

Compiler Design

Unit-3

Bottom-up Parsing :-

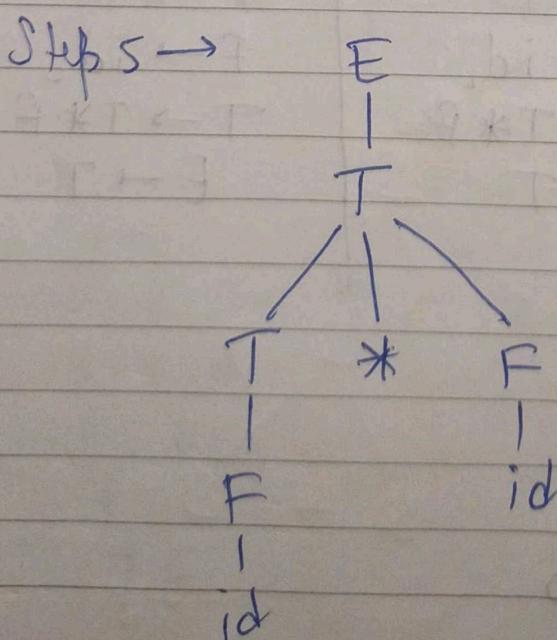
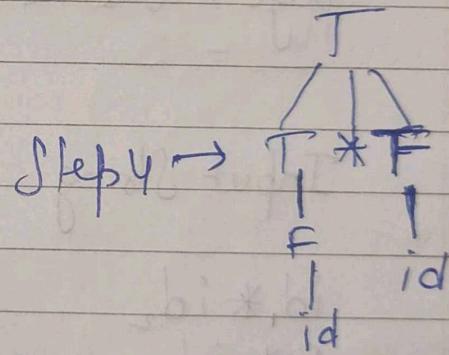
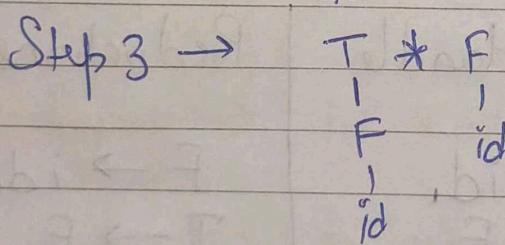
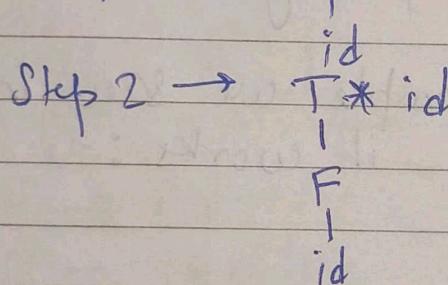
$$E \rightarrow E + T \mid T$$

$$T \rightarrow T * F \mid F$$

$$F \rightarrow (E) \mid id$$

Generate Parse tree for $id * id$

$$\Rightarrow \text{Step 1} \rightarrow F * id$$



\Rightarrow Reductions:- $id * id$,
 $F * id$, $T * id$, $T * F$, F , E

Handle Punning :-

The handle is the substring that matches the body of a production whose reduction represents one step along with the reverse of the rightmost derivation.

→ Removing the children of left-hand side non-terminal from parse tree is called Handle punning.

A rightmost derivation in reverse can be obtained by handle punning.

For the same example we saw in last page, let's see how it works :-

Input String	Handle	Reductions
$id_1 * id_2$	id_1	$F \rightarrow id_1$
$F * id_2$	F	$T \rightarrow F$
$T * id_2$	id_2	$F \rightarrow id_2$
$T * F$	$T * F$	$T \rightarrow T * F$
T	T	$E \rightarrow T$
E (start)		
↑ aapt		

$$\text{Q. } S \rightarrow aABe \\ \text{A} \rightarrow Abc / b \\ \text{B} \rightarrow d$$

Input string abbcde

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Page _____

Input String	Handle	Reduction
abbcde	b	$A \rightarrow b$
aAbcde	Abc	$A \rightarrow Abc$
aAde	d	$B \rightarrow d$
aABe	aABe	$S \rightarrow aABC$
- S (Start)		
↑ accept		

Shift Reduce Parsing :-

Q

$$\begin{aligned} E &\rightarrow E + T \mid T \\ T &\rightarrow T * F \mid F \\ F &\rightarrow (E) \mid id \end{aligned}$$

Input String \rightarrow id, * id₂.

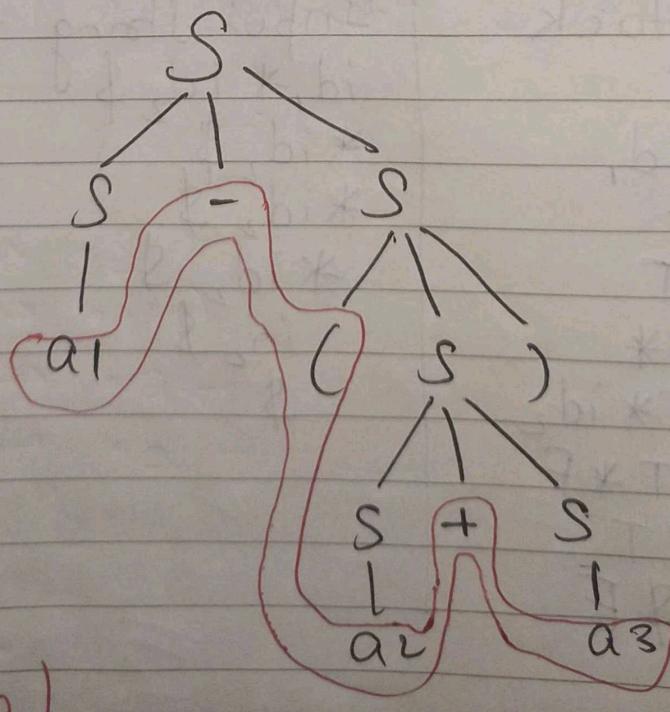
Stack	Input String	Action
\$	id, * id ₂ \$.	Shift id,
\$ id,	* id ₂ \$	Replace by F \rightarrow id
\$ F	* id ₂ \$	Replace by T \rightarrow F
\$ T	* id ₂ \$	Shift *
\$ T *	id ₂ \$	Shift id ₂
\$ T * id ₂	\$	Replace by F \rightarrow id
\$ T * F		Replace T \rightarrow T * F
\$ T		Replace E \rightarrow T
\$ E		Accept string.

$$\text{Q. } S \rightarrow S + S \mid S - S$$

$$S \rightarrow (S) \mid a$$

$$\text{String} \rightarrow a_1 - (a_2 + a_3)$$

\Rightarrow Stack	Input String	Action
\$	$a_1 - (a_2 + a_3) \$$	Shift a_1
\$ a_1	$- (a_2 + a_3) \$$	Replace $S \rightarrow a_1$
\$ a_1 \$	$- (a_2 + a_3) \$$	Shift -
\$ a_1 \$ -	$(a_2 + a_3) \$$	Shift (
\$ a_1 \$ - ($(a_2 + a_3) \$$	Shift a_2
\$ a_1 \$ - (a_2	$+ a_3) \$$	Replace $S \rightarrow a_2$
\$ a_1 \$ - (a_2 \$	$+ a_3) \$$	Shift +
\$ a_1 \$ - (a_2 \$ +	$a_3) \$$	Shift a_3
\$ a_1 \$ - (a_2 \$ + a_3	$) \$$	Replace $S \rightarrow a_3$
\$ a_1 \$ - (a_2 \$ + a_3)	$) \$$	Replace $S \rightarrow S + S$
\$ a_1 \$ - (a_2 \$ + a_3))	$) \$$	Shift)
\$ a_1 \$ - (a_2 \$ + a_3)) \$	$\$$	Replace $S \rightarrow (S)$
\$ a_1 \$ - (a_2 \$ + a_3)) \$ \$	$\$$	Replace $S \rightarrow S - S$
\$ a_1 \$ - (a_2 \$ + a_3)) \$ \$	$\$$	Accept string.



$$a_1 - (a_2 + a_3) =$$

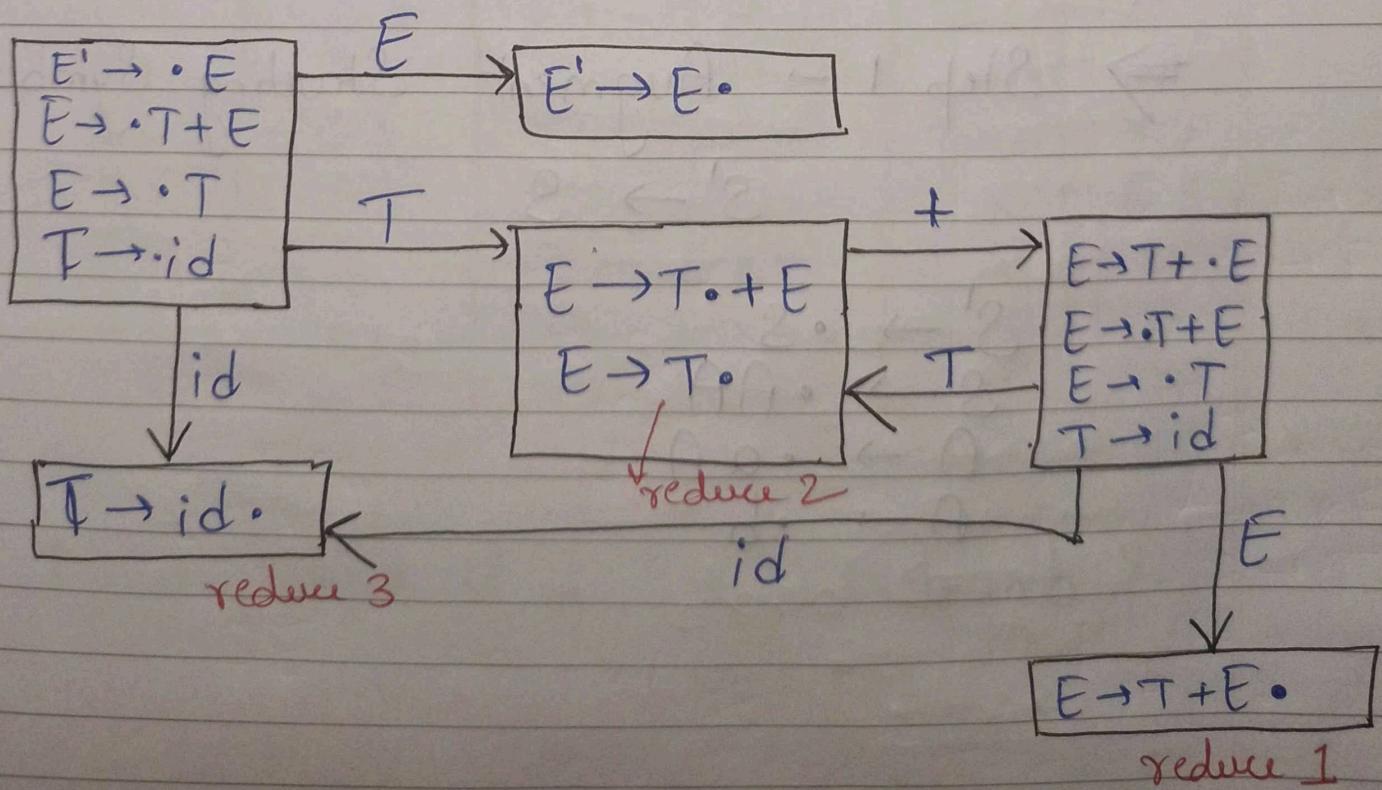
LR(0) Parsing

↓ to right ↑ Rightmost symbol ↓ 0 lookahead

Q. Create LR(0) Parsing table for
 $E \rightarrow T + E / T$
 $T \rightarrow id$

\Rightarrow Step 1 \rightarrow Augment starting symbol.
 $E' \rightarrow E$

$$\begin{aligned} E' &\rightarrow \cdot E \\ E &\rightarrow \cdot T + E \\ E &\rightarrow \cdot T \\ T &\rightarrow \cdot id \end{aligned}$$



States	Actions			Go to	
	+	id	\$	E	T
I0		s3		1	2
I1			accept		
I2	s4/ γ_2	γ_2	γ_2		
I3	γ_3	γ_3	γ_3		
I4		s3		5	2
I5	γ_1	γ_1	γ_1		

↓

For terminals only, we use

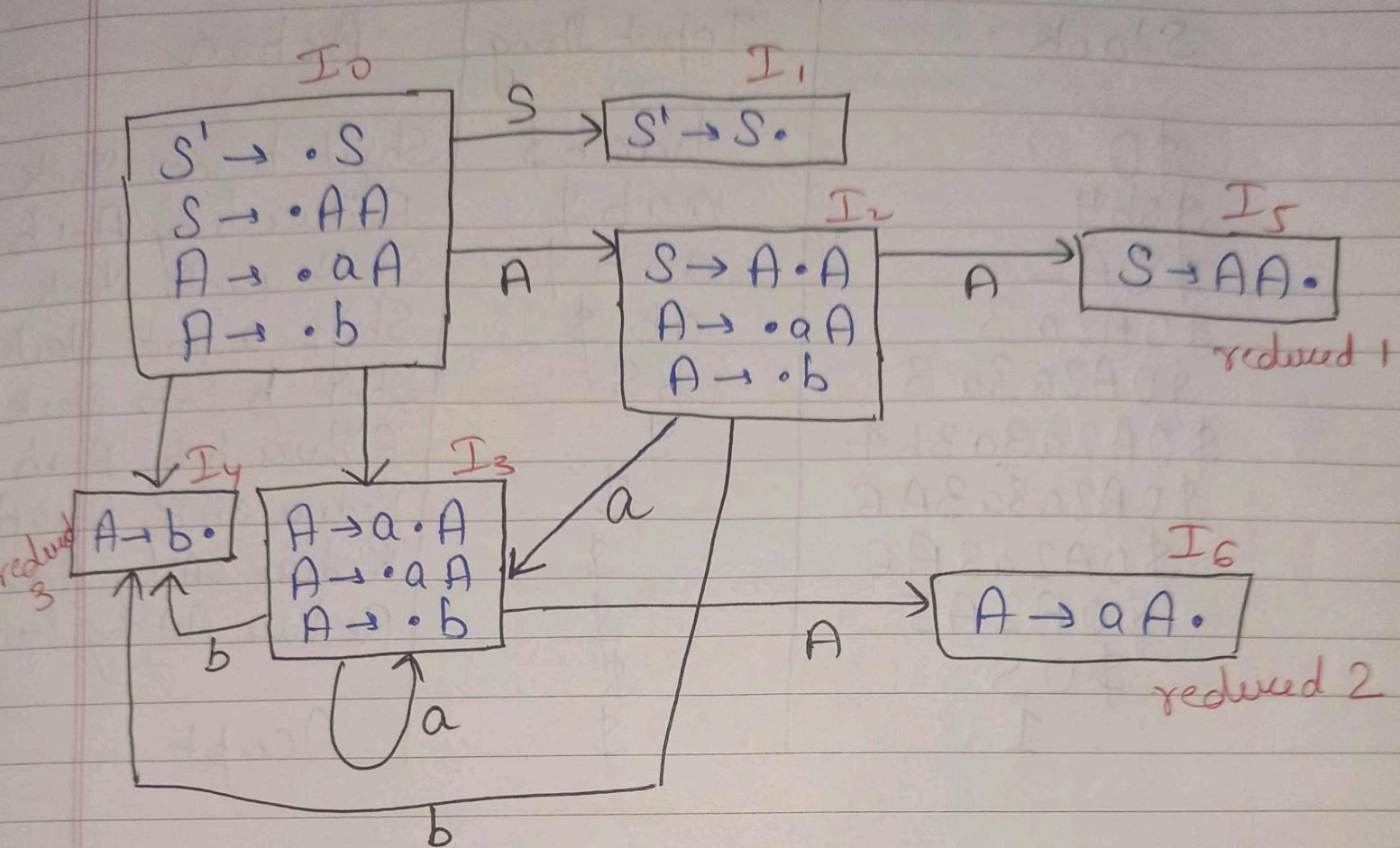
- 1) Shift (S)
- 2) Reduce (γ)

Q. $S \rightarrow AA$
 $A \rightarrow aA/b$

\Rightarrow Step 1 \rightarrow Augment Starting Symbol

$$S' \rightarrow S$$

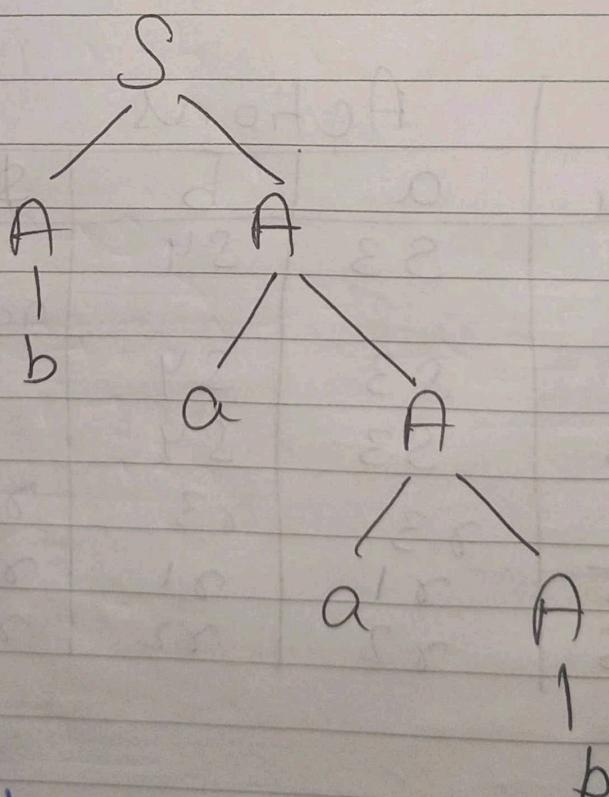
$$\begin{aligned} S' &\rightarrow \cdot S \\ S &\rightarrow \cdot AA \\ A &\rightarrow \cdot aA \\ A &\rightarrow \cdot b \end{aligned}$$



States	Actions				S	A
	a	b	\$			
I_0	s_3	s_4			1	2
I_1				accept		
I_2	s_3	s_4			5	
I_3	s_3	s_4			6	
I_4	γ_3	γ_3	γ_3			
I_5	γ_1	γ_1	γ_1			
I_6	γ_2	γ_2	γ_2			

Now, check if it accept baab?

Stack	Input String	Action
\$0	baab\$	Shift b into Stack
\$0b4	aab\$	Reduce b by A→b
\$0A2	aab\$	Shift a into Stack
\$0A2a3	ab\$	Shift a into stack
\$0A2a3a3	b\$	Shift b into Stack
\$0A2a3a3b4	\$	Reduce b by A→b
\$0A2a3a3A6	\$	Reduce AA by A→AA
\$0A2a3A6	\$	Reduce AA by A→AA
\$0A2A5	\$	Reduce AA by A→AA
\$0S	\$	
1	\$	
		Accept



⇒ baab

Question to Practise :-

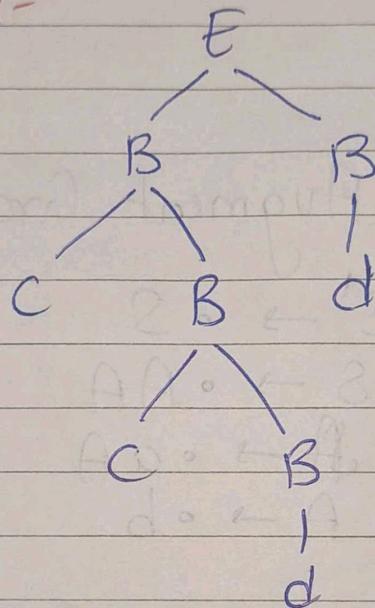
$$1) E \rightarrow BB$$

$$B \rightarrow cB/d$$

Generate string ccdd.

→ It is similar to last quest.

Ans to match :-

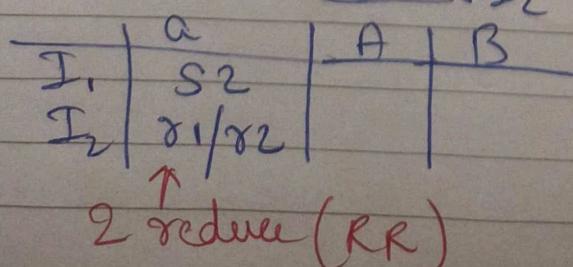
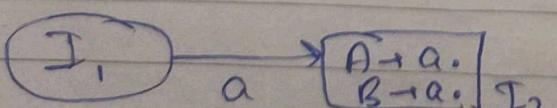


Not → While using LR(0), there are 2 types of conflict that may arise.

1) RR Conflict

$$A \rightarrow \cdot a$$

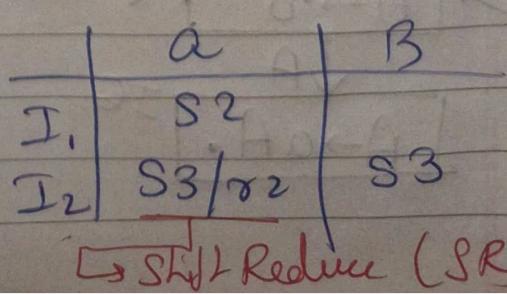
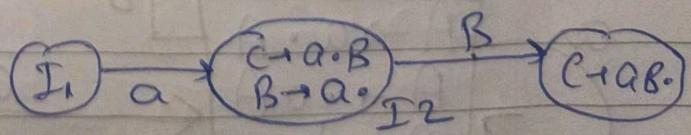
$$B \rightarrow \cdot a$$



2) SR Conflict

$$C \rightarrow \cdot aB$$

$$B \rightarrow \cdot a$$



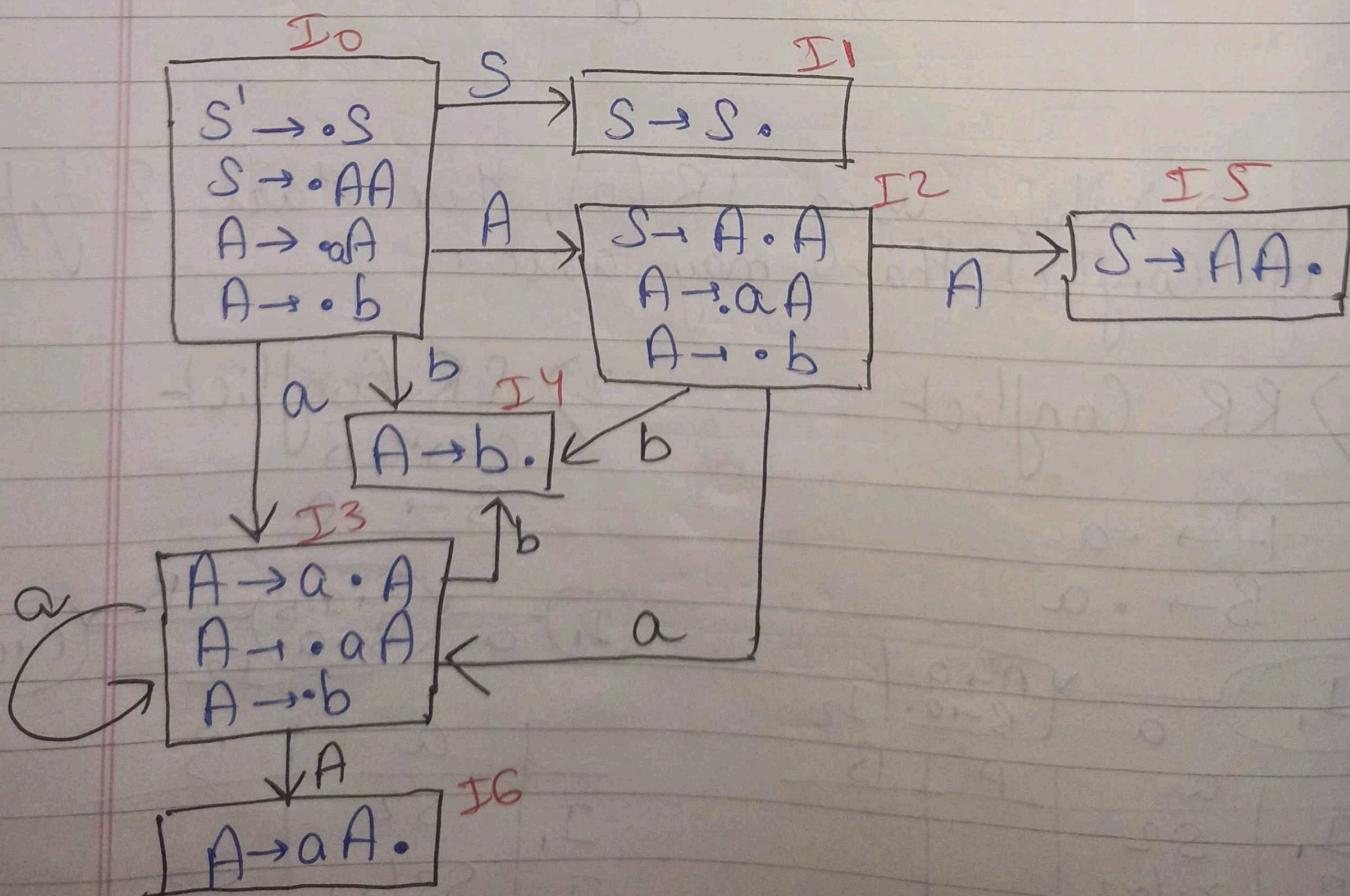
To overcome these conflicts, we use SLR parser (Simple LR Parser).

SLR(1) Parser

Q. $S \rightarrow AA$
 $A \rightarrow aA/b$.

\Rightarrow Step 1 \rightarrow Augment Grammar.

$$\begin{aligned} S' &\rightarrow \cdot S \\ S &\rightarrow \cdot AA \\ A &\rightarrow \cdot aA \\ A &\rightarrow \cdot b \end{aligned}$$



Step 2 → Calculate follow of S and A.

$$\text{Follow}(S) = \{\$\}$$

$$\text{Follow}(A) = \{a, b, \$\}$$

Stage	Action			Goto	
	a	b	\$	S	A
I0	s3	s4		1	2
I1			accept		
I2	s3	s4			5
I3	s3	s4			6
I4	r3	r3	r3		
I5			r1		
I6	r2	r2	r2		

Q. $E \rightarrow T + E/T$
 $T \rightarrow id$

$$\Rightarrow \text{Follow}(E) = \{\$\}$$

$$\text{Follow}(T) = \{+, \$\}$$

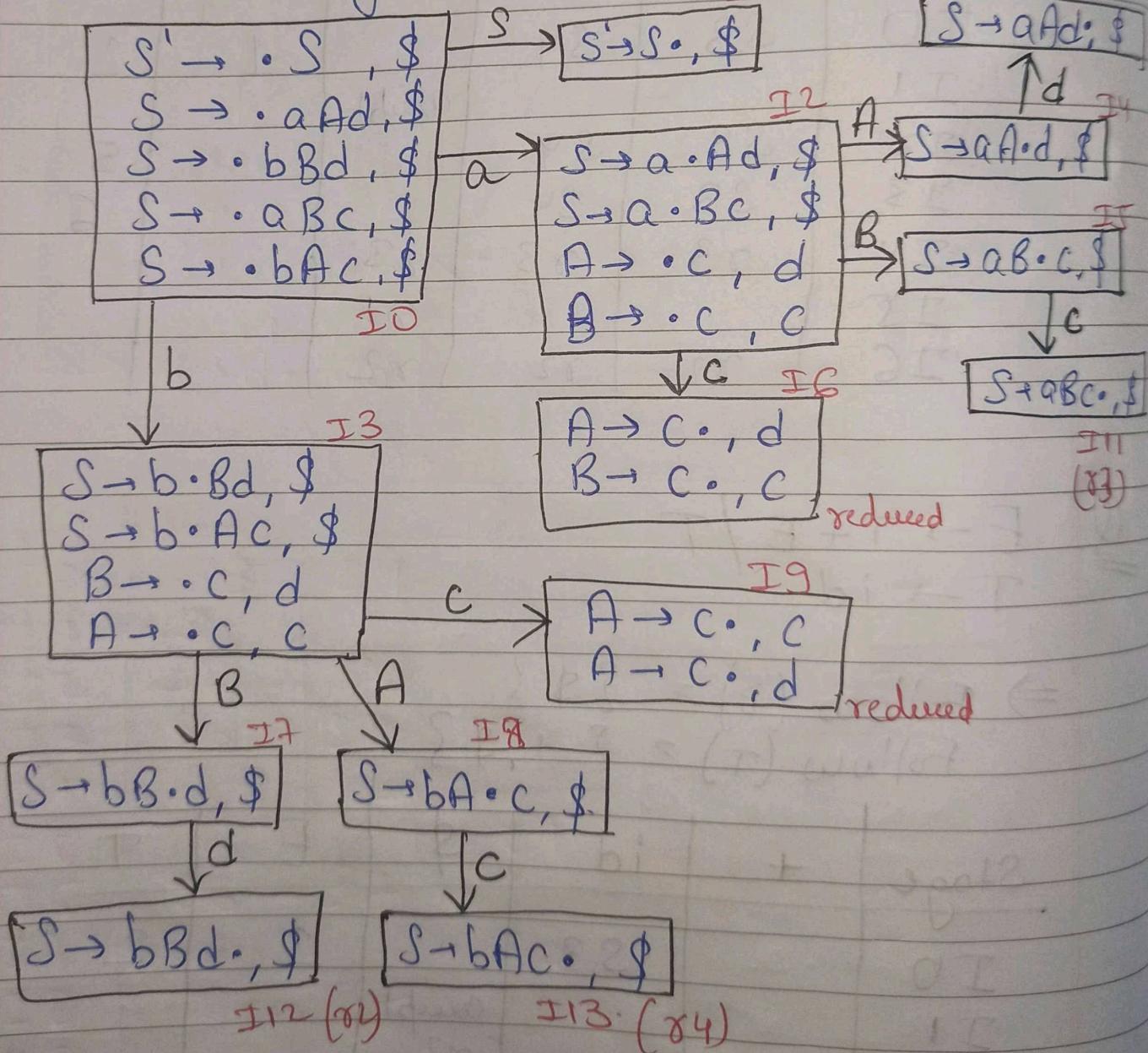
Stage	+	id	\$	E	T
I0		s3		1	2
I1			accept		
I2	<u>s4</u>				
I3	<u>r3</u>				
I4		s3	r3		5
I5			r1		2

no conflict this time using SLR.

Canonical Left Right Parser (CLR(1)) :-

Q. $S \rightarrow aAd / bBd / aBc / bAc$
 $A \rightarrow c$
 $B \rightarrow c$

\Rightarrow Step 1 → Augment Starting Symbol.



Stage	a	b	c	d	\$	S	A	B
I0	S2	S3					1	
I1				:	,	accept		
I2			S6				4	5
I3			S9				8	7
I4				.	S10			
I5			S11					
I6			γ6	γ5				
I7				S12				
I8			S13					
I9			γ5	γ6				
I10				γ1				
I11				γ3				
I12				γ2				
I13				γ4				

Look-ahead Left-Right (LALR)

⇒ similar to CLR But, we ~~can~~ merge the production on same reduction. Like in this question, I6 and I9 is reducing same production. So, we can merge it.

	a	b	c	d	\$	S	A	B
$S_1 \rightarrow I_6 + I_9$			γ_6, γ_5	γ_5, γ_6				
$I_6 \rightarrow$			γ_6	γ_5				
$I_9 \rightarrow$			γ_5	γ_6				

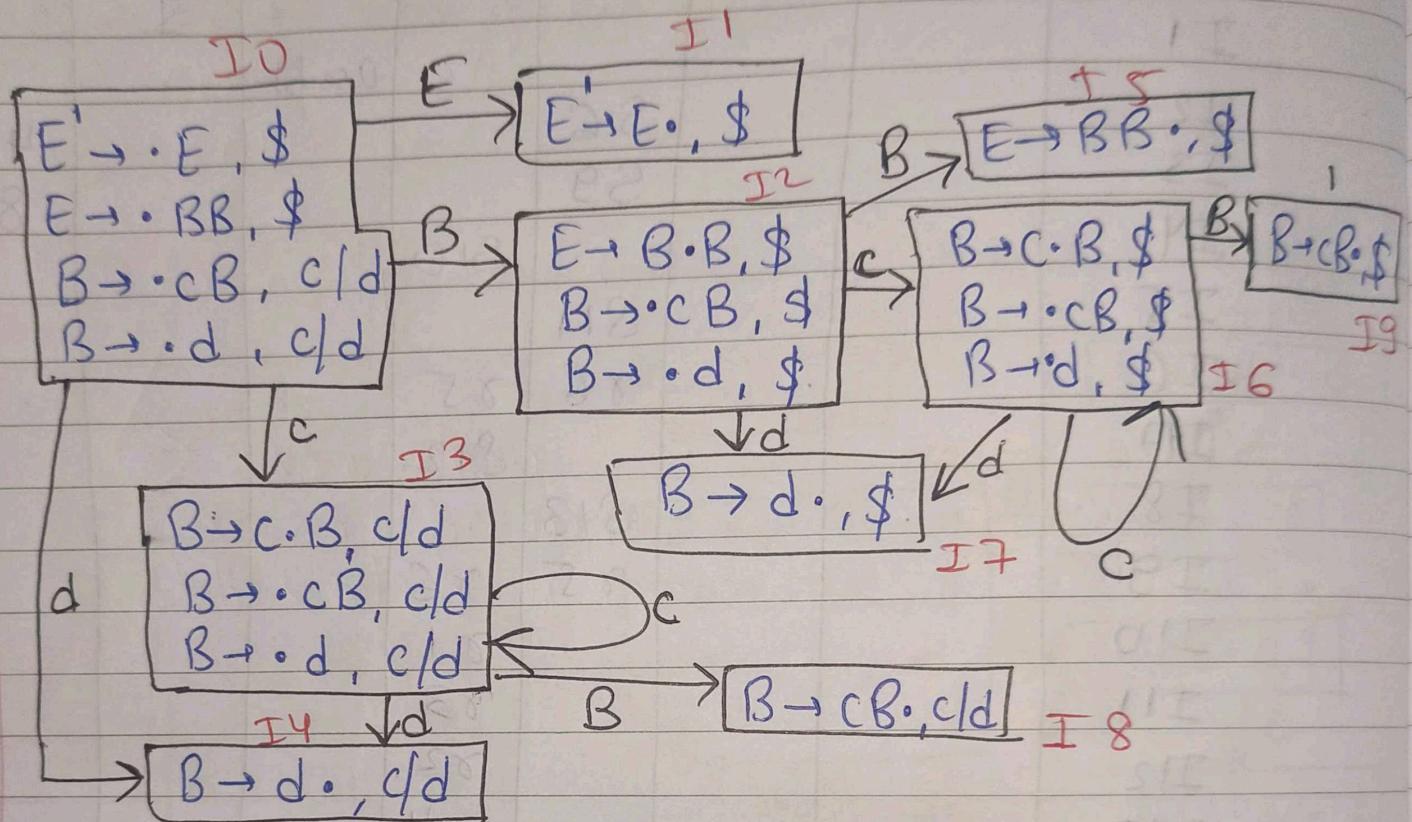
2 production in one block

So, it is not LALR Parser. It is only CLR Parser

Q.

$$E \rightarrow BB$$

$$B \rightarrow cB/d$$



Stages	Action			Count.	
	C	D	\$	E	B
I0	S3	S4		1	2
I1	S3				
I2	S6	S7			5
I3	S3	S4			8
I4	S3	S3			
I5			S1		
I6	S6	S7			9
I7			S3		
I8	S2	S2			
I9			S2		

For LALR :-

$$I_3 \& I_6 \rightarrow I_{36}$$

$$I_4 \& I_7 \rightarrow I_{47}$$

$$I_8 \& I_9 \rightarrow I_{89}$$

	C	d	\$	E	B
I ₀	S ₃₆	S ₄₇		1	2
I ₁			accept		
I ₂	S ₃₆	S ₄₇			5
→ I ₃	S ₃₆	S ₄₇			89
Copy → I ₄	γ ₃	γ ₃	γ ₃		
Copy → I ₅			γ ₁		
Copy → I ₆	S ₃₆	S ₄₇			89
Copy → I ₇	γ ₃	γ ₃	γ ₃		
Copy → I ₈	γ ₂	γ ₂	γ ₂		
Copy → I ₉	γ ₂	γ ₂	γ ₂		

Now, I₃&I₆ having same values so we can ignore 1 of them. Same we can do with (I₄&I₇) & (I₈&I₉).

	C	d	\$	E	B
I ₀	S ₃₆	S ₄₇		1	2
I ₁			accept		
I ₂	S ₃₆	S ₄₇			5
I ₃₆	S ₃₆	S ₄₇			89
I ₄₇	γ ₃	γ ₃	γ ₃		
I ₅			γ ₁		
I ₈₉	γ ₂	γ ₂	γ ₂		

⇒ Both CLR (1) & LALR

Question to Practise :-

Q.

Check if the following production is

(a) CLR or not

(b) LALR or not.

Ans to verify - (a) Y₄,
(b) Y₃,

→ Isme dono table me difference nahi
ayega because, there is no 2
state which is providing same reduction.

Operator Precedence Parser :-

↳ Based on operator precedence grammar.
Rules :-

1. It should not have ϵ in RHS of production

$$S \rightarrow E \times$$

2. Two Non-terminal should not be adjacent

$$E \rightarrow A C \times$$

$$E \rightarrow a C \checkmark$$

$$E \rightarrow C a \checkmark$$

Q. $E \rightarrow EA E \mid (E) \mid id$

$$A \rightarrow + \mid - \mid * \mid ,$$

$$E \rightarrow E + E \mid E - E \mid E * E \mid E / E \mid (E) \mid id$$

Precedence Table Rules:-

1. id, a, b, c = higher precedence
terminal

2. operator & { $(+, +)$
 $(*, *)$
 $(/, /)$
 $(-, -)$ } \rightarrow Left side has more precedence.

3. $\$ \rightarrow$ lowest precedence

4. $\$\$ \rightarrow$ accept

	+ ↓	*	(↓) byt	id	\$
+	>	<	<	>	<	>
*	>	>	<	>	<	>
(>	>	<	=	<	X
)	>	<	X	>	X	>
id	>	>	X	>	X	>
\$	<	<	<	X	<	accept

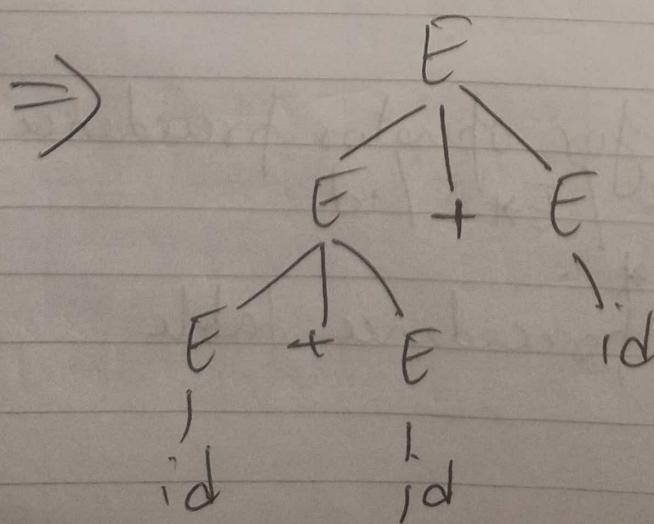
Q. $E \rightarrow EA E / id$
 $A \rightarrow +/*$.

\Rightarrow Step 1 \rightarrow Check for operators precedence grammar
 $E \rightarrow E+E / E*E / id$
 $A \rightarrow +, *$.
 Step 2 \rightarrow Draw precedence table

	id	+	*	\$	
id	= < <	> > >	> < >	> > >	
+	< <	> > >	> < >	> > >	
*	< <	> > >	> < >	> > >	accept
\$					

Input String \rightarrow id + id + id.
 $\Rightarrow \$ id + id + id \$$

Stack	Relation	String	Action
\$.	<	\$ id + id + id \$	Shift \$
\$ id	>	id + id + id \$	Shift id
\$ id E	>	id + id + id \$	Reduce by E → id
\$ id E +	>	+ id + id \$	Shift +
\$ id E + id	>	id + id \$	Shift id
\$ id E + E	>	+ id \$	Reduce E → id
\$ id E E	<	+ id \$	Shift +
\$ id E E +	>	id \$	Shift id
\$ id E E + id	>	id \$	Reduce E → id
\$ id E E	>	id \$	Reduce E → E + E
\$	>	id \$	Accept



Q for Practice :-

$$E \rightarrow EAE / id$$

$$A \rightarrow * / -$$

Generate Parse tree for $id - id * id$.

Computation of Leading & Trailing :-

Leading :- ϵ or single non-terminal

Rule 1 :- $A = \overset{\epsilon \text{ or single}}{\underset{\text{terminal}}{\alpha a Y}} \rightarrow \text{anything}$

$$\text{Leading}(A) = a$$

Rule 2 :- $A = B\alpha \rightarrow \text{anything}$
 \downarrow
 single Non-terminal

$$\text{Leading}(A) = \text{Leading}(B)$$

Trailing :-

Rule 1 :- $A = \overset{\text{terminal}}{\underset{\epsilon \text{ or single N-T}}{\alpha a Y}} \rightarrow \text{anything}$

$$\text{Trailing}(A) = \{a\}$$

Rule 2 :-

$A = \boxed{\alpha B \alpha} \rightarrow \text{anything}$
 \downarrow
 single N.T

$$\text{trailing}(A) = \text{trailing } \{B\}$$

$$\begin{array}{l}
 \text{Q} . \quad E \rightarrow E + T \quad | \quad T \\
 \quad\quad\quad T \rightarrow T * F \\
 \quad\quad\quad F \rightarrow (E) \quad | \quad \text{id}
 \end{array}
 \quad \text{①} \quad \text{②} \quad \text{③} \quad \text{④} \quad \text{⑤} \quad \text{⑥}$$

$\Rightarrow 1. E \rightarrow E + T$
 $\text{Leading}(E) = \{ + \}, \text{Leading}(E)$
 $\text{Trailing}(E) = \{ + \}, \text{trailing}(T).$

Leading :-

$$\text{Rule 1} \rightarrow E \rightarrow \underbrace{E + T}_{\alpha \quad \alpha} \quad Y$$

$$\text{Leading}(E) = \{ + \}$$

$$\text{Rule 2} \rightarrow E \rightarrow \underbrace{E + T}_{B \quad \alpha}$$

$$\text{Leading}(E) = \text{Leading}\{ E \}$$

Trailing :-

$$\text{Rule 1} \rightarrow E \rightarrow \underbrace{E + T}_{\alpha \quad \alpha} \quad Y$$

$$\text{Trailing}(E) = \{ + \}$$

$$\text{Rule 2} \rightarrow E \rightarrow \underbrace{E + T}_{\alpha \quad B}$$

$$\text{Trailing}(E) = \text{Trailing}(T)$$

2. $E \rightarrow T$

$$\begin{array}{l}
 \text{Leading}(E) = \text{Leading}(T) \\
 \text{Trailing}(E) = \text{Trailing}(T).
 \end{array}$$

Rule 1 not applicable as $E \rightarrow T$ does not in form $\alpha \gamma$

Similarly, $T \rightarrow T * F$
 $\Rightarrow \text{Leading}(T) = \{\ast\}, \text{Leading}(F)$
 $\Rightarrow \text{Trailing}(T) = \{\ast\}, \text{Trailing}(F)$

$T \rightarrow F$
 $\Rightarrow \text{Leading}(T) = \text{Leading}(F)$
 $\text{Trailing}(T) = \text{Trailing}(F).$

$F \rightarrow (E)$
 $\Rightarrow \text{Leading}(F) = \{(\}$
 $\text{Trailing}(F) = \{)\}.$

$F \rightarrow id$
 $\Rightarrow \text{Leading}(F) = \{id\}$
 $\text{Trailing}(F) = \{id\}$

Operator Precedence Relation Table Algo :-

1. Calculate Leading & Trailing for each N.T.

2. (a) $S \rightarrow ab$
 $a = b$

(b) $S \rightarrow aAb$
(i) $a = b$

(ii) $a < \text{Leading}(A)$

(iii) $\text{Trailing}(A) > b$

(c) If $S = \text{Starting N.T.}$, $\$ \rightarrow \text{end terminal}$
(i) $\$ < \text{Leading}(S)$
(ii) $\text{Trailing}(S) > \$$

By using that let's find Table of above question

<u>N.T</u>	Leading	Trailing
E	{+, *, id, ()}	{+, *, id, ()}
T	{*, id, ()}	{*, id, ()}
F	{id, ()}	{id, ()}

	+	*	()	id	\$
+	>	<	<	>	<	>
*	>	>	<	>	<	>
(<	<	<	=	<	X
)	>	>	X	>	X	>
id	>	X	>	X	X	>
\$	<	<	X <	X	<	Acept

$$E \rightarrow E + T$$

$$\downarrow \quad E \rightarrow E +$$

form of S $\rightarrow A b$

$$\& \quad E \rightarrow + T$$

form of S $\rightarrow a A$

Trailing (E) $> \{ +, *, id, () \} > \{ + \}$ (i) $+ < \text{Leading } (T)$ $+ < \{ *, id, () \}$ (ii)

$E \rightarrow T \Rightarrow$ ignored

$$\underline{3. T \rightarrow T * F}$$

$\hookleftarrow T \rightarrow T *$

↓

Trailing (T) $> *$
 $\{ *, id,) \} > *$
(iii)

$$T \rightarrow * F$$

$* < \text{Leading}(F)$

$* < \{ id, C \}$
(iv)

$$\underline{4. T \rightarrow F \Rightarrow \text{ignored}}$$

$$\underline{\underline{5. F \rightarrow (E)}}$$

$\hookrightarrow (=) \quad \text{---} \quad \text{(v)}$

$\hookrightarrow (< \text{Leading}(E))$
 $\Rightarrow (< \{ +, *, id, C \}) \quad \text{---} \quad \text{(vi)}$

→ Trailing $(E) >)$
 $\Rightarrow \{ +, *, id,) \} >) \quad \text{---} \quad \text{(vii)}$

$$\underline{\underline{6. F \rightarrow id \Rightarrow \text{ignored}}}$$

$$\$ < \{ +, *, id, C \} \quad \text{---} \quad \text{(viii)}$$

$$\{ +, id, id,) \} > \$ \quad \text{---} \quad \text{(ix)}$$

Now, just fill f_u table using these equations.