GROUP WORK PROJECT #2
GROUP NUMBER: 5224

MScFE 620: Derivative Pricing

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Statement of integrity: By typing the names of all group members in the text boxes below, you confirm that the assignment submitted is original work produced by the group (excluding any non-contributing members identified with an “X” above).	
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Use the box below to explain any attempts to reach out to a non-contributing member. Type (N/A) if all members contributed. **Note:** You may be required to provide proof of your outreach to non-contributing members upon request.

Step 1

1. The European options parameters:

   The initial stock price is: S0 = 100

   Risk free IR: r = 5%

   Vol: σ=20%

   Expiration: T = 3 months (3/12 year)

a. BS Model and ATM European Call pricing

   Black-Scholes Formula

   The option is ATM, so the strike price K = S0 = 100. For a European call the BS formula is:

$$C(S_0, T, K, r, \sigma) = S_0 * \phi(d_1) - K * e^{-rT}\phi(d_2)$$

   For a European put option the BS formula is given by:

$$P(S_0, T, K, r, \sigma) = K * e^{-rT}\phi(-d_2) - S_0 * \phi(-d_1)$$

   where:

$$d_1 = \frac{ln(\frac{S_0}{K}) + (r + \frac{\sigma^2}{2}) * T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$\phi() - \textit{Cumulative distribution function of Standard Normal Distribution}$$

b. Rationale for Black-Scholes Model

   The Black-Scholes model is selected due to analytical tractability and widespread acceptance within the financial computations area. It equips us with a closed-form formula and provides us with computationally efficient solutions for European options.

a. Delta Formulas

delta of a European call option (Δ C):

$$\Delta C = \phi(d_1)$$

And for a put option (Δ P):

$$\Delta P = \phi(d_1) - 1$$

c. Interpretation and Comparison

The term "delta" describes how sensitive an option's price is to even a tiny shift in the value of the underlying asset. It indicates how much an option's price should fluctuate in response to a one-unit change in the underlying asset's price. A call option's positive delta signifies that the option price rises in tandem with the stock price. On the other hand, the put option's delta is negative, indicating that the stock price and option price are correlated.

Vega - Sensitivity to Volatility

a. Vega Computation

Vega quantifies the sensitivity of the option's price to an infinitesimal change in the underlying asset's volatility. For both types of options (put and call), Vega is:

Vega=S0 √TΦ(d1)

Where $\phi()$ is a PDF of the standard normal distribution. It is possible to calculate Vega for the initial volatility (σ=20%) and the increased volatility (σ=25%), and compute the change in option prices.

b. Impact of Volatility Change

Typically, the prices of both call and put options increase in response to heightened volatility due to the rising probability of the option expiration ITM. The impact may vary between calls and puts due to additional factors like the time remaining until expiration and the strike price.

Based on the BS model, the results can be found below:

Pricing ATM European Options:

Call Option Price is equal to $4.61

Put Option Price is equal to $3.37

Delta for European Options at T0:

Delta of Call Option (Δ C) is equal to 0.569 (Positive)

Delta of Put Option (Δ P) is equal to -0.431 (Negative)

Vega - Sensitivity to Volatility:

Same Vega for both types of options: 19.64

Change in Call Price with 5% Increase in Volatility: $0.98 increase

Change in Put Price with 5% Increase in Volatility: $0.98 increase

2. Initial options' parameters:

S0=100

K=100

r=5%

sigma=20%

T=3 months

N=100000

The Jupyter notebook includes Python code to solve the problem

Option Type	Exce	Method	Price	delta	vega
Call	European	MC	4.66	-0.58	0.00
Put	European	MC	3.42	0.00	0.01

3.

a. In this section we are going to check for put-call parity and see if its holds.

Put-call parity is $S_0 + P = Ke^{-rt} + C$.

So, put-call parity check for BS model

100 + 3.37 = 100 * $e^{-0.05*0.25}$ + 4.61

103.37 = 98.76 + 4.61 = 103.37

=> Put-call parity holds

Put-call parity check for monte-carlo simulations

100 + 3.42 = 100 * $e^{-0.05*0.25}$ + 4.66

103.42 = 98.76 + 4.66 = 103.42

=> Put-call parity holds for Monte-carlo simulations

b. Yes, the value obtained in both the cases converse to a value. Both monte-carlo and BS models are predicting or conversing to the same value. In monte-carlo simulations we use the same model implementation and choose the size to n (simulations number) to be very high, which is responsible for the value to converse.

Step 2

4. The answer (Python code) to this problem is available in our Jupyter notebook. These are the answers that we got to the given problem:

Option Type	Exce	Method	Price	delta	vega
Call	American	MC	4.73	0.56	1.65

5. The Jupyter notebook includes Python code to solve the problem. The answers to the given problem are:

Option Type	Exce	Method	Price	delta	vega
Put	American	MC	3.27	-0.44	54.59

6.

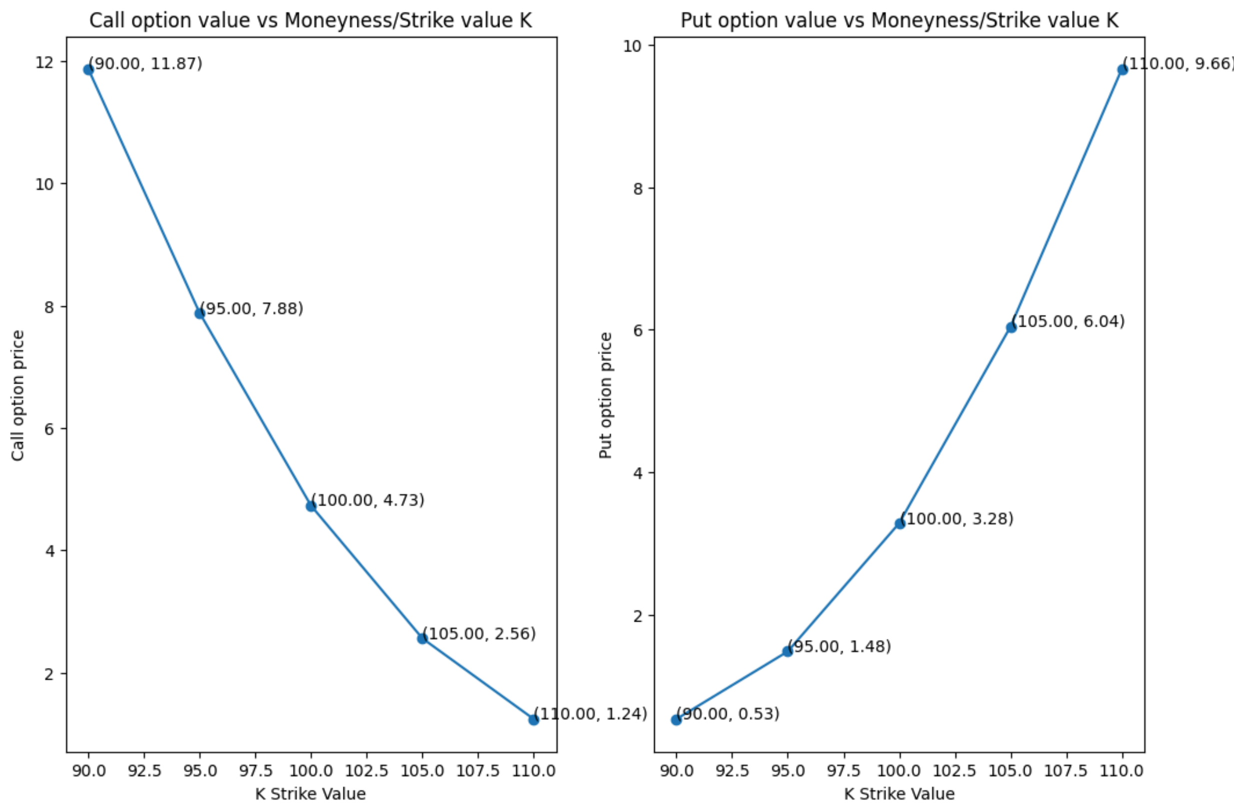
a.

Strike Price (K)	In or Out of Money	moneyness = $\frac{Strike\ Price\ K}{Stock\ Price\ S}\%$	Call Price (using MC Model)
90	Deep ITM	90%	11.87
95	ITM	95%	7.88
100	ATM	100%	4.73
105	OTM	105%	2.56
110	Deep OTM	110%	1.24

Strike Price (K)	In or Out of Money	moneyness = $\frac{Strike\ Price\ K}{Stock\ Price\ S}\%$	Put Price (using MC Model)
90	Deep OTM	90%	0.53
95	OTM	95%	1.48
100	ATM	100%	3.28

105	ITM	105%	6.18
110	Deep ITM	110%	9.66

b.



Step 3

7. Our Jupyter notebook has the python code for the given problem.

a.

The call option price is 1.19

The put option price is 1.53

b.

The delta of portfolio 1 (buy call and put) is -0.027

c.

The delta of portfolio 2 (buy call and sell put) is 0.46

8. Our Jupyter notebook has the python code for this problem.

We will use the equation given below for stimulating the underlying stock price paths:

S_t = S_0 * exp((r - sigma^2 / 2) * t + sigma * sqrt(t) * Z)

Here, S_t denotes stock price at time t

S_0 is the initial stock price

r denotes the rate of interest

t denotes the time step

Z denotes the standard normal random variable

sigma means volatility

If the option is not knocked out, the payoff is calculated by using the following:

If S_T < B, the payoff is max(S_T - K, 0), otherwise it is 0 in a UAO call option.

The payoff is max(K - S_T, 0) if S_T < B, else the payoff will be 0 in a UAO put option.

Here,

B denotes the barrier level

K denotes the strike price

S_T denotes the stock price at maturity

Expected payoff value is calculated by the following equation:

Price = (1 / N) * sum(payoffs)

Here, N denotes the no of stimulated stock price paths.

And **the UAO call option price = 2.65**

9.

a. In this part we are going to calculate the value related to barrier options with Up-and-In (UAI) barrier option. In the case UAI, we are only going the execute the options which have stock price greater than the barrier value i.e., if we are talking about the call options and we have the stock price greater than the barrier we will execute the option or we will use this value stock value at this node to calculate the call option price at initial time.
    i. by using the python code we have calculated the call option is 12.05.
b. Now we are going to calculate the call price for the Vanilla stocks. price = 14.70

c. The relation between the Vanilla, UAI and UAO is -

$$Option\ Price_{Vanilla} = Option\ Price_{UAI} + Option\ Price_{UAO}$$

Option Type	Vanilla option price	UAI Price	UAO Price
Call	14.70	12.05	2.65
Put	9.62	0.0	9.62

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