VISVESVARAYA TECHNOLOGICAL UNIVERSITY

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Design and Analysis of Algorithms Assignment
on

"Hypercube: Odd-Even Merge Algorithm"

Submitted by

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INTRODUCTION TO THE ALGORITHM

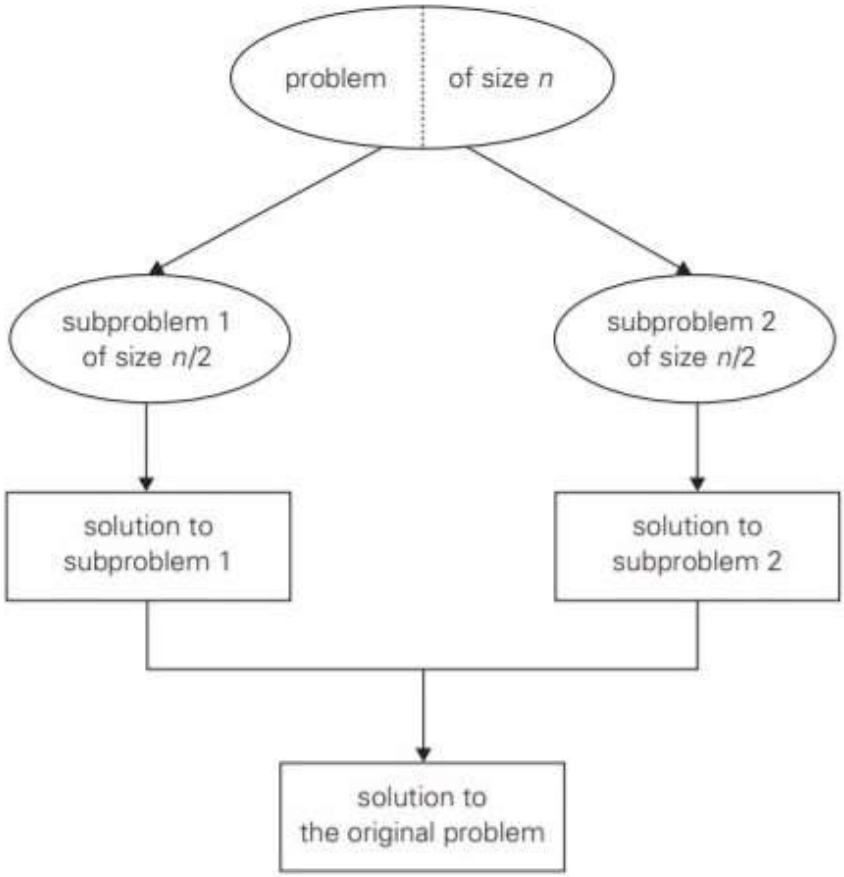
Odd-Even Mergesort:

The odd-even mergesort algorithm was developed by K.E. Batcher. It is based on a merge algorithm that merges two sorted halves of a sequence to a completely sorted sequence.

In contrast to mergesort, this algorithm is not data-dependent, i.e. the same comparisons are performed regardless of the actual data. Therefore, odd-even mergesort can be implemented as a sorting network.

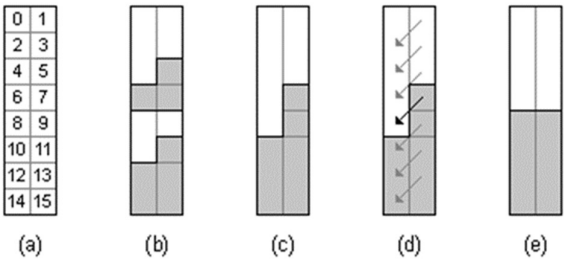
DESIGNING TECHNIQUE

Odd-EvenMergesort is one of the application of Divide and Conquer Technique. Where the problem is divided into sub problems and find their respective solution. Later combine these solutions to get solution of main problem.



Correctnessof the algorithm

The correctness of the merge algorithm is proved using induction and the 0-1-principle.
If $n = 2^1$ the sequence is sorted by the comparison [0 : 1]. So let $n = 2^k$, $k > 1$ and assume the algorithm is correct for all smaller k (induction hypothesis).
Consider the 0-1-sequence $a = a_0, ..., a_{n-1}$ to be arranged in rows of an array with two columns.
The corresponding mapping of the index positions is shown in Figure (a), here for $n = 16$.
Then Figure (b) shows a possible situation with a 0-1-sequence.
Each of its two sorted halves starts with some 0's (white) and ends with some 1's (gray).



In the left column the even subsequence is found, i.e. all a_i with i even, namely a_0, a_2, a_4 etc. ;in the right column the odd subsequence is found, i.e. all a_i with i odd, namely a_1, a_3, a_5 etc.
Just like the original sequence the even as well as the odd subsequence consists of two sorted halves.

By induction hypothesis, the left and the right column are sorted by recursive application of *odd-even merge*(n/2) in step 1 of the algorithm. The right column can have at most two more 1's than the left column (Figure (c)).

After performing the comparisons of step 2 of the algorithm (Figure (d)), in each case the array is sorted (Figure (e)

ALGORITHM

Algorithm: *Odd-Even Merge*(*n*)

INPUT : Sequence $a_0, ..., a_{n-1}$ of length $n>1$ whose two halves $a_0, ..., a_{n/2-1}$ and $a_{n/2}, ..., a_{n-1}$ are sorted (n a power of 2)

OUTPUT : the sorted sequence

1. if $n>2$ then
    1. apply *odd-even merge*(n/2) recursively to the even subsequence $a_0, a_2, ...,a_{n-2}$ and to the odd subsequence $a_1, a_3, , ..., a_{n-1}$
    2. comparison [i : i+1] for all i {1, 3, 5, 7, ..., n-3}
2. else
    1. comparison [0 : 1]

Algorithm : O*dd-Even Mergesort*(*n*)

INPUT : sequence a_0, ..., a_{n-1} (*n* a power of 2)

OUTPUT : the sorted sequence

if *n*>1 then

1. apply odd-even mergesort(n/2) recursively to the two halves a_0 ,...,$a_{n/2-1}$ and $a_{n/2}$, ..., a_{n-1} of the sequence

2. *Odd-Even Merge(n)*

IMPLEMENTATION

An implementation of odd-even mergesort in Java is given in the following. The algorithm is encapsulated in a class *OddEvenMergeSorter*. Its method *sort* passes the array to be sorted to array *a* and calls function *oddEvenMergeSort*.

Function *oddEvenMergeSort* recursively sorts the two halves of the array. Then it merges the two halves with *oddEvenMerge*.

Function *oddEvenMerge* picks every 2*r*-th element starting from position *lo* and *lo*+*r*, respectively, thus forming the even and the odd subsequence. According to the recursion depth *r* is 1, 2, 4, 8, ...

With the statements

Sorter s=new OddEvenMergeSorter();
s.sort(b);

an object of type *OddEvenMergeSorter* is created and its method *sort* is called in order to sort array *b*. The length *n* of the array must be a power of 2.

CODE :

```
import java.util.Arrays;
import java.util.Scanner;

interface Sorter
{
        void sort(int[] a);
}

public class OddEvenMergesort {
        static int[] a ;
```

```
        public static void  main(String[] args)
        {
                Scanner input = new Scanner(System.in);
                System.out.print("Enter Max array size in powers of 2 (i.e 2, 4, 8, 16...) : ");
                int n = input.nextInt();
                a = new int[n];
                System.out.println("Enter the array elements : ");
                for(int i=0 ; i <n ;i++)
                {
                        a[i] = input.nextInt();
                }

                Sorter s = new OddEvenMergeSorter();
                s.sort(a);

                System.out.println("Sorted Array :");
                for(int i = 0;i<n;i++)
                        System.out.print(a[i]+ "  ");

                input.close();
    }
}

class OddEvenMergeSorter implements Sorter
{
   private int[] a;

   public void sort(int[] a)
   {
      this.a=a;
      oddEvenMergeSort(0, a.length);

   }

  /* * sorts a piece of length n of the array  starting at position low   */

   private void oddEvenMergeSort(int low, int n)
   {
      if (n>1)
      {
         int m=n/2;
         oddEvenMergeSort(low, m);
         oddEvenMergeSort(low+m, m);
         oddEvenMerge(low, n, 1);
      }
  }
```

```
/* * low is the starting position and n is the length of the piece to be merged,
   r is the distance of the elements to be compare  */

  private void oddEvenMerge(int low, int n, int r)
  {
     int m=r*2;
     if (m<n)
     {
        oddEvenMerge(low, n, m);       // even subsequence
        oddEvenMerge(low+r, n, m);    // odd subsequence
        for (int i=low+r; i+r<low+n; i+=m)
           compare(i, i+r);
     }
     else
        compare(low, low+r);
  }

  private void compare(int i, int j)
  {
    if (a[i]>a[j])
        exchange(i, j);
  }

  private void exchange(int i, int j)
  {
     int t=a[i];
     a[i]=a[j];
     a[j]=t;
  }
}
```

TIME ANALYSIS

Let T(n) be the number of comparisons performed by odd-even merge(n).

Then we have for n>2

$T(n) = 2 \cdot T(n/2) + n/2-1.$

With T(2) = 1 we have

$T(n) = n/2 \cdot (\log(n)-1) + 1$ element

.'. $T(n) \in O(n \cdot \log(n))$.

Time Analysis :

* Using Back tracking method:

We have,

$$T(n) = 2T(n/2) + \frac{n}{2} - 1 \quad \& \quad T(2) = 1$$

Let $n = 2^k$

$$\Rightarrow T(2^k) = 2T\left(\frac{2^k}{2}\right) + \frac{2^k}{2} - 1$$

$$T(2^k) = 2T\left(2^{k-1}\right) + 2^{k-1} - 1$$

$$= 2\left[2T\left(2^{k-2}\right) + 2^{k-2} - 1\right] + 2^{k-1} - 1$$

$$= 2^2 T\left(2^{k-2}\right) + 2^{k-1} - 2 + 2^{k-1} - 1$$

$$= 2^2 T\left(2^{k-2}\right) + 2 \times 2^{k-1} - (1+2)$$

$$= 2^2\left[2T\left(2^{k-3}\right) + 2^{k-3} - 1\right] + 2\left(2^{k-1}\right) - (1+2^1)$$

$$= 2^3 T\left(2^{k-3}\right) + 2^{k-1} - 2^2 + 2\left(2^{k-1}\right) - (1+2)$$

$$= 2^3 T\left(2^{k-3}\right) + 3\left(2^{k-1}\right) - \left(1 + 2^1 + 2^2\right)$$

$$\vdots$$

for i^{th} value

$$T(2^k) = 2^i T\left(2^{k-i}\right) + i\left(2^{k-1}\right) - \left(1 + 2^1 + 2^2 + \ldots 2^{i-1}\right)$$

$$= 2^i \cdot T\left(2^{k-i}\right) + i\left(2^{k-1}\right) - \left(2^i - 1\right)$$

We have $T(2) = 1$

$$\text{i.e. } i = k-1$$

$$T(2^k) = 2^{k-1} T\left(2^1\right) + (k-1)\left(2^{k-1}\right) - \left(2^{k-1} - 1\right)$$

$$= \left(2^{k-1}\right)(1) + (k-1-1)\left(2^{k-1}\right) + 1$$

$$T(2^k) = \left(2^{k-1}\right)(k-2+1) + 1$$

$$\Rightarrow T(n) = \frac{n}{2}\left(\log_2(n) - 1\right) + 1$$

$$T(n) \in O(n \cdot \log n)$$

* Using Masters Theorem:

We have:

$$T(n) = 2 \cdot T\left(\frac{n}{2}\right) + \frac{n}{2} - 1$$

$a = 2$, $b = 2$ $\quad f(n) = \left(\frac{n}{2} - 1\right)$

$f(n) \in n$

$\therefore d = 1$

$b^d = 2^1 = 2$

$a = b$

from Masters theorem, when $a = b$

$$T(n) \in \Theta\left(n^d \log n\right)$$

$$T(n) \in \Theta(n \log n)$$

APPLICATIONS

- The single-processor algorithm like bubblesort are generally simple but not very efficient.
- Batcher algorithm is a related but more efficient sort and merge algorithm.
- It uses perfect-shuffle operations and compare-exchange operations.
- It is usually more efficient on parallel processors with long range connections.
- Overcall this algorithm is fast than their counterparts proposed so far.

OUTPUT