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Answer

1.

a)

Given parity check matrix:

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Parity check matrix is represneted as **H = [P : I_m]**

Thus, By comparing, m = 3

$$P = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

It is a (6,3) LBC code

Thus, n = 6, k = 3

Genrerator matrix can be developed as: **G = [I_k : P^T]**

$$P^T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

It is a (6,3) LBC code

Thus, n = 6, k = 3

For k = 3, three message bits or 8 distinct message blocks are possible.

The codeword table can be developed as:

D (Mesage block)	Codeword: C = DG	Hamming weight: W(C)
0 0 0	0 0 0 0 0 0	0
0 0 1	0 0 1 0 1 1	3
0 1 0	0 1 0 1 0 1	3
0 1 1	0 1 1 1 1 0	4
1 0 0	1 0 0 1 1 1	4
1 0 1	1 0 1 1 0 0	3
1 1 0	1 1 0 0 1 0	3
1 1 1	1 1 1 0 0 1	4

Hamming weight is the number of 1s in a codeword.

Minimum hamming distance is given by the smallest possible non-zero hamming weight.

The above table has smallest possible non - zero hamming weight as 3.

Thus, minimum hamming distance is 3.

$$\Rightarrow d_{min} = 3$$

A Linear Block Code can detect upto 't' errors provided:

$$d_{min} \geq t + 1$$

$$\Rightarrow 3 \geq t + 1$$

$$\Rightarrow t \leq 2$$

Thus, All single bit errors can be detected in transmission.

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TO DECODE THE RECEIVED CODEWORDS:

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$H^T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

a)

Received codeword: r =111000

Syndrome: S

$$S = rH^T = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S = 111 + 101 + 011 = 001$$

All additons are XOR addition.

S = [0 0 1] matches with the last row of H^T matrix. Thus error is in the last bit of 'r'.

Corrected codeword, r = [1 1 1 0 0 1]

....

b)

Received codeword: r =100100

Syndrome: S

$$S = rH^T = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S = 111 + 100 = 011$$

S = [0 1 1] matches with the 3rd row of H^T matrix. Thus error is in the third bit of 'r'.

Corrected codeword, r = [1 0 1 1 0 0]

....

b)

Given generator matrix:

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Now, performing elementary operation: C2 = C2 + C3

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Genrerator matrix is represented as: **G = [I_k : P^T]**

$$P^T = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow P = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

It is a (6,3) LBC code

Thus, n = 6, k = 3, m = n - k = 3

Parity check matrix can be developed as **H = [P : I_m]**

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Received codeword: r =111101

Syndrome: S

$$S = rH^T = \begin{bmatrix}1 & 1 & 1 & 1 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 1 & 0\\0 & 1 & 1\\0 & 0 & 1\\1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$$S = 110 + 011 + 001 + 100 + 001 = 001$$

All additons are XOR addition.

S = [0 0 1] matches with the 3rd and last row of H^T matrix. Thus error is in the 3rd or last bit of 'r'.

Corrected codeword, r = [1 1 1 1 0 0] or r = [1 1 0 1 0 1]

....

Received codeword: r =100100

Syndrome: S

$$S = rH^T = \begin{bmatrix}1 & 0 & 0 & 1 & 0 & 0\end{bmatrix}\begin{bmatrix}1 & 1 & 0\\0 & 1 & 1\\0 & 0 & 1\\1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$$S = 110 + 100 = 010$$

S = [0 1 0] matches with the 5th row of H^T matrix. Thus error is in the 5th bit of 'r'.

Corrected codeword, r = [1 0 0 1 1 0]

....

Received codeword: r =111100

Syndrome: S

$$S = rH^T = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S = 110 + 011 + 001 + 100 = 000$$

As syndrome S = 0. There is no error in this received codeword.

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