

UNIT - IIntroduction to Computer Vision

It is a field of AI that enables computers and systems to derive meaningful information from digital images, videos and other visual inputs and take actions or make recommendations based on that information.

Applications of CV

1. Optical character recognition (OCR)
  - Reading handwritten postal codes on letters and automatic number plate recognition (ANPR)
2. Machine Inspection.
  - ① Machine parts inspection for quality assurance using Stereo Vision → process of extracting 3D information from digital images taken by 2 cameras displaced horizontally from one another to obtain 2 different views of the same scene.

2) looking for defects in steel castings using X-ray vision.

3. Retail: Object recognition for automated Checkout lanes. (A place where a product or service can be purchased).

4. 3D Model Building (Photogrammetry)

→ Fully automated construction taken from any photograph taken by aerial photographs using rocket maps (allows aircraft to create that layer photo-based images).

5. Medical Imaging: registering pre-operative and intra-operative imagery. or performing long-term studies of people's brain morphology. (Study of size, shape & structure).

6. Automotive Safety: Detecting unexpected obstacles in fully automated driving.

## 7. Match more.

Merging computer-generated imagery (CGI) with live action footage by tracking feature points in the source video to estimate 3D camera motion and shape of the environment.

## 8. Motion Capture (MoCap)

Using retro-reflective markers (which reflect light back in the direction it came) viewed from multiple cameras or other vision-based techniques to capture actors for computer animation.

## 9. Surveillance

Monitoring for intruders, traffic and monitoring highway drowning victims.

Analyzing pools for fingerprint recognition and Biometrics: For automatic access authentication.

## 11. stitching

turning overlapping photos into single  
seamless view.

## 12. Exposure bracketing:

Merging multiple exposures taken  
under challenging lighting conditions into a  
single perfectly exposed image.

## 13. Morphing:

Turning a picture of one face  
into another.

## 14. 3D Modeling

Converting one or more snapshots into  
a 3D model of the object

## 15. Face detection:

For improved camera focusing  
as well as more relevant image searching.

## 16. Visual authentication

for automatic log in into the  
system.

## 17. Video Match move & stabilization

Inserting 2D pictures or 3D models  
into videos by automatically tracking nearby  
reference points

- ### Image Formation
- Geometric Primitives and transformations.
  - Geometric primitives form the basic building blocks used to approximate 3D shapes.
  - Basic \* Points, lines & planes. \* Curves, surfaces & volumes.
  - Pixel denoted by a pair of values denoted by a point in an image.
  - 2D points.
  - Point at infinity denoted by a pair of values denoted by a point in an image both magnitudes at the end of luminosity point sanitizer ideal point real world.
  - $x = \begin{bmatrix} x \\ y \end{bmatrix}$   $\rightarrow$  column vector. co-ordinates  $\bar{x} = (x, y, w) \in \mathbb{P}^2$ .
  - Also represented using homogeneous coordinates to divide a bulk into an inhomogeneous converted back into an inhomogeneous by to divide a bulk element by the last element.
  - A homogeneous vector  $\bar{x}$  can be represented as  $\bar{x} / w$  height/affinity.

$$x_5 = \tilde{w}(x, y, z) = (x, y, z)$$

where  $\bar{x} = (x_1, y_1)$  is the augmented point or

If  $B = 0$  one called ideal points at infinity (pm will look).

2D Lines

- $\rightarrow$  represented using homogeneous co-ordinates  $\vec{r} = (a, b, c)$  and  $\vec{m} = (m, n, l)$

Coefficients

a/b

۲۱۷

1

$\chi$  intercept is (0,0)

$\alpha$  intercept is found by setting  $x=0$  in the equation of the line. The  $y$ -intercept is found by point where a line crosses the  $y$ -axis.  $\therefore y = mx + c$

$y = C_1 b$  gives the equation

~~the~~ contains  $y = mx + c$

Two are variables  $x$  &  $y$ .

Y-axis intercept is found by setting  $x = 0$ .  
 X-intercept is found by setting  $y = 0$ .  
 Line crosses the x-axis at  $x = -\frac{c}{m}$ .

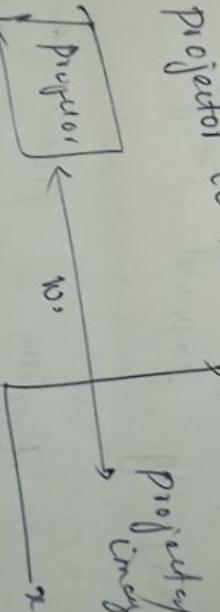
## Face detection

Visual authentication.

for improved camera security -  
image resizing  $\rightarrow$  to allow ~~more~~ <sup>more</sup> inside  
the room by detecting the face.

## Homogeneous Co-ordinates

Co-ordinates in Projective space  
Co-ordinates in Projective space (extra dimension w)  
are homogeneous co-ordinates (extra dimension w)  
w dimension is the distance from  
the projector to the screen



If you move the projector closer to  
the screen, the whole 2D image becomes  
smaller. If you move the projector away  
from the screen, the 2D image becomes  
larger. The value of w affects the size  
of the image.

For  $NO3D$  in 3D  $w$  is always = 1.

## operations

1. **Stitching** → turning overlapping photos into a single seamlessly stitched panorama (wide angle picture)  
Panorama (wide position of an area)

### 2. **Exposure bracketing**

→ merging multiple exposures taken under challenging lighting conditions.  
Strong sunlight and shadows) into a perfectly exposed image.

### 3. **Morphing**

turning a picture ~~into~~ into another picture, using a seamless morph transition.

### 4. **3D modeling**

Converting one or more snapshots into a 3D model of the object

### 5. **Video matching & Stabilization**

Inserting 2D pictures or 3D models into the videos by automatically tracking nearby reference points. Or using motion estimate to remove shake from the video.

2D Conics.

Conic Sections  $\rightarrow$  intersection of a plane and a 3D Cone

$$\tilde{x}^T Q \tilde{x} = 0$$

2D Plane

It is a flat, 2D surface that extends indefinitely (lengthwidth)

3D points.

In homogeneous co-ordinates

$$\tilde{x} = (x_1, y_1, z_1) \in \mathbb{R}^3$$

Homogeneous co-ordinates

$$\tilde{x} = (\tilde{x}, \tilde{y}, \tilde{z}, \tilde{w}) \in \mathbb{P}^3$$

$$\tilde{x} = (x_1, y_1, z_1, 1) \text{ with } \tilde{x} = \tilde{w}^{-1} x$$

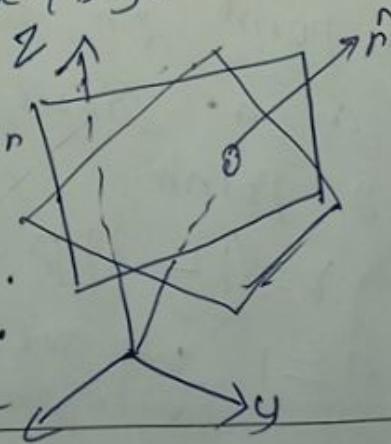
3D planes.

Homogeneous co-ordinate  
 $\tilde{m} = (a_1, b_1, c_1, d)$

Plane equation

$$\tilde{x} \cdot \tilde{m} = ax + by + cz + d = 0.$$

- 3 axes which intersects at the origin
- $x_1, y_1, z_1$  and  $s$  are mutually perpendicular to each other
- Called as hyperplane

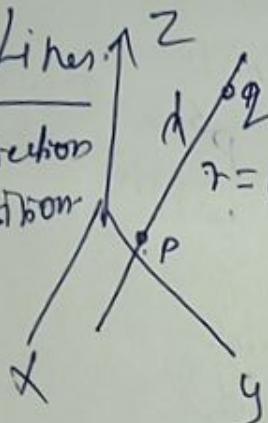


3D plane

3 axes which intersect at the origin.  $x \& y \& z$  axes are mutually perpendicular each other. 3D plane is called a hyperplane.

3D Lines

- need direction
- need position



$$r = (1-d)p + dq \quad \vec{r} = \vec{p} + \lambda \vec{q}$$

A special case if  $q$  is at infinity  
 $\vec{q} = (d_x^1, d_y^1, d_z^1, 0) = (d, 0)$

$\lambda$  is the direction of the line.

$$\vec{r} = \vec{p} + \lambda \vec{d}$$

Consider 2 points on the line  $(P, q)$

Any other point on the line can be expressed as a linear combination of these 2 points.

$$r = (1 - \lambda)p + \lambda q$$

$0 \leq \lambda \leq 1$  we get line segment  
 covering the points joining  $P \& q$ .

$\lambda$  is called as parameter

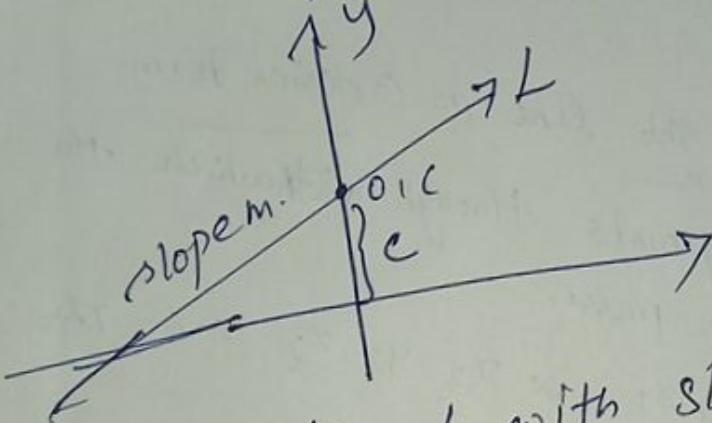
It takes the value  $\frac{1}{2}$  to the indicates any point is left side or right for one point side of the given point

Overall  
the points  
in the line

Line equation

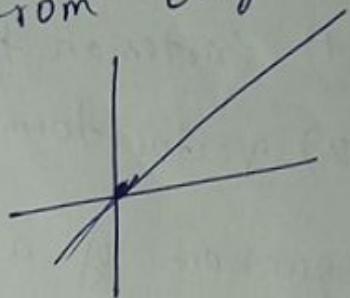
$$y = mx + c$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



slope-intercept form

:  $y = mx$  means  
the line starts  
from origin.

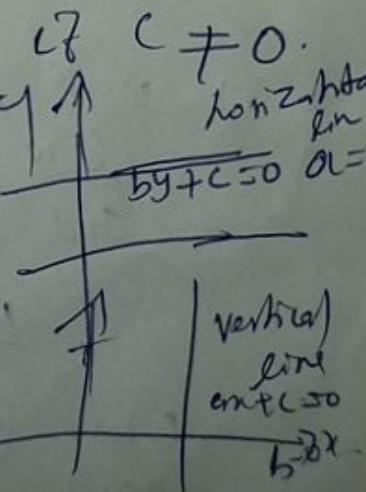
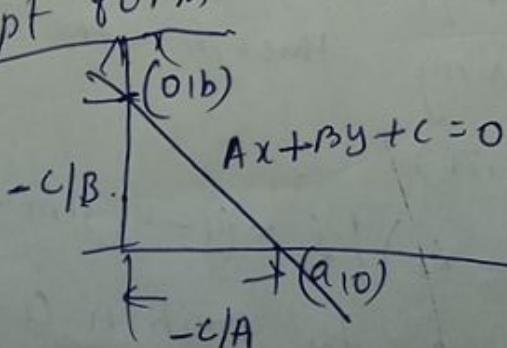


Line  $L$  with slope  $m$  cuts the  $x$ -axis at a distance  $c$  from the origin where the distance  $c$  is called the  $y$ -intercept of the line  $L$ .

Therefore the point  $(x, y)$  on the line with slope  $m$  and  $y$ -intercept  $c$  lies on the line if and only if

$$y = mx + c$$

Intercept form



## 3D Line

3D lines are represented in 2 forms.

1) Cartesian form

2) Vector form.

Equation of a straight line in Cartesian form

+ segment 2 points through which the straight line passes

$x_1, y_1, z_1 \propto x_2, y_2, z_2$  are the

Position Co-ordinates.

To obtain equation

1) Find the Direction Ratios (DR's) by taking the difference of the corresponding position co-ordinates of the 2 given points

$$l = (x_2 - x_1) \quad m = (y_2 - y_1) \quad n = (z_2 - z_1)$$

l, m, n are the DR's.

2) Choose either of the 2 given points say we choose  $(x_1, y_1, z_1)$ .

3) write the required equation of the straight line passing through the points  $(x_1, y_1, z_1)$  and  $(x_2, y_2, z_2)$

$$L : (x - x_1) / l = (y - y_1) / m = (z - z_1) / n$$

Where  $(x, y, z)$  are the position co-ordinates of any variable point lying on the straight line.

## 2D Transformations.

Translation  $\rightarrow$  moves an object to a different position on the screen.

$$\bar{x}^1 = \bar{x} + t$$

$$\bar{x}^1 = [I \quad t] \bar{x}$$

where  $I$  is the  $(2 \times 2)$  identity matrix.

$$\begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \\ 1 \end{bmatrix}$$

Rotation + translation

\* also known as 2D rigid body motion  
or 2D Euclidean transformation

$$\bar{x}^1 = R\bar{x} + t$$

$$\bar{x}^1 = [R \quad t] \bar{x}$$

where  $R = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$

Scaled rotation.

+ Known as Similarity transform

$$x' = S R x + t \rightarrow \text{where } S \text{ is an arbitrary scale factor.}$$

- Scaling may be used to increase or reduce the size of object.
- If scaling factor  $> 1$ , then the object size is increased.
- If scaling factor  $< 1$ , then the object size is reduced.

Affine.

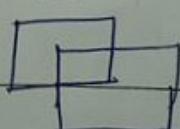
## Affine Translation

$$x' = A \bar{x}$$

$$x' = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ d_x + e_x & d_y + e_y & f \end{bmatrix} \bar{x}$$

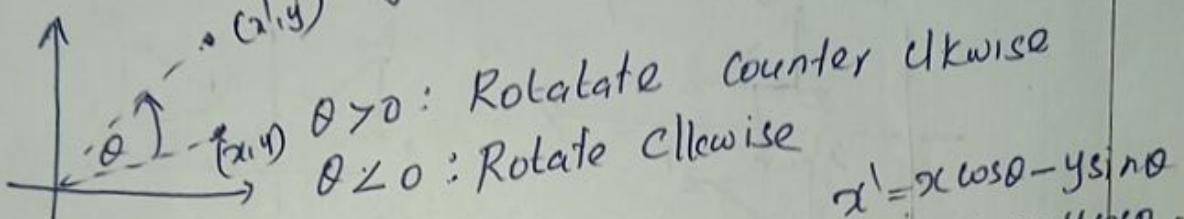
Parallel lines remain parallel under affine transformations, used to correct for geometric distortions or deformations that occur with non-ideal camera angles.

Translation



Rotation  $\rightarrow$  Circular Path.

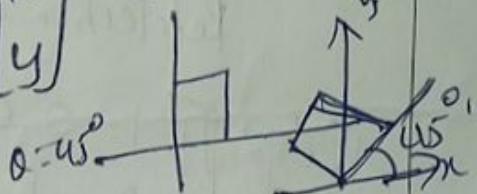
- Rotation axis
- Rotation direction - clockwise or counter clockwise
- Rotation angle



$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$x' = x \cos\theta - y \sin\theta$$

$$y' = x \sin\theta + y \cos\theta$$



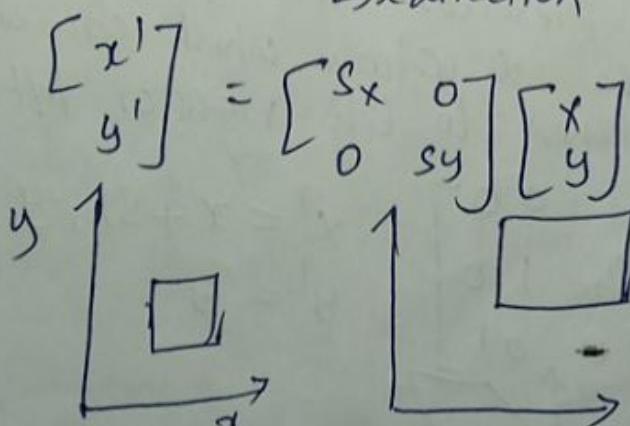
Scaling

- Changes the size of the object

$$x' = x \cdot S_x \quad y' = y \cdot S_y$$

$\hookrightarrow$  x direction

$\hookrightarrow$  y direction

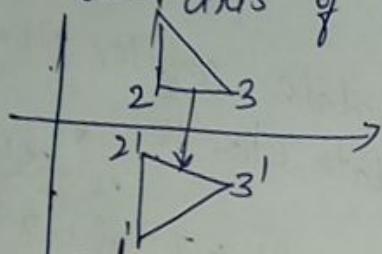


For uniform scaling  
 $S_x$  &  $S_y$  should have same value.

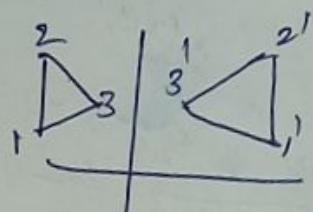
Value  $< 1$  | Reduce the size of object  
 Value  $> 1$  | Produce enlargement

Reflection:

- ^ Produces a mirror image relative to an axis of reflection.



Reflection about x-axis



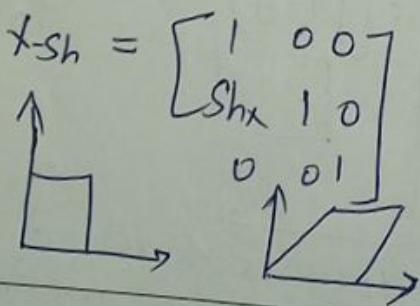
Reflection about y-axis

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Shear:

- ^ X Shear preserves the y coordinates changing the x values which causes vertical lines to tilt right or left



$$x' = x + S_{hx}y$$

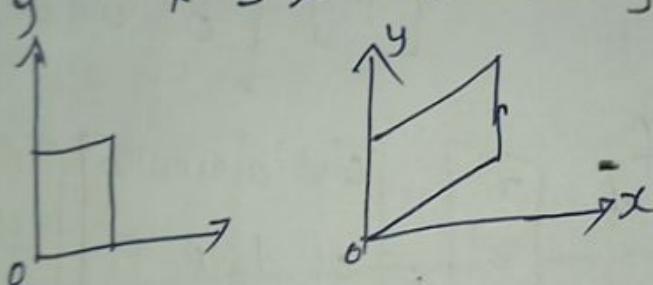
$$y' = y$$

y Shear

\* preserves the x-coordinates, but changes the y values.

$$y_{sh} = \begin{bmatrix} 1 & sh_y & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

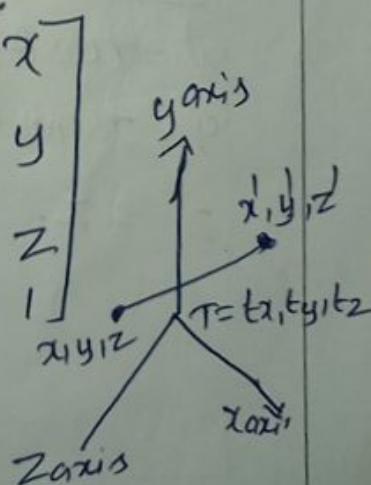
$$x' = x \quad y' = y + sh_y \cdot x$$

3D Transformation1. 3D Translation  $P \rightarrow \text{point}$ 

$$P' = T \cdot P$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & tx \\ 0 & 1 & 0 & ty \\ 0 & 0 & 1 & tz \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$x' = x + tx \quad y' = y + ty \quad z' = z + tz$$



3D RotationX-axis Rotation

$$x' = x$$

$$y' = y \cos\theta - z \sin\theta$$

$$z' = y \sin\theta + z \cos\theta$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Y-axis rotation

$$x' = x \cos\theta + z \sin\theta$$

$$y' = y$$

$$z' = z \cos\theta - x \sin\theta$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Z-axis Rotation

$$x' = x \cos\theta - y \sin\theta$$

$$y' = x \sin\theta + y \cos\theta$$

$$z' = z$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

3D Scaling

$$x' = x \cdot s_x$$

$$y' = y \cdot s_y$$

$$z' = z \cdot s_z$$

3D Reflection

Reflection about x-axis

$$x' = x \quad y' = -y \quad z' = -z$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Reflection about y-axis

$$y' = y \quad x' = -x \quad z' = -z$$

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Reflection about z-axis

$$x' = -x \quad y' = -y \quad z' = z$$

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Course Code: 18CSE390T

Subject Name: COMPUTER VISION

3D Shearing

Shearing x-direction

$$x' = x$$

$$y' = y + s_x \cdot x$$

$$z' = z + s_z \cdot x$$

Shearing y-direction

$$x' = x + s_y \cdot y$$

$$y' = y$$

$$z' = z + s_z \cdot y$$

Shearing z-direction

$$x' = x + s_x \cdot z$$

$$y' = y + s_y \cdot z$$

$$z' = z$$

3D to 2D Projection

Projection is a technique which is used to transform a 3D object into a 2D plane.

Projection requires

1. Projection Plane
2. Projection Reference Point (PRP)

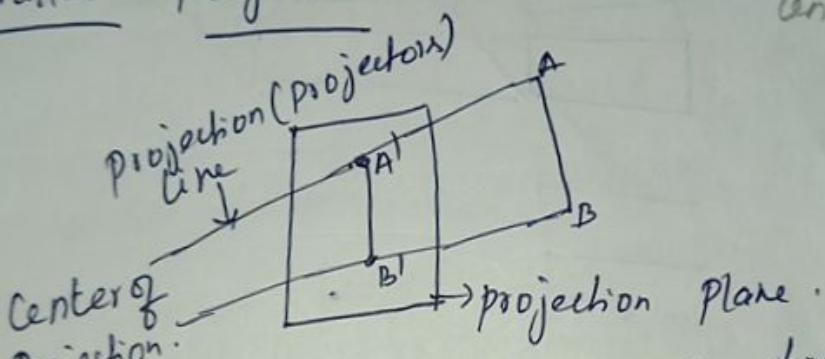
or center of projection (CRP)

Projected View  $\rightarrow$  Calculating the intersection of projection lines with the view plane

Types

- 1) Parallel Projection
- 2) Perspective Projection

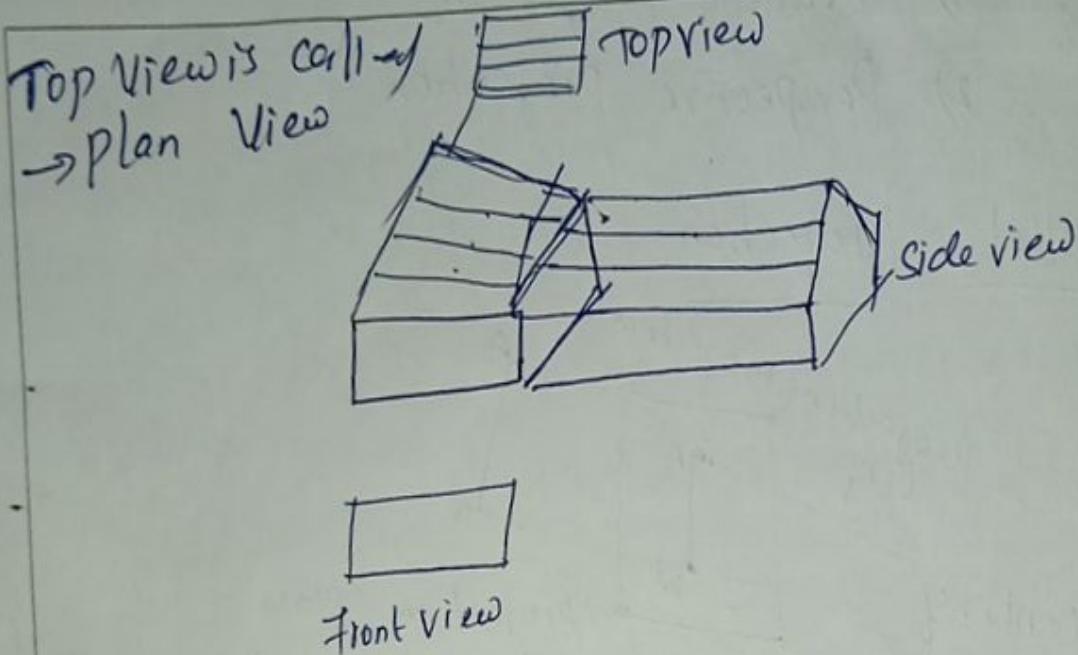
Transform  
3D to 2D plane.  
Topic  
\* projection plane  
\* projection  
reference point  
center of projection

1. Parallel Projection

- \* Co-ordinate positions are transformed to the view plane along parallel lines.
- \* Projection lines are parallel to each other.
- \* Projection lines are extended from the object & intersect the view plane.

Types:1. Orthographic

- \* Projection lines are parallel to each other & also perpendicular to the plane.
- \* Used to create different views of given object. Produces front, top and side views.



## 2. OBLIQUE

Projection lines are parallel to each other but not perpendicular to view planes.

### CAVALIER:

In this projection lines makes angle of 30 degree with the view plane & there is no change in length of projection.

### CABINET:

In this projection lines makes angle of 45 degree with the view plane & length of projected line will reduce.

Orthography Projection  $\rightarrow$  simply drops Z component of 3D co-ordinate but we keep w component.

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \\ 1 \end{bmatrix}$$

$$x' = x$$

$$y' = y$$

$$w' = w$$

$$x = [I_{2 \times 2} | 0] P$$

2D point

$\downarrow$   
3D point

\* It is an approximate model for long focal length lenses and objects whose depth is shallow relative to their distance to the camera.

Long focal length lens.

The focal length of a lens is determined when the lens is focused to infinity. Lens focal length tells us the angle of view  $\rightarrow$  (i.e.) how much of the scene will be captured and the magnification - how large individual elements will be.

The longer the focal length the narrower the angle of view and for the higher the magnification.

Shorter the focal length  $\rightarrow$  the wider the angle of view and the greater the area captured.

Longer the focal length (55mm) the smaller the angle & the larger the subject appears to be.

In practice world co-ordinates (which may measure dimensions in meters) need to be scaled to fit onto an image sensor (measured in millimetres (or in <sup>millimetre</sup> pixels)).

So scaled orthography is used.

$$x = \begin{bmatrix} S & I \\ 0 & 1 \end{bmatrix} p.$$

\* The scaling can be the same for all parts of the scene.  
\* or different for objects that are being modeled independently.

\* The scaling can vary from frame to frame when estimation structure from motion  $\rightarrow$  as the object approaches the camera.

Scaled Orthography used for reconstructing the 3D shape of objects for a way from the camera.

## Perspective

In homogeneous co-ordinates,

$$\tilde{x} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tilde{p}$$

We drop the  $w$  component of  $\tilde{p}$ .  
After projection, it is not possible  
to recover the distance of the 3D point  
from the image.

Perspective projection is done  
in 2 step projection process.

1) First projects 3D co-ordinates  
into normalized device co-ordinates.

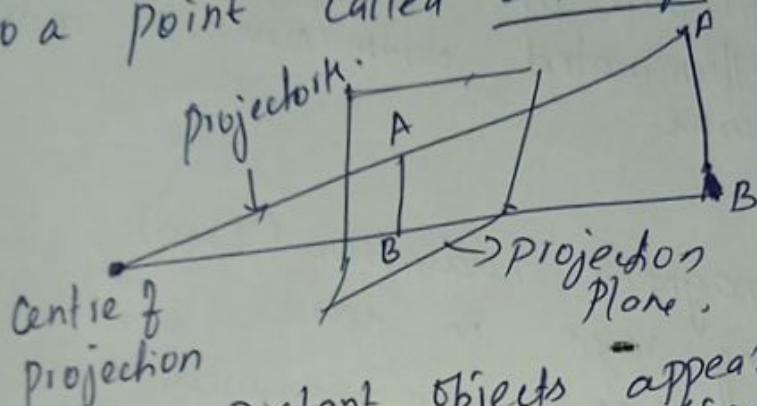
(2) 3D coordinates represent the  
2D position on the screen with  
 $x \& y$  in  $[-1, +1]$  and  
depth  $z$   $[0, 1]$   $x \rightarrow$  right  
 $y \rightarrow$  up

3) use scaling factor:  $z \rightarrow$  forward/backward.  
rescales these co-ordinates  
to integer pixel co-ordinates using  
Video-port transformation

↳ area on the screen  $\rightarrow$  device co-ordinates  
where graphics (S to be displayed)

Perspective Projection

\* Object Position are transformed to the view plane along lines that converge to a point called center of projection.



\* Distant objects appear smaller than nearer objects.

Type:

- 1) One point perspective projection  $\rightarrow$  x, y axis parallel to projection plane.
- 2) Two point perspective projection  $\rightarrow$  z axis remains parallel to projection plane.
- 3) Three point perspective projection  $\rightarrow$  no any principle axis which is parallel to projection plane  $\rightarrow$  all three axis intersects with projection plane.

\* Points are projected onto the image plane by dividing them by their z component. Using Inhomogeneous

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = P_z(P) = \begin{bmatrix} x/z \\ y/z \end{bmatrix}$$

## Lighting, Reflectance & shading.

To produce an image, the scene must be illuminated with one or more light sources.

### Light

Electromagnetic radiation (EMR) moving along rays in space.

+  $R(\lambda)$  is EMR measured in units of power (watts)  $\lambda \rightarrow$  wavelength

### Light field

+ Radiation arriving at every point in space and from every direction.

Light sources can be divided into

### point

+ area light sources

+ A point light source originates at a single location in space. (light bulb)

+ In addition to location  $\rightarrow$  It has

In Electromagnetic spectrum  $\lambda$  & intensity  $\rightarrow$  amount of light. range wavelength  $\rightarrow$  color spectrum. (distribution over

human eye can detect wavelengths  $L(\lambda)$ . Eg table lamp

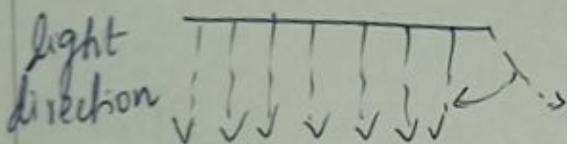
+ Modelled using single location  $(x_1, y_1, z_1)$

from 380 to 700 nanometer.

Area light source

The light source comes from a rectangular area and projects light from one side of the rectangle.

\* fluorescent light fixture in a ceiling panel.



$(dx, dy, dz, 0)$

In incident illumination on an object sitting in an outdoor courtyard, can be represented using an environment map → also called reflection map

$L(\vec{v}; \lambda)$  -  $\vec{v}$  → light directions  
 $\lambda$  - color values or wavelength

- \* Light sources are at infinity ( $\rightarrow$  sun (call direction)) Environment map can be represented on a collection of cubical faces on a single longitude-latitude map, or as the image of a reflecting sphere.

Reflectance and Shading

When light hits an object's surface, it is scattered and reflected. Types of Models to explain reflectance are 1) Bidirectional reflectance distribution function, 2) Diffuse, 3) Specular 4) Phong Shading Model

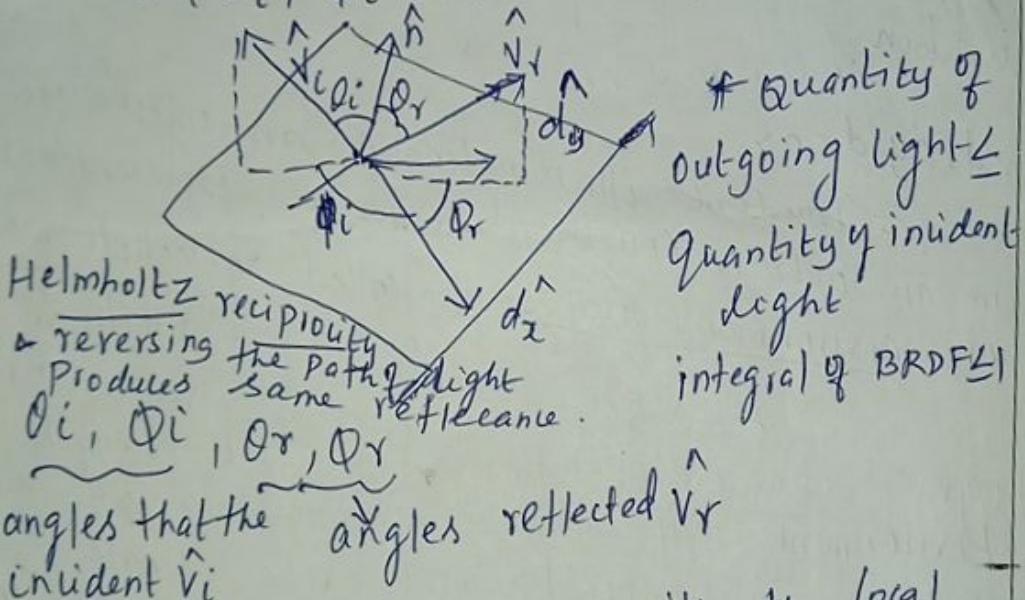
Course Code: 18CSE390T

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## Bidirectional Reflectance Distribution function(BRDF)

It is a 4 dimensional function that describes how much of each wavelength arriving at an incident direction  $\hat{v}_i$  is emitted in a reflected direction  $\hat{v}_r$ .

$$fr(\theta_i, \phi_i, \theta_r, \phi_r; d)$$



Light ray directions make with the local surface co-ordinate frame ( $dx, dy, \hat{n}$ ).

Amount of light exiting a surface point P in a direction  $\hat{v}_r$  can be calculated using

$$L_r(\hat{v}_r; \lambda) = \int L_i(\hat{v}_i; \lambda) f_r(\hat{v}_i, \hat{v}_r, \hat{n}; d) \cos \theta_i d\hat{v}_i$$

$\cos \theta_i = \max(0, \cos \theta_i)$

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An Isotropic Surface →

reflectance depends on

the light orientation relative to the direction of scratch

Isotropic Surface - There are no preferred directions on the surface for light transport

on light transport

is concerned, brush scratches aluminum

Diffuse reflection

The diffuse component (Lambertian or) matte reflection scatters light uniformly in all directions.

$$\text{A surface normal } \mathbf{f}_d(\mathbf{v}_i, \mathbf{v}_r, \mathbf{n}; \mathbf{l}) = f_d(\mathbf{l}) \quad \text{Eq (smooth nonshiny surface)}$$

\* the amount of light depends on the angle b/w the incident light direction and the surface normal &

\* Dull, matte surfaces like chalk or latex paint reflected equally in all directions.

Specular reflection

+ depends strongly on the direction of the outgoing light.  $\rightarrow$  reflects into a single outgoing direction.

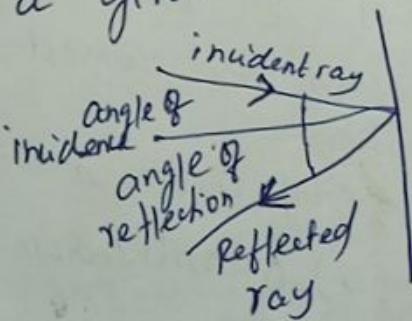
\* reflection seen on smooth shiny

objects. (mirror surface).

\* depends on surface orientation, viewer location.

\* Reflects all light which arrives at the same angle.

- from a given direction at the same angle.



angle of incidence = angle of reflection

Phong Shading

- \* Combine diffuse & specular component  
 ↳ rough surface      mirror
- ↳ called as ambient illumination.

- Objects are illuminated not only by point light source but also by a general diffuse illumination corresponding to inter-reflection (eg. the walls in a room) or distant sources such as the blue sky.

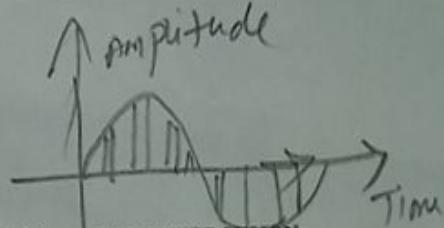
- \* the ambient light depends on surface orientation color of both ambient illumination  $L_a(\lambda)$  and the object  $k_a(\lambda)$

$$f_a(\lambda) = k_a(\lambda) L_a(\lambda)$$

Sampling and aliasing

- \* Signals are analog in nature (eg. speech, weather signals).

- \* To process the analog signal by digital means, it is essential to convert them to discrete-time signals, and then convert them to a sequence of numbers.
- \* Analog to digital conversion steps.
  - 1) Sampling 2) Quantization 3) Coding



## Sampling

- A continuous-time signal is converted into discrete-time signals by taking samples of continuous-time signals at discrete time intervals.

$$x[n] = x(nT_s)$$

$T \rightarrow$  sampling interval       $x[n] \rightarrow$  samples of the  
analog input signal  $x(t)$

## Sampling Criteria

- Sampling frequency must be at least twice the highest frequency.

$$f_s = 2W$$

$f_s \rightarrow$  sampling frequency

$\omega \rightarrow$  higher frequency content

$2W$  known as Nyquist rate

- Shannon's Sampling Theorem: Minimum sampling rate required to reconstruct a signal from its instantaneous samples must be at least twice the highest frequency.

$$f_s \geq 2f_{\max}$$

The maximum frequency in a signal is known as Nyquist frequency and the inverse of the minimum sampling frequency  $r_s = 1/T_s$  is Nyquist rate.

Nyquist rate

\* minimum sampling rate for the perfect reconstruction of the continuous time signals from samples.

Aliasing

\* It is an effect that causes different signals to become indistinguishable when sampled.  
↳ appearing to be the same.

\* The distortion that results when a signal reconstructed from samples is different from the original continuous signal.

Temporal aliasing → occurs in signals sampled in time.  
Spatial aliasing → occurs in spatially sampled signals.  
(Video/audio signals)  
(more pattern) in a brickwall

If we sample at a rate which is less than the Nyquist rate aliasing will occur.

Shannon's theorem tells us that if we have at least 2 samples per period of a sinusoid, we have enough information to reconstruct the sinusoid.

## Image processing Point operators

\* Image processing is an application of CN.  
Image processing is to preprocess the image and convert it into a form suitable for analysis. Some operations are

- 1) exposure correction, color balancing, reduction of image noise, increasing sharpness, straightening the image by rotating it.

\* Image processing operators map pixel values from one image to another. Point operators or point process transforms image, that manipulate each pixel independently of its neighbors.

\* Fourier Transform <sup>tool to analyze</sup> neighborhood operators, where each new pixels value depends on a small number of neighboring input values.

### Point operators

The simplest kind of image processing transforms are point operators, where each output pixels value depends on only the corresponding input pixel value and brightness and contrast adjustments as well as color correction & transformation. This operation is known as point processes.

### Simple point operators

- 1) Brightness scaling
- 2) Image addition

### Pixel transforms

A general image processing operator is a function that takes one or more IP images.

and produces an output image

$$g(x) = h(f(x)) \text{ or}$$

$$g(x) = h(f_0(x) \dots f_n(x)).$$

$x$  is ~~in~~ in the  $D$ -dimensional domain of the functions ( $D=2$  for images). and the fns  $f$  and  $g$  operate over some range, which can be either scalar or vector valued.

For discrete (sampled) images the domain consists of a finite no. of pixel locations

$$x = (i, j) \text{ and}$$

$$g(i, j) = h(f(i, j)).$$

2 commonly used point processes are multiplication and addition with a constant.

$$g(x) = a f(x) + b.$$

$a > 0$  and  $b$  are called the gain & bias parameters used to control contrast and brightness. The bias and gain parameters can also be spatially varying  $g(x) = a(x)f(x) + b(x)$

Another commonly used dyadic ( $2 \rightarrow 1$ ) operator is the linear blend operator

$$g(x) = (1-\alpha)f_0(x) + \alpha f_1(x) \quad \begin{matrix} \text{to } f_1 \rightarrow f_2 \\ \text{image} \end{matrix}$$

$\alpha \rightarrow 1$ , this operator can be used to perform a temporal cross-dissolve between two images

2 images or videos. It's a post production video editing technique in which we gently increase the opacity of one scene over the previous one.  $\hookrightarrow$  not allowing light.

## Color transforms

In an RGB model, the 3 values are red, green and blue components of the color of the pixel. In an HSI model, the 3 values are hue, saturation and intensity of the color of the pixel.

• Color transformation is a transformation of these values.

$$s_i = T_i(r_1, r_2, r_3) \quad i = 1, 2, 3.$$

$r_1, r_2, r_3$  represents the color component of the input image  $f(m, n)$  and  $s_1, s_2, s_3$  represent the color component of the output image  $g(m, n)$ .

A color transformation can be performed in any color space model.

Eg simple transformation by Modifying intensity

$$g(\underline{m, n}) = k f(\underline{m, n})$$

$0 < k < 1$  is a scaling factor.

In HSI

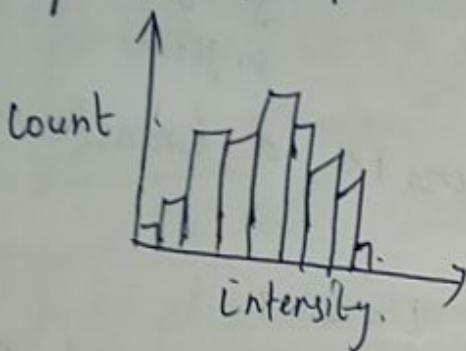
$$s_1 = r_1 \quad s_2 = r_2 \quad s_3 = k r_3.$$

In RGB

$$s_1 = k r_1 \quad s_2 = k r_2 \quad \text{and} \quad s_3 = k r_3.$$

## Histogram Equalization

- Used to enhance contrast. A histogram of an image is the graphical interpretation of the image's pixel intensity values & frequency of all the pixel intensity levels in the image.

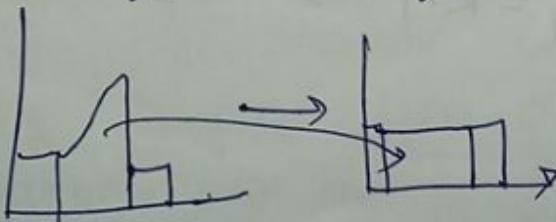


- intensity level is from 0-255.
- For RGB colored image 3 - 2D histograms are there - one for each color.
- For gray-scale only one histogram.
- X-axis represents the pixel intensity levels.
- Y-axis indicates the no. of pixels that have intensity values.

• Histogram equalization is an image processing technique that adjusts the contrast of an image using its histogram.

• To enhance the image's contrast, it spreads out the most frequent intensity values or stretches out the intensity range of the image.

• This allows the image's areas with lower contrast to gain a higher contrast.



## Color Transforms

### H S I

Hue  $\rightarrow$  Color itself. (Yellow, orange & red)  
Saturation  $\rightarrow$  how much the color is  
(mixed) polluted with white colors.  
range  $\in [0,1]$ .

intensity  $\rightarrow$  range  $[0,1]$  0 means black  
1 means white.

### Color balancing:

It is a method of shifting the colors in a photograph. we can change the color balance in the camera before taking a photo or in an editing program during post-processing.

→ Performed either by multiplying each channel with a different scale factor or by changing the nominal white point and adjust the other colors, until the target object appears white.  $\rightarrow$  piece of paper or a wall

Eg take a picture with a rainbow in it, and enhance the strength of the rainbow.

~~Shannon's theorem tell us that if we have at least 2 samples per period of a sinusoid, we have enough information to reconstruct the sinusoid.~~

### Image noise (unwanted signal)

1) random variation of brightness or color information in images, produced by image sensor in digital camera

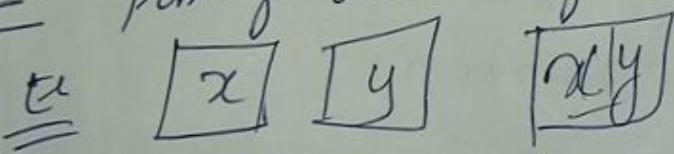
point In pixel transforme.

Multiplicative gain is a linear operation, since it obeys the superposition principle.

superposition f in eight states that when 2 or more waves overlap in space the resultant disturbance is equal to the algebraic sum of the individual disturbances

$$h(f_0 + f_1) = h(f_0) + h(f_1)$$

Blending putting one image to another,



## Advanced histogram equalization Techniques

### 1. Adaptive Histogram Equalization

### 2. Contrastive Limited Adaptive Equalization.

#### 1. Adaptive Histogram Equalization (AHE)

• Utilizes the adaptive method to compute several histograms, each corresponding to a distinct section of the image. Using these histograms, this technique spreads the pixel intensity values of the image to improve contrast.

• Used to improve the local contrast and enhance the edges in specific regions of the image.

#### 2. Contrastive Limited Adaptive Equalization (CLAHE)

• The contrast implication is limited by clipping the histogram at a predefined value. This clip limit depends on the normalization of the histogram or the size of the neighbourhood region.

• The value below 3 and 4 is normally used.

• Operates on small region in the image  $\rightarrow$  tiles  $8 \times 8$ .

- 1) Steps  
Calculate the PMF (Probability Mass Function) density of all pixels in the image.  
→ PMF is calculated by divide the count of each bar from vertical axis and divide it by Total Count
- 2) Calculate CDF (Cumulative distributive function)  
→ simply keep the first value as it is, and then in the 2nd value, we will add the first one and so on. 40, 30, 40, 70, 110, ...
- 3) Multiply the CDF value with Gray levels (minus 1).
- 4) Map the new gray level values onto noisy pixels.
- 5) Map the new values onto histogram.

### Linear Filtering

Filtering is a technique used for modifying or enhancing an image like highlight certain features or remove other features.

Image filtering include  
1) Smoothing 2) Sharpening 3) edge enhancement.

## Linear Filtering

Linear filter is a filter which operate the pixel value in the support region in linear manner. (i.e) output pixels value is determined as a weighted sum of input pixel values.

The support region is specified by the 'filter matrix' can be represented as  $H(i,j)$ . The size of  $H$  is call filter region and filter matrix has its own co-ordinate system,  $i$  is column index and  $j$  is row index. The center of it is the origin location and it is called the hot spot.

+ Images are stored in the form of a matrix of numbers where the numbers are known as pixel values.

+ These pixel values represent the intensity of each pixel.

+ 0 represents black and 255 represents white.

+ For color three Matrix one for  $R$  & another for  $G$  and another for  $B$

$$g(i,j) = \sum_{k,l} f(i+k, j+l) h(k, l).$$

The entries in the weight kernel or mask  $h(k, l)$  are often called as filter coefficients.

$$g = f \otimes h \rightarrow \text{correlation operator}.$$

~~the~~

A common variant on this formula is

$$\begin{aligned} g(i,j) &= \sum_{k,l} f(i-k, j-l) h(k, l) \\ &= \sum_{k,l} f(k, l) h(i-k, j-l). \end{aligned}$$

This is called convolution operator.

$g = f * h$  and  $h$  is then called the impulse response function.

Steps

+ ~~Multiply the filter matrix over the image  $I$  and  $H(0,0)$~~

super position (addition) of shifted impulse response functions  $\sum h(i-k, j-l)$  multiplied by the input pixel values  $f(k, l)$ .

Histogram equalization. Some pictures have excess of dark values, light values. It is better to darken some light value, & lighten some dark values.  
 To find an intensity mapping function  $f(I)$  such that the resulting histogram is flat.

histogram  $\rightarrow$  count the no. of pixels at each gray level Value.

$h(i) \xrightarrow{\text{intensity value}}$  distribution of pixels.

Cumulative distribution:

$$C(i) = \sum_{i=0}^N h(i)$$

$N$  is the no. of pixels in the image

Linear filtering

Output pixels value is determined as a weighted sum of input pixel values.

$$g(i,j) = \sum_{k,l} f(i+k, j+l) h(k, l)$$

~~kernel~~

$h(k, l) \rightarrow$  kernel matrix.

~~filter mask~~

$f(k, l) \rightarrow$  filter coefficients.

$$g = f \otimes h$$

\* In image processing, a kernel/convolutional mask is a small matrix used for blurring, sharpening, embossing, edge detection, etc.

\* This is done by convolution between the Kernel & the image.

Identity

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Sharpen the image.

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix} * \begin{bmatrix} 2 & 2 & 2 \\ 2 & 3 & 2 \\ 2 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 7 \end{bmatrix}$$

(i.e.)  $3 \times 5 + 2 \times -1 + 2 \times -1 + 2 \times -1 + 2 \times -1$

Box blur

$$\frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Both Correlation and convolution are linear shift invariant (LSI) which obey both the superposition & shift invariance principle.

Shift invariance principle

LSI is a system which has the property of linearity / superposition & shift invariance.

$$\text{linearity} \rightarrow f(ax+by) = af(x)+bf(y)$$

1) Shift invariance  $\rightarrow$  If we shift the input in time then the O/P is shifted by the same amount.

$$\text{superposition} \rightarrow \text{LSI operator called "behaves the same everywhere"} \\ h_o(f_0 + f_1) = h_0 f_0 + h_0 f_1$$

Shift invariance.

$$g(i,j) = f(i+k, j+l) \Leftrightarrow (h \circ g)(i,j) \\ = (h \circ f)(i+k, j+l)$$

## Histogram Equalization

### Algorithm

1. Convert the i/p image into a grayscale image
2. Find frequency of occurrence for each pixel value. (ie) histogram of an image Values lie in the range  $[0, 255]$  for any grayscale image.
3. Calculate PMP (probability mass function of all pixels in the image). Calculate Cumulative frequency of all pixel values.
4. Divide the cumulative frequencies by total no. of pixels and multiply them by maximum gray count (pixel value) in the image.

Equalize the given histogram

gray level ( $g_r$ )	0	1	2	3	4	5	6	7
no. of pixels	790	1023	850	656	329	245	124	89

Gray level ( $g_{ik}$ )	no. of pixels $n_k$	PDF $P_g(g_{ik}) = n_k / n$	CDF $s_k = \sum P_k(g_{ik})$	$L-1(8)$ $= 7 \times s_k$	Round off val
0	790	0.19	0.19	1.33	1
1	1023	0.25	0.44	3.08	3
2	850	0.21	0.65	4.55	5
3	656	0.16	0.81	5.67	6
4	829	0.08	0.89	6.25	6
5	245	0.06	0.95	6.65	7
6	122	0.03	0.98	6.85	7
7	81	0.02	1	7	7

$$n = 4096$$

Equating Gray levels to no. of pixels.

$$0 \rightarrow 0 \quad 5 \rightarrow 850$$

$$1 \rightarrow 790 \quad 6 \rightarrow 985$$

$$2 \rightarrow 0 \quad 7 \rightarrow 448$$

$$3 \rightarrow 1023$$

$$4 \rightarrow 0$$

## Non-linear Filtering

A non-linear combination of neighboring pixels.

### Median Filtering

It selects the median value from each pixel's neighborhood. (ie) the result is the middle value after input values have been sorted.

Even order median filter, the output is the average of the central 2 samples after sorting.

Median filtering can be useful to reduce noise.

- \* Removing noise using the median filter does not reduce the difference in brightness of images since the intensity values of the filtered image are taken from the original image.

- \* Median Filter does not shift the edges of images.

## Fourier Transforms

It is a mathematical function that takes a time based pattern as input and determines the overall cycle offset, rotation speed and strength for every possible cycle in the given pattern.

The Fourier Transform is applied to waveforms which are basically a function of time, space or some other variable. The Fourier transform decomposes a waveform into a sinusoid and thus provides another way to represent a waveform.

The result produced by the Fourier Transform is a complex value function of frequency. The absolute value of the Fourier transform represents the frequency present in the original function and its complex argument represents the phase offset of the basic sinusoidal in that frequency.

The Fourier transform of a fn  $f(x)$  is given by

$$f(x) = \int_{-\infty}^{\infty} F(k) e^{2\pi j k x} dk$$

$$F(k) = \int_{-\infty}^{\infty} f(x) e^{-2\pi j k x} dx$$

(4)

$F(\omega)$  can be obtained using inverse Fourier transform.

### Properties

#### Superposition

1) It is a linear transform. The Fourier transform of a sum of signals is the sum of their Fourier transforms. Thus the Fourier transform is called as Superposition.  $f_1(x) + f_2(x) = F_1(\omega) + F_2(\omega)$

#### Shift

The Fourier transform of a shifted signal is the transform of the original signal multiplied by a linear transform  $e^{j\omega x_0}$ .

#### Convolution

The Fourier transform of a pair of convolved signal is the product of their transforms.  $f(x) * h(x) = F(\omega) H(\omega)$

#### Correlation

It is the product of the first transform times the complex conjugate of the second one.  $F(x) \oplus h(x) = F(\omega) H^*(\omega)$

5) Reversal:  $f(-x) = F^*(\omega)$  Fourier transform of the reversed signal is the complex conjugate of the signals transform changing the sign of the imaginary part.

## Two dimensional Fourier Transform, Wiener filtering

2 dimensional Fourier transforms are

$$H(\omega_x, \omega_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x,y) e^{-j(\omega_x x + \omega_y y)} dx dy$$

$\omega_x, \omega_y \rightarrow$  horizontal & vertical frequency

In the discrete domain-

$$H(k_x, k_y) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} h(x,y) e^{-j \frac{2\pi}{NN} k_x x + k_y y}$$

M, N are the width and height of the image.

## Wiener Filtering

To derive wiener filter, we analyze each frequency component of a signals Fourier transform independently. The noisy image formation process can be written as

$$O(x,y) = S(x,y) + n(x,y)$$

$S(x,y)$  is the image we are trying to recover,  $n(x,y)$  is the additive noise signal,

$O(x,y)$  is the observed noisy image.  
Because of the linearity of the Fourier transform we can write

$$O(\omega_x, \omega_y) = S(\omega_x, \omega_y) + N(\omega_x, \omega_y)$$

where each quantity in the above equation is the Fourier transform of the corresponding image.

At each frequency  $(\omega_x, \omega_y)$  we know from our image spectrum that the unknown transform component  $S(\omega_x, \omega_y)$  has a prior distribution which is a zero mean Gaussian with Variance  $P_s(\omega_x, \omega_y)$

we also have noisy measurement  $O(\omega_x, \omega_y)$  whose Variance is  $P_n(\omega_x, \omega_y)$  (ie) Power spectrum of the noise which is normally assumed to be constant (white)

$$P_n(\omega_x, \omega_y) = \sigma_n^2$$

According to Bayes Rule

The posterior estimate of  $s$  can be written as

$$P(s|o) = \frac{P(o|s)p(s)}{P(o)}$$

where  $P(o) = \int P(o|s)p(s)$  is a normalizing constant used to make the  $P(s|o)$  distribution proper (integrate to 1).

The prior distribution  $p(s)$  is given by

$$p(s) = e^{-\frac{(s-\mu)^2}{2P_s}}$$

$\mu$  is the expected mean at the frequency ( $o$ ) everywhere except at the origin) and the measurement distribution  $P(o|s)$  is given by

$$P(o|s) = e^{-\frac{(o-s)^2}{2P_n}}$$

$$W(\omega_x, \omega_y) = \frac{1}{1 + \sigma_n^2 / P_s(\omega_x, \omega_y)}$$

$W(\omega_x, \omega_y)$  is the Fourier transform of optimum Wiener filtered to remove the noise from an image whose power spectrum is  $P_s(\omega_x, \omega_y)$ .

## Non-linear filtering

### Median Filtering

- \* Used to remove noise from images
- \* removing noise while preserving edges.
- \* Removing salt & pepper type noise

### Salt & Pepper noise

Randomly occurring white & black pixels in the image.

- \* also known as impulse noise

\* Median filter works by moving through the image pixel by pixel, replacing each value with the median value of neighbouring pixels.

\* The pattern of neighbours is called the window which slides pixel by pixel over the entire image.

\* The median is calculated by first sorting all the pixel values from the window into numerical order, and then replacing the pixel being considered with the middle pixel value.

Simple one dimensional signal

→ window size 3 used.

$$x = 3 \quad \boxed{3 \mid 9 \mid 4 \mid 5 \mid 2 \mid 3 \mid 8 \mid 6 \mid 2 \mid 2 \mid 9} \quad 9$$

$$y[0] = \text{median}[3, 3, 9] = 3 \quad y[5] = M[3, 6, 8] = 6$$

$$y[1] = \text{median}[3, 4, 9] = 4 \quad y[6] = M[2, 6, 8] = 6$$

$$y[2] = " [4, 9, 52] = 9 \quad y[7] = M[2, 2, 6] = 2$$

$$y[3] = " [3, 8, 52] = 8 \quad y[8] = M[2, 2, 9] = 2$$

$$y[9] = M[2, 9, 9] = 9.$$

$$y = \boxed{3 \mid 4 \mid 9 \mid 8 \mid 6 \mid 6 \mid 2 \mid 2 \mid 9}.$$

For  $y[0]$  and  $y[9]$  - extend the left most or right most value outside the boundaries of the image. Same as leaving left-most & right most value unchanged after 1-D median.

2D Median Filtering example using 3x3 Sampling window

\* Keeping border Value unchanged -

1	4	0	1	3	1
2	2	4	2	2	3
1	0	1	0	1	0
1	2	1	0	2	2
2	5	3	1	2	5
1	1	4	2	3	0

Sorted  
00110  
2244  
0001122  
44

1	4	0	1	3	1
2	1	1	1	1	3
1	1	1	1	1	2
1	1	1	1	1	2
2	2	2	2	2	5
1	1	4	2	3	0

2D Median Filtering using 3x3 sampling  
 window - extending border Values outside  
 with Values at boundary

	1	1	4	0	1	3	1	1	Sorted
1	1	1	4	0	1	3	1	0	0 0 1 1 0
2	2	2	4	2	2	3	3	3	2 2 4 4
1	1	0	1	0	1	0	0	0	
1	1	2	1	0	2	2	2	2	
1	2	5	3	1	2	5	5	5	
2	1	1	4	2	3	0	0	0	
1	1	1	4	2	3	0	0	0	

2	2	2	2	2	2
1	1	1	1	1	1
1	1	1	1	2	2
1	1	1	1	1	2
1	2	2	2	2	2
1	2	2	3	2	2

Input

Output

3) Extending border Values outside with 0's.

0	0	0	0	0	0	0	0	0	0
0	1	4	0	1	3	1	0	0	0
0	2	2	4	2	2	3	0	0	0
0	1	0	1	0	1	0	0	0	0
0	1	2	1	0	2	2	0	0	0
0	2	5	3	1	2	5	0	0	0
0	1	1	4	2	3	0	0	0	0
0	0	0	0	0	0	0	0	0	0

Sorted  
0 0 0 0 1 2 2 4 4

0	2	1	1	1	0
0	1	1	1	1	1
0	1	1	1	2	2
0	1	1	1	1	1
1	2	2	2	2	2
0	1	1	1	1	0

The Fourier transform is simply  
a tabulation of the magnitude

\* Phase response at each frequency,

$$H(\omega) = F \{ h(x) \} = Ae^{j\phi}$$

(ie) it is the response to a complex sinusoid of frequency  $\omega$  passed through a filter  $h(x)$ . The Fourier Transform exist both in continuous domain

Fourier  
Transform  
Pair is written

$$H(\omega) = \int_{-\infty}^{\infty} h(x) e^{-j\omega x} dx$$

on  $h(x) \xrightarrow{H(\omega)}$  in discrete domain (DFT) Discrete Fourier Transform

$$H(k) = \frac{1}{N} \sum_{n=0}^{N-1} h(n) e^{-j \frac{2\pi k n}{N}}$$

$N$  is the length of the signal or region of analysis.  $K \xrightarrow{\text{sample}} \left[ -\frac{N}{2}, \frac{N}{2} \right]$

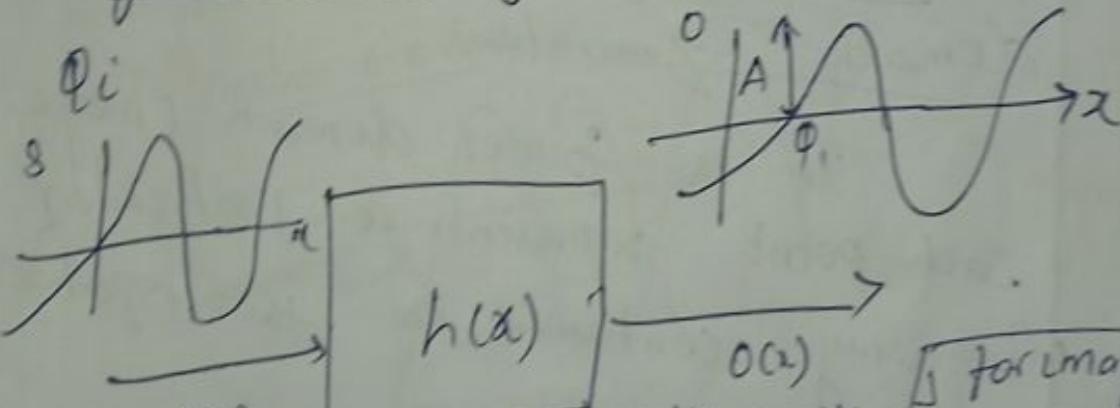
These formulas apply both to filters such as  $h(x)$  and to signals or images such as  $s(x)$  or  $g(x)$ .

If  $k$  is large aliasing occur for lower frequencies.

Fourier Transform → ① Image Transform  
 1) Fourier analysis → used to analyze the frequency  
 2) How can we analyze what a given filter does to high, medium and low frequencies.

The soln is to simply pass a sinusoid of known frequency through the filter and to observe by how much it is attenuated (weaker).

Sine wave  
Amplitude 2) Frequency 3) Phase  
 $s(x) = \sin(2\pi f x + \phi_i) = \sin(\omega x + \phi_i)$   
 $s(x)$  is input sinusoid whose frequency is  $f$ , angular frequency is  $\omega = 2\pi f$  and Phase is  $\phi_i$



If we convolve the sine wave  $s(x)$  with a filter  $h(x)$  for imaginary  $h_0$ , we get another wave  $o(x) = h(x) * s(x) = A \sin(\omega x + \phi_o)$  with same frequency but different magnitude and phase.

The new magnitude  $A$  is called gain or magnitude of the filter while the phase difference  $\Delta\phi = \phi_o - \phi_i \rightarrow$  shift or phase.

## Fourier Series

(2)

- \* periodic signals can be represented into sum of sines and cosines when multiplied with a certain weight.

## Fourier Transform.

- \* used to decompose an image into its sine and cosine components.

The DFT of the transformation represents the image in the Fourier or frequency domain, while the original image is the spatial domain equivalent.

In the Fourier domain image each point represents a particular frequency contained in the spatial domain image.

The compact notation to represent sinusoid is to use the complex-value sinusoid

$$S(x) = e^{j\omega x} = \cos \omega x + j \sin \omega x.$$

$$O(x) = h(x) * S(x) = A e^{j\omega x + \phi}$$

Sine wave.  
amplitude → the peak deviation of the ~~to~~ from zero  
 $f \rightarrow$  no. of cycles that occur each second  
 $f \text{ time}$   
 $\omega = 2\pi f \rightarrow$  angular frequency, the rate of change of the function argument per second

$\phi \rightarrow$  Phase - specifies in radians where in its cycle the oscillation is at  $t=0$

sinusoidal refers to both sine waves and cosine waves with any phase offset

