Twenty-sixth International Olympiad,1985

1. A circle has center on the side AB of the cyclic quadrilateral $ABCD$. The other three sides are tangent to the circle. Prove that $AD + BC = AB$.

2. Let n and k be given relatively prime natural numbers $k < n$. Each number in the set $M = 1, 2, ...n - 1$ is colored either blue or white. It is given that
(*i*) for each $i\epsilon M$, both i and $n - i$ have the same color;
(*ii*) for each $i\epsilon M$, $i \neq k$, both i and $\left|i - k\right|$ have the same color. Prove that all numbers in M must have the same color.

3. For any polynomial $P(x) = a_0 + a_1 x + ... + a_k x^k$ with integer coefficients, the number of coefficients which are odd is denoted by $w(P)$. For $i = 0, 1, ...$,let $Q_i(x) = (1 + x)^i$. Prove that if $i_1 i_2, ..., i_n$ are integers such that $0 \leq i_1 < i_2 < ...... < i_n$, then

$$w(Q_{i1} + Q_{i2}, +.... + Q_{in}) \geq w(Q_{i1})$$

4. Given a set M of 1985 distinct positive integers, none of which has a prime divisor grater than 26. Prove that M contains at least one subset of four distinct elements whose product is the fourth power of an integer.

5. A circle with center O passes through the vertices A and C of triangle ABC and intersects the segments AB and BC again at distinct points K and N respectively. The circumscribed circle of the triangle ABC and EBN intersect at exactly two distinct points B and M. Prove that angle OMB is a right angle.

6. For every real number x_1, construct the sequence $x_1, x_2, ...$by setting

$$x_{n+1} = x_n \left(x_n + \frac{1}{4}\right)$$

for each $n \geq 1$ Prove that there exists exactly one value of x_1 for which

$$0 < x_n < x_{n+1} < 1$$

for every n.