GROUP WORK PROJECT #1
GROUP NUMBER: 5224

MScFE 620: Derivative Pricing

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Statement of integrity: By typing the names of all group members in the text boxes below, you confirm that the assignment submitted is original work produced by the group (excluding any non-contributing members identified with an "X" above).

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Note: You may be required to provide proof of your outreach to non-contributing members upon request.

N/A

Step 1:

1. Yes, Put-Call Parity holds for the European options for underlying assets. This reason is very simple: we cannot exercise the European option before maturity. The Put-Call parity equation is -

$$C + Ke^{-rt} = P + S_0$$

where C is Call Price

P is Put Price

K is Strike Price

S_0 is Initial Stock Price

t is time to maturity from Initial time

r risk-free interest rate

e exponential function

If we ignore any dividends or any change in the values of risk-free interest rate then we can say that the above equation holds every time for European option prices.

And one more reason being is that if this doesn't hold which means we have an arbitrage condition and which is very less likely.

2. We are going to solve for Call option from Put-Call Parity-

$$C + Ke^{-rt} = P + S_0$$

$$C = P + S_0 - Ke^{-rt}$$

where C is Call Price

P is Put Price

K is Strike Price

S_0 is Initial Stock Price

t is time to maturity from Initial time

r risk-free interest rate

e exponential function

3. We are going to solve for Put price from the Put-Call Parity equation

$$C + Ke^{-rt} = P + S_0$$

$$P = C - S_0 + Ke^{-rt}$$

where C is Call Price

P is Put Price

K is Strike Price

S_0 is Initial Stock Price

t is time to maturity from Initial time

r risk-free interest rate

e exponential function

4.

Put-call parity generally does not hold for American options because we can exercise the American options before the maturity of option, which brings all the difference between the call and put values of the American option. This choice of exercising at any time before maturity brings inequality in put-call parity.

Put-call parity can sometimes hold for American options if we don't exercise them before maturity.

5.

a) When determining the number of stages in a binomial tree, accuracy and computational complexity are the two main factors to consider. Our results are more accurate the more steps we select. However, the calculations get more complex the more steps there are. Thus, between 100 and 200 steps would be a good amount.

b) The procedure entails determining the number of steps, building the binomial tree, and then applying the binomial option pricing model to determine option prices at each node. The accuracy required and the available processing resources are taken into consideration while determining the number of stages in the binomial tree. More stages equate to enormous computational resources and more precise estimations. Reducing the number of steps results in decreased accuracy and usage of computer resources. Counting steps is a function of need.

6.

a) Delta is an indicator of an option's sensitivity to shifts in the underlying asset's price. The difference between a European ATM call and put will be around 0.5 at time zero. This is due to the fact that they both have about 50% of the money. As a result, the delta for the two alternatives is about equal.
b) The sign is the primary distinction between a call delta and a put delta. The delta of a call option is positive, while that of a put option is negative. This implies that when the value of the underlying asset rises, so does the call option's price. This implies that when the value of the underlying asset rises, the put option's price falls. Therefore, a larger probability of the call option completing in the money is indicated by a positive delta, while a higher probability of the put option finishing in the money is indicated by a negative delta.

7.

a) The sensitivity of the call and put option prices to a five percent rise in volatility, or a change in volatility from twenty to twenty-five percent, can be calculated using an options pricing model. We may apply one such options pricing model to the Black-Scholes model. Our ability to compute the change in option pricing with greater volatility will be aided by the model. The prices of call and put options will typically rise in tandem with an increase in volatility.
b) The call and put option prices will react differently to a five percent rise in volatility. Although the rate of increase in call and put option prices may differ, they will both rise. Generally speaking, as volatility rises, the price of the call option will rise more since increased volatility suggests that the price of the underlying asset may rise. Increased volatility may potentially be advantageous to the put option, but it is unlikely to have a significant influence as the put option gains from the underlying asset's decline in value.

8.

The pricing procedure for American options that use a binomial tree is comparable to that of European options. The main distinction is that we must choose at each node if it would be better to execute the choice at an earlier time or to keep onto it.

For an American call option:

$$C(i,j) \ = \ max([S_0 * (u * d^{i-j}) \ - \ K], \frac{q_u * C(i+1,j+1) + q_d * C(i+1,j)}{e^{r\Delta t}})$$

For an American put option:

$$P(i,j) \ = \ max([K \ - \ S_0 * (u * d^{i-j})], \frac{q_u * P(i+1,j+1) + q_d * P(i+1,j)}{e^{r\Delta t}})$$

Where:

$$q_u \ is\ the\ risk - neutral\ probability\ of\ an\ upward\ move.$$

$$q_d = 1 - q_u \ is\ the\ risk - neutral\ probability\ of\ a\ downward\ move.$$

Though the number of steps in the tree might vary, a bigger number in the tree usually offers a more accurate estimate. Using one hundred or two hundred stages is practically achievable. More steps have been selected in order to more accurately depict the potential early exercise feature of American options.

9.

American options' deltas are computed in a manner akin to those of European options. It is the difference between the price of the option and the minor price movement of the underlying asset:

For a call,

$$\Delta = \frac{C(i,j+1)-C(i,j)}{S*u-S}$$

For a put,

$$\Delta = \frac{P(i,j+1)-P(i,j)}{S*u-S}$$

*Code answer in jupyter notebook

For American call options, the delta will be positive, meaning that the price of the call option will rise in tandem with the stock price. On the other hand, American put options have a negative delta, which means that the price of the put option will fall as the stock price rises. Delta can be understood as the equivalent stake in the underlying stock required to hedge the option position, or as the hedge ratio.

10.

a. The price of American call and put options will climb if volatility increases from 20% to 25%. A binomial tree can be used to pinpoint the exact change in pricing with the additional volatility. American options, especially puts, could be more volatile due to the early exercise feature.
b. Elevated volatility implies more potential swings in the stock price, boosting the worth of both call and put options. Because American options might exercise early, especially when dividends are included, the effect might be larger for puts. Puts with dividends are more likely to execute early when there is increased volatility since it raises the possibility that the option will be deep in the money.

11.

The answer for Q1 is “Yes”, which states that the put-call parity holds for European Options.

For example, for values S = 100, K = 100, r = 0.05, σ = 20% , T = 3 months, and steps taken in binomial model = 100, we calculated call and put option price -

C =4.61 and P = 3.36

and if we calculate put-call parity as

C-P = S- Ke^(-rt)

4.61 - 3.36 = 100 - 100*$e^{-0.05*3/12}$

1.25 = 1.25 => put-call parity holds for european options.

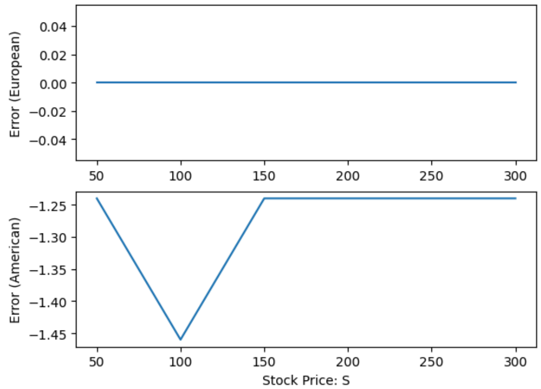
12.

Answer for the question is “No” which means put-call parity does not hold for American option prices.

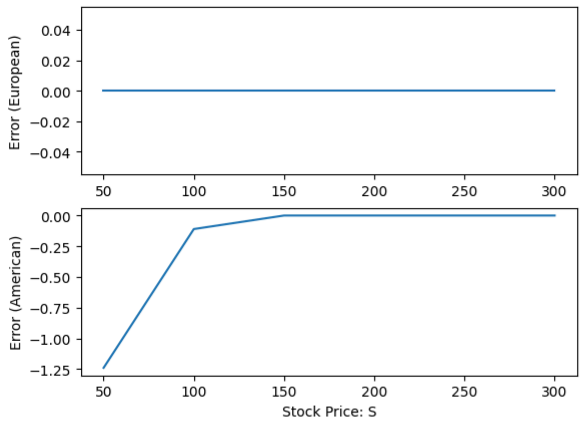
Reason as stated in the answer for question 2, we can exercise the american options before expiration date which introduce maximum payoff function in the equation which brings inequality in the put-call parity equations

Graphs for question 11 and 12- (also useful for question 13 and 14)

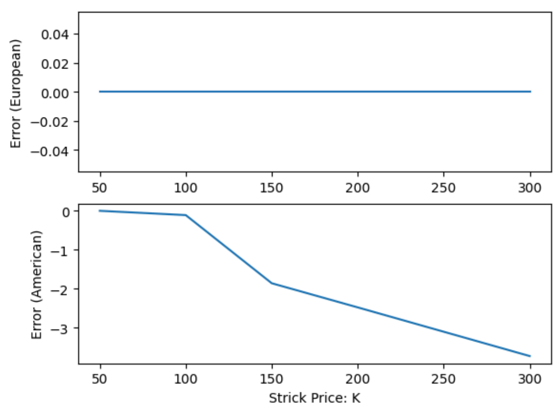
European and American Put-Call Parity Error using Trinomial Model Vs Stock Price S

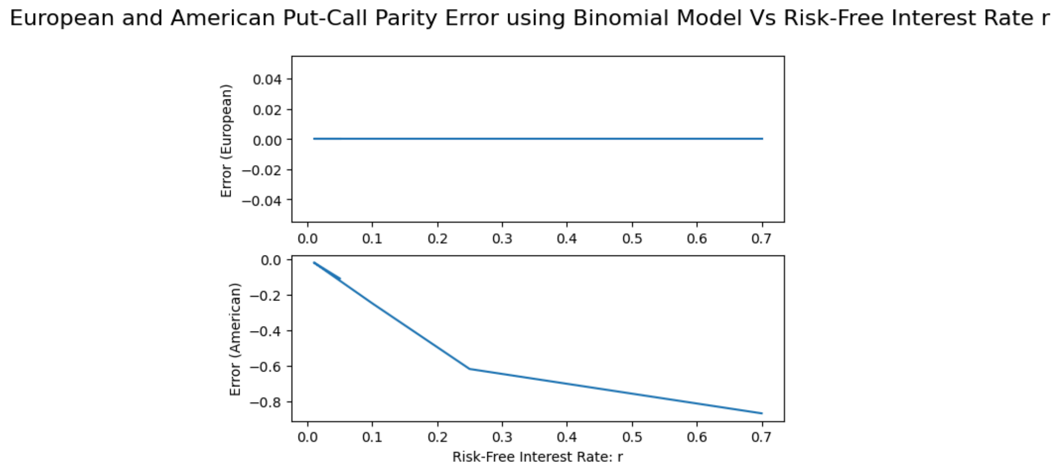


European and American Put-Call Parity Error using Binomial Model Vs Stock Price S



European and American Put-Call Parity Error using Binomial Model Vs Strick Price K





13.

The European Call option will always be less than or equal to the American call option, and they will only equal if we don't exercise the American option before maturity. The reason for European call option being less than because in reality we have dividends, interest rate, etc which affect the price for option. And the power to exercise American options before maturity gives flexibility to option owners which increase the price.

14.

Same as call options before, the American put option is greater than or equal to the European put option. The reason being the same as before, the flexibility provided by American put options to exercise them before maturity, increases their market value.

Step 2

15. European Call Option

a. Trinomial Tree Pricing for Calls

Stock Price (S) = 100

Time to maturity = 3 months (3/12)

Risk-free interest rate (r) = 5% = 0.05

Volatility σ = 20% = 0.2

Model = Trinomial Model

Strike Price (K)	In or Out of Money	moneyness = $\frac{Strike\ Price\ K}{Stock\ Price\ S}\%$	Call Price (using Trinomial Model)
90	Deep ITM	90%	11.67
95	ITM	95%	7.72
100	ATM	100%	4.61
105	OTM	105%	2.48
110	Deep OTM	110%	1.19

b. As we can see in table from the answer (a) that the call price tends to drop as we move from in the money to deep out of the money. The reason for this behavior is that if we are out of the money we have less probability of making money from the option since the strike price is more than the underlying stock price, which makes S - K < 0 .

16. European Put Option

a. Trinomial Tree Pricing for Calls

Stock Price (S) = 100

Time to maturity = 3 months (3/12)

Risk-free interest rate (r) = 5% = 0.05

Volatility σ = 20% = 0.2

Model = Trinomial Model

Strike Price (K)	In or Out of Money	moneyness = $\frac{Strike\ Price\ K}{Stock\ Price\ S}\%$	Put Price (using Trinomial Model)
90	Deep OTM	90%	0.55
95	OTM	95%	1.54
100	ATM	100%	3.37
105	ITM	105%	6.18

110	Deep ITM	110%	9.83

*Code answer in jupyter notebook

b. As we can see in table from the answer (a) that the put price tends to increase as we move from out the money to deep in the money. The reason for this behavior is that if we are out of the money we have less probability of making money from the option since the strike price is less than the underlying stock price, which makes K - S < 0 .

17. American Call Option

a. Trinomial Tree Pricing for Calls

Stock Price (S) = 100

Time to maturity = 3 months (3/12)

Risk-free interest rate (r) = 5% = 0.05

Volatility σ = 20% = 0.2

Model = Trinomial Model

The up probability, pu=0.333, middle probability, pm=0.333 , down probability, pd=0.333

Strike Price (K)	In or Out of Money	moneyness = $\frac{Strike\ Price\ K}{Stock\ Price\ S}\%$	Call Price (using Trinomial Model)
90	Deep ITM	90%	10.50
95	ITM	95%	6.69
100	ATM	100%	3.84
105	OTM	105%	1.97
110	Deep OTM	110%	0.90

b)As seen from the above table from part one

18.American Put Option

a. Trinomial Tree Pricing for Calls

Stock Price (S) = 100

Time to maturity = 3 months (3/12)

Risk-free interest rate (r) = 5% = 0.05

Volatility σ = 20% = 0.2

Model = Trinomial Model

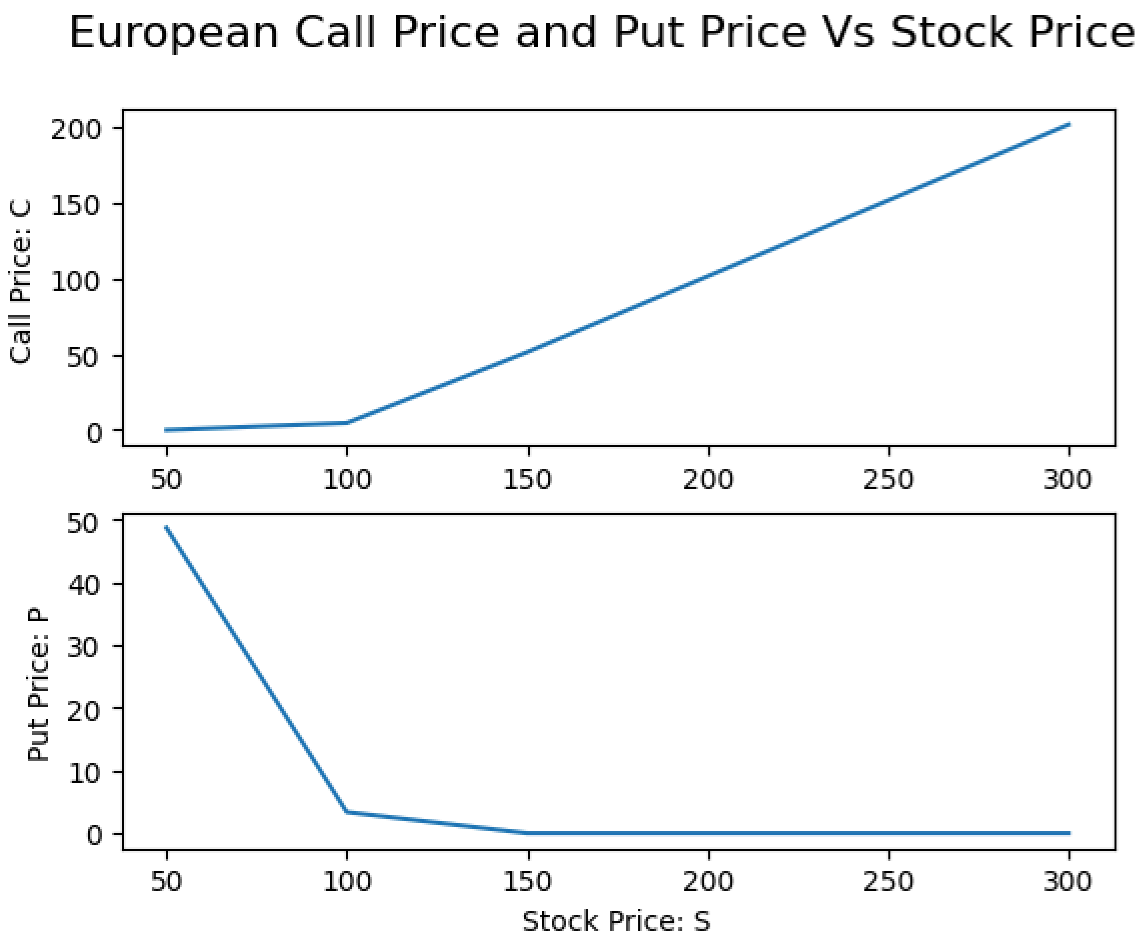
using above values calculating probabilities

pu= 0.333, pm=0.333, pd=0.333

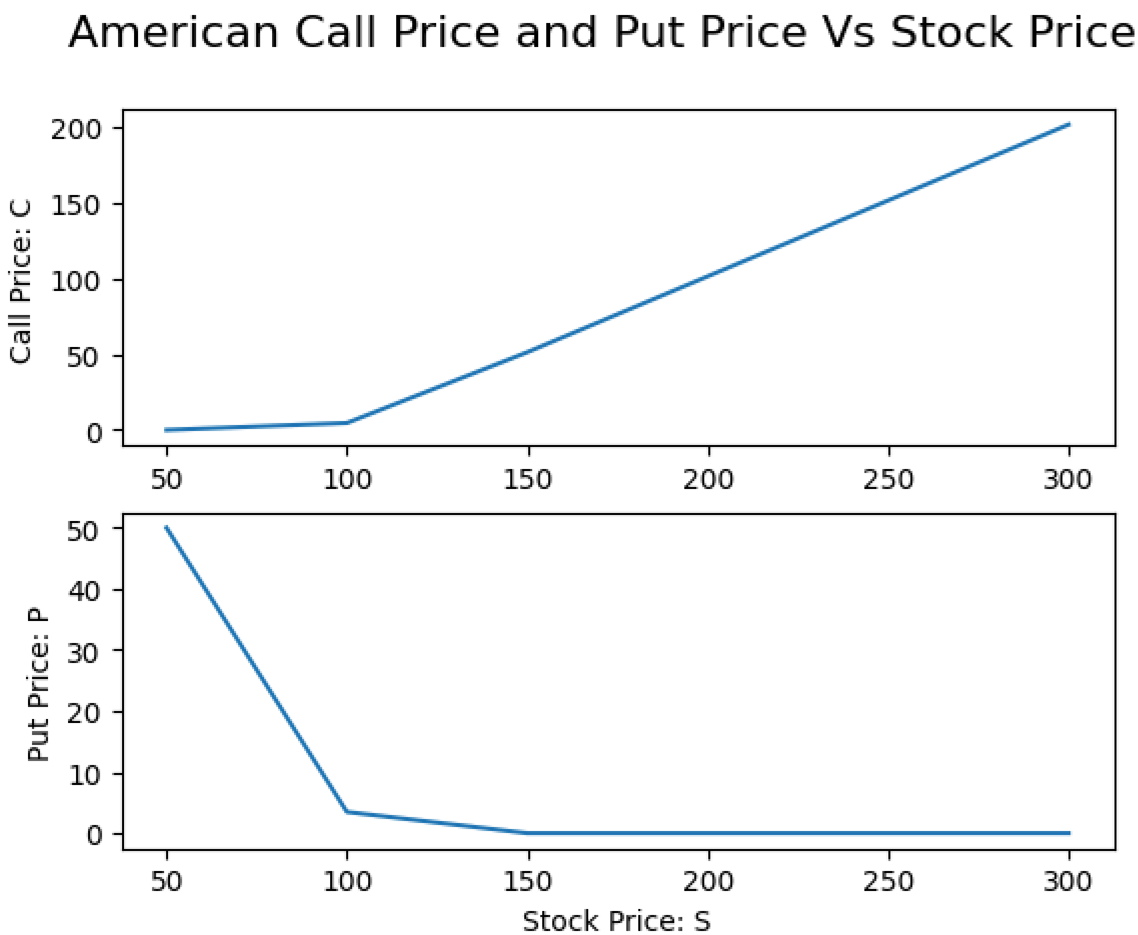
Strike Price (K)	In or Out of Money	moneyness = $\frac{Strike\ Price\ K}{Stock\ Price\ S}\%$	Put Price (using Trinomial Model)
90	Deep OTM	90%	0.74
95	OTM	95%	1.94
100	ATM	100%	4.06
105	ITM	105%	7.14
110	Deep ITM	110%	11.02

b) With an increase in moneyness comes an increase in put option prices. This makes perfect sense because a put option's likelihood of being exercised early increases with its market value. As a result, the put option's price rises.

19.


European Call Price and Put Price Vs Stock Price
Call Price: C
Put Price: P
Stock Price: S
0
50
100
150
200
10
20
30
40
250
300

20.


American Call Price and Put Price Vs Stock Price
Call Price: C
200
150
100
50
0
50
100
150
200
250
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Put Price: P
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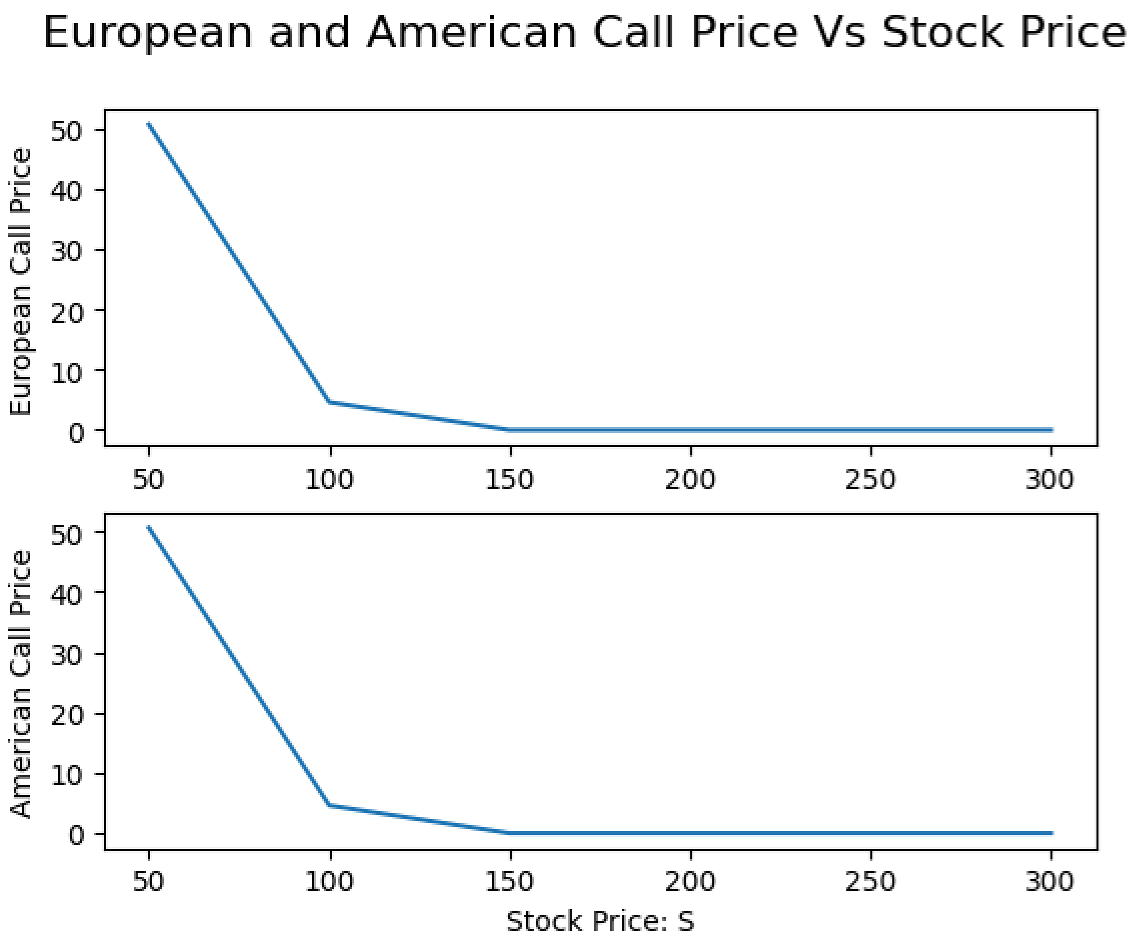
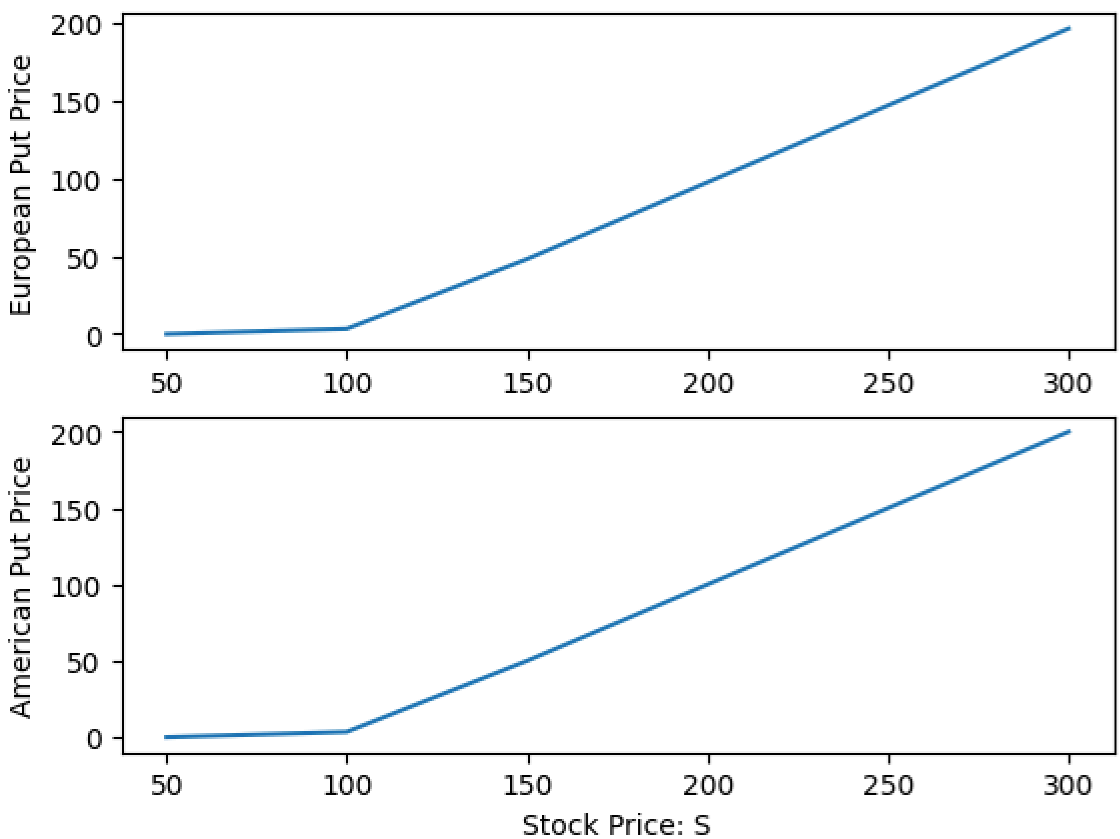
200
250
300
Stock Price: S

21.


European and American Call Price Vs Stock Price
European Call Price
American Call Price
50
40
30
20
10
0
50
100
150
200
250
300
Stock Price: S

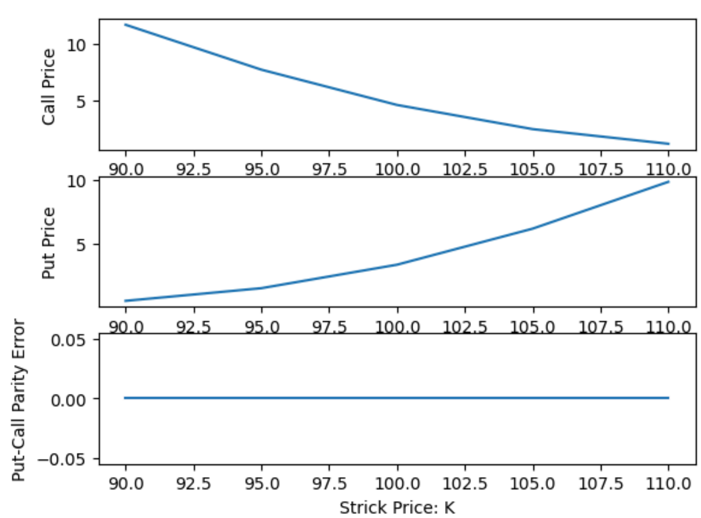
22.

European and American Put Price Vs Stock Price



23.

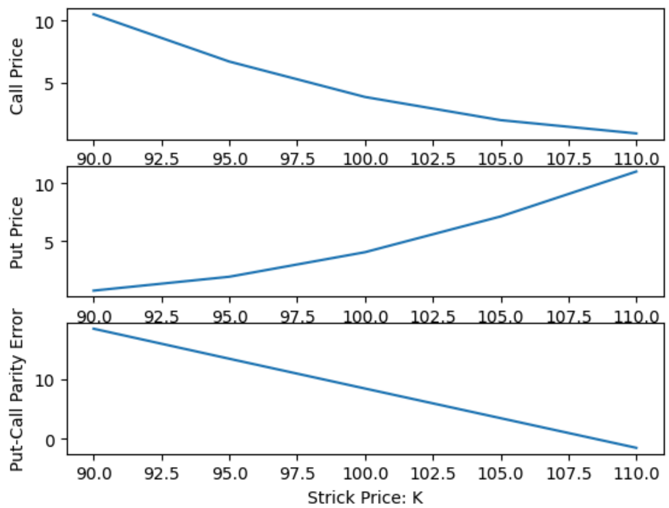
European Call Price, Put Price using Trinomial Model and Put-Call Parity Error Vs Strick Price



As we can see in the above graph, the put-call parity error is zero for European options.

24.

American Call Price, Put Price using Trinomial Model and Put-Call Parity Error Vs Strick Price



As we can see in the above graph, the put-call parity error is **not** always zero for European options.

Step 3:

25.

a) This is the given data:
S0=180

T=6 months

sigma=25%

r =2%

K = 182

The following steps can be used to price European put option with the data that is given by using a 3-step binomial tree:

- A binomial tree with 3 steps should be constructed.
- The risk neutral probability of up and down moves should be calculated.

- The option value at each node should be calculated
- The option value should be discounted to the present day

b) i) One of the example path may be Up Down Up

For this, delta hedging process is

Step	Cash account	Delta	Shares to buy/sell
0	-182	-0.50	91 shares
1	91	-0.625	29 shares
2	120	-0.50	91 shares

ii)

Step	Cash account	Change in cash account
0	-182	-182
1	91	273
2	120	29

26. We have the following data: S0=180, r =2%, sigma=25%, T=6 months, K = 182:

a. The code for this answer is in the jupyter notebook
b. The code for this answer is in the jupyter notebook
c. For American options, the delta hedging is more intricate when compared to the European options, this has a reason which is that there are two choices: exercising the option or continuing the hedge, these two options apply for every node in the tree. Early exercising of American options might change the delta or hedge ratio at each step.

   The price movements of underlying assets pose a risk which is mitigated by delta hedging. This process is completely iterative and the process also adjusts itself at every step depending upon the underlying asset's new price.

   The American option's delta hedging strategy should take this decision into account because of the early exercise potential. Unlike European options which never allow for early exercise of the option. American options are more dynamic.

In real life, while delta hedging an American option, there might be more frequent modifications which are needed when compared to the number of modifications needed while hedging an European option, this makes the strategy more expensive because of the transaction costs.

27. The following is the data which has been given:

S0=180

T=6 months

K = 182

r =2%

sigma=25%

Asian ATM Put option can be priced by using a trinomial tree in the following way:

Setup: For finding the average, we need an auxiliary tree which will track the cumulative stock price as well as we will need a single trinomial tree for calculating the stock price.

Leaf Nodes: The value of the Asian option at each node at the final step is determined by the average of the stock prices up until that point.

Backward Induction: A method which is similar to the American option approach can be used to calculate the anticipated value of option at every node by proceeding through the tree but in backward motion. This will be more difficult for Asian options because the average stock price until that point has to be accounted for too.

a. The code for this answer is in the jupyter notebook
b. The code for this answer is in the jupyter notebook
c. Average price options or Asian options have a separate payoff structure when compared to American or European options because the American and European options are based on the underlying asset's average price over a time frame which is predetermined whereas the Asian option is based on price at maturity. Because of this, Asian options are cheaper than American and European options.

   The delta hedging methods used in Asian options is more complex than the one used in American options and European options. This is because Asian options are less susceptible to fluctuations in price in the short term because they have an averaging feature which considerably reduces the impact of underlying asset's huge swings in prices.

Comment on Asian ATM Put Option vs. American Put Option:

1. **Price Sensitivity:** Because Asian options have an averaging element, they are usually less sensitive to abrupt price swings in the underlying asset than American options. Because of this, the delta of Asian options—the rate at which the option price fluctuates in reaction to changes in the price of the underlying asset—may be more stable.
2. **Cost:** Asian alternatives are usually less expensive than American options due to their averaging method. This is because the volatility of the option's inherent value is reduced by the averaging.
3. **Hedging Frequency:** Delta hedging for Asian options may require fewer modifications due to their smoother payment profile than American options. This could lead to a decrease in transaction fees.

*This answer's code is provided in the jupyter notebook

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