

18CSE390T
Computer Vision

Incremental Refinement

Incremental Refinement

- Nearest pixel – integer pixel
- Higher accuracy is required for stabilization or stitching
- Sub-pixel estimates
 - Evaluate several values (u,v) around the best value
 - Interpolate the matching score to find the analytic minimum
- Gradient descent on SSD energy function

Incremental Refinement

- SSD energy and Taylor series expansion Lucas and Kanade (1981)

$$\begin{aligned} E_{\text{LK-SSD}}(\mathbf{u} + \Delta\mathbf{u}) &= \sum_i [I_1(\mathbf{x}_i + \mathbf{u} + \Delta\mathbf{u}) - I_0(\mathbf{x}_i)]^2 \\ &\approx \sum_i [I_1(\mathbf{x}_i + \mathbf{u}) + \mathbf{J}_1(\mathbf{x}_i + \mathbf{u})\Delta\mathbf{u} - I_0(\mathbf{x}_i)]^2 \\ &= \sum_i [\mathbf{J}_1(\mathbf{x}_i + \mathbf{u})\Delta\mathbf{u} + e_i]^2, \end{aligned}$$

Image gradient or Jacobian at $(\mathbf{x}+\mathbf{u})$

$$\mathbf{J}_1(\mathbf{x}_i + \mathbf{u}) = \nabla I_1(\mathbf{x}_i + \mathbf{u}) = \left(\frac{\partial I_1}{\partial x}, \frac{\partial I_1}{\partial y} \right) (\mathbf{x}_i + \mathbf{u})$$

Current intensity error (residual error)

$$e_i = I_1(\mathbf{x}_i + \mathbf{u}) - I_0(\mathbf{x}_i)$$

Incremental Refinement

$$E_{\text{LK-SSD}}(\mathbf{u} + \Delta\mathbf{u}) \approx \sum_i [\mathbf{J}_1(\mathbf{x}_i + \mathbf{u})\Delta\mathbf{u} + e_i]^2$$



$$I_x u + I_y v + I_t = 0$$

Spatial derivative

temporal derivative

Optical flow constraint or brightness constancy constraint

Incremental Refinement

$$E_{\text{LK-SSD}}(\mathbf{u} + \Delta \mathbf{u}) \approx \sum_i [\mathbf{J}_1(\mathbf{x}_i + \mathbf{u}) \Delta \mathbf{u} + e_i]^2$$

$$\Rightarrow \mathbf{A} \Delta \mathbf{u} = \mathbf{b}$$

$$\mathbf{A} = \sum_i \mathbf{J}_1^T(\mathbf{x}_i + \mathbf{u}) \mathbf{J}_1(\mathbf{x}_i + \mathbf{u}) \quad \mathbf{b} = - \sum_i e_i \mathbf{J}_1^T(\mathbf{x}_i + \mathbf{u})$$

Gaussian-Newton approximation of the Hessian

Gradient-weighted residual vector

Incremental Refinement

- For efficiency
 - $J_1(x_i + u) \approx J_0(x_i)$
 - Precompute the Hessian and Jacobian image: save significant computation
 - Precompute the inner product between the gradient field and shifted version of I_1 allows the iterative re-computation of e_i to be performed in constant time (independent of the number of pixels)

Incremental Refinement

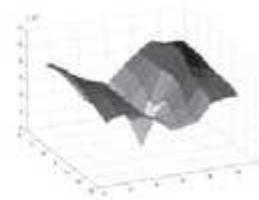
- Iterations
 - The effectiveness relies on the quality of Taylor series approximation
 - When far away from the true displacement (say, 1–2 pixels), several iterations may be needed
 - It is possible to estimate a value for J_1 using a least squares fit to a series of larger displacements in order to increase the range of convergence (Jurie and Dhome 2002) or to “learn” a special-purpose recognizer for a given patch

Incremental Refinement

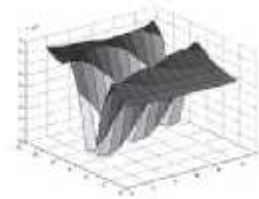
- Stopping criterion
 - monitor the magnitude of the displacement correction $|u|$ and to stop when it drops below a certain threshold (say, 1/10 of a pixel)
- For larger motions
 - combine the incremental update rule with a hierarchical coarse-to-fine search strategy

Incremental Refinement

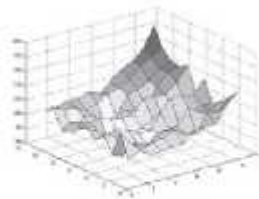
- Poorly conditioned because of lack of two-dimensional texture in the patch being aligned



(a)



(b)



(c)