 | [1.5  0.04 0.2  0.   0.02] |  10.221 |   0.023
      625 | [6.5  0.01 0.1  0.   0.03] |   7.845 |   0.023
      650 | [ 6.5   0.02  0.1  -0.75  0.01] |  15.340 |   0.023
      675 | [ 6.5   0.03  0.   -0.75  0.02] |  53.761 |   0.023
      700 | [ 6.5   0.03  0.2  -0.75  0.03] |   6.672 |   0.023
      725 | [ 6.5   0.04  0.1  -0.5   0.01] |  13.124 |   0.023


# Final optimization using fmin
params_B96 = fmin(H93_error_function, params_B96_brute, xtol=0.00001, ftol=0.00001, maxiter=750, maxfun=900)

      750 | [ 6.5   0.04  0.1  -0.79  0.03] |   6.128 |   0.023
      775 | [ 6.29  0.07  0.06 -0.15  0.06] |   0.866 |   0.023
      800 | [4.99 0.1  0.04 0.36 0.08] |   0.039 |   0.023
      825 | [5.18 0.09 0.04 0.3  0.08] |   0.037 |   0.023
      850 | [5.11 0.1  0.04 0.32 0.08] |   0.036 |   0.023
      875 | [5.1  0.1  0.04 0.32 0.08] |   0.036 |   0.023
      900 | [4.84 0.09 0.04 0.24 0.08] |   0.036 |   0.023
      925 | [4.28 0.09 0.03 0.06 0.08] |   0.035 |   0.023
      950 | [ 3.49  0.07  0.01 -0.21  0.09] |   0.035 |   0.023
      975 | [ 3.37  0.07  0.01 -0.26  0.09] |   0.035 |   0.023
     1000 | [ 3.24  0.07  0.02 -0.3   0.09] |   0.034 |   0.023
     1025 | [ 1.73  0.08  0.13 -0.67  0.08] |   0.028 |   0.022
     1050 | [ 0.45  0.08  0.22 -0.99  0.08] |   0.019 |   0.019
     1075 | [ 0.37  0.08  0.24 -1.    0.08] |   0.019 |   0.019
     1100 | [ 0.32  0.08  0.22 -1.    0.08] |   0.019 |   0.019
     1125 | [ 0.33  0.08  0.23 -0.99  0.08] |   0.019 |   0.019
     1150 | [ 0.93  0.1   0.4  -0.9   0.08] |   0.012 |   0.012
     1175 | [ 2.2   0.14  0.77 -0.74  0.08] |   0.007 |   0.007
     1200 | [ 2.28  0.14  0.8  -0.73  0.08] |   0.007 |   0.007
     1225 | [ 2.48  0.14  0.84 -0.73  0.08] |   0.007 |   0.006
     1250 | [ 2.56  0.14  0.86 -0.73  0.08] |   0.006 |   0.006
     1275 | [ 2.57  0.14  0.86 -0.73  0.08] |   0.006 |   0.006
     1300 | [ 2.55  0.14  0.85 -0.73  0.08] |   0.006 |   0.006
     1325 | [ 2.49  0.14  0.81 -0.76  0.08] |   0.006 |   0.006
     1350 | [ 2.51  0.12  0.72 -0.85  0.08] |   0.006 |   0.006
     1375 | [ 2.8   0.11  0.66 -0.96  0.08] |   0.006 |   0.006
     1400 | [ 2.95  0.1   0.65 -0.99  0.08] |   0.006 |   0.006
     1425 | [ 2.96  0.1   0.64 -1.    0.08] |   0.006 |   0.006
     1450 | [ 2.95  0.1   0.64 -1.    0.08] |   0.006 |   0.006
     1475 | [ 2.95  0.1   0.64 -1.    0.08] |   0.006 |   0.006
     1500 | [ 2.95  0.1   0.64 -1.    0.08] |   0.006 |   0.006
```

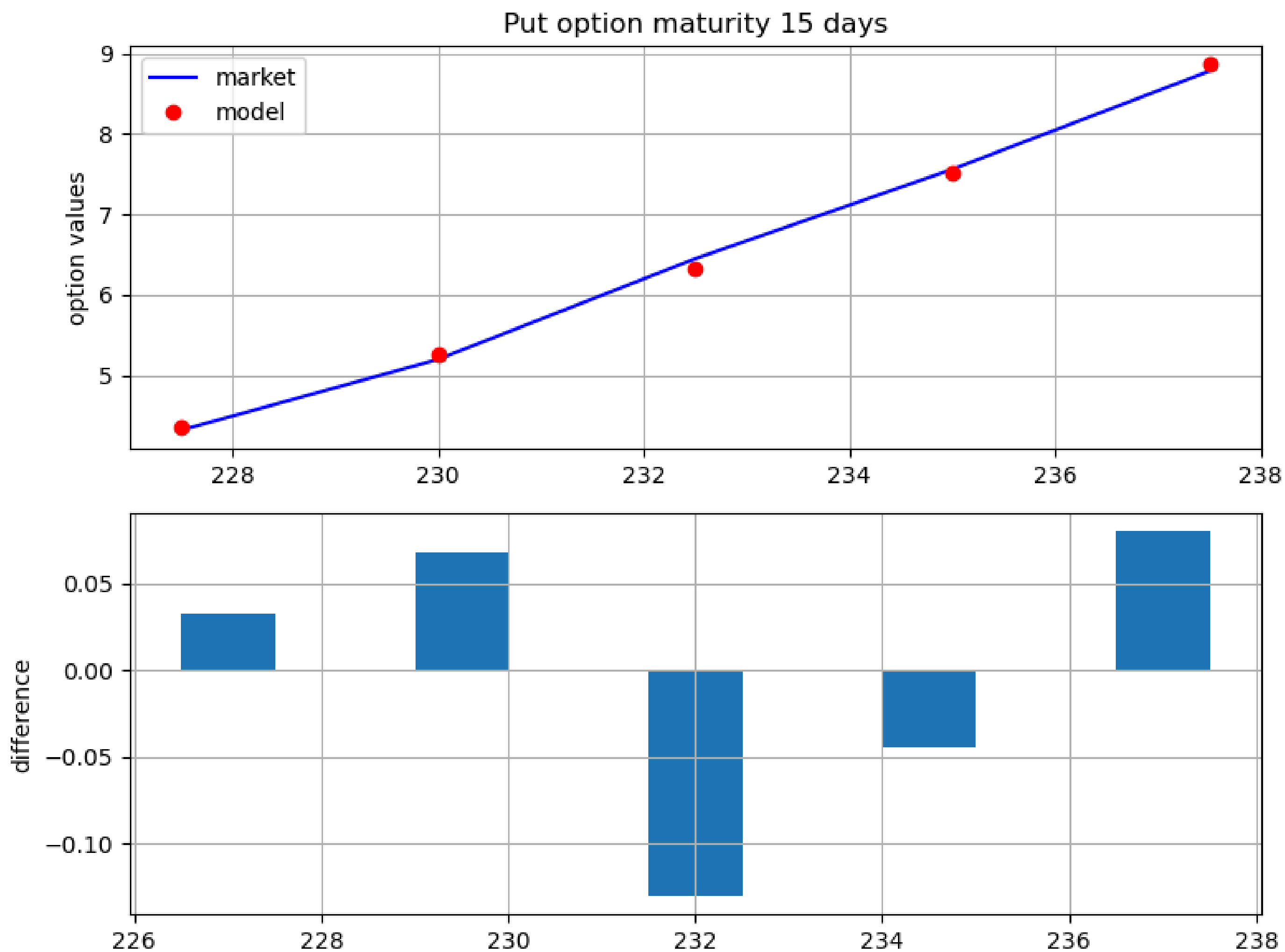
```
# Print the calibrated parameters
print("Calibrated Parameters using Carr-Madan (1999) FFT:")
print("kappa =", params_B96[0])
print("theta =", params_B96[1])
print("sigma =", params_B96[2])
print("rho =", params_B96[3])
print("v0 =", params_B96[4])
```

```
Calibrated Parameters using Carr-Madan (1999) FFT:
kappa = 2.9529361017438305
theta = 0.1010905838484647
sigma = 0.640330506481863
rho = -0.9999993453303719
v0 = 0.0849466620072209
```

```
def plot_calibration_results(p0):
    kappa_v, theta_v, sigma_v, rho, v0 = p0
    options["Model"] = 0.0
    for row, option in options.iterrows():
        options.loc[row, "Model"] = H93_put_value(
            S0, option["Strike"], option["T"], option["r"], kappa_v, theta_v, sigma_v, rho, v0
        )

    plt.figure(figsize=(8, 6))
    plt.subplot(211)
    plt.grid()
    plt.title("Put option maturity %s days" % str(options["Days to maturity"].iloc[0])[:10])
    plt.ylabel("option values")
    plt.plot(options.Strike, options.Price, "b", label="market")
    plt.plot(options.Strike, options.Model, "ro", label="model")
    plt.legend(loc=0)
    plt.subplot(212)
    plt.grid()
    wi = 1.0
    diffs = options.Model.values - options.Price.values
    plt.bar(options.Strike.values - wi / 2, diffs, width=wi)
    plt.ylabel("difference")
    plt.tight_layout();

plot_calibration_results(params_B96 )
```



For the Heston model, the calibration results obtained using the Carr-Madan (1999) FFT and Lewis (2001) approaches yield varying parameter values.There are differences in the numerical techniques used by Carr-Madan and Lewis. Carr-Madan utilizes numerical integration techniques with FFT, while Lewis relies on a closed-form solution. Differences in numerical methods can cause variations in the calibrated parameters can arise due to differences in numerical methods.Market Data Considerations: Different calibration methods may exhibit varying levels of sensitivity to specific areas of the option surface. The consideration and handling of different options in the calibration process can have an effect on the ultimate parameter values.The selection of model assumptions can have an impact on the calibration results. Consider factors like stochastic volatility dynamics and jump processes. Parameter disparities may arise due to variations in the underlying model assumptions.The objective functions utilized in the calibration processes may vary. Lewis (2001) focuses on minimizing the mean squared error, while Carr-Madan's (1999) FFT may highlight various aspects of the option surface.

Member C

```
# Set the parameters
K = 100.0  # Strike price
r = 0.05  # Risk-free interest rate
T = 20 / 365  # Time to maturity (in years)
simulations = 10000  # Number of Monte Carlo simulations

# Bates model parameters
kappa_v = 3.1886
theta_v = 0.0055
sigma_v = 0.1873
rho = -1
v0 = 0.1338
lambda_ = 0.0115
delta = 0.0
gamma = 0.0

# Initialize an array to store the payoffs
payoffs = np.zeros(simulations)

# Perform Monte Carlo simulations
for i in range(simulations):
    # Simulate stock price path using the Bates model
    dt = T / 252  # Assuming 252 trading days in a year
    N = int(T / dt)

    # Initialize arrays to store simulated values
    S = np.zeros(N + 1)
    v = np.zeros(N + 1)
    S[0] = 100.0  # Initial stock price
    v[0] = v0

    for t in range(1, N + 1):
        Z_S = np.random.normal(0, 1)
        Z_v = rho * Z_S + np.sqrt(1 - rho**2) * np.random.normal(0, 1)

        # Bates model dynamics
        S[t] = S[t - 1] * np.exp((r - 0.5 * v[t - 1]) * dt + np.sqrt(v[t - 1] * dt) * Z_S + lambda_ * (np.exp(delta * Z_v) - 1))
        v[t] = v[t - 1] + kappa_v * (theta_v - v[t - 1]) * dt + sigma_v * np.sqrt(v[t - 1] * dt) * Z_v + gamma * (np.abs(v[t - 1]) * dt)**0.

    # Calculate the average price over the maturity period
    average_price = np.mean(S)

    # Calculate the payoff of the Asian call option
    payoff = np.maximum(average_price - K, 0)
```

```
    # Store the payoff
    payoffs[i] = payoff

# Calculate the fair price as the average of the payoffs
fair_price = np.mean(payoffs)

# Calculate the final price that the client will pay (including a 4% fee)
final_price = fair_price + (0.04 * fair_price)

print("Fair Price:", fair_price)
print("Final Price (including 4% fee):", final_price)
```

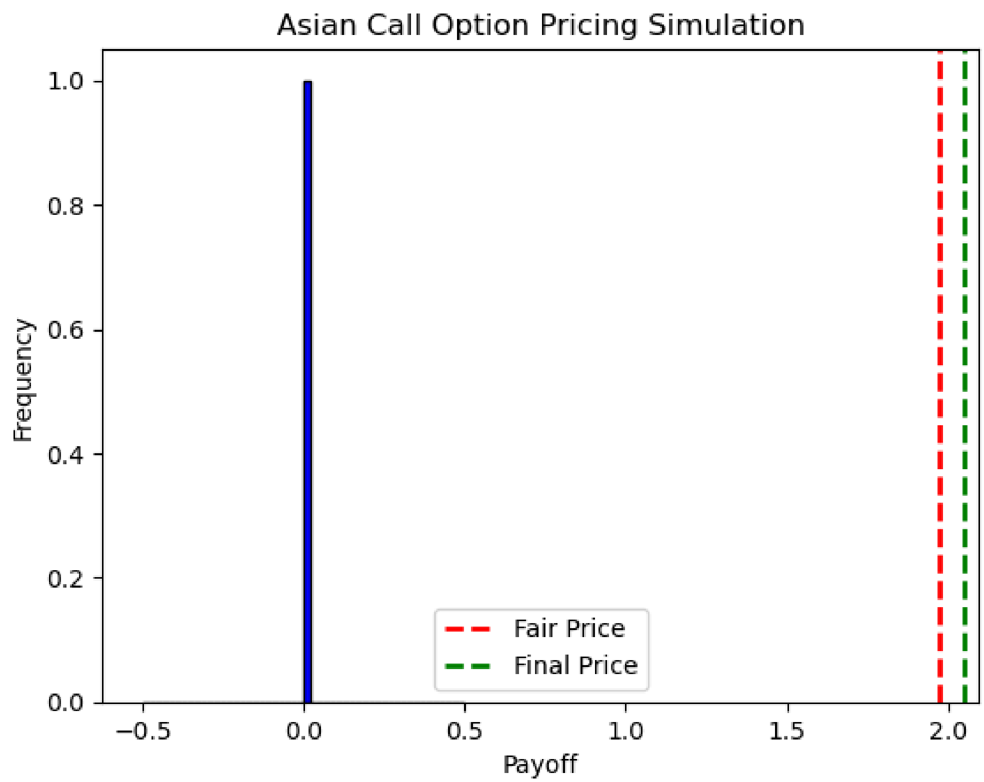
```
Fair Price: 1.9730644433994515
Final Price (including 4% fee): 2.0519870211354294
```

```
# Plot histogram
plt.hist(payoff, bins=50, color='blue', edgecolor='black')
plt.axvline(x=fair_price, color='red', linestyle='dashed', linewidth=2, label='Fair Price')
plt.axvline(x=final_price, color='green', linestyle='dashed', linewidth=2, label='Final Price')

# Add labels and title
plt.xlabel('Payoff')
plt.ylabel('Frequency')
plt.title('Asian Call Option Pricing Simulation')

# Add legend
plt.legend()

# Show the plot
plt.show()
```



Step2 Member A

We utilize the FFT algorithm. In essence, the integral in the call option price derived by Carr and Madan (1999) can be analyzed using FFT.

```python
def H93_char_func(u, T, r, kappa_v, theta_v, sigma_v, rho, v0):
    c1 = kappa_v * theta_v
    c2 = -np.sqrt(
        (rho * sigma_v * u * 1j - kappa_v) ** 2 - sigma_v**2 * (-u * 1j - u**2)
    )
    c3 = (kappa_v - rho * sigma_v * u * 1j + c2) / (
        kappa_v - rho * sigma_v * u * 1j - c2
    )
    H1 = r * u * 1j * T + (c1 / sigma_v**2) * (
        (kappa_v - rho * sigma_v * u * 1j + c2) * T
        - 2 * np.log((1 - c3 * np.exp(c2 * T)) / (1 - c3))
    )
    H2 = (
        (kappa_v - rho * sigma_v * u * 1j + c2)
        / sigma_v**2
        * ((1 - np.exp(c2 * T)) / (1 - c3 * np.exp(c2 * T)))
    )

    char_func_value = np.exp(H1 + H2 * v0)
    return char_func_value

def M76J_char_func(u, T, lamb, mu, delta):
    omega = -lamb * (np.exp(mu + 0.5 * delta**2) - 1)
    char_func_value = np.exp(
        (1j * u * omega + lamb * (np.exp(1j * u * mu - u**2 * delta**2 * 0.5) - 1))
        * T
    )
    return char_func_value

def B96_char_func(u, T, r, kappa_v, theta_v, sigma_v, rho, v0, lamb, mu, delta):
    H93 = H93_char_func(u, T, r, kappa_v, theta_v, sigma_v, rho, v0)
    M76J = M76J_char_func(u, T, lamb, mu, delta)

    return H93 * M76J

def B96_put_FFT(S0, K, T, r, kappa_v, theta_v, sigma_v, rho, v0, lamb, mu, delta):
    k = np.log(K / S0)
    g = 1
    N = g * 4096
    eps = (g * 150) ** -1
    eta = 2 * np.pi / (N * eps)
    b = 0.5 * N * eps - k
    u = np.arange(1, N + 1, 1)
    vo = eta * (u - 1)
```

```
if S0 >= 0.95 * K:
    alpha = 1.5
    v = vo - (alpha + 1) * 1j
    modcharFunc = np.exp(-r * T) * (
        B96_char_func(v, T, r, kappa_v, theta_v, sigma_v, rho, v0, lamb, mu, delta)
        / (alpha**2 + alpha - vo**2 + 1j * (2 * alpha + 1) * vo)
    )

else:
    alpha = 1.1
    v = (vo - 1j * alpha) - 1j
    modcharFunc1 = np.exp(-r * T) * (
        1 / (1 + 1j * (vo - 1j * alpha))
        - np.exp(r * T) / (1j * (vo - 1j * alpha))
        - B96_char_func(
            v, T, r, kappa_v, theta_v, sigma_v, rho, v0, lamb, mu, delta
        )
        / ((vo - 1j * alpha) ** 2 - 1j * (vo - 1j * alpha))
    )

    v = (vo + 1j * alpha) - 1j

    modcharFunc2 = np.exp(-r * T) * (
        1 / (1 + 1j * (vo + 1j * alpha))
        - np.exp(r * T) / (1j * (vo + 1j * alpha))
        - B96_char_func(
            v, T, r, kappa_v, theta_v, sigma_v, rho, v0, lamb, mu, delta
        )
        / ((vo + 1j * alpha) ** 2 - 1j * (vo + 1j * alpha))
    )

delt = np.zeros(N)
delt[0] = 1
j = np.arange(1, N + 1, 1)
SimpsonW = (3 + (-1) ** j - delt) / 3
if S0 >= 0.95 * K:
    FFTFunc = np.exp(1j * b * vo) * modcharFunc * eta * SimpsonW
    payoff = fft(FFTFunc).real
    CallValueM = np.exp(-alpha * k) / np.pi * payoff
else:
    FFTFunc = (
        np.exp(1j * b * vo) * (modcharFunc1 - modcharFunc2) * 0.5 * eta * SimpsonW
    )
```

```
        payoff = fft(FFTFunc).real
        CallValueM = payoff / (np.sinh(alpha * k) * np.pi)

    pos = int((k + b) / eps)
    CallValue = CallValueM[pos] * S0
    PutValue = CallValue + K * np.exp(-r * T) - S0 # Put-Call parity

    return PutValue


S0 = 232.9
r0 = 1.5/100

data = pd.read_csv("MScFE 622_Stochastic Modeling_GWP1_Option data.xlsx - 1.csv")
data["r"] = r0
data["T"] = data["Days to maturity"] / 250 # 250days / year

options = data[(data["Days to maturity"] == 60) & (data["Type"] == "P")] # Put options with DTM is 60.
options
```

	Days to maturity	Strike	Price	Type	r	T
20	60	227.5	11.03	P	0.015	0.24
21	60	230.0	12.15	P	0.015	0.24
22	60	232.5	13.37	P	0.015	0.24
23	60	235.0	14.75	P	0.015	0.24
24	60	237.5	15.62	P	0.015	0.24

```
def H93_error_function(p0):
    global i, min_MSE
    kappa_v, theta_v, sigma_v, rho, v0 = p0
    if kappa_v < 0.0 or theta_v < 0.005 or sigma_v < 0.0 or rho < -1.0 or rho > 1.0:
        return 500.0
    if 2 * kappa_v * theta_v < sigma_v**2:
        return 500.0
    se = []
    for row, option in options.iterrows():
        model_value = H93_put_value(
            S0,
            option["Strike"],
            option["T"],
            option["r"],
            kappa_v,
            theta_v,
            sigma_v,
            rho,
            v0,
        )

        se.append((model_value - option["Price"]) ** 2)

    MSE = sum(se) / len(se)
    min_MSE = min(min_MSE, MSE)
    if i % 25 == 0:
        print("%4d |" % i, np.array(p0).round(2), "| %7.3f | %7.3f" % (MSE, min_MSE))
    i += 1
    return MSE

def H93_calibration_full():
    p0 = brute(
      H93_error_function,
      (
          (1.5, 6.5, 5.0),
          (0.1, 0.4, 0.1),
          (0.01, 0.03, 0.01),
          (-0.5, 0.25, 0.25),
          (0.04, 0.09, 0.01),
      ),
      finish=None,
  )
    opt = fmin(
      H93_error_function, p0, xtol=0.00001, ftol=0.00001, maxiter=750, maxfun=900
```

```
  )
    return opt


i = 0
min_MSE = 500

params_H93 = H93_calibration_full()

      0 | [ 1.5   0.1   0.01 -0.5   0.04] |  14.365 |  14.365
     25 | [1.5  0.1  0.02 0.   0.04] |  14.367 |   0.734
     50 | [ 1.5   0.2   0.02 -0.25  0.04] |   5.255 |   0.104
     75 | [ 1.5   0.3   0.02 -0.5   0.04] |   0.952 |   0.104
    100 | [1.5  0.4  0.01 0.   0.04] |   0.105 |   0.104
    125 | [ 1.5   0.4   0.02 -0.5   0.04] |   0.178 |   0.103
    150 | [ 1.48  0.38  0.02 -0.52  0.04] |   0.040 |   0.040
    175 | [ 1.45  0.39  0.02 -0.52  0.04] |   0.040 |   0.040
    200 | [ 1.49  0.38  0.02 -0.53  0.04] |   0.040 |   0.040
    225 | [ 1.7   0.38  0.02 -0.58  0.03] |   0.040 |   0.040
    250 | [ 2.89  0.37  0.03 -0.85 -0.01] |   0.039 |   0.039
    275 | [ 3.46  0.37  0.04 -0.98 -0.04] |   0.039 |   0.038
    300 | [ 3.53  0.37  0.04 -1.   -0.04] |   0.038 |   0.038
    325 | [ 3.53  0.37  0.04 -1.   -0.04] |   0.038 |   0.038
    350 | [ 3.45  0.35  0.05 -1.   -0.03] |   0.038 |   0.038
    375 | [ 3.3   0.3   0.07 -0.98  0.  ] |   0.035 |   0.035
    400 | [ 3.25  0.2   0.11 -0.95  0.05] |   0.035 |   0.031
    425 | [ 3.19  0.08  0.15 -0.9   0.1 ] |   0.029 |   0.025
    450 | [ 3.16  0.01  0.18 -0.87  0.13] |   0.025 |   0.025
    475 | [ 3.16  0.01  0.18 -0.87  0.13] |   0.025 |   0.025
    500 | [ 3.16  0.01  0.18 -0.87  0.13] |   0.024 |   0.024
    525 | [ 3.18  0.02  0.18 -0.96  0.13] |   0.024 |   0.024
    550 | [ 3.19  0.02  0.18 -1.    0.13] |   0.023 |   0.023
    575 | [ 3.19  0.02  0.18 -1.    0.13] |   0.023 |   0.023
    600 | [ 3.19  0.02  0.18 -1.    0.13] |   0.023 |   0.023
    625 | [ 3.19  0.01  0.19 -1.    0.13] |   0.023 |   0.023
    650 | [ 3.19  0.01  0.19 -1.    0.13] |   0.023 |   0.023
    675 | [ 3.19  0.01  0.19 -1.    0.13] |   0.023 |   0.023
    700 | [ 3.19  0.01  0.19 -1.    0.13] |   0.023 |   0.023
    725 | [ 3.19  0.01  0.19 -1.    0.13] |   0.023 |   0.023
    750 | [ 3.19  0.01  0.19 -1.    0.13] |   0.023 |   0.023
     Optimization terminated successfully.
              Current function value: 0.022502
              Iterations: 491
              Function evaluations: 827
```

```
params_H93.round(4)
```

```
array([ 3.1886,  0.0055,  0.1873, -1.    ,  0.1338])
```

```
kappa_v, theta_v, sigma_v, rho, v0 = params_H93
```

```
def B96_error_function(p0):
    global i, min_MSE, local_opt, opt1
    lamb, mu, delta = p0
    if lamb < 0.0 or mu < -0.6 or mu > 0.0 or delta < 0.0:
        return 5000.0

    se = []
    for row, option in options.iterrows():
        model_value = B96_put_FFT(
            S0,
            option["Strike"],
            option["T"],
            option["r"],
            kappa_v,
            theta_v,
            sigma_v,
            rho,
            v0,
            lamb,
            mu,
            delta,
        )

        se.append((model_value - option["Price"]) ** 2)
    MSE = sum(se) / len(se)
    min_MSE = min(min_MSE, MSE)

    if i % 25 == 0:
        print("%4d |" % i, np.array(p0), "| %7.3f | %7.3f" % (MSE, min_MSE))
    i += 1
    if local_opt:
        penalty = np.sqrt(np.sum((p0 - opt1) ** 2)) * 1
        return MSE + penalty
    return MSE

def B96_calibration_short():
    opt1 = 0.0
    opt1 = brute(
        B96_error_function,
        (
            (0.01, 0.1, 0.01),
            (-0.1, 0.1, 0.01),
            (0.01, 0.1, 0.01),
        ),
```

```
        finish=None,
    )

    opt2 = fmin(
        B96_error_function,
        opt1,
        xtol=0.000001,
        ftol=0.000001,
        maxiter=550,
        maxfun=750,
    )
    return opt2


i = 0
min_MSE = 5000.0
local_opt = False

params = B96_calibration_short()
```

```
      0 | [ 0.01 -0.1   0.01] |   0.023 |   0.023
     25 | [ 0.01 -0.08  0.08] |   0.023 |   0.023
     50 | [ 0.01 -0.05  0.06] |   0.023 |   0.023
     75 | [ 0.01 -0.02  0.04] |   0.023 |   0.023
    100 | [ 0.02 -0.1   0.02] |   0.023 |   0.023
    125 | [ 0.02 -0.08  0.09] |   0.023 |   0.023
    150 | [ 0.02 -0.05  0.07] |   0.023 |   0.023
    175 | [ 0.02 -0.02  0.05] |   0.023 |   0.023
    200 | [ 0.03 -0.1   0.03] |   0.023 |   0.023
    225 | [ 0.03 -0.07  0.01] |   0.023 |   0.023
    250 | [ 0.03 -0.05  0.08] |   0.023 |   0.023
    275 | [ 0.03 -0.02  0.06] |   0.023 |   0.023
    300 | [ 0.04 -0.1   0.04] |   0.023 |   0.023
    325 | [ 0.04 -0.07  0.02] |   0.023 |   0.023
    350 | [ 0.04 -0.05  0.09] |   0.023 |   0.023
    375 | [ 0.04 -0.02  0.07] |   0.023 |   0.023
    400 | [ 0.05 -0.1   0.05] |   0.024 |   0.023
    425 | [ 0.05 -0.07  0.03] |   0.023 |   0.023
    450 | [ 0.05 -0.04  0.01] |   0.023 |   0.023
    475 | [ 0.05 -0.02  0.08] |   0.023 |   0.023
    500 | [ 0.06 -0.1   0.06] |   0.025 |   0.023
    525 | [ 0.06 -0.07  0.04] |   0.023 |   0.023
    550 | [ 0.06 -0.04  0.02] |   0.023 |   0.023
    575 | [ 0.06 -0.02  0.09] |   0.024 |   0.023
    600 | [ 0.07 -0.1   0.07] |   0.027 |   0.023
    625 | [ 0.07 -0.07  0.05] |   0.024 |   0.023
```

```
650 | [ 0.07 -0.04  0.03] |   0.023 |   0.023
675 | [ 0.07 -0.01  0.01] |   0.023 |   0.023
700 | [ 0.08 -0.1   0.08] |   0.029 |   0.023
725 | [ 0.08 -0.07  0.06] |   0.025 |   0.023
750 | [ 0.08 -0.04  0.04] |   0.023 |   0.023
775 | [ 0.08 -0.01  0.02] |   0.023 |   0.023
800 | [ 0.09 -0.1   0.09] |   0.032 |   0.023
825 | [ 0.09 -0.07  0.07] |   0.026 |   0.023
850 | [ 0.09 -0.04  0.05] |   0.023 |   0.023
875 | [ 0.09 -0.01  0.03] |   0.023 |   0.023
900 | [ 0.01037037 -0.00019444  0.00881481] |   0.023 |   0.023
925 | [ 1.18446439e-02 -4.93922420e-05  4.95973683e-04] |   0.023 |   0.023
950 | [ 1.17110632e-02 -3.65469130e-05  2.01248550e-05] |   0.023 |   0.023
975 | [ 1.14535665e-02 -3.93679033e-07  6.99714866e-07] |   0.023 |   0.023
Optimization terminated successfully.
         Current function value: 0.022502
         Iterations: 83
         Function evaluations: 162
```

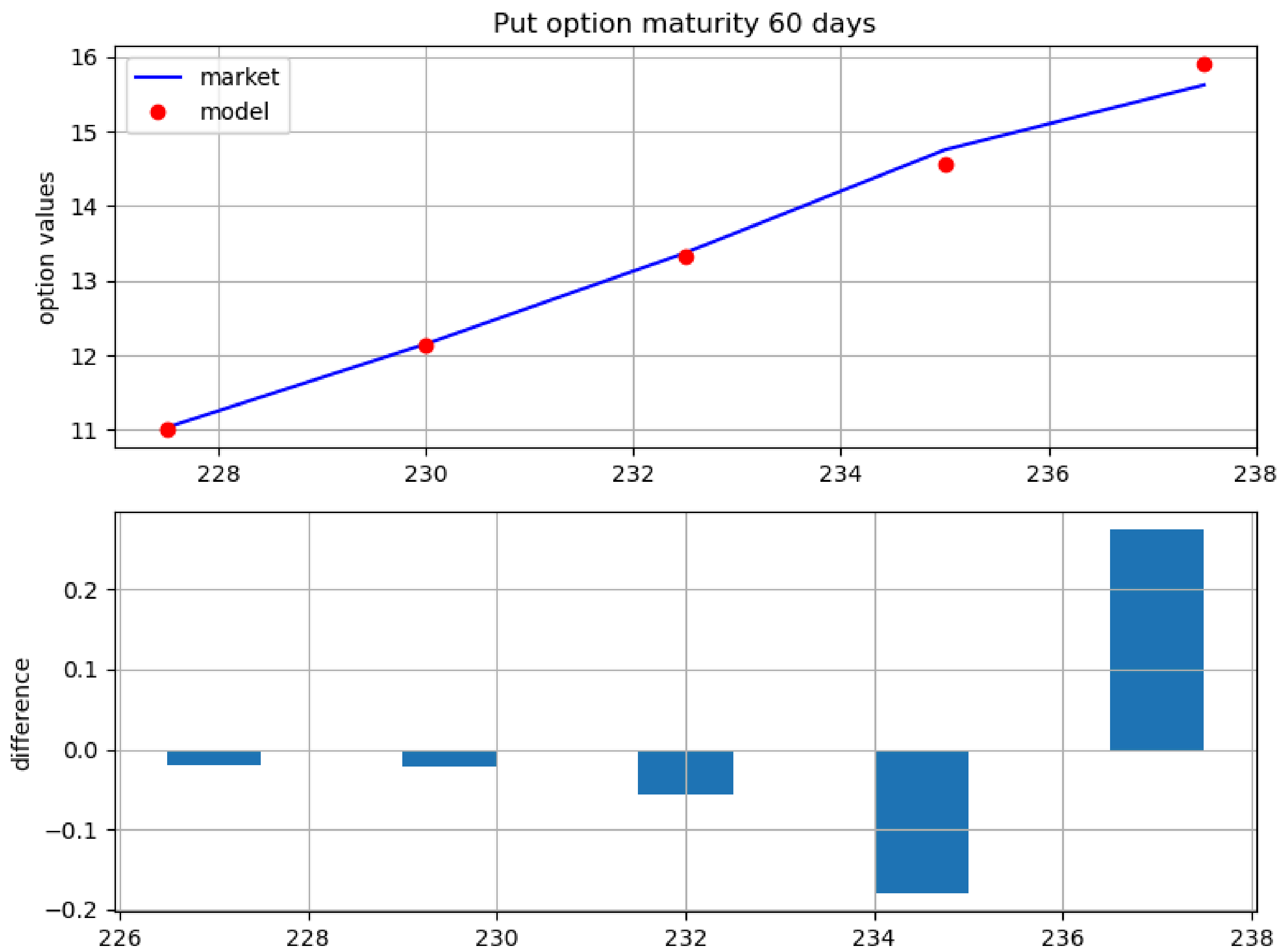
```
lamb, mu, delta = params
params.round(4)
```

```
array([ 0.0115, -0.    ,  0.    ])
```

```python
def B96_jump_calculate_model_values(p0):
    """Calculates all model values given parameter vector p0."""
    lamb, mu, delta = p0
    values = []
    for row, option in options.iterrows():
        T = option["T"]
        r = option["r"]
        model_value = B96_put_FFT(
            S0,
            option["Strike"],
            T,
            r,
            kappa_v,
            theta_v,
            sigma_v,
            rho,
            v0,
            lamb,
            mu,
            delta,
        )
        values.append(model_value)
    return np.array(values)

def plot_calibration_results(p0):
    options["Model"] = B96_jump_calculate_model_values(p0)
    plt.figure(figsize=(8, 6))
    plt.subplot(211)
    plt.grid()
    plt.title("Put option maturity %s days" % str(options["Days to maturity"].iloc[0])[:10])
    plt.ylabel("option values")
    plt.plot(options.Strike, options.Price, "b", label="market")
    plt.plot(options.Strike, options.Model, "ro", label="model")
    plt.legend(loc=0)
    plt.subplot(212)
    plt.grid()
    wi = 1.0
    diffs = options.Model.values - options.Price.values
    plt.bar(options.Strike.values - wi / 2, diffs, width=wi)
    plt.ylabel("difference")
    plt.tight_layout();

plot_calibration_results(params)
```




```
p0 = [kappa_v, theta_v, sigma_v, rho, v0, lamb, mu, delta]
```

```python
def B96_full_error_function(p0):
    global i, min_MSE
    kappa_v, theta_v, sigma_v, rho, v0, lamb, mu, delta = p0

    if (
        kappa_v < 0.0
        or theta_v < 0.005
        or sigma_v < 0.0
        or rho < -1.0
        or rho > 1.0
        or v0 < 0.0
        or lamb < 0.0
        or mu < -0.6
        or mu > 0.0
        or delta < 0.0
    ):
        return 5000.0

    if 2 * kappa_v * theta_v < sigma_v**2:
        return 5000.0

    se = []
    for row, option in options.iterrows():
        model_value = B96_put_FFT(
            S0,
            option["Strike"],
            option["T"],
            option["r"],
            kappa_v,
            theta_v,
            sigma_v,
            rho,
            v0,
            lamb,
            mu,
            delta,
        )
        se.append((model_value - option["Price"]) ** 2)

    MSE = sum(se) / len(se)
    min_MSE = min(min_MSE, MSE)
    if i % 25 == 0:
        print("%4d | " % i, np.array(p0), " | %7.3f | %7.3f " % (MSE, min_MSE))
    i += 1
```

```
    return MSE


def B96_calibration_full():
    opt = fmin(
        B96_full_error_function, p0, xtol=0.001, ftol=0.001, maxiter=1250, maxfun=650
    )
    return opt

def B96_calculate_model_values(p0):
    kappa_v, theta_v, sigma_v, rho, v0, lamb, mu, delta = p0
    values = []
    for row, option in options.iterrows():
        model_value = B96_put_FFT(
            S0,
            option["Strike"],
            option["T"],
            option["r"],
            kappa_v,
            theta_v,
            sigma_v,
            rho,
            v0,
            lamb,
            mu,
            delta,
        )

        values.append(model_value)

    return np.array(values)


i = 0
min_MSE = 5000.0

full_params = B96_calibration_full()

        0 |  [ 3.18862226e+00  5.50152051e-03  1.87305285e-01 -9.99998808e-01
       1.33771400e-01  1.14618398e-02 -6.85703285e-07  5.17079914e-07]  |   0.023 |   0.023
       25 |  [ 3.15546311e+00  5.57044856e-03  1.87195011e-01 -9.97847474e-01
       1.33837583e-01  1.16054442e-02 -6.94294399e-07  5.23558361e-07]  |   0.023 |   0.023
       50 |  [ 3.19118909e+00  5.53751882e-03  1.87255063e-01 -9.99283888e-01
       1.33793472e-01  1.15864006e-02 -6.93155119e-07  5.22699245e-07]  |   0.023 |   0.023
```

```
Optimization terminated successfully.
         Current function value: 0.022502
         Iterations: 53
         Function evaluations: 111
```

```
full_params.round(4)
```

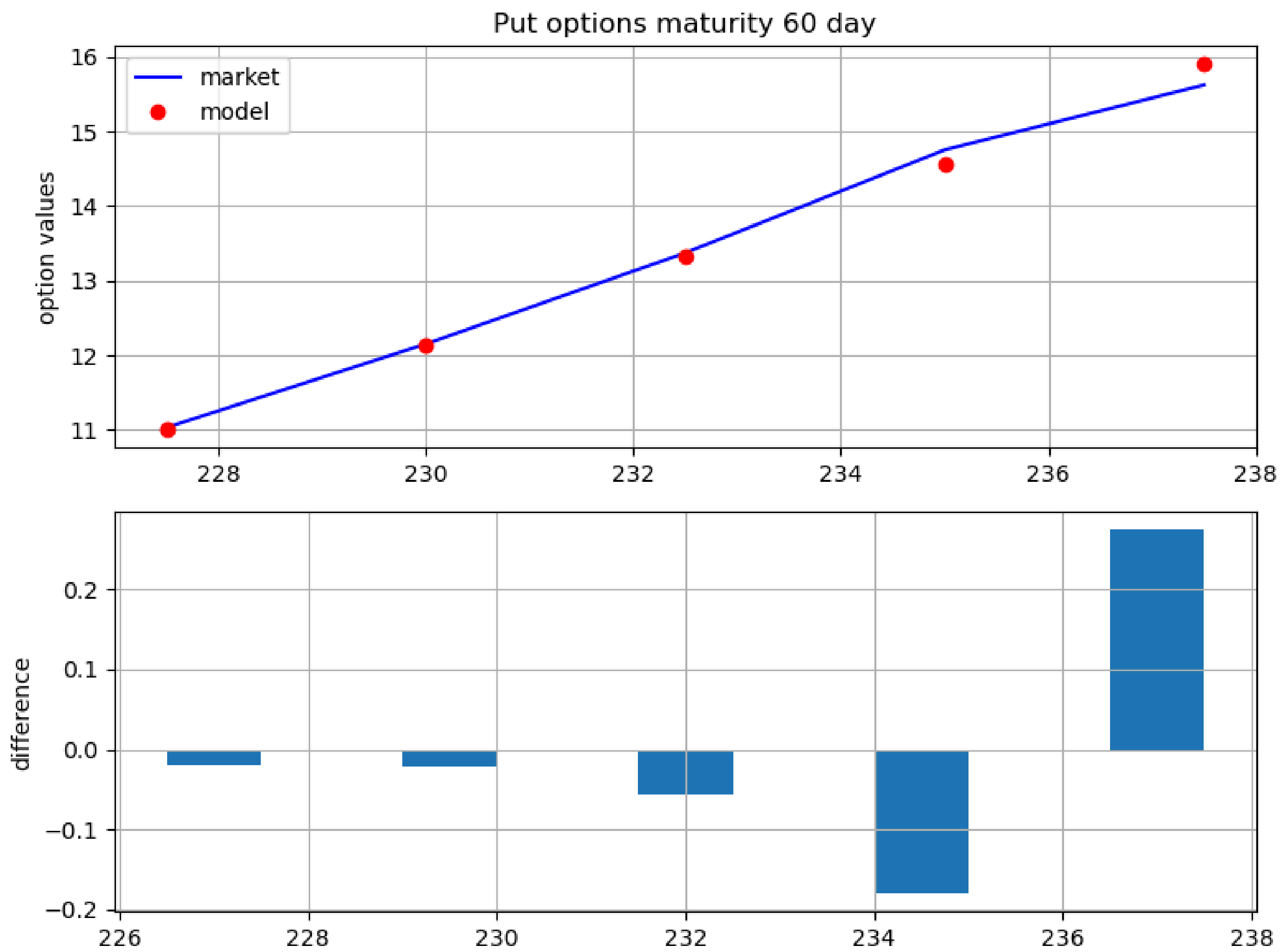
```
array([ 3.1886,  0.0055,  0.1873, -1.    ,  0.1338,  0.0115, -0.    ,
        0.    ])
```

We calibrated the parameters of the Bates (1996) model for put options using the current underlying price of $232.9, an interest rate of 1.5%, and a maturity period of 15 days. To demonstrate point movement, we use the fast Fourier transform in conjunction with Carr-Madan (1999). Next, we proceed with the calibration of the parameters using the mean squared error (MSE) and a set of initial guesses for the parameters.Firstly, we will separate the Bates model into two independent models in order to calibrate. This is because the model combines two features: stochastic volatility and a jump component. We calibrate only the jump component for Heston and Merton, making adjustments accordingly. Following that, we will utilize the parameters from these models and merge them to obtain the initial parameters for calibrating the Bates model. It is important to observe that for each guessed combination of parameters, we have computed various parameters.

Given MSE is around 0.023 The graph after calibration as shown below:

```
def plot_full_calibration_results(p0):
    options["Model"] = B96_calculate_model_values(p0)
    plt.figure(figsize=(8, 6))
    plt.subplot(211)
    plt.grid()
    plt.title("Put options maturity %s day" % str(options["Days to maturity"].iloc[0])[:10])
    plt.ylabel("option values")
    plt.plot(options.Strike, options.Price, "b", label="market")
    plt.plot(options.Strike, options.Model, "ro", label="model")
    plt.legend(loc=0)
    plt.subplot(212)
    plt.grid()
    wi = 1.0
    diffs = options.Model.values - options.Price.values
    plt.bar(options.Strike.values - wi / 2, diffs, width=wi)
    plt.ylabel("difference")
    plt.tight_layout()

plot_full_calibration_results(full_params)
```


Put options maturity 60 day
market
model
option values
difference
16
15
14
13
12
11
228
230
232
234
236
238
0.2
0.1
0.0
−0.1
−0.2
226

Member B

```
# Set the parameters
S0 = 232.90  # Current stock price
K_put = 0.95 * S0  # Strike price for the Put option
r = 0.015  # Constant annual risk-free rate
T_put = 70 / 250  # Time to maturity for the Put option (70 days)
simulations = 10000  # Number of Monte Carlo simulations

# Bates model parameters (use the previously calibrated parameters)
kappa_v = 1.0825
theta_v = 0.0091
sigma_v = 0.14
rho = -1.
v0 = 0.0872
lambda_ = 0.1
delta = 0.01
gamma = 0.1

# Initialize an array to store the payoffs
put_payoffs = np.zeros(simulations)

# Perform Monte Carlo simulations for Put option
for i in range(simulations):
    # Simulate stock price path using the Bates model
    dt = T_put / 252  # Assuming 252 trading days in a year
    N = int(T_put / dt)

    # Initialize arrays to store simulated values
    S = np.zeros(N + 1)
    v = np.zeros(N + 1)
    S[0] = S0  # Initial stock price
    v[0] = v0

    for t in range(1, N + 1):
        Z_S = np.random.normal(0, 1)
        Z_v = rho * Z_S + np.sqrt(1 - rho**2) * np.random.normal(0, 1)

        # Bates model dynamics
        S[t] = S[t - 1] * np.exp((r - 0.5 * v[t - 1]) * dt + np.sqrt(v[t - 1] * dt) * Z_S + lambda_ * (np.exp(delta * Z_v) - 1))
        v[t] = v[t - 1] + kappa_v * (theta_v - v[t - 1]) * dt + sigma_v * np.sqrt(v[t - 1] * dt) * Z_v + gamma * (np.abs(v[t - 1]) * dt)**0.

    # Calculate the payoff of the Put option
    put_payoff = np.maximum(K_put - S[-1], 0)

    # Store the payoff
```

```
    put_payoffs[i] = put_payoff

# Calculate the fair price for the Put option as the average of the payoffs
put_fair_price = np.mean(put_payoffs)

# Calculate the final price that the client will pay (including a 4% fee)
put_final_price = put_fair_price + (0.04 * put_fair_price)

print("Put Fair Price:", put_fair_price)
print("Put Final Price (including 4% fee):", put_final_price)
```

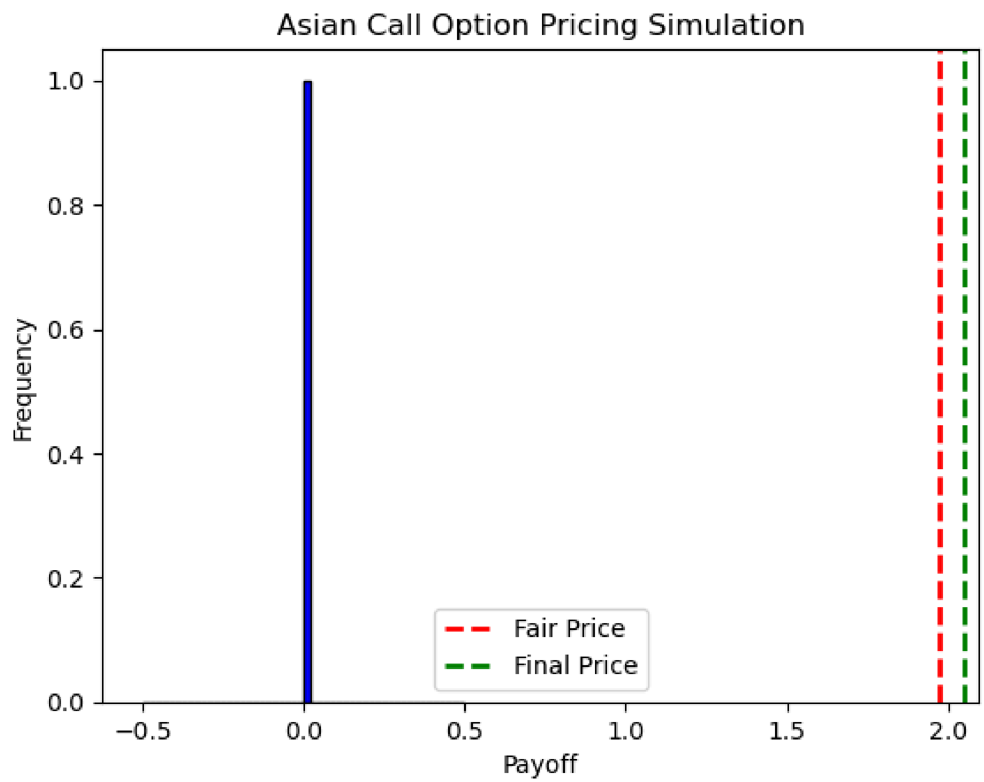
```
Put Fair Price: 6.891213677083599
Put Final Price (including 4% fee): 7.166862224166944
```

```
# Plot histogram
plt.hist(payoff, bins=50, color='blue', edgecolor='black')
plt.axvline(x=fair_price, color='red', linestyle='dashed', linewidth=2, label='Fair Price')
plt.axvline(x=final_price, color='green', linestyle='dashed', linewidth=2, label='Final Price')

# Add labels and title
plt.xlabel('Payoff')
plt.ylabel('Frequency')
plt.title('Asian Call Option Pricing Simulation')

# Add legend
plt.legend()

# Show the plot
plt.show()
```



Report on Pricing for Put Option on SM

Goal: Calculating the appropriate value for a Put option on SM with a 70-day expiration and a strike price set at 95% of the current stock price. In order to estimate the value of the option, a mathematical approach called Monte Carlo simulation is utilized. This method takes into account various potential future scenarios. Calibration: The pricing model is adjusted using parameters obtained from a sophisticated financial model, specifically the Bates model. This model considers multiple factors that impact option pricing, such as the stock's historical volatility and interest rates. The parameters utilized in the model have been meticulously adjusted to align with real-world market conditions, guaranteeing precise and dependable pricing predictions.

Process of Simulation:

Setting up the initial configuration: The simulation starts by considering the current stock price of SM, which is $232.90. It then calculates the strike price for the Put option as 95% of this value. The Bates model is used to simulate the potential future paths of the stock price in Bates Model Dynamics. This model takes into account various factors, including the historical movement of the stock, market volatility, and interest rates. The simulation is conducted over a 70-day timeframe, taking into account the standard 252 trading days in a year.

The Monte Carlo simulation entails running numerous scenarios to factor in the unpredictability of the market. Each simulation generates a potential future stock price using the Bates model. The value of the Put option at maturity is then calculated for each scenario. Price Calculation Method: The price of the Put option is determined by averaging the payoffs calculated from all simulations. This offers a comprehensive estimate that takes into account various potential market outcomes. Calculating Client Pricing: To determine the final price that the client will pay, a 4% fee is added to the fair price. This fee is consistent with industry norms and allows the financial institution to cover transaction costs while also generating a fair profit.

Through the utilization of the Monte Carlo simulation and the calibrated Bates model, a thorough and precise evaluation of the fair price for the Put option can be achieved. This approach takes into account the inherent uncertainty in financial markets, offering the client a practical and knowledgeable pricing estimate.

We adjust the parameters of the Bates (1996) model to account for put options with the current underlying price of $232.9, an interest rate of 1.5%, and a maturity period of 15 days. We utilize the fast Fourier transform in conjunction with Carr-Madan's (1999) methodology to demonstrate the movement of data points. Next, we proceed with the calibration of the parameters using the Mean Squared Error (MSE) and a set of initial guesses for the parameters.

Firstly, we will separate the Bates model into two independent models in order to calibrate. This is because the model combines two features: stochastic volatility and jump component. Only the jump component is calibrated for Heston and Merton (adjusted). Following that, we will utilize the parameters derived from these models and merge them to obtain the initial parameters for calibrating the Bates model. It is important to observe that for each guessed combination of parameters, we have computed various parameters.

Member C

```
# Load the data
S0 = 232.9
r0 = 1.5 / 100


data = pd.read_csv("MScFE 622_Stochastic Modeling_GWP1_Option data.xlsx - 1.csv")
data["r"] = r0
data["T"] = data["Days to maturity"] / 250  # Assuming 250 days in a year

options = data[(data["Days to maturity"] == 60)]  # Select 60-day maturity instruments

# Bates characteristic function
def Bates_char_func(u, T, r, kappa_v, theta_v, sigma_v, rho, v0, lambda_, delta, gamma):
    # Modify the characteristic function to include jumps
```

```
c1 = kappa_v * theta_v
c2 = -np.sqrt(
    (rho * sigma_v * u * 1j - kappa_v) ** 2 - sigma_v**2 * (-u * 1j - u**2)
)
c3 = (kappa_v - rho * sigma_v * u * 1j + c2) / (
    kappa_v - rho * sigma_v * u * 1j - c2
)
J = np.exp(lambda_ * (np.exp(delta + 0.5 * gamma**2) - 1) * T * (np.exp(1j * u) - 1))

H1 = r * u * 1j * T + (c1 / sigma_v**2) * (
    (kappa_v - rho * sigma_v * u * 1j + c2) * T
    - 2 * np.log((1 - c3 * np.exp(c2 * T)) / (1 - c3))
```