SRMIST, Ramapuram Campus.
Department of Mathematics
18MAB302 – Discrete Mathematics for Engineers.
CT-3 Answer Key.

SET-1

Exam Date: 16/11/2022.

## Part-A:

1. c   2. b   3. d   4. c   5. a
6. b   7. a   8. c   9. c   10. c

## Part-B.

11. (i) Abelian group.

A group G is called abelian, if the commutative law holds. i.e., $\forall a,b \in G$, $a*b = b*a$. (2M)

(ii) cyclic group.

A group G is cyclic if $\forall x \in G$, $\exists$ some $a \in G$ $\ni$ $x = a^n$ for some $n \in Z^+$. (2M)

Where a is the generator.

12.

Suppose $e_1$ and $e_2$ be two identity elements in G. (1M)

Since $e_1$ is an identity,

$e_1 * e_2 = e_2 * e_1 = e_2$ — (1) (2M)

Since $e_2$ is an identity.

$e_1 * e_2 = e_2 * e_1 = e_1$ — (2)

from (1) and (2). (1M)

$e_1 = e_1 * e_2 = e_2$, unique.

13. Ring:

$(R, +, \cdot)$ is called a ring if the binary operations + and $\cdot$ on R satisfy the following. (2M)

(i) $(R, +)$ is abelian.

(ii) $(R, \cdot)$ is a semi group.

(iii) $a \cdot (b+c) = (a \cdot b) + (a \cdot c)$, $\forall a,b,c \in R$.

Integral Domain:

A commutative ring is an integral domain if it has no zero divisor. (2M)

14. Let $G = (V, E)$ be an undirected graph.

$\sum_i \deg(v_i) = \sum_{\text{Even}} \deg(v_j) + \sum_{\text{odd}} \deg(v_k)$ (2M)

$\sum_{\text{odd}} \deg(v_k) = \sum_i \deg(v_i) - \sum_{\text{even}} \deg(v_j)$

$= 2e - \sum_{\text{even}} \deg(v_j)$

$=$ even   $\because$ (Hand shaking Theorem)

(2M)

15. Let T be an undirected tree

$\Leftrightarrow$ T is connected.

$\Leftrightarrow$ There is a simple path b/w every pair of vertices, say $v_i$ and $v_j$ (2M)

$\Leftrightarrow$ If possible, let there be two paths b/w $v_i$ and $v_j$.

$\Leftrightarrow$ which forms circuit, contradicts T is a tree.

$\Leftrightarrow$ unique simple path b/w every pair of vertices. (2M)

16. (i) Graph Colouring:

An assignment of colours to the vertices of a graph so that no two adjacent vertices get the same colour is called a colouring of the graph. (2M)

(ii) Chromatic number of a graph:

The chromatic number $\chi(G)$ of a graph G is the minimum number of colours needed to colour G. (2M)

17 (a) Here,

$$H = [A^T \mid I_{n-m}] = \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \quad 2M$$

The generator matrix $\quad n=6, m=3$

$$G = [I_m \mid A] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right] \quad 2M$$

Now, $B^3 = \{000, 001, 010, 100, 011, 101, 110, 111\}$ with $e(w) = wG$

$$\left.\begin{array}{l} e(000) = (000)G = (000000) \\ e(001) = (001)G = (001011) \\ e(010) = (010)G = (010101) \\ e(100) = (100)G = (100111) \end{array}\right\} 4M$$

$$\left.\begin{array}{l} e(011) = (011)G = (011110) \\ e(101) = (101)G = (101100) \\ e(110) = (110)G = (110010) \\ e(111) = (111)G = (111001) \end{array}\right\} 4M$$

(b) Let $G$ be the cyclic group generated by the element $a$.

Let $H$ be a subgroup of $G$.

To prove: $H$ is cyclic. $\rightarrow$ 2M

Case (i) Suppose $H = G$ or $\{e\}$.

When $H = G$, Then $H$ is cyclic.

When $H = \{e\}$, $e$ itself is the generator of $H$.

$\therefore H$ is cyclic. $\rightarrow$ 2M

Case (ii) The elements of $H$ are non-zero integral powers of $a$.

Let $m$ be the least positive integer for which $a^m \in H$. $\rightarrow$ 2M

Let $a^n \in H$

Now, $(a^m)^q \in H$. $\quad \left.\begin{array}{l} \text{N.K.T.,} \\ n = mq + r,\ 0 \le r < m \end{array}\right] \rightarrow$ 2M

$\Rightarrow a^{-mq} \in H$ also $a^n \in H$

$\left.\Rightarrow a^{n-mq} \in H \Rightarrow a^r \in H \Rightarrow \boxed{r=0} \Rightarrow n = mq \right\}$ 1M

$\therefore a^n = a^{mq} = (a^m)^q$ $\quad \therefore H$ is cyclic generated by $a^m$.

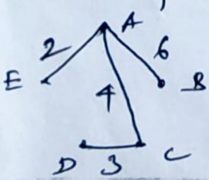
18. (a)

| Edge | Weight | Selected |
|---|---|---|
| AE | 2 | Yes ✓ |
| DC | 3 | Yes ✓ |
| AC | 4 | Yes ✓ |
| EC | 4 | No ✗ |
| AB | 6 | Yes ✓ |
| BE | 6 | — |
| BC | 6 | — |
| ED | 7 | — |
| BD | 8 | — |
| AD | 8 | — |

6M

Since 5 vertices $\Rightarrow$ 4 edges needed.

Minimum spanning tree,




5M

The length is $2+3+4+6 = 15.$ 1M

(b) Let $n_1, n_2, \ldots, n_k$ be the vertices in each of $k$ components of the graph $G$.

Then $\displaystyle\sum_{i=1}^{k} n_i = n$ — (1), $n_i \ge 1$. } 2M

Hence, $\displaystyle\sum_{i=1}^{k} (n_i - 1) = n - k$

$$\left.\begin{array}{l} \left[\displaystyle\sum_{i=1}^{k} (n_i - 1)\right]^2 = n^2 + k^2 - 2nk \\ \displaystyle\sum_{i=1}^{k} (n_i - 1)^2 \le n^2 + k^2 - 2nk. \end{array}\right\} 4M$$

$$\Rightarrow \sum_{i=1}^{k} n_i^2 \le n^2 + k^2 - 2nk + 2n - k \quad \text{— (2)}$$

$$\left.\begin{array}{l} \text{Maximum number of edges of } G. \\ = \frac{1}{2} \displaystyle\sum_{i=1}^{n} n_i(n_i - 1) \\ = \frac{1}{2}\left[\displaystyle\sum_{i=1}^{n} n_i^2 - \displaystyle\sum_{i=1}^{n} n_i\right] \end{array}\right\} 4M$$

$\le \frac{1}{2}(n-k)(n-k+1)$, from (1) and (2) } 2M

Hence proved.

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