Gaussian Process

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1 Introduction

Gaussian process is a stochastic process widely used in machine learning. It involves computing a measure of similarity between all possible combinations of points from the training data to predict the function value of an unseen point. Along with an estimate of the function value of the unseen point, it also provides a measure of uncertainty about the prediction. Every prediction made by a Gaussian process is not a single value but a normal distribution of the function value. It is this flexibility that makes Gaussian process one of the most powerful and popular model fitting approaches. In this report, we use Gaussian process to model the kinematic behaviour of a person performing in a 3D reaching task. Two different model fitting approaches, (1) global fitting and (2) local fitting, have been used in this work to analyse which approach provides better representation of the motion trajectory. We also discuss if it is possible to predict muscle co-contraction, dynamic human body movement behaviour using just the kinematic information and Gaussian process.

2 Methods

In this work, the motion behaviour has been modeling using a Gaussian process associated with white noise as shown in eq (1).

$$y = f + \epsilon \tag{1}$$

The likelihood for such a noise model is give by eq(2).

$$p(y|f) = \mathcal{N}(f, \sigma^2\mathbf{I}) \tag{2}$$

Upon integrating over the function variables, we get the marginal likelihood given by eq(3).

$$p(y) = \mathcal{N}(0, \mathbf{K} + \sigma^2\mathbf{I}) \tag{3}$$

Consider the joint training(N) and test(T) marginal likelihood given in eq (4).

$$p(y, y_T) = \mathcal{N}(0, \mathbf{K}_{N+T} + \sigma^2\mathbf{I}) \tag{4}$$

$$K_{\mathrm{N+T}} = \begin{bmatrix} K_{\mathrm{N}} & K_{\mathrm{NT}} \\ K_{\mathrm{TN}} & K_{\mathrm{T}} \end{bmatrix}$$

The condition on training output is given as follows.

$$p(y_T|y) = \mathcal{N}(\mathbf{K}_{TN}[\mathbf{K}_N + \sigma^2\mathbf{I}]^{-1}y, \mathbf{K}_T - \mathbf{K}_{TN}[\mathbf{K}_N + \sigma^2\mathbf{I}]^{-1}\mathbf{K}_{NT} + \sigma^2\mathbf{I} \tag{5}$$

In order to obtain optimal hyper parameters for the chosen kernel, we maximize the log-likelihood of the above equation with respect to the kernel parameters.

The data used for this work is a 3 dimensional movement data of multiple joints on a person performing a arm reaching task in a VR motion capture setup. The dataset consisted of three dimensional movement trajectories of 50 markers with five repeats and 1030 time-steps per trajectory. For simplicity, we choose a single marker of a single trajectory in this work. This results in using five repeats of x,y,z trajectories of a single marker. Only 1/5Th of the overall time steps have been used to fit the model. At a given time-step, data point is randomly chosen for one of the five trails to generate the training data.

We used two types of fitting methods, global and local. Global fitting is a straight forward approach where we model the entire trajectory as a single Gaussian process. This means, its kernel involves calculating similarity between all points from start to end of the trajectory. The second approach is a local fitting method where we model the entire trajectory as a combination of multiple local Gaussian process sliding through the trajectory with a limited window size. This allows each time point in the trajectory to be part of multiple overlapping processes and enhances the representation of local behaviour.

3 Results

Figure 1 shows five repeats of trajectory used for the present analysis in all three directions. The reason for choosing this particular trajectory is its limited inter-trail variability. Since all 5 repeats not much far apart and there are no sudden spikes in movement, this trajectory would be a good candidate to discuss muscle co-contractions. Figure 2 shows the predictions made by the global fitting method in all three directions. It can be observed that this type of fitting already provides a decent prediction of the trajectory.

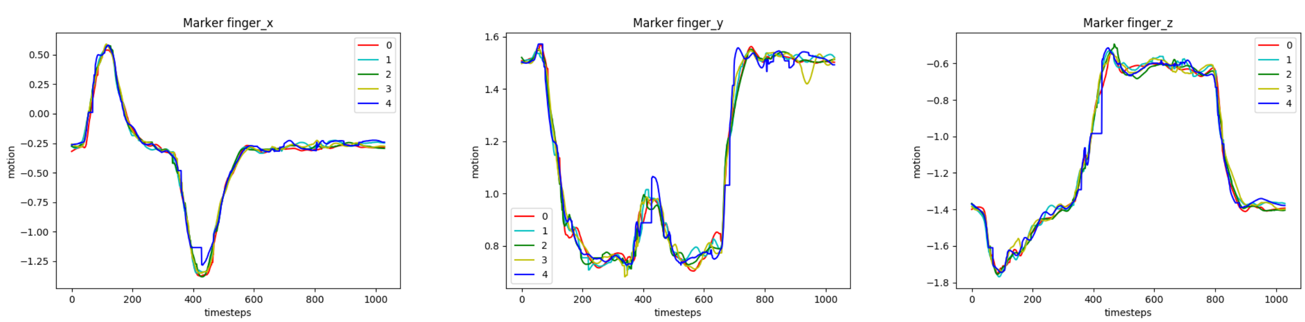

Figure 1: Trajectories of marker "finger" for 5 repeats.

Figure 3 shows the predictions made by the local fitting approach. After testing with several combinations, the window size and the slide were chosen to be 100 and 10 time steps respectively. It is important to note that each time-step can be a part of multiple windows ranging from anywhere between 1-10. To account for this, at a given time-step, an average of prediction made by all the overlapping windows is used to compute the final prediction which is shown in Figure 3. It can be observed that the trajectories are discontinuous and not as smooth as the predictions made by the global fitting case. Since we are performing local fitting of data in each window, this is an expected

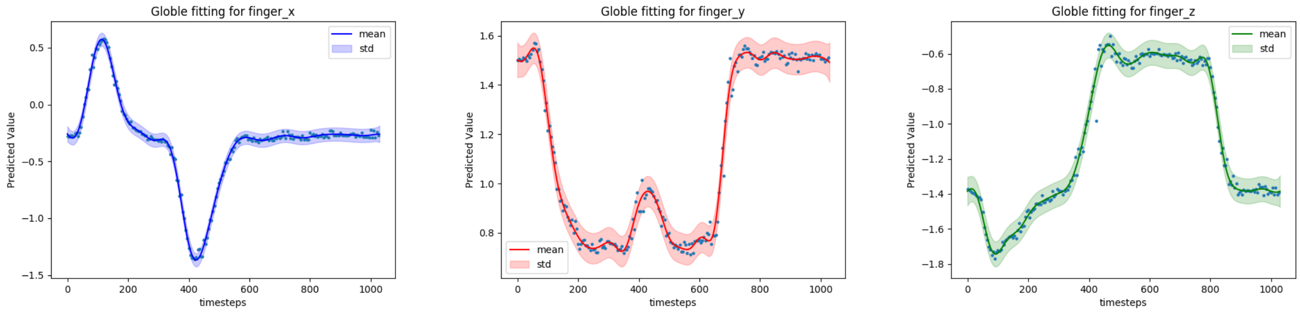

Figure 2: Posteriors in all three directions for global fitting.

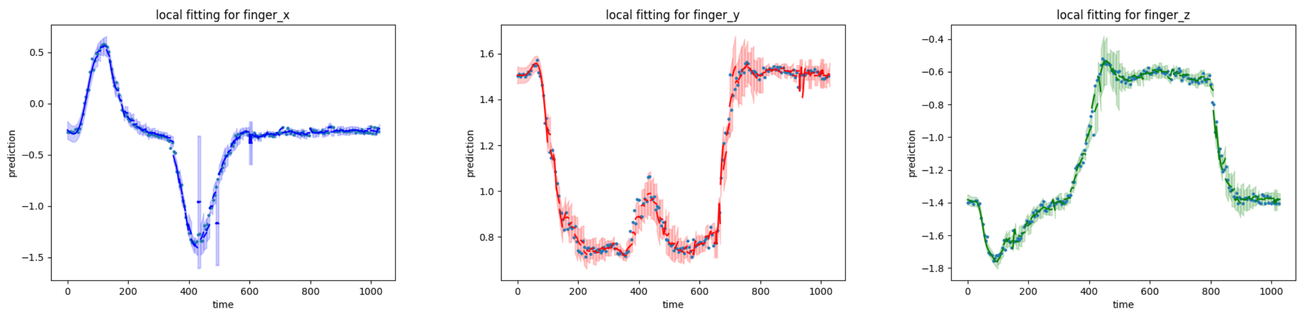

Figure 3: Posteriors in all three directions for local fitting.

outcome and one of the disadvantages of the local fitting method. Further implications of this type of fitting are discussed in the following sections.

Figure 4,5,6 shows the comparison of kernel parameters in global and local fitting methods across all the times-steps. Though not entirely straight forward, it can be sensed that in the time-steps where the value of parameter l is increases, the value of *sigma y* along all three dimensions which suggests a small *negative correlation* between l and *sigma y*.

Upon observing the variation of l among all three dimensions, a common increment in the parameters can be seen towards the end of the trajectory. This kind of common variation across dimension might be correlated to the muscle co-contraction. The purpose of a muscle co-contraction is to increase the joint stiffness which would result in lowered movement variability. Intuitively, when a person is performing a reaching task, the tendency is to decrease the movement speed and increase the joint stiffness as they reach towards the end of the movement to avoid overshoot. An increased value of l and decreased value of sigma y towards the end of the trajectory suggests a prediction with relatively less variance.

4 Conclusion

Based on these results, we can hypothesis that if a similar variation of a given hyper parameter is observed across all 3 dimensions of a particular trajectory resulting in low movement variance then it could be an indication of the presence of a muscle co-contraction. To test this hypothesis, first

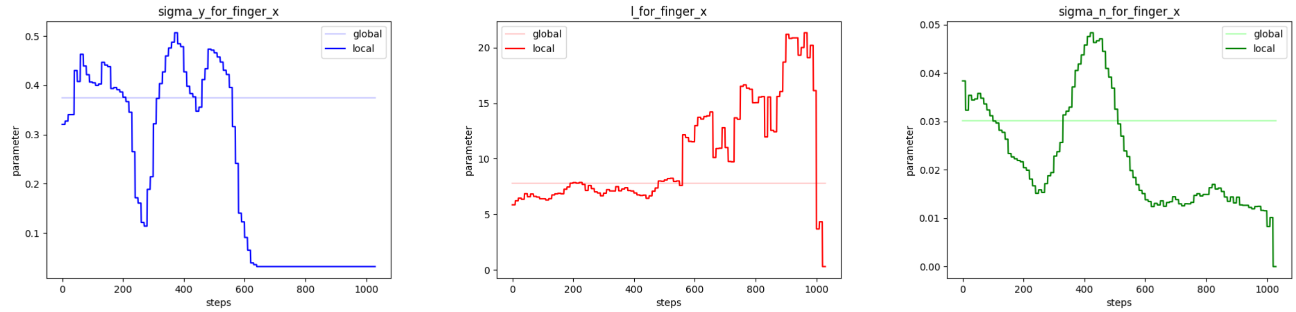

Figure 4: Comparison of kernel hyper parameters of x trajectory in global and local fitting across time-steps.

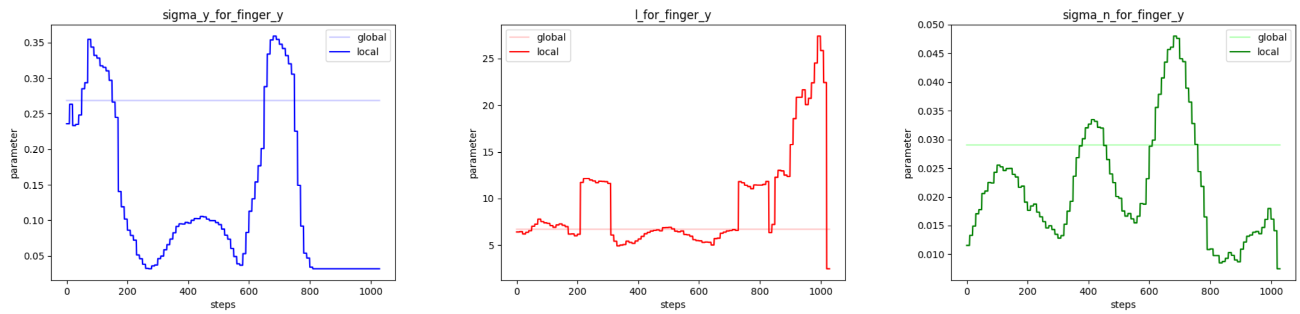

Figure 5: Comparison of kernel hyper parameters of y trajectory in global and local fitting across time-steps.

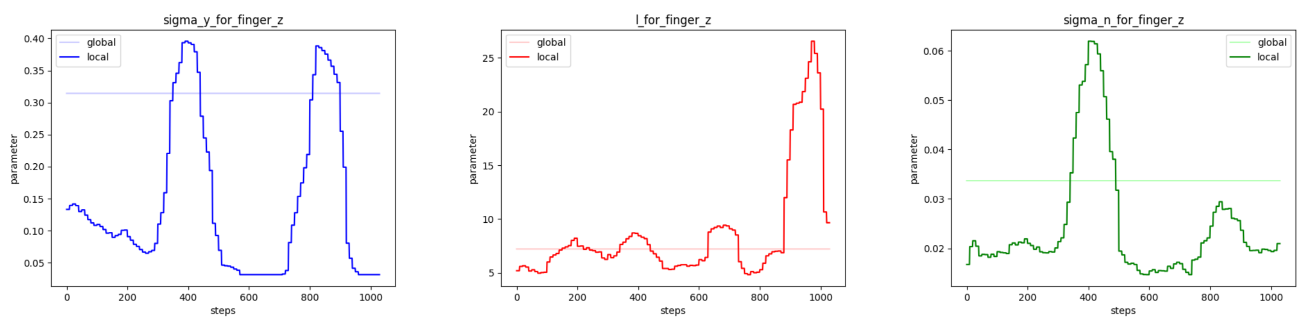

Figure 6: Comparison of kernel hyper parameters of x trajectory in global and local fitting across time-steps.

we need to evaluate the joint stiffness variation corresponding to the trajectory of interest using a dynamic body model to identify true muscle co-contractions. Then using local fitting method of Gaussian process we need to localize the time-steps with variations in hyper parameters showing lower movement variation across all three dimensions and compare these time-steps with true co-contractions observed using the body model.

Overall, if the goal is to only predict the location at an unknown point, the global fitting method seems to be the right candidate. On the other hand, if the target is to study muscle co-contractions, then local-fitting method shows great promise. A more deeper analysis into local-fitting using a dynamic body model can definitely verify whether the Gaussian process can be used to predict muscle co-contractions.