GROUP WORK PROJECT #3 MScFE 620: Derivative Pricing
GROUP NUMBER: 5224

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Statement of integrity: By typing the names of all group members in the text boxes below, you confirm that the assignment submitted is original work produced by the group (excluding any non-contributing members identified with an "X" above).	
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Use the box below to explain any attempts to reach out to a non-contributing member. Type (N/A) if all members contributed. **Note:** You may be required to provide proof of your outreach to non-contributing members upon request.

Overview of Heston Model

In financial engineering, the Heston model is used widely to price options. It is a stochastic volatility model. In Black Scholes model, it is assumed that the volatility of the asset price follows a random process. Whereas in the Heston model, the volatility of the asset price is assumed to follow a random process

Given:

Mean reversion rate (κv) = 1.85

Initial stock price (S0) = 80

Risk-free rate (r) = 5.5%

Initial variance (v0) = 3.2%

Time to maturity = 3 months

Long-term variance (θv) = 0.045

Volatility of volatility (sigma) =35%

5.

Monte Carlo Simulation: Using the Heston model dynamics and the stated correlation of -0.30 between the asset and its volatility, generate routes for the underlying asset price while accounting for volatility's stochastic character.

To ensure accuracy, use a sufficient number of time steps and simulations.

ATM pricing European Call and Put: Determine the mean return on all simulated paths for the call and put options.

Utilizing the risk-free rate, reduce these payouts to their current value.

The Heston model is used by the Python software to simulate asset routes, after which ATM European call and put options are priced.

Option	Option Type	Moneyness	Model	Option Price
Call	European	ATM	Heston	2.8794
Put	European	ATM	Heston	2.8547

6.

Delta (∆):

To approximate Delta numerically, raise the stock price by a small amount (say, 1%), then recalculate the option price.

The change in option price divided by the change in stock price is a rough way to calculate delta.

Gamma (Γ):

Gamma can also be approximated using a similar method. A tiny price increase in the stock is applied; the new Delta is then computed and contrasted with the initial Delta.

Gamma is calculated by dividing the change in stock price by the change in Delta.

In order to price an ATM European call and put option using the Heston model and a different correlation value of -0.70, the Python script modifies the earlier Python script.

Option	Option Type	Moneyness	Model	Option Price
Call	European	ATM	Heston	2.0886
Put	European	ATM	Heston	3.4973

7. The python code for this is provided in the Jupyter notebook. It will give the functions which are needed to calculate delta and gamma.

Correlation: -0.3

Option	Option Type	Moneyness	Model	Delta	Gamma	Option Price
Call	European	ATM	Heston	0.0108	0.0565	2.8794
Put	European	ATM	Heston	0.0092	0.0275	2.8547

Correlation: -0.7

Option	Option Type	Moneyness	Model	Delta	Gamma	Option Price
Call	European	ATM	Heston	0.0052	0.0670	2.0886
Put	European	ATM	Heston	0.1451	-0.1228	3.4973

8.

The Merton model might be defined:

$$dS(t) = \mu S(t)dt + \sigma S(t)dW(t) + J(t)S(t)dN(t)$$

The price of a stock: S(t)

The Wiener process: W(t)

The size of a jump in Merton model: J(t)

The Time variable: t

The Poisson process: N(t)

The exp return: μ

Volatility: σ

$\mu = -0.5$

$\delta = 0.22$

Option	Option Type	Moneyness	Option Price
Call	European	ATM	31.4514
Put	European	ATM	26.5743

9. Given Values-

$\mu = -0.5, \delta = 0.22$

Option	Option Type	Moneyness	Option Price
Call	European	ATM	8.8362
Put	European	ATM	3.9591

10. The calculation of delta for call/put options is performed using the following expressions:

$$\Delta_{call} = N(d_1)$$

$$\Delta_{put} = N(d_1) - 1$$

The Gamma value is equal for call and put options.

Option	Option Type	Moneyness	Gamma	Delta
Call	European	ATM	0.0107	0.1524
Put	European	ATM	0.0107	-0.9327

11. Checking Put-Call Parity for question 5,6,8 and 9

Please check the calculation in shared python code.

Put Call Parity Error: C + K*exp(-r*t) - (P + S0)

where,

C = Call Price

P= PutPrice

K = Strike Price

r = risk-free interest rate

t = Time to Maturity

S0 = initial Stock price/asset price

Error from put-call parity equation for question 5 : **-0.988**

Error from put-call parity equation for question 6 : **-2.502**

Error from put-call parity equation for question 8 : **0.0**

Error from put-call parity equation for question 9 : **0.0**

12. Below are the relative graphs asked in question, please check these in shared python code for more.

Call Option price for different moneyness with Heston Model					

S0 = 100 # Initial Stock'/Asset price
r = 0.055 # Risk-free rate
v0 = 0.032 # Initial variance
kappa = 1.85 # Mean reversion rate
theta = 0.045 # Long-term variance
sigma = 0.35 # Volatility of volatility
rho = -0.3 # Correlation
T = 0.25 # Time Period					

Option	**Option Type**	**Moneyness (K/S0)**	**Strike Price (K)**	**Model**	**Option value**
Call	European	0.85 - OTM	85	Heston	15.0827
Call	European	0.90 - OTM	90	Heston	10.5975
Call	European	0.95 - OTM	95	Heston	6.6496
Call	European	1.0 - ATM	100	Heston	3.5993
Call	European	1.05 - ITM	105	Heston	1.6692
Call	European	1.10 - ITM	110	Heston	0.6697
Call	European	1.15 - ITM	115	Heston	0.2402

Put Option price for different moneyness with Heston Model

S0 = 100 # Initial Stock'/Asset price
r = 0.055 # Risk-free rate
v0 = 0.032 # Initial variance
kappa = 1.85 # Mean reversion rate
theta = 0.045 # Long-term variance
sigma = 0.35 # Volatility of volatility

rho = -0.3 # Correlation
T = 0.25 # Time Period
[0.2566, 0.7031, 1.6869, 3.5683, 6.5699, 10.5021, 15.0044]					

Option	**Option Type**	**Moneyness (K/S0)**	**Strike Price (K)**	**Model**	**Option value**
Put	European	0.85 - ITM	85	Heston	0.2566
Put	European	0.90 - ITM	90	Heston	0.7031
Put	European	0.95 - ITM	95	Heston	1.6869
Put	European	1.0 - ATM	100	Heston	3.5993
Put	European	1.05 - OTM	105	Heston	6.5699
Put	European	1.10 - OTM	110	Heston	10.5021
Put	European	1.15 - OTM	115	Heston	15.0044

Call Option price for different moneyness with Merton Model					

# Given parameters 	
S0 = 100 # Initial stock price
r = 0.055 # Risk-free rate
T = 0.25 # Time to maturity
mu = -0.5 # Expected return
sigma = 0.35 # Volatility
lambda_param = 0.25 # Jump intensity parameter					

Option	**Option Type**	**Moneyness (K/S0)**	**Strike Price (K)**	**Model**	**Option value**
Call	European	0.85 - OTM	85	Merton	24.8788
Call	European	0.90 - OTM	90	Merton	23.9595
Call	European	0.95 - OTM	95	Merton	23.1379
Call	European	1.0 - ATM	100	Merton	21.992
Call	European	1.05 - ITM	105	Merton	20.2932
Call	European	1.10 - ITM	110	Merton	18.0415

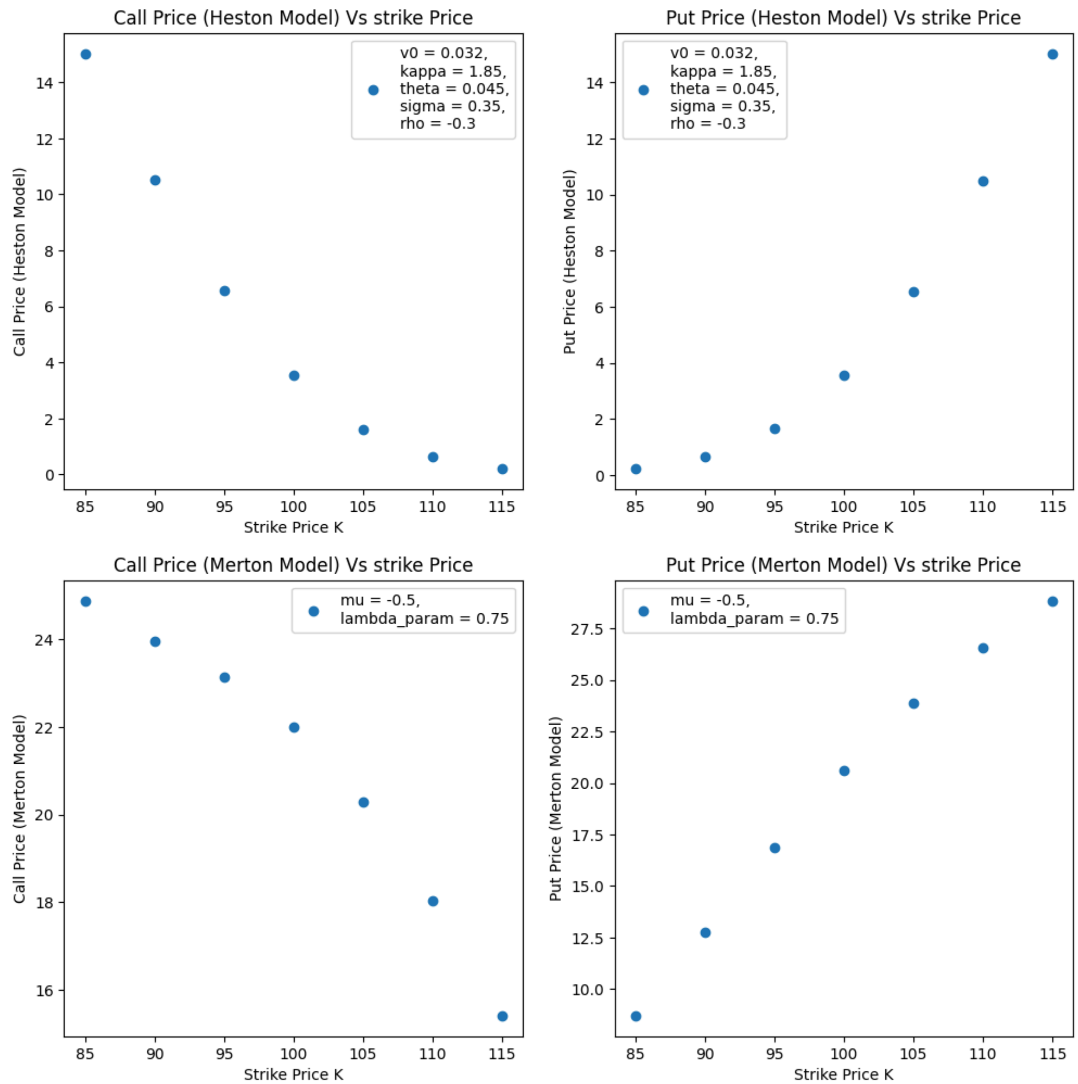
Call	European	1.15 - ITM	115	Merton	15.4096

Put Option price for different moneyness with Merton Model					

# Given parameters 	
S0 = 100 # Initial stock price
r = 0.055 # Risk-free rate
T = 0.25 # Time to maturity
mu = -0.5 # Expected return
sigma = 0.35 # Volatility
lambda_param = 0.25 # Jump intensity parameter					

Option	**Option Type**	**Moneyness (K/S0)**	**Strike Price (K)**	**Model**	**Option value**
Put	European	0.85 - ITM	85	Merton	8.718
Put	European	0.90 - ITM	90	Merton	12.7305
Put	European	0.95 - ITM	95	Merton	16.8406
Put	European	1.0 - ATM	100	Merton	20.6264
Put	European	1.05 - OTM	105	Merton	23.8593
Put	European	1.10 - OTM	110	Merton	26.5394
Put	European	1.15 - OTM	115	Merton	28.8392

Call Price and Put price calculated with different Strike price for Heston and Merton Model with S0 = 100, r=5%, T = 0.25,sigma = 35%



Step 2:

13.

American option pricing is much more complicated than European option pricing, especially when using techniques like the Heston model and Monte Carlo simulation. This is so that the value is further complicated by the fact that American options can be exercised at any point prior to expiration.

Nonetheless, the Least Squares Monte Carlo (LSM) technique is a widely applied technique for American options pricing using Monte Carlo simulation in practice. The continuation value of retaining the option at each time step is estimated using this method, which was first presented by Longstaff and Schwartz. It is based on regression analysis and is contrasted with the immediate exercise value.

Several important factors need to be addressed when discussing how pricing European options (Question 5 and 6) and American options (modified Questions 5 and 6) differ in terms of how the Heston model is used, as well as how these options' Delta and Gamma are calculated (Question 7 and adapted Question 7).

S.no.	Topic	European Option	American Option
1	Option Pricing Methodology	Only exercisable upon reaching maturity. This makes it possible to use the Heston model in a straightforward way to apply the Monte Carlo approach. In this case, the option's value is calculated using the discounted expected payment at maturity.	You can exercise them whenever you want up until they expire. A more intricate strategy, such as the Least Squares Monte Carlo (LSM) method, is needed for this feature. LSM requires more computing power because it calculates the ideal stopping time for exercise.
2	Computational Complexity and Time	Because of their smaller payout structure, they are typically less computationally demanding.	Because of the early exercising feature, they are more complex. The LSM approach lengthens calculation time by requiring extra procedures at each time step, such as regression analysis.
3	Pricing Dynamics	The early exercise feature in American options can add value, particularly for call options on dividend-paying equities, which is why European options are frequently less expensive than their American equivalents	Because they provide for early exercising flexibility, they typically have greater pricing. This is especially true in situations (such high rewards compared to the risk-free rate) where it may be advantageous to exercise early.
4	Delta and Gamma	Since the payout of the Greeks occurs only at maturity, they may be computed rather easily with numerical techniques.	Because American options depend on a path, calculating Greeks for them is more difficult. Rerunning the simulation with perturbed parameters is necessary for the numerical approximation, making it more complicated than for European choices.
5	Model Sensitivity	Sensitivity to variables such as interest rate and volatility is part of the pricing mechanism and is usually easier to understand.	The early exercise feature increases the sensitivity of these characteristics, especially with regard to their impact on the

			relative time value of the option versus its intrinsic value. As a result, sensitivity levels may differ from those of European choices.
6	Risk Management and Hedging	American and European options differ in terms of price and Greeks, which suggests that hedging and risk management approaches would also be different. Because American options have the potential to exercise early, more dynamic hedging may be necessary.	
7	Numerical Stability and Accuracy	Because of their simpler pricing structure, they are usually more numerically stable.	The number of simulations, time steps, and regression model selected can all have an impact on the LSM approach's numerical stability and accuracy.

In conclusion, there are extra levels of complexity, computational effort, and sensitivity to the underlying models and parameters when switching from pricing and Greek calculations for European options to American options. For financial engineers and experts in option pricing and risk management, these distinctions are essential.

The Python script handling this task is given in the Jupyter Notebook.

14. The Python script handling this task is given in the Jupyter Notebook.

The payoff structure for an UAI Call is as follows:

Payoff UAI Call=max(ST-X, 0)

The price of a stock: St

The strike price: X

The European UAI call option's price is as follows:

Price UAI Call= e^-rT integral of (ST-X) with lower limit H and upper limit infinity . f(ST)dST

The barrier level: H

The PDF of a stock price at option's maturity f(ST)

The European UAI Call price: nan

The European Call price (from Question 6): 8.8362

15. The Python script handling this task is given in the Jupyter Notebook.

The payoff structure for a DAI Put is as follows:
Payoff DAI Put=max(0, X-ST)

The stock price: ST

The strike price: X

The European DAI Put price: -7.9311

The European Put price (from Question 8): 3.9591

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