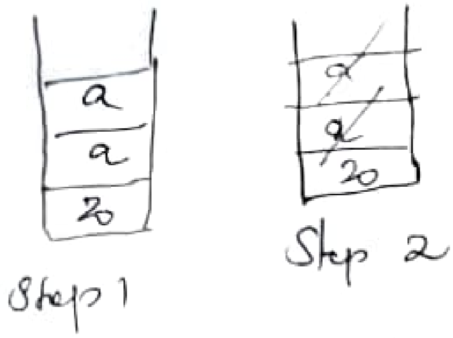
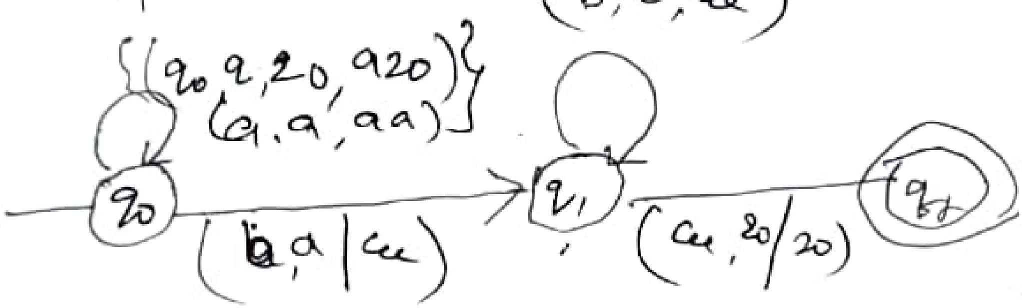
Ex:

$a^n b^n \mid n \geq 1$

ex:

aabb





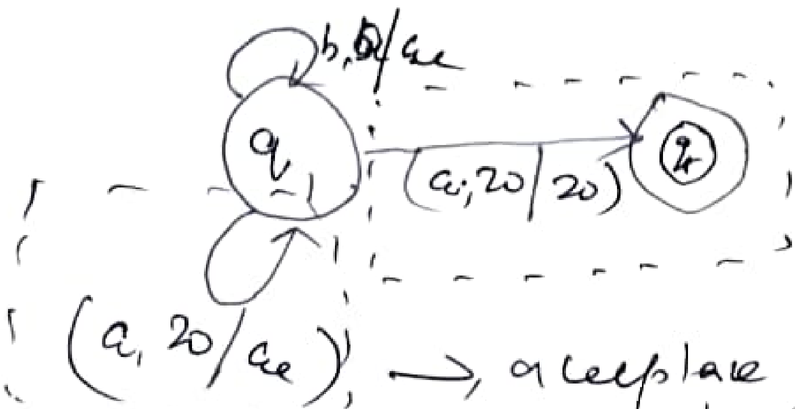
$\delta(q_0, a, Z_0) = (q_0, aZ_0)$

$\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, b, a) = (q_1, \epsilon)$

$\delta(q_1, b, a) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, Z_0) = (q_f, Z_0)$ (or) (q_1, ϵ)



→ acceptance by final state

→ acceptance by empty stack.

Equivalance of Acceptance

From empty stack to Final State.

Theorem

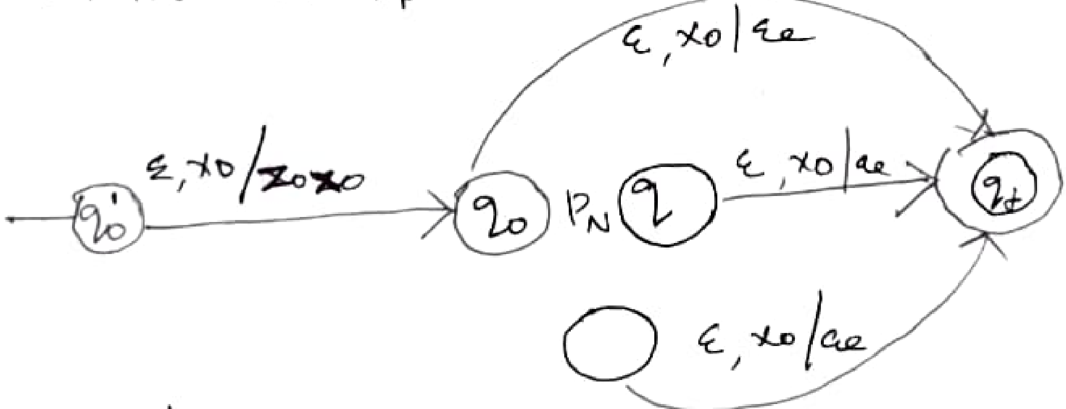
If $L = N(P_N)$ for some PDA $P_N = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$ then there is a PDA P_F such that $L = L(P_F)$

Proof :-

The theorem states that if there is a PDA which has acceptance by empty stack then there should be a PDA which also has acceptance by final state.

$P_N = \{Q, \Sigma, \Gamma, \delta_N, q_0, z_0, F)$ be a PDA

It is planned to have simulate P_N & find when P_F empties its stack.



2

(i) $\delta'(q_0', \varepsilon, x_0) = \{(q_0, z_0 x_0)\}$

(ii) For all q in Q, a in $\Sigma \cup \{\varepsilon\}$ and Z in Γ,

$\delta'(q, a, Z) = \delta(q, a, Z)$

(iii) For all q in Q, $\delta'(q, \varepsilon, x_0)$ contain (q_f, ε)

To Prove

w is in $L(P_F)$ if and only if w is in $N(P_N)$

If Part

$$(q_0', w, X_0) = \vdash_{P_F} (q_0, w, Z_0 X_0) \quad \text{Rule 1}$$

$$= \vdash (q, \varepsilon, X_0) \quad \text{Rule 2}$$

$$= \vdash_{P_F} (q_f, \varepsilon, \varepsilon) \quad \text{Rule 3}$$

Since X_0 is the bottom of the stack,

$$(q_0', w, X_0) \vdash (q, \varepsilon, X_0)$$

This is P_F accept w by final state

Ex:

Design a PDA that Process the if else loops using the acceptance equivalence of empty stack to final state.

Sol:

In PL having equal no. of if and else.

$Z \rightarrow$ in the stack symbol used to count the no. of if

if $\rightarrow$ i

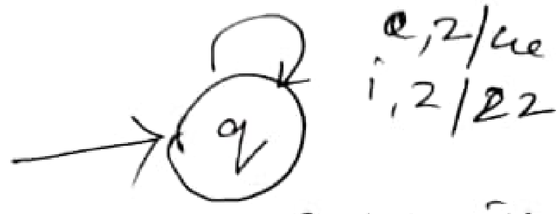
else $-$ e.

Let $P_N = (\{q\}, \{i, e\}, (Z), \delta, q, Z)$

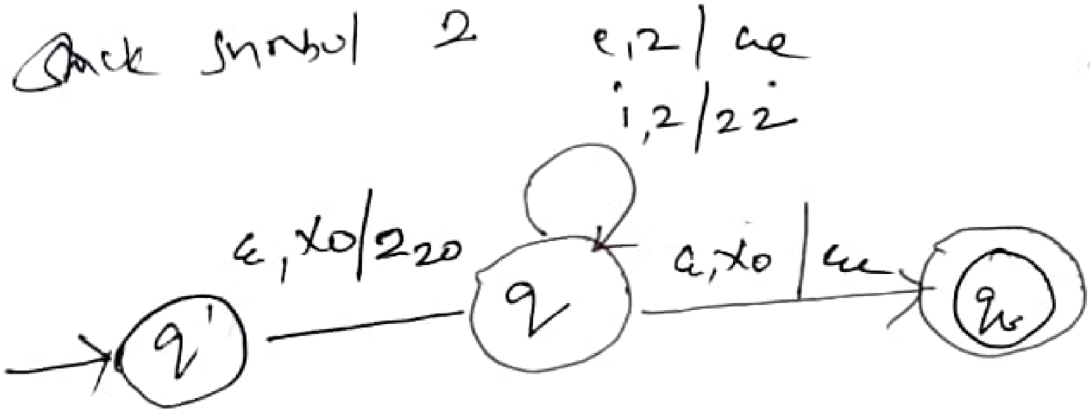
δ is given by

$$\delta(q, i, z) = \{q, iz\}$$

$$\delta(q, e, z) = (q, \varepsilon)$$



If i is encountered Pushed it on the stack
when e is encountered pop the stack, then
i is on the stack till read the initial
stack symbol z



Transition func

1. $\delta'(q', \varepsilon, x_0) = \{q, zx_0\}$ → bottom of stack

2. $\delta'(q, i, z) = \{q, zz\}$ → P, Pushes

3. $\delta'(q, e, z) = \{q, \varepsilon\}$ → P, Pops z

4. $\delta'(q', a, x_0) = \{q_f, \varepsilon, \varepsilon\}$