Unit - 3

Push Down Automata

↓

Comparision

\+

Memory → $\boxed{\text{Stack}}$

← $a^n b^m$ $m = n$

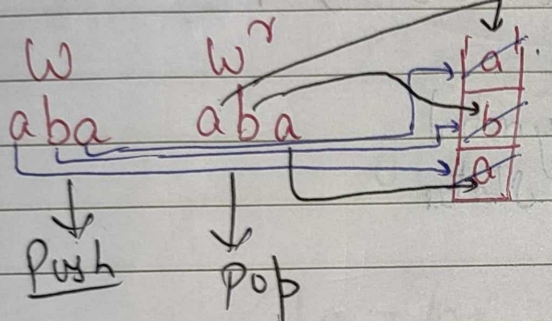
ww^r

$a^n b^{2n}$

wcw^r

↑

PDA



PDA :-

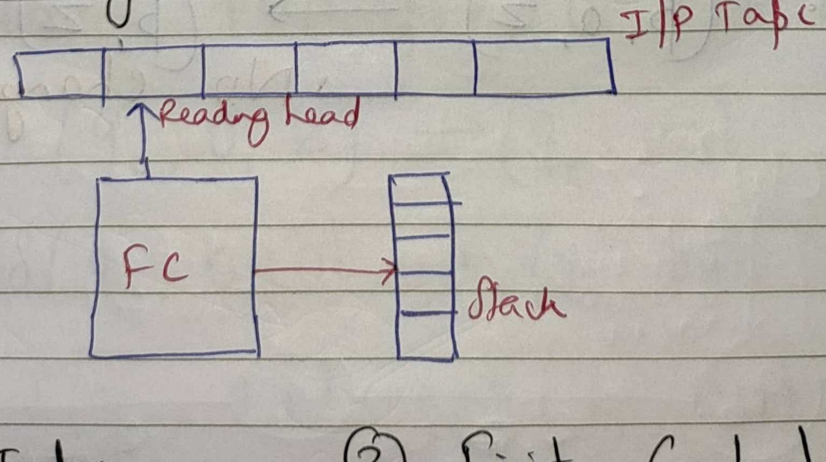
* CFG

* FA + memory

* Comparision

* Stack

Block Diagram :-



① I/P Tape

② Reading Head

③ Finit Control

④ Stack

Tuple :-

$Q, \Sigma, \delta, q_0, \Gamma, Z_0, F$

$Q \rightarrow$ finite set of states
$\Sigma \rightarrow$ finite set of input symbols
$\Gamma \rightarrow$ finite set of stack symbols
$\delta \rightarrow$ Transition function
$Q \times (\Sigma \cup \{\epsilon\}) \times \Gamma$

$q_0 \rightarrow$ Start state
$Z_0 \rightarrow$ initial stack symbol
$F \rightarrow$ final or accept states

Transition :-

$\delta(q_0, a, b) \rightarrow q_1$

1. $\delta(q_0, a, z) \rightarrow (q_0, az)$
   $\hookrightarrow$ Push
2. $\delta(q_0, a, \epsilon) \rightarrow (p, a)$
   $\hookrightarrow$ Push into empty stack
3. $\delta(q, a, z) \rightarrow (p, \epsilon)$
   $\hookrightarrow$ Pop
4. $(p, a, z) \rightarrow (p, z)$
   $\hookrightarrow$ No change

Example :- Construct PDA for $a^n b^n$ $n > 0$.

Logic → a ko push karo, and then pop out a for input 'b'.

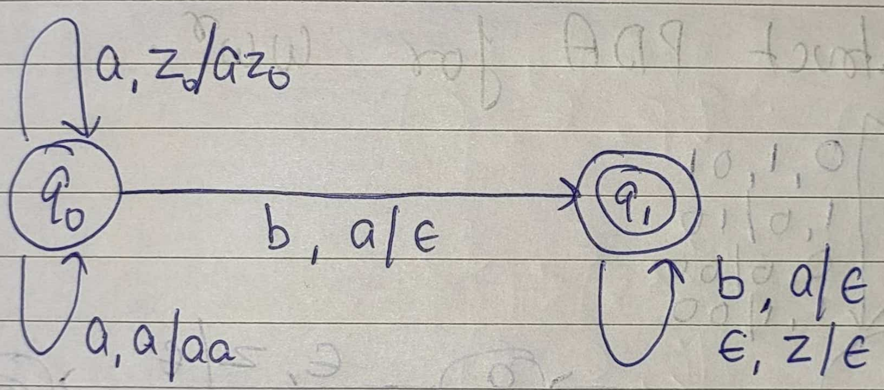
$\Rightarrow$ $\delta(q_0, a, z) \rightarrow (q_0, az)$

$\delta(q_0, a, a) \rightarrow (q_0, aa)$

$\delta(q_0, b, a) \rightarrow (q_1, \epsilon)$

$\delta(q_1, b, a) \rightarrow (q_1, \epsilon)$

$\delta(q_1, \epsilon, z) \rightarrow (q_1, \epsilon)$



Q. Construct PDA for $0^n 1^{2n}$ $n > 0$

$\Rightarrow$ $\delta(q_0, 0, z) \rightarrow (q_0, 0z)$

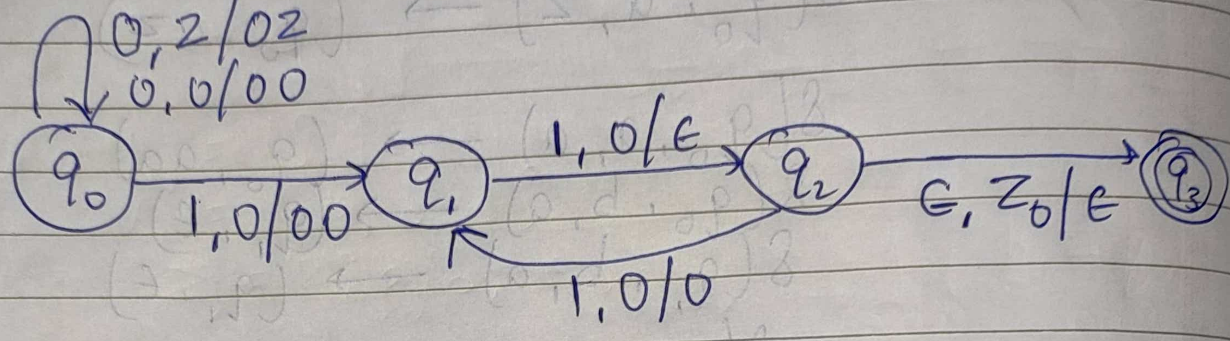
$\delta(q_0, 0, 0) \rightarrow (q_0, 00)$

$\delta(q_0, 1, 0) \rightarrow (q_1, 00)$

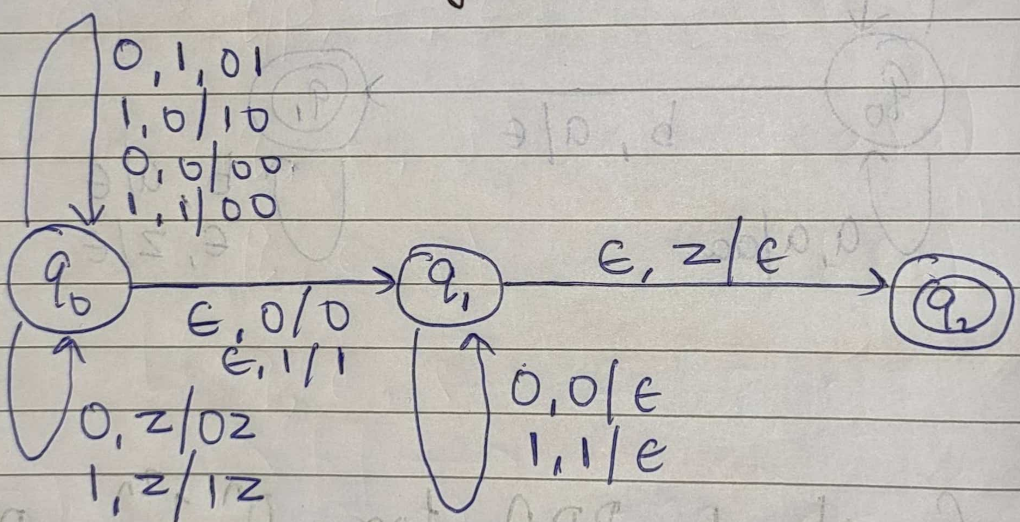
$\delta(q_0, 1, 0) \rightarrow (q_2, \epsilon)$

$$\delta(q_2, 1, 0) \rightarrow (q_1, 0)$$

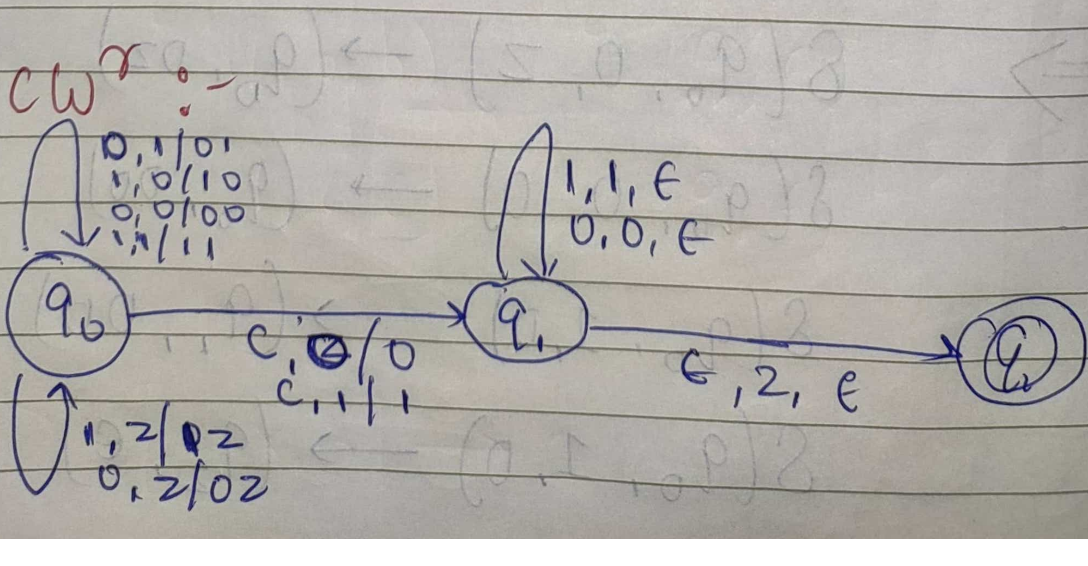
$$\delta(q_2, \epsilon, z) \rightarrow (q_3, \epsilon)$$



Q. Construct PDA for ww^r.



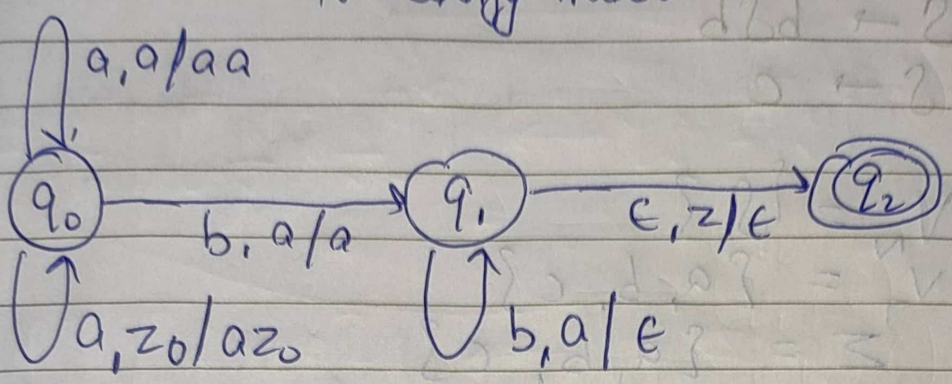
Q wcw^r :-



Q. $a^n b^{n+1}$:-

$a^n \underline{b}\ b^n$

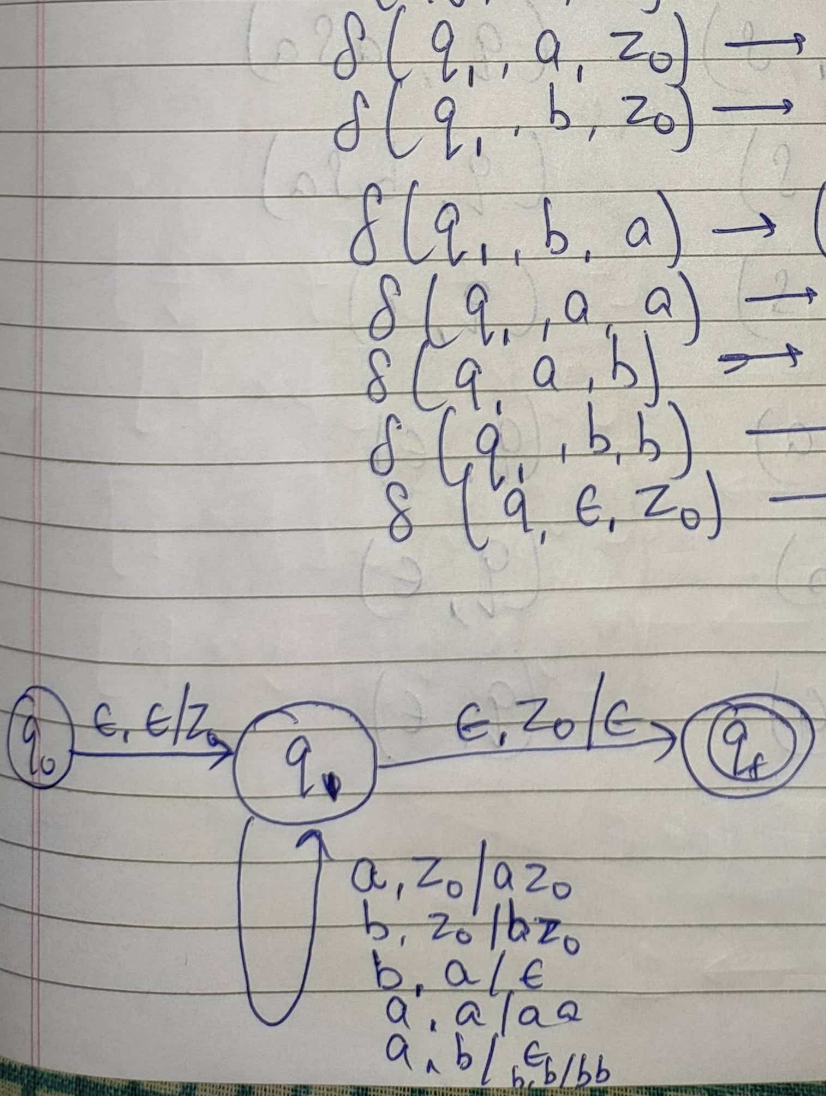
↑ to identify middle



Q. Equal no. of a & b :-

$\delta(q_0, \epsilon, \epsilon) \rightarrow (q_1, z_0)$
$\delta(q_1, a, z_0) \rightarrow (q_1, z_0 a)$
$\delta(q_1, b, z_0) \rightarrow (q_1, z_0 b)$

$\delta(q_1, b, a) \rightarrow (q_1, \epsilon)$
$\delta(q_1, a, a) \rightarrow (q_1, aa)$
$\delta(q_1, a, b) \rightarrow (q_1, \epsilon)$
$\delta(q_1, b, b) \rightarrow (q_1, bb)$
$\delta(q_1, \epsilon, z_0) \rightarrow (q_f, \epsilon)$.



Conversion of CFG to PDA

Q. $S \rightarrow aSa$ $\longrightarrow$ wcw^r

$S \rightarrow bSb$

$S \rightarrow c$

$\Rightarrow$ $V_N = \{S\}$

$V_T = \{a, b, c\}$

$\Sigma = \{a, b, c\}$

$P = \{a, b, c, S\}$

Common for all question — ① (p, ϵ, ϵ) (q, S)

② (q, ϵ, S) (q, aSa)

Push — (q, ϵ, S) (q, bSa)

(q, ϵ, S) (q, c)

③ (q, a, a) (q, ϵ)

Pop — (q, b, b) (q, ϵ)

(q, c, c) (q, ϵ)

Conversion of PDA to CFG :-

$$G = (V, T, P, S)$$

V → Variable (Non-terminal)

T → Terminal

P → Production Rule

$\delta(q, \alpha, \gamma)$ $(\gamma\ y_1\ y_2 - - y_\alpha$

q → State

α → input

$$[q, a, \gamma] \longrightarrow a[- - -]$$

$$P = (\{q, p\}, \{0, 1\}, \{x, z\}, \delta, q, z).$$

$\{q, p\}$ → Q

$\{0, 1\}$ → Σ

$\{x, z\}$ → Γ

δ → T.F

q → initial

z → Stack

1. $\delta(q, 1, z)$ (q, xz)
2. $\delta(q, 1, x)$ (q, xx)
3. $\delta(q, \epsilon, x)$ (q, ϵ)
4. $\delta(q, 0, x)$ (p, x)
5. $\delta(p, 1, x)$ (p, ϵ)
6. $\delta(p, 0, z)$ (q, z)

$\Rightarrow$ $V = S, [pxp], [pxq], [qxp], [qxq]$
$[pzp], [pzq], [q,zp], [qzq]\}$

$S \rightarrow [q z q] / [q z p]$.

① $\delta(q, 1, z)\ (q, xz)$.

$[q z q] \rightarrow 1[q x q][q z q]$

$[q z q] \rightarrow 1[q x p][p z q]$

$[q z p] \rightarrow 1[q x p][p z p]$

$[q z p] \rightarrow 1[q x q][q z p]$

② $\delta(q, 1, x)\ (q, xx)$

$[q x q] \rightarrow 1[q x q][q x q]$

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$[q x q] \rightarrow 1[q x p][p x q]$

$[q x p] \rightarrow 1[q x q][q x p]$

⑤ $\delta(p, 1, x)\ (p, e)$

$\Rightarrow [p . x . p] \rightarrow 1$

③ $\delta(q, \epsilon, x) \rightarrow (q, \epsilon)$

$\Rightarrow [q, x, q] \rightarrow \epsilon$

④ $\delta(q, 0, x) \; (p, x)$

$\Rightarrow [q, x\, p] = 0\,[p\, x\, p]$

$[q\, x\, q] = 0\,[p\, x\, q]$

⑥ $\delta(p, 0, z) \; (q, z)$

$[p\, z\, q] \rightarrow 0\,[q\, z\, q]$

$[p\, z\, p] \longrightarrow 0\,[q\, z\, p]$

Q Convert the PDA into CFG.

$M = (\{p, q\}, \{0, 1\}, \{x, Z_0\}, \delta, q, Z_0)$

δ is given as

$\delta(q, 1, Z_0) = (q, xZ_0)$

$\delta(q, 0, Z_0) = (q, \wedge)$

$\delta(q, 1, Z_0) = (p, \wedge)$

$\Rightarrow$ Let G be the grammar equivalent to M

$G = (V, \Sigma, P, S)$ where $\Sigma = \{0, 1\}$,

S is starting symbol.

Now,

$V = \{S, [p, n, p], [p, n, q], [q, n, p], [q, n, q], [p, z_0, p], [p, z_0, q], [q, z_0, p], [q, z_0, q]\}$

Now, $P : S \to [q, z_0, p] / [q_0, z_0, q]$

Now, $\delta(q, 1, z_0) = (q, n z_0)$ will give rise to

$[q, z_0, q] = 1 [q\ n\ q] [q\ z_0\ q]$

$[q, z_0, p] = 1 [q\ n\ p] [q\ z_0\ p]$

$[q, z_0, q] = 1 [q\ n\ p] [p\ z_0\ q]$

$[q, z_0, p] = 1 [q\ n\ p] [p\ z_0\ p]$

$\delta(q, 0, z_0) = (q, \wedge)$ will give rise to

$[q\ z_0\ q] \to 0$

$\delta(q, 1, z_0) = (p, \wedge)$ will give rise to

$[q\ z_0\ p] \to 1$