Unit 2.

Attenuation - Dispersion - Introduction

Signal attenuation is one of the important properties of an optical fiber because it determine the maximum unamplified or repeaterless separation between a transmitter and receiver. Since amplifiers and repeaters are expensive to fabricate, install and maintain, the degree of attenuation in a fiber has a large influence on system cost. The dispersion mechanism in a fiber cause optical pulses to broaden as they travel along a fiber. If ~~they~~ pulses travel sufficiently far, they eventually overlap with neighbouring pulses, thereby creating Errors in the receiver output. The dispersion thus limit the information carrying capacity of a fiber.

Attenuation

Attenuation is the signal loss when ~~as~~ the signal propogate along the fiber. It determine the transmission distance between a transmitter and a receiver or an in line amplifier.

The basic attenuation mechanisms in a fiber are
- Absorption
- Scattering
- Radiative losses of the optical energy

→ Absorption is related to fiber material and with Scattering is due to fiber material and with optical wave guide structural imperfection in the optical wave guide. Radiative losses is due to perturbations of the fiber geometry.

Attenuation units.

As light travels along a fiber, its power decreases exponentially with distance. If P(0) is the optical power in a fiber at the origin (z=0), then power at distance z farther down the fiber is

$$P(z) = P(0)e^{-\alpha_p z}$$

where α_p is the attenuation coefficient

$$\alpha_p = \frac{1}{z}\ln\left[\frac{P(0)}{P(z)}\right]$$ and its unit is km^{-1}

Or dB/km $$\alpha(dB/km) = \frac{10}{z}\log\left[\frac{P(0)}{P(z)}\right] = 4.343\alpha_p \; km^{-1}$$

Absorption

Absorption is caused by three different mechanisms.

1. Absorption by atomic defects in the glass Composition
2. Extrinsic absorption by impurity atoms in the glass material
3. Intrinsic absorption by the basic constituent atoms of the fiber material.

1. Atomic Defects are imperfections in the atomic Structure of the fiber material
egs: missing molecules, high density clusters of atom groups, oxygen defects in the glass structure
Absorption losses arising from these are negligible Compared with intrinsic and impurity absorption effects. But it is significant if fiber is exposed to ionizing radiation.

Radiation damages a material by changing its internal structure. The damage effects depend on the energy of the ionizing particles or rays, the radiation flux and the fluence (particles/cm^2)
The total dose a material receives is expressed in Units of rad (Si) [radiation absorbed in bulk Silicon]

Problems

Problem 1

Consider a 30 km long optical fiber that has an attenuation of 0.4 dB/km at 1310 nm. Calculate the output power if 200 μW of optical input power is launched into the fiber.

[optical powers are commonly expressed in units of dBm which is the decibel power level referred to 1 mW]

$$P_{in}(dBm) = 10 \log \left[\frac{P_{in}(W)}{1mW}\right]$$

Given

$\alpha = 0.4\, dB/km$

$z = 30\, km$

$P_{in}(W) = 200\, \mu W$

$$= 10 \log \left[\frac{200 \times 10^{-6}\, W}{1 \times 10^{-3}\, W}\right] = -7.0\, dBm$$

$$P_{out}(dBm) = 10 \log \left[\frac{P_{out}(W)}{1mW}\right]$$

$$= 10 \log \left[\frac{P_{in}(W)}{1mW}\right] - \alpha z$$

$$= -7.0\, dBm - (0.4\, dB/km)\, 30\, km$$

$$\boxed{P_{out}(dBm) = -19.0\, dBm}$$

SRMIST,RAMAPURAM
Department of Electronics and Communication Engineering

Course Code: 18ECO107T   Subject Name: Fibre Optics and Optoelectronics

$\boxed{1 \text{ rad(Si)} = 100 \text{ erg/g} = 0.01 \text{ J/Kg}}$

The ~~ionig~~ basic response of a fiber to ionizing radiation is an increase in attenuation owing to the creation of ~~atmoi~~ atomic defects or attenuation centers that absorb optical energy. The higher the radiation level, larger the attenuation as shown in figure (a). However the attenuation centers will relax with time as shown in fig (b). The degree of radiation effects depends on the dopant materials used in the fiber. Pure Si fibers or fiber with a low Ge doping and no other dopants have the lowest radiation induced losses.

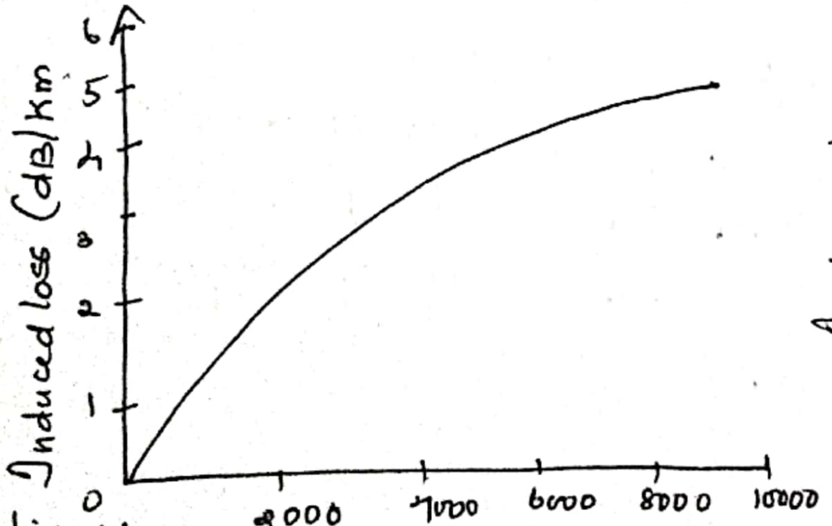


figure (a)
Loss increase during steady irradiation to a total dose of 10^4 rad

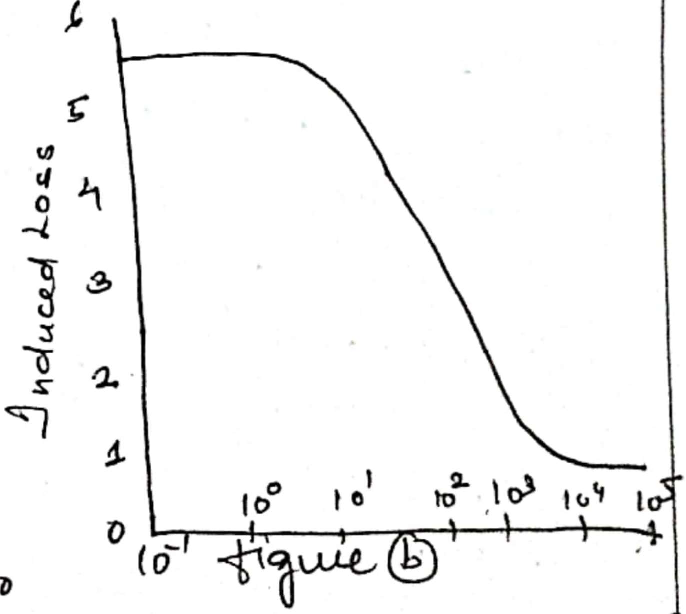


figure (b)
Subsequent recovery as a function of time after radiation has stopped

Prepared by: NAME/DESIGNATION/DEPT/SRMIST

The dominant absorption factor in silica fibers is the presence of minute quantities of impurities in the fiber material. These impurities include OH^- ions that are dissolved in the glass and transition metal ions such as iron, copper, chromium and vanadium. Transition metal impurity levels were around 1 part per million (ppm) in glass fibers made in 1970s which resulted in losses ranging from 1 to 4 dB/km. Impurity absorption losses ... either because of electron transitions between the energy levels within these ions or because of charge transitions between ions. The absorption peaks of the various transition metal impurities tend to be broad and several peaks may overlap, which further broadens the absorption in a specific region.

Modern vapour phase fiber techniques for producing a fiber perform have reduced the transition metal impurity levels by several orders of magnitude. Such low impurity levels allow the fabrication of low loss fibers.

The presence of OH ion impurities in a fiber perform results mainly from the oxyhydrogen flame used in the hydrolysis reaction of the $SiCl_4$, $GeCl_4$ and $POCl_3$ starting materials. Water impurity concentrations of less than few parts per billion are required if the attenuation is to be less than 20dB/Km. The high levels of OH ions in early fibers resulted in large absorption peaks at 725, 950, 1240 and 1380nm. Regions of low attenuation lie between these absorption peaks.

The peaks and valleys in the attenuation curves resulted in the various transmission windows shown in fig. By reducing the residual OH content of fibers to below 1ppb, standard commercially available single mode fibers have nominal attenuations of 0.4dB/km at 1310nm (in the O band) and less than 0.25dB/km at 1550nm (in the C band). Further elimination of water ions diminishes the absorption peak around 1440nm and thus opens up the E band for data transmission. Optical fibers that can be used in E band are known as low water peak or full spectrum fibers

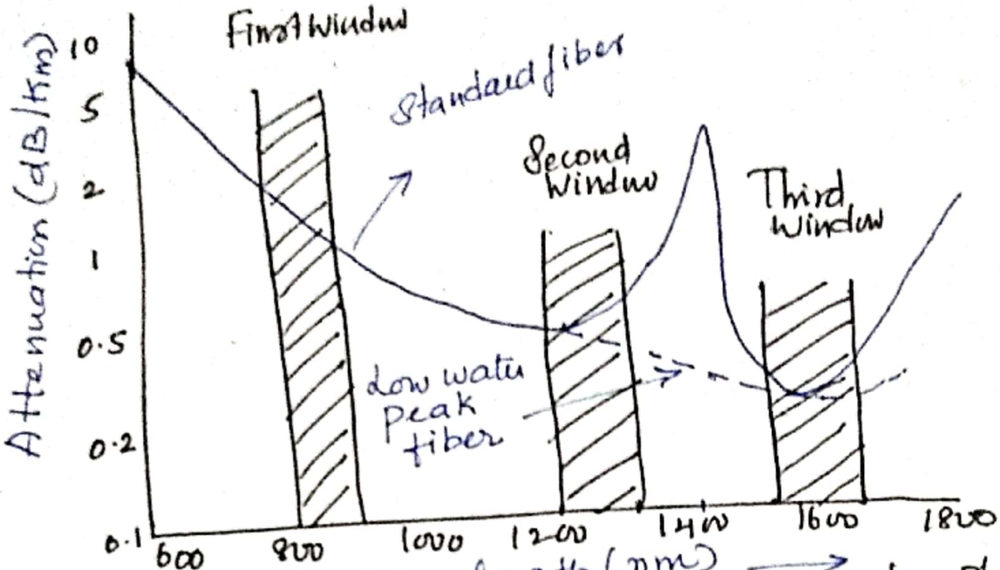


fig shows optical fiber attenuation as a fn of wavelength

Instrinsic Absorption is due to dechomic absorption bands in U.V region and from atomic vibration bands in near Infra red region. The elechomic absorption is due to band gaps of amorphous glass material. The ultraviolet edge of the electron absorption bands of amorphous and crystalline materials have the relationship

$$\boxed{\alpha_{uv} = Ce^{E/E_0}} \Rightarrow \text{Urbach's rule}$$

C, E_0 are Empirical constant and E is the photon energy. Since $\boxed{E \propto \frac{1}{\lambda}}$ > U.V absorption decays exponentially as λ increases. U.V absorption loss at any wave length can be expressed as a fn of mole fraction x of GeO_2

$$\boxed{\alpha_{uv} = \frac{154.2x \times 10^{-2}}{46.6x+60} e^{4.63/\lambda}}$$

An empirical (derived from observation or experiment) expression for the infrared absorption in dB/Km for $GeO_2 - SiO_2$ glass with λ given in μm

$$\boxed{\alpha_{IR} = 7.81 \times 10^{11} \exp\left(\frac{-48.48}{\lambda}\right)}$$

Problem

Consider two silica fibers that are doped with 6 percent and 18 percent mole fraction of GeO_2 respectively. Compare the Ultraviolet absorption at wavelengths of 0.7 μm and 1.3 μm

Solution.

(a) for the fiber with $x = 6\% = 0.06$ and $\lambda = 0.7$ μm

$$\alpha_{uv} = \frac{154.2 \times .06}{46.6(0.06)+60} e^{\left(\frac{4.63}{.7}\right)} = 1.10 \text{ dB/Km}$$

b) for the fiber with $x = 0.06$ and $\lambda = 1.3$ μm

$$\alpha_{uv} = \frac{154.2 \times .06}{46.6(0.06)+60} \times 10^{-2} e^{\left(\frac{4.63}{1.3}\right)}$$

$$= 0.07 \text{ dB/Km}$$

c) for the fiber with $x = 0.18$ and $\lambda = 0.7\,\mu m$

$$\alpha_{uv} = \frac{1.542(0.18)}{46.6(0.18)+60} e^{\left(\frac{4.63}{0.7}\right)} = 3.03\,dB/km$$

d) for the fiber with $x = 0.18$ and $\lambda = 1.3\,\mu m$

$$\alpha_{uv} = \frac{1.542(0.18)}{46.6(0.18)+60} e^{\left(\frac{4.63}{1.3}\right)} = 0.19\,dB/km$$

Scattering Losses

Scattering losses in glass is due microscopic variations in the material density, fluctuation and structural inhomogeneities or defects occuring during fiber manufacture. Fluctuation in molecular density and compositional fluctuation (SiO_2, GeO_2 P_2O_5) give rise to refractive index variations in glass over distance. These index variation cause a Rayleigh type scattering of light. For single component glass the scattering loss is given by

$$\boxed{\alpha_{scat} = \frac{8\pi^3}{3\lambda^4}(n^2-1)^2 k_B T_f B_T}$$

where n is the refractive index

k_B Boltzmann constant

B_T isothermal compressibility of the material

T_f fictive temperature T_f is the temperature at which the density fluctuation are frozen into the glass as it solidifies.

(Or) $\boxed{\alpha_{scat} = \frac{8\pi^3}{3\lambda^4} n^8 p^2 k_B T_f \beta_T}$ where p is the photo elastic coefficient.

Problem:

For silica the fictive temperature T_f is 1400K, the isothermal compressibility β_T is $6.8\times10^{-12}\ cm^2/dyn$ $= 6.8\times10^{-11}\ m^2/N$ and the photo elastic coefficient is 0.286. Estimate the scattering loss at a 1.3 μm wavelength where n = 1.450

$\beta_T = 6.8\times10^{-12}\ cm^2/dyn$

$T_f = 1400\,k$

$\beta_T = 6.8\times10^{-11}\ m^2/N$

$P = 0.286$

Solution $\alpha = \frac{8\pi^3}{3\lambda^4} n^8 p^2 k_B T_f \beta_T$

$= \frac{8\times(3.14)^3}{3\times(1.3\times10^{-6})^4}\times(1.45)^8\times(0.286)^2 \times 1.38\times10^{-23}\times1400\times 6.8\times10^{-12}$

$= 6.68\times10^{-2}$ neper/km or 0.26 dB/km

Dr Christeena Joseph / Associate Professor
ECE Dept / SRMIST

Scattering Losses

It is due to microscopic variations in the material density, from compositional fluctuations and from structural inhomogeneities occuring during fiber manufacture.

Scattering losses contd.....

For multicomponent glasses the scattering at a wavelength λ is given by

$$\boxed{\alpha = \frac{8\pi^3}{3\lambda^4} (\delta n^2)^2 \delta V}$$

Where the square of the mean square refractive index fluctuation $(\delta n^2)^2$ over a volume of δV is

$$\boxed{(\delta n^2)^2 = \left(\frac{\partial n^2}{\partial p}\right)^2 (\delta p)^2 + \sum_{i=1}^{m} \left(\frac{\partial n^2}{\partial C_i}\right)^2 (\delta C_i)^2}$$

δp is density fluctuation

δC_i is concentration fluctuation of i^{th} glass component.

Their values are determined from experimental data.

The factor $\partial n^2/\partial p$ and $\partial n^2/\partial C_i$ are the variation of the square of the index w.r.to density and the i^{th} glass component.

Structural inhomogeneities and defects during fabrication can also cause scattering. The defects may be in the form of trapped gas bubbles, unreacted starting materials and crystallized regions in the glass. latest or developments in the manufacturing process have minimized these extrinsic scattering effects.

Since Rayleigh scattering is λ^{-4} dependent it decreases with increasing wavelength. For wavelengths below 1μm, Rayleigh scattering is very dominant. At wavelengths longer than 1μm Infrared absorption effects tend to dominate optical signal attenuation. The losses of multimode fibers are generally higher than those of single mode fibers. This is due to higher dopant concentrations and the accompanying larger scattering loss due to greater compositional fluctuation in multimode fibers. Also multimode fibers are subject to higher order mode losses owing to perturbation at core cladding interface.

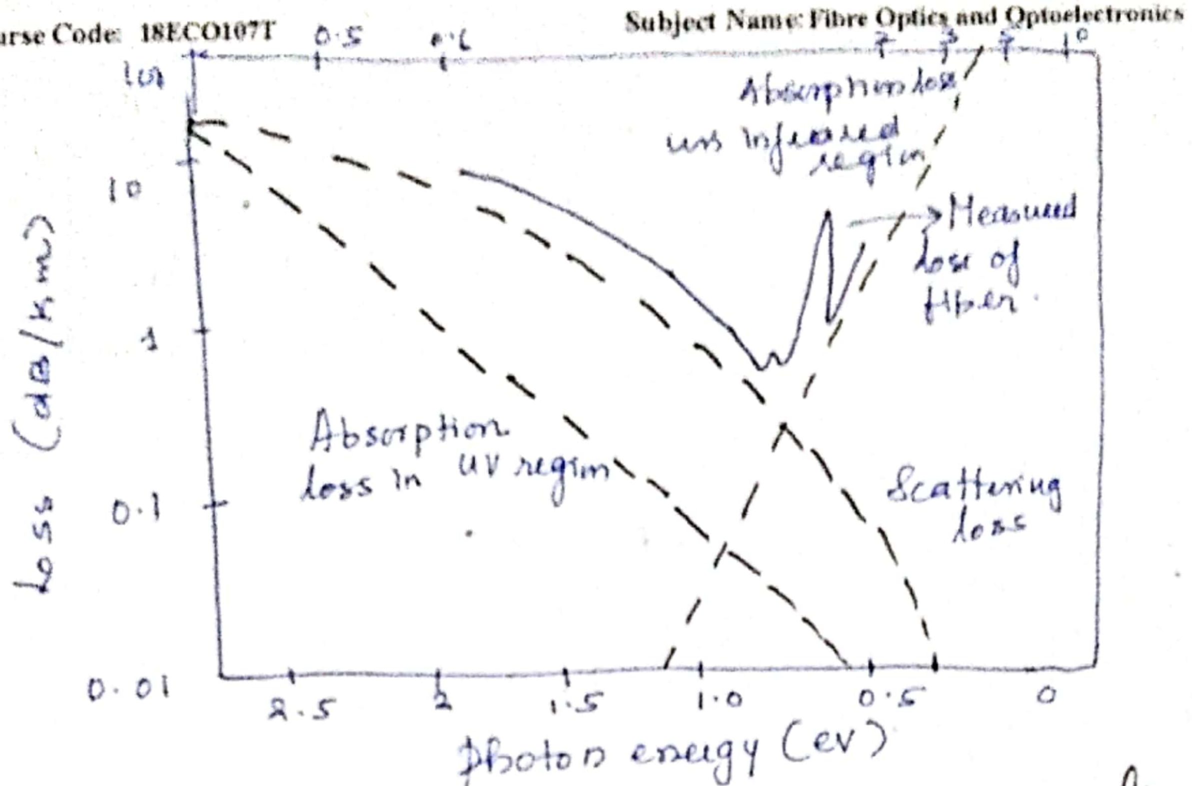


fig shows Optical fiber attenuation characteristic and their limiting mechanisms for a GeO_2 doped low loss low water content silica fibres.

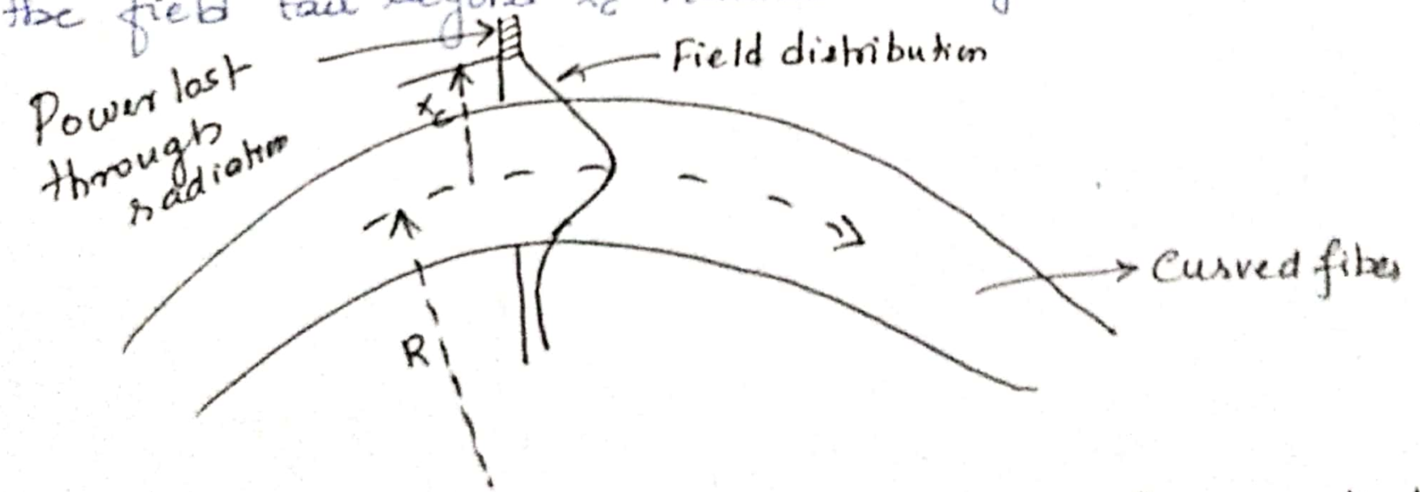
Bending Losses

Radiative losses occur whenever an optical fiber undergoes a bend of finite radius of curvature

* macroscopic bends having radii that are large compared with the fiber diameter such as those that occur when a fiber cable turns a corner.
* random microscopic bends of the fiber axis that can arise when the fibers are incorporated into cables.

For slight bends the excess loss is extremely small and is unobservable. As the radius of curvature decreases, the loss increases exponentially until at a certain critical radius the curvature loss becomes observable. If the bend radius is made a bit smaller once this threshold point has been reached the losses

The curvature loss effects can be explained by modal electric field distribution. Any bound core mode has an evanescent field tail in cladding decay exponentially as a function of distance from the core. Since this field tail moves along with the field in the core part of the energy of a propagating mode travels in the fiber cladding. When a fiber is bent, the field tail on the far side of the center of curvature must move faster to keep up with the field in the core for the lowest order fiber mode. At a certain critical distance x_c from the center of the fiber, the field tail would have to move faster than the speed of light to keep up with the core field. Since this is not possible, the optical energy in the field tail beyond x_c radiates away.



Sketch of the fundamental mode field in a curved optical waveguide

The amount of optical radiation from a bent fiber depends on the field strength at x_c and on the radius of curvature R. Since higher-order modes are bound less tightly to the fiber core than lower order modes, the higher order modes will radiate out of fiber first.

The total number of modes that can be supported by a curved fiber is less than in a straight fiber.

The effective number of modes M_{eff} that are guided by a curved multimode fiber of radius a

$$\boxed{M_{eff} = M_\infty \left\{ 1 - \frac{\alpha+2}{2\alpha\Delta}\left[\frac{2a}{R} + \left(\frac{3}{2n_2 k R}\right)^{2/3}\right]\right\}}$$

$\alpha \rightarrow$ graded indexed profile

$\Delta \rightarrow$ core cladding index difference

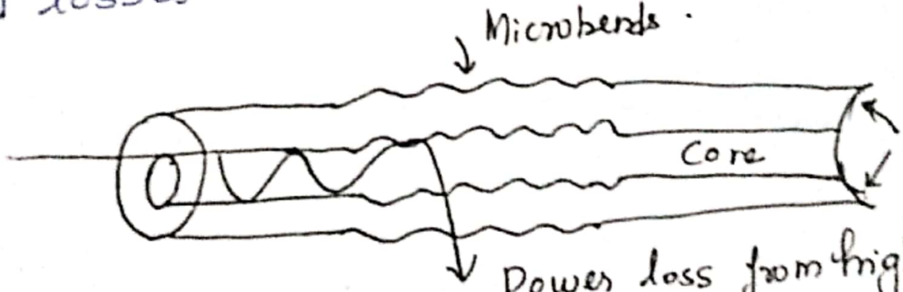
$n_2 \rightarrow$ cladding refractive index

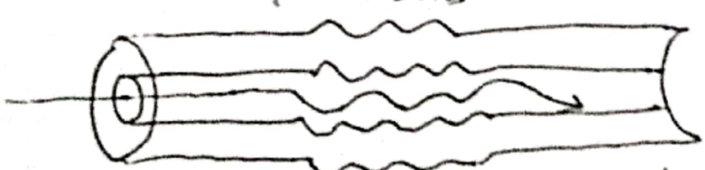
$k = 2\pi/\lambda$ wave propagation constant

where
$$\boxed{M_\infty = \frac{\alpha}{\alpha+2}(n_1 k a)^2 \Delta}$$

M_∞ is the total number of modes in a straight fiber.

Another form of radiation loss in optical waveguide results from mode coupling caused by random microbends of the optical fiber. Microbends are repetitive small scale fluctuations in the radius of curvature of the fiber axis. They are caused by non uniformities in the manufacturing of the fiber or by non uniform lateral pressure created during the cabling of the fiber. An increase in attenuation results from microbending because the fiber curvature causes repetitive coupling of energy between the guided modes and the leaky or non guided modes in the fiber. The figure shows Microbend losses





Small scale fluctuations in the radius of curvature of the fiber axis lead to microbending losses

One method of minimizing microbending losses is by extruding a compressible jacket over the fiber when external forces are applied, the jacket will be deformed but the fiber will tend to stay relatively straight.

For a multimode graded index fiber having a core radius a, outer radius b, the index difference Δ the microbending loss α_M of a jacketed fiber is reduced from that of an unjacketed fiber by a factor

$$\boxed{F(\alpha_M) = \left[1 + \pi\Delta^2 \left(b/a\right)^4 \frac{E_f}{E_j}\right]^{-2}}$$

E_j and E_f are the Young's Moduli of the jacket and fiber respectively

Core and Cladding Losses

Since the core and cladding have different indices of refraction and therefore differ in composition, the core and cladding have different indices of refraction and therefore differ in composition. If α_1 and α_2 are the attenuation coefficients of core and cladding, The loss for a mode of order (ν, m) for a step index waveguide is

$$\boxed{\begin{aligned} \alpha_{\nu m} &= \alpha_1 \frac{P_{core}}{P} + \alpha_2 \frac{P_{clad}}{P} \quad \text{or} \\ \alpha_{\nu m} &= \alpha_1 + (\alpha_2 - \alpha_1) \frac{P_{clad}}{P} \end{aligned}}$$

For the case of a graded index fiber the situation is complicated.
At a distance 'r' from the core axis the loss is

$$\boxed{\alpha(r) = \alpha_1 + (\alpha_2 - \alpha_1)\frac{n^2(0) - n^2(r)}{n^2(0) - n_2^2}}$$

where α_1 and α_2 are the axial and cladding attenuation coefficients

Then the loss encountered by a given mode is

$$\boxed{\alpha = \frac{\int_0^\infty \alpha(r)\, p(r)\, r\, dr}{\int_0^\infty p(r)\, r\, dr}}$$

Signal dispersion in fibers

An optical signal weakens from attenuation mechanisms and broadens due to dispersion effects as it travels along a fiber. Eventually these two factors will cause neighbouring pulses to overlap. After a certain amount of overlap occurs, the receiver can no longer distinguish the individual adjacent pulses and errors arise when interpreting the received signal.

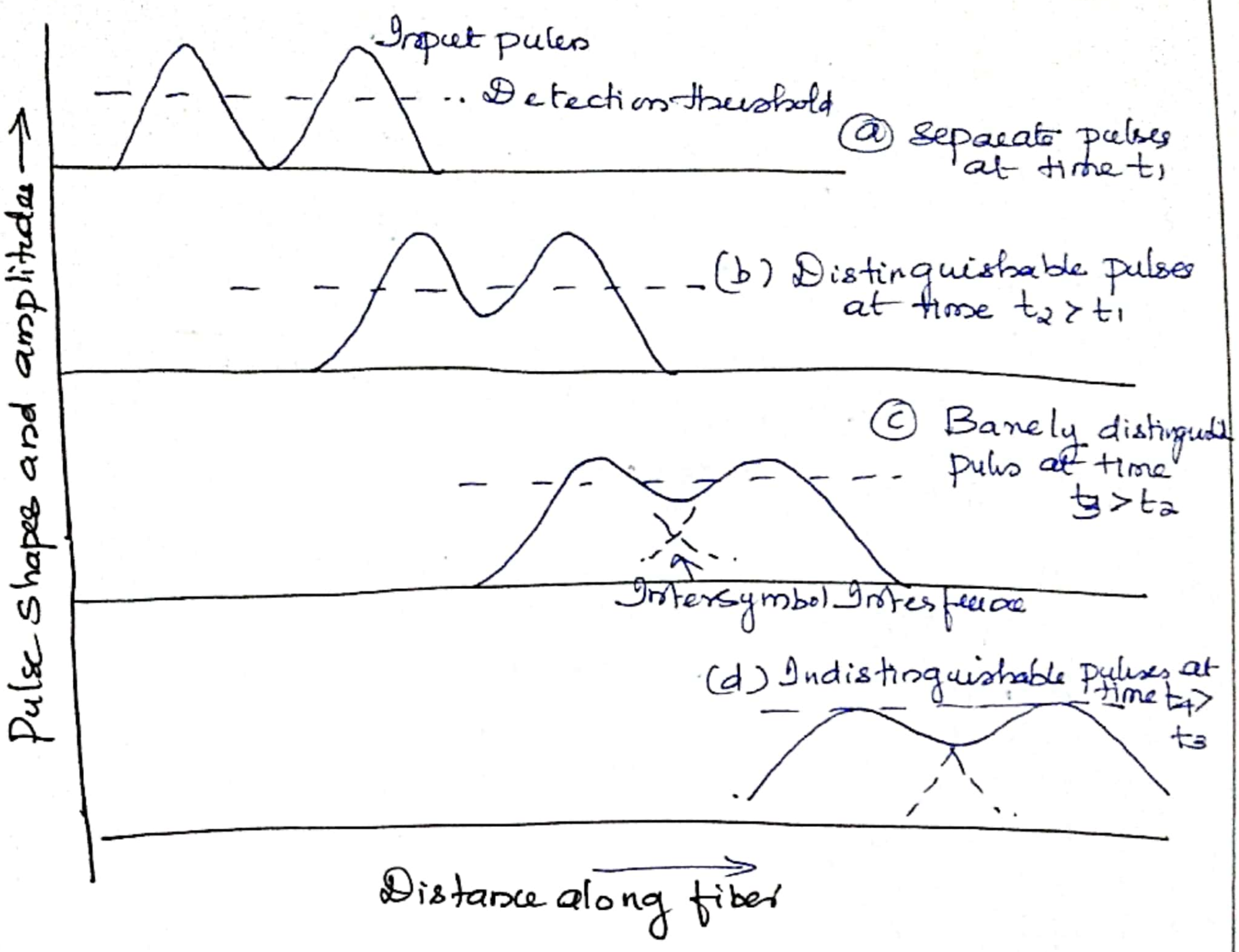


fig Broadening and attenuating of two adjacent pulses as they travel along a fiber

Overview of Dispersion Origins.

Signal dispersion is a consequence of factors such as

- intermodal delay
- Intramodal dispersion
- Polarization mode dispersion
- higher order dispersion.

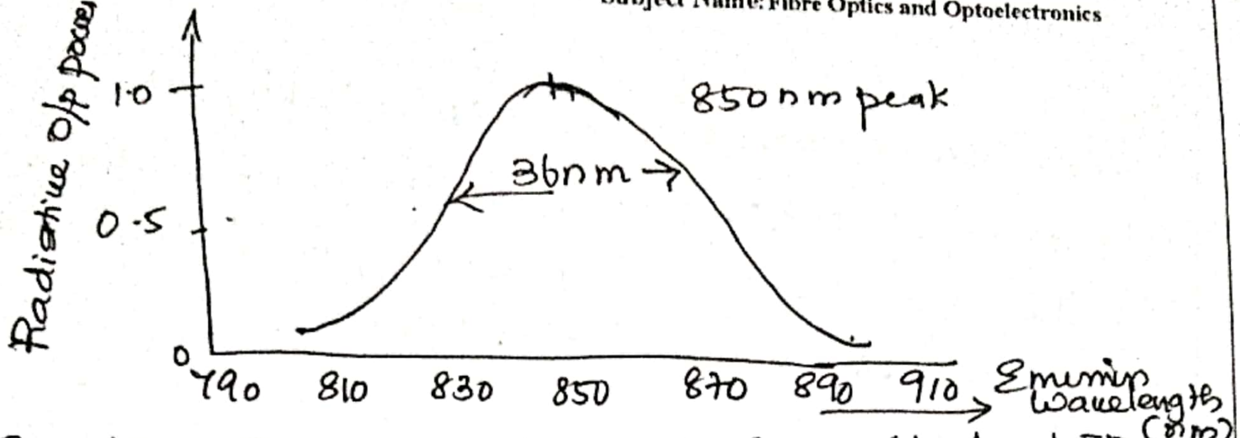
[These distortions can be explained by the group velocities of the guided modes where group velocity is the speed at which energy in a particular mode travels along the fiber.]

Intermodal Delay (Modal delay or Intermodal dispersion)

It appears in multimode fibers. It is a result of each mode having a different value of group velocity at a single frequency. It has direct impact on the information carrying capacity of a multimode fiber.

Intramodal ~~Delay~~ Dispersion or Chromatic Dispersion

It appears in single mode fibers. This spreading is due to finite spectral emission width of an optical source. It also depends on the wavelength and hence its effect on signal distortion increases with the spectral width of the light source. The spectral width is the band of wavelengths over which light source emits light. This wavelength band is represented by rms spectral width σ_λ. Depending on the device structure of a LED, the spectral width is $\sim$ 4 to 9% of central wavelength.



Spectral Emission pattern of $Ga_{1-x}Al_xAs$ LED

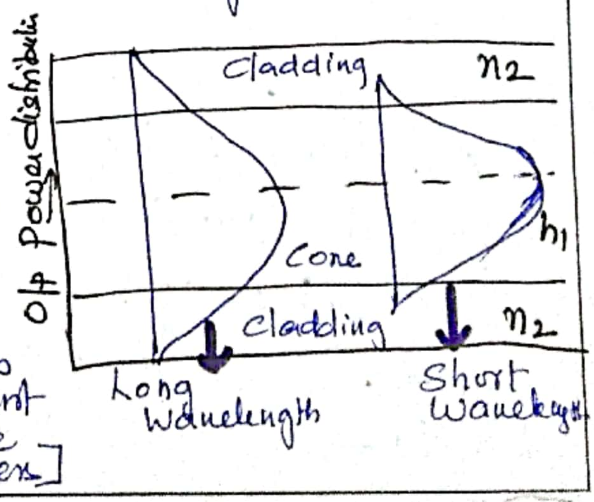
Intra modal dispersion is due to

→ Material Dispersion (Chromatic dispersion)

→ Waveguide Dispersion.

Material Dispersion: Due to variations of the refractive index of the core material as a function of wavelength and also depends on group velocity of a given mode.

Wave guide Dispersion: Causes pulse spreading as only part of the optical power propagation along a fiber is confined to the core. Shorter wavelengths are confined to fiber core and longer wavelengths propagate in the cladding. The refractive index of cladding is less compared to the core so light propagating in cladding travels faster than the light confined to the core. Thus spectral components have different speeds and this causes dispersion.



Degree of wave guide depends on fiber design [very significant in single mode fibers]

Polarization Mode Dispersion

Light signal energy at a given wavelength in a single mode fiber occupies two orthogonal polarization states or modes. At the start of the fiber, the two polarization states are aligned. However since fiber material is not perfectly uniform throughout its length each polarization mode will encounter a slightly different refractive index. Consequently each mode will travel at a slightly different velocity. The resulting difference in propagation times between the two orthogonal polarization modes will cause pulse spreading.

Intermodal dispersion (Modal Delay)

Intermodal dispersion or modal delay appears in multimode fibers. The signal distorting mechanism is a result of each mode having a different velocity at a single frequency. This variation in the group velocities of the different modes results in a group delay spread known as intermodal dispersion. The maximum pulse broadening arising from the modal delay is the difference between the travel time Tmax of the longest ray congruence paths (highest order modes) and the travel time Tmin of the shortest ray congruence paths. This broadening is obtained by Ray tracing and for a fiber of length L is given by

$$\Delta T = T_{max} - T_{min} = \frac{n_1}{c}\left(\frac{L}{\sin\phi_c} - L\right) = \frac{L n_1^2}{c n_2}\Delta$$

$$\simeq \frac{L n_1 \Delta}{c}$$

$\sin\phi_c = n_2/n_1$ and Δ is the index difference.

The fiber capacity is specified in terms of the bit rate - distance product BL, the bit rate times the possible transmission distance L. In order for neighbouring signal pulses to remain distinguishable at the receiver the pulse spread should be less than 1/B which is the width of a bit period.

$$BL < \frac{n_2}{n_1^2}\frac{c}{\Delta}$$

Problem

Consider a 1 km long multimode step index fiber in which $n_1 = 1.480$ and $\Delta = 0.01$, so that $n_2 = 1.465$. What is the modal delay per length in this fiber? Also find the capacity

Solution

given $n_1 = 1.480$

$n_2 = 1.465$

$\Delta = 0.01$

$$\Delta T = \frac{L n_1^2 \Delta}{c n_2}$$

$$\frac{\Delta T}{L} = \frac{n_1^2 \Delta}{c n_2}$$

$$\frac{\Delta T}{L} = \frac{(1.48)^2 \times .01}{3\times10^8 \times 1.465} = 50\,ns/Km$$

Capacity $BL = \frac{n_2}{n_1^2}\frac{c}{\Delta}$

$$BL = \frac{1.465}{(1.480)^2} \times \frac{3\times10^8}{0.01} = 20\,Mb/s\text{-}km$$

The root mean square (rms) value of the time delay is a useful parameter for assessing the effect of modal delay in a multimode fiber. If the light rays are uniformly distributed over the acceptance angles of a fiber, then the rms impulse response σ_s due to intermodal dispersion in a step index multimode fiber

$$\sigma_s = \frac{L n_1 \Delta}{2\sqrt{3}c} = \frac{L(NA)^2}{4\sqrt{3}n_1 c}$$

Here L is the fiber length and NA is the numerical aperature.

A succesful technique for reducing modal delay in multimode fibers is through a graded refractive index of the fiber core. In any multimode fiber the ray paths associated with higher order modes are concentrated near the edge of the core and thus follow a longer path through the fiber than low order modes. If the core has a graded index, then the higher order modes encounter a lower refractive index near the core edge. Since the speed of light in a material depends on the refractive index value, the higher order mode travel faster in the outer core region than those modes that propagate through a higher refractive index along the fiber center. This reduces the delay difference between the fastest and slowest modes then

$$\sigma_s = \frac{L n_1 \Delta^2}{20\sqrt{3}c}$$

Problem

Consider the following two multimode fibers (a) a step index fiber with a core index $n_1 = 1.458$ and a core cladding index difference $\Delta = 0.01$ (b) a parabolic-profile graded index fiber with the same values of n_1 and Δ. Compare the rms pulse broadening per kilometer for these two fibers.

Solution:

given
$n_1 = 1.458$
$\Delta = 0.01$

(a)

$$\frac{\sigma_s}{L} = \frac{n_1 \Delta}{2\sqrt{3}c} = \frac{1.458 \times .01}{2 \times 1.73 \times 3 \times 10^8}$$

$$= 14.0 ns/Km$$

(b) for a graded index fiber

$$\frac{\sigma_s}{L} = \frac{n_1 \Delta^2}{20\sqrt{3}c} = \frac{1.458 \times (0.01)^2}{20 \times 1.73 \times 3 \times 10^8}$$

$$= 14.0 ps/Km$$

Intramodal dispersion (Chromatic dispersion)

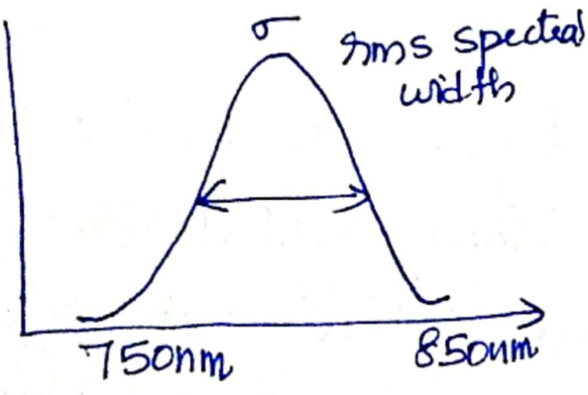
Material dispersion + Wave guide Dispersion = Chromatic Dispersion.

[Arises from the variation of refractive index with wavelength]

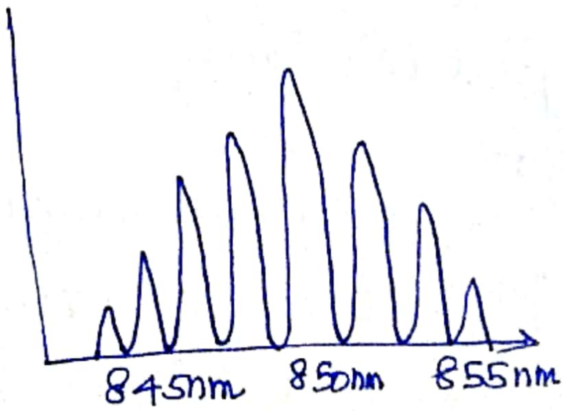
[Arises from the dependence of the fiber's waveguide properties on wavelength]

Material dispersion

It results from different group velocities of the various spectral components launched into the fiber by the source. The optical source has an optical output that spreads over a range of wavelengths



LED Typical spectral width is 75-125nm



Conventional LASER Multimode operation spectral width 2-5nm

Variety of waves are possible – plane, spherical etc and it is distinguished by their wave fronts. Wave front is a point of constant phase AND constant amplitude

Phase Velocity

For a monochromatic light the points of constant phase propagate with a velocity called the phase velocity V_p (V_p is the velocity at which the phase of any one frequency component of the wave will propagate) phase velocity $v_p = \frac{c}{n}$ $\left\{ \begin{array}{l} c - \text{velocity of light} \\ n \rightarrow \text{refractive index} \end{array} \right.$

* In free space $\lambda = c/f$

* In medium of refractive index $n > 1$, the velocity changes and as frequency is a constant, then wavelength in the medium $\lambda_m = \lambda/n$

As $n > 1$, $\lambda_m < \lambda$

Then v_p can be written as

$$v_p = \frac{f\lambda}{n} = f\lambda_m$$

Propagation constant (β)

It determines how the phase and amplitude of that light with a given frequency varies along the propagation direction z

$$A(z) = A(0)\,\mathrm{Exp}^{i\beta z}$$

β is a complex, the real part represents phase delay per unit propagation distance where as the imaginary part represents optical gain or loss.

The propagation constant depends on the optical frequency of the light. This frequency dependence determines the group delay and the chromatic dispersion of the waveguide.

For a plane wave in a medium

$$\beta = \frac{2\pi}{\lambda_m}$$

The angular frequency ω is $2\pi f$

then $V_p = \lambda_m f$ $\left\{ \lambda_m = \frac{2\pi}{\beta} \right.$

$$V_p = \frac{2\pi f}{\beta} = \frac{\omega}{\beta}.$$

Group velocity

The wave packets propagates in the direction of travel of the plane wave. The wave packets travel with a group velocity V_g and $V_g = \frac{d\omega}{d\beta}$. If information is modulated on the optical signal as a pulse then many wave packets with closely similar frequencies propagate

Group velocity is sometimes called modulation velocity

$$V_g = \frac{d\omega}{d\beta} \quad \text{— (1)}$$

The time delay per unit length is called group delay τ_g and can be given by

$$\tau_g = \frac{-\lambda^2 L}{2\pi c} \frac{d\beta}{d\lambda} \quad \text{— (2)}$$

~~This equation~~ In a medium that is susceptible to material dispersion, the refractive index is a fn of $n(\lambda)$ (wavelength)

The free space propagation constant k is given by $2\pi/\lambda$

The propagation constant in the medium is given by $\beta = k\,n(\lambda) = \frac{2\pi n(\lambda)}{\lambda}$ — (3)

Using equation (2) and (3) it is possible to determine the gp delay as a function of wavelength and refractive index in a medium where refractive index is a function of wavelength. If the propagation distance is L then group delay is given by

$$\tau_g = \frac{L}{c}\left[n - \lambda \frac{dn}{d\lambda}\right] \quad \text{— (4)}$$

The time τ_m taken for a pulse to propagate a distance L in a fiber is given by $\tau_m = \frac{L}{c}\left[n_1 - \lambda \frac{dn_1}{d\lambda}\right]$ (5)

If an impulse source with RMS optical spectral width of σ_λ and a mean wavelength of λ, then each spectral component will arrive at a different point in time so each τ_m value will be different.

Then the pulse broadening due to spectral broadening (material dispersion) σ_m is given by

$$\sigma_m = \sigma_\lambda \frac{d\tau_m}{d\lambda} + \sigma_\lambda{}^2 \frac{d^2\tau_m}{d\lambda^2}$$

The first term is very dominating

The I derivative of τ_m with respect to λ can be found by differentiating equ (4) w.r.to λ

$$\frac{d\tau_m}{d\lambda} = \frac{L\lambda}{c}\left[\frac{dn_1}{d\lambda} - \frac{d^2 n_1}{d\lambda^2} - \frac{dn_1}{d\lambda}\right]$$

$$= -\frac{L\lambda}{c}\left[\frac{d^2 n}{d\lambda^2}\right]$$

$$\sigma_m = \sigma_\lambda \frac{L}{c}\left|-\lambda \frac{d^2 n_1}{d\lambda^2}\right|$$

where $-\lambda \frac{d^2 n_1}{dn^2} = Y_m$ $\Big\{$ dimensionless dispersion coefficient

$$\sigma_m = \sigma_\lambda \frac{L}{c\lambda}|Y_m|$$