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Q1

Given: $x_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $x_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $y = 1$

$x_3 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$ $x_4 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ $y = -1$

$w = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ bias $= 0$ $\alpha = 0.5$

epoch 1

$n(1) = w_1^T x_1 + b = \begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = 0$

$f(n(1)) = 1$

$e(1) = y - f(n(1)) = 1 - 1 = 0$

$w_2 = w_1 + (0.5)(e(1)\ x(1))$

$= \begin{bmatrix} 0 \\ 0 \end{bmatrix} + 0.5(0) \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$b_2 = b_1 + 0.5 e_1 = 0$

$n(2) = w(2)^T x(2) + b(2)$

$\begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 0 = 0$

$f((n(2)) = 1$

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$e(2) = y - f(n(2)) = 1 - 1 = 0$

$w(3) = w(2) + 0.5\,(x(2)\,e^T(2))$

$= \begin{bmatrix} 0 \\ 0 \end{bmatrix} + 0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$b(3) = 0$

$n(3) = w(3)^T x(3) + b(3)$

$\begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix} + 0 = 0$

$f(n(3)) = 1$

$e(3) = y - f(n(3)) = -1 - 1 = 0$

$w(4) = w(3) + 0.5\,x(3)\,e^T(3)$

$\begin{bmatrix} 0 \\ 0 \end{bmatrix} + 0.5(-2)\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

$= \begin{bmatrix} 0 \\ 0 \end{bmatrix} + (-1)\begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$

$b(4) = b(3) + 0.5\,(e(3))$

$0 + (0.5)(-2) = -1$

$n(4) = w(4)^T x(4) + b(4)$

$\begin{bmatrix} -2 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix} + (-1) = -4 - 1$

$= \underline{\underline{-5}}$

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Q1. $f(n(4)) = -1$

$e(4) = -1 - (-1) = 0$

$\therefore \quad w = \begin{bmatrix} -2 \\ 0 \end{bmatrix} \qquad b = -1$

<u>Epoch 2</u>

$w(1) = \begin{bmatrix} -2 \\ 0 \end{bmatrix} \qquad b = -1$

$n(1) = w(1)^T x(1) + b(1)$

$\begin{bmatrix} -2 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} + (-1) = -1$

$f(n(1)) = -1$

$e(1) = +1 - (-1) = 2$

$w(2) = w(1) + 0.5\, x(1) e(1)$

$\begin{bmatrix} -2 \\ 0 \end{bmatrix} + 0.5(2) \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$

$b(2) = b(1) + 0.5 e(1)$

$= -1 + 0.5(2) = -1 + 1 = 0$

$n(2) = w(2)^T x(2) + b(2)$

$\begin{bmatrix} -2 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 9 \end{bmatrix} + 0 = 0$

$f(n(2)) = 1$
$e(2) = 1 - 1 = 0$

$w(3) = w(2) + 0.5\, e(2)\, (x(2))$
$\begin{bmatrix} -2 \\ 0 \end{bmatrix} + (0.5)(0) \begin{bmatrix} -2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$

$b(3) = b(2) + 0.5\, e(2)$
$= 0$

$n(3) = w(3)^T x(3) + b(3)$
$\begin{bmatrix} -2 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix} + 0 = -4$

$f(n(3)) = -1$
$e(3) = -1 - (-1) = 0$
$\therefore w(4) = w(3)$ & $b(4) = b(3)$

$n(4) = w_4^T x(4) + b(4)$
$\begin{bmatrix} -2 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix} + 0 = -4$

$f(n(4)) = -1$ $\quad e(4) = 0$
$\therefore w = w(4)$ & $b = b(4)$

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for epoch 2 $w = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$ $b = [0]$

Third epoch

$w(1) = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$ $b = [0]$

$n(1) = w_1^T (x_1) + b_1$

$\begin{bmatrix} -2 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} + (0) = [0]$

$f(n(1)) = 1$ $e(1) = 0$

$w(2) = w(1)$ $b(2) = b(1)$

$n(2) = w(2)^T x(2) + b(2)$

$\begin{bmatrix} -2 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 0 = 0$

$f(n(2)) = 1$

$e(n(2)) = 0$

$w(3) = w(2)$ $b(3) = b(2)$

$n(3) = w_3^T x_3 + b(3)$

$\begin{bmatrix} -2 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix} + 0 = -4$

$f(n(3)) = -1 \Rightarrow e(3) = 0$

$f(n(3)) = 1 \quad e(3) = 0$

$w(4) = w(3) \quad b(4) = b(3)$

$n(4) = w^T x(4) + b(4)$

$\begin{bmatrix} -2 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix} + 0 = -4$

$f(n(4)) = 1$ & $e(4) = 0$

$\therefore$ final weights are

$w = \begin{bmatrix} -2 \\ 0 \end{bmatrix} \quad b = 0.$

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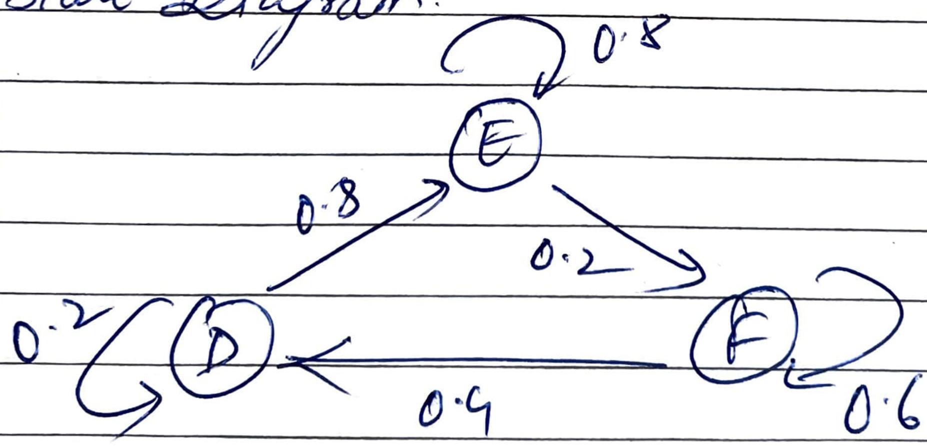
Q2

(a)

$$A = \begin{array}{c|ccc} & D & E & F \\ \hline D & 0.2 & 0.8 & 0.0 \\ E & 0 & 0.8 & 0.2 \\ F & 0.4 & 0.0 & 0.6 \end{array}$$

$$B = \begin{array}{c|ccc} & \alpha & \beta & \gamma \\ \hline D & 0.8 & 0.2 & 0 \\ E & 0.0 & 0.6 & 0.4 \\ F & 0.2 & 0.0 & 0.8 \end{array}$$

State Diagram:



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A

(b) $O = (\alpha, \beta, \gamma, \alpha)$

All possible states with $P(X, O) \neq 0$

⇒ (1) DDEF
(2) DEEF
(3) DEFF
(4) DEFD

(c) $P(O) = ?$

$P(DDEF) = 1 * 0.8 * 0.2 * 0.2 * 0.8 * .4 * .2 * 0.2$
$= 0.0004$

$P(DEEF) = 1 * 0.8 \times 0.2 \times 0.8 \times 0.6 \times 0.8 \times 0.2 \times 0.2$
$= 0.0025$

$P(DEFF) = 1 * 0.8 \times 0.8 \times 0.6 \times 0.2 \times 0.8 \times 0.6 \times 0.2$
$= 0.0074$

$P(DEFD) = 1 * 0.8 \times 0.8 \times 0.6 \times 0.2 \times 0.8 \times 0.4 \times 0.8$
$= \cancel{0.0079} = 0.0197$

$$P(O) = P(DDEF) + P(DEEF) + P(DEFF) + P(DEFD)$$

$$= \underline{\underline{0.03}}$$

(d) The most likely path is DEFD

$$P(DEFD) = 0.8 \times 0.8 \times 0.6 \times 0.2 \times 0.8 \times 0.4 \times 0.8$$

$$= \underline{\underline{0.0197}}$$

$= \cancel{0.0197} = 0.0197$

Question 3

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Q3 Given

(a) $\left\{ x_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad x_2 = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \right\} y = 1$

$\left\{ x_3 = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \quad x_4 = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \right\} y = -1$

(a) $f(x) = \beta_0 + \beta_1 x_1 + \beta_2 x_2$

Subs x_1 & $y = 1$ we get

$1 = \beta_0 + \beta_2$

subs x_2 & $y = 1$ we $\Rightarrow$ $1 = \beta_0 - \beta_1$

subs x_3 & $y = -1$ & x_4 & $y = -1$

we get $\beta_0 + \beta_1 = -1$ & $\beta_0 - \beta_2 = -1$

solving we get $\beta_0 = 0$, $\beta_1 = -1$, $\beta_2 = +1$

decision boundary: $f(x) = x_2 - x_1$

(b) The support vectors are x_1 x_2 x_3 x_4

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Q4 $x_1 = (0,0)$ $x_2 = (0,1)$ $x_3 = (2,0)$ $x_4 = (2,2)$

(a) initialy $x_1, x_4 \to 1$ $x_2, x_3 \to 2$

centroid of cluster 1

$x_c = \frac{0+2}{2} = 1$, $\frac{0+2}{2} = 1$

$C_1 = (1,1)$

centroid of cluster 2 $C_2 = \left(1, \frac{1}{2}\right)$

assigning now :- x_1 to $C_1 = \sqrt{1+1} = \sqrt{2}$

x_1 to $C_2 = \sqrt{0+\frac{1}{2}} = \sqrt{1/2}$

after iteration

$C_2 = (x_1, x_3)$

x_2 to $C_1 = \sqrt{\cancel{2}} = 1$

x_2 to $C_2 = \sqrt{1+1/2}$

$C_1 = (x_2, x_4)$

x_3 to $C_1 = \sqrt{2}$

x_3 to $C_2 = \sqrt{1+1/2}$

$= \sqrt{3/2}$

x_4 to $C_1 = \sqrt{2}$

x_4 to $C_2 = \sqrt{1+3/2}$

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$C_2 = (x_1, x_3)$

$C_1 = (x_2, x_4)$

~~Iteration 2:~~

~~C_1 centroid $= \frac{0+2}{2}, \frac{0+0}{2} = (1, 0)$~~

~~C_2 centroid $= \frac{0+2}{2}, \frac{1+2}{2} = (1, 3/2)$~~

~~Assigning clusters:~~

~~$x_1 = (0,0)$ $C_1 = \sqrt{1+} = 1$, $C_2 = \sqrt{1+\frac{3}{2}}$~~

~~$x_2 = (0,1)$ $C_1 = \sqrt{2}$ $C_2 = \sqrt{1+1/2} = \sqrt{3/2}$~~

~~$x_3 = (2,0)$ $C_1 = \sqrt{1+0} = 1$ $C_2 = \sqrt{1+3/2}$~~

~~$x_4 = (2,2)$ $C_1 = \sqrt{1+2^2} = \sqrt{5}$ $C_2 = \sqrt{1+1/2}$~~

~~$C_1 = (x_1, x_3)$ $C_2 = (x_2, x_4)$~~

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Iteration 3

~~$C_1 = (x_1, x_3)$ $C_2 = (x_2, x_4)$~~

centroid of cluster 1 :- (x_2, x_4)

$$= \left(\left(\frac{2+0}{2}\right), \left(\frac{2+1}{2}\right)\right) = \left(1, \frac{3}{2}\right)$$

cluster 2 $C = (x_1, x_3)$

$$= \left(\left(\frac{0+2}{2}\right), \left(\frac{0+0}{2}\right)\right) = (1, 0)$$

$C_1 = (1, 1.5)$ $C_2 = (1, 0)$

New Assign:

$(1, 1.5)$

for cluster 1 distances:

x_1 (0, 0)	x_2 (0, 1)	x_3 (2, 0)	x_4 (2, 2)
1.8	$\sqrt{1+(0.5)^2}$	$\sqrt{1+(1.5)^2}$	$\sqrt{1+(1/2)^2}$
	$= 1.118$	1.8	$= 1.118$

Cluster 1 = (x_2, x_4)

cluster 2 distance: $(1, 0)$

x_1 (0, 0)	x_2 (0, 1)	x_3 (2, 0)	x_4 (2, 2)
$\sqrt{1}$	$\sqrt{2}$	$\sqrt{1}$	$\sqrt{1+2^2}$

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cluster 2 : (x_1, x_3)

cluster 1 : (x_2, x_4)

Since cluster assignments dont change, the above cluster is final.

(b) Single Linkage

	x_1	x_2	x_3	x_4
x_1	0	1	2	2.833
x_2	1	0	2.24	2
x_3	2	2.24	0	2
x_4	2.83	2	2	0

minimum is $x_1\,x_2$ hence first join

	$x_1 x_2$	x_3	x_4
$x_1 x_2$	0	2	2
x_3	2	0	2
x_4	2	2	0

Since all are same, we take x_3 & x_4

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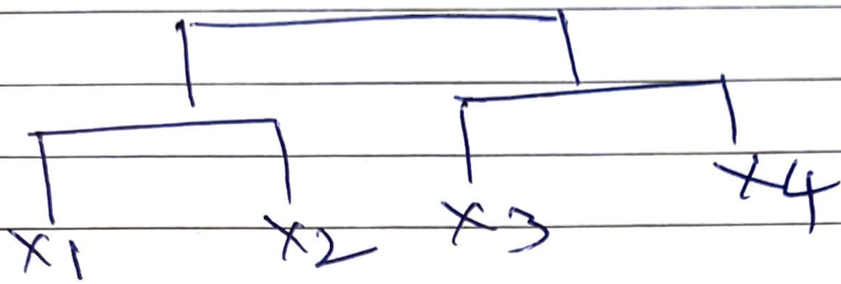
	x_1x_2	x_3x_4
x_1x_2	0	2
x_3x_4	2	0

Hence final dendrogram:



Q5

$K=3$

$GI[R_m] = \sum_{K=1}^{K} \hat{p}_{mk}(1-\hat{p}_{mk})$

$R_m = 4$ regions

$(-\infty, 1.5)$ $(1.5, 2.5)$ $(2.5, 3.5)$ $(3.5, \infty$

if $X_{split} = 1.5$

Gini index $[x < x_{split}] = 0$

$GI\,[x > x_{split}]$

$= \frac{1}{4} \times \frac{3}{4} + \frac{2}{4} \times \frac{2}{4} + \frac{1}{4} \times \frac{3}{4}$

$= \frac{5}{8} = \boxed{0.625}$

if $X_{split} = 2.5$

$GI\,(x < x_{split}) = \frac{2}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{2}{3} + 0$

$= \frac{4}{9}$

$GI\,(x > x_{split}) = \frac{1}{3} \times \frac{2}{3} \times 3$

$= \frac{2}{3}$

$= \boxed{1.11}$

$= \underline{\underline{1.11}}$

if X split $= 3.5$

$GI(x < x_{split}) = \frac{3}{4} + \frac{3}{4} \times \frac{1}{4} + 0 = \frac{3}{8}$

$GI(x > x_{split}) = \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2}$

$= \boxed{0.875}$

Best X split $= 1.5$

2nd level

if $X_S = 2.5$ $\quad GI(x < x_S) = 0$

$GI(x > x_S) = \frac{1}{3} \times \frac{2}{3} \times 3$

$\boxed{= 0.666}$

if $X_S = 3.5$

$GI(x < x_S) = \cancel{0}$ $\quad \frac{1}{2} \times \frac{1}{2} \times 2 = \frac{1}{2}$

$GI(x > x_S) = \frac{1}{2}$

$= \boxed{1}$

Best $X_S = \underline{\underline{2.5}}$

Decision Tree:-