## Twenty-fourth International Olympiad,1983

1. Find all functions $f$ defined on the set of positive real numbers which take positive real values and satisfy the conditions:
(*i*) $f\left(xf(y)\right) = yf(x)$ for all positive $x, y$;
(*ii*) $f(x) \to 0$ as $x \to \infty$.

2. let $A$ be one of the two distinct points of intersection of two unequal coplanar tangents to the circles $C_1$ and $C_2$ with centers $O_1$ and $O_2$, respectively. One of the common tangents to the circles touches $C_1$ at $P_1$ and $C_2$ at $P_2$, while the other touches $C_1$ at $Q_1$ and $C_2$ at $Q_2$. Let $M_1$ be the midpoint of $P_1Q_1$, $M_2$ be the midpoint of $P_2Q_2$ prove that $\angle O_1AO_2 = \angle M_1AM_2$.

3. Let $a, b$ and $c$ be positive integers, no two of which have a common divisor grater than 1. Show that $2abc-ab-bc-ca$ is the largest integer which cannot be expressed in the form $xbc + yca + zab$, where $x, y$ and $z$ are non-negative integers.

4. Let $ABC$ be an equilateral triangle and $\epsilon$ the set of all points contained in the three segments $AB$, $BC$, and $CA$ (including $A$, $B$, and $C$). Determine whether for every partition of $\epsilon$ into two disjoint subsets, at least one of the two subsets that contains the vertices of a right-angled triangle. Justify your answer.

5. Is it possible to choose 1983 distinct positive integers, all less than or equal to $10^5$, no three of which are consecutive terms of an arithmetic progression ? justify your answer.

6. Let $a, b$ and $c$ be the lengths of the sides of a triangle. Prove that.

$$a^2b(a-b) + b^2c(b-c) + c^2a(c-a) \geq 0$$

Determine when quality occurs.