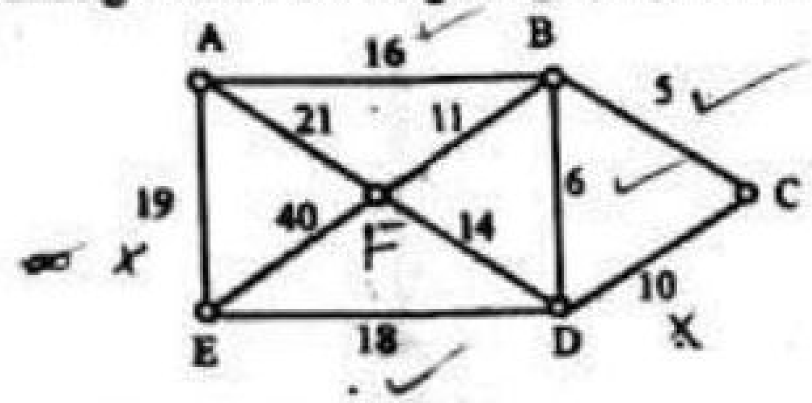
ii. Find the code words generated by the parity check matrix

$$H = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ when the encoding function is $C: B^3 \to B^6$.

45 5 6 11 18 19 35 51

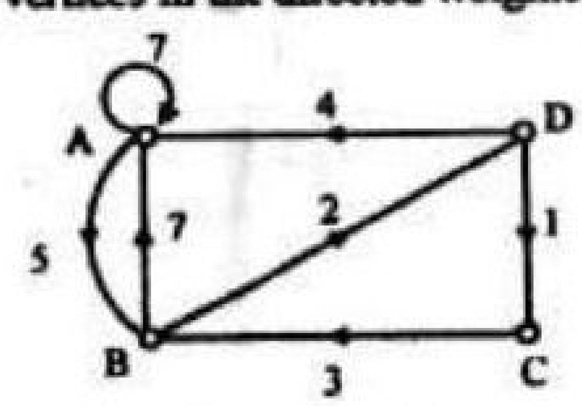
31. a.i. Find the minimum spanning tree for the weighted graph given in figure




ii. Prove that a tree with n vertices has (n–1) edges.

(OR)

b. Using Warshall's algorithm, find the shortest distance matrix and the corresponding shortest path matrix for all the pairs of vertices in the directed weighted graph given below




32. a. Draw the state transition diagram for the NFA for which the state transition table is given in table. Characterize the strings accepted by this NFA for which the accepting states are $S_1$ and $S_2$.

Table

| S \ I | f: a | f: b |
|---|---|---|
| $S_0$ | $S_0, S_1$ | $S_2$ |
| $S_1$ | $\phi$ | $S_1$ |
| $S_2$ | $S_1, S_2$ | $\phi$ |

(OR)

b. Find the DFA equivalent to the NFA for which the state transition table is given in table and the accepting state is $S_2$

Table

| S \ I | f: a | f: b |
|---|---|---|
| $S_0$ | $\phi$ | $S_0, S_1$ |
| $S_1$ | $\phi$ | $S_2$ |
| $S_2$ | $S_0, S_1, S_2$ | $\phi$ |

*****



Reg. No.

## B.Tech. DEGREE EXAMINATION, NOVEMBER 2016
Third Semester

## 15MA203 – DISCRETE MATHEMATICS FOR INFORMATION TECHNOLOGY
*(For the candidates admitted during the academic year 2015 – 2016 onwards)*

**Note:**
(i) Part - A should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) Part - B and Part - C should be answered in answer booklet.

Time: Three Hours | Max. Marks: 100

### PART – A (20 × 1 = 20 Marks)
Answer ALL Questions

1. Which of the following statement is the negation of the statement, "2 is even and –3 is negative"?
   (A) 2 is even and –3 is not negative
   (B) 2 is odd and –3 is not negative
   (C) 2 is even or –3 is not negative
   (D) 2 is odd or –3 is not negative

2. Which one is the contrapositive of $Q \to P$?
   (A) $P \to Q$
   (B) $\neg P \to \neg Q$
   (C) $\neg Q \to \neg P$
   (D) $Q \to P$

3. What is the dual of $(P \to Q) \to (\neg Q \to \neg P) \equiv T$?
   (A) $\neg(\neg P \wedge Q) \wedge (\neg Q \wedge \neg P) \equiv F$
   (B) $\neg(P \wedge Q) \wedge (Q \wedge P) \equiv T$
   (C) $((\neg P \wedge Q) \wedge (Q \wedge \neg P) \equiv F$
   (D) $(\neg P \wedge Q) \wedge (Q \wedge P) \equiv F$

4. A premise may be introduced at any point in the derivation is called ________
   (A) Rule P
   (B) Rule P and rule T
   (C) Rule T
   (D) Rule CP

5. If 100 pigeons are accomated in 90 pigeon holes, then one of the pigeon hole must contain
   (A) $\left[\frac{99}{90}\right]+1$
   (B) $\left[\frac{100}{90}\right]+1$
   (C) $\left[\frac{101}{100}\right]+1$
   (D) $\left[\frac{99}{100}\right]+1$

6. Given the sequence 1,3,9,….., the recurrence relation is
   (A) $a_n = 3a^n$
   (B) $a_n = 3a_{n-1}$
   (C) $a_n = (a_{n-1})^2$
   (D) $3a^{n+1}$

7. The generating function of the sequence 1,1,1, ….. is given by
   (A) $\frac{1}{1+x}$
   (B) $\frac{1}{1-x}$
   (C) $\frac{1}{(1-x)^2}$
   (D) $\frac{1}{x}$

8. The characteristic equation of $s(k) - 4s(k-1) + 4s(k-2) = 0$ is
   (A) $R^2 - 4R - 4 = 0$
   (B) $R^2 - 4R + 4 = 0$
   (C) $R^2 + 4 = 0$
   (D) $R^2 - 4 = 0$

9. If (G, $*$) is an Abelian group then $(A*B)^N$ is equal to
(A) $B^N * A^N$ (B) $B*A$
(C) $A^N * B^N$ (D) $A*B$

10. In a group $(Q^+,*)$ where $Q^+$ is set of all positive rational numbers and $A*B=\frac{1}{2}AB;\ A,B\in Q^+$, the identity element is
(A) E = 2 (B) E = 4
(C) E = 6 (D) E = 8

11. In the cyclic group $(G=\{1,-1,I,-I\})$, where $*$ is the usual multiplication the generators are ______
(A) 1, –1 (B) 1, I
(C) –1, –I (D) I, –I

12. The minimum distance of a code {10110, 11110, 10011} is ______
(A) 1 (B) 2
(C) 3 (D) 4

13. A circuit of a graph G is called ______ circuit if it includes each edge of G exactly once
(A) Hamiltonian (B) Konisberg
(C) Closed (D) Eulerian

14. If a vertex V of a tree has no children it is called ______
(A) Pendant vertex (B) Non-terminal
(C) Descendant (D) Root

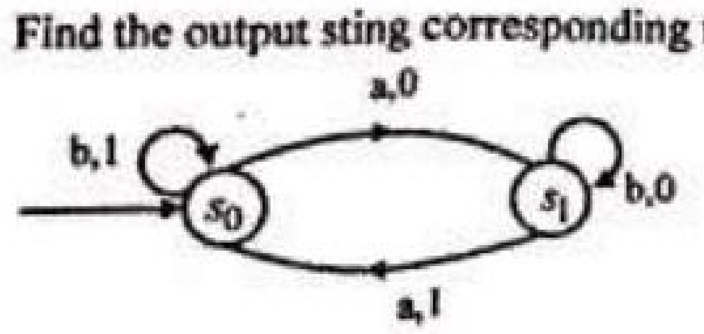
15. If every internal vertex of a rooted tree has exactly 2 children the tree is called ______
(A) Full binary tree (B) Binary tree
(C) Tree (D) Circuit

16. A connected graph without any circuit is called ______
(A) Directed graph (B) Tree
(C) Leaf (D) Loop

17. The transition function in DFA assigns ______ next state to every pair of state and input
(A) All (B) Unique
(C) Two (D) Several

18. If a language L is accepted by non-deterministic finite automata, then there exists a ______ that also accepts L
(A) Deterministic finite automata (B) Push down automata
(C) Context free grammar (D) Turing machine

19. Find the output sting corresponding to the input string for *ababa*, whose state diagram is




(A) 01100 (B) 01010
(C) 00110 (D) 10011





20. The language accepted by a finite state automaton is called ______
(A) Regular language (B) Context free language
(C) Context sensitive language (D) Formal language

## PART – B (5 × 4 = 20 Marks)
Answer ANY FIVE Questions

21. Prove that $\neg(q\to r)\wedge r\wedge(p\to q)$ is a contradiction using truth table.

22. Use mathematical induction to show that $n!\geq 2^{n-1}$ *for* $n\geq 1$.

23. If there are 5 points inside a square of side length 2, prove that two of the points are within a distance of $\sqrt{2}$ of each other.

24. Prove that the intersection of two subgroups is a subgroup and the union of two subgroups need not necessarily be a subgroup.

25. Prove that the number of vertices of odd degree in an undirected graph is even.

26. Find the language generated by the grammar G =({ S,A,B}, {a,b,}, S,P) where P is set of productions {S→AB, S→AA, A→aB, A→ab, B→b}.

27. Construct a finite state automaton that accepts the set of all strings over {a,b} starting with the prefix ab.

## PART – C (5 × 12 = 60 Marks)
Answer ALL Questions

28. a.i. Show that a→b, c→b, d→(a∨c), d⇒b by indirect method.

ii. Show that the following premises are inconsistent.
- If Raja misses many classes, then he fails in the final examination.
- If Raja fails in the final examination then he is uneducated
- If Raja reads a lot of books, then he is not uneducated
- Raja misses many classes and reads a lot of books.

(OR)

b.i. Without constructing the truth tables prove that $\neg p\to(q\to r)\equiv q\to(p\vee r)$.

ii. Using the rule CP, derive $p\to(q\to s)$ from the premises $p\to(q\to r)$ and $q\to(r\to s)$.

29. a. Solve $a_{n+2}-6a_{n+1}+9a_n=3(2^n)+7(3^n)$ where n ≥ 0 given that $a_0=1$ *and* $a_1=4$.

(OR)

b.i. Solve $F_{n+2}=F_{n+1}+F_n;\ n\geq 0$ *with* $F_0=0$ *and* $F_1=1$.

ii. Use the method of generating functions to solve $a_n=3a_{n-1}+1$ where n ≥1 given that $a_0=1$.

30. a. State and prove the Lagrange's theorem.

(OR)

b.i. Show that every subgroup of a cyclic group is cyclic.

Dr. D. K. SHEENA CHRISTY,

# ANSWER KEY

B.Tech Degree Examination, Nov-2016.

15MA203- Discrete Maths for IT. III sem

## PART-A.

| | | | |
|---|---|---|---|
| 1) B | 6) B | 11) B | 16) B |
| 2) B | 7) B | 12) A | 17) B |
| 3) C | 8) B | 13) D | 18) A |
| 4) A | 9) A | 14) A | 19) (C) |
| 5) A | 10) A | 15) A | 20) A. |

## Part-B

21)

| $p$ | $q$ | $r$ | $p \to q$ | $\neg(q \to r)$ | $\neg(q\to r) \wedge r \wedge (p \to q)$ |
|---|---|---|---|---|---|
| T | T | T | T | F | F |
| T | T | F | T | T | F |
| T | F | T | F | F | F |
| T | F | F | F | F | F |
| F | T | T | T | F | F |
| F | T | F | T | T | F |
| F | F | T | T | F | F |
| F | F | F | T | F | F |

$\longrightarrow (4m)$

22) $S_n: n! \geq 2^{n-1}$

$S_1 : 1! \geq 2^0$, which is true.

Let $S_k$ : $k! \geq 2^{k-1}$ be true. $\longrightarrow$ (2m)

$(k+1)! = (k+1) \cdot k!$

$\geq (k+1)\ 2^{k-1}$

$= 2^k$ $\longrightarrow$ (2m)

23) Divide the square into 4 squares of side 1 cm.

$\therefore$ No. of pigeon holes = 4; No. of pigeons = 5.

$\therefore$ Atleast two points in one square. maximum distance = length of diagonal $= \sqrt{2}$. Hence by PHP, atleast 2 pts are within a distance of $\sqrt{2}$ of each other (4m).

24. Proof:- Let

$H_1$ and $H_2$ be any two subgroups of $G$. Let $a \in H_1 \cap H_2$ then $a \in H_1$ and $a \in H_2$ as $H_1 \cap H_2$ is a nonempty set.

Let $b \in H_1 \cap H_2$. Since $H_1$ is a subgp of $G$, $a * b^{-1} \in H_1$ and $H_2$ is a subgp of $G$, $a * b^{-1} \in H_2$. Hence $a * b^{-1} \in H_1 \cap H_2$. → (2m)

$\therefore$ $H_1 \cap H_2$ is a subgp. of $G$

$H_1 \cup H_2$ is not a subgroup of $G$. Let $H_1 = \{ \dots -6, -4, -2, 0, 2, 4 \dots \}$

and $H_2 = \{ \dots -9, -6, -3, 0, 3, 6, 9 \dots \}$.

$H_1 \cup H_2 = \{ \dots -2 \dots 0, 2, 3, 4, 6, 9, \dots \}$ → (2m)

As $2+3 = 5 \notin H_1 \cup H_2$.

25) Let $G=(V,E)$ be the undirected graph.

$V_1$ – set of vertices of $G$ of even degree.

$V_2$ – " odd degree.

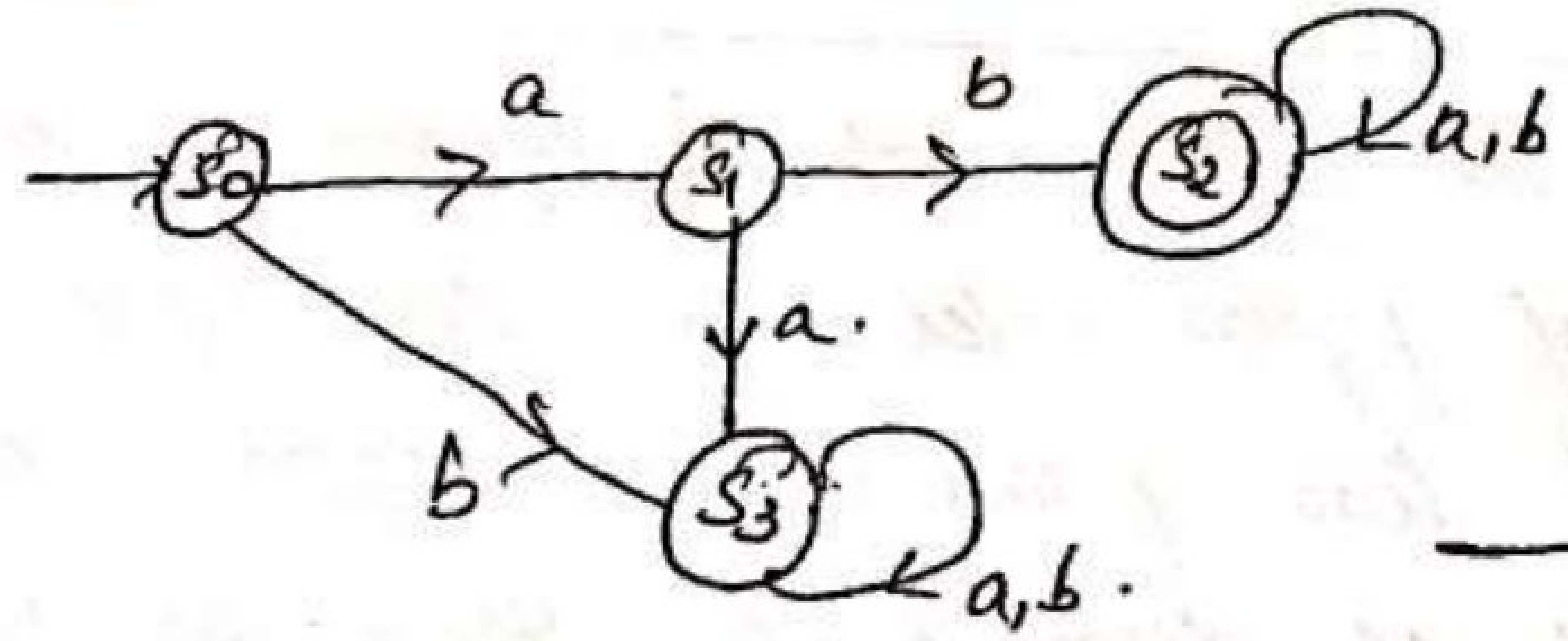
WKT $2e = \sum_{v_i \in V_1} \deg(v_i) + \sum_{v_j \in V_2} \deg(v_j)$

$\Rightarrow \sum_{v_j \in V_2} \deg(v_j) = 2e - \sum \deg(v_i) = \text{even} - \text{even}$

$= \text{even}$. → (4m)

26) $L(G) = \{ abb, abab \}$ → (4m)

27)




→ (4m)

3).

## Part-C

28) a) i)

| S.No | Statement | Reason: |
|---|---|---|
| 1. | $a \rightarrow b$ | P |
| 2. | $c \rightarrow b$ | P |
| 3. | $(a \vee b) \rightarrow b$ | T (1,2) equivalence. |
| 4. | $d \rightarrow (a \vee c)$ | P. |
| 5. | $d \rightarrow b$ | T (3,4) HS. |
| 6. | $d$ | P |
| 7. | $b$ | T (5,6). MP. |
| 8. | $\neg b$ | A.P |
| 9. | F | T (7,8). |

1) $\neg b$ AP
2) $a \rightarrow b$ P
3) $c \rightarrow b$ P
4) $d \rightarrow (a \vee c)$ P
5) $d$ P
6) $a \vee c$ T(4,5)
7) $\neg b \rightarrow \neg c$ T(3)
8) $\neg c$ T(1,7)
9) $a$ T(6,8)
10) $b$ (9,2)

⟶ (6m)

a) ii)

C: Raja misses many classes

E: He fails in the final Exam

U: He is uneducated.

B: He reads a lot of Books.

⟶ (1m)

Symbolic Form:-

$C \rightarrow E$, $E \rightarrow U$, $B \rightarrow \neg U$ and $C \wedge B$

⟶ (1m)

| S.NO | Statement | Rules |
|---|---|---|
| 1. | $C \rightarrow E$ | P |
| 2. | $E \rightarrow U$ | P |
| 3. | $C \rightarrow U$ | T(1,2) HS. |
| 4. | $C \wedge B$ | P |
| 5. | $C$ | T (4) Simplification |
| 6 | $B$ | T (4) " |
| 7. | $B \rightarrow \neg U$ | P |
| 8. | $\neg U$ | T (6,7) MP. |
| 9. | $U$ | T (5, 3) MP. |
| 10. | F | T (8,9). ⟶ (4m) |

④

28) b) $\neg p \to (q \to r) \equiv q \to (p \vee r)$

i) $\underline{\text{LHS}}$: $\neg p \to (q \to r) \equiv p \vee (\neg q \vee r) \equiv \neg q \vee (p \vee r)$

$\equiv q \to (p \vee r) \longrightarrow$ (4m).

ii)

| S.No | Statement | Rules. |
|---|---|---|
| 1. | $p$ | P (A.P) |
| 2. | $p \to (q \to r)$ | P |
| 3. | $q \to r$ | T (1,2) MP. |
| 4. | $\neg q \vee r$ | T (3) |
| 5. | $q \to (r \to s)$ | P |
| 6. | $\neg q \vee (r \wedge (r \to s))$ | T (4,5) |
| 7. | $\neg q \vee s$ | T (6) MP |
| 8. | $q \to s$ | T (7) |
| 9. | $p \to (q \to s)$ | T (1,8) CP rule |

$\longrightarrow$ (8m)

29) a) $a_{n+2} - 6a_{n+1} + 9a_n = 3\cdot 2^n + 7\cdot 3^n$, $n \geq 0$, $a_0 = 1$, $a_1 = 4$, $\longrightarrow$ ①

$\boxed{a_n^{(h)} = (c_1 + c_2 n)3^n} \longrightarrow$ (2m)

$\underline{\text{Particular}}$ $\underline{\text{Soln:}}$ $\boxed{a_n^{(p)} = A_0 \cdot 2^n + A_1 \cdot n^2 \cdot 3^n}$

① $\Rightarrow \left(A_0 \cdot 2^{n+2} - A_1 (n+2)^2 \cdot 3^{n+2}\right) - 6\left(A_0 2^{n+1} + A_1 (n+1)^2 \cdot 3^{n+1}\right)$

$+ 9a_n = 3\cdot 2^n + 7\cdot 3^n$

$\longrightarrow$ ②

Comparing like terms $A_0 = 1$ and $A_1 = 7/18 \longrightarrow$ (5m)

$a_n^{(p)} = 2^n + \frac{7}{18} n^2 \cdot 3^n.$

$\therefore a_n = (c_1 + c_2 \cdot n)\,3^n + 2^n + 7/18\, n^2 \cdot 3^n.$

Using $a_0 = 1$, $a_1 = 4 \Rightarrow c_1 = 0$, $c_2 = 5/18$.

$\therefore \boxed{a_n = \frac{5n}{18} \cdot 3^n + 2^n + \frac{7n^2}{18} \cdot 3^n} \longrightarrow$ (5m)

29 b) i) $F_{n+2} - F_{n+1} - F_n = 0$ ; $n \geq 0$ ; $F_0 = 0$, $F_1 = 1$.

CH. Eqn is $r^2 - r - 1 = 0 \Rightarrow r = \frac{1 \pm \sqrt{5}}{2}$

$$\therefore F_n = c_1\left[\frac{1+\sqrt{5}}{2}\right]^n + c_2\left[\frac{1-\sqrt{5}}{2}\right]^n \longrightarrow (3m)$$

$F_0 = 0$ and $F_1 = 1$ gives $c_1 = 1/\sqrt{5}$ and $c_2 = -1/\sqrt{5}$.

$$\therefore F_n = \frac{1}{\sqrt{5}}\left[\frac{1+\sqrt{5}}{2}\right]^n - \frac{1}{\sqrt{5}}\left[\frac{1+\sqrt{5}}{2}\right]^n \longrightarrow (3m)$$

---

29) b) (ii) $a_n = 3a_{n-1} + 1$ ; $n \geq 1$ gives $a_0 = 1$.

$$\therefore \sum_{n=1}^{\infty} a_n x^n = \sum_{n=1}^{\infty} 3a_{n-1} x^n + \sum_{n=1}^{\infty} x^n.$$

$$\Rightarrow (G(x) - a_0) = 3x\,G(x) + \frac{x}{1-x} \longrightarrow (2m)$$

$$\Rightarrow G(x) = \frac{1}{(1-x)(1-3x)} = \frac{-1}{2(1-x)} + \frac{3}{2(1-3x)} \longrightarrow (2m)$$

$$\Rightarrow G(x) = \frac{-1}{2}\sum_{n=0}^{\infty} x^n + \frac{3}{2}\sum_{n=0}^{\infty} 3^n x^n.$$

$$a_n = \text{coeff of } x^n \text{ in } G(x) = \frac{1}{2}(3^{n+1} - 1) \longrightarrow (2m)$$

---

30) a) Lagrange's Thm:

The order of a subgroup of a finite group is a divisor of the order of the group. $\longrightarrow$ (2m)

Proof $\longrightarrow$ (10m).

---

b) i) Every subgroup of a cyclic group is cyclic $\longrightarrow$ (8m).

30) b) i) Taking $H = \left[\begin{array}{ccc|ccc} 1&1&0&1&0&0 \\ 1&0&1&0&1&0 \\ 1&1&1&0&0&1 \end{array}\right] = [A^T \mid I_{n-m}]$

$G = [I_m \mid A] = \left[\begin{array}{ccc|ccc} 1&0&0&1&1&1 \\ 0&1&0&1&0&1 \\ 0&0&1&0&1&1 \end{array}\right] \longrightarrow (2m)$

Now $B^3 = \{000, 001, 010, 100, 011, 101, 110, 111\}$.

$e(000) = [000]\, G = [000000]$

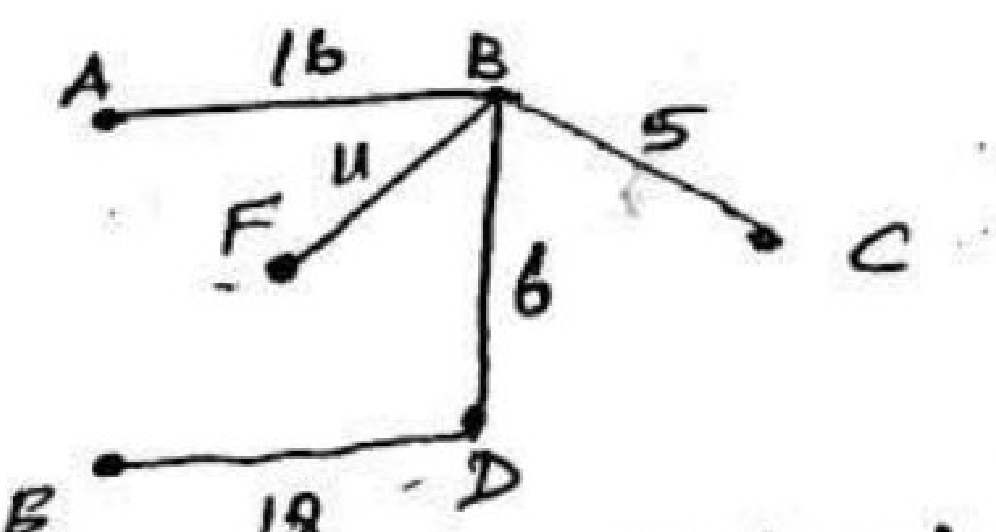
$e(001) = [001011]$ ; $e(010) = [010101]$

$e(100) = [100111]$ ; $e(011) = [011110]$

$e(101) = [101100]$ ; $e(110) = [110010]$ $\longrightarrow (4m)$

$e(111) = [111001]$

31) a) i) MST:




$\longrightarrow (4m)$

List out the increasing order of weight $\longrightarrow (2m)$

total weight of MST = 56. $\longrightarrow (2m)$

a) ii) The property is true for $n = 1, 2, 3$.

Let us assume that it is true for $n = k$. i.e.,

$k$ vertices of a tree will have $(k-1)$ edges.

$T_{k+1}$ : $k+1$ vertices.

$= (k-1) + 1$ edges $\longrightarrow (4m)$.

$= k$ edges.

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|---|---|---|---|---|---|---|---|---|---|---|

# B.Tech. DEGREE EXAMINATION, NOVEMBER 2010

Third Semester

## MA0213 – DISCRETE MATHEMATICS

*(For the candidates admitted from the year 2007-2008 onwards)*

Time: Three hours Max.Marks:100

### PART – A ($10 \times 2 = 20$ Marks)

Answer **ALL** Questions

1. Define: Conditional and Bi-conditional propositions.
2. When is a set of premises said to be inconsistent?
3. State Pigeon-hole principle.
4. Using mathematical induction, show that $(1+2+3+...n) = \frac{n(n+1)}{2}$.
5. Define a Sub group. Give an example.
6. What is a group code?
7. Define a Complete graph. Give an example.
8. When is a graph said to be an Eulerian graph? Give an example.
9. Define: Phase structure grammar.
10. Explain Finite State Machine with an example.

### PART – B ($5 \times 16 = 80$ Marks)

Answer **ANY FIVE** Questions

11. i. Prove the equivalence, using truth table $(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$.

ii. By indirect method, show that $r \to 7q,\ r \vee s,\ s \to 7q,\ p \to q \Rightarrow 7p$

12. i. Prove by mathematical induction

$$\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+....+\frac{1}{n(n+1)}=\frac{n}{n+1}.$$

ii. Solve the recurrence relation

$a_n = 4a_{n-1} - 4a_{n-2} + (n+1)2^n$.

13. i. Prove that subgroup of a cyclic group is again cyclic.

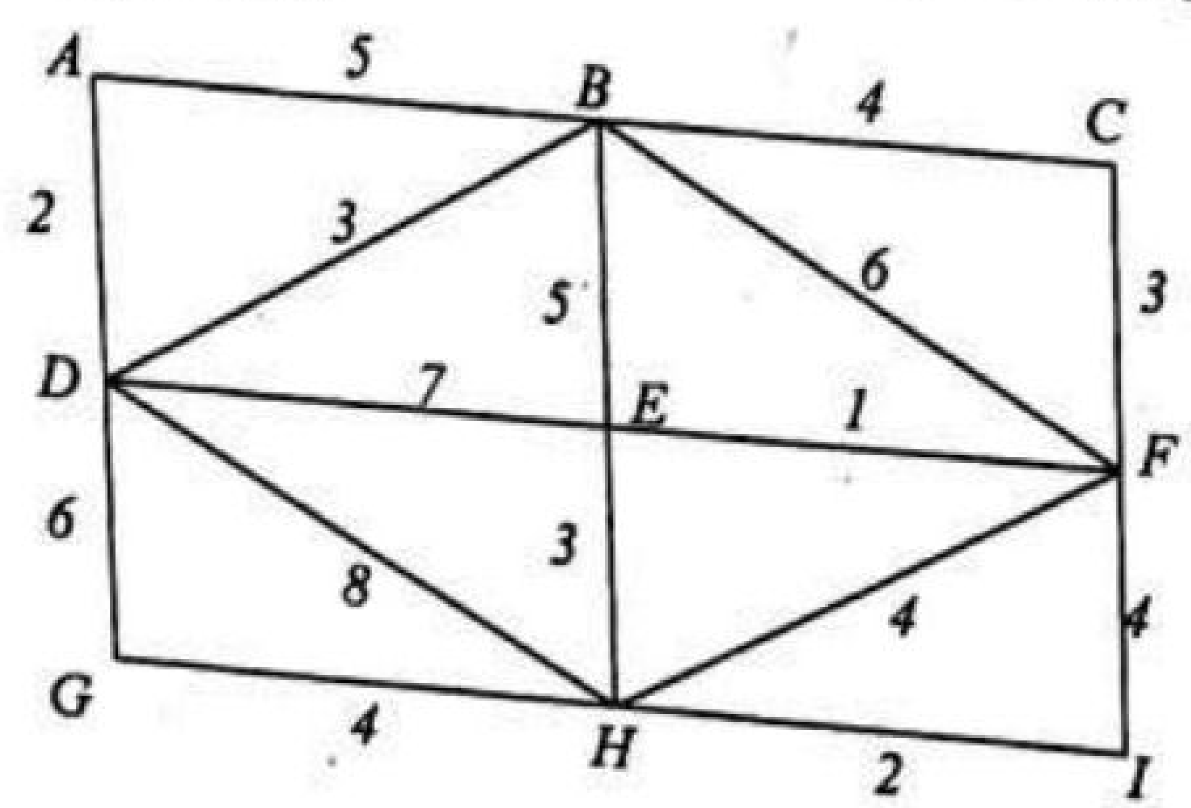
ii. If $\varphi: G \to G'$ is a group homomorphism, prove that

(a) $\varphi(e) = e'$

(b) $\varphi(x^{-1}) = [\varphi(x)]^{-1}$

14. i. Prove that number of odd degree vertices in an undirected graph is even.

ii. Find minimum spanning tree for the weighted graph, using Kruskal's algorithm.




15. i. Find a regular grammar $G$ generating language $L$ which consists of all words in '$a$' and '$b$' with an even number of $a$'s.

ii. Construct a Finite State Automata (FSA) that accepts all strings over $\{a,b\}$ which begin with $a$ and end with $b$.

16. i. Prove the following implication without using truth tables:

$(p \vee q) \wedge (p \to r) \wedge (q \to r) \Rightarrow r$.

ii. Find the general term of Fibonacci sequence using recurrence relation.

17. i. Find the code words generated by the parity check matrix.

$$H = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

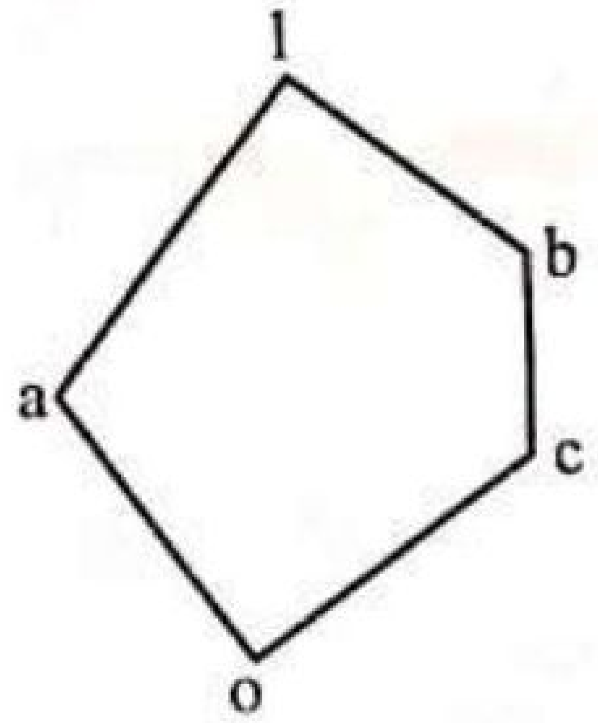
when the coding function is $e: B^3 \to B^6$.

ii. Show that the maximum number of edges in a simple disconnected graph $G$ with n vertices and $k$ components is $\frac{(n-k)(n-k+1)}{2}$.

****

ii. Prove that the number of vertices of a full binary tree is odd and the number of pendant vertices of a tree is equal to $\frac{n+1}{2}$.

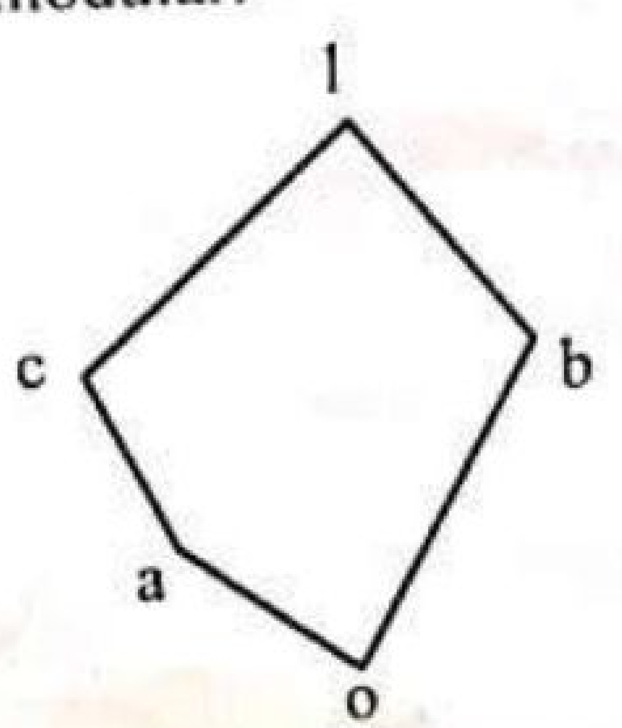
15.a.i Prove that in any lattice L, $a \oplus (b * c) \leq (a \oplus b) * (a \oplus c)$. Also verify whether the following lattice is distributive or not.




ii. Design a FSM that performs binary addition.

**(OR)**

b. i. Define: Modular lattice. Prove the following lattice is not modular.




ii. Find a regular grammar that generates the language, $L = \{a^m b^n c^p / m, n, p \geq 1\}$

* * * *



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|---|---|---|---|---|---|---|---|---|---|---|

## B.Tech. DEGREE EXAMINATION, NOVEMBER 2014

Fifth Semester

### MA0321 – DISCRETE MATHEMATICS

*(For the candidates admitted from the academic year 2007-2008 to 2012-2013)*

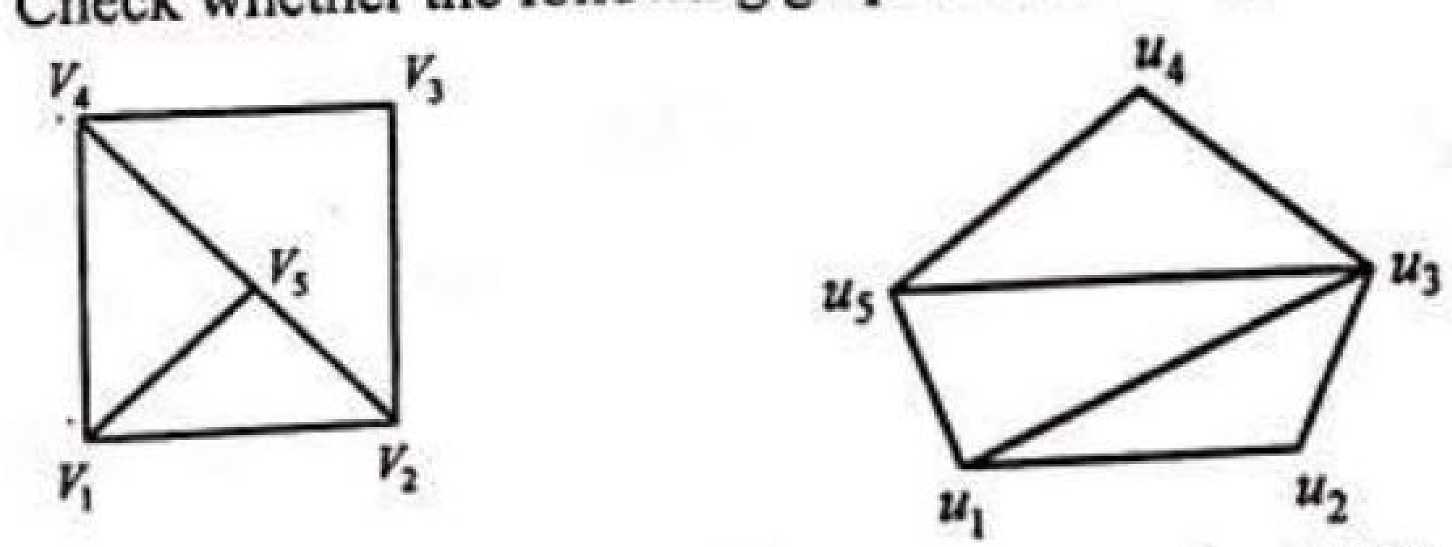
(Statistical table is to be provided)

Time: Three hours | Max. Marks: 100

Answer **ALL** Questions

**PART – A (10 × 2 = 20 Marks)**

1. Write the dual of $(p \vee q) \wedge (\neg p \wedge q) \wedge F$.
2. Write in symbolic form.
   "Everyone who takes a fruit daily is healthy".
3. Give an example of a relation which is not symmetric.
4. Give a partition of {1,2,3,4,5,6} into minsets generated by $B_1$ = {1,2,3} and $B_2$ = {3,4,5}.
5. Solve $a_n - 2a_{n-1} + 2a_{n-2} = 0$, $n \geq 2$ with $a_0 = 1$ and $a_1 = 1$.
6. Define: Order of an element in a group G.
7. Prove that the number of vertices of odd degree in an undirected graph is even.
8. Check whether the following graphs are isomorphic or not.




9. Show that in a lattice if $a \leq b$ and $c \leq d$, then $a*c \leq b*d$
10. Find the language generated by the grammar G = ({S,A,B}, {a,b}, P, S}) where P = {S→ 11S, S→ 0}.

## PART – B (5 × 16 = 80 Marks)

11.a.i Construct a truth table for
$\neg(p \vee (q \wedge r)) \rightleftarrows (p \vee q) \wedge (p \rightarrow r)$

ii. Check the validity of the following argument:
If A works hard then B or C will enjoy himself. Then A will not work hard. If D enjoys himself then C will not. Therefore if A works hard, D will not enjoy himself.

**(OR)**

b. i. Disprove the implication
$(\exists x)P(x) \wedge (\exists x)Q(x) \Rightarrow (\exists x)(P(x) \wedge Q(x))$ by giving a suitable example.

ii. Using mathematical induction, prove that $3^n+7^n-2$ is divisible by 8 for $n \geq 1$.

12.a.i If A, B, C are any 3 sets, prove analytically,
$A-(B \cap C)=(A-B) \cup (A-C)$.

ii. Let $A=\{1,2,3,4,5\}$ and
$R=\left\{\begin{matrix}(1,1),(1,3),(1,5),(2,3),(2,4),(3,3),(3,5),\\(4,2),(4,4,)(5,4)\end{matrix}\right\}$.
Using Warshall's algorithm, find the transitive closure of R.

**(OR)**

b. i. If R is the relation on the set of ordered pairs of +ve integers such that (a,b) R(c,d) whenever ad = bc, show that R is an equivalence relation.

ii. If $A=\{x \in \mathbb{R} \mid x \neq 1/2\}$ *and* $f: A \rightarrow \mathbb{R}-\{2\}$ defined by
$f(x)=\dfrac{4x}{2x-1}$ for every $x \in$ A.

(A) Is f 1-1? (B) Is f onto?
(C) Find a formula for $f^{-1}$
(D) Dom $f(^{-1})$ = ? (E) Range of $(f^{-1})$ = ?



13.a.i Solve the recurrence relation:
$a_{n+2}-6a_{n+1}+9a_n=3(2)^n+7(3)^n, n \geq 0$ given
$a_0=1$ and $a_1=4$

ii. Prove that every subgroup of a cyclic group is cyclic.

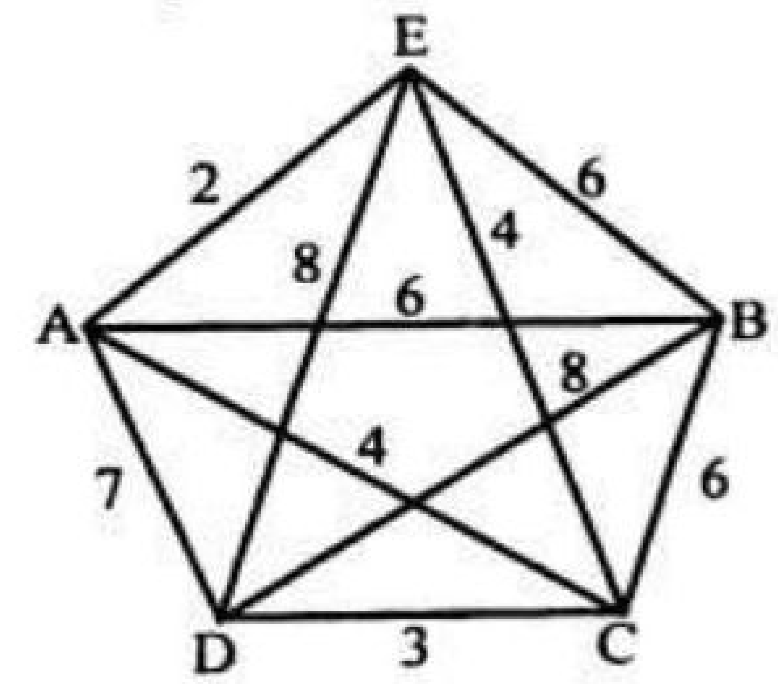
**(OR)**

b. i. Solve by using generating function,
$a_n+3a_{n-1}-4a_{n-2}=0$ for $n \geq 2$ given $a_0$= 3 and $a_1$ = −2.

ii. State and prove Lagrange's theorem.

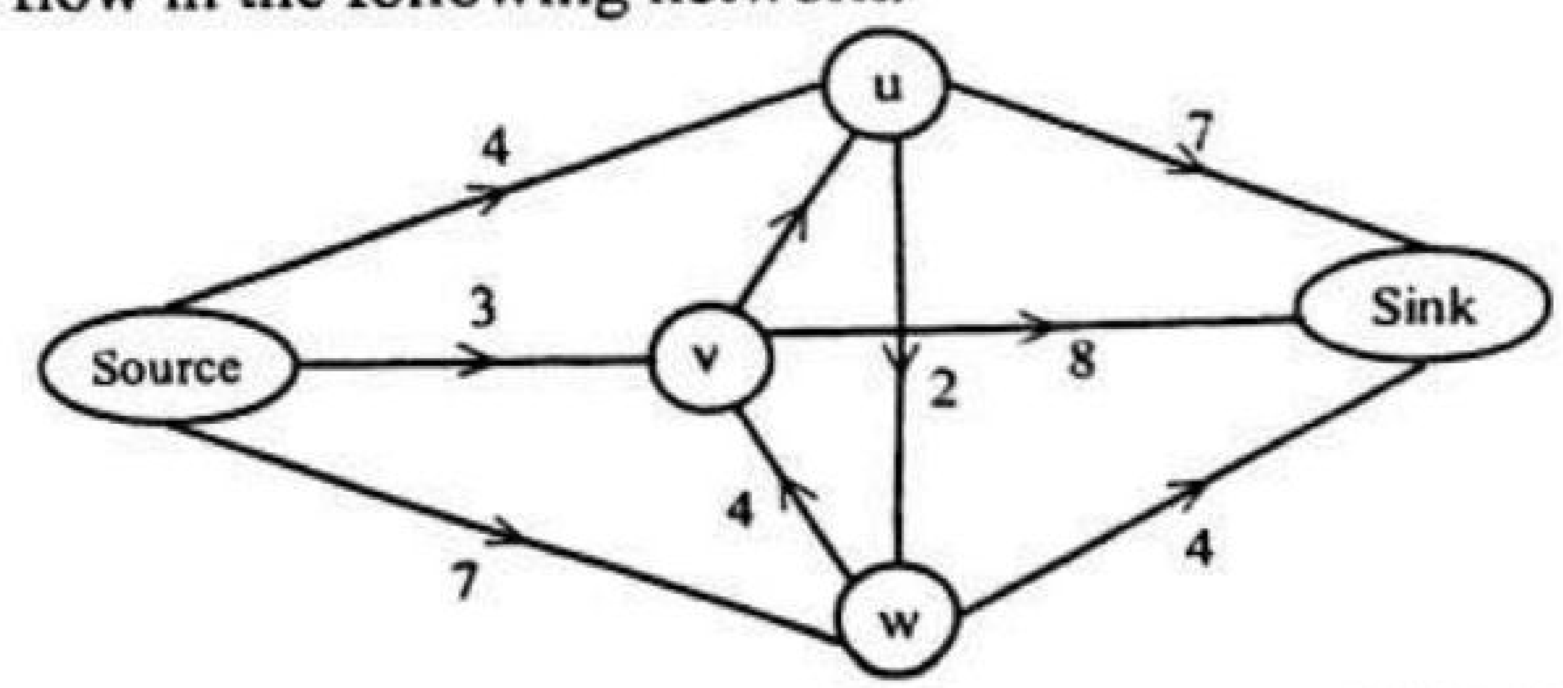
14.a.i Prove that the number of edges in a bipartite graph with n vertices is atmost $\left(\dfrac{n}{2}\right)^2$.

ii. Find the minimum spanning tree for the weighted graph.




**(OR)**

b. i. Use Ford and Funkerson algorithm to find a maximum flow in the following network.



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**B.Tech. DEGREE EXAMINATION, NOVEMBER 2009**
Third Semester

# MA0213 – DISCRETE MATHEMATICS

*(For the candidates admitted from the year 2007-2008 onwards)*

Time: Three hours Max.Marks:100

## PART – A (10 × 2 = 20 Marks)

Answer **ALL** Questions

1. Define: Conditional and biconditional propositions.
2. Explain tautology and contradiction with simple examples.
3. State the principle of mathematical induction.
4. State the generalized pigeonhole principle.
5. Define: Dihedral group.
6. Define: Group homomorphism.
7. Prove that in a graph with $e$ edges, $\sum deg(v_i)=2e$.
8. Define: (i) Complete graph (ii) Regular graph.
9. Determine the type of following grammar: $S\to aAB;\ S\to AB;\ A\to a;\ B\to b$.
10. Define: FSM with an example.

## PART – B (5 × 16 = 80 Marks)

Answer **ANY FIVE** Questions

11. i. Prove the implication $((P\vee 7P)\to Q)\to((P\vee 7P)\to R)\Rightarrow Q\to R$.

ii. Using indirect method, show that $R\to 7Q,\ R\vee S,\ S\to 7Q,\ P\to Q\Rightarrow 7P$.

12. i. Prove by mathematical induction that $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+....+\frac{1}{n\cdot(n+1)}=\left(\frac{n}{n+1}\right)$.

ii. Show that the following set of premises is inconsistent:
If Rama gets his degree, he will go for a job
If he goes for a job, he will get married soon.
If he goes for higher study, he will not get married.
Rama gets his degree and goes for higher study.

13. State the Tower of Hanoi problem and solve it by using recurrence relation.

14. Prove that order of a subgroup divides order of the group.

15. i. Prove that subgroup of a cyclic group is cyclic.

ii. Prove that the number of edges in a bipartite graph with n vertices is atmost $(n^2/4)$.

16. i. Explain the terms with examples:
(a) Eulerian path
(b) Eulerain circuit
(c) Hamiltonian path
(d) Hamiltonian circuit

ii. Prove that a graph is a tree iff there exists unique simple path between every pair of vertices.

17. i. Find a phrase-structure grammar G to generate the language $L$ that consists of equal number of $a$'s and $b$'s.

ii. Design an $FSM$ that outputs $1$ if $k$ 1's have been input where $k$ is a multiple of $3$ and output $0$ otherwise.

* * * *

13. i. Solve: $a_{n+2}+3a_{n+1}+2a_n=3^n (n\geq 0)$ *given* $a_0=0,\ a_1=1$.

ii. Prove that $(3^n+7^n-2)$ is divisible by 8, $(n\geq 1)$.

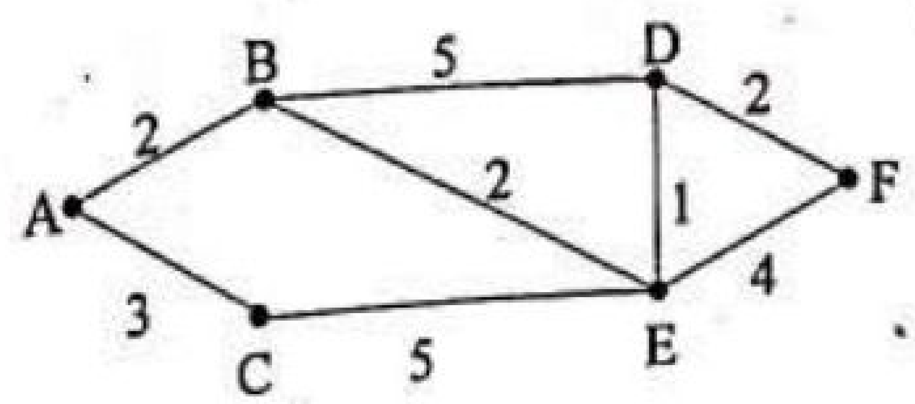
14. i. Prove that Kernel of group homorphism is a normal subgroup.

ii. Find the code words generated by the encoding function $e: B^2 \to B^5$ with respect to parity check matrix.

$$H=\begin{pmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

15. State and prove Lagrange's theorem.

16. i. Find shortest path from vertex A to F in the following graph by Dijktra's algorithm.




ii. Prove that a tree with n vertices has (n-1)edges.

17. i. Find a regular grammar that generates the language $L=(a^m b^n c^p / m, n, p\geq 1)$.

ii. Draw the state diagram of the FSM, the state table of which is given below. Show that this FSM is an FSA and redraw the transition diagram as the diagram of an FSA. What is the characteristic of the string accepted by the FSA?

| | f | | g | |
|---|---|---|---|---|
| S \ I | a | b | a | b |
| $S_0$ | $S_1$ | $S_3$ | 1 | 0 |
| $S_1$ | $S_1$ | $S_2$ | 1 | 1 |
| $S_2$ | $S_3$ | $S_4$ | 0 | 0 |
| $S_3$ | $S_1$ | $S_0$ | 0 | 0 |
| $S_4$ | $S_3$ | $S_4$ | 0 | 0 |

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B.Tech DEGREE EXAMINATION, NOVEMBER 2008

Third Semester

MA0213 – DISCRETE MATHEMATICS

*(For the candidates admitted from the year 2007-2008 onwards)*

Time: Three hours Max.Marks:100

**PART – A (10 × 2 = 20 Marks)**

Answer **ALL** Questions

1. Explain the connectives conjunction and disjunction.
2. Define Tautological implication and give an example.
3. State Pigeon hole principle.
4. Find particular solution of recurrence relation. $a_n - 2a_{n-1} = 3^n$, with $a_1=5$.
5. Define abelian group and give an example.
6. Define permutation group.
7. Define: Simple graph, Digraph with examples.
8. Define isomorphic graphs.
9. Define phase structure grammar.
10. What is Backus-Naur form? Give an example.

**PART – B (5 × 16 = 80 Marks)**

Answer **ANY FIVE** Questions

11. i. Determine whether the compound proposition $7(q \rightarrow r) \wedge r \wedge (p \rightarrow q)$ is Tautology or Contradiction.

ii. Without using truth tables, prove the equivalence. $(7p \vee q) \wedge (p \wedge (p \wedge q)) \equiv p \wedge q$.

12. i. Prove the implication $p \rightarrow (q \rightarrow r) \Rightarrow (p \rightarrow q) \rightarrow (p \rightarrow r)$.

ii. Prove by mathematical induction, that $1^2 + 2^2 + 3^2 + 5^2 + \ldots + (2n-1)^2 = \frac{1}{3}n(2n-1)(2n+1)$.

**B.Tech DEGREE EXAMINATION, NOVEMBER 2008**
Third Semester

MA0213 – DISCRETE MATHEMATICS
*(For the candidates admitted from the year 2007-2008 onwards)*

Time: Three hours Max.Marks:100

**PART – A (10 × 2 = 20 Marks)**
Answer **ALL** Questions

1. Explain the connectives conjunction and disjunction.
2. Define Tautological implication and give an example.
3. State Pigeon hole principle.
4. Find particular solution of recurrence relation. $a_n - 2a_{n-1} = 3^n$, with $a_1=5$.
5. Define abelian group and give an example.
6. Define permutation group.
7. Define: Simple graph, Digraph with examples.
8. Define isomorphic graphs.
9. Define phase structure grammar.
10. What is Backus-Naur form? Give an example.

**PART – B (5 × 16 = 80 Marks)**
Answer **ANY FIVE** Questions

11. i. Determine whether the compound proposition $\neg(q \to r) \wedge r \wedge (p \to q)$ is Tautology or Contradiction.

ii. Without using truth tables, prove the equivalence. $(\neg p \vee q) \wedge (p \wedge (p \wedge q)) \equiv p \wedge q$.

12. i. Prove the implication $p \to (q \to r) \Rightarrow (p \to q) \to (p \to r)$.

ii. Prove by mathematical induction, that $1^2 + 2^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \frac{1}{3}n(2n-1)(2n+1)$.

13. i. Solve: $a_{n+2}+3a_{n+1}+2a_n=3^n (n\geq 0)$ *given* $a_0=0,\ a_1=1$.

ii. Prove that $(3^n+7^n-2)$ is divisible by 8, $(n\geq 1)$.

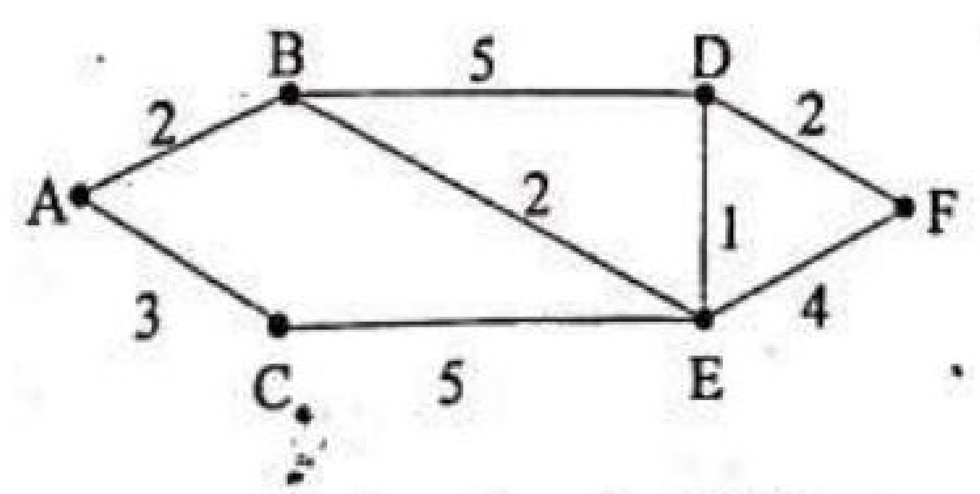
14. i. Prove that Kernel of group homorphism is a normal subgroup.

ii. Find the code words generated by the encoding function $e: B^2 \to B^5$ with respect to parity check matrix.

$$H=\begin{pmatrix}0&1&1\\0&1&1\\1&0&0\\0&1&0\\0&0&1\end{pmatrix}$$

15. State and prove Lagrange's theorem.

16. i. Find shortest path from vertex A to F in the following graph by Dijktra's algorithm.




ii. Prove that a tree with n vertices has (n-1)edges.

17. i. Find a regular grammar that generates the language $L=(a^m b^n c^p / m, n, p \geq 1)$.

ii. Draw the state diagram of the FSM, the state table of which is given below. Show that this FSM is an FSA and redraw the transition diagram as the diagram of an FSA. What is the characteristic of the string accepted by the FSA?

| | f | | g | |
|---|---|---|---|---|
| s \ I | a | b | a | b |
| $S_0$ | $S_1$ | $S_3$ | 1 | 0 |
| $S_1$ | $S_1$ | $S_2$ | 1 | 1 |
| $S_2$ | $S_3$ | $S_4$ | 0 | 0 |
| $S_3$ | $S_1$ | $S_0$ | 0 | 0 |
| $S_4$ | $S_3$ | $S_4$ | 0 | 0 |

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15.a.i Prove that "If $(L,\leq)$ is a lattice in which $\vee$ and $\wedge$ denote operations of join and meet, then for $a, b \in L$, $a \leq b \Leftrightarrow a \vee b = b \Leftrightarrow a \wedge b = a$

ii. Find the DFA equivalent to the NFA for which the state table is given in table below and $S_2$ is the final state.

| S \ I | f: a | f: b |
|---|---|---|
| $s_0$ | $s_0\ s_1$ | $s_2$ |
| $s_1$ | $s_0$ | $s_1$ |
| $s_2$ | $s_1$ | $s_0, s_1$ |

(OR)

b. i. In Boolean algebra, if $a+b = 1$ and $a.b = 0$, show that $a = a'$ ; $a'$ is complement of a.

ii. Prove that $D_{42} = (S_{42}, D)$ is a complemented lattice, where $S_{42} = \{1,2,3,6,7,14,21,42\}$ and D is divisibility.

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# B.Tech. DEGREE EXAMINATION, MAY 2015

Fifth Semester

## MA0321 – DISCRETE MATHEMATICS

*(For the candidates admitted from the academic year 2007-2008 to 2012-2013)*

Time: Three hours | Max. Marks: 100

Answer **ALL** Questions

### PART – A (10 × 2 = 20 Marks)

1. Symbolize the statement: "All men are mortal".
2. Define: Rule conditional proof.
3. Define: Minset.
4. Draw the Hasse diagram of the Poset $(P,\leq)$, where $P = \{1, 3, 5, 9, 15, 45\}$ and $\leq$ is the relation divisibility.
5. Solve $a_n = 2(a_{n-1} - a_{n-2})$ ; $n \geq 2$.
6. Show that every cyclic group is Abelian.
7. Define: Euler and Hamiltonian graphs.
8. Define: Binary tree with an example.
9. Define: Boolean algebra.
10. Derive the string abab using the grammar $G = (\{S,A,B\}, \{a,b\}, S,P)$ where P is the set of productions $S \rightarrow AB$, $S \rightarrow AA$, $A \rightarrow aB$, $B \rightarrow b$.

### PART – B (5 × 16 = 80 Marks)

11.a.i Using truth table, show that $7(q \rightarrow r) \wedge r \wedge (p \rightarrow q)$ is a contradiction.

ii. Show that

$$(p \rightarrow q) \wedge (r \rightarrow s), (q \rightarrow t) \wedge (s \rightarrow u), 7(t \wedge u), p \rightarrow r \Rightarrow 7p$$

(OR)

b. i. Use mathematical induction to prove that $3^n+7^n-2$ is divisible by 8 for all $n \geq 1$.

ii. Use indirect method to prove that $\forall(P(x) \rightarrow Q(x)), \exists x P(y) \Rightarrow \exists z Q(z)$.

12.a.i Find the transitive closure of the relation

$$R = \begin{Bmatrix} (1,1),(1,3),(1,5),(2,3),(2,4),(3,3),(3,5),(4,2), \\ (4,4,)(5,4) \end{Bmatrix}$$

defined on a set A = {1, 2, 3, 4, 5}using Warshall's algorithm.

ii. Prove that "If $f{:}A \rightarrow B$ and $g{:}B \rightarrow A$ are functions such that $fog = I_B$ and $gof = I_A$, then $f^{-1} = g$ and $g^{-1}$ exist and $f^{-1} = g$ and $g^{-1} = f$.

**(OR)**

b. i. If R and S be relations on a set A represented by matrices

$$M_R = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \text{ and } M_S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Find the matrices that represent

(1) $R \cup S$
(2) $R \cap S$
(3) $R \oplus S$
(4) $R \circ S$

ii. If R is a relation on the set of ordered pairs of positive integers such that (a,b), (c,d) ∈ R whenever ad = bc, then show that R is an equivalence relation.

13.a.i State and prove the Lagrange's theorem.

ii. Solve $F_{n+2} = F_{n+1} + F_n$ ; $n \geq 0$ with $f_0 = 0$ and $F_1 = 1$.

**(OR)**



b. i. Using generating functions, solve $a_n = 3a_{n-1}+1$ ; $n \geq 1$ given that $a_0 = 1$.

ii. If the permutations of the elements of {1,2,3,4,5} are

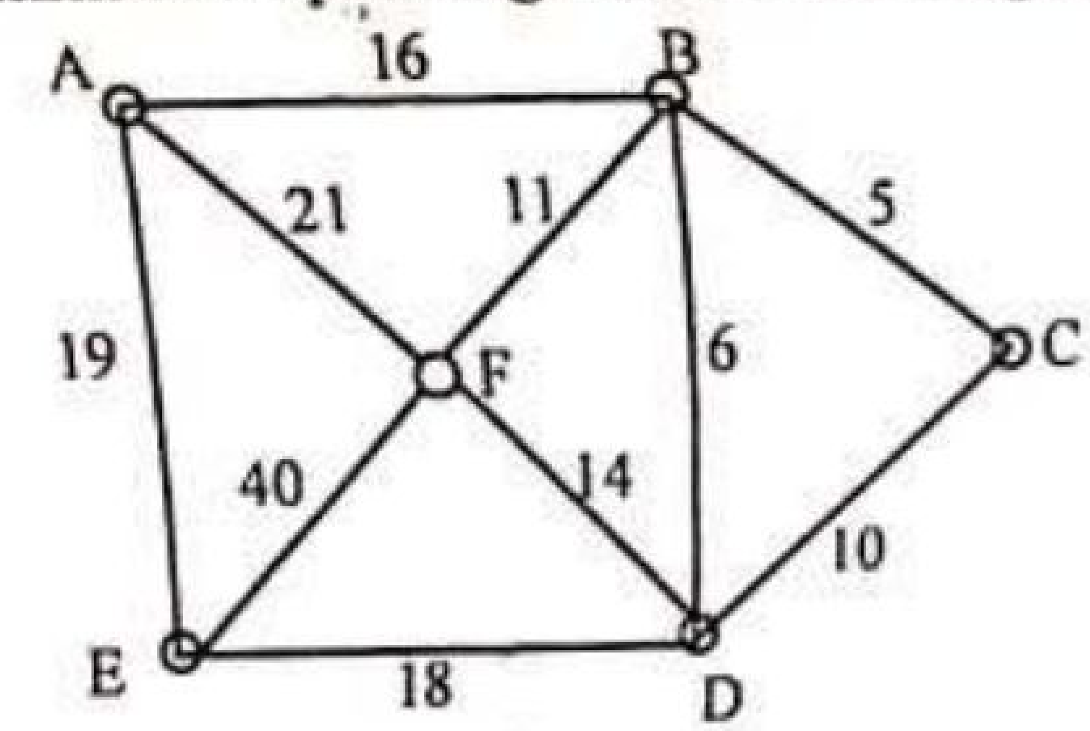
$$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 4 & 5 \end{pmatrix} \quad \beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 5 & 4 \end{pmatrix}$$

$$\gamma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 3 & 1 & 2 \end{pmatrix} \quad \delta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 2 & 1 & 5 & 4 \end{pmatrix}$$

Find $\alpha\beta\gamma$, $\gamma^2$ and $\delta^{-1}$.

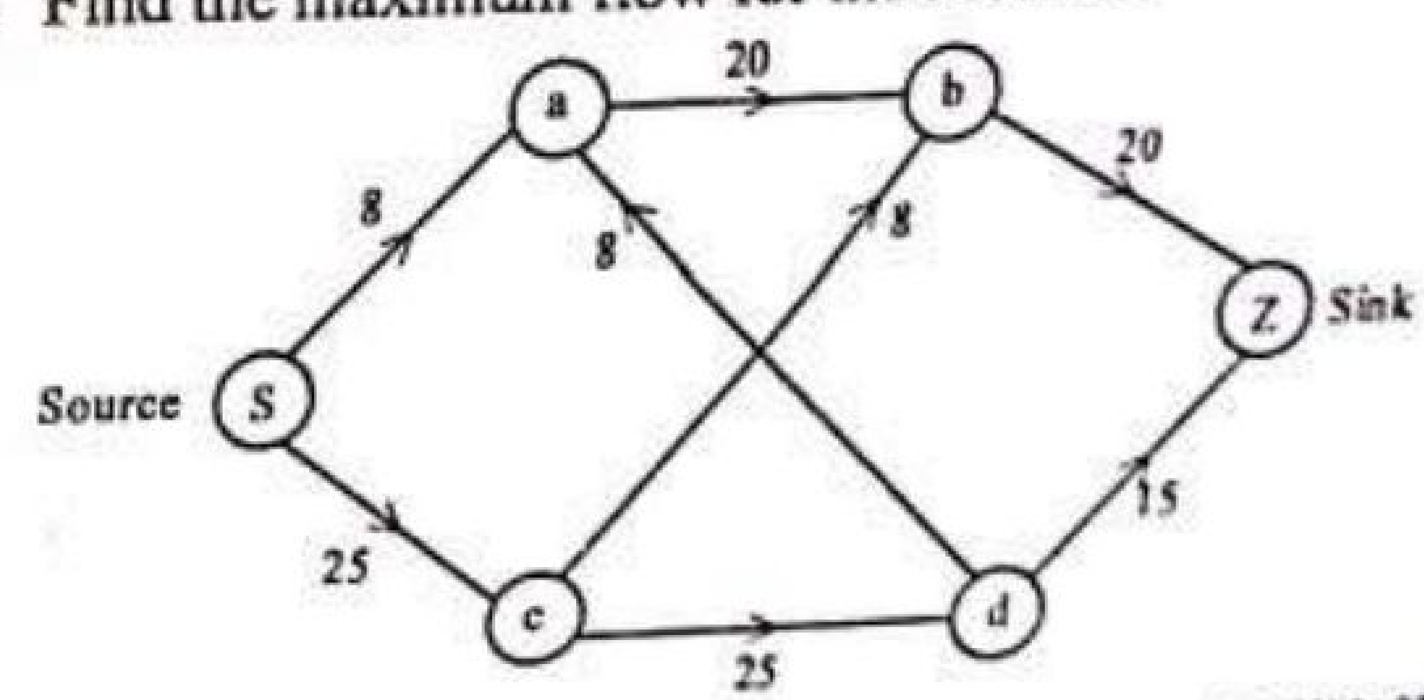
14.a.i Prove that a tree with n vertices has n–1edges.

ii. Find the minimum spanning tree for the weighted graph.




**(OR)**

b. Find the maximum flow for the network.



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**B.Tech. DEGREE EXAMINATION, MAY 2015**
Seventh Semester

MA0421 – DISCRETE MATHEMATICS
*(For the candidates admitted from the academic year 2007-2008 to 2012-2013)*

Time: Three hours | Max. Marks: 100

Answer **ALL** Questions
**PART – A (10 × 2 = 20 Marks)**

1. Show that $P \rightarrow (P \vee Q)$ is a tautology.
2. Write the following statement in symbolic form: every engineering student needs a course in mathematics.
3. Let X = {1, 2, 3, 4}, and R={($x$, $y$): $x > y$}. Show that the relation R is antisymmetric.
4. Define Composition of two functions.
5. How many ways can you arrange the letters in the word LAPTOP?
6. Find the recurrence relation for the sequence $a^n = 2(4)^n - 5(-3)^n$.
7. Explain multigraph and pseudo graph with examples.
8. Define path and circuit in a graph.
9. State principle of duality for lattices.
10. Explain Finite state machine.

**PART – B (5 × 16 = 80 Marks)**

11. a. Show that $R \wedge (P \vee Q)$ is valid conclusion for the premises $P \vee Q, Q \rightarrow R, P \rightarrow M, \neg M$.

(OR)

b. Using principle of mathematical induction, prove that $\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+....+\frac{1}{n(n+1)}=\frac{n}{n+1}$.

12. a. Using Warshall's algorithm, find the transitive closure of the relation $R\{=(1,4),(2,1),(2,2),(2,3),(3,2),(4,3),(4,5),(5,1)\}$ on the set $A=\{1,2,3,4,5\}$.

**(OR)**

b. Let $A=\{1,2,3,4,5,6\}$ with the subsets $B_1=\{1,3,5\}$ and $B_2=\{1,2,3\}$. Find the maxsets generated by $B_1$ and $B_2$. Also show that minsets from a partition of A.

13. a. Solve $T(k)-7T(k-1)+10T(k-2)=6+8k$ with T(0) = 1, T(1) = 2.

**(OR)**

b.i. State and prove Lagrange's theorem.

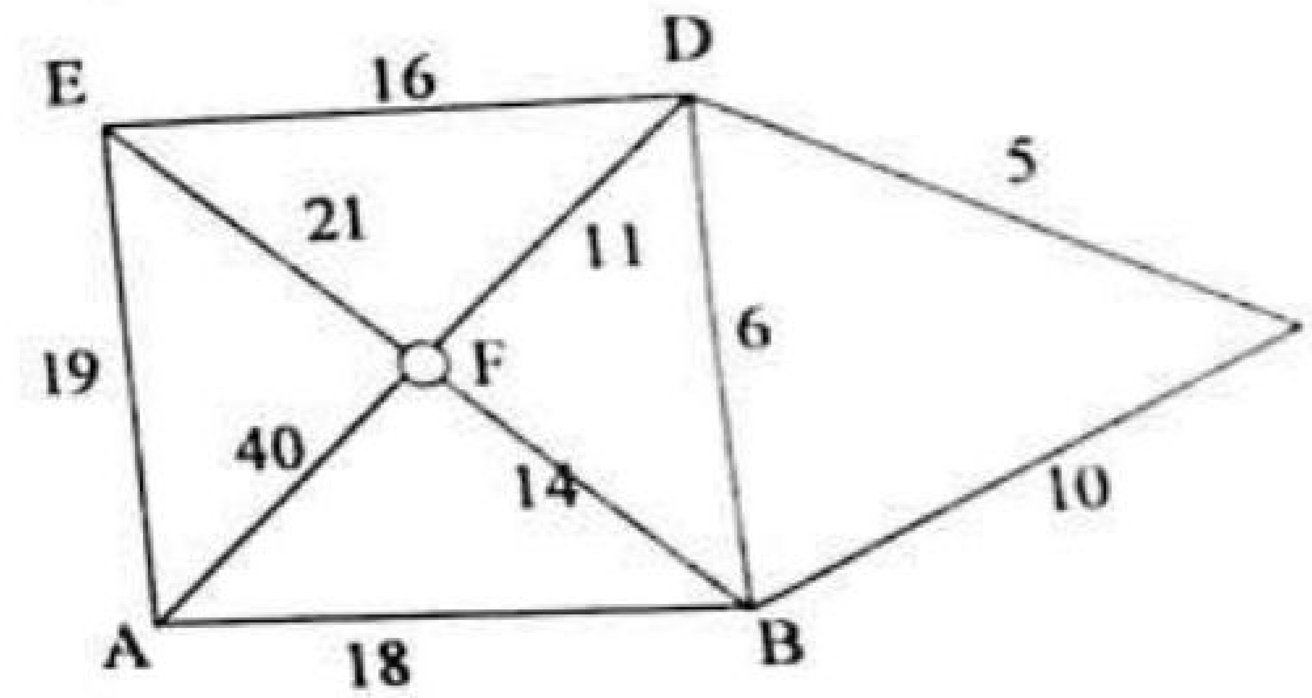
ii. Prove that kernel of a homomorphism of G is the normal subgroup of G.

14. a. Give an example of a graph which contains
   (i) A Eulerian circuit that is also a Hamiltonian circuit
   (ii) An Eulerian circuit and a Hamiltonian circuit that are distinct
   (iii) An Eulerian circuit, but not a Hamiltonian circuit
   (iv) A Hamiltonian circuit, but not an Eulerian circuit

**(OR)**



b. Find minimum spanning tree for the weighted graph, by Kruskal's algorithms.




15. a. State and prove distributive in equalities for a lattice $\{L, \leq\}$.

**(OR)**

b. From the state transition table given below:

| | 0 | 1 |
|---|---|---|
| $S_0$ | $S_0$, 1 | $S_4$, 1 |
| $S_1$ | $S_0$, 0 | $S_3$, 1 |
| $S_2$ | $S_0$, 0 | $S_2$, 0 |
| $S_3$ | $S_1$, 1 | $S_1$, 1 |
| $S_4$ | $S_1$, 1 | $S_0$, 0 |

Compute
   (i) Input set and output set.
   (ii) Draw the state diagram of finite state automata.
   (iii) Obtain the output of the string 0010110.

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# B.Tech. DEGREE EXAMINATION, JUNE 2011
Third Semester

## MA0213 – DISCRETE MATHEMATICS
*(For the candidates admitted from the year 2007-2008 onwards)*

Time: Three hours                                        Max.Marks:100

### PART – A (10 × 2 = 20 Marks)
Answer **ALL** Questions

1. Define: Connectives conjunction and disjunction.
2. Define: Tautological implication with an example.
3. State the generalized Pigeon hole principle.
4. Define: Generating function of a sequence and give an example.
5. Define: Cyclic group. Give an example.
6. What is meant by the hamming distance between two code words?
7. What are isomorphic graphs? Give an example.
8. Explain connected and disconnected graphs with example.
9. What is Backus-Naur form? Give an example.
10. Explain Finite Stat Automata (FSA) with an example.

### PART – B (5 × 16 = 80 Marks)
Answer **ANY FIVE** Questions

11. i. Using truth table, prove the implication $\left(\left(p \vee 7\ (q \wedge r)\right) \wedge 7p\right) \Rightarrow (7q \vee 7r)$.

ii. Prove that the premises $p \rightarrow q$, $q \rightarrow r$, $s \rightarrow 7r$ and $q \wedge s$ are inconsistent.

12. i. Using mathematical induction, show that $\left(1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\ldots+\frac{1}{\sqrt{n}}\right)>\sqrt{n},\ (n \geq 2)$.

ii. Solve the recurrence relation $a_{n+2}-6a_{n+1}+9a_n=3(2^n),\ (n \geq 0)$ given that $a_0=1,\ a_1=4$.

13. i. Prove in any group $(G,*)$

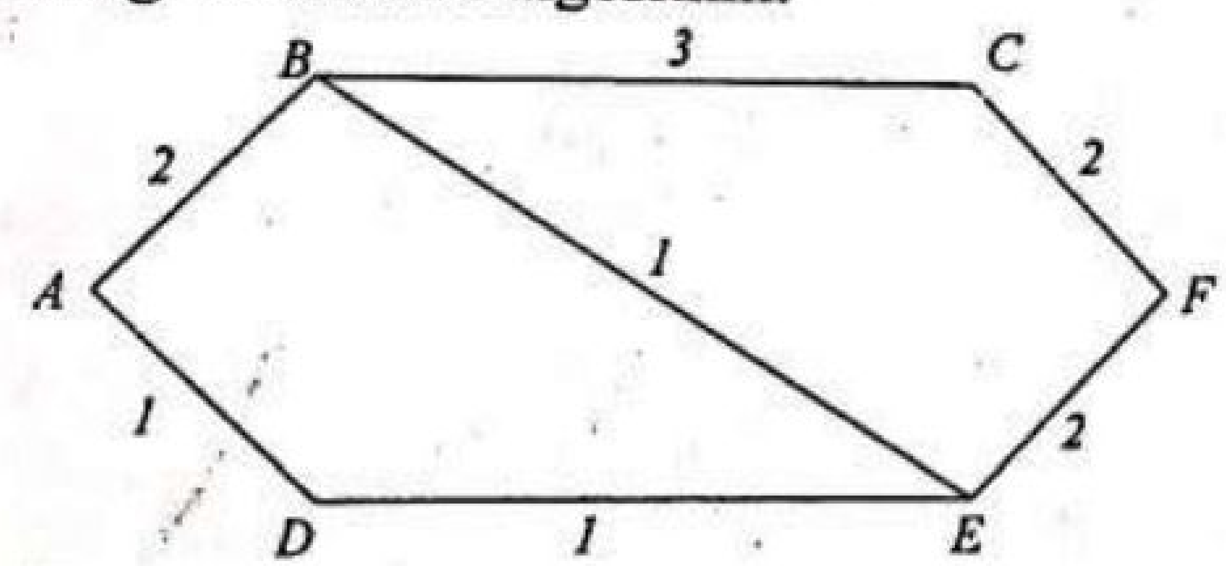
(a) identity element is unique

(b) inverse of each element $g \in G$ is unique

ii. Prove that kernel of a group homomorphism is the normal subgroup of $(G,*)$.

14. i. Prove that number of edges in a bipartite graph with $n$ vertices is atmost $(n/2)^2$.

ii. Find the shortest path between all pairs of vertices of the graph, using Warshall's algorithm.




15. i. Draw the state diagram for Finite State Automata (FSA) having $s_0$ and $s_3$ accepting states and the state table is given below.

| S \ I | 0 | 1 |
|---|---|---|
| $s_0$ | $s_0$ | $s_1$ |
| $s_1$ | $s_3$ | $s_2$ |
| $s_2$ | $s_2$ | $s_2$ |
| $s_3$ | $s_3$ | $s_3$ |

Also find the language accepted by the FSA.

16. i. Show that $b$ can be derived from the premises $a \to b$, $c \to b$, $d \to (a \vee c)$, $d$ by the indirect method.

ii. State and solve the tower of Hanoi problem, using recurrence relation.

17. i. Find the code words generated by the encoding function $e: B^2 \to B^5$ with respect to the parity check matrix

$$H=\begin{pmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

ii. Prove that an undirected graph is a tree if and only if there is a unique simple path between every pair of vertices.

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**B.Tech. DEGREE EXAMINATION, DECEMBER 2014**

Third Semester

MA0213 – DISCRETE MATHEMATICS

Time: Three hours

Max. Marks: 100

Answer **ALL** Questions

**PART – A (10 × 2 = 20 Marks)**

1. Define: Tautology and Contradiction. Give example.
2. Demonstrate that R is a valid inference from the premises $P \to Q, Q \to R$ and $p$.
3. Prove by mathematical induction $1+2+.....+n = \frac{n(n+1)}{2}$
4. Define: Pigeon – Hole principle.
5. Prove that the Kernal of a Homomorphism g from a group (G,*) to (H,Δ) is a sub group of (G,*).
6. Define: Group endomorphism.
7. Define: Isomorphic graphs.
8. Define: Spanning Tree.
9. Define: Types of grammers.
10. Define: Finite State Automation.

**PART – B (5 × 16 = 80 Marks)**

11.a.i Construct the truth table of $P \to ((P \to (Q \to P). \to P))$.

ii Show that R→S can be derived from the premises $P \to (Q \to S), 7R \vee P \, and \, Q$.

**(OR)**

b. i. Show that the following premises $E \rightarrow S, S \rightarrow H, A \rightarrow 7H$ and $E \wedge A$.

ii Prove the following implication without using truth table. $((P \vee 7P) \rightarrow Q) \rightarrow ((P \vee 7P) \rightarrow R) \Rightarrow (Q \rightarrow R)$

12.a.i Show that $(n^3+2n)$ is divisible by 3.

ii If $a_{n-2}\ a_{n-1} = 2^n$ and $a_0=2$ solve by recurrence relation.

**(OR)**

b. i. Prove that $\frac{1}{1.2}+\frac{1}{2.3}+\ldots\ldots+\frac{1}{n(n+1)}=\frac{n}{n+1}$.

ii Solve:

$$a_{n+2}-6a_{n+1}+9a_n=3(2^n)+7(3^n), n \geq 0, a_0=1 \text{ and } a_1=4$$

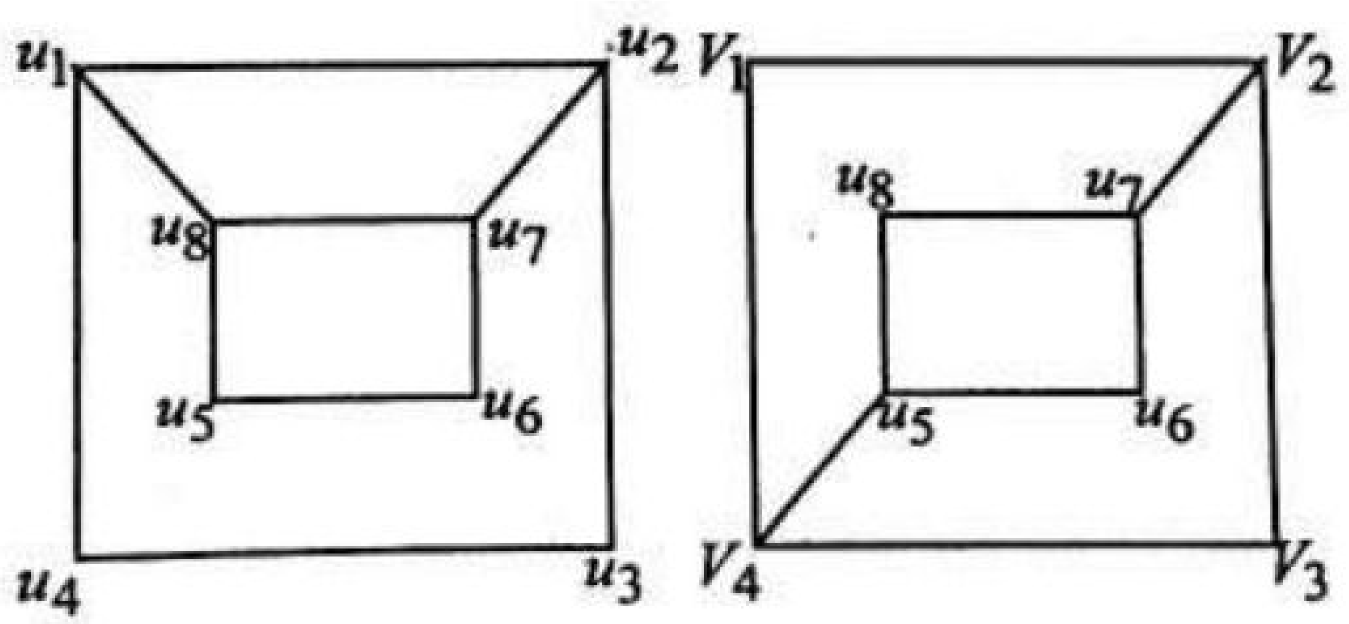
13. a. State and Prove Lagrange's Theorem.

**(OR)**

b. i. Show that in a group (G, A), If for any a, b$\in$ G, $(a * b)^2 = a^2 * b^2$, then (G, *) must be Abelian.

ii Prove that every sub group of a cyclic group is cyclic.

14.a.i Show that the given two graphs are isomorphic.



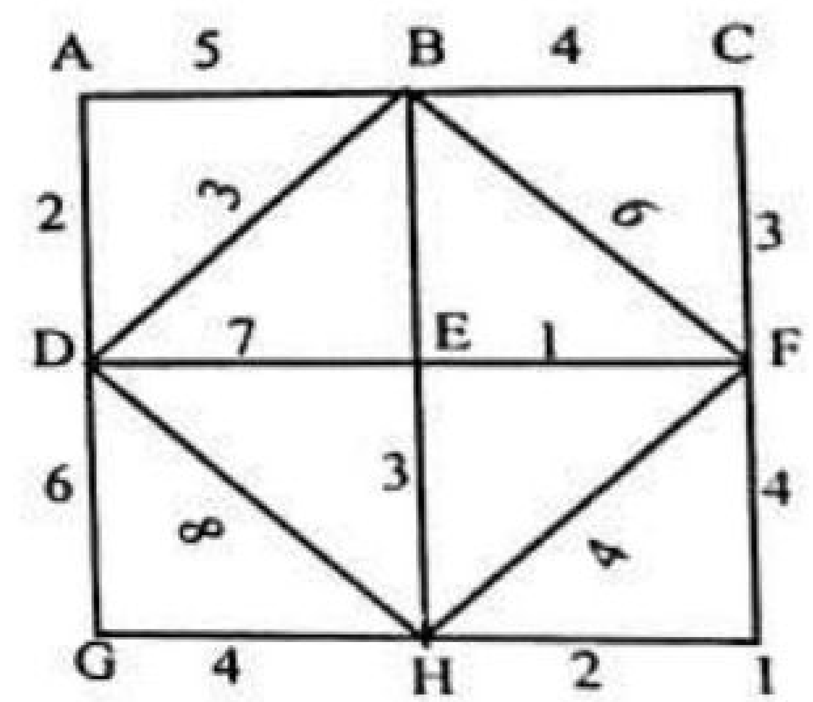



ii Prove that the number of odd degree vertices are even in a graph G(V,E).

**(OR)**

b. i. Prove that the number of edges in a bipartite graph with n vertices is at most $\left(\frac{n^2}{4}\right)$.

ii Use Kruskal's algorithm to find a minimal spanning tree for the weighted graph shown in




15.a.i Find a regular grammar G will generate the Language L which consists of all words in $a$ and $b$ with an even number of $a$'s.

ii Design an FSM that outputs 1 whenever it sees 101 as consecutive input bits and output 0 otherwise.

**(OR)**

b. i Find a grammar that generates the language $L=\left\{a^m b^n c^p \mid m,n,p \geq 1\right\}$.

ii Construct an FSA that accepts all strings over $\{a, b\}$ that contain $m$ $a$'s where $m$ is a multiple of 3.

* * * * *

(i) Find the input set, the state set, output set and the initial state.

(ii) Draw the state diagram of M.

(iii) Find the output of the word $w = a^2bab^2a$

* * * *

D. Thanga Rajathi.

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**B.Tech. DEGREE EXAMINATION, NOVEMBER 2013**

Third Semester

MA0213 – DISCRETE MATHEMATICS

*(For the candidates admitted from the academic year 2007-2008 to 2012-2013)*

Time: Three hours Max. Marks: 100

Answer ALL Questions

**PART – A (10 × 2 = 20 Marks)**

1. Define Dual of compound proposition with an example.
2. When is a set of premises said to be inconsistent?
3. What is the minimum number of students required in a class to be sure that atleast 6 will receive the same grade, if there are 5 possible grades.
4. Using mathematical induction, prove that $n! \geq 2^{n-1}$ for $n \geq 1$.
5. Define Group with an example.
6. Define Hamming code.
7. State and prove the handshaking theorem.
8. Draw all the spanning trees of $K_3$.
9. Find the language generated by the grammar $G = \{(S,A,B), (a,b), (S,P)\}$, where P is the set of production $\{S \to AB, S \to AA, A \to aB, A \to ab, B \to b\}$
10. Define an FSA.

## PART – B (5 × 16 = 80 Marks)

11.a.i Prove that using truth table, $(p \to q) \wedge (q \to r) \Rightarrow (p \to r)$

ii. Prove that without using truth table, $7(p \wedge q) \to (7p \vee (7p \vee q))$

(OR)

b. i. Show that $(s \vee r)$ is tautologically implied by $(p \vee q) \wedge (p \to r) \wedge (q \to s)$

ii. Show that the premises $a \to (b \to c)$, $d \to (b \wedge 7c)$ and $(a \wedge d)$ are inconsistent.

12.a.i Prove that in any group of six people, atleast three must be mutual friends or atleast three must be mutual strangers.

ii. Prove by mathematical induction that $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots + \frac{1}{n(n+1)} = \frac{n}{n+1}$.

(OR)

b. Solve $a_{n+2} - 6a_{n+1} + 9a_n = 3(2^n) + 7(3^n)$, given that $a_0 = 1$ and $a_1 = 4$.

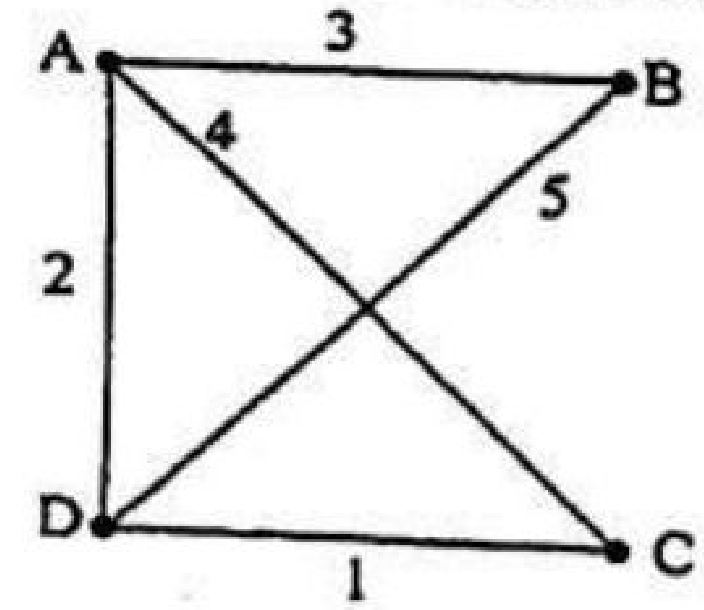
13. a. State and prove Lagrange's theorem.

(OR)

b. Find the code words generated by the encoding function $e : B^2 \to B^5$ with respect to the parity check matrix.

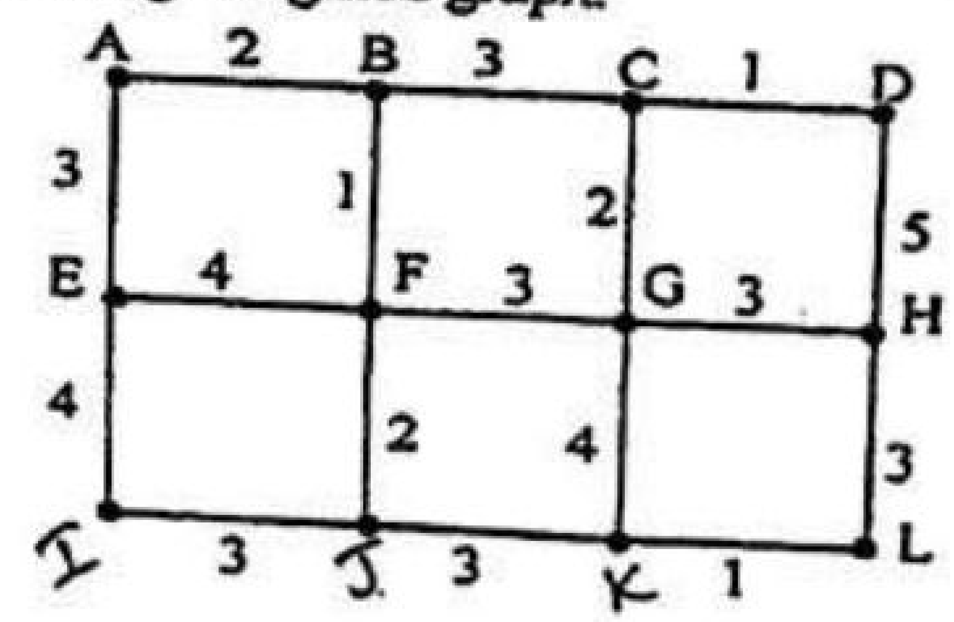
$$H = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

14. a. Find the shortest distance matrix and the corresponding shortest path matrix for all the pairs of vertices in the undirected graph given below, using Warshall's algorithm.




(OR)

b. Use Kruskal's algorithm to find a minimum spanning tree for the following weighted graph.




15. a. Find a grammar that generates the set of words $\{a^n b^n c^n / n \geq 1\}$.

(OR)

b. The state table of a finite state machine M is given below:

| f, g | a | b |
|---|---|---|
| $S_0$ | $S_0$, b | $S_4$, b |
| $S_1$ | $S_0$, a | $S_3$, b |
| $S_2$ | $S_0$, a | $S_2$, a |
| $S_3$ | $S_1$, b | $S_1$, b |
| $S_4$ | $S_1$, b | $S_0$, a |

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**B. Tech. DEGREE EXAMINATION, NOVEMBER 2015**

Seventh Semester

**MA0421 – DISCRETE MATHEMATICS**

*(For the candidates admitted from the academic year 2007-2008 to 2012-2013)*

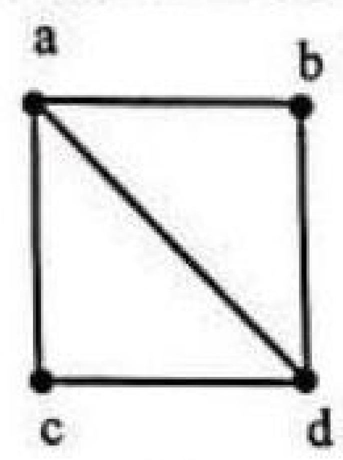
Time: Three Hours

Max. Marks: 100

Answer **ALL** Questions

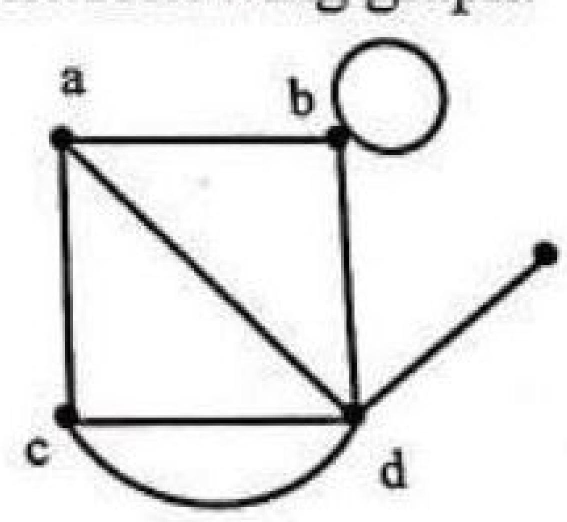
**PART – A (10 × 2 = 20 Marks)**

1. Write the dual of $(p \to r) \wedge (q \to r) \Leftrightarrow (p \vee q) \to r$.
2. Symbolise the expression:
   (i) All cats are animals (ii) Some cats are black.
3. Define an equivalent relation on a set.
4. Let $f(x) = x^2; g(x) = 3x$ where $f:\mathbb{R} \to \mathbb{R}$ and $g:\mathbb{R} \to \mathbb{R}$ ($\mathbb{R}$ is the set of real numbers). Find $f \circ g$ and $g \circ f$
5. Solve the recurrence relation $a_{n+2} - 6a_{n+1} + 9a_n = 0$
6. If (G,*) is a group, prove that identify element of G is unique.
7. Write the adjacency matrix of the graph given below:




x

8. Verify hand shaking theorem for the following graph:




9. Determine the type of grammar G which consists of the productions:
   (i) $S \to aAB,\ S \to AB,\ A \to a,\ B \to b,$
   (ii) $S \to aB,\ B \to bB,\ B \to bA,\ A \to a,\ B \to b.$
10. Define: Finite State Automata.

**PART – B (5 × 16 = 80 Marks)**

11. a.i. Using truth table, prove $p \to (q \to r) \Rightarrow (p \to q) \to (p \to r)$.

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ii. Without using truth table, prove the following:
$(7p \vee q) \wedge (p \wedge (p \wedge q)) \Leftrightarrow p \wedge q$.

**(OR)**

b.i Show that $((t \wedge s)$ can be derived from the premises $p \to q,\ q \to 7r,\ r, p \vee (t \wedge s)$.

ii. Show that $\forall x(p(x) \vee Q(x)) \Rightarrow \forall x(p(x)) \vee \exists x(Q(x))$ by indirect method.

12.a.i. A={1,2,3,4,5,6,7,8,9} $B_1$={1,3,4}, $B_2$={1,2,4,5,6}, $B_3$={2,3,5,7,8,9}. Find the minset generated by $B_1, B_2, B_3$. Give the partition of A using the minsets.

ii. Using Warshall's algorithm, find the transitive closure of the relation R whose matrix is

$$M_R = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

**(OR)**

b.i Draw the Hasse diagram representing the partial ordering P = {(a,b)/a divides b} on {1,2,3,4,6,8,12}.

ii. Let $A=\{x \in \mathbb{R} / x \neq 2\}$ and $B=\{x \in \mathbb{R} / x \neq 1\}$. Define $f : A \to B$ by $f(x) = \frac{x}{x-2}$. Prove f is one-one and onto. find $f^{-1}$.

13.a.i. Solve the recurrence relation $a_{n+1} - a_n = 3n^2 - n$.

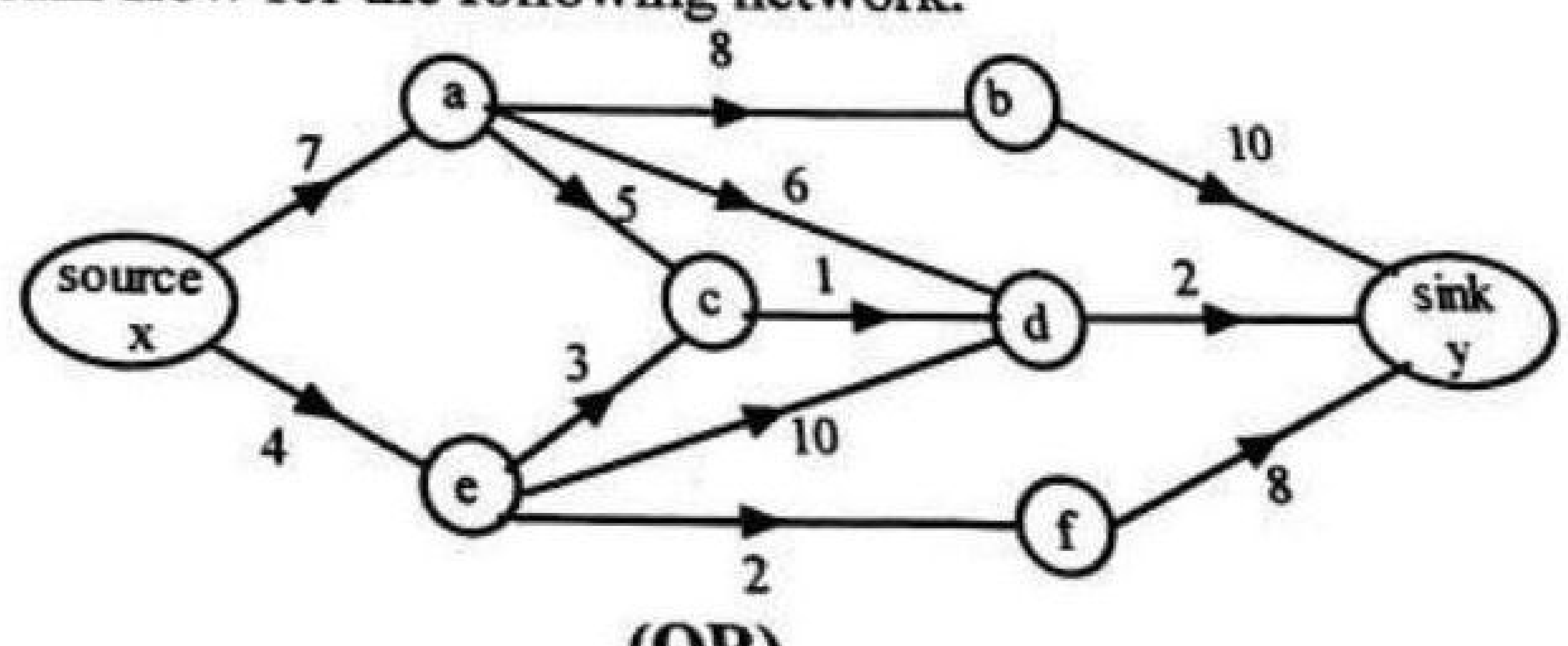
ii. Use the method of generating function to solve the recurrence relation $a_n = 3a_{n-1} + 1, n \geq 1$, given $a_0 = 1$

**(OR)**

b.i. Let(G,*) be a group and let H be a subgroup of G satisfying $a^{-1} * h * a \in H$ for all $a \in G$ and $h \in H$. Prove that H is a normal subgroup of G.

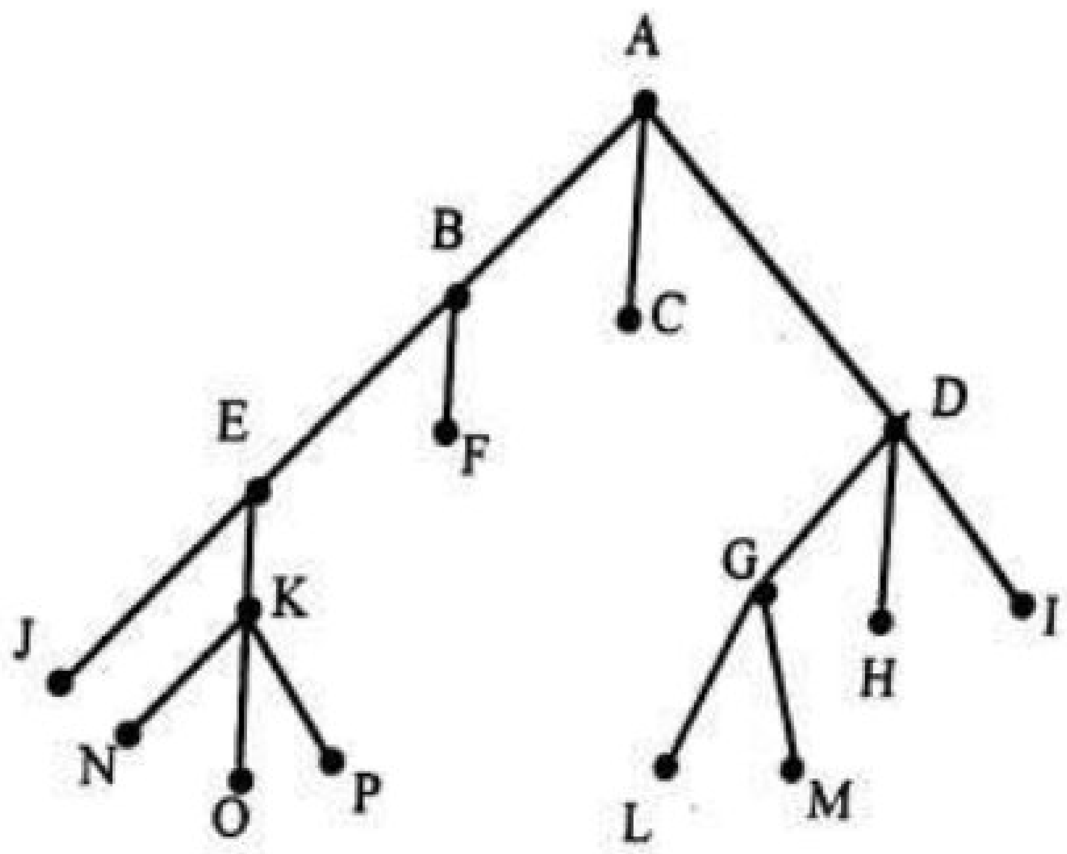
ii. Let(G,*) and $(G', \Delta)$ be two groups with identity elements e and $e'$ respectively. Let f: $G \to G'$ be a group homomorphism. Prove (i) $f(e) = e'$ (ii) $f(a^{-1}) = [f(a)]^{-1}$ (iii) Kernel of *f*, ker(f) is a subgroup of (G,*).

14.a. Find the maximum flow for the following network.

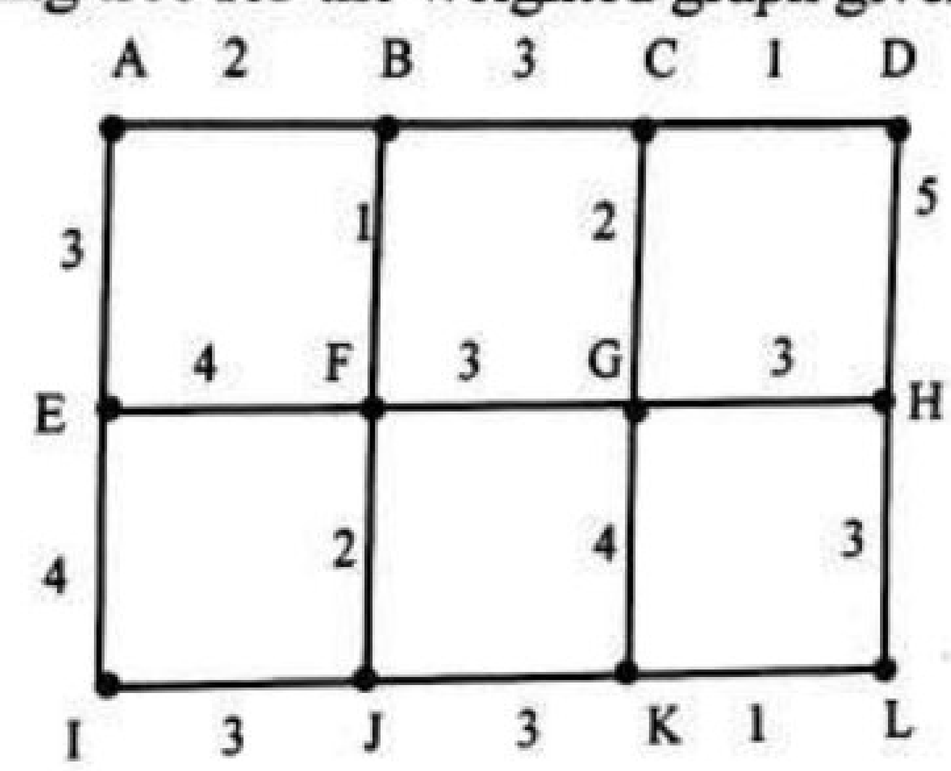



**(OR)**

b.i. List the order in which the vertices of the tree given below are processed using preorder, inorder and postorder traversal.




ii. Find the minimum spanning tree for the weighted graph given below:




15.a.i. Find the language generated by the following grammars:

(1) $G = (\{S, A, B\}, \{a, b\}, S, P)$ where $P = \{S \to AB, S \to AA, A \to aB, A \to ab, B \to b\}$

(2) $G = (\{S, A\}, \{a, b, c\}, S, P)$ where $P = \{S \to aSb, Sb \to bA, abA \to c\}$

ii. Find a phrase structure grammar that generates the language L consisting of equal number of a's and b's.

**(OR)**

b.i. Draw the state diagram for the FSA for which state table is given below. The accepting states are $s_1$ and $s_3$. Find whether the string aaababbab is accepted by the FSA.

| | f | |
|---|---|---|
| S \ I | a | b |
| $s_0$ | $s_1$ | $s_2$ |
| $s_1$ | $s_2$ | $s_1$ |
| $s_2$ | $s_2$ | $s_3$ |
| $s_3$ | $s_1$ | $s_0$ |

ii. Design an FSM that outputs 1 whenever it sees 101 as consecutive input bits and outputs '0' otherwise.

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**B.Tech. DEGREE EXAMINATION, NOVEMBER 2012**
Third Semester

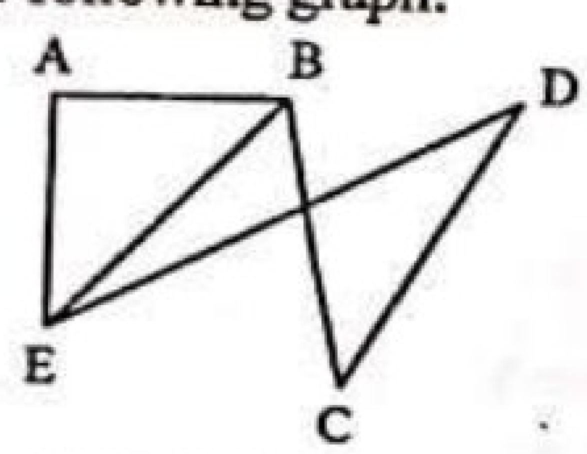
MA0213 – DISCRETE MATHEMATICS
*(For the candidates admitted from the year 2007-2008 onwards)*

Time: Three hours

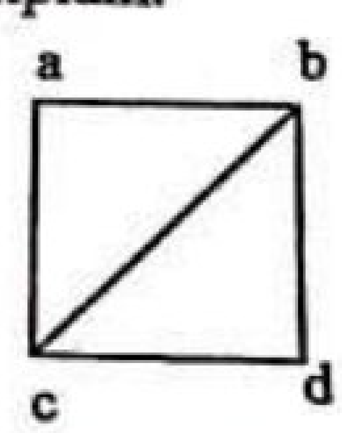
Max. Marks: 100

Answer ALL Questions
**PART – A (10 × 2 = 20 Marks)**

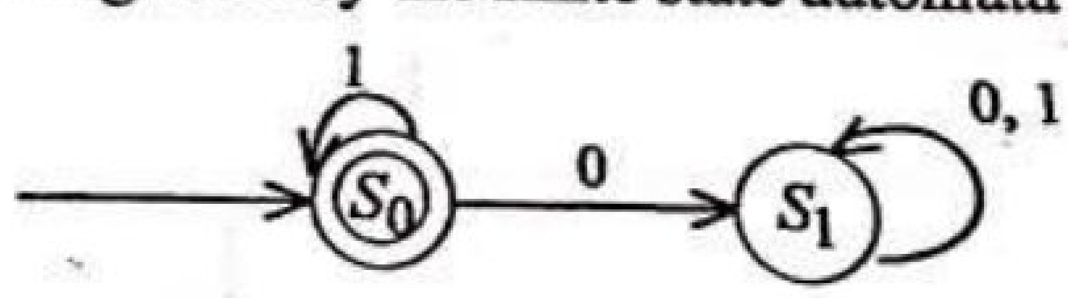
1. Construct the truth table for $(p \to q) \to (q \to p)$.
2. Write in symbolic form: 'you can access the internet from campus only if you are a computer science major or you are not a fresher'.
3. Show that in any group of 27 English words, there must be atleast two that begin with the same letter.
4. Find the solution to the recurrence relation $a_n = 3a_{n-1}$, $a_0 = 2$.
5. Define Hamming Codes.
6. If a, b, c are elements of a group (G, *) and a*b = a*c, prove that b = c.
7. Give the adjacency matrix of the following graph:



8. Is the graph given below Eulerian? Explain.



9. Determine the language recognized by the finite state automata $M_1$ given by



10. Let A = {0, 11}, B = {1, 10, 110}. Find AB and BA.

**PART – B (5 × 16 = 80 Marks)**

11. a. Without using Truth tables, prove
    (i) $p \to (q \to p) \Leftrightarrow \neg p \to (p \to q)$
    (ii) Using laws of logic, show that $(p \wedge q) \to (p \to q)$ is a tautology.

**(OR)**

b.i. Show by direct method that $r \to s$ can be derived from the premises $p \to (q \to s)$, $(\neg r \vee p)$, $q$.

ii. Show that the following premises are inconsistent $p \to q,\ p \to r,\ q \to \neg r, p$.

12. a.i. Use mathematical induction to prove that $n^3+2n$ is divisible by 3 for $n \geq 1$.

ii. A cricket team plays atleast one 20 – 20 match a day but not more than 45 matches during a month with 30 days. Show that there must be a period of some consecutive days during which period the team plays exactly 14 matches.

(OR)

b.i. Find a recurrence relation and give initial conditions to find the number of n-bit strings that do not contain two consecutive zeros. How many such 5-bit strings are there? (6 Marks)

ii. Solve the recurrence relation $S(k)-4S(k-1)+4S(k-2)=3k+2^k, k \geq 2$, $S(0)=1, S(1)=1$. (10 Marks)

13. a.i. State Lagrange's theorem for a finite group G. Deduce the following

(1) If a is any element of G, then 0(a) divides 0(G)
(2) If 0(G) = n, then $a^n$ =e for any element a in G
(3) Every group G of prime order is cyclic. (12 Marks)

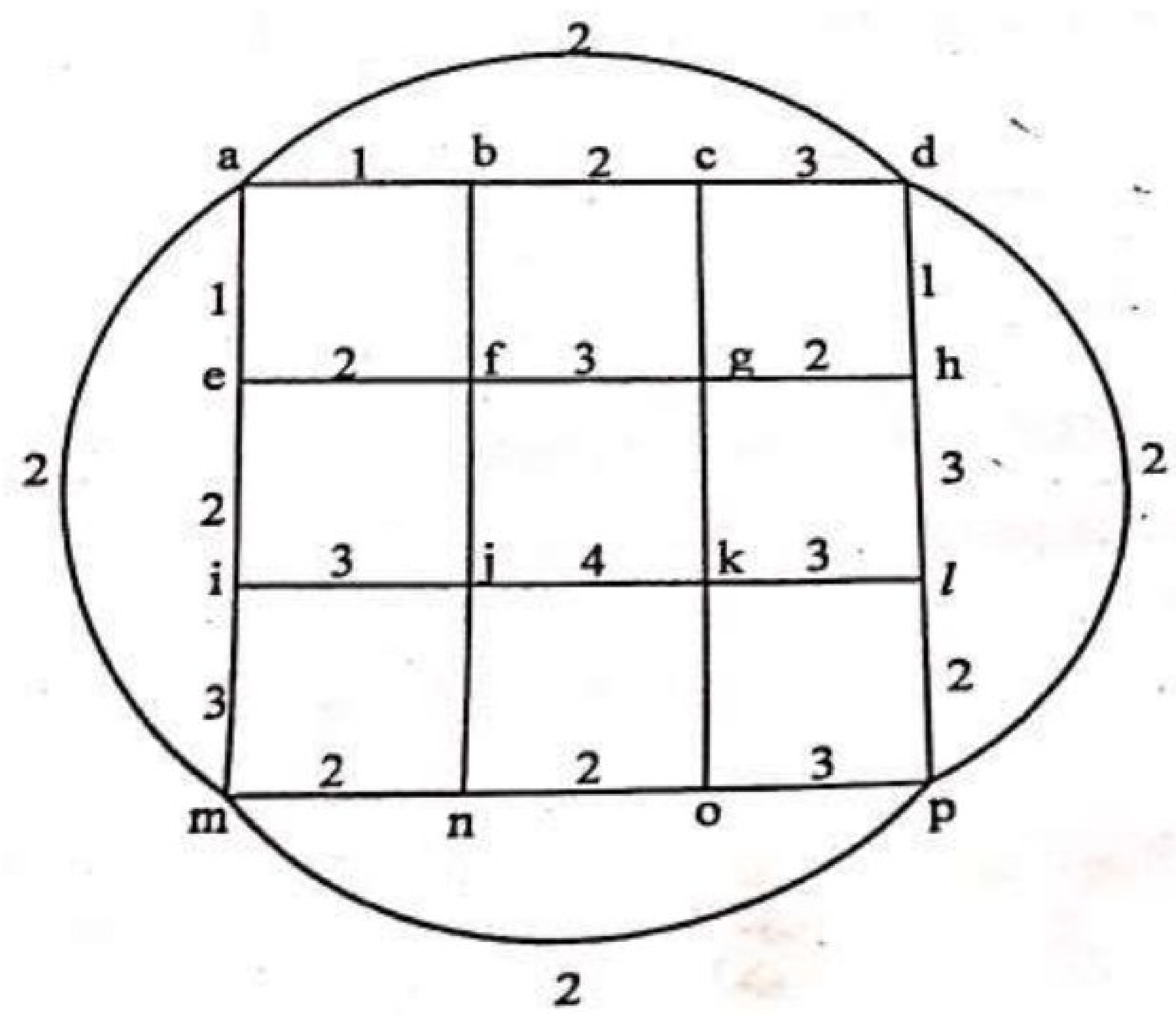
ii. If $f:(G,*) \rightarrow (G',\Delta)$ is a homomorphism with kernel k, then prove that k is a normal subgroup of G. (4 Marks)

(OR)

b.i. Find the code words generated by the parity check matrix $H=\begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (10 Marks)

ii. Decode the following received words corresponding to the encoding function $e:B^3 \rightarrow B^6$ given by
e(000) = 000 000, e (001) = 001 011
e(010) = 010 101, e(100) = 100 111,
e(011) = 011 110, e(101) = 101 100,
e(110) = 110 010 and E(111) = 111 001. Assuming that no error or single error has occurred (1) 011110 (2) 110 111 (3) 110 000 (4) 111 000 (5) 011 111. (6 Marks)

14. a.i. Use Kruskal's algorithm to find a minimum spanning tree for the following weighted graph.

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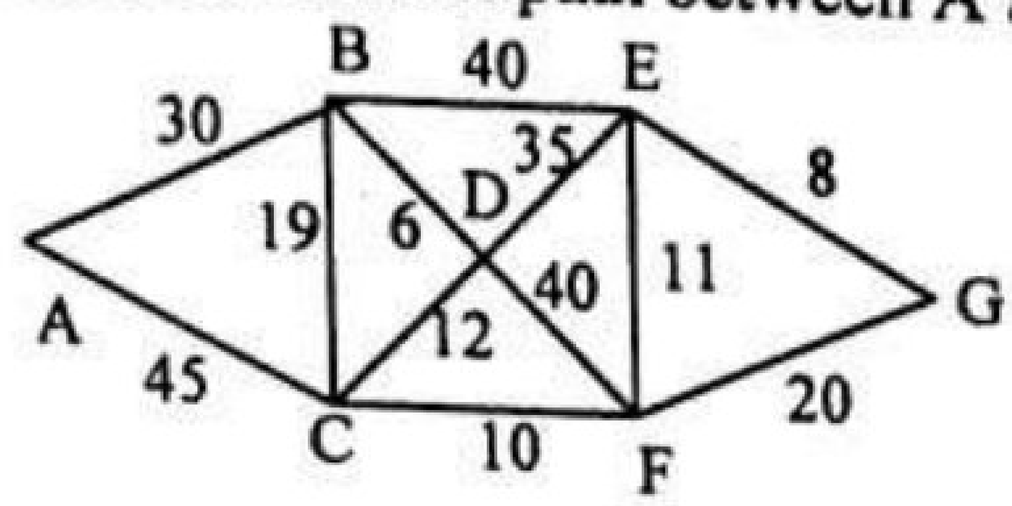
ii. Prove that an undirected graph is a tree if and only if every pair of vertices has a unique path between them.

(4 Marks)

**(OR)**

b. Use Dijkstra's algorithm to find the shortest path between A and G




15. a.i. Find the language generated by each of the following grammar.

(1) G = { S, A, B), (a, b), S, P } where P is the set of productions { s → AB, S→ AA, A→aB, A→ab, B→b}

(2) G = {A, B, C, S), (a, b, c), S, P} where P = { S→AB, A→Ca, B→Cb, B→b, C = cb)}

ii. Find a regular grammar that generates the language $L = (a^m b^n c^p / m, n, p \geq 1)$.

**(OR)**

b.i. The state table of a finite state machine M is given in table.

| f, g | a | B |
|---|---|---|
| $s_0$ | $s_0$, b | $s_4$, b |
| $s_1$ | $s_0$, a | $s_3$, b |
| $s_2$ | $s_0$, a | $s_2$, a |
| $s_3$ | $s_1$, b | $s_1$, b |
| $s_4$ | $s_1$, b | $s_0$, a |

(i) Find the input set I, the state set S, the output set O and the initial state of M

(ii) Draw the state diagram of M

(iii) Find the output of the word $w = a^2bab^2a$

(10 Marks)

ii. Construct a finite state automation that accepts those strings {a, b} which begin with an a and followed by $b^n (n \geq 0)$.

(6 Marks)

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## B.Tech. DEGREE EXAMINATION, NOVEMBER 2015
Third Semester

## MA1023 – DISCRETE MATHEMATICS
*(For the candidates admitted during the academic year 2013 – 2014 and 2014 -2015)*

**Note:**
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of $45^{th}$ minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours — Max. Marks: 100

### PART – A (20 × 1 = 20 Marks)
Answer **ALL** Questions

1. Which of the following statement is the negation of the statement "2 is even and -3 is negative"
   (A) 2 is even and –3 is not negative
   (B) 2 is odd and –3 is not negative
   (C) 2 is even or –3 is not negative
   (D) 2 is odd or –3 is not negative

2. Which one is the contrapositive of $q \to p$?
   (A) $p \to q$
   (B) $7p \to 7q$
   (C) $7q \to 7p$
   (D) $7p \vee q$

3. The statement $(p \wedge q) \Rightarrow p$ is a
   (A) Contradiction
   (B) Tautology
   (C) Inconsistent
   (D) Consistent

4. The dual of $7(p \wedge q) \vee T$ is
   (A) $(P \vee Q) \wedge F$
   (B) $(P \vee Q) \wedge T$
   (C) $(P \wedge Q) \vee F$
   (D) $7(P \vee Q) \wedge F$

5. In a group of 100 people, the minimum number of people having birthday in the same month is
   (A) 7
   (B) 8
   (C) 9
   (D) 10

6. The order of the recurrence relation $S(k) - 4\,S(k-1) - 11\,S(k-2) + 30\,S(k-3) = 0$ is
   (A) 0
   (B) 3
   (C) 1
   (D) 2

7. The solution of $a_n + 5a_{n-1} = 0$, $a_0 = 6$ is
   (A) $5(6^n)$
   (B) $6(5^n)$
   (C) $5(6^{n-1})$
   (D) $6(-5)^n$

8. The generating function for the recurrence relation $a_n = 2\,a_{n-1}$, $a_0 = 1$ is
   (A) $\frac{1}{1+2x}$
   (B) $\frac{1}{1-2x}$
   (C) $\frac{1}{1-x}$
   (D) $\frac{1}{1+x}$

20. Find the ...

9. Every group of prime order is ----------
(A) Cyclic and hence abelian
(B) Abelian and hence cyclic
(C) Not cyclic & abelian
(D) Not abelian and cyclic

10. What are the generators of the group (Z, +)?
(A) 1 and 0
(B) -1 and 0
(C) 0 alone
(D) 1 and -1

11. The necessary condition that a non-empty subset H of a group G to be a subgroup is
(A) $a, b \varepsilon H \Rightarrow a^{-1}, b^{-1} \in H$
(B) $a, b \varepsilon H \Rightarrow (a * b^{-1}) \in H$
(C) $a, b \varepsilon H \Rightarrow (a * b) \in H$
(D) $a, b \varepsilon H \Rightarrow (a * b)^{-1} \in H$

12. Let G be a group. If $a, b \in G$, then the inverse of $(a*b)$ is
(A) $a^{-1} * b^{-1}$
(B) $a*b^{-1}$
(C) $a^{-1} * b$
(D) $b^{-1} * a^{-1}$

13. How many edges are there in a graph with 10 vertices each of degree 6?
(A) 30
(B) 60
(C) 15
(D) 16

14. The maximum number of edges in a simple graph with n vertices is
(A) $\frac{n(n-1)}{2}$
(B) $\frac{n(n+1)}{2}$
(C) $\frac{(n-1)(n+1)}{2}$
(D) $\frac{n}{2}$

15. A simple graph with n vertices and K components can have atmost --------- edges
(A) $\frac{(n-k)(n-k-1)}{2}$
(B) $\frac{(n-k)(n-k+1)}{2}$
(C) $\frac{(n+k)(n+k-1)}{2}$
(D) $\frac{(n+k)(n-k+1)}{2}$

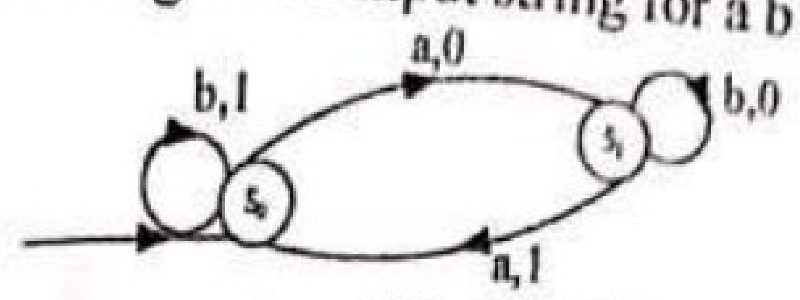
16. The complete graph on a vertices Kn where $n \geq 3$, is
(A) Hamiltonian
(B) Eulerian
(C) Both Hamiltonian & Eulerian
(D) Neither Hamiltonian nor Eulerian

17. The transition function in NFA assigns ---------- next state to every pair of state and input.
(A) All
(B) Unique
(C) Two
(D) Several

18. If a language L is accepted by an NFA, then there exists a ---------- that also accepts L
(A) DFA
(B) Grammar
(C) FSM
(D) Language

19. Difference between FSA and FSM
(A) FSA does not produce output
(B) FSM does not produce output
(C) FSA does not produce input
(D) FSM does not produce input

20. Find the output string corresponding to the input string for a b a b a, whose diagram is




(A) 01100
(B) 01010
(C) 00110
(D) 10011

## PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

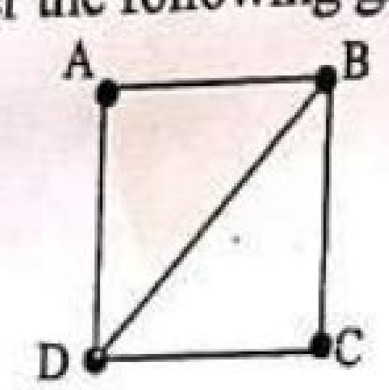
21. Prove that $P \rightarrow (Q \rightarrow R) \Leftrightarrow (P \wedge Q) \rightarrow R$, without using truth table.

22. Prove that, in any group of 6 people atleast 3 are mutual friends or atleast 3 are mutual strangers

23. Let I = {+1, -1, i, -i} and (I, .) be a group. Find the order of each element of this group.

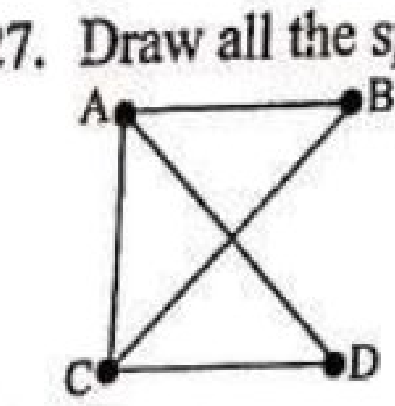
24. State and prove Handshaking theorem

25. Check whether the following graph is Eulerian and / or Hamiltonian Graph.




26. Find the type of grammar G which consists of the following productions.
$S \rightarrow aAB$, $S \rightarrow AB$, $A \rightarrow a$, $B \rightarrow b$

27. Draw all the spanning trees for the following graph.




## PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28.a. i. Without using truth table, prove that $(7P \vee Q) \wedge (P \wedge (P \wedge Q)) \Leftrightarrow (P \wedge Q)$

ii. Show that $R \rightarrow S$ can be derived from the premises,
$P \rightarrow (Q \rightarrow S)$, 7RVP and Q

**(OR)**

b. i. State the duality law and find the dual of $(P \rightarrow Q) \rightarrow s\ (7Q \rightarrow 7P)$

ii . Prove that the following premises $a \rightarrow (b \rightarrow c)$, $d \rightarrow (b \wedge 7C)$ and $(a \wedge d)$ are inconsistent.

29.a. i. Prove, by mathematical induction that, $(6 \times 7^n) - (2 \times 3^n)$ is divisible by 4, for $n \geq 1$

ii. What is the minimum number of students required in a class to be sure that atleast six will receive the same grade, if there are five possible grades?

**(OR)**

b. i. Solve $a_{n+2} - 6a_{n+1} + 9a_n = 3(2^n) + 7(3^n)$

ii. Use the method of generating function to solve the recurrence relation.
$a_n = 3a_{n-1} + 1, n \geq 1, a_0 = 1$

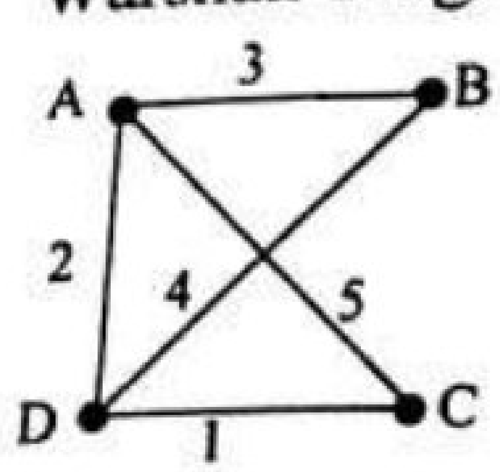
30. a. State and prove Lagrange's theorem on group theory.

**(OR)**

b. i. Prove that the subgroup of cyclic group is cyclic

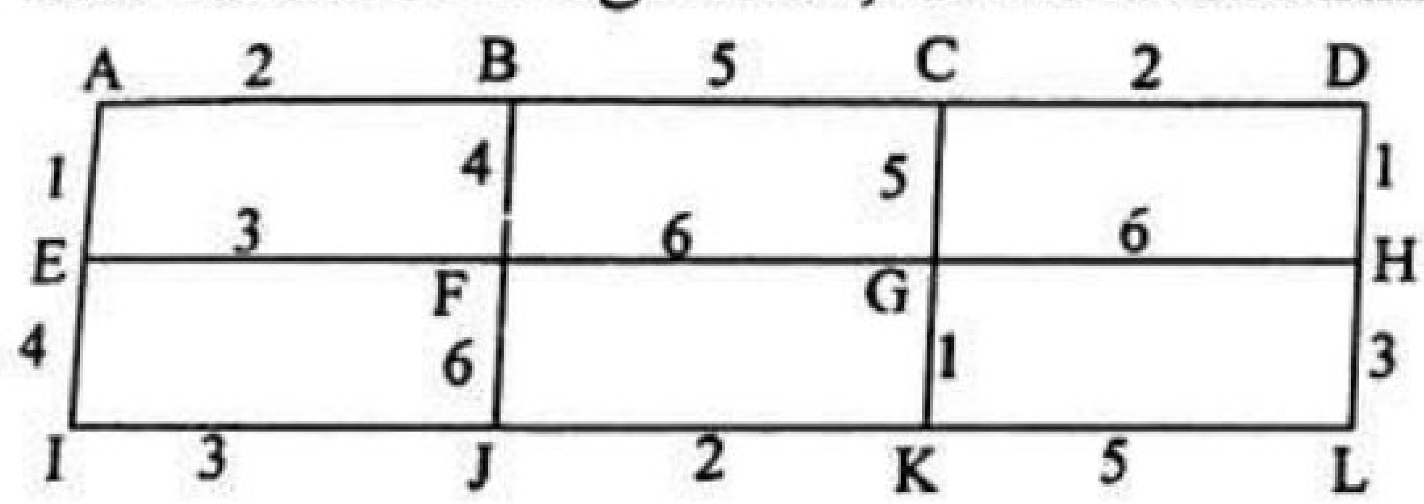
ii. Prove that every group of prime order is cyclic.

31. a. Find the shortest distance matrix for all pairs of vertices in the graph given below, using Warshall's algorithm




**(OR)**

b. Use Krushkal's algorithm, to find the minimum spanning tree in the graph given below.




32. a. Find a grammar that generates the set of words $\{a^n b^n c^n / n \geq 1\}$

**(OR)**

b. i. Design an FSM that outputs 1 whenever it sees 101 as consecutive input bits and outputs 0 otherwise.

ii. Construct a finite state automaton that accepts all strings over {a,b} in which every a is followed by b.

*****

| Reg. No. | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|

# B.Tech. DEGREE EXAMINATION, NOVEMBER 2015
Third Semester

## MA1023 – DISCRETE MATHEMATICS
*(For the candidates admitted during the academic year 2013 – 2014 and 2014 -2015)*

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of $45^{th}$ minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours — Max. Marks: 100

### PART – A (20 × 1 = 20 Marks)
Answer ALL Questions

1. Which of the following statement is the negation of the statement "2 is even and -3 is negative"
   (A) 2 is even and –3 is not negative
   (B) 2 is odd and –3 is not negative
   (C) 2 is even or –3 is not negative
   (D) 2 is odd or –3 is not negative

2. Which one is the contrapositive of $q \to p$?
   (A) $p \to q$
   (B) $7p \to 7q$
   (C) $7q \to 7p$
   (D) $7p \vee q$

3. The statement $(p \wedge q) \Rightarrow p$ is a
   (A) Contradiction
   (B) Tautology
   (C) Inconsistent
   (D) Consistent

4. The dual of $7(p \wedge q) \vee T$ is
   (A) $(P \vee Q) \wedge F$
   (B) $(P \vee Q) \wedge T$
   (C) $(P \wedge Q) \vee F$
   (D) $7(P \vee Q) \wedge F$

5. In a group of 100 people, the minimum number of people having birthday in the same month is
   (A) 7
   (B) 8
   (C) 9
   (D) 10

6. The order of the recurrence relation $S(k) - 4\,S(k-1) - 11\,S(k-2) + 30\,S(k-3) = 0$ is
   (A) 0
   (B) 3
   (C) 1
   (D) 2

7. The solution of $a_n + 5a_{n-1} = 0$, $a_0 = 6$ is
   (A) $5(6^n)$
   (B) $6(5^n)$
   (C) $5(6^{n-1})$
   (D) $6(-5)^n$

8. The generating function for the recurrence relation $a_n = 2\,a_{n-1}$, $a_0 = 1$ is
   (A) $\frac{1}{1+2x}$
   (B) $\frac{1}{1-2x}$
   (C) $\frac{1}{1-x}$
   (D) $\frac{1}{1+x}$

9. Every group of prime order is ----------

(A) Cyclic and hence abelian
(B) Abelian and hence cyclic
(C) Not cyclic & abelian
(D) Not abelian and cyclic

10. What are the generators of the group (Z, +)?

(A) 1 and 0
(B) -1 and 0
(C) 0 alone
(D) 1 and -1

11. The necessary condition that a non-empty subset H of a group G to be a subgroup is

(A) $a, b \in H \Rightarrow a^{-1}, b^{-1} \in H$
(B) $a, b \in H \Rightarrow (a * b^{-1}) \in H$
(C) $a, b \in H \Rightarrow (a * b) \in H$
(D) $a, b \in H \Rightarrow (a * b)^{-1} \in H$

12. Let G be a group. If $a, b \in G$, then the inverse of $(a*b)$ is

(A) $a^{-1} * b^{-1}$
(B) $a*b^{-1}$
(C) $a^{-1} * b$
(D) $b^{-1} * a^{-1}$

13. How many edges are there in a graph with 10 vertices each of degree 6?

(A) 30
(B) 60
(C) 15
(D) 16

14. The maximum number of edges in a simple graph with n vertices is

(A) $\frac{n(n-1)}{2}$
(B) $\frac{n(n+1)}{2}$
(C) $\frac{(n-1)(n+1)}{2}$
(D) $\frac{n}{2}$

15. A simple graph with n vertices and K components can have atmost --------- edges

(A) $\frac{(n-k)(n-k-1)}{2}$
(B) $\frac{(n-k)(n-k+1)}{2}$
(C) $\frac{(n+k)(n+k-1)}{2}$
(D) $\frac{(n+k)(n-k+1)}{2}$

16. The complete graph on a vertices Kn where $n \geq 3$, is

(A) Hamiltonian
(B) Eulerian
(C) Both Hamiltonian & Eulerian
(D) Neither Hamiltonian nor Eulerian

17. The transition function in NFA assigns ---------- next state to every pair of state and input.

(A) All
(B) Unique
(C) Two
(D) Several

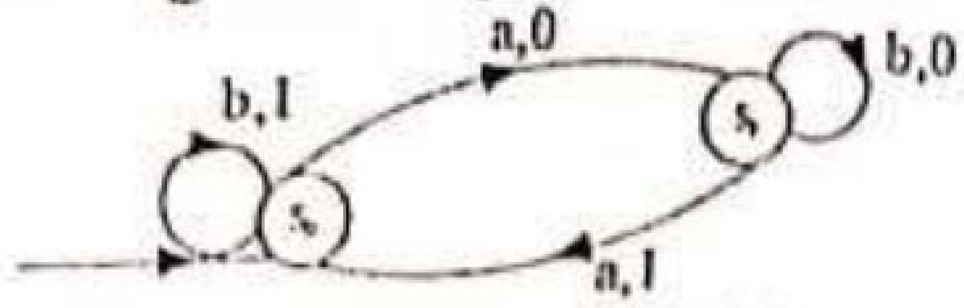
18. If a language L is accepted by an NFA, then there exists a ---------- that also accepts L

(A) DFA
(B) Grammar
(C) FSM
(D) Language

19. Difference between FSA and FSM

(A) FSA does not produce output
(B) FSM does not produce output
(C) FSA does not produce input
(D) FSM does not produce input

20. Find the output string corresponding to the input string for a b a b a, whose diagram is




(A) 01100

(B) 01010

(C) 00110

(D) 10011

## PART – B (5 × 4 = 20 Marks)
Answer ANY FIVE Questions

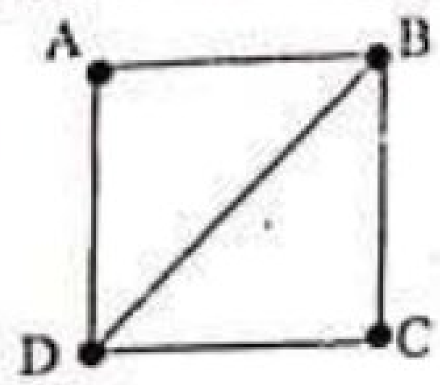
21. Prove that $P \rightarrow (Q \rightarrow R) \Leftrightarrow (P \wedge Q) \rightarrow R$, without using truth table.

22. Prove that, in any group of 6 people atleast 3 are mutual friends or atleast 3 are mutual strangers

23. Let I = {+1, -1, i, -i} and (I, .) be a group. Find the order of each element of this group.

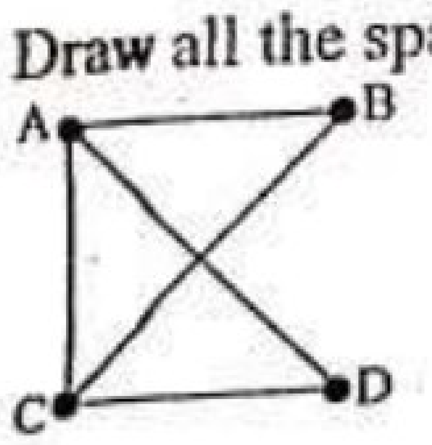
24. State and prove Handshaking theorem

25. Check whether the following graph is Eulerian and / or Hamiltonian Graph.




26. Find the type of grammar G which consists of the following productions.
$S \rightarrow aAB$, $S \rightarrow AB$, $A \rightarrow a$, $B \rightarrow b$

27. Draw all the spanning trees for the following graph.




## PART – C (5 × 12 = 60 Marks)
Answer ALL Questions

28.a. i. Without using truth table, prove that $(7P \vee Q) \wedge (P \wedge (P \wedge Q)) \Leftrightarrow (P \wedge Q)$

ii. Show that $R \rightarrow S$ can be derived from the premises,
$P \rightarrow (Q \rightarrow S)$, 7RVP and Q

(OR)

b. i. State the duality law and find the dual of $(P \rightarrow Q) \rightarrow s\ (7Q \rightarrow 7P)$

ii . Prove that the following premises $a \rightarrow (b \rightarrow c)$, $d \rightarrow (b \wedge 7C)$ and $(a \wedge d)$ are inconsistent.

29.a. i. Prove, by mathematical induction that, $(6 \times 7^n) - (2 \times 3^n)$ is divisible by 4, for $n \geq 1$

ii. What is the minimum number of students required in a class to be sure that atleast six will receive the same grade, if there are five possible grades?

(OR)

b. i. Solve $a_{n+2} - 6a_{n+1} + 9a_n = 3(2^n) + 7(3^n)$

ii. Use the method of generating function to solve the recurrence relation.
$a_n = 3a_{n-1} + 1, n \geq 1, a_0 = 1$

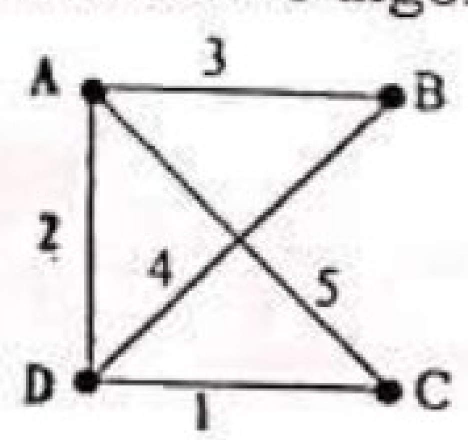
30. a. State and prove Lagrange's theorem on group theory.

(OR)

b. i. Prove that the subgroup of cyclic group is cyclic

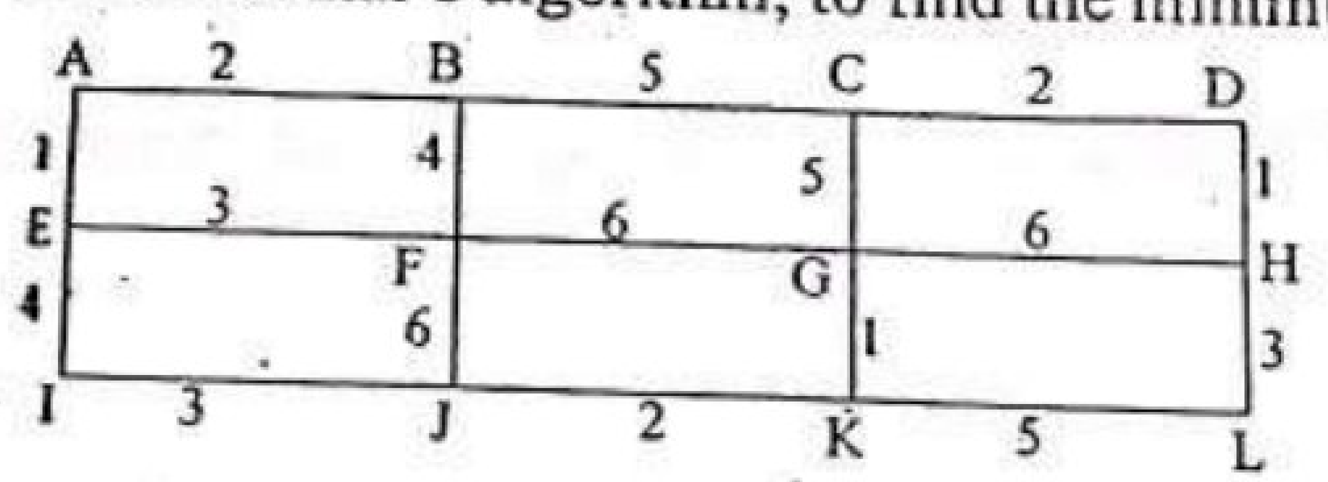
ii. Prove that every group of prime order is cyclic.

31. a. Find the shortest distance matrix for all pairs of vertices in the graph given below, using Warshall's algorithm




(OR)

b. Use Krushkal's algorithm, to find the minimum spanning tree in the graph given below.




32. a. Find a grammar that generates the set of words $\{a^n b^n c^n / n \geq 1\}$

(OR)

b. i. Design an FSM that outputs 1 whenever it sees 101 as consecutive input bits and outputs 0 otherwise.

ii. Construct a finite state automaton that accepts all strings over {a,b} in which every a is followed by b.

* * * * *

30. a. Solve the recurrence relation $a_n - 4a_{n-1} + 4a_{n-2} = (n+1)2^n$ with $a_0 = 0$ and $a_1 = 1$.

(OR)

b.i Solve $a_n - 3a_{n-1} = 1, n \geq 1$ and $a_0 = 1$ using generating function.

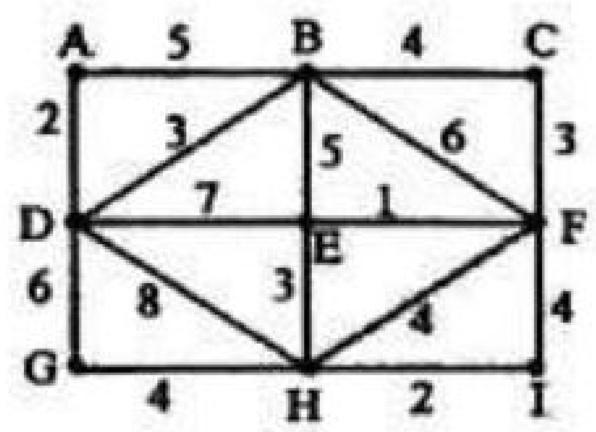
ii. State and prove Lagrange's theorem.

31.a.i Prove that the number of edges in a bipartite graph with n vertices is atmost $\left(\frac{n}{2}\right)^2$.

ii. Construct the binary tree whose in order and post order traversals are DCEBFAHGI and DECFBHIGA respectively.

(OR)

b. Find the minimum spanning tree for the following weighted graph using Kruskal's algorithm.




32.a.i State and prove distributive inequalities in a lattice.

ii. If $S_n$ is the set of all divisors of the positive integer and D is the relation defined by aDb if and only if a divides b, prove that $D_{42} = \{S_{42}, D\}$ is a complemented lattice by finding the complements of all the elements.

(OR)

b.i State and prove DeMorgan's law in Boolean algebra.

ii. Simplify the Boolean expression $f(x,y,z) = x\left[y + z\left(\overline{xy + xz}\right)\right]$.

* * * * *



C. Palanivani

Reg. No.

# B.Tech. DEGREE EXAMINATION, NOVEMBER 2017

Third/ Fourth/ Fifth Semester

## 15MA302 – DISCRETE MATHEMATICS

*(For the candidates admitted during the academic year 2015 – 2016 onwards)*

Note:

(i) Part - A should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.

(ii) Part - B and Part - C should be answered in answer booklet.

Time: Three Hours | Max. Marks: 100

### PART – A (20 × 1 = 20 Marks)
Answer ALL Questions

1. $p \to q$ is logically equivalent to
(A) $\neg p \to \neg q$ (B) $\neg q \to \neg p$
(C) $p \to \neg q$ (D) $\neg p \to q$

2. $p \to (q \to p) =$
(A) T (B) F
(C) $p \to q$ (D) $p \wedge q$

3. The dual of $(\neg p \vee q) \wedge F$ is
(A) $(p \vee q) \wedge T$ (B) $(\neg p \wedge q) \vee F$
(C) $(p \to q) \wedge F$ (D) $(\neg p \to q) \vee T$

4. The negation of "every city in Tamilnadu is clean".
(A) Every city in Tamilnadu is not clean (B) Some city in Tamilnadu are not clean
(C) Every city in Tamilnadu is clean (D) Every city in Tamilnadu are dirty

5. A relation R from a non-empty set A to a non - empty set B is a
(A) Subset of A×B (B) Subset of A×A
(C) Subset of B×B (D) Subset of B×A

6. In a poset, the greatest and least element, if they exist, are
(A) Exactly two (B) Zero
(C) Unique (D) More than one

7. If $A = \{1,2,3\}$ and $B = \{a,b,c\}$ and $f = \{(1,a),(2,b),(3,c),(4,b)\}$, then the function is
(A) Neither 1 – 1 nor onto (B) 1 – 1 but not onto
(C) Onto but not 1 – 1 (D) Both 1 – 1 and onto

8. Assuming that repetitions are not permitted, how many four-digit numbers can be formed from 1, 2, 3, 4, 5, 6.....
(A) 360 (B) 15
(C) 20 (D) 460

9. The generating function of the sequence 1, 1, 1..... is given by

(A) $(1+x)^{-1}$ (B) $(1+x)^{-2}$

(C) $(1-x)^{-1}$ (D) $(1-x)^{-2}$

10. The recurrence relation of Fibonacci sequence is

(A) $F_n = F_{n-1} + F_{n-2}, n \geq 0$ (B) $F_n = F_{n-1} - F_{n-2}, n \geq 0$

(C) $F_n = F_{n-1} + F_{n-2}, n \geq 2$ (D) $F_n = F_{n-1} - F_{n-2}, n \geq 2$

11. The generator of a cyclic group $\{1,-1,i,-i\}$ is

(A) $1, i$ (B) $-1, -i$

(C) $i, -i$ (D) $1, -i$

12. The inverse element of any element "a" in the group of integers Z with the operator * defined by $a * b = a + b + 2 \forall a, b \in Z$ is

(A) $-2$ (B) 2

(C) a+4 (D) $-a-4$

13. A vertex with zero indegree is called as

(A) Sink (B) Source

(C) Terminal (D) Out degree

14. The value of the prefix expression $+-\uparrow 32 \uparrow 23/8-42$ is

(A) 0 (B) 5

(C) $-5$ (D) 2

15. A connected graph without any circuit is called as

(A) Leaf (B) Flower

(C) Tree (D) Loop

16. A maximum height of a 11 vertex binary tree is

(A) 4 (B) 5

(C) 3 (D) 6

17. All Boolean algebras of order $2^n$ are

(A) Isomorphic to each other (B) Homorphic to each other

(C) Non-isomorphic to each other (D) Non-homomorphic to each other

18. Dominance laws are

(A) $a+1=0$ and $a.0=1$ (B) $a+1=1$ and $a.0=0$

(C) $a+1=a$ and $a.0=a$ (D) $a+a=2a$ and $a.0=0$

19. In a lattice $\{L, \leq\}$ $a \vee b = b$

(A) if and only if $a \leq b$ (B) if and only if $a=b$

(C) if and only if $a \geq b$ (D) if and only if $b \leq b$

20. Every finite lattice is

(A) Bounded (B) Unbounded

(C) Infinite lattice (D) Uncountable lattice



## PART – B (5 × 4 = 20 Marks)
### Answer ANY FIVE Questions

21. Construct the truth table for
$(\neg p \to \neg q) \leftrightarrow (q \leftrightarrow r)$

22. Show that 'r' can be derived from the premises $p \vee q, p \to r, q \to r$.

23. Prove that $(A-C) \cap (C-B) = \phi$ analytically.

24. If R is the relation of $A = \{1, 2, 3\}$ such that $(a,b) \in R$ if and only if $a+b = even$, find the relational matrix $M_R, M_{R^{-1}}$ and $M_{R^2}$.

25. Show that $\{1,3,5,7\}$ is an abelian group under multiplication modulo 8.

26. If any disconnected graph has exactly two vertices of odd degree, show that there is a path joining these two vertices.

27. In a Boolean algebra, show that $(a+b)' = a'.b'$.

## PART – C (5 × 12 = 60 Marks)
### Answer ALL Questions

28. a. Show that the premises "one student in this class knows how to write programs in JAVA" and "everyone who known how to write programs in JAVA can get a high-paying job" imply the conclusion "someone in this class can get a high-paying job".

(OR)

b.i. Using CP-rule, derive $p \to (q \to s)$ form the premises $p \to (q \to r)$ and $q \to (r \to s)$. (8 Marks)

ii. Using mathematical induction, show that $\angle n \geq 2^{n-1}$ for $n \geq 1$. (4 Marks)

29.a.i Let $R = \{(1,1),(1,3),(1,5),(2,3),(2,4),(3,3),(3,5),(4,2),(4,4),(5,4)\}$ be a relation on a set $A = \{1,2,3,4,5\}$. Find transitive closure using Warshall's algorithm. (8 Marks)

ii. Draw the Hasse diagram for the partial ordering relation $\{(A,B) / A \subseteq B\}$ on a power set $P(S)$ where $S = \{a,b,c\}$. (4 Marks)

(OR)

b.i Show that composition of invertible functions is invertible.

ii. If we select 10 points in the interior of an equilateral triangle of side 1, show that there must be atleast two points whose distance apart is less than $\frac{1}{3}$.

## PART A – ANSWERS

1) B 2) A 3) D 4) B 5) A 6) C 7) C 8) A 9) C 10) C

11) C 12) D 13) B 14) B 15) C 16) B 17) A 18) B

19) A 20) A.

## PART B

21)

| $p$ | $q$ | $r$ | $\neg p$ | $\neg q$ | ① $\neg p \to \neg q$ | ② $q \leftrightarrow r$ | ① $\leftrightarrow$ ② |
|---|---|---|---|---|---|---|---|
| T | T | T | F | F | T | T | T |
| T | T | F | F | F | T | F | F |
| T | F | T | F | T | T | F | F |
| T | F | F | F | T | T | T | T |
| F | T | T | T | F | F | T | F |
| F | T | F | T | F | F | F | T |
| F | F | T | T | T | T | F | F |
| F | F | F | T | T | T | T | T |

2m 2m

22)

| | | |
|---|---|---|
| {1} | (1) $p \vee q$ | Rule P. |
| {1} | (2) $\neg p \to q$ | (1), T, $p \vee q \Leftrightarrow \neg p \to q$ |
| {3} | (3) $q \to r$ | Rule P |
| {1,3} | (4) $\neg p \to r$ | (2), (3), T, $p \to q, q \to r \Rightarrow p \to r$ |
| {5} | (5) $p \to r$ | Rule P |
| {5} | (6) $\neg r \to \neg p$ | (5), T, $p \to q \Leftrightarrow \neg q \to \neg p$ |
| {1,3,5} | (7) $\neg r \to r$ | (4), (6), T, $p \to q, q \to r \Rightarrow p \to r$ |
| {1,3,5} | (8) $r \vee r$ | (7), T, $p \to q \Leftrightarrow \neg p \vee q$ |
| {1,3,5} | (9) $r$ | (8), T, $p \vee p \Leftrightarrow p$. |

4m

Marks can be given if student proved this by using truth table method.

m

23) Let $x \in (A-C) \cap (C-B)$

$\Leftrightarrow x \in (A-C)$ and $x \in (C-B)$

$\Leftrightarrow (x \in A$ and $x \notin C)$ and $(x \in C$ and $x \notin B)$

$\Leftrightarrow x \in A$ and $(x \notin C$ and $(x \in C$ and $x \notin B)$

$\Leftrightarrow x \in A$ and $((x \notin C$ and $x \in C)$ and $x \notin B)$

$\Leftrightarrow x \in A$ and $(x \in \phi$ and $x \notin B)$

$\Leftrightarrow x \in A$ and $x \in \phi$.

$\Leftrightarrow x \in \phi$.

$\therefore (A-C) \cap (C-B) = \phi$.

4m

---

24) $R = \{(1,1)(1,3),(2,2),(3,1),(3,3)\}$ 1m

$$M_R = \begin{array}{c|c} & 1\ 2\ 3 \\ \begin{array}{c}1\\2\\3\end{array} & \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{array} \text{ (1m)} \quad M_{R^{-1}} = \begin{array}{c|c} & 1\ 2\ 3 \\ \begin{array}{c}1\\2\\3\end{array} & \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{array} \text{ (1m)} \quad M_{R^2} = M_{R \circ R} = \begin{array}{c|c} & 1\ 2\ 3 \\ \begin{array}{c}1\\2\\3\end{array} & \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{array} \text{ (1m)}$$

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25)

| $\times_8$ | [1] | [3] | [5] | [7] |
|---|---|---|---|---|
| [1] | [1] | [3] | [5] | [7] |
| [3] | [3] | [1] | [7] | [5] |
| [5] | [5] | [7] | [1] | [3] |
| [7] | [7] | [5] | [3] | [1] |

1m

closure & associativity, Commutative property → 1m

[1] – is the id elt as

$a * [1] = a$ for $a \in (Z_8, \times_8)$ 1m

$[1]^{-1} = [1]$ $[3]^{-1} = [3]$ $[5]^{-1} = [5]$, $[7]^{-1} = [7]$ 1m

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26) Let G be a disconnected graph. Let $v_1$ and $v_2$ be the vertices of odd degree. 1m since the graph is disconnected there will be components of the graph G which are connected. 1m Since the no of odd degree vertices in a connected graph is even, $v_1$ & $v_2$ belong to the same component of G. 1m since the component is connected there will be a path between $v_1$ and $v_2$ 1m

---

27) $(a+b) + a' \cdot b' = (a+b+a') \cdot (a+b+b')$
$= 1 \cdot 1 = 1$ 2m

$(a+b) \cdot (a' \cdot b') = (a \cdot a' \cdot b') + (b \cdot a' \cdot b')$
$= 0 + 0 = 0$ 2m

m

PART B

28) a)

$C(x)$: $x$ is a student in the class.

$J(x)$: $x$ knows Java programming.

$H(x)$: $x$ can get a high paying job. (3m)

$y \to$ a student of the class.

Premises are $C(y) \wedge J(y)$, $(x)(J(x) \to H(x))$.

Conclusion is $(\exists x)(C(x) \wedge H(x))$ (2m)

| | | | |
|---|---|---|---|
| {1} | (1) $C(y) \wedge J(y)$ | Rule P | |
| {1} | (2) $C(y)$ | (1), T, $P \wedge Q \Rightarrow P$ | |
| {3} | (3) $(x)(J(x) \to H(x))$ | P | |
| {3} | (4) $J(y) \to H(y)$ | US, (3) | (7m) |
| {1} | (5) $J(y)$ | (1), T, $P \wedge Q \Rightarrow Q$ | |
| {1, 3} | (6) $H(y)$ | (4), (5), T, $P, P \to Q \Rightarrow Q$ | |
| {1, 3} | (7) $C(y) \wedge H(y)$ | (2), (6), $P, Q \Rightarrow P \wedge Q$ | |
| {1, 3} | (8) $(\exists x)(C(x) \wedge H(x))$ | EG. | |

b)(i)

| | | | |
|---|---|---|---|
| {1} | (1) $p$ | Assumed premise | |
| {2} | (2) $p \to (q \to r)$ | Rule P | |
| {1, 2} | (3) $q \to r$ | (1), (2), $P, P \to Q \Rightarrow Q$ | (4m) |
| {4} | (4) $q \to (r \to s)$ | Rule P | |
| {4} | (5) $r \to (q \to s)$ | (4), T, $\neg P \vee (\neg Q \vee S) \Rightarrow \neg Q \vee (\neg P \vee S)$ | |
| {1, 2, 4} | (6) $q \to (q \to s)$ | (3), (5), T, $P \to Q, Q \to R \Rightarrow P \to R$. | |
| {1, 2, 4} | (7) $\neg q \vee (\neg q \vee s)$<br>$\neg q \vee s$ | (6), T, $P \to Q \Leftrightarrow \neg P \vee Q$ & associative property. | (4m) |
| {1, 2, 4} | (8) $q \to s$. | (7), T, $P \to Q \Leftrightarrow \neg P \vee Q$. | |
| {2, 4} | (9) $p \to (q \to s)$ | Rule CP | |

b) (ii) $P(n)$: $n! \geq 2^{n-1}$

$P(1)$: $1! \geq 2^0 = 1$ true. Assume $P(k)$ true $k! \geq 2^{k-1}$ (2m)

Consider $(k+1)! = (k+1)k! \geq (k+1)2^{k-1}$

$= 2 \cdot 2^{k-1} = 2^k$ (as $k+1 \geq 2$) (2m).

m2

29) a)

$$W_0 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 1 & 0 & 1 & 0 & 1 \\ 2 & 0 & 0 & 1 & 1 & 0 \\ 3 & 0 & 0 & 1 & 0 & 1 \\ 4 & 0 & 1 & 0 & 1 & 0 \\ 5 & 0 & 0 & 0 & 1 & 0 \end{array} \longrightarrow \text{(2M)}$$

$W_1 = W_0 \longrightarrow$ (1m)

(2) 4 3, 4

$$W_2 = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \text{(1m)}$$

(3) 1, 2, 3 3, 5

$$W_3 = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \text{(1m)}$$

(4) 2, 4, 5 2, 4

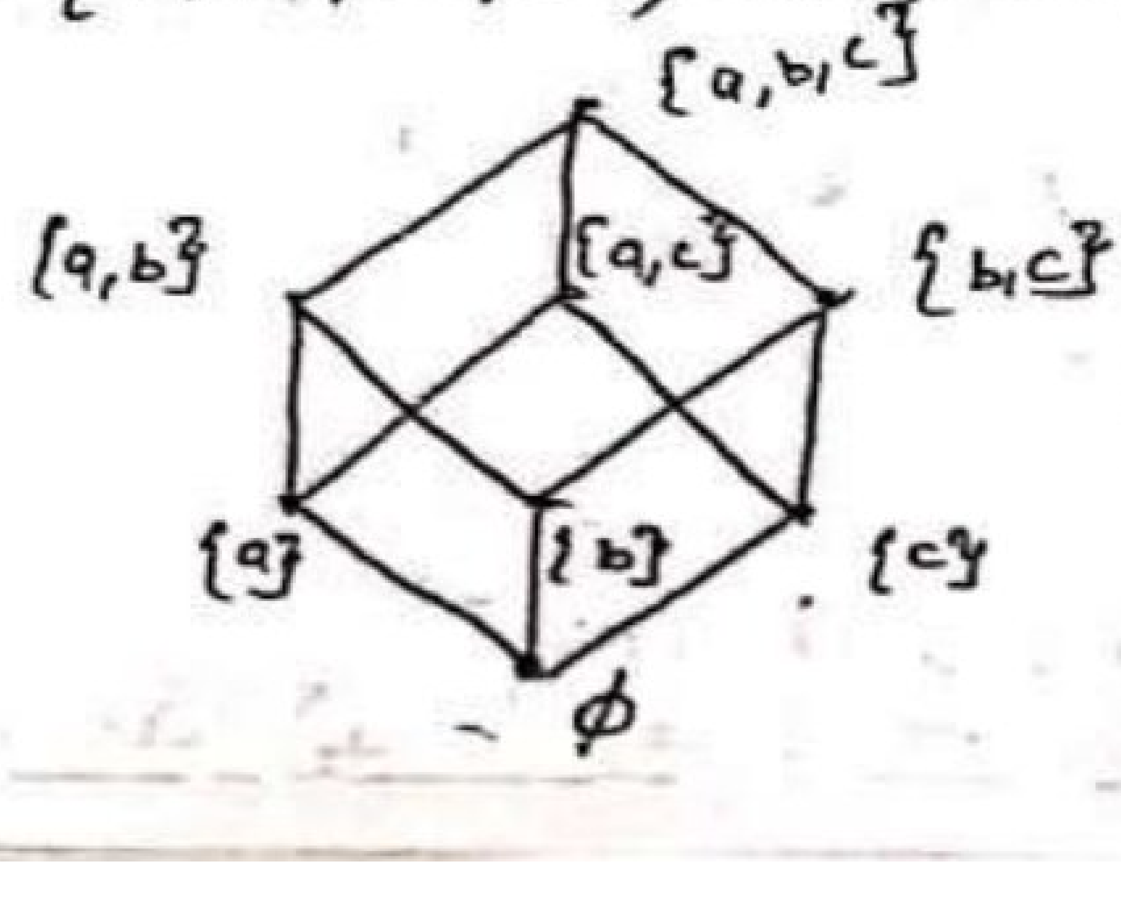
$$W_4 = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \text{(1m)}$$

(5) 1, 2, 3 2, 4

$$W_5 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & \phi & 1 & \phi \\ 0 & 1 & \phi & 1 & q \end{bmatrix} \text{(1m)}$$

Transitive closure = {(1,1) (1,2) (1,3) (1,4) (1,5) (2,2) (2,3) (2,4) (2,5) (3,2) (3,3) (3,4) (3,5) (4,2) (4,4) (5,2) (5,4)} (1m)

29) b) ii) $P(S) = \{\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\} \longrightarrow$ (1m)




(3m)

4

m

b) i) Let $f$ and $g$ be two invertible fns

$\therefore$ $f: A \to B$, $g: B \to C$ are both bijective fns

To show that $g \circ f : A \to C$ is invertible. (i.e) we have to show that $g \circ f$ is 1-1 and onto. $\longrightarrow$ 1m

Let $a_1, a_2 \in A$, such that $(g \circ f)(a_1) = (g \circ f)(a_2)$

$$g(f(a_1)) = g(f(a_2)) \quad (g \text{ is } 1\text{-}1)$$
$$f(a_1) = f(a_2)$$
$$a_1 = a_2 \quad (\text{as } f \text{ is } 1\text{-}1)$$

$\therefore$ $g \circ f$ is 1-1. 2m

Let $c \in C$.

Since $g$ is onto, there is an element $b \in B$ such that $c = g(b)$

Since $f$ is onto, there is an element $a \in A$ such that $b = f(a)$.

$$c = g(b) = g(f(a)) = (g \circ f)(a).$$

$\therefore$ for $c \in C$ $\exists$ a preimage $a$ in $A$ under $g \circ f$.

$\therefore$ $g \circ f$ is onto. 3m

$\therefore$ $g \circ f$ is invertible.

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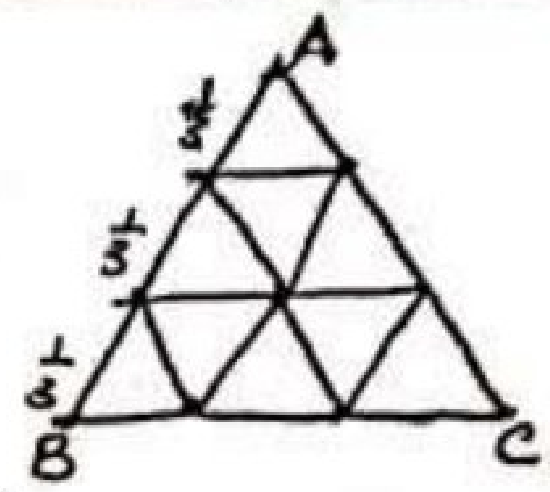
b ii) Divide the $\Delta ABC$ into 9 subtriangles of sidelength $1/3$.




2m

The 9 subtriangles are treated as 9 pigeonholes and 10 interior pts are treated as pigeons. 2m

Thus by pigeonhole principle, one hole must contain two interior points. Thus the distance b/w any two interior points in the triangle is less than $1/3$. 2m

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30) a) Characteristic eqn is $r^2 - 4r + 4 = 0$.

$$r = 2, 2.$$

$$a_n^{(h)} = (c_1 + c_2 n)(2)^n$$ 3m

Assume $a_n^{(p)}$ be $n^2(An+B)2^n = (An^3+Bn^2)2^n$. $\longrightarrow$ 1m

$$(An^3+Bn^2)2^n - 4\left(A(n-1)^3 + B(n-1)^2\right)2^{n-1} + 4\left[A(n-2)^3 + B(n-2)^2\right]2^{n-2} = (n+1)2^n$$

$\underline{m}$

$$2^n\left\{An^3+Bn^2-2\left[A(n-1)^3+B(n-1)^2\right]+\left[A(n-2)^3+B(n-2)^2\right]\right\}=(n+1)\,2^n$$

$$\cancel{An^3}+\cancel{Bn^2}-\cancel{2An^3}+2A-6An+\cancel{6An^2}-\cancel{2Bn^2}+4Bn-2B+\cancel{An^3}$$

$$-8A+12An-\cancel{6An^2}+\cancel{Bn^2}-4Bn+4B=n+1$$

$$\left[-6A+4B+12A-4B\right]n+(2A-2B-8A+4B)=n+1$$

$$6A=1 \Rightarrow A=1/6 \qquad -6A+2B=1 \Rightarrow B=1.$$

4m

$$a_n^{(P)}=\left(n^2+\frac{n^3}{6}\right)2^n.$$

Hence $a_n=(c_1+c_2n)\,2^n+\left(n^2+\frac{n^3}{6}\right)2^n \longrightarrow$ 1m

$a_0=0 \Rightarrow \boxed{0=c_1}$

$a_1=1 \Rightarrow 1=2c_2+2\left(1+\frac{1}{6}\right)$

$1=2c_2+2\left(\frac{7}{6}\right) \Rightarrow 2c_2=1-\frac{7}{3}=-\frac{4}{3}$

$\boxed{c_2=-\frac{2}{3}}$

2m

$$\therefore a_n=\left(-\frac{2}{3}n+n^2+\frac{n^3}{6}\right)2^n \longrightarrow \text{1m}$$

---

30) b)(i) $a_n-3a_{n-1}=1,\quad n\geq 1\quad a_0=1$

$$\sum_{n=1}^{\infty}a_nz^n-3\sum_{n=1}^{\infty}a_{n-1}z^n=\sum_{n=1}^{\infty}z^n$$

$$(G(z)-a_0)-3z\,G(z)=\frac{z}{1-z}$$

$$G(z)-1-3z\,G(z)=\frac{z}{1-z}$$

$$G(z)(1-3z)=\frac{z}{1-z}+1$$

$$=\frac{1}{1-z}$$

2m

$$G(z)=\frac{1}{(1-z)(1-3z)}=\frac{-\frac{1}{2}}{1-z}+\frac{3/2}{1-3z}$$

$$a_n=-\frac{1}{2}(1)^n+\frac{3}{2}(3)^n$$

2m