FLA

CT3 Answer key

1. a) Design a Turing machine to add the number of red and blue color pens owned by a school kid in his bag in single digits. The result should be the addition of two digit numbers (solution in words (2), State transition diagram (4) Table (2), example parsing (4))

Turing Machine as Adder

Example parsing (4 marks)

Take pens as 0, and separator as 1

a=3 000

then the Result should have

b=4 0000 a+b, 3+4=7 0's

B	B	0	0	0	~~1~~	0	0	0	~~0~~	B	B
		↑			0				B		

(2 marks)

take 1 as separator

1. The control should be moved towards right until it reaches 1 that is separator.

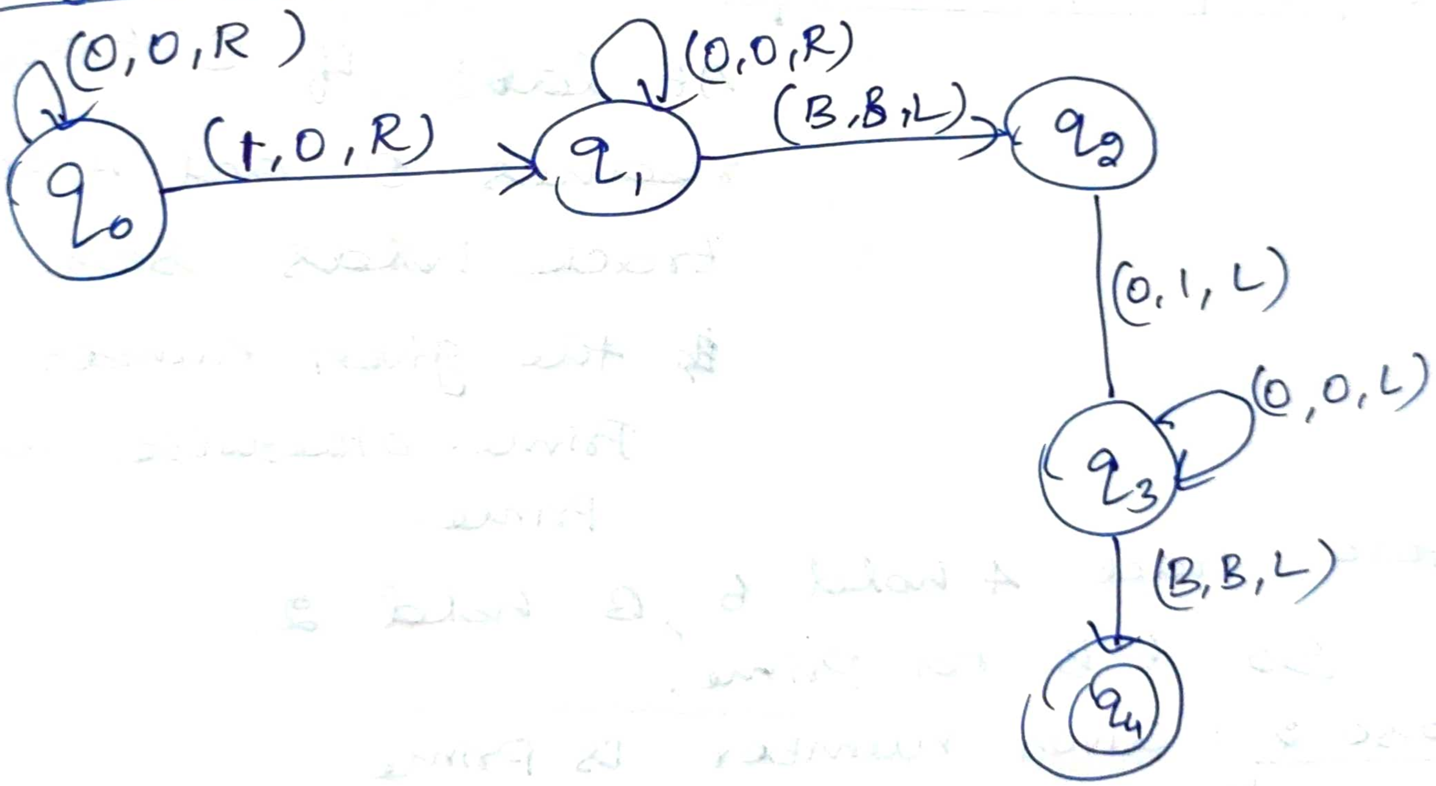
2. The 1 should be replaced with 0

3. Then control should be moved towards right until it reaches B, when it reach it should move 1 position left and replace last 0 by B.

State transition table (2 marks)

	0	1	B
q_0	$(q_0, 0, R)$	$(q_1, 0, R)$	–
q_1	$(q_1, 0, R)$	–	(q_2, B, L)
q_2	$(q_3, 1, L)$	–	–
q_3	$(q_3, 0, L)$	–	(q_4, B, L)
q_4	–	–	–

Diagram (4 marks)



b) A simple robot was designed by a set of interns in a research lab which asks everyone their age and simply tells whether it is prime or no. Design a multiple track turing machine which performs this task (10)

<u>Two cases</u>

case 1: Given number is not Prime

take $n = 6$

A	6	6	6	6
B	2	2	2	2
C	6	4	2	0

At last if C track reaches 0 and A and B track holds same number the given number is Prime. Otherwise its not Prime.

Here track A hold 6, B hold 2.

So 6 is not Prime.

<u>case 2</u> : Given number is Prime

take $n = 7$

A	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
B	2	2	2	2	3	3	3	4	4	5	5	6	6	7	7
C	7	5	3	1	7	4	1	7	3	7	2	7	1	7	0

The track A and B holds same number when the track C has 0.

Given number is ~~not~~ prime.

take three symbols as x, y, z

⇒ First x = given number, y = 2, z = x

⇒ then x = x y = y z = x - y

⇒ Repeat until z reaches 0.

⇒ If z gives negative number means increment y by one and write z as x value then repeat.

⇒ when z reaches 0, x and y is same means x is prime otherwise not.

c) Name some of the computational functions for which we can design a Turing Machine.

⇒ Addition $f(n1, n2) = n1 + n2$

⇒ Subtraction $f(n1, n2) = n1 - n2$

⇒ Multiplication $f(n1, n2) = n1 * n2$

d) Recursively Enumerable Language is a subset of context Free language. True or False.

False. Context free language is a subset of Recursively enumerable language

) write a corresponding binary code for the given Turing Machine (12)

A single Turing Machine M has six states ($q_0, q_1, q_2, q_3, q_4, q_5$) of q_0 and q_5 are initial states and final state respectively. The tape alphabet of M is {a, b, B} and its input alphabet is {a, b}. The symbol B is the blank symbol used to indicate end of an input string. The transition function of M is given below.

a → x_1, b → x_2, B → x_3
L → D_1, R → D_2, change $q_0 \to q_1$, $q_1 \to q_2$ ...

$\delta(q_0, a) = (q_1, a, R) \Rightarrow \delta(q_1, x_1) = (q_2, x_1, D_2)$
↳ 0101001010 0

$\delta(q_1, a) = (q_2, a, R) \Rightarrow \delta(q_2, x_1) = (q_3, x_1, D_2)$
↳ 00101000101 00

$\delta(q_2, a) = (q_3, a, R) \Rightarrow \delta(q_3, x_1) = (q_4, x_1, D_2)$
↳ 00010 100001 0100

$\delta(q_3, a) = (q_3, a, R) \Rightarrow \delta(q_4, x_1) = (q_5, x_1, D_2)$
00001 0 10000010100

$\delta(q_3, b) = (q_4, b, R) \Rightarrow \delta(q_4, x_2) = (q_5, x_2, D_2)$
000010010000010 0100

$\delta(q_3, B) = (q_5, B, L)$
$\Rightarrow \delta(q_4, x_3) = (q_6, x_3, D_1)$
00001000100000010001 0

$\delta(q_4, b) = (q_4, b, R)$
$\Rightarrow \delta(q_5, x_2) = (q_5, x_2, D_2)$
00000100 1 00000100100

$\delta(q_4, B) = (q_5, B, L)$
$\Rightarrow \delta(q_5, x_3) = (q_6, x_3, D_1)$
000001 000 10000001 00010

General Format

$\delta(q_i, x_j) = (q_k, x_l, D_m)$ then code

$0^i 1 0^j 1 0^k 1 0^l 1 0^m$

Code

$C_1 \ 11 \ C_2 \ 11 \ C_3 \ldots$

Final Binary code for given TM

0101001010011 00101 00010100 11 0001010000 10100

11 0000101 000001 0100 11 0000100 1000000100100

11 00001000100000001000 10 11 000001 001000000 1001

0011 000001 0001 0000000 100010

b) Every non trivial property of Turing Machine is undecidable. Define the theorem with its correct name. Write the proof for the same.

Rice's Theorem: $\Rightarrow$ Refer page No 398 Theorem 9.11

c) When can you say that the problem P1 reduces to P2? Write the proof (7)

Refer page No 393 Theorem 9.7.

There are two set of computer in a lab. The detail are give below

DELL	ASUS
1	10
110	0
0	11

we need to find the correct sequence with which they need to be arranged, so that both the computers are uniformly distributed in the lab.

This is a PCP problem.

$$\frac{1}{10}, \quad \frac{110}{0}, \quad \frac{0}{11}$$

Now arrange it as Numerator and denomina should be same.

$$\cancel{\frac{0}{11}} \quad \cancel{110} \qquad \frac{1}{10}\,\frac{0}{11}\,\frac{110}{0} \Rightarrow \frac{10110}{10110}$$

So the Dell computer should be in the order of 1, 3, 2 and ASUS should be in the order of 1,3,2

b) Is it a PCP problem. If yes define it.

An instance of Post's correspondence Problem (PCP) consists of two lists of strings over some alphabet Σ, the two lists must be equal length. We generally refer to the A and B lists and write $A = w_1, w_2 \ldots w_k$ and $B = x_1, x_2 \ldots x_k$ for some integer k. For each i pair (w_i, x_i) is said to be corresponding pair.

We say this instance of PCP has a solution if there is a sequence of one or more integers $i_1, i_2 \ldots i_m$ that when interpreted as indexes for string in in the A and B lists, yield the same string.

In the given proble two list given with equal length that can solve with the correct solution also.

Thus the given problem is PCP

c) If it is modified PCP. Justify it.

In the modified PCP, there is additional requirement on a solution that the first pair on the A and B lists must be the first pair in the solution.

Thus the given problem is modified PCP.

d) Prove the following

1) The intersection of two recursively enumerable language is also recursively enumerable (5)

ii) The language is recursive if it and it's complement are recursively enumerable (5)

Refer page No : 384
section : 9.2.2

e) write the difference b/w NP - complete and NP hard problem.