Chapter 4

Step-index fibers (STU)

This chapter discusses the wave propagation in step-index fibers. The field calculation in the step-index fiber as well as the chromatic dispersion are explained.
The simplest form of an optical waveguide consists of a light-conducting fiber core with the refractive index n_1 and a fiber cladding with the refractive index $n_2 < n_1$, where the difference is about a few percent or even lower.
The fiber core diameter is typically of the order $2a \approx 10 - 100\,\mu\text{m}$ and the diameter of the core and cladding is $D \approx 125\,\mu\text{m}$.

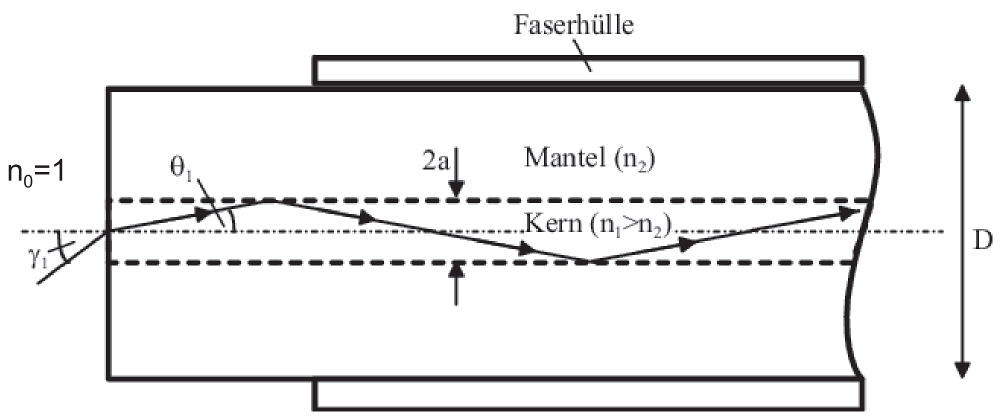

Figure 4.1: Schematic of a step index fiber

Typically the step-index fiber has a refractive index profile of

$$n(r) = \begin{cases} n_1 & \text{für } r \leq a \\ n_2 & \text{für } a < r \leq \dfrac{D}{2} \end{cases} \tag{4.1}$$

θ_1 is defined as the angle of the incident wave to the fiber axis in the fiber and γ_1 is the angle of the incident wave coupling from the free space ($n_0 = 1$). The wave is then guided through the core, if $\theta_1 < \theta_{1g}$ applies.

This can be obtained:

$$\sin(\theta_{1g}) = \sqrt{1 - \cos^2(\theta_{1g})} = \frac{1}{n_1}\sqrt{n_1^2 - n_2^2} \tag{4.2}$$

and with Snell's Law the result is:

$$\sin(\gamma_{1g}) = n_1 \sin(\theta_{1g}) = \sqrt{n_1^2 - n_2^2} = A_N \tag{4.3}$$

A_N is called the *numerical aperture* and specifies the maximum angle γ_1, for which the wave remains in the core. A typical value is $A_N = 0,2$ leading to a maximum angle of incidence $\gamma_{1g} = 11,5°$.

4.1 Field calculation in a step-index fiber

Since the beam examination only describes the wave propagation correct for $\lambda \to 0$, the question arises about the calculation of the field in the step-index fiber. Therefore the so-called *eigenmodes (or normal modes)* are determined. They are given by assuming that a transverse field distribution $\underline{\vec{E}}(x, y)$, propagates unaltered in z-direction with the propagation constant β. The refractive index is assumed to be independent of z. The field approach is then

$$\underline{\vec{E}}(x, y, z) = \underline{\vec{E}}(x, y) \exp(-j\,\beta\, z) \tag{4.4}$$

Firstly the order of β should be estimated. From the beam examination in fig. 4.1 it follows:

$$\beta = k_0 n_1 \cos(\theta_1) \tag{4.5}$$

For the guided wave $0 < \theta_1 < \theta_{1g}$ can be determined, hence:

$$k_0 n_2 < \beta < k_0 n_1 \tag{4.6}$$

For the Cartesian field components $\underline{E}_x$ and $\underline{E}_y$ the wave equation applies to both the core and the cladding.

$$\triangle \underline{E}_{x,y} + k_0^2 n_i^2 \underline{E}_{x,y} = 0 \tag{4.7}$$

with $i = 1, 2$. The use of eq. (4.4) yields

$$\frac{\partial^2 \underline{E}_{x,y}}{\partial z^2} = -\beta^2 \underline{E}_{x,y} \tag{4.8}$$

and consequently with eq. (4.7) the wave equation can be determined as

$$\triangle_t \underline{E}_{x,y} + \left(k_0^2 n_i^2 - \beta^2\right) \underline{E}_{x,y} = 0 \tag{4.9}$$

with the transverse Laplace operator $\triangle_t = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$. In the following, we assume a weakly guiding waveguide with

$$\frac{n_1 - n_2}{n_1} \ll 1 \tag{4.10}$$

thus leading to the following equations:

$$|k_0^2 n_i^2 - \beta^2| \ll \beta^2 \qquad \text{with} \qquad \beta = k_0 \cdot n_{eff} \qquad \text{and} \qquad n_2 < n_{eff} < n_1 \tag{4.11}$$

and therefore

$$|\triangle_t| \ll \beta^2 \tag{4.12}$$

Hence

$$\left|\frac{\partial}{\partial x}\right|, \left|\frac{\partial}{\partial y}\right| \ll \beta \tag{4.13}$$

Considering the boundary conditions, the fields of the propagable waves from eq. (4.7) are applied. The field components $\underline{E}_z$, $\underline{H}_z$ and $\underline{E}_\varphi$, $\underline{H}_\varphi$ are continuous for $r = a$, where a is the core radius (fig. 4.2).

4.1.1 Field components of the eigenmodes of a weakly guiding step-index fiber

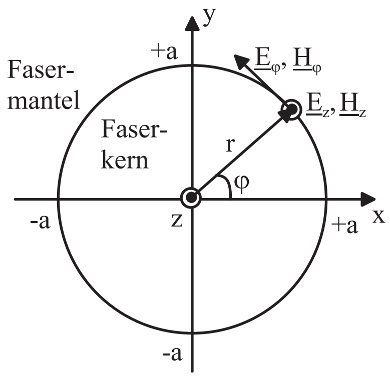

Figure 4.2: Boundary conditions of the step-index fiber

Let us assume that eq. (4.9) was solved for $\underline{E}_x$ and $\underline{E}_y$. The question then arises as on how to compute the other field components. By using the Maxwell equation $-\nabla \times \underline{\vec{E}} = j\omega\mu\underline{\vec{H}}$, we can obtain:

$$\underline{H}_z = -\frac{1}{j\omega\mu_0}\left(\frac{\partial \underline{E}_y}{\partial x} - \frac{\partial \underline{E}_x}{\partial y}\right) \tag{4.14}$$

From the Maxwell equation $\nabla \times \underline{\vec{H}} = j\omega\varepsilon_0 n_i^2 \underline{\vec{E}}$ it results for $\underline{E}_x$ and $\underline{E}_y$:

$$\underline{E}_x = \frac{1}{j\omega\varepsilon_0 n_i^2}\left(\frac{\partial \underline{H}_z}{\partial y} + j\beta\underline{H}_y\right) \tag{4.15}$$

$$\underline{E}_y = \frac{1}{j\omega\varepsilon_0 n_i^2}\left(-\frac{\partial \underline{H}_z}{\partial x} - j\beta\underline{H}_x\right) \tag{4.16}$$

Hence, for the known $\underline{E}_x$, $\underline{E}_y$ and $\underline{H}_z$ from eq. (4.14), $\underline{H}_x$ and $\underline{H}_y$ can be determined. Assuming a weakly guiding fiber (see eq. (4.13)) $\partial \underline{H}_z/\partial y$ and $\partial \underline{H}_z/\partial x$ in eq. (4.15) and eq. (4.16) can be neglected. Therefore $\underline{H}_x$ and $\underline{H}_y$ can be expressed as

$$\underline{H}_x \approx -\frac{\omega\varepsilon_0 n_i^2}{\beta}\underline{E}_y \tag{4.17}$$

$$\underline{H}_y \approx \frac{\omega\varepsilon_0 n_i^2}{\beta}\underline{E}_x \tag{4.18}$$

$\underline{E}_z$ is thus given by

$$\underline{E}_z = \frac{1}{j\omega\varepsilon_0 n_i^2}\left(\frac{\partial \underline{H}_y}{\partial x} - \frac{\partial \underline{H}_x}{\partial y}\right) \approx \frac{1}{j\beta}\left(\frac{\partial \underline{E}_x}{\partial x} - \frac{\partial \underline{E}_y}{\partial y}\right) \tag{4.19}$$

This shows that all six field components can be deduced from $\underline{E}_x$ and $\underline{E}_y$. For weakly guiding fibers, n_i can be approximated by n_1.

4.1.2 Continuity of the tangential field components

The tangential field components of the fiber $\underline{E}_\varphi$, $\underline{H}_\varphi$, $\underline{E}_z$ and $\underline{H}_z$ have to be continuous at $r = a$. $\underline{E}_\varphi$ and $\underline{H}_\varphi$ are thus of the form

$$\underline{E}_\varphi = \underline{E}_y \cos(\varphi) - \underline{E}_x \sin(\varphi) \tag{4.20}$$

$$\underline{H}_\varphi = \underline{H}_y \cos(\varphi) - \underline{H}_x \sin(\varphi) \tag{4.21}$$

As a result of eq.(4.17), eq. (4.18), eq. (4.20) and eq. (4.21) with $n_i \approx n_1$, it can be deduced that $\underline{E}_x$ and $\underline{E}_y$ have to be continuous (at $r = a$), if $\underline{E}_\varphi$ and $\underline{H}_\varphi$ are also to be continuous. Since this continuity should apply for all φ, $\frac{\partial \underline{E}_x}{\partial \varphi}$ and $\frac{\partial \underline{E}_y}{\partial \varphi}$ at $r = a$ must also be continuous.
As already shown in eq. (4.14) and eq. (4.19) $\underline{E}_z$ and $\underline{H}_z$ can be determined from $\underline{E}_x$ and $\underline{E}_y$, in which $\underline{E}_x$ and $\underline{E}_y$ are derived with respect to x and y, respectively. The derivatives of $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$ are in cylindrical coordinates

$$\frac{\partial}{\partial x} = \cos(\varphi)\frac{\partial}{\partial r} - \frac{1}{r}\sin(\varphi)\frac{\partial}{\partial \varphi} \tag{4.22}$$

$$\frac{\partial}{\partial y} = \sin(\varphi)\frac{\partial}{\partial r} + \frac{1}{r}\cos(\varphi)\frac{\partial}{\partial \varphi} \tag{4.23}$$

Inserted into eq. (4.14) and eq. (4.19) yields

$$\underline{E}_z = \frac{1}{j\beta}\left(\cos(\varphi)\frac{\partial \underline{E}_x}{\partial r} - \frac{1}{r}\sin(\varphi)\frac{\partial \underline{E}_x}{\partial \varphi} + \sin(\varphi)\frac{\partial \underline{E}_y}{\partial r} + \frac{1}{r}\cos(\varphi)\frac{\partial \underline{E}_y}{\partial \varphi}\right) \tag{4.24}$$

$$\underline{H}_z = -\frac{1}{j\omega\mu_0}\left(\cos(\varphi)\frac{\partial \underline{E}_y}{\partial r} - \frac{1}{r}\sin(\varphi)\frac{\partial \underline{E}_y}{\partial \varphi} - \sin(\varphi)\frac{\partial \underline{E}_x}{\partial r} - \frac{1}{r}\cos(\varphi)\frac{\partial \underline{E}_x}{\partial \varphi}\right) \tag{4.25}$$

Since $\frac{\partial \underline{E}_x}{\partial \varphi}$ and $\frac{\partial \underline{E}_y}{\partial \varphi}$ at $r = a$, due to the continuity of $\underline{E}_\varphi$ and $\underline{H}_\varphi$, have to be continuous, according to eq. (4.24) and eq. (4.25), $\frac{\partial \underline{E}_x}{\partial r}$ and $\frac{\partial \underline{E}_y}{\partial r}$ at $r = a$ also have to be continuous.
The condition of continuity of $\underline{E}_\varphi$, $\underline{H}_\varphi$, $\underline{E}_z$ and $\underline{H}_z$ can be replaced by the requirement of continuity of $\underline{E}_x$, $\underline{E}_y$, $\frac{\partial \underline{E}_x}{\partial r}$ and $\frac{\partial \underline{E}_y}{\partial r}$.

4.1.3 Linearly polarized LP-waves

Since $\underline{E}_x$ and $\underline{E}_y$ for weakly guided fibers are <u>neither</u> coupled by the wave equation (4.9) nor for the boundary condition at $r = a$, eigenmodes can be found, where e.g. $\underline{E}_y = 0$. These eigenmodes are called *linearly polarized LP-wave*, which are defined as e.g.

$$\underline{E}_x = \psi(r, \varphi)\exp(-j\beta z) \tag{4.26}$$

and

$$\psi(r,\varphi) = \begin{cases} \psi_1(r,\,\varphi) & \text{für} \quad r \le a \\ \psi_2(r,\,\varphi) & \text{für} \quad r > a\,. \end{cases} \tag{4.27}$$

From the wave equation (4.9), the scalar wave equation can be deduced

$$\triangle_t \psi_i + \left(k_0^2 n_i^2 - \beta^2\right)\psi_i = 0 \tag{4.28}$$

With $i = 1, 2$ and the boundary conditions

$$\psi_1|_{r=a} = \psi_2|_{r=a} \tag{4.29}$$

$$\frac{\partial \psi_1}{\partial r}|_{r=a} = \frac{\partial \psi_2}{\partial r}|_{r=a}\,. \tag{4.30}$$

Initially, the following normalizations are introduced:

Fiber parameter: $$V = k_0 a\sqrt{n_1^2 - n_2^2} = k_0 \cdot a \cdot A_N \tag{4.31}$$

Normalized propagation constant: $$B = \frac{\frac{\beta^2}{k_0^2} - n_2^2}{n_1^2 - n_2^2} = \frac{\left(\frac{\beta}{k_0} - n_2\right)\cdot\left(\frac{\beta}{k_0} + n_2\right)}{(n_1 - n_2)\cdot(n_1 + n_2)} \approx \frac{\frac{\beta}{k_0} - n_2}{n_1 - n_2} \tag{4.32}$$

Core parameter: $$u = V\sqrt{1 - B} = a\sqrt{k_0^2 n_1^2 - \beta^2} \tag{4.33}$$

Cladding parameter: $$v = V\sqrt{B} = a\sqrt{\beta^2 - k_0^2 n_2^2} \tag{4.34}$$

Inserting these normalizations in eq. (4.28), the result is

$$a^2 \triangle_t \psi_1 + u^2 \psi_1 = 0 \qquad \text{for } r \le a \tag{4.35}$$

$$a^2 \triangle_t \psi_2 - v^2 \psi_2 = 0 \qquad \text{for } r \ge a \tag{4.36}$$

Solutions of these differential equations are

$$\psi_1(r,\varphi) = A_1 J_l\left(r\frac{u}{a}\right)\begin{Bmatrix}\cos(l\cdot\varphi)\\ \sin(l\cdot\varphi)\end{Bmatrix} \tag{4.37}$$

$$\psi_2(r,\varphi) = A_2 K_l\left(r\frac{v}{a}\right)\begin{Bmatrix}\cos(l\cdot\varphi)\\ \sin(l\cdot\varphi)\end{Bmatrix} \tag{4.38}$$

Here J_l is a Bessel function and K_l is a modified Hankel function of integer order. For low orders, they are shown in figure 4.3 and 4.4. Their derivatives with respect to the argument are given by

$$\frac{dJ_l(x)}{dx} = J_l'(x) = -J_{l+1}(x) + \frac{l}{x}J_l(x) = J_{l-1}(x) - \frac{l}{x}J_l(x) \tag{4.39}$$

$$\frac{dK_l(x)}{dx} = K_l'(x) = -K_{l+1}(x) + \frac{l}{x}K_l(x) = -K_{l-1}(x) - \frac{l}{x}K_l(x) \tag{4.40}$$

Hence, following from eq. (4.29) and eq. (4.30), they can be written as

$$A_1 J_l(u) = A_2 K_l(v) \tag{4.41}$$

$$A_1 \frac{u}{a} J_l'(u) = A_2 \frac{v}{a} K_l'(v) \tag{4.42}$$

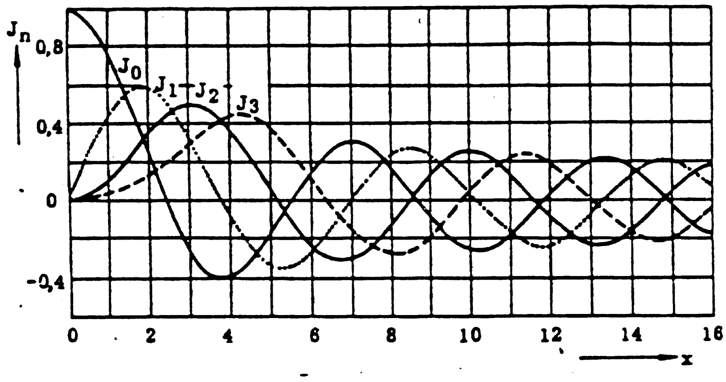

Figure 4.3: Bessel function of integer order

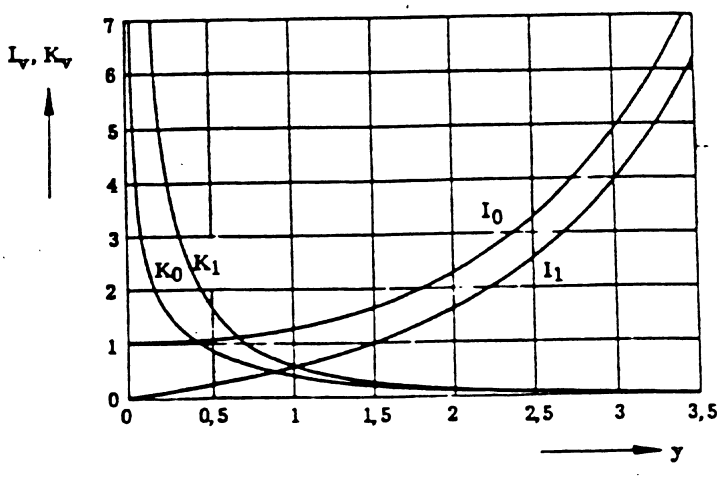

Figure 4.4: Modified Bessel- and Hankel function of 0th and 1st order

Dividing now the two equations by themselves, the characteristic equation to determine the propagation constants β is obtained (u and v contain only β as unknown variable).

$$\frac{u \cdot J_l'(u)}{J_l(u)} = \frac{v \cdot K_l'(v)}{K_l(v)} \tag{4.43}$$

By using eq. (4.39) and eq. (4.40), it yields

$$-\frac{u \cdot J_{l+1}(u)}{J_l(u)} + \frac{v \cdot K_{l+1}(v)}{K_l(v)} = 0 \tag{4.44}$$

with $V^2 = u^2 + v^2$.

For a given V and a given circumferential order l, the propagation constant from eq. (4.44) can be determined numerically. The equation generally has multiple solutions, which are numbered with $p = 1, 2, 3....$ p denotes the number of the field maxima in the radial direction. Therefore, the term LP_{lp}-wave is chosen, with l for the circumferential and p for the radial order. (LP field distributions of some LP$_{lp}$-waves are shown in figure 4.5.)

With the given dimension (a, λ, n_1, n_2) V is implied. Thus u and v can be deduced from eq. (4.44). With eq. (4.33) and eq. (4.34) β and therefore from eq. (4.37) and eq. (4.38) the field distribution $\psi(r, \varphi)$ can be derived. The solution of eq. (4.44) is shown in fig. 4.6. It shows the normalized phase constant B (see eq. (4.32)) as a function of V.

4.1.4 Single mode range and condition

In fig. 4.6 it is obvious, that only the LP_{01}-wave for arbitrarily small V (V $\sim$ frequency) is capable of propagation. This wave is also called the *fundamental mode*.

The characteristic equation of the LP_{01}-wave is, in accordance with eq. (4.44):

$$\frac{u \cdot J_1(u)}{J_0(u)} - \frac{v \cdot K_1(v)}{K_0(v)} = 0 \tag{4.45}$$

This equation leads to solutions even for arbitrarily small Vs. In this case, however, the wave propagates essentially in the fiber cladding since $B \approx 0$ and therefore $\beta \approx k_0 n_2$. Hence the wave is badly guided. For a better guidance V should be something bigger, e.g. $V > 1.5$.

For the fiber parameter $1.5 < V < 2.5$ eq. (4.45) is approximately solved by:

$$v = 1,1428 \cdot V - 0,996 \tag{4.46}$$

In the single-mode range the LP_{11}-wave (compare with fig. 4.6) is not guided. It follows that the characteristic equation of the LP_{11}-wave from eq. (4.43) with $l = 1$ is:

$$\frac{u \cdot J_0(u)}{J_1(u)} + \frac{v \cdot K_0(v)}{K_1(v)} = 0 \tag{4.47}$$

The limit of the propagation ability of the LP_{11}-wave is reached when the cladding parameter is $v = 0$. Taking this condition the core parameter u_c can be determined from eq. (4.47).

$$\frac{u_c J_0(u_c)}{J_1(u_c)} = 0 \quad \Rightarrow \quad J_0(u_c) = 0 \tag{4.48}$$

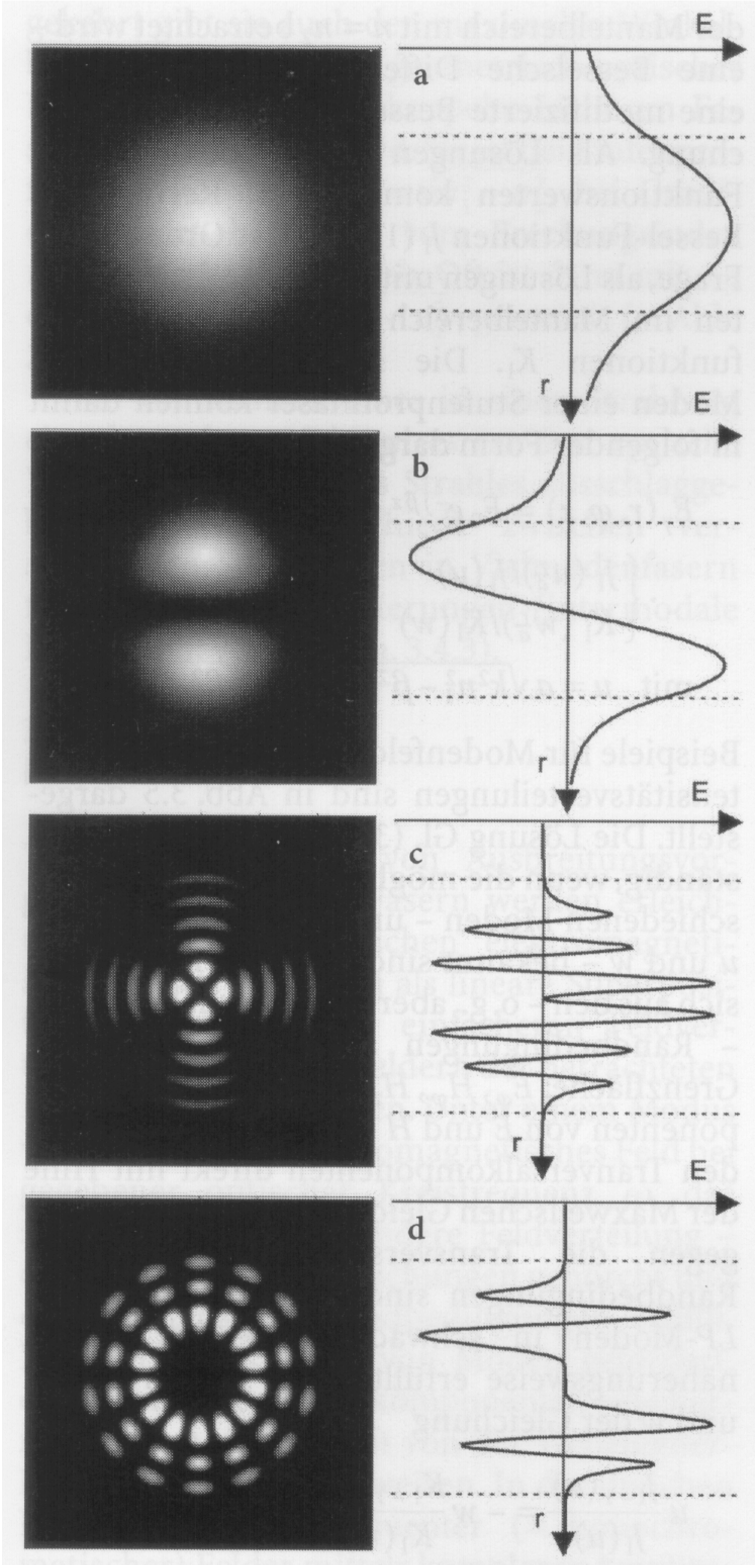

Figure 4.5: Field distributions of some LP modes. From the top: LP_{01}, LP_{11}, LP_{25} and LP_{73} (from: Voges/Petermann, Handbuch Optische Kommunikationstechnik)

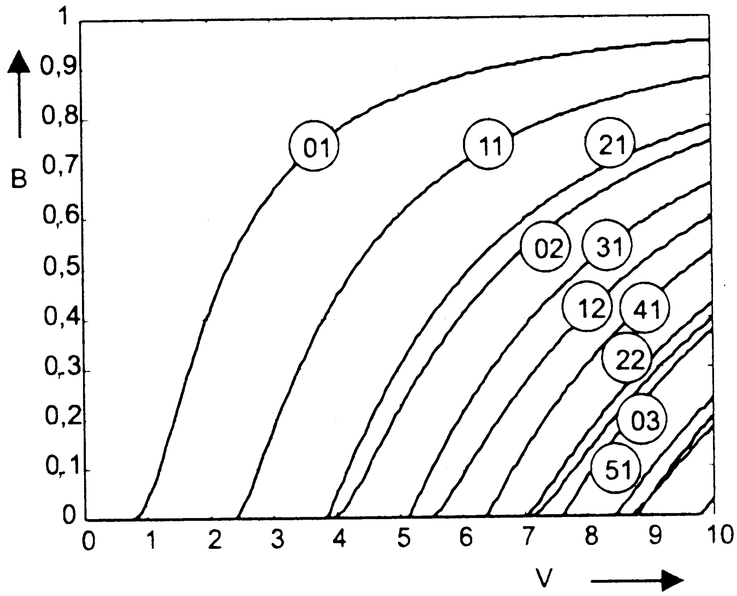

Figure 4.6: Normalized phase constant B of LP_{lp}-wave in weakly guided step-index fibers (from: Voges/Petermann, Handbuch Optische Kommunikationstechnik)

The core parameter u_c corresponds to the first zero of the Bessel function $J_0(x)$. Hence the core parameter is calculated to $u_c(LP_{11}) = 2.405$. With $V = \sqrt{u^2 + v^2}$ it follows that $V_c = 2.405$. This means, that for a fiber parameter $V < 2.405$, a single-mode fiber exits (although the remaining LP_{01}-wave is still propagable in two polarizations ($\underline{E}_x$ and $\underline{E}_y$)). Typically, a fiber parameter $V > 1.5$ is chosen because otherwise the wave is too weakly centered on the fiber core.
For example these fiber parameters are given:

Fiber diameter:	$2a = 8\,\mu\text{m}$
Relative refractive index difference:	$\Delta = \frac{n_1 - n_2}{n_1} = 3 \cdot 10^{-3}$
Numerical aperture:	$A_N = 0,116$
Fiber parameter:	$V = 2,9\,\frac{\mu\text{m}}{\lambda}$

With this fiber a single-mode operation is possible for the wavelength range of $1,2\,\mu\text{m} < \lambda < 1,9\,\mu\text{m}$.
$\psi(r)$ of the fundamental mode is often approximated by a Gaussian distribution:

$$\psi(r) = A_0 \exp\left(-\frac{r^2}{w^2}\right). \tag{4.49}$$

Here w corresponds to the spot radius.
Assuming a step-index fiber with $V > 1.2$, then $\frac{w}{a} \approx 0.65 + \frac{1.619}{V^{3/2}} + \frac{2.879}{V^6}$. Thus with increasing V the spot radius decreases. This corresponds to an increasing concentration of the field in the fiber core.

4.2 Chromatic dispersion

Even with a single-mode fiber it has to be taken into account that the group delay of the LP_{01}-wave is wavelength dependent (chromatic dispersion), which affects the transmission characteristics. The group delay of the fundamental mode per length is:

$$\tau = \frac{\mathrm{d}\beta}{\mathrm{d}\omega} \tag{4.50}$$

With eq. (4.32) the propagation constant β can be described as:

$$\beta = k_0(B(n_1 - n_2) + n_2) \tag{4.51}$$

$$\Rightarrow \quad \tau = \frac{\mathrm{d}(k_0(B(n_1 - n_2) + n_2))}{\mathrm{d}\omega} \tag{4.52}$$

For simplicity, the assumption is made, that the refraction indices n_1 and n_2 are equally dependent upon ω:

$$\frac{\mathrm{d}n_1}{\mathrm{d}\omega} = \frac{\mathrm{d}n_2}{\mathrm{d}\omega} \tag{4.53}$$

$$\Rightarrow \quad \frac{\mathrm{d}(n_1 - n_2)}{\mathrm{d}\omega} = 0 \tag{4.54}$$

$$\Rightarrow \quad \frac{\mathrm{d}A_N}{\mathrm{d}\omega} = \frac{\mathrm{d}\left(\sqrt{n_1^2 - n_2^2}\right)}{\mathrm{d}\omega} \approx 0 \tag{4.55}$$

Thus the equation of the group delay eq. (4.52) can be simplified to:

$$\tau = (n_1 - n_2)\frac{\mathrm{d}(k_0 B)}{\mathrm{d}\omega} + \frac{\mathrm{d}(k_0 n_2)}{\mathrm{d}\omega} = \frac{n_1 - n_2}{c} \cdot \frac{\mathrm{d}(V \cdot B)}{\mathrm{d}V} + \frac{1}{c}N_2 \tag{4.56}$$

The chromatic dispersion is the derivative of the group delay with respect to the wavelength:

$$\frac{\mathrm{d}\tau}{\mathrm{d}\lambda} = \underbrace{-\frac{n_1 - n_2}{c \cdot \lambda} \cdot \frac{V\mathrm{d}^2(V \cdot B)}{\mathrm{d}V^2}}_{D_W \hat{=} \text{wave guide dispersion}} \quad \underbrace{-\frac{\lambda}{c} \cdot \frac{\mathrm{d}^2 n_2}{\mathrm{d}\lambda^2}}_{D_M \hat{=} \text{material dispersion}} \tag{4.57}$$

Thus the chromatic dispersion consists essentially of two parts (fig. 4.7):

1. the waveguide dispersion D_W and
2. the material dispersion D_M.

While the material dispersion was discussed in the chapter GRU in detail, the waveguide dispersion arises mainly from the curvature of the $B(V)$ characteristics. For illustration, the normalized phase constant B, the term $\mathrm{d}(V \cdot B)/\mathrm{d}V$, and the term $V \cdot \mathrm{d}^2(V \cdot B)/\mathrm{d}V^2$ are shown in fig. 4.8 as a function of V for the LP_{01}-wave of a single-mode fiber.

For single mode fibers with the fiber parameter $V < 2,4$ the derivative $V \cdot \mathrm{d}^2(V \cdot B)/\mathrm{d}V^2$ becomes positive and therefore the wave guide dispersion is negative. At wavelengths of $\lambda > 1,3\,\mu\mathrm{m}$ the material dispersion D_M is positive. This can be used to adjust the dimensions of the fiber, so that the zero of the total dispersion is adjusted to the wavelengths $\lambda > 1,3\,\mu\mathrm{m}$. In particular, the zero point of the entire chromatic dispersion for $\lambda \approx 1,55\,\mu\mathrm{m}$ can be moved to the point, where the minimum attenuation is achieved.

Examples:

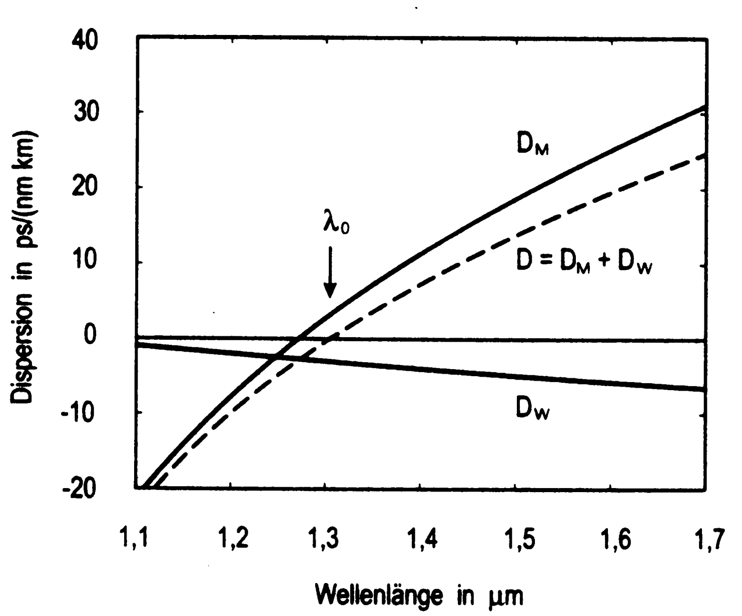

Figure 4.7: waveguide dispersion D_W and material dispersion D_M of a standard single mode fiber (from: Voges/Petermann, Handbuch Optische Kommunikationstechnik)

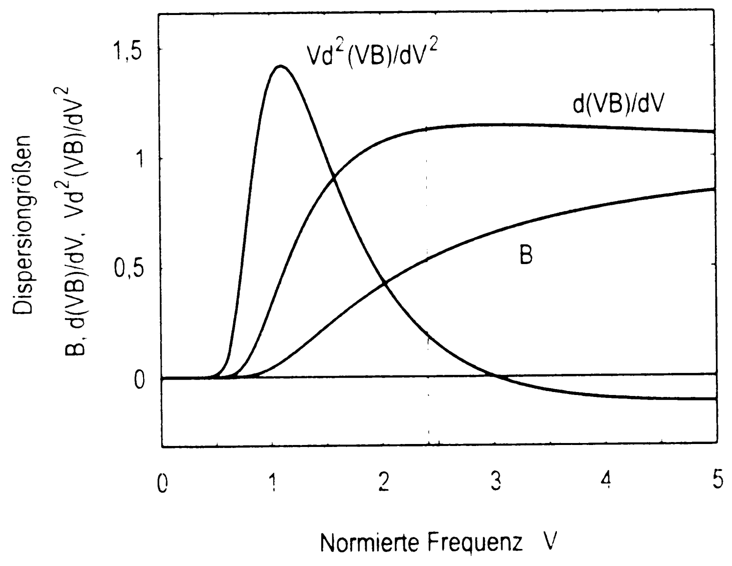

Figure 4.8: Dispersion values of the LP_{01}-fundamental wave with weakly guided step-index fiber (from: Voges/Petermann, Handbuch Optische Kommunikationstechnik)

1. Standard single-mode fiber
   for example a standard single-mode fiber has the following dimensions: $a = 4\,\mu\text{m}$, $\Delta = \frac{n_1 - n_2}{n_1} = 3 \cdot 10^{-3}$, respectively $n_1 - n_2 = 4.5 \cdot 10^{-3}$. For a wavelength of $\lambda = 1.55\,\mu\text{m}$, a value of $V = 1.88$ follows. According to fig. 4.8 $V \cdot \mathrm{d}^2(V \cdot B)/\mathrm{d}V^2 = 0.58$. Using these values and eq. (4.57), a waveguide dispersion of $D_W = -5.6\,\frac{\text{ps}}{\text{km}\cdot\text{nm}}$ follows. This dispersion can compensated partially by the material dispersion. For illustration the single components of the chromatic dispersion of such a fiber are shown in fig. 4.7.

2. Dispersion-shifted (single-mode) fibers
   By varying the fiber parameters, higher fiber dispersion values can be obtained, for example to compensate and even to overcompensate for the material dispersion $D_M = 20\,\frac{\text{ps}}{\text{km}\cdot\text{nm}}$ at the given wavelength $\lambda = 1,55\,\mu\text{m}$ (e.g. for a so called dispersion-compensating fibers). According to eq. (4.57), higher values of the waveguide dispersion for an increased refractive index difference and lower V-values are achievable. For example with the fiber dimensions $a = 2.4\,\mu\text{m}$, $\Delta = \frac{n_1 - n_2}{n_1} = 5 \cdot 10^{-3}$, respectively $n_1 - n_2 = 7.5 \cdot 10^{-3}$ with $\lambda = 1.55\,\mu\text{m}$ $V = 1.46$ and $V \cdot \mathrm{d}^2(V \cdot B)/\mathrm{d}V^2 = 1.13$ is obtained. Thus a significantly larger magnitude of the fiber dispersion results in $D_W = -18,2\,\frac{\text{ps}}{\text{km}\cdot\text{nm}}$, with which the material dispersion at $\lambda = 1.55\,\mu\text{m}$ can be compensated substantially in order to achieve an improved wave guidance for the required small V-values. The highly light-guiding fiber core is, again, usually surrounded by a refractive index-ring, as shown schematically in fig. 4.9.

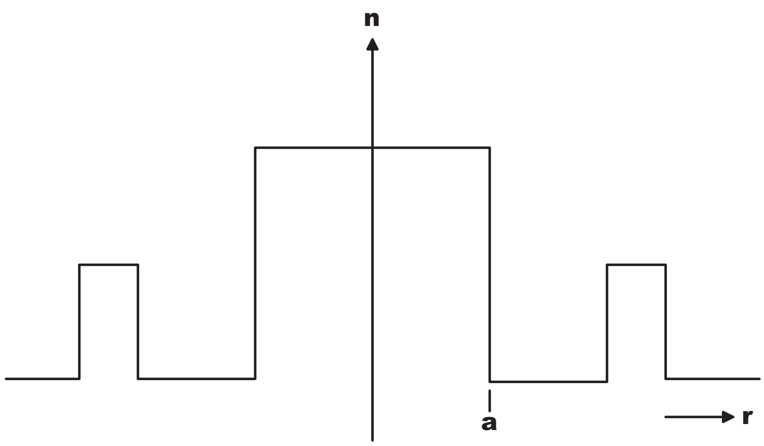

Figure 4.9: Schematic of the refraction profile for a dispersion displaced fiber