RESOURCE MANAGEMENT TECHINQUES
5 MARKS

14. Discuss any three characteristics of assignment model.

16. Explain simplex method of solving a linear programming problem.

17. Explain Vogel's approximation method in detail.

18. Discuss about travelling sales man problem.

19. Explain PERT and CPM method.

13. Discuss the essential characteristics of O.R.

14. How O.R. can be used as a tool for decision making?

16. Discuss any four reasons for solving O.R. problems by simulation

18. Discuss Monte Carlo techniques for simulation.

14. Describe the general rule for writing the dual of a LPP.

17. Draw the network for the following information

19. Write the algorithm for dual simplex method.

13. Write the scope of Operation Research.

14. Write the properties of Primal and Dual Optimal Solution.

19. Write about characteristics of Game Theory.

10MARKS

21. Explain the steps involved in Monte-Carto simulation.

22. Explain the difference between a Transportation problem and an assignment problem.

22. Discuss in detail about PERT Estimation Techniques.

14. Discuss any three characteristics of assignment model.

A model assignment maintains a clear goal toward accomplishing a course objective. For adult online learners, course goals relate less to theory or original research and more to practical approaches for day-to-day application or career advancement. More details equals higher quality of student final product.

1. **Create assignments which directly relate to accomplishing the course objective.**
   A model assignment maintains a clear goal toward accomplishing a course objective. For adult online learners, course goals relate less to theory or original research and more to practical approaches for day-to-day application or career advancement.
2. **More details equals higher quality of student final product.**
   Since adult online learners come from diverse backgrounds, do not assume students will understand the purpose of the assignment. Be prepared to tell students what you expect (e.g. word count, citation format, number of sources, etc.) and how it should be done (e.g. upload to Moodle versus email attachment).
3. **Give incremental due dates.**
   Large comprehensive assignments due at the course finality leads to unfocused, or even plagiarized, writing. Break down a large assignment into several smaller assignments due sporadically throughout the term. In turn, students receive valuable feedback incrementally as they progress throughout the course.
4. **Allow students to brainstorm for topics.**
   Allow students to brainstorm topics or share with other students using the Moodle Discussion Board form. Or consider offering students a choice among 3-4 essay questions, case scenarios, or case studies. By allowing student choice, students will find a greater connection in their writing which in turn will lead to better final submissions.
5. **Give examples.**
   In addition to clear directions, students also appreciate a visual piece of the final product. If you decide to use another student's work, be sure to ask permission to use from the student. Post model assignments on your Moodle course shell.
6. **Share student evaluation tools.**
   Share rubrics, or other evaluation tool, early in the assignment rather than at the end so students may clarify expectations first-hand. Post rubrics or evaluation tools on your Moodle course shell so students may refer to it when necessary.

16. Explain simplex method of solving a linear programming problem.

Simplex Method
The Simplex method is an approach for determining the optimal value of a linear program by hand. The method produces an optimal solution to satisfy the given constraints and produce a maximum zeta value. To use the Simplex method, a given linear programming model needs to be in standard form, where slack variables can then be introduced. Using the tableau and pivot variables, an optimal solution can be reached.

<u>Slack Variable</u>
Slack variables are additional variables that are introduced into the linear constraints of a linear program to transform them from inequality constraints to equality constraints.
<u>Surplus Variable</u>
Surplus variables are variables subtracted into the linear constraints of a linear program to transform them from inequality constraints to equality constraints.

If the inequality is ≤ (less than or equal), then we add a *slack* variable + S to change ≤ to =.
For example: $2x_1 + x_2 \leq 3$ is an inequality.
Then, $2x_1 + x_2 + s = 3$; s is the slack variable

If the inequality is ≥(greater than or equal), then we subtract a *surplus* variable - S to change ≥ to =.
For example: $2x_1 + 3x_2 \geq 5$ is an inequality.
Then, $2x_1 + 3x_2 - s = 5$; s is the surplus variable

Standard Form of a maximization problem in two variables
Standard form is the baseline format for all linear programs before solving for the optimal solution and has three requirements: (1) must be a maximization problem, (2) all linear constraints must be in a less-than-or-equal-to inequality, (3) all variables are non-negative.
Example:
$Z = 7x_1 + 5x_2$
subject to
$x_1 + 2x_2 \leq 6$
$4x_1 + 3x_2 \leq 12$
$x_1, x_2 \geq 0$

Basic Solution
Given a system of m linear equations with n variables (m < n). Any solution which is obtained by solving for m variables keeping the remaining (n–m) variables zero is called a basic solution.

Basic feasible Solution
A basic solution, which also satisfies the non-negative constraints, is called a basic feasible solution.

Bounded, Unbounded, Empty Solutions
If the value of objective function Z has both a maximum value and minimum value, such a solution is a bounded solution.
If the value of the objective function Z can be increased or decreased indefinitely, such solutions are called unbounded solutions. An unbounded solution has minimum values but no maximum value.
An empty solution will have no maximum or minimum value.

Fundamental Theorem of LP
The fundamental theorem of linear programming says that if there is a solution, it occurs on the boundary of the feasible region, not inside the region.

Basic Variables
Basic variables are variables that are non-negative in terms of the optimal solution.
Non-Basic Variables
Non-basic variables are variables that are zero in terms of the optimal solution.

Simplex Tableau
Simplex tableau is used to perform row operations on the linear programming model as well as for checking optimality.

Optimality Check
Optimal solutions of a maximization linear programming model are the values assigned to the variables in the objective function to give the largest zeta value. The optimal solution would exist on the corner points of the graph of the entire model.

Exercise 1 (Step-wise explanation)

Use the simplex method to find the optimal solutions of the following LP Problem.

Max. $Z = 7x_1 + 5x_2$

subject to

$x_1 + 2x_2 \leq 6$

$4x_1 + 3x_2 \leq 12$

$x_1, x_2 \geq 0$

Solution:

Step 1: Standard form

Standard form is necessary because it creates an ideal starting point for solving the Simplex method as efficiently as possible.

Max. $P = 7x_1 + 5x_2$

subject to

$x_1 + 2x_2 \leq 6$

$4x_1 + 3x_2 \leq 12$

$x_1, x_2 \geq 0$

Note:

To transform a minimization linear program model into a maximization linear program model, simply multiply both the left and the right sides of the objective function by -1.

$-1 \times [-Z = -8x_1 - 10x_2 - 7x_3]$

$Z = 8x_1 + 10x_2 + 7x_3$

Maximize: $Z = 8x_1 + 10x_2 + 7x_3$

Transforming linear constraints from a greater-than-or-equal-to inequality to a less-than-or-equal-to inequality can be done similarly as what was done to the objective function. By multiplying by -1 on both sides, the inequality can be changed to less-than-or-equal-to.

$-1 \times [x_1 - 5x_2 - x_3 \geq -8]$

$x_1 + 5x_2 + x_3 \leq 8$

Step 2: Determine Slack Variables

Let x_3 and x_4 be non-negative slack variables,

$x_1 + 2x_2 + x_3 = 6$

$4x_1 + 3x_2 + x_4 = 12$

$7x_1 + 5x_2 = P$

Now, the given LP problem in its standard form is,

$1.x_1 + 2.x_2 + 1.x_3 + 0.x_4 + 0.P = 6$

$4.x_1 + 3.x_2 + 0.x_3 + 1.x_4 + 0.P = 12$

$-7.x_1 + -5.x_2 + 0.x_3 + 0.x_4 + 1.P = 0$

Step 3: Setting up the Tableau

The tableau consists of the coefficient corresponding to the linear constraint variables and the coefficients of the objective function**.**

The equations in initial simplex tableau is as follows:

Basic Variables	x_1	x_2	x_3	x_4	P	RHS (b)
x_3	1	2	1	0	0	6
x_4	4	3	0	1	0	12
	-7	-5	0	0	1	0

Step 4: Check Optimality

To check optimality using the tableau, all values in the last row must contain values greater than or equal to zero. If a value is less than zero, it means that variable has not reached its optimal value. As seen in the previous tableau, two negative values exist in the bottom row indicating that this solution is not optimal. If a tableau is not optimal, the next step is to identify the pivot element to base a new tableau on.

Step 5: Identify Pivot Element

The pivot element can be identified by looking at the bottom row of the tableau and the indicator. Pick the smallest negative value in the bottom row. That column containing the smallest negative value would be the pivot column. One of the values lying in the pivot column will be the pivot element. To find the indicator, divide the beta values of the linear constraints by their corresponding values from the pivot column.

Basic Variables	x_1	x_2	x_3	x_4	P	RHS (b)
x_3	1	2	1	0	0	6
x_4	$\boxed{4}$	3	0	1	0	12
	-7	-5	0	0	1	0

∵ -7 is the most -ve value(smallest value), so, the first column is the pivot column.

$\because \frac{6}{1} = 6$ and $\frac{12}{4} = 3$(min) [3<6]

∴ 4 is the pivot element.

Step 6: Create the New Tableau

1) To optimize the pivot variable, it will need to be transformed into a unit value (value of 1). To transform the value, multiply the row containing the pivot variable by the reciprocal of the pivot value. In the example below, the pivot variable is originally 4, so multiply the entire row by 1/4.

$R_2 \rightarrow R_2/4$

Basic Variables	x_1	x_2	x_3	x_4	P	RHS (b)
x_3	1	2	1	0	0	6
x_2	1	¾	0	¼	0	3
	-7	-5	0	0	1	0

2) After the unit value has been determined, the other values in the column containing the unit value will become zero. This is because the x_2 in the second constraint is being optimized, which requires x_2 in the other equations to be zero.

$R_1 \rightarrow R_1 - R_2$

Basic Variables	x_1	x_2	x_3	x_4	P	RHS (b)
x_1	0	5/4	1	-1/4	0	3
x_2	1	3/4	0	1/4	0	3
	-7	-5	0	0	1	0

$R_3 \rightarrow 7R_2 + R_3$

Basic Variables	x_1	x_2	x_3	x_4	P	RHS (b)
x_1	0	5/4	1	-1/4	0	3
x_2	1	3/4	0	1/4	0	3
	0	1/4	0	7/4	1	21

Once the new tableau has been completed, the model can be checked for an optimal solution

∴ All the entries in the last row are non-negative.

So, the optimal solution is obtained.

So, maximum P=21 when x_1= 3 and x_2= 0

Finally, Max P = $7x_1 + 5x_2$ = 7.3 + 0 = 21

Step 8: Identify New Pivot Variable

If the solution has been identified as not optimal, a new pivot element will need to be determined. Steps are repeated from Step 5 and optimality is checked until optimal values can be obtained.

Exercise 2

Use the simplex method to find the optimal solutions of the following LP Problem.

Max. $Z = 3x_1 + 5x_2$

subject to

$3x_1 + 2x_2 \leq 18$

$x_1 \leq 4$

$x_2 \leq 6$

$x_1, x_2 \geq 0$

Solution:

Let x_3, x_4 and x_5 be non-negative slack variables,

$3x_1 + 2x_2 + x_3 = 18$

$x_1 + x_4 = 4$

$x_2 + x_5 = 6$

$3x_1 + 5x_2 = Z$

Now, the given LP problem in its standard form is,

$3.x_1 + 2.x_2 + 1.x_3 + 0.x_4 + 0.x_5 + 0.Z = 18$

$1.x_1 + 0.x_2 + 0.x_3 + 1.x_4 + 0.x_5 + 0.Z = 4$

$0.x_1 + 1.x_2 + 0.x_3 + 0.x_4 + 1.x_5 + 0.Z = 6$

$3.x_1 + 5.x_2 + 1.x_3 + 0.x_4 + 0.x_5 + 0.Z = 0$

The equations in initial simplex tableau is as follows:

Basic Variables	x_1	x_2	x_3	x_4	x_5	Z	RHS (b)
x_3	3	2	1	0	0	0	18
x_4	1	0	0	1	0	0	4
x_5	0	$\boxed{1}$	0	0	1	0	6
	-3	-5 ↑	0	0	0	1	0

∵ -5 is the most -ve value(smallest value), so, the second column is the pivot column.

∵ $\frac{18}{2}$ = 9 and $\frac{5}{1}$ = 5(min) [5<18]

∴ 1 is the pivot element.

$R_1 \rightarrow R_1 - 2R_3$

$R_4 \rightarrow R_4 + 5R_3$

Basic Variables	x_1	x_2	x_3	x_4	x_5	Z	RHS (b)
x_3	$\boxed{3}$	0	1	0	-2	0	6
x_4	1	0	0	1	0	0	4
x_2	0	1	0	0	1	0	6
	-3 ↑	0	0	0	0	1	30

∵ -3 is the most -ve value(smallest value), so, the first column is the pivot column.

∵ $\frac{6}{3}$ = 2(min) and $\frac{4}{1}$ = 4 [2<4]

∴ 3 is the pivot element.

Basic Variables	x_1	x_2	x_3	x_4	x_5	Z	RHS (b)
x_1	1	0	1/3	0	-2/3	0	2
x_4	1	0	0	1	0	0	4
x_2	0	1	0	0	1	0	6
	-3	0	0	0	0	1	30

$R_1 \rightarrow R_1/3$

$R_2 \rightarrow R_2 - R_1$

Basic Variables	x_1	x_2	x_3	x_4	x_5	Z	RHS (b)
x_1	1	0	1/3	0	-2/3	0	2
x_4	0	0	-1/3	1	2/3	0	2
x_2	0	1	0	0	1	0	6
	0	0	1	0	-2	1	36

$R_4 \rightarrow R_4 + 3R_1$

∴ All the entries in the last row are non-negative.
So, the optimal solution is obtained.
So, maximum Z=36 when x_1= 2 and x_2= 6
Finally, Max Z= $3x_1$+ $5x_2$ = 3.2 + 5.6 = 36
Dual of LP Problem

If the LP problem is of maximization type then it can be solved by simplex method.

But if the given LP Problem is of the minimization type then it can be solved after changing itself into the maximization problem, this is known as the dual problem of the given LP Problem.

Dual of a Minimization LP Problem

Min. C= ax + by
subject to
$a_1x + b_1y \le c$
$a_2x + b_2y \le c_2$
$c_{1,} c_2 \ge 0$
$x, y \ge 0$
Here,
A= Augmented matrix formed from the constraints and the objective function

$$A = \left(\begin{array}{cc|c} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ \hline a & b & 0 \end{array}\right)$$

The corresponding dual problem of the given minimization problem is as follows:
Max. $P = c_1x + c_2y$
subject to
$a_1u + b_1v \le a$
$a_2u + b_2v \le b$
$c_{1,} c_2 \ge 0$
$u, v \ge 0$

$$B = \left(\begin{array}{cc|c} a_1 & a_2 & a \\ b_1 & b_2 & b \\ \hline c_1 & c_2 & 0 \end{array}\right)$$

B is the transposed matrix of A i.e. $B = A^T$
By duality theorem, Minimize C = Maximize P
Exercise 3
Minimize $P = 7x_1 + x_2$
subject to
$3x_1 - 2x_2 \le -6$
$x_1 + 3x_2 \ge 15$
$x_1, x_2 \ge 0$
Solution:
Standard Form:
Minimize $P = 7x_1 + x_2$
subject to

$3x_1 - 2x_2 \le -6$ or,$-3x_1 + 2x_2 \ge 6$
$x_1 + 3x_2 \ge 15$
$x_1, x_2 \ge 0$

Let A be the augmented matrix formed by the coefficients of constraints with objective function at the bottom.

$$A = \left(\begin{array}{cc|c} -3 & 2 & 6 \\ 1 & 3 & 15 \\ \hline 7 & 1 & 0 \end{array}\right)$$

$$A^T = \left(\begin{array}{cc|c} -3 & 1 & 7 \\ 2 & 3 & 1 \\ \hline 6 & 15 & 0 \end{array}\right)$$

Now, corresponding dual LP is,
Max $P^* = 6y_1 + 15y_2$
subject to
$-3y_1 + y_2 \le 7$
$2y_1 + 3y_2 \le 1$
$y_1, y_2 \ge 0$
Let x_1 and x_2 be non-negative slack variables,
$-3y_1 + y_2 + x_1 = 7$
$2y_1 + 3y_2 + x_2 = 1$
$-6y_1 + -15y_2 + P^* = 0$
Now, the given LP problem in its standard form is,
$-3.y_1 + 1.y_2 + 1.x_1 + 0.x_2 + 0.P^* = 7$
$2.y_1 + 3.y_2 + 0.x_1 + 1.x_2 + 0.P^* = 1$
$-6y_1 + -15y_2 + 0.x_1 + 0.x_2 + 1.P^* = 0$

The equations in initial simplex tableau is as follows:

Basic Variables	y_1	y_2	x_1	x_2	P^*	RHS (b)
x_1	-3	1	1	0	0	7
x_2	2	$\boxed{3}$	0	1	0	1
	-6	-15 ↑	0	0	1	0

∵ -15 is the most -ve value(smallest value), so, the second column is the pivot column.

∵ $\frac{7}{1}$ = 7 and $\frac{1}{3}$ = 0.33 (min) [0.33<7]
∴ 3 is the pivot element.
$R_2 \rightarrow R_2/3$

Basic Variables	y_1	y_2	x_1	x_2	P^*	RHS (b)
x_1	-3	1	1	0	0	7
y_2	2/3	1	0	1/3	0	1/3
	-6	-15	0	0	1	0

$R_1 \rightarrow R_1 - R_2$
$R_3 \rightarrow R_3 + 15R$

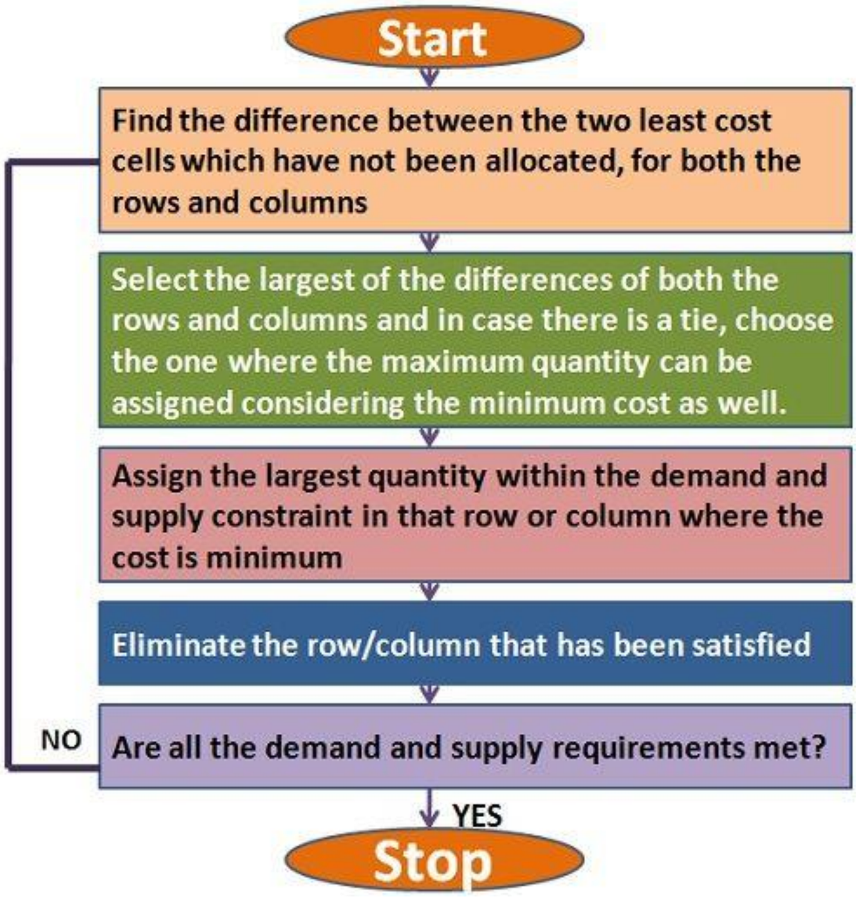
Basic Variables	y_1	y_2	x_1	x_2	P^*	RHS (b)
x_1	-1/3	0	1	-1/3	0	20/3
y_2	2/3	1	0	1/3	0	1/3
	4	0	0	5	1	5

∴ All the entries in the last row are non-negative.
So, the optimal solution is obtained.
So, maximum $P^*=5$ when $x_1= 0$ and $x_2= 5$
Finally, Max $P^* = 7x_1 + x_2 = 0 + 5 = 5$.

17. Explain Vogel's approximation method in detail.

The **Vogel's Approximation Method** or **VAM** is an iterative procedure calculated to find out the initial feasible solution of the transportation problem. Like Least cost Method, here also the shipping cost is taken into consideration, but in a relative sense.

The following is the flow chart showing the steps involved in solving the transportation problem using the Vogel's Approximation Method:



The concept of Vogel's Approximation Method can be well understood through an illustration given below:

1. First of all the difference between two least cost cells are calculated for each row and column, which can be seen in the iteration given for each row and column. Then the largest difference is selected, which is 4 in this case. So, allocate 20 units to cell BD, since the minimum cost is to be chosen for the allocation. Now, only 20 units are left with the source B.

To / From	D	E	F	Supply	Iteration-I
A	6	4	1	50	3
B	3 (20)	8	7	40	4
C	4	4	2	60	2
Demand	20	95	35	150	
Iteration-I	1	0	1		

2. Column D is deleted, again the difference between the least cost cells is calculated for each row and column, as seen in the iteration below. The largest difference value comes to be 3, so allocate 35 units to cell AF and 15 units to the cell AE. With this, the Supply and demand of source A and origin F gets saturated, so delete both the row A and Column F.

To / From	E	F	Supply	Iteration-II
A	4 (15)	1 (35)	50	3
B	8	7	20	1
C	4	2	60	2
Demand	95	35	150	
Iteration-II	0	1		

3. Now, single column E is left, since no difference can be found out, so allocate 60 units to the cell CE and 20 units to cell BE, as only 20 units are left with source B. Hence the demand and supply are completely met.

To / From	E	Supply
B	8 (20)	20
C	4 (60)	60
Demand	80	150

Now the total cost can be computed, by multiplying the units assigned to each cell with the cost concerned. Therefore,

Total Cost = 20*3 + 35*1 + 15*4 + 60*4 + 20*8 = Rs 555

Note: Vogel's Approximation Method is also called as **Penalty Method** because the difference costs chosen are nothing but the penalties of not choosing the least cost routes.

18. Discuss about travelling sales man problem.

In the traveling salesman Problem, a salesman must visits n cities. We can say that salesman wishes to make a tour or Hamiltonian cycle, visiting each city exactly once and finishing at the city he starts from. There is a non-negative cost c (i, j) to travel from the city i to city j. The goal is to find a tour of minimum cost. We assume that every two cities are connected. Such problems are called Traveling-salesman problem (TSP).

We can model the cities as a complete graph of n vertices, where each vertex represents a city.

It can be shown that TSP is NPC.

If we assume the cost function c satisfies the triangle inequality, then we can use the following approximate algorithm.

Triangle inequality

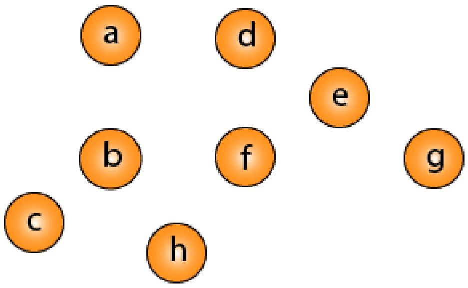
Let u, v, w be any three vertices, we have

$$\boxed{c\,(u, w) \leq c\,(u, v) + c\,(v, w)}$$

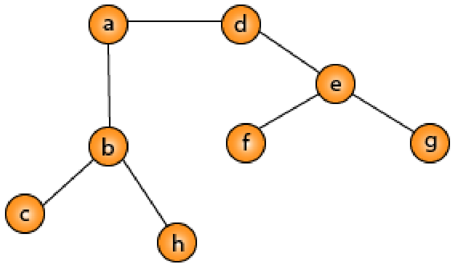
One important observation to develop an approximate solution is if we remove an edge from H*, the tour becomes a spanning tree.

```
Approx-TSP (G= (V, E))
{
 1. Compute a MST T of G;
 2. Select any vertex r is the root of the tree;
 3. Let L be the list of vertices visited in a preorder tree walk of T;
 4. Return the Hamiltonian cycle H that visits the vertices in the order L;
}
```

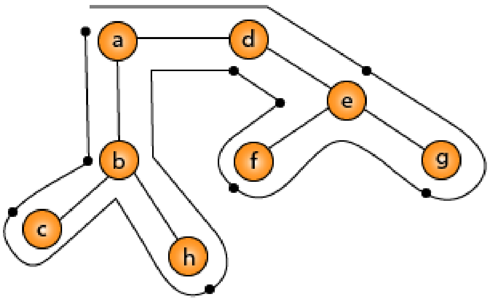
Traveling-salesman Problem



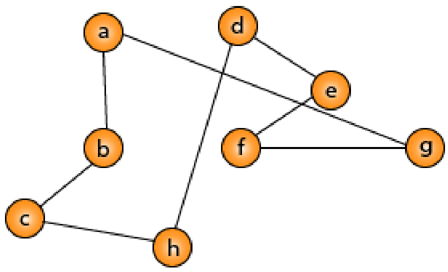
(1) A given set of points



(2) MST T



(3) Full tree walk on T.



(4) A preorder sequence gives a tour H.

Intuitively, Approx-TSP first makes a full walk of MST T, which visits each edge exactly two times. To create a Hamiltonian cycle from the full walk, it bypasses some vertices (which corresponds to making a shortcut)

19. Explain PERT and CPM method.

Project management can be defined as a structural way of planning, scheduling, executing, monitoring and controlling various phases of a project. To achieve the end goal of a project on time, PERT and CPM are two project management techniques that every management should implement. These techniques help in displaying the progress and series of actions and events of a project.

Meaning of PERT

Program (Project) Evaluation and Review Technique (PERT) is an activity to understand the planning, arranging, scheduling, coordinating and governing of a project. This program helps to understand the technique of a study taken to complete a project, identify the least and minimum time taken to complete the whole project. PERT was developed in the 1950s, with the aim of the cost and time of a project.

Meaning of CPM

Critical Path Method or CPM is a well-known project modelling technique in project management. It is a resource utilising algorithm that was developed in the 1950s by James Kelly and Morgan Walker.

CPM is mainly used in projects to determine critical as well as non-critical tasks that will help in preventing conflicts and reduce bottlenecks.

In essence, CPM is about choosing the path in a project that will help in calculating the least amount of time that is required to complete a task with the least amount of wastage.

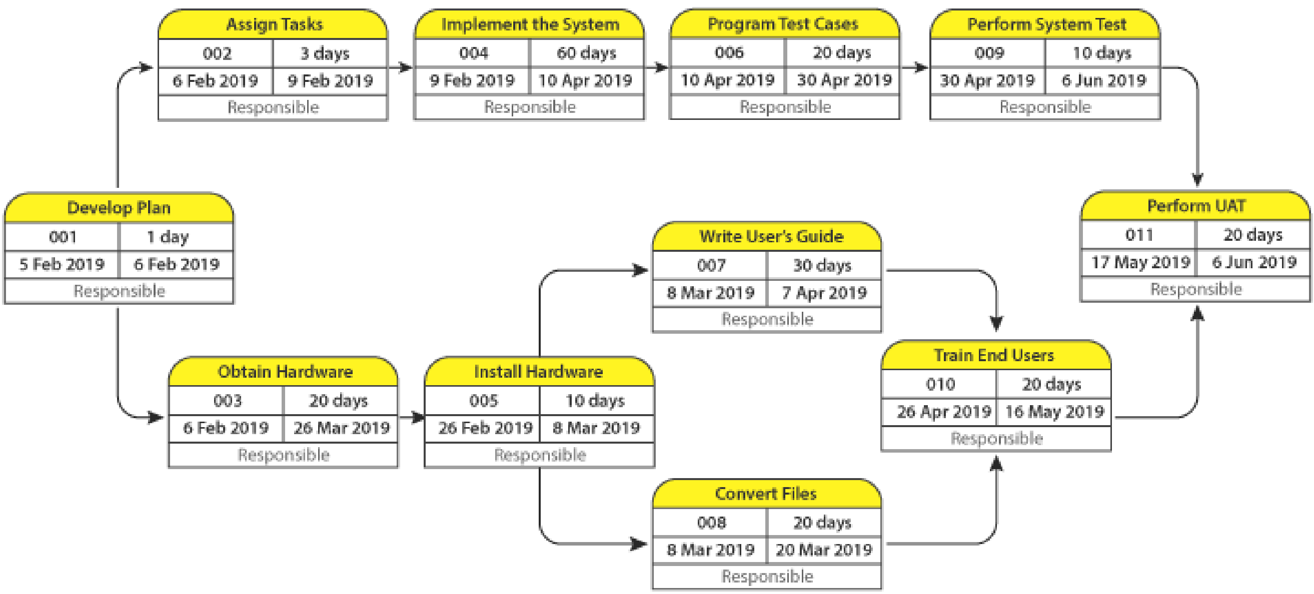
The Critical Path Method or CPM has been used in many industries starting from defence, construction, software, aerospace, etc.

PERT vs CPM

Abbreviation

PERT – Project Evaluation and Review Technique	CPM – Critical Path Method
What does It Mean?	
PERT – PERT is a popular project management technique that is applicable when the time required to finish a project is not certain	CPM – CPM is a statistical algorithm which has a certain start and end time for a project
Model Type	
PERT – PERT is a probabilistic model	CPM – CPM is a deterministic model
Focus	
PERT – The main focus of PERT is to minimise the time required for completion of the project	CPM – The main focus of CPM is on a trade-off between cost and time, with a major emphasis on cost-cutting.
Orientation type	
PERT – PERT is an event-oriented technique	CPM – CPM is an activity-oriented technique

PERT Example



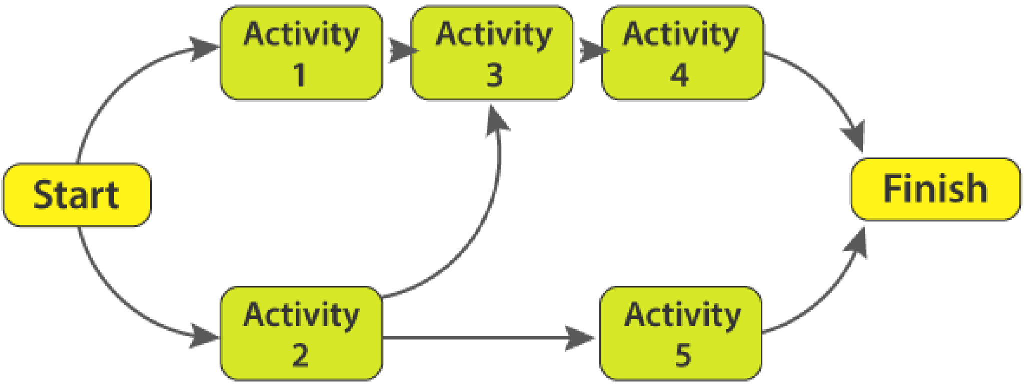
Advantages of CPM

- Provides an outline for long term coordination and planning of a project
- Recognizes critical activities
- Easy to plan, schedule and control project
- It improves productivity
- Manages the resource needed

Disadvantages of CPM

- For beginners its difficult to understand
- Software too expensive
- Sometimes, to structure CPM is too time-consuming
- It cannot control and form the schedule of a person involved in the project
- Allocation of resources cannot be monitored properly

CPM Example



How to Make a PERT Chart?

To prepare a PERT Chart, the following steps need to be followed.

- Recognize particular projects and milestones.
- Decide the precise sequence of the project.
- Create a network diagram.
- Determine the time needed for each project activity.
- Manage the critical path.
- Update the PERT chart as the project progresses.

CPM in Project Management

The Critical Path Method in project management is a step-by-step technique used in the planning process that explains the critical and non-critical activities of a project. CPM goals are to check time-bound issues and process that causes blockage in the project. The CPM is preferably applicable to projects that involve various activities that are associated with a complex method. Once CPM is applied, it will help you keep your projects on track.

- Helps you recognize the action that needs to be performed on time so that the whole project is completed on time.
- Indicates which responsibilities can be delayed and for how long without affecting the overall project plan.
- Determines the least amount of time it will take to accomplish the project.
- Tells you the newest and latest time each activity can start on in order to manage the schedule.

The term of each action is listed above each joint in the diagram. For an individual path, insert the duration of each node to ascertain the total duration. The critical path is the one that has the longest duration.

13**. Discuss the essential characteristics of O.R.**

Three essential characteristics of operations research are a systems orientation, the use of interdisciplinary teams, and the application of scientific method to the conditions under which the research is conducted.

1. Decision making: Operations research is a decision science which helps management to make better decisions.

2. Use of Information Technology (IT): O.R. often requires a computer to solve the complex mathematical model or to perform a large number of computations that ae involved. Use of digital computer has become an integral part of the operations research approach to decision making.

3. Quantitative solution: Operations research provides the managers with a quantitative basis for decision making. OR attempts to provide a systematic and rational approach for quantitative solution to the various managerial problems.

4. Human factors: In deriving quantitative solution we do not consider human factors, which doubtlessly plays a great role in the problems. So study of the OR is incomplete without a study of human factors.

5. System orientation: O.R. study the situation or problem as a whole. This means that an activity by any part of an organization has some effect on the activity of every other part. The optimum result of one part of a system may not be the optimum for some other part. Therefore, to evaluate a decision, one must identify all possible interactions and determine their impact on the organization as a whole.

6. Scientific approach: O.R. uses scientific methods to solve the problems. Most of the scientific studies such as chemistry, physics, biology etc. can be carried out in the laboratories, without much interference form the outside world. Bust same is not true in the systems under study by OR teams. So, OR is an formalized process of reasoning. Under OR the problem is to be analysed ad defined clearly. Observations are made under different conditions to study the behavior of the system. On the basis of these observations a hypothesis describing how the various factors involved are believed to interact and the best solution to the problem is formulated. To test the hypothesis experiment is designed and executed. Observations are made and measurement s are recorded. Finally results of the experiments are studied and the hypothesis is accepted or rejected. So, OR is the use of scientific method to solve the problem under study.

7. Inter-disciplinary team approach: O.R. is performed by a team of scientists whose individual members have been drawn from different scientific and engineering disciplines. For example, one may find a mathematician, statistician, physicist, psychologist, economist and an engineer working together on an OR problem. 8. Uncovering new problems: Solution of an OR problem may uncover a number of new problems. In order to derive the maximum benefit each one of them must be solved. OR is not effectively used if It is restricted to one shot problems only.

14. How O.R. can be used as a tool for decision making?

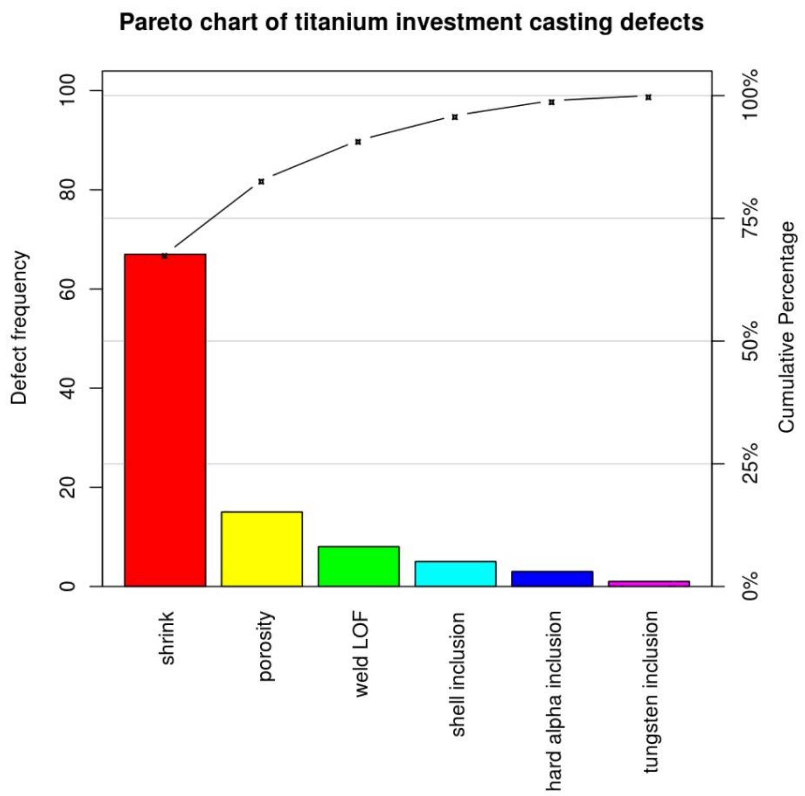
One thing almost everyone has in common is that we all want to make good decisions. That's been the motivation of many a decision making tool. Over time, people have created and refined tools for every type of decision.

Almost all charts, diagrams and reports lend themselves to decision making in one way or another. But here in this section, we're going to take a look at some of the tools that are most specifically devoted to the decision making process.

Pareto Diagram

What it does for you: It identifies beneficial opportunities and issues.

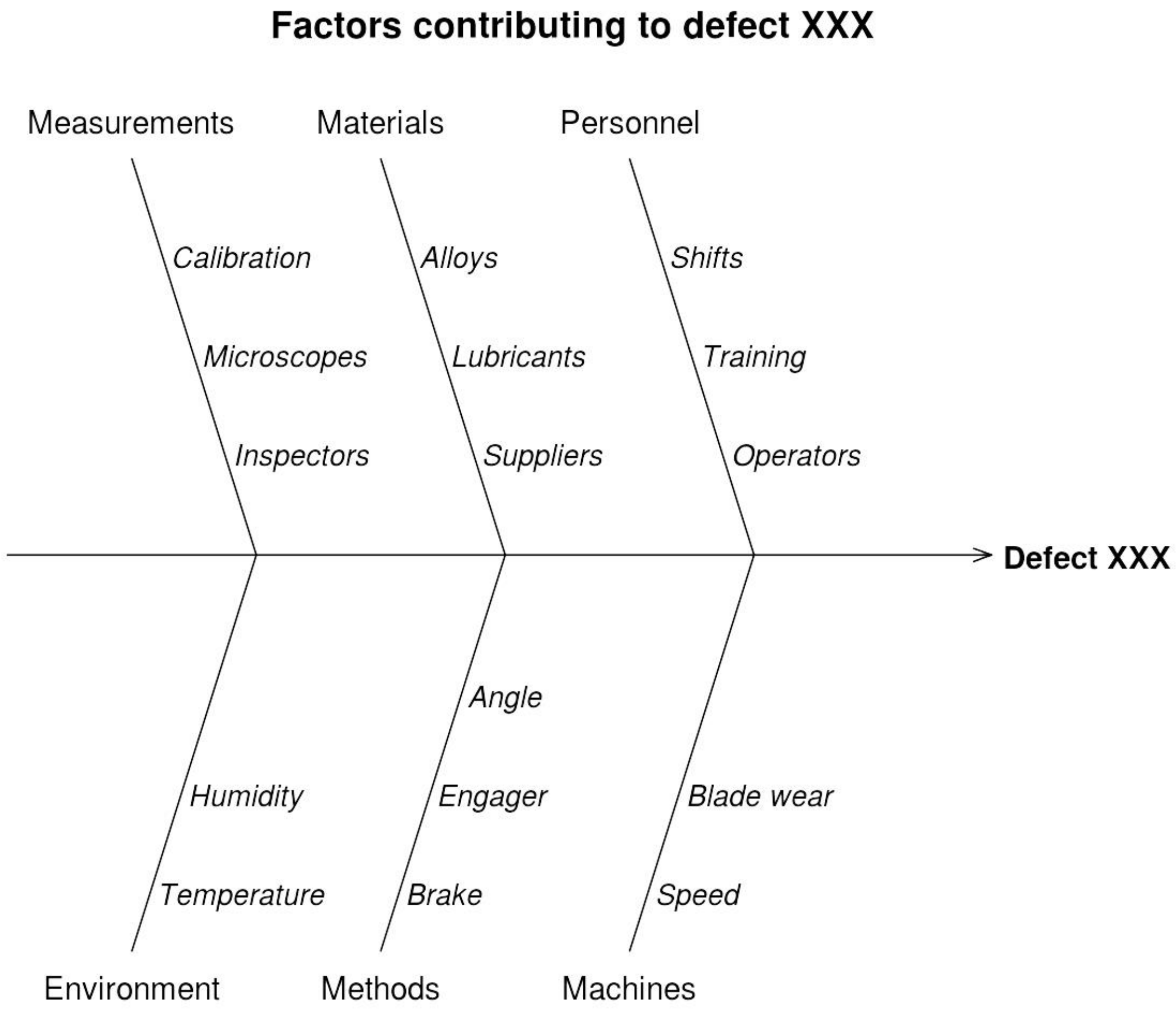
A Pareto chart contains both bars and a line graph, where individual values are represented in descending order by bars, and the cumulative total is represented by the line. The chart below outlines titanium investment casting defects. The chart is meant to highlight the most important among a list of factors. The bar chart is the frequency of occurrence, and the line graph shows the cumulative percentage of time these issues occur. Looking at this bar chart, if we wanted to decrease titanium investment casting defects by 80%, we'd need to tackle the first two issues on the chart. The Pareto chart helps you define and tackle the issues that have the most impact on your problem.



Cause and Effect or Ishikawa Diagram

What It Does For You: Helps you hone in on the exact cause of an issue.

As you can see from the cause and effect diagram below (or fishbone diagram, because it looks very much like the bones of a fish), the user needs to list all the possible causes of a particular issue, by category. Each category is a “bone” of the fish. The issue is listed in the fish’s head. In this instance, the group is looking at why so much staff is required for a particular process. The answers are divided into categories, like “policies” and “procedures” below.



This is commonly used with product design and quality issues, and, as a very visual brainstorming tool, can spark many more ideas for cause/effect issues. On the other hand, bigger issues can start to look cluttered, and interrelationships between causes are hard to identify using this method.

Feasibility Reporting

What it does for you: It lets you know the rate of return on the investment of your project.

Now we’re doing math! (It was inevitable). Feasibility reporting, packed with things like cost-benefit analysis and payback calculations, allows an organization to see all the details of a particular project: when it will start paying back, what the rate of return is on the investment, and so on.

Benefit cost ratio and payback are just a portion of a feasibility report, but this short video shows you the kind of information you’re likely to see and thus, the kinds of decisions it can help you make.

SWOT Analysis

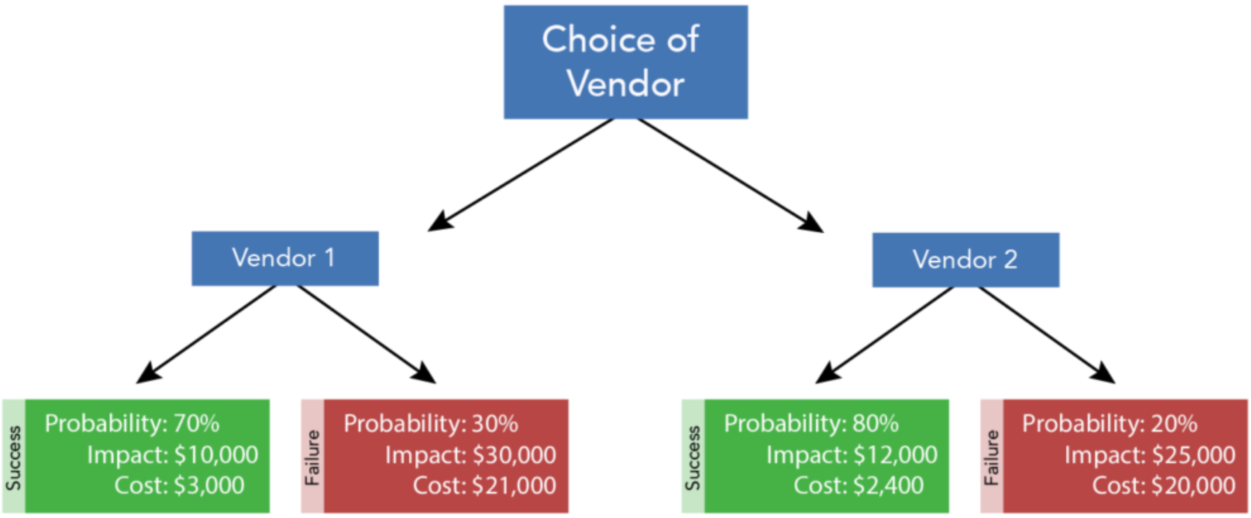
SWOT stands for strengths, weaknesses, opportunities and threats. SWOT analysis helps you identify the internal strengths and weaknesses of your organization that give you an advantage over others of your kind, and reminds you to look for external opportunities and threats at the same time. It helps an organization identify its objectives and determine which environmental and non-environmental factors are favorable to that success.

This sample SWOT analysis shows the considerations of a particular organization as they went through their strategic planning process.

A SWOT analysis can be used any time a business or individual wants to determine if a particular objective is achievable. Limitations of the SWOT analysis have also been noted, chief among them that the list weighs heavily on perception rather than actual assessment of strengths and weaknesses.

Decision Making Diagrams

What they do for you: They help you see all the alternatives and the associated costs.



The decision making diagram allows you to map out all the possible alternatives to each decision, their costs and even chances of success or failure. In the diagram above, an individual is trying to decide between Vendor 1 and Vendor 2. As you can see in the diagram, Vendor 2's probability of failure is only 20%, at a cost of $2,400, but would have an impact of $12,000 total, compared with Vendor 1's $10,000 total impact in the case of failure. Looks like Vendor 1 is a bigger risk with a bigger payoff. Which would you choose?

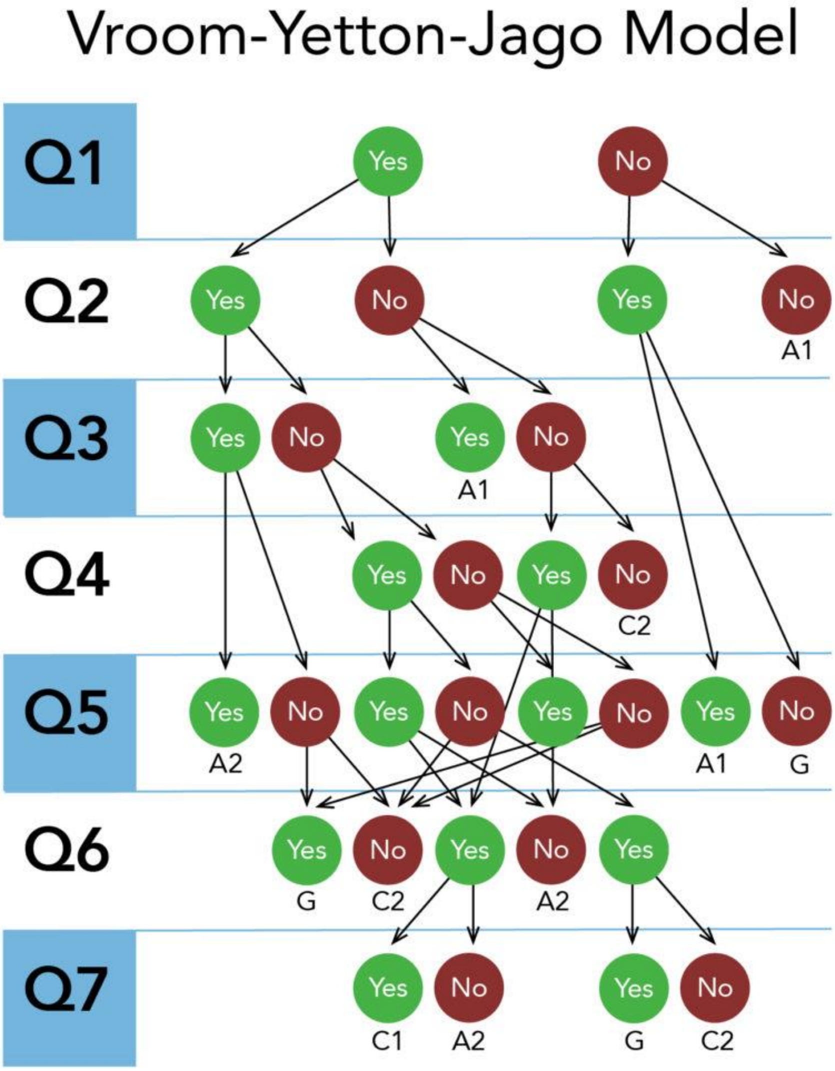
Decision Making Software

What it does for you: Allows for collaborative decision making and parsing large amounts of data.

There are a variety of decision making software solutions on the market today to help in any situation you might find yourself in. These software solutions allow for collaborative decision making, analysis and reporting of substantial amounts of data.

Vroom-Yetton-Jago Decision Making Model[1]

What it does for you: Helps you figure out the best method to make a decision and who to involve.



Vroom, Yetton, and Jago created a decision model to help you decide how you were going to make a decision. Should you make a decision individually or assemble the group and make a decision together? These are the questions you should ask yourself, according to Victor Vroom and his compatriots:

1. Is the quality of the decision important?
2. Is team commitment important for the decision?
3. Do you have enough information to make the decision on your own?
4. Is the problem well-structured?
5. Would the team support you if you made the decision alone?
6. Does the team share the organizational goals?
7. Is conflict among the team over the decision likely?

Note the significance of the annotations on the chart:

- **Autocratic (A1):** The leader makes the decision by themself using existing information without any communication with the team.

- **Autocratic (A2):** The leader consults with team members to get information, but makes the decision by himself or herself without informing the group.
- **Consultative (C1):** The leader consults the team members to get their opinion about the situation, but he or she makes the decision for themselves.
- **Consultative (C2):** The leader consults the team members seeking opinions and suggestions, but he or she makes the decision for himself or herself. In this type of leadership style, the leader is open to suggestions and ideas.
- **Collaborative (G):** The leader shares the decision making process with team members. He or she supports the team in making the decision and finding an answer that everyone agrees on.

The model doesn't allow for the personality characteristics of the leader, allow for large group use, or provide questions that are precise enough. That said, it's very flexible and allows the leader the ability to make a good decision in a variety of different situations. It can also be shared and duplicated.

16. Discuss any four reasons for solving O.R. problems by simulation

- risk-free environment. Simulation modeling provides a safe way to test and explore different "what-if" scenarios. ...
- save money and time. ...
- visualization. ...
- insight into dynamics. ...
- increased accuracy. ...
- handle uncertainty.

Simulation modeling solves real-world problems safely and efficiently. It provides an important method of analysis which is easily verified, communicated, and understood. Across industries and disciplines, simulation modeling provides valuable solutions by giving clear insights into complex systems.

18. Discuss Monte Carlo techniques for simulation.

A Monte Carlo simulation is used to model the probability of different outcomes in a process that cannot easily be predicted due to the intervention of random variables. It is a technique used to understand the impact of risk and uncertainty.

A Monte Carlo simulation is used to tackle a range of problems in many fields including investing, business, physics, and engineering.

It is also referred to as a multiple probability simulation.

- A Monte Carlo simulation is a model used to predict the probability of a variety of outcomes when the potential for random variables is present.
- Monte Carlo simulations help to explain the impact of risk and uncertainty in prediction and forecasting models.
- A Monte Carlo simulation requires assigning multiple values to an uncertain variable to achieve multiple results and then averaging the results to obtain an estimate.
- Monte Carlo simulations assume perfectly efficient markets.

Monte Carlo Simulation Steps

Microsoft Excel or a similar program can be used to create a Monte Carlo simulation that estimates the probable price movements of stocks or other assets.

There are two components to an asset's price movement: drift, which is its constant directional movement, and a random input, which represents market volatility.

By analyzing historical price data, you can determine the drift, standard deviation, variance, and average price movement of a security. These are the building blocks of a Monte Carlo simulation.

The 4 Steps in a Monte Carlo Simulation

Step 1: To project one possible price trajectory, use the historical price data of the asset to generate a series of periodic daily returns using the natural logarithm (note that this equation differs from the usual percentage change formula):

Periodic Daily Return=��(Day's PricePrevious Day's Price)
*Periodic Daily Return=ln(*Previous Day's PriceDay's Price
Step 2: Next use the AVERAGE, STDEV.P, and VAR.P functions on the entire resulting series to obtain the average daily return, standard deviation, and variance inputs, respectively. The drift is equal to:

Drift=Average Daily Return−Variance2where:Average Daily Return=Produced from Excel'sAVERAGE function from periodic daily returns seriesVariance=Produced from Excel'sVAR.P function from periodic daily returns seriesDrift=Average Daily Return−2Variance
where:*Average Daily Return=Produced from Excel'sAVERAGE function from periodic daily returns seriesVariance=Produced from Excel'sVAR.P function from periodic daily returns series*

Alternatively, drift can be set to 0; this choice reflects a certain theoretical orientation, but the difference will not be huge, at least for shorter time frames.

Step 3: Next, obtain a random input:

Random Value=�×NORMSINV(RAND())

where:�=Standard deviation, produced from Excel'sSTDEV.P function from periodic daily returns seriesNORMSINV and RAND=Excel functionsRandom Value=σ×NORMSINV(RAND())

where:*σ=Standard deviation, produced from Excel'sSTDEV.P function from periodic daily returns seriesNORMSINV and RAND=Excel functions*

The equation for the following day's price is:

Next Day's Price=Today's Price×�(Drift+Random Value)
Next Day's Price=Today's Price×e(Drift+Random Value)

Step 4: To take *e* to a given power *x* in Excel, use the EXP function: EXP(x). Repeat this calculation the desired number of times. (Each repetition represents one day.) The result is a simulation of the asset's future price movement.

By generating an arbitrary number of simulations, you can assess the probability that a security's price will follow a given trajectory.

14. Describe the general rule for writing the dual of a LPP.

Definition: The **Duality in Linear Programming** states that every linear programming problem has another linear programming problem related to it and thus can be derived from it. The original linear programming problem is called **"Primal,"** while the derived linear problem is called **"Dual."**

Before solving for the duality, the original linear programming problem is to be formulated in its standard form. Standard form means, all the variables in the problem should be non-negative and "≥," "≤" sign is used in the minimization case and the maximization case respectively.

The concept of Duality can be well understood through a problem given below:

Maximize

$Z = 50x_1+30x_2$

Subject to:
$4x_1 + 3x_2 \leq 100$
$3x_1 + 5x_2 \leq 150$
$X_1, x_2 \geq 0$

The duality can be applied to the above original linear programming problem as:

Minimize

$G = 100y_1+150y_2$

Subject to:

$4y_1 + 3y_1 \geq 50$
$3y_1 +5y_2 \geq 30$
$Y_1, y_2 \geq 0$

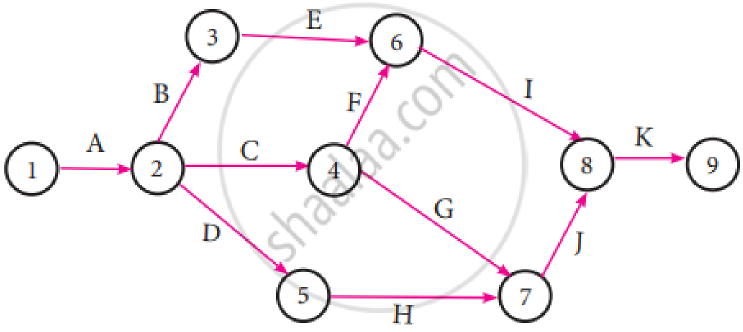
The following observations were made while forming the dual linear programming problem:

1. The primal or original linear programming problem is of the maximization type while the dual problem is of minimization type.
2. The constraint values 100 and 150 of the primal problem have become the coefficient of dual variables y_1 and y_2 in the objective function of a dual problem and while the coefficient of the variables in the objective function of a primal problem has become the constraint value in the dual problem.
3. The first column in the constraint inequality of primal problem has become the first row in a dual problem and similarly the second column of constraint has become the second row in the dual problem.
4. The directions of inequalities have also changed, i.e. in the dual problem, the sign is the reverse of a primal problem. Such that in the primal problem, the inequality sign was "≤" but in the dual problem, the sign of inequality becomes "≥".

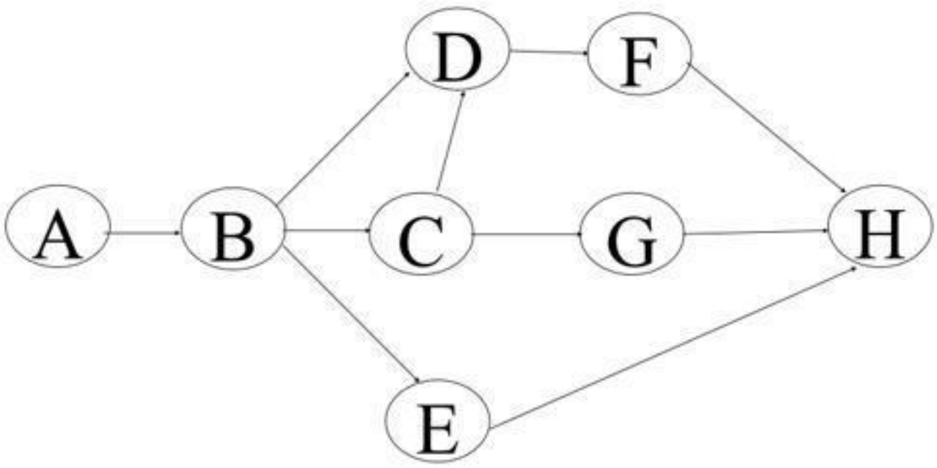
Note: The dual of a dual problem is the primal problem.

17. Draw the network for the following information

SOLUTION



Draw the project network diagram as shown below:



An activity is said to be a burst activity if that activity is followed by two or more than two activities. In the above network diagram activity B is followed by activities D, C, and E and activity C is followed by activities D and G.

Hence, the burst activities are $\boxed{\textbf{B and C}}$.

An activity is said to be a merge activity that cannot be started before completion of two or more than two activities. In the above network diagram, activity D can be started if activities B and C are completed. Activity H can be started if activities E, F, and G are completed.

Hence, the merge activities are $\boxed{\textbf{D and H}}$.

19. Write the algorithm for dual simplex method.

The **Dual Simplex method** is used in situations where the optimality criterion (i.e., $z_j c_j \geq 0$ in the maximization case and $z_j c_j \leq 0$ in minimization case) is satisfied, but the basic solution is not feasible because under the X_B column of the simplex table there are one or more negative values.

What are the reasons for studying the dual simplex method?

- Sometimes it allows to easily select an initial basis without having to add any artificial variable.
- It aids in certain types of sensitivity testing.
- It helps in solving integer programming problems.

Algorithm, Example

The **dual simplex algorithm** proceeds in this way:

Steps of Dual Simplex Algorithm

1. Formulate the Problem

Formulate the **mathematical model** of the given linear programming problem.

"The model is a vehicle for arriving at a well-structured view of reality." -Anonymous

Convert every inequality constraint in the LPP into an equality constraint, so that the problem can be written in a standard from.

2. Find out the Initial Solution

Calculate the initial basic feasible solution by assigning zero value to the decision variables. This solution is shown in the initial dual simplex table.

3. Determine an improved solution

If all the values under X_B column ≥ 0, then don't apply dual simplex method because optimal solution can be easily obtained by the simplex method.
On the contrary, if any value under X_B column < 0, then the current solution is infeasible so move to step 4.

4. Determine the key row

Select the smallest (most) negative value under the X_B column. The row that indicates the smallest negative value is the key row.

5. Determine the key column

Select the values of the non basic variables in the index row ($z_j c_j$), and divide these values by the corresponding values of the key row determined in the previous step. Specifically,

$$\text{Key column} = \text{Min}\left\{ \left| \frac{z_j\, c_j}{a_{ij}} \right| : a_{ij} < 0 \right\}$$

7. Revise the Solution

If all basic variables have non-negative values, an optimal solution has been obtained. If there are basic variables having negative values, then go to step 3.

The rules for determining a key column and key row differentiate the dual simplex method from the standard simplex method.

13. Write the scope of Operation Research.

Scope of Operation Research

In recent years of organized development, OR has entered successfully in many different areas of research. It is useful in the following various important fields

In agriculture

With the sudden increase of population and resulting shortage of food, every country is facing the problem of

- Optimum allocation of land to a variety of crops as per the climatic conditions
- Optimum distribution of water from numerous resources like canal for irrigation purposes

Hence there is a requirement of determining best policies under the given restrictions. Therefore a good quantity of work can be done in this direction.

In finance

In these recent times of economic crisis, it has become very essential for every government to do a careful planning for the economic progress of the country. OR techniques can be productively applied

- To determine the profit plan for the company
- To maximize the per capita income with least amount of resources
- To decide on the best replacement policies, etc

In industry

If the industry manager makes his policies simply on the basis of his past experience and a day approaches when he gets retirement, then a serious loss is encounter ahead of the industry. This heavy loss can be right away compensated through appointing a young specialist of OR techniques in business management. Thus OR is helpful for the industry director in deciding optimum distribution of several limited resources like men, machines, material, etc to reach at the optimum decision.

In marketing

With the assistance of OR techniques a marketing administrator can decide upon

- Where to allocate the products for sale so that the total cost of transportation is set to be minimum

- The minimum per unit sale price
- The size of the stock to come across with the future demand
- How to choose the best advertising media with respect to cost, time etc?
- How, when and what to buy at the minimum likely cost?

In personnel management

A personnel manager can utilize OR techniques

- To appoint the highly suitable person on minimum salary
- To know the best age of retirement for the employees
- To find out the number of persons appointed in full time basis when the workload is seasonal

In production management

A production manager can utilize OR techniques

- To calculate the number and size of the items to be produced
- In scheduling and sequencing the production machines
- In computing the optimum product mix
- To choose, locate and design the sites for the production plans

In L.I.C

OR approach is also applicable to facilitate the L.I.C offices to decide

- What should be the premium rates for a range of policies?
- How well the profits could be allocated in the cases of with profit policies?

<u>Role of Operations Research in Decision-Making</u>

The Operation Research may be considered as a tool which is employed to raise the efficiency of management decisions. OR is the objective complement to the subjective feeling of the administrator (decision maker). Scientific method of OR is used to comprehend and explain the phenomena of operating system.

The benefits of OR study approach in business and management decision making may be categorize as follows

Better control

The management of large concerns finds it much expensive to give continuous executive supervisions over routine decisions. An OR approach directs the executives to dedicate their concentration to more pressing matters. For instance, OR approach handles production scheduling and inventory control.

Better coordination

Sometimes OR has been very helpful in preserving the law and order situation out of disorder. For instance, an OR based planning model turns out to be a vehicle for coordinating marketing decisions with the restrictions forced on manufacturing capabilities.

Better system

OR study is also initiated to examine a particular problem of decision making like setting up a new warehouse. Later OR approach can be more developed into a system to be employed frequently. As a result the cost of undertaking the first application may get better profits.

Better decisions

OR models regularly give actions that do enhance an intuitive decision making. Sometimes a situation may be so complex that the human mind can never expect to assimilate all the significant factors without the aid of OR and computer analysis.

14. Write the properties of Primal and Dual Optimal Solution.

In linear programming, the primal problem refers to the problem of maximizing or minimizing an objective function subject to a set of linear constraints, while the dual problem refers to the problem of minimizing or maximizing a different objective function subject to a different set of linear constraints that are related to the primal problem.

Here are the properties of primal and dual optimal solutions:

1. Primal Optimal Solution: Suppose we have the following linear programming problem:

Maximize 3x + 4y

Subject to:

2x + y <= 5 x + 3y <= 8 x, y >= 0

A primal optimal solution to this problem could be (x = 2, y = 1), which maximizes the objective function while satisfying all the constraints.

2. Dual Optimal Solution: The dual problem of the example above would be:

Minimize 5a + 8b

Subject to:

2a + b >= 3 a + 3b >= 4 a, b >= 0

A dual optimal solution to this problem could be (a = 3, b = 0), which minimizes the objective function while satisfying all the constraints.

3. Duality: Using the example above, we can see that the optimal value of the primal problem (11) is equal to the optimal value of the dual problem (11), which confirms the duality principle.
4. Complementary Slackness: For example, if we have the following primal problem:

Maximize 2x + 3y

Subject to:

x + y <= 4 2x + y <= 5 x, y >= 0

and its dual problem:

Minimize 4a + 5b

Subject to:

a + 2b >= 2 a + b >= 3 a, b >= 0

Suppose we have a primal optimal solution of (x = 1, y = 3) and a dual optimal solution of (a = 0, b = 1). We can see that the product of the corresponding variables in the primal and dual solutions is:

x*(a+2b) = 1*(0+2*1) = 2 y*(a+b) = 3*(0+1) = 3

Both of these values are non-zero, but they sum up to the optimal value of the primal problem, which satisfies complementary slackness.

5. Strong Duality: In the example above, we can see that the primal and dual problems have optimal solutions and that they satisfy complementary slackness, which confirms the strong duality property.
6. Unboundedness: For example, if we have the following primal problem:

Maximize x + y

Subject to:

x >= 0 y >= 0 x - y >= 1

We can see that there is no upper limit to the objective function, as the constraints only limit the variables to be non-negative and enforce a lower bound on the difference between x and y.

7. Infeasibility: For example, if we have the following primal problem:

Maximize x + y

Subject to:

x >= 2 y <= 1 x + y <= 1

We can see that the constraints are conflicting and cannot be satisfied simultaneously, which makes the problem infeasible.

8. No Feasible Solution: For example, if we have the following primal problem:

Maximize x + y

Subject to:

x <= -1 y >= 2

There are no feasible solutions to this problem, as both constraints cannot be satisfied by any non-negative values of x and y. Therefore, there is no primal optimal solution.

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19. Write about characteristics of Game Theory.

Game theory is a kind of decision theory in which one's alternative action is determined after taking into consideration all possible alternatives available to an opponent playing the similar game, rather than just by the possibilities of various outcome results. Game theory does not insist on how a game must be played but tells the process and principles by which a particular action should be chosen.

Characteristics of Game Theory

1. Competitive game

A competitive situation is known as **competitive game** if it has the four properties

1. There are limited number of competitors such that $n \geq 2$. In the case of $n = 2$, it is known as **two-person game** and in case of $n > 2$, it is known as **n-person game**.
2. Each player has a record of finite number of possible actions.
3. A play is said to takes place when each player selects one of his activities. The choices are supposed to be made simultaneously i.e. no player knows the selection of the other until he has chosen on his own.
4. Every combination of activities finds out an outcome which results in a gain of payments to every player, provided each player is playing openly to get as much as possible. Negative gain means the loss of same amount.

2. Strategy

The strategy of a player is the determined rule by which player chooses his strategy from his own list during the game. The two types of strategy are

1. Pure strategy
2. Mixed strategy

Pure Strategy

If a player knows precisely what another player is going to do, a deterministic condition is achieved and objective function is to maximize the profit. Thus, the pure strategy is a decision rule always to choose a particular startegy.

Mixed Strategy

If a player is guessing as to which action is to be chosen by the other on any particular instance, a probabilistic condition is achieved and objective function is to maximize the expected profit. Hence the mixed strategy is a choice among pure strategies with fixed probabilities.

Repeated Game Strategies

- In repeated games, the chronological nature of the relationship permits for the acceptance of strategies that are dependent on the actions chosen in previous plays of the game.
- Most contingent strategies are of the kind called as "trigger" strategies.
- For Example trigger strategies

- In prisoners' dilemma: At start, play doesn't confess. If your opponent plays Confess, then you need to play Confess in the next round. If your opponent plays don't confess, then go for doesn't confess in the subsequent round. This is called as the "tit for tat" strategy.

- In the investment game, if you are sender: At start play Send. Play Send providing the receiver plays Return. If the receiver plays keep, then never go for Send again. This is called as the "grim trigger" strategy.

3. Number of persons

When the number of persons playing is 'n' then the game is known as 'n' person game. The person here means an individual or a group aims at a particular objective.

Two-person, zero-sum game

A game with just two players (player A and player B) is known as 'two-person, zero-sum game', if the losses of one player are equal to the gains of the other one so that the sum total of their net gains or profits is zero.

Two-person, zero-sum games are also known as rectangular games as these are generally presented through a payoff matrix in a rectangular form.

4. Number of activities

The activities can be finite or infinite.

5. Payoff

Payoff is referred to as the quantitative measure of satisfaction a person obtains at the end of each play.

6. Payoff matrix

Assume the player A has 'm' activities and the player B has 'n' activities. Then a payoff matrix can be made by accepting the following rules

- Row designations for every matrix are the activities or actions available to player A
- Column designations for every matrix are the activities or actions available to player B
- Cell entry V_{ij} is the payment to player A in A's payoff matrix when A selects the activity i and B selects the activity j.
- In a zero-sum, two-person game, the cell entry in the player B's payoff matrix will be negative of the related cell entry V_{ij} in the player A's payoff matrix in order that total sum of payoff matrices for player A and player B is finally zero.

7. Value of the game

Value of the game is the maximum guaranteed game to player A (maximizing player) when both the players utilizes their best strategies. It is usually signifies with 'V' and it is unique.

21. Explain the steps involved in Monte-Carto simulation.

Monte Carlo simulation is a computational technique used to model the probability of different outcomes in a process that involves random variables. Here are the steps involved in Monte Carlo simulation with an example:

1. Define the problem and determine the random variables: The first step is to define the problem and determine the random variables that affect the outcome of the problem. For example, consider the problem of estimating the probability of rolling a sum of 7 or 11 when rolling two fair six-sided dice. The random variables in this problem are the values on each die.

2. Generate random values: The next step is to generate a large number of random values for each of the random variables. For example, we can use a random number generator to generate 1000 pairs of random values for the two dice.

3. Evaluate the function of interest: Using the random values generated in step 2, evaluate the function of interest. In this example, the function of interest is the probability of rolling a sum of 7 or 11. We can calculate this by counting the number of times the sum of the two dice is 7 or 11 and dividing by the total number of rolls (i.e., 1000 in this case).

4. Repeat steps 2 and 3: Repeat steps 2 and 3 a large number of times to obtain a distribution of the function of interest. In this example, we can repeat steps 2 and 3 10,000 times to obtain a distribution of the probability of rolling a sum of 7 or 11.

5. Analyze the results: Finally, analyze the results of the Monte Carlo simulation. In this example, we can use the distribution obtained in step 4 to estimate the mean probability of rolling a sum of 7 or 11, as well as the uncertainty associated with this estimate (e.g., by calculating the standard deviation of the distribution).

For instance, let's say we generate 1000 pairs of random values for two six-sided dice, and we get the following sums:

2, 8, 4, 11, 6, 7, 9, 7, 5, 10, ...

Using these sums, we can evaluate the function of interest, which is the probability of rolling a sum of 7 or 11. We can count the number of times the sum is 7 or 11 and divide by the total number of rolls (i.e., 1000):

P(sum = 7 or 11) = (number of 7's + number of 11's) / 1000

Suppose we obtain 320 rolls that resulted in a sum of 7 or 11. We can repeat steps 2 and 3 10,000 times to obtain a distribution of the probability of rolling a sum of 7 or 11. Finally, we can analyze the results to estimate the mean probability of rolling a sum of 7 or 11 and the uncertainty associated with this estimate.

22. Explain the difference between a Transportation problem and an assignment problem.

selected 2 days ago by faiz

Transportation Problem	**Assignment Problem**
It is used to optimize the transportation cost.	It is about assigning finite source to finite destination (one source is alloted to one destination).
Number of Source and demand may or may not be equal.	Number of source and number of destination must be equal.
If demand and supply are not equal, then transportation problem is known as Unbalanced Transportation Problem.	If number of rows and number of columns are not equal, then the assignment problem is known as Unbalanced Assignment Problem.
It requires to step to solve: Find Initial Solution using North West, Least Cost or Vogel Approximation Find Optimal Solution using MODI method.	It requires only one step to solve. Hungarian Method is sufficient to find the optimal solutions.

22. Discuss in detail about PERT Estimation Techniques.

Before any activity begins related to the work of a project, every project requires an advanced, accurate time estimate. Without an accurate estimate, no project can be completed within the budget and the target completion date.

Developing an estimate is a complex task. If the project is large and has many stakeholders, things can be more complex.

Therefore, there have been many initiatives to come up with different techniques for estimation phase of the project in order to make the estimation more accurate.

PERT (Program Evaluation and Review Technique) is one of the successful and proven methods among the many other techniques, such as, CPM, Function Point Counting, Top-Down Estimating, WAVE, etc.

PERT was initially created by the US Navy in the late 1950s. The pilot project was for developing Ballistic Missiles and there have been thousands of contractors involved.

After PERT methodology was employed for this project, it actually ended two years ahead of its initial schedule.

The PERT Basics

At the core, PERT is all about management probabilities. Therefore, PERT involves in many simple statistical methods as well.

Sometimes, people categorize and put PERT and CPM together. Although CPM (Critical Path Method) shares some characteristics with PERT, PERT has a different focus.

Same as most of other estimation techniques, PERT also breaks down the tasks into detailed activities.

Then, a Gantt chart will be prepared illustrating the interdependencies among the activities. Then, a *network* of activities and their interdependencies are drawn in an illustrative manner.

In this map, a *node* represents each event. The activities are represented as arrows and they are drawn from one event to another, based on the sequence.

Next, the Earliest Time (TE) and the Latest Time (TL) are figured for each activity and identify the slack time for each activity.

When it comes to deriving the estimates, the PERT model takes a statistical route to do that. We will cover more on this in the next two sections.

Following is an example PERT chart:

The Three Chances

There are three estimation times involved in PERT; Optimistic Time Estimate (TOPT), Most Likely Time Estimate (TLIKELY), and Pessimistic Time Estimate (TPESS).

In PERT, these three estimate times are derived for each activity. This way, a range of time is given for each activity with the most probable value, TLIKELY.

Following are further details on each estimate:

1. TOPT

This is the fastest time an activity can be completed. For this, the assumption is made that all the necessary resources are available and all predecessor activities are completed as planned.

2. TLIKELY

Most of the times, project managers are asked only to submit one estimate. In that case, this is the estimate that goes to the upper management.

3. TPESS

This is the maximum time required to complete an activity. In this case, it is assumed that many things go wrong related to the activity. A lot of rework and resource unavailability are assumed when this estimation is derived.

The PERT Mathematics

BETA probability distribution is what works behind PERT. The expected completion time (E) is calculated as below:

```
E = (TOPT + 4 x TLIEKLY + TPESS) / 6
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At the same time, the possible variance (V) of the estimate is calculated as below:

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V = (TPESS - TOPT)^2 / 6^2
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Now, following is the process we follow with the two values:

- For every activity in the critical path, E and V are calculated.
- Then, the total of all Es are taken. This is the overall expected completion time for the project.
- Now, the corresponding V is added to each activity of the critical path. This is the variance for the entire project. This is done only for the activities in the critical path as only the critical path activities can accelerate or delay the project duration.
- Then, standard deviation of the project is calculated. This equals to the square root of the variance (V).
- Now, the normal probability distribution is used for calculating the project completion time with the desired probability.