Ex:

Convert the PDA $P = \{(p,q), \{0,1\}, \{x,z_0\}, \delta, q, z_0)$, to CFG.

It δ is given by

$$\delta(q,1,z_0) = \{(q,x,z_0)\} \qquad \delta(p,\epsilon,z_0) = \{(q,\epsilon)\}$$

$$\delta(q,1,x) = \{(q,xx)\} \qquad \delta(p,1,x) = \{(p,\epsilon)\}$$

$$\delta(q,0,x) = \{(p,x)\} \qquad \delta(p,0,z_0) = \{(q,z_0)\}$$

Sol:

$V = \{ S, [q,x,q], [q,x,p], [p,x,q], [p,x,p]$
$[q,z_0,q], [q,z_0,p], [p,z_0,q], [p,z_0,q] \}$

$$S \to [q, z_0, q]$$
$$[q, z_0, p]$$

(i) $\delta(q,1,z_0) = \{(q,xz_0)\}$

$$[p, z_0, q] \to 1 [q,x,q] [q, z_0, q]$$
$$| [q,x,p] [q, z_0, q]$$

$$[p, z_0, p] \to 1 [q,x,q][q, z_0, p]$$
$$| [q,x,p] [q, z_0, p]$$

(ii) $\delta(q,1,x) = \{(q,xx)\}$

$$[q,x,q] \to 1 [q,x,q] [q,x,q]$$
$$| [q,x,p] [p,x,q]$$

$$[q,x,p] \to 1 [q,x,q] [q,x,p]$$
$$| [q,x,p] [p,x,p]$$

(iii) $\delta(q,0,x) = \{(p,x)\}$

$$[q,x,q] \to 0 [p,x,q]$$
$$[q,x,p] \to 0 [p,x,p]$$

(iv) $\delta(q,\varepsilon,z_0) = \{(q,\varepsilon e)\}$

$$(q,z_0,q) \to \varepsilon e$$

(v) $\delta(p,1,x) = \{(p,\varepsilon e)\}$

$$(p,x,p) \to 1$$

(vi) $\delta(p,0,z_0) = \{(q,z_0)\}$

$$[p,z_0,q] \to 0 [q,z_0,q]$$
$$[p,z_0,p] \to 0 [q,z_0,p]$$

Final CFG is

$$S \to [q,z_0,q]$$
$$[q,z_0,p] \to \{1 [q,x,p] [p,z_0,q], \varepsilon e\}$$
$$[q,x,p] \to \{1 [q,x,p], [p,x,p], 0 [p,x,p]$$
$$[p,x,p] \to 1$$
$$[p,z_0,q] \to 0 [q,z_0,q]$$

Pumping lemma (For CFL) ③
$\rightarrow$ CFG $\rightarrow$ [V, Σ, s, P]

It is used to Prove that a language in not
a Contex free.

Assume L is a CFL, there in a Pumping
length n such that any string w ∈ L of length
≥ n can be written as

$$|w| \geq n \quad —①$$

we can break w into 5 stings.

w = uvxyz , such that

$$\rightarrow |vxy| \leq n$$

$$\rightarrow |vy| \neq \varepsilon$$

$\rightarrow$ For all $k \geq 0$ the string $\boxed{u v^k x y^k z \in L}$

steps

① Assume that L is Context free
② Pumping length say n
③ All stings longer than n can be Pumped
$|w| \geq n$
④ now find a sting " w' in L such that
$|w| \geq n$
⑤ Divide w into uvxyz ⑥ show that $u v^k w y^k z \notin L$
⑦ Then Consider the ways that w can be
divided into uvwxyz.
⑧ show that none of these Can satisfy all the
3 Pumping Conditions at same time
⑨ we cannot be Pumped (Contradiction)

Ex:

Find wethethy $L = \{\{x^n y^n z^n\} \mid n \geq 1\}$ in   ④
context free or not

Sol:

Let L is context free

& now we can take a string such that

$$s = x^n y^n z^n$$

& we divide s into 5 Parts $u v x y z$

case(i) $n = 4$ $x^4 y^4 z^5$

v & y each contain only one type of symbols

$$\underbrace{\overset{u}{\big|}\underset{v}{\underbrace{x}}x\underset{x}{\underbrace{xxx}}\underset{4}{\underbrace{4444}}\underset{4}{\big|}\underset{2}{\overset{2222}{\frac{}{2}}}} \rightarrow \text{divide into 5 Parts}$$
$$[u, v, x, y, z]$$

$\Rightarrow u v^k x y^k z$ [Let $k = 2$) $u = x$
$\Rightarrow u v^2 x y^2 z$ $v = xx$
$\Rightarrow xxxxxxyyyy2 2\overset{④}{2} 22.$ $x = x4442$
$\Rightarrow x^6 y^4 z^5$ $y = 2$
$\Rightarrow x^6 y^4 z^5 \notin L$ $z = 22$

the given languge is not context fre.

Case(ii)   ⑤

Either v or y has more than one kind of symb

$\Rightarrow x^n y^n z^n \implies$ 5 Parts

$\qquad\qquad\qquad u\,v\,x\,y\,z$

$n=4$, So $x^4 y^4 z^4 \left[u\, v\, \overset{k}{x}\, \overset{k}{y}\, z\, \ne L \right]$

$\Rightarrow \underbrace{xxxx}_{u}\underbrace{yyyy}_{}\underbrace{2222}_{} \implies$ 5 Parts

$\qquad\qquad\qquad\qquad\qquad\qquad\quad z$

$\qquad\qquad v\qquad\quad|\quad y$

$\qquad\qquad\qquad\qquad x$

$\qquad u = xx$

$\qquad v = xxyy$

$\qquad x = 4$

$\Rightarrow$ Let anuue $\boxed{k=2}$

$\qquad y = y$

$\qquad 2 = 2222$

$\Rightarrow u\,v^2\,x\,y^2\,z$

$\Rightarrow xx\,xx\,xx\,yy\,xx\,yyy\,yy\,2222$

$\Rightarrow x^4 y^2 x^2 y^5 2^4$

$\Rightarrow$ It is not belong to L

$\qquad$ This is not context free

Ex:
$$L = a^i b^j c^i d^j \mid i, j \geq 1 \ \& \text{ is not CFL}$$

① Assume L is context free

② now we can't take a String such that
$$S = a^i b^j c^i d^j$$

we divide into 5 Parts $uvxyz$

Case (i) $i = 2 \quad j = 2$
$$= a^2 b^2 c^2 d^2$$

$$\underbrace{aa}_{u} \ \underbrace{bb}_{v} \ \underset{x}{c} \ \underset{y}{c} \ \underbrace{dd}_{z}$$

$$\begin{aligned} u &= aa \\ v &= bb \\ x &= c \\ y &= c \\ z &= dd \end{aligned}$$

③ $k = 2$

$$\Rightarrow u v^k x y^k z$$

$$\Rightarrow u v^2 x y^2 z$$

$$\Rightarrow aa \ bbbb \ c \ cc \ dd$$

$$\Rightarrow a^2 b^4 c^3 d^2$$

$$\Rightarrow \notin L$$