Automata

Unit-1

Automata is abstract machine

- lexical analysis
- semantic analysis (logic processing)

Automata works on complexity and conductability.
Basic symbols, strings, length of string, languages, power of string.

Symbols:

$w, x, y, z, \Sigma, \epsilon$

Basically any character can be a symbol

strings:

In automata we use 0 or 1

Eg: $W = \{0, 1\}$

Generate the valid string with n number of 0's and n no of 1's where $n \geq 0$

sol:

0011

0

null string = ϵ

→ non-required transistion

Length of string:

Represented by Σ → collection of string.

$W = 1100110011$

length of string = $|W| = 10$

→ modulus.

Power of string : Σ^N

Suppose $W = \{0, 1\}$

$\Sigma^0 = \epsilon$ (null string)
$\Sigma^1 = 0, 1$ (either 0 or 1)
$\Sigma^2 = 00, 01, 10, 11$
⋮

$\Sigma^* =$ deals with universal set of substring
$\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \ldots)$
$\Sigma^+ =$ should not include empty set.
$(\Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \ldots)$

$$\boxed{\Sigma^* = \Sigma^0 \cup \Sigma^+}$$

Languages : L

collection of strings represented by L
(cumulative collection of string)

$L = \{01, 10, 110, 011\}$.
It includes null strings

Automata

- State finite machine
- Infinite machine (lot of ambiguities)

Finite state machine

- with o/p
  - DFA
  - NFA
  - Є NFA
    - (epsilon)
- without o/p (no use in real time)

NFA has Є but DFA never ever has Є.

Question:
mostly NFA to DFA

Q → Set of all inputs
q_0 → Start state
F → Set of final States
δ → Transition state
Σ → Set of states.

When Σ in NFA in final state is to avoid ambiguity.

∇ Applications
∇ Transition diag
∇ List of transition
∇ Transition Table

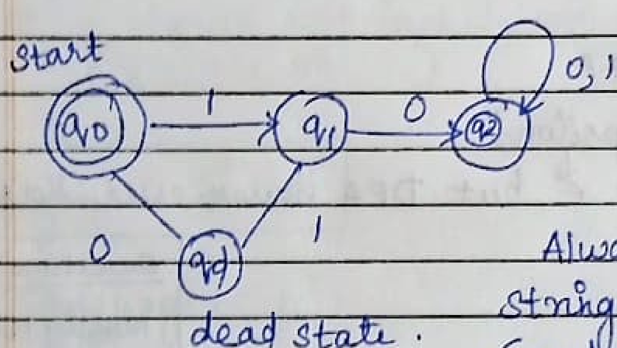
DFA :
Deterministic Finite Automata

Applications are :
→ Designing circuit diagram
→ Lexical analyser.

⊙ Transition Diagram:

start & end state both double circle

Q: Construct a DFA which accepts all '11' starting with 10.



Always there is a string called dead state. (need not to draw just for understanding)

⊙ List of transition

$\Sigma = \{\{q_0, q_1, q_2\}, \{0,1\}, \delta, \{q_2\}\}$

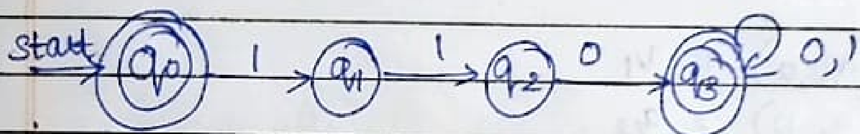
$\delta(q_0, 1) = q_1$
$\delta(q_1, 0) = q_2$
$\delta(q_2, 0) = q_2$
$\delta(q_2, 1) = q_2$

⊙ Transition Table

	0	1
q_0	-	q_1
q_1	q_2	-
q_2	q_2	q_2

Q: Construct a DFA accepts all strings starting with 110.

DFA will never ever accepts null state so no null state.



List of Transition

$\Sigma = \{ \{q_0, q_1, q_2, q_3\}, \{0,1\}, \delta, \{q_3\} \}$

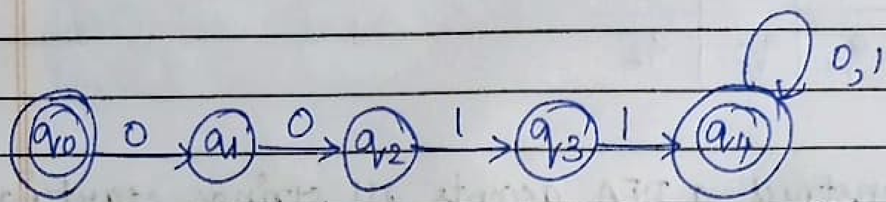
$\delta(q_0,1) = q_1$
$\delta(q_1,1) = q_2$
$\delta(q_2,0) = q_3$
$\delta(q_3,0) = q_3$
$\delta(q_3,1) = q_3$

Transition Table

	0	1
q_0	-	q_1
q_1	-	q_2
q_2	q_3	-
q_3	q_3	q_3

Q: Construct a DFA that accepts n no of 0's and followed by n no of 1's where $n=2$

0011



List of transitions

$\Sigma = \{\{q_0, q_1, q_2, q_3, q_4\}, \{0,1\}, \delta, \{q_4\}\}$

$\delta(q_0, 0) = q_1$
$\delta(q_1, 0) = q_2$
$\delta(q_2, 1) = q_3$
$\delta(q_3, 1) = q_4$
$\delta(q_4, 0) = q_4$
$\delta(q_4, 1) = q_4$

Transition Table

	0	1
q_0	q_1	-
q_1	q_2	-
q_2	-	q_3
q_3	q_4	-
q_4	q_4	q_4

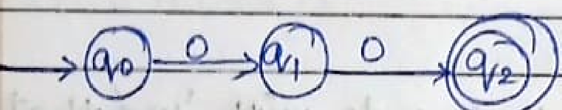
Construction of NFA

DFA → only one state.
NFA → for single i/p → transmit to multiple states

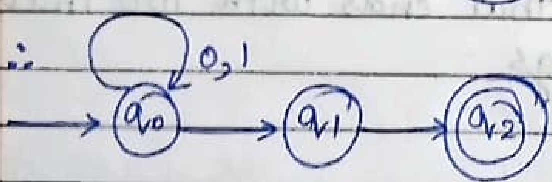
How to draw for a NFA :

construct a NFA that accepts all strings ends with 00.
↳ any no of zeroes and 1's before that.

eg. 10100
100100
0000100.



∴



$q_0 - \{q_0, q_1\}$

List of transitions

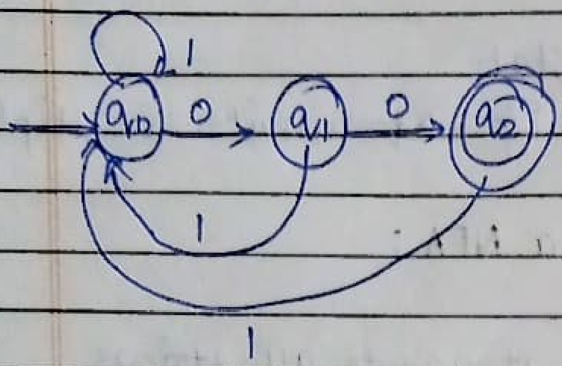
$\Sigma = \{\{q_0, q_1, q_2\}, \{0,1\}, \delta, \{q_0, q_2\}\}$

Transition table.

	0	1
q_0	$\{q_0, q_1\}$	q_0
q_1	q_2	–
q_2	–	–

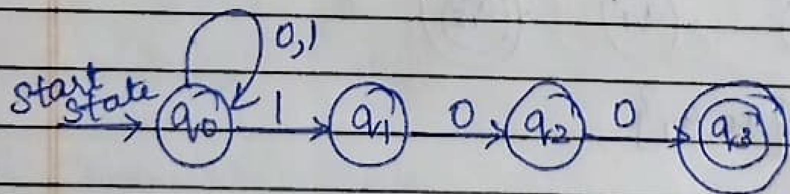
NFA results in ambiguity.

By using this we can draw DFA



	0	1
q_0	q_1	q_0
q_1	q_2	q_0
q_2	q_2	q_0

Q: NFA and DFA that ends with '100' that accepts all strings

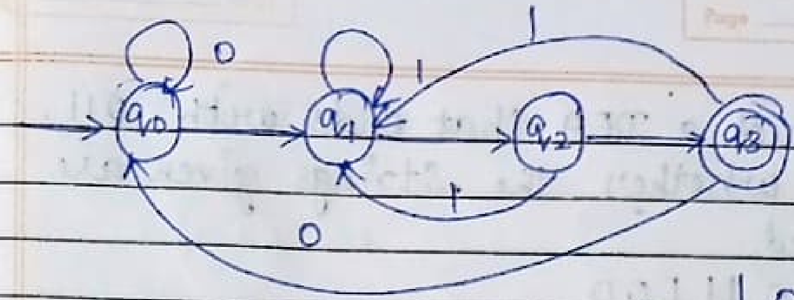


List of transitions

$$S = \{\{q_0, q_1, q_2, q_3\}, \{0, 1\}, \delta, \{q_0, q_3\}\}$$

Transition Table

	0	1
q_0	q_0	$\{q_0, q_1\}$
q_1	q_2	–
q_2	q_3	–
q_3	–	–



	0	1
q_0	q_0	q_1
q_1	q_2	q_1
q_2	q_3	q_1
q_3	q_0	q_1

write the list of transitions for

i) 000100111100

ii) 1110110001 1

pass character by character and find out list of states.

If not reached final state the string is rejected.

i) $\delta(q_0, 0) = q_0$
$\delta(q_0, 0) = q_0$
$\delta(q_0, 0) = q_0$
$\delta(q_0, 1) = q_1$
$\delta(q_1, 0) = q_2$
$\delta(q_2, 0) = q_3$
$\delta(q_3, 1) = q_1$
$\delta(q_1, 1) = q_1$
$\delta(q_1, 1) = q_1$
$\delta(q_1, 0) = q_2$
$\delta(q_2, 0) = \boxed{q_3}$

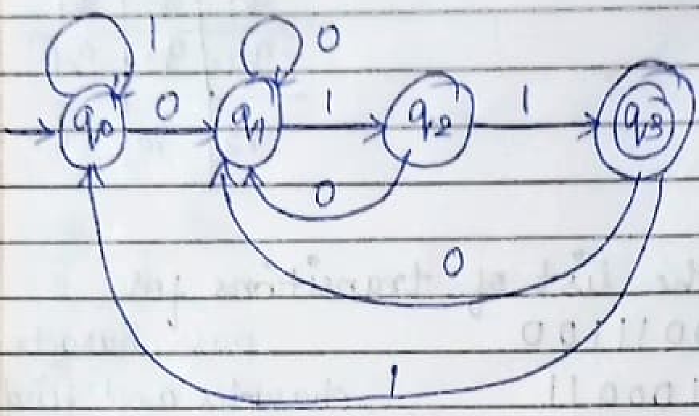
111011000 11

(ii) $\delta(q_0, 1) = q_1$
$\delta(q_1, 1) = q_1$
$\delta(q_1, 1) = q_1$
$\delta(q_1, 0) = q_2$
$\delta(q_2, 1) = q_1$
$\delta(q_1, 1) = q_1$
$\delta(q_1, 0) = q_2$
$\delta(q_2, 0) = q_3$
$\delta(q_3, 0) = q_0$
$\delta(q_0, 1) = q_1$
$\delta(q_1, 1) = \boxed{q_1}$ ×

String rejected

Q: construct a DFA that ends with 011. check whether the strings given are accepted.

i) 0010011100
ii) 11101100011



	0	1
q_0	q_1	q_0
q_1	q_1	q_2
q_2	q_1	q_3
q_3	q_1	q_0

i) 0010011100

$\delta(q_0, 0) = q_1$

$\delta(q_1, 0) = q_1$

$\delta(q_1, 1) = q_2$

$\delta(q_2, 0) = q_1$

$\delta(q_1, 0) = q_1$

$\delta(q_1, 1) = q_2$

$\delta(q_2, 1) = q_3$

$\delta(q_3, 1) = q_0$

$\delta(q_0, 0) = q_1$

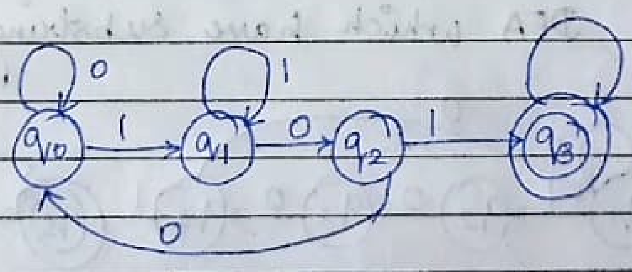
$\delta(q_1, 0) = q_1 \longrightarrow$ String rejected

111 0 11 00011

$\delta(q_0, 1) = q_0$
$\delta(q_0, 1) = q_0$
$\delta(q_0, 1) = q_0$
$\delta(q_0, 0) = q_1$
$\delta(q_1, 1) = q_2$
$\delta(q_2, 1) = q_3$
$\delta(q_3, 0) = q_1$
$\delta(q_1, 0) = q_1$
$\delta(q_1, 0) = q_1$
$\delta(q_1, 1) = q_2$
$\delta(q_2, 1) = \boxed{q_3}$ string accepted

Q: construct a DFA that accepts a substring of 101

it may be ending as
101000
000101
000101111

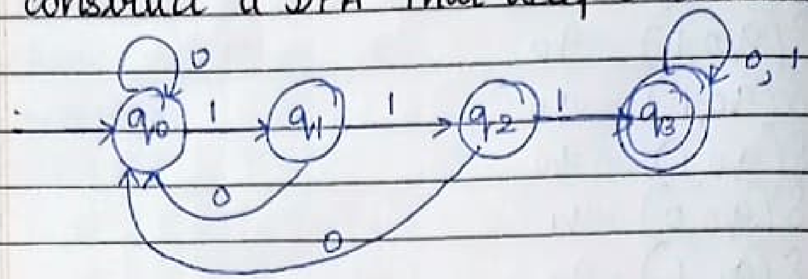


$\Sigma \{\{q_0, q_1, q_2, q_3\}, \{0, 1\}, \delta, \{q_0, q_3\}\}$

$\delta(q_0, 0) = q_0$
$\delta(q_0, 1) = q_1$
$\delta(q_1, 0) = q_2$
$\delta(q_2, 0) = q_0$
$\delta(q_1, 1) = q_1$
$\delta(q_2, 1) = q_3$
$\delta(q_3, 0) = q_3$
$\delta(q_3, 1) = q_3$

	0	1
q_0	q_0	q_1
q_1	q_2	q_1
q_2	q_0	q_3
q_3	q_3	q_3

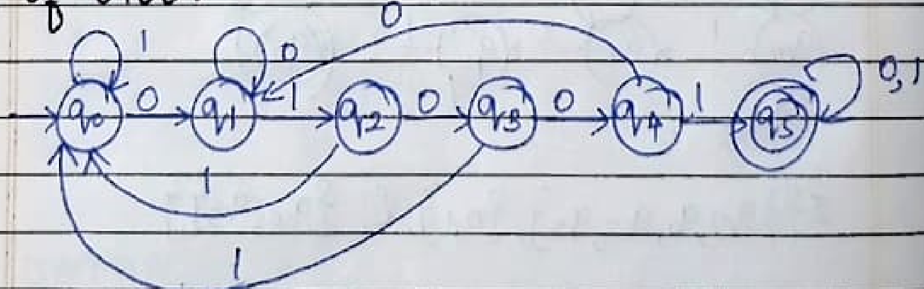
Q: construct a DFA that accepts substring 111



List of transition

$\delta(q_0, 0) = q_0$
$\delta(q_0, 1) = q_1$
$\delta(q_1, 0) = q_0$
$\delta(q_1, 1) = q_2$
$\delta(q_2, 0) = q_0$
$\delta(q_2, 1) = q_3$
$\delta(q_3, 0) = q_3$
$\delta(q_3, 1) = q_3$

Q: construct DFA which have substring of 01001

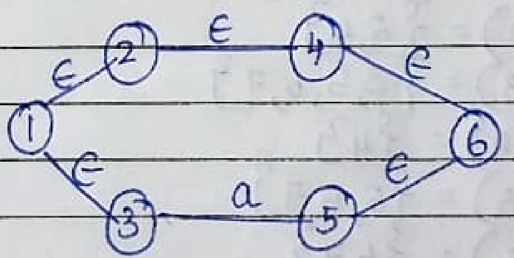


$\delta(q_0, 0) = q_1$
$\delta(q_0, 1) = q_0$
$\delta(q_1, 0) = q_1$
$\delta(q_1, 1) = q_2$
$\delta(q_2, 0) = q_3$
$\delta(q_2, 1) = q_0$
$\delta(q_3, 0) = q_4$
$\delta(q_3, 1) = q_0$
$\delta(q_4, 0) = q_1$
$\delta(q_4, 1) = q_5$
$\delta(q_5, 0) = q_5$
$\delta(q_5, 1) = q_5$.

	0	1
q_0	q_1	q_0
q_1	q_1	q_2
q_2	q_3	q_0
q_3	q_4	q_0
q_4	q_1	q_5
q_5	q_5	q_5

ϵ - closure (epsilon)

ϵ closure with no proper i/p the NFA will transfer to one state to another state.

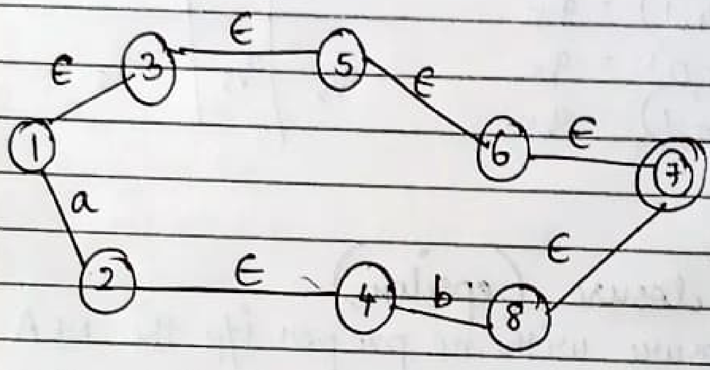


ϵ closure $(1) = \{1, 2, 3, 4\}$
ϵ closure $(2) = \{2, 4\}$
ϵ closure $(3) = \{3\}$
ϵ closure $(4) = \{4\}$
ϵ closure $(5) = \{5, 6\}$
ϵ closure $(6) = \{6\}$

If any reverse transition, it will be rep using arrow otherwise alway forward direction.

	a	b	ε
1	φ	φ	{1,2,3,4}
2	φ	φ	{2,4}
3	{5}	φ	{3}
4	φ	{0}	{4}
5	φ	φ	{5,6}
6	φ	φ	{6}

Q:



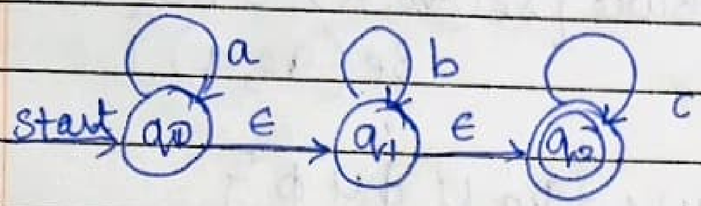
ϵclosure(1) = {1,3,5,6,7}
ϵclosure(2) = {2,4}
ϵclosure(3) = {3,5,6,7}
ϵclosure(4) = {4}
ϵclosure(5) = {5,6,7}
ϵclosure(6) = {6,7}
ϵclosure(7) = {7}
ϵclosure(8) = {8,7}

	a	b	ε-closure
1	2	ϕ	{1, 3, 5, 6, 7}
2	ϕ	ϕ	{2, 4}
3	ϕ	ϕ	{3, 5, 6, 7}
4	ϕ	8	{4}
5	ϕ	ϕ	{5, 6, 7}
6	ϕ	ϕ	{6, 7}
7	ϕ	ϕ	{7}
8	ϕ	ϕ	{8, 7}

3	ϕ	ϕ	{3, 5, 6, 7}
4	ϕ	8	{4}
5	ϕ	ϕ	{5, 6, 7}
6	ϕ	ϕ	{6, 7}
7	ϕ	ϕ	{7}
8	ϕ	ϕ	{8, 7}

ϵ-NFA to DFA / conversion of NFA with null to DFA

Q:



δ	a	b	c	ϵ
q_0	q_0	ϕ	ϕ	q_1
q_1	ϕ	q_1	ϕ	q_2
q_2	ϕ	ϕ	q_2	ϕ

ϵ closure of q_0 = $\{q_0, q_1, q_2\}$
ϵ closure of q_1 = $\{q_1, q_2\}$
ϵ closure(q_2) = $\{q_2\}$

Transition table for DFA

δ	a	b	c
$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_2\}$
$\{q_1, q_2\}$	ϕ	$\{q_1, q_2\}$	$\{q_2\}$
$\{q_2\}$	ϕ	ϕ	$\{q_2\}$

all have q_2 as final states

✓ $\delta((q_1, q_2, q_3), a) = \epsilon \text{ closure}\{\delta_e(q_0, q_1, q_2), a\}$

$= \epsilon \text{ closure}\{\delta_e(q_0, a) \cup \delta_e(q_1, a) \cup \delta_e(q_2, a)\}$

$= \epsilon \text{ closure}\{q_0 \cup \phi \cup \phi\}$

$= \epsilon \text{ closure}(q_0)$

$= q_0, q_1, q_2$

✓ $\delta((q_1, q_2, q_3), b) = \epsilon \text{ closure}\{\delta(q_0, q_1, q_2), b\}$

$= \epsilon \text{ closure}\{\delta_e(q_0, b) \cup \delta_e(q_1, b) \cup \delta_e(q_2, b)\}$

$= \epsilon \text{ closure}\{\phi \cup q_1 \cup \phi\}$

$= \epsilon \text{ closure}\{q_1\}$

$= \{q_1, q_2\}$

⋮

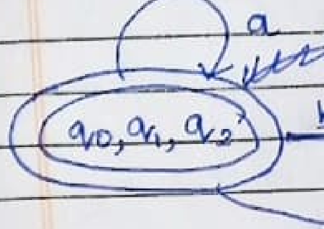
repeat for entire table

DFA diagram

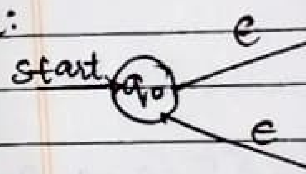
Before dra

which sets

ques tioue.



Q:



δ	0
q_0	ϕ
q_1	$\{q_3\}$
q_2	ϕ
q_3	ϕ
q_4	ϕ

ϵ closur

ϵ closur

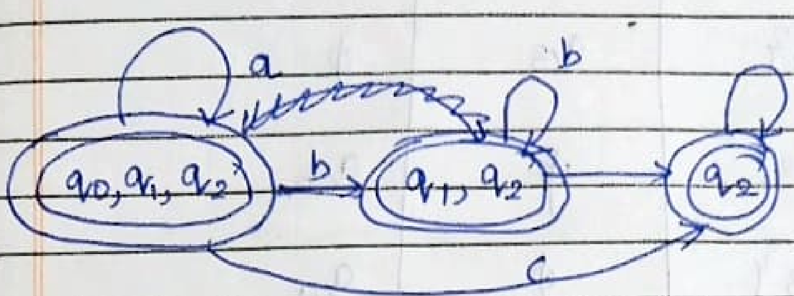
ϵ closur

ϵ closur

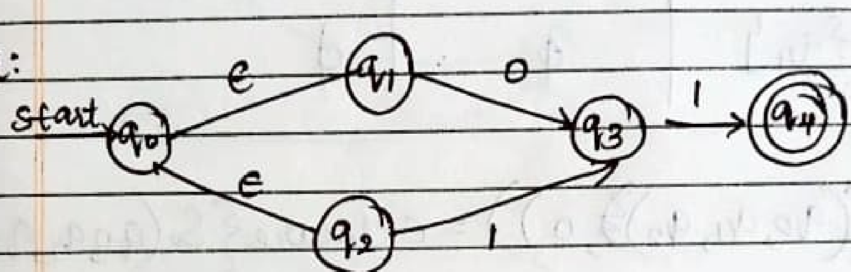
ϵ closur

DFA diagram:

l states Before drawing check whether which sets have q_2 has final states (w.r.t question.



Q:



δ	0	1	ε
q_0	ϕ	ϕ	$\{q_1, q_2\}$
q_1	$\{q_3\}$	ϕ	ϕ
q_2	ϕ	$\{q_3\}$	ϕ
q_3	ϕ	$\{q_4\}$	ϕ
q_4	ϕ	ϕ	ϕ

ϵ closure $(q_0) = \{q_0, q_1, q_2\}$

ϵ closure $(q_1) = \{q_1\}$

ϵ closure $(q_2) = \{q_2\}$

ϵ closure $(q_3) = \{q_3\}$

ϵ closure $(q_4) = \{q_4\}$

Transition table for DFA

δ	0	1
$\{q_0, q_1, q_2\}$	q_3	q_3
$\{q_1\}$	q_3	ϕ
$\{q_2\}$	ϕ	q_3
$\{q_3\}$	ϕ	q_4
$\{q_4\}$	ϕ	ϕ

✓ $\delta((q_0, q_1, q_2), 0) = \epsilon\text{ closure}\{\delta_0(q_0, q_1, q_2), 0\}$

$= \epsilon\text{ closure}\{\delta_e(q_0, 0) \cup \delta_e(q_1, 0) \cup \delta_e(q_2, 0)\}$

$= \epsilon\text{ closure}\{\phi \cup q_3 \cup \phi\}$

$= \epsilon\text{ closure}\{q_3\}$

$= q_3$

✓ $\delta((q_0, q_1, q_2), 1) = \epsilon\text{ closure}\{\delta_0(q_0, q_1, q_2), 1\}$

$= \epsilon\text{closure}\{\delta_e(q_0, 1) \cup \delta(q_1, 1) \cup \delta(q_2, 1)\}$

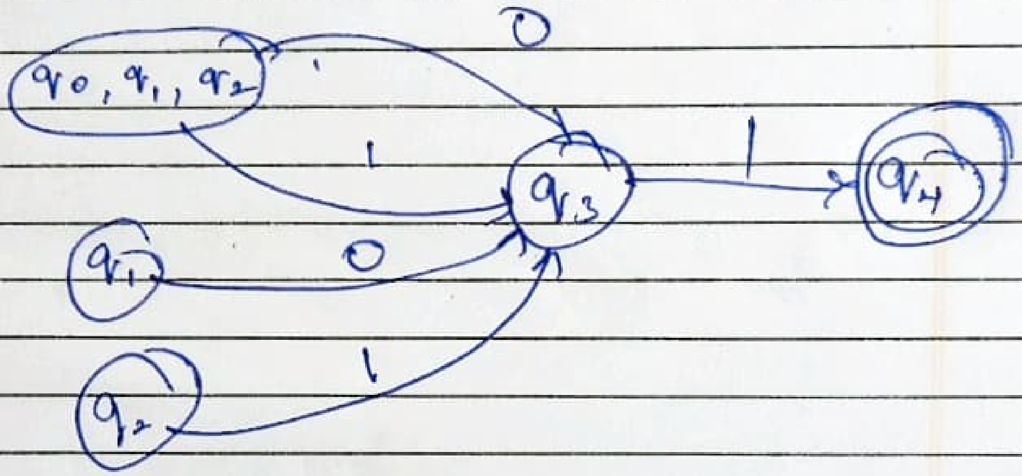
$= \epsilon\text{ closure}\{\phi \cup \phi \cup q_3\}$

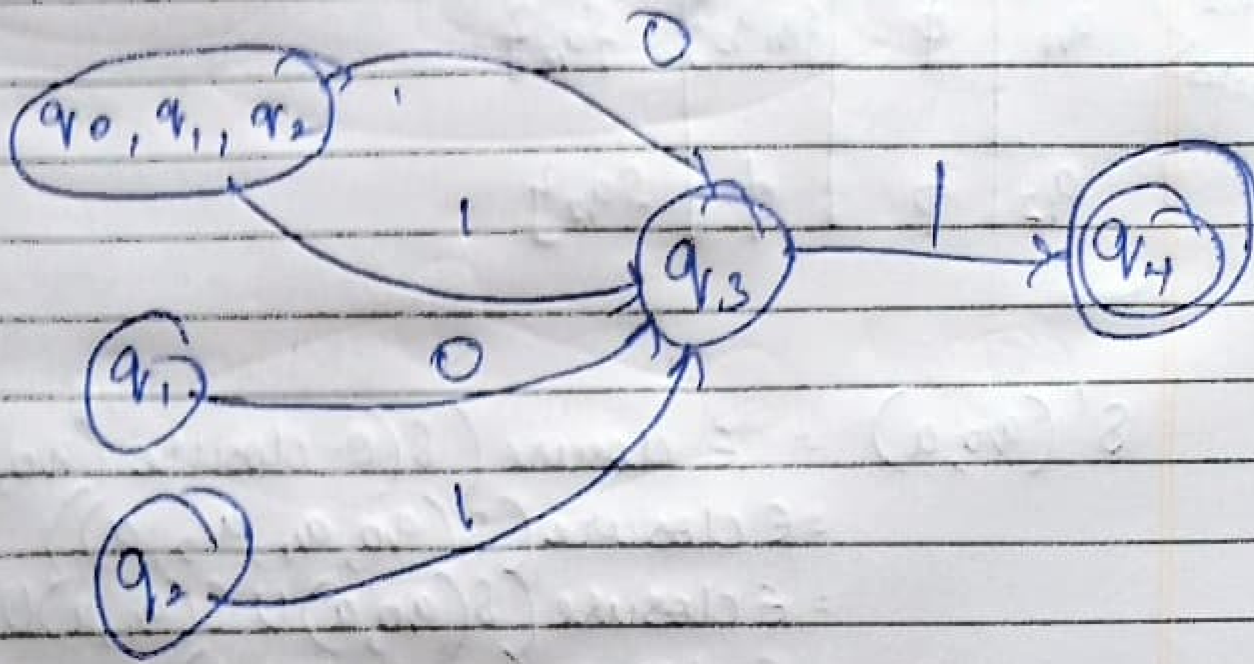
$= \epsilon\text{ closure}\{q_3\}$

$= q_3$

✓ $\delta(q_1, 0) = \epsilon\ closure\ \{\delta_0(q_1), 0\}$
$= \epsilon\ closure\ \{q_3\}$
$= q_3$

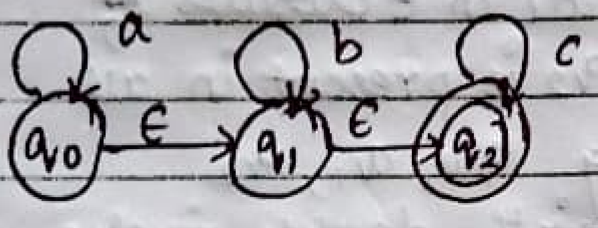
DFA diagram:-





ENFA to NFA

↓ withnull ↓ without null



soln:

Transition table

δ	a	b	c	ϵ
q_0	q_0	ϕ	ϕ	q_1
q_1	ϕ	q_1	ϕ	q_2
q_2	ϕ	ϕ	q_2	ϕ

ϵ closure $(q_0) = \{q_0, q_1, q_2\}$
ϵ closure $(q_1) = \{q_1, q_2\}$
ϵ closure $(q_2) = \{q_2\}$

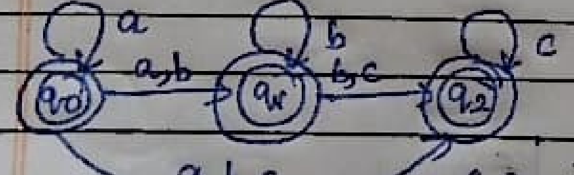
δ'	a	b	c
q_0	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_2\}$
q_1	ϕ	$\{q_1, q_2\}$	$\{q_2\}$
q_2	ϕ	ϕ	$\{q_2\}$

Here ϵ closure will not be the new states.

$\delta'(q_0, a) = \epsilon$ closure $(\delta(\epsilon\text{-closure}(q_0, a))$
$= \epsilon$ closure $(\delta(q_0, q_1, q_2, a))$
$= \epsilon$ closure $(\delta(q_0, a) \cup \delta(q_1, a) \cup \delta(q_2, a))$
$= \epsilon$ closure $(q_0 \cup \phi \cup \phi)$
$= q_0, q_1, q_2$

(q_2)

Check for final state in ϵ closures of q_0, q_1, q_2 since q_2 is present in all we are considering q_0, q_1, q_2 as final states with double circles.



~~Without null~~