SRM INSTITUTE OF SCIENCE AND TECHNOLOGY

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Department of Mathematics

Sub Title: DISCRETE MATHEMATICS FOR ENGINEERS

Sub Code: 18 MAB 302 T –Unit-3- Mathematical Logic

Syllabus:

Propositions and Logical operators- Truth values and truth tables.- Propositions generated by a set-Symbolic writing using conditional and bi-conditional connectives.- Writing converse inverse and contra positive of a given conditional.- Tautology, contradiction and contingency examples.- Proving tautology and contradiction using truth table method.- Equivalences – truth table method to prove equivalences.- Implications- truth table method to prove implications- Laws of logic and some equivalences.- Proving equivalences and implications using laws of logic.- Rules of inference – Rule P, Rule T and Rule CP - Direct proofs - Problems using direct method.- Problems using CP rule.- Inconsistency and indirect method of proof.- Inconsistent premises and proof by contradiction (indirect method).- Principle of mathematical induction. - Problems based on Mathematical Induction - Applications of sets ,relations and functions in Engineering.

1. **Express the statement "Good food is not cheap" in symbolic form.**

   Ans:

   P: Food is good   Q:Food is cheap

   Symbolic form : $P \to \neg Q$

2. **Define simple statement function.**

   Ans: A statement is a sentence which can either be True (T) or False (F).

   For example, The door is kept opened is a simple statement.

3. **How many rows are needed for the truth table of the formula: $(P \wedge \neg Q) \leftrightarrow ((\neg R \wedge S) \to T)$.**

   Ans: There are 5 variable P,Q,R,S,T . So, 2^5 =32 rows (different combinations of truth values) are needed.

4. **Negate the statement: " John is playing football" in two different forms.**

   Ans: Form 1: John is not playing football

   Form 2: It is not the case that John is playing football.

5. **State the truth value of " If tigers have wings then the earth travels round the sun".**

   Ans: P: Tigers have wings which is a false statement

   Q: Earth travels round the sun which is a true statement

We have $P \to Q$ So we have a combination of $F \to T$ which is True (T)

So the truth value of the given statement is T.

6. Write the symbolic representation of " if it rains today,then I buy an umbrella".

Ans: P: It rains today,

Q: I buy an umbrella

symbolic representation: $P \to Q$

7. Find the truth table for $p \to q$.

Ans:

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

8. Construct the truth table for $P \to \sim Q$.

Ans:

P	Q	$\sim Q$	$P \to \sim Q$
T	T	F	F
T	F	T	T
F	T	F	T
F	F	T	T

9. Construct the truth table for $(p \to q) \to (q \to p)$.

Ans:

P	Q	$(P \to Q)$	$(Q \to P)$	$(P \to Q) \to (Q \to P)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

10. Give the truth value of $T \leftrightarrow T \wedge F$.

Ans: $T \leftrightarrow T \wedge F \equiv T \leftrightarrow F \equiv F$.

11. Express $p \to q$ in terms of the connectives $\{\vee, \neg\}$.

Ans: $p \to q \Leftrightarrow \neg p \vee q$.

12. Express $A \leftrightarrow B$ in terms of the connectives $\{\wedge, \neg\}$.

Ans: $A \to B \Leftrightarrow (A \to B) \wedge (B \to A) \Leftrightarrow (\neg A \vee B) \wedge (\neg B \vee A)$

$\Leftrightarrow \neg(A \wedge \neg B) \wedge \neg(B \wedge \neg A)$.

13. When do you say that two compound propositions are equivalent?

Ans: If the two compound propositions take same truth value for (irrespective of) all possible combinations of truth values then they are said to be equivalent.

14. Define Tautology with an example.

Ans: A compound proposition $P = P(p_1, p_2, p_3, \ldots\ldots, p_n)$ where $p_1, p_2, \ldots\ldots, p_n$ are variables is called a tautology, if it is true for every truth assignment for $p_1, p_2, \ldots\ldots, p_n$.

Example: $p \vee \neg p$ is a tautology.

15. Show that the propositions $p \to q$ and $\neg p \vee q$ are logically equivalent.

Ans:

P	Q	$(P \to Q)$	$\neg P$	$\neg P \vee Q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

From 3rd and 5th columns of truth table (since they are identical) we conclude that $p \to q$ and $\neg p \vee q$ are logically equivalent.

11.Show that $(p \to r) \wedge (q \to r)$ *and* $(p \vee q) \to r$ are logically equivalent.

Ans.

p	q	r	$p \to r$	$q \to r$	$(p \to r) \wedge (q \to r)$ 6	$p \vee q$	$(p \vee q) \to r$ 8
T	T	T	T	T	T	T	T
T	T	F	F	F	F	T	F
T	F	T	T	T	T	T	T
T	F	F	F	T	F	T	F
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	F
F	F	T	T	T	T	F	T
F	F	F	T	T	T	F	T

From 6th and8th columns of truth table (since they are identical) we conclude that $(p \to r) \wedge (q \to r)$ and $(p \vee q) \to r$ are logically equivalent.

16. Show that $P \wedge (P \rightarrow Q) \Leftrightarrow P \wedge Q$.

Ans:

P	Q	$(P \rightarrow Q)$	$P \wedge (P \rightarrow Q)$	$P \wedge Q$
T	T	T	T	T
T	F	F	F	F
F	T	T	F	F
F	F	T	F	F

From the last two columns of the truth table (since they are identical) we conclude that $P \wedge (P \rightarrow Q) \Leftrightarrow P \wedge Q$

17. Show that the formula $Q \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$ is a tautology.

Ans:

P	Q	$\neg P$	$\neg Q$	$P \wedge \neg Q$	$\neg P \wedge \neg Q$	$Q \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$
T	T	F	F	F	F	T
T	F	F	T	T	F	T
F	T	T	F	F	F	T
F	F	T	T	F	T	T

The truth value of the given statement is T irrespective of the truth values of the variables. So it is a tautology.

18. Express the bi-conditional $P \leftrightarrow Q$ in any form using only disjunction $(\vee)$, conjunction $(\wedge)$ and negation $(\sim)$.

Ans:

$$P \leftrightarrow Q \quad \Leftrightarrow \quad (P \rightarrow Q) \wedge (Q \rightarrow P)$$
$$\Leftrightarrow \quad (\neg P \vee Q) \wedge (\neg Q \vee P)$$

19. Verify whether the statement $(P \vee Q) \rightarrow P$ is a tautology.

Ans:

P	Q	$P \vee Q$	$(P \vee Q) \rightarrow P$
T	T	T	T
T	F	T	T
F	T	T	F

F	F	F	T

The last column has both T and F truth values. So the given statement is not a tautology.

20. Determine whether the conclusion C follows logically from the premises H_1 , H_2 or not $H_1 : P \to Q$, $H_2 : P$, $C : Q$.

Ans:

Statement No	**Statement**	**Rule**	**Identities used**	**Statements used**
1	P	P		
2	$P \to Q$	P		
3	Q	T	Modus ponens	1,2

So C is a valid conclusion from the two premises H_1,H_2

21. State any two rules of inference with explanation.

Ans:

Rule P : This rule is used to introduce any given premise at the time of derivation

Rule T : This rule is derive a statement from the previous set of premises, statements using basic and known laws, equivalences .

22. When a set of formulae is consistent and inconsistent?

Ans:

If the set of formulae logically conclude F then they are said to be inconsistent.

Otherwise they are said to be consistent.

23. Give an indirect proof of the theorem " If $3n + 2$ is odd then n is odd".

Ans: If n is even then 3 n is also even when an even number 2 is added 3n+2 is also even number. Hence the theorem.

24. Give the converse and contrapositive of the interpolation " If it is raining then I get wet".

Ans: The Converse is If I got wet then it was raining

The contra positive is If I don't get wet then it is not raining.

25. Give the contrapositive statement of the statement "If there is rain, then I buy an umbrella".

Ans: Let P : There is rain

Q : I buy umbrella

The contra positive is $\neg Q \to \neg P$

If I don't buy umbrella then there is no rain.

26. Construct the truth table for $(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R)$.

Solution:

Take $S : (\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R)$

P	Q	R	$(Q \wedge R)$	$(P \wedge R)$	$\neg P$	$\neg Q$	$(\neg Q \wedge R)$	$(\neg P \wedge (\neg Q \wedge R))$	S
T	T	T	T	T	F	F	T	F	F
T	T	F	F	F	F	F	F	F	F
T	F	T	F	T	F	T	T	F	F
T	F	F	F	F	F	T	F	F	F
F	T	T	T	F	T	F	F	F	F
F	T	F	F	F	T	F	F	F	F
F	F	T	F	F	T	T	T	T	F
F	F	F	F	F	T	T	F	F	F

Since all the truth values of the given statement are F irrespective of the values taken by its components, the given statement is a contradiction.

27. Show that the expression $[(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow R)] \rightarrow R$ is a tautology by using truth table.

Solution:

Take $S_1 : [(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow R)]$

P	Q	R	$(P \vee Q)$	$(P \rightarrow R)$	$(Q \rightarrow R)$	S_1	$S_1 \rightarrow R$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	T	T	T	T	T
T	F	F	T	F	T	F	T
F	T	T	T	T	T	T	T
F	T	F	T	T	F	F	T
F	F	T	F	T	T	F	T
F	F	F	F	T	T	F	T

Since all the truth values of the given statement are T irrespective of the values taken by its components, the given statement is a Tautology.

28. Construct truth table for $(\neg P \rightarrow Q) \wedge (Q \leftrightarrow P)$.

Solution:

P	Q	$\neg P$	$(\neg P \rightarrow Q)$	$(Q \leftrightarrow P)$	$(\neg P \rightarrow Q) \wedge (Q \leftrightarrow P)$
T	T	F	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	F	T	F

29. Without using truth table, prove that $\neg P \rightarrow (Q \rightarrow R) \cong Q \rightarrow (P \vee R)$.

Solution:

Step No	Statement	Equivalence used
1	$\neg P \rightarrow (Q \rightarrow R)$	Given LHS
2	$\neg P \rightarrow (\neg Q \vee R)$	$(A \rightarrow B) \Leftrightarrow (\neg A \vee B)$
3	$\neg(\neg P) \vee (\neg Q \vee R)$	$(A \rightarrow B) \Leftrightarrow (\neg A \vee B)$
4	$P \vee (\neg Q \vee R)$	Double negation
5	$\neg Q \vee (P \vee R)$	Commutative and associative laws
6	$Q \rightarrow (P \vee R)$	$(A \rightarrow B) \Leftrightarrow (\neg A \vee B)$

30. Prove that $(P \rightarrow Q) \wedge (Q \rightarrow R) \Rightarrow (P \rightarrow R)$.

Solution:

To prove $(P \rightarrow Q) \wedge (Q \rightarrow R) \rightarrow (P \rightarrow R)$ is a tautology

Step No	Statement	Equivalence used
1	$(P \rightarrow Q) \wedge (Q \rightarrow R) \rightarrow (P \rightarrow R)$	Given
2	$\neg[(P \rightarrow Q) \wedge (Q \rightarrow R)] \vee (P \rightarrow R)$	$(A \rightarrow B) \Leftrightarrow (\neg A \vee B)$
3	$\neg[(\neg P \vee Q) \wedge (\neg Q \vee R)] \vee (\neg P \vee R)$	$(A \rightarrow B) \Leftrightarrow (\neg A \vee B)$
4	$\neg(\neg P \vee Q) \vee \neg(\neg Q \vee R) \vee (\neg P \vee R)$	De Morgan's Law
5	$(\neg\neg P \wedge \neg Q) \vee (\neg\neg Q \wedge \neg R) \vee (\neg P \vee R)$	De Morgan's Law
6	$(P \wedge \neg Q) \vee (Q \wedge \neg R) \vee (\neg P \vee R)$	Double Negation
7	$\begin{bmatrix} (P \vee Q) \wedge (P \vee \neg R) \wedge \\ (\neg Q \vee Q) \wedge (\neg Q \vee \neg R) \end{bmatrix} \vee (\neg P \vee R)$	Distributive law
8	$\begin{bmatrix} (P \vee Q) \wedge (P \vee \neg R) \wedge \\ T \wedge (\neg Q \vee \neg R) \end{bmatrix} \vee (\neg P \vee R)$	$A \vee \neg A \Leftrightarrow T$
9	$\begin{bmatrix} (P \vee Q) \wedge (P \vee \neg R) \wedge \\ (\neg Q \vee \neg R) \end{bmatrix} \vee (\neg P \vee R)$	Identity law
10	$\begin{bmatrix} (P \vee Q \vee (\neg P \vee R)) \wedge (P \vee \neg R \vee (\neg P \vee R)) \wedge \\ (\neg Q \vee \neg R \vee (\neg P \vee R)) \end{bmatrix}$	Distributive law
11	$[T \wedge T \wedge T]$	$A \vee \neg A \Leftrightarrow T$
12	T	Idempotent law

31. Determine whether the compound proposition $\neg(Q \rightarrow R) \wedge R \wedge (P \rightarrow Q)$ is a tautology or contradiction.

Solution:

Step No	Statement	Equivalence used
1	$\neg(Q \to R) \wedge R \wedge (P \to Q)$	Given
2	$\neg(\neg Q \vee R) \wedge R \wedge (\neg P \vee Q)$	$(A \to B) \Leftrightarrow (\neg A \vee B)$
3	$(\neg\neg Q \wedge \neg R) \wedge R \wedge (\neg P \vee Q)$	De Morgan's Law
4	$(Q \wedge \neg R) \wedge R \wedge (\neg P \vee Q)$	Double Negation
5	$(Q \wedge \neg R \wedge R) \wedge (\neg P \vee Q)$	Associative Law
6	$F \wedge (\neg P \vee Q)$	$A \wedge \neg A \Leftrightarrow F$
7	F	Identity Law

So, the given statement is a contradiction.

32. Define tautology and contradiction. Check whether $Q \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$ is a tautology or contradiction without using truth table.

Solution:

If the truth value of a statement formulas is T(True) irrespective of the truth values of its components, then it is called Tautology.

If the truth value of a statement formulas is F(False) irrespective of the truth values of its components, then it is called Contradiction.

Step No	Statement	Equivalence used
1	$Q \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$	Given
2	$[(Q \vee P) \wedge (Q \vee \neg Q)] \vee (\neg P \wedge \neg Q)$	Distributive law
3	$[(Q \vee P) \wedge T] \vee (\neg P \wedge \neg Q)$	$A \vee \neg A \Leftrightarrow T$
4	$(Q \vee P) \vee (\neg P \wedge \neg Q)$	Identity law
5	$(Q \vee P \vee \neg P) \wedge (Q \vee P \vee \neg Q)$	Distributive Law
6	$T \wedge T$	$A \vee \neg A \Leftrightarrow T$
7	T	Identity Law

So the given statement is a Tautology.

33. Show that $P \leftrightarrow Q \Leftrightarrow (P \wedge Q) \vee (\neg P \wedge \neg Q)$.

Solution:

Step	Statement	Equivalence used

No		
1	$(P\wedge Q)\vee(\neg P\wedge\neg Q)$	Given RHS
2	$(P\vee\neg P)\wedge(P\vee\neg Q)\wedge\ (Q\vee\neg P)\wedge(Q\vee\neg Q)$	Distributive law
3	$T\wedge(P\vee\neg Q)\wedge\ (Q\vee\neg P)\wedge T$	$A\vee\neg A\Leftrightarrow T$
4	$(P\vee\neg Q)\wedge\ (Q\vee\neg P)$	Identity law
5	$(\neg Q\vee P)\wedge\ (\neg P\vee Q)$	Commutative Law
6	$(Q\to P)\wedge(P\to Q)$	$(A\to B)\Leftrightarrow(\neg A\vee B)$
7	$P\leftrightarrow Q$	Biconditional definition

34. Show that $(P\to Q)\wedge(R\to Q)$ and $(P\vee R)\to Q$ are equivalent formula.

Solution:

Step No	Statement	Equivalence used
1	$(P\to Q)\wedge(R\to Q)$	Given LHS
2	$(\neg P\vee Q)\wedge(\neg R\vee Q)$	$(A\to B)\Leftrightarrow(\neg A\vee B)$
3	$(\neg P\wedge\neg R)\vee Q$	Distributive law
4	$\neg(P\vee R)\vee Q$	De Morgan's Law
5	$(P\vee R)\to Q$	$(A\to B)\Leftrightarrow(\neg A\vee B)$

35. Without using truth table, prove the following implication $(P\vee Q)\wedge(P\to R)\wedge(Q\to R)\Rightarrow R$.

Solution:

To Prove that $[(P\vee Q)\wedge(P\to R)\wedge(Q\to R)]\to R$ is a tautology

Step No	Statement	Equivalence used
1	$[(P\vee Q)\wedge(P\to R)\wedge(Q\to R)]\to R$	Given
2	$\neg[(P\vee Q)\wedge(P\to R)\wedge(Q\to R)]\vee R$	$(A\to B)\Leftrightarrow(\neg A\vee B)$
3	$\neg[(P\vee Q)\wedge(\neg P\vee R)\wedge(\neg Q\vee R)]\vee R$	$(A\to B)\Leftrightarrow(\neg A\vee B)$
4	$\neg[(P\vee Q)\wedge([\neg P\wedge\neg Q]\vee R)\ ]\vee R$	Distributive law
5	$\neg[(P\vee Q)\wedge(\neg[P\vee Q]\vee R)\ ]\vee R$	Distributive law
6	$\neg[\{(P\vee Q)\wedge\neg[P\vee Q]\}\vee([P\vee Q]\wedge R)\ ]\vee R$	Distributive law
7	$\neg[F\vee([P\vee Q]\wedge R)\ ]\vee R$	$A\wedge\neg A\Leftrightarrow F$
8	$[\neg([P\vee Q]\wedge R)]\vee R$	Identity Law
9	$[(\neg[P\vee Q]\vee\neg R)]\vee R$	De Morgan's Law
10	$\neg[P\vee Q]\vee[\neg R\vee R]$	Associative Law
11	$\neg[P\vee Q]\vee T$	$A\vee\neg A\Leftrightarrow T$
12	T	Identity Law

36. Show that $J \wedge S$ logically follows from the premises $P \to Q$, $Q \to \neg R$, R, $P \vee (J \wedge S)$.

Solution:

Statement No	Statement	Rule	Identities used	Statements used
1	R	P		
2	$Q \to \neg R$	P		
3	$\neg Q$	T	Modus Tollens	1,2
4	$P \to Q$	P		
5	$\neg P$	T	Modus Tollens	3,4
6	$P \vee (J \wedge S)$	P		
7	$\neg P \to (J \wedge S)$	T	$(A \to B) \Leftrightarrow (\neg A \vee B)$	6
8	$(J \wedge S)$	T	Modus Ponens	5,7

37. Show that the set of premises $(A \to B) \wedge (A \to C)$, $(\neg(B \wedge C))$ and $(D \vee A)$ yields a conclusion D.

Solution:

Statement No	Statement	Rule	Identities used	Statements used
1	$(\neg(B \wedge C))$	P		
2	$(A \to B) \wedge (A \to C)$	P		
3	$(\neg A \vee B) \wedge (\neg A \vee C)$	T	$(A \to B) \Leftrightarrow (\neg A \vee B)$	2
4	$(\neg A \vee (B \wedge C))$	T	Distributive Law	3
5	$(A \to [B \wedge C])$	T	$(A \to B) \Leftrightarrow (\neg A \vee B)$	4
6	$\neg A$	T	Modus Tollens	1,5
7	$(D \vee A)$	P		
8	$\neg D \to A$	T	$(A \to B) \Leftrightarrow (\neg A \vee B)$	7
9	D	T	Modus Tollens	6,8

38. Show that the premises $P \to Q$, $P \to R$, $Q \to \neg R$, P are inconsistent.

Solution:

Statement No	Statement	Rule	Identities used	Statements used

1	P	P		
2	$P \to Q$	P		
3	Q	T	Modus Ponens	1,2
4	$P \to R$	P		
5	R	T	Modus Ponens	1,4
6	$Q \to \neg R$	P		
7	$\neg R$	T	Modus Ponens	3,6
8	$R \wedge \neg R$	T		5,7
9	F	T	$A \wedge \neg A \Leftrightarrow F$	8

39. Using conditional proof, prove that $\neg P \vee Q$, $\neg Q \vee R$, $R \to S \Rightarrow P \to S$.

Solution:

To prove the conclusion S from the set of premises $\neg P \vee Q$, $\neg Q \vee R$, $R \to S$, P

Statement No	Statement	Rule	Identities used	Statements used
1	P	CP		
2	$\neg P \vee Q$	P		
3	$P \to Q$	T	$(A \to B) \Leftrightarrow (\neg A \vee B)$	2
4	Q	T	Modus Ponens	1,3
5	$\neg Q \vee R$	P		
6	$Q \to R$	T	$(A \to B) \Leftrightarrow (\neg A \vee B)$	5
7	R	T	Modus Ponens	4,6
8	$R \to S$	P		
9	S	T	Modus Ponens	7,8

40. Show that $P \to Q$, $Q \to R$, $P \vee R \Rightarrow R$ by using indirect method.

Solution:

Use $\neg R$ as one of the premises

Statement No	Statement	Rule	Identities used	Statements used

1	$\neg R$	P		
2	$Q \to R$	P		
3	$\neg Q$	T	Modus Tollens	1,2
4	$P \vee R$	P		
5	$R \vee P$	T	Commutative law	4
6	$(\neg R \to P)$	T	$(A \to B) \Leftrightarrow (\neg A \vee B)$	5
7	P	T	Modus Ponens	1,6
8	$P \to Q$	P		
9	Q	T	Modus Ponens	7,8
10	$Q \wedge \neg Q$	T		3,9
11	F	T	$A \wedge \neg A \Leftrightarrow F$	10

41. Using derivation process prove that $S \to \neg Q$, $S \vee R$, $\neg R$, $(P \leftrightarrow Q) \Rightarrow \neg P$ **.**

Solution:

Statement No	Statement	Rule	Identities used	Statements used
1	$\neg R$	P		
2	$S \vee R$	P		
3	$R \vee S$	T	Commutative law	2
4	$\neg R \to S$	T	$(A \to B) \Leftrightarrow (\neg A \vee B)$	3
5	S	T	Modus Ponens	1,4
6	$S \to \neg Q$	P		
7	$\neg Q$	T	Modus Ponens	5,6
8	$P \leftrightarrow Q$	P		
9	$(P \to Q) \wedge (Q \to P)$	T	Definition	8
10	$P \to Q$	T	Simplification	9
11	$\neg P$	T	Modus Tollens	7,10

42. Test the validity of the following argument
If I study then I will pass in the examination.
If I watch TV then I will not study.
I failed in the examination
Therefore, I watched TV.

Solution:

Take P: I study

Q: I pass in the examination

R: I watch TV

In symbolic form $P \to Q, \quad R \to \neg P, \quad \neg Q \quad \Rightarrow R$

Statement No	Statement	Rule	Identities used	Statements used
1	$\neg Q$	P		
2	$P \to Q$	P		
3	$\neg P$	T	Modus Tollens	1,2
4	$R \to \neg P$	P		
5	$\neg R \vee \neg P$	T	$(A \to B) \Leftrightarrow (\neg A \vee B)$	4
6	$\neg P \wedge (\neg R \vee \neg P)$	T		3,5
7	$\neg P$	T	Absorption law	6

So The given arguments is not valid.

43. If there was rain then traveling was difficult.
If they had an umbrella, then traveling was not difficult .
They had umbrella.
Therefore, there was no rain.
Show that these statements constitute a valid argument

Solution:

Take P: There was rain
Q: Traveling was difficult
R: They had an umbrella

To Prove, $P \to Q, \quad R \to \neg Q, \quad R \quad \Rightarrow \neg P$

Statement No	Statement	Rule	Identities used	Statements used
1	R	P		
2	$R \to \neg Q$	P		
3	$\neg Q$	T	Modus Ponens	1,2
4	$P \to Q$	P		
5	$\neg P$	T	Modus Tollens	3,4

44. State the principle of mathematical induction.

Ans: Let P(n) be a statement defined in the set of positive integers.
If P(n) is true for an initial value $n = n_0$ and
P(k+1) is true whenever P(k) is true $(P(k) \to P(k+1))$
Then P(n) is true for all the values $n \geq n_0$

45. Use mathematical induction to show that $n! \geq 2^{n-1}, n = 1, 2, 3, \ldots\ldots.$

Ans:

$1! = 1 = 2^0 = 2^{1-1}$ So the statement is true for the initial value n=1

Assume P(n) is true for n =k that is $k! \geq 2^{k-1}$,

$$(k+1)! = (k+1)k!$$
$$\geq (k+1)2^{k-1} \quad by\ \ Induction\ hypothesis$$
$$\geq 2\,2^{k-1} \qquad \sin ce\ k \geq 1 \quad k+1 \geq 2$$
$$= 2^k$$

P(k+1) : (k+1) ≥ 2^k is true.

Hence by mathematical principle we have $n! \geq 2^{n-1}$ $, n = 1, 2, 3, ...$

46. Show that $2^n > n^3$, for $n \geq 10$ using induction principle.

Ans:

$2^{10} = 1024 > 1000 = 10^3$ So the statement is true for the initial value n =10

Assume P(n) is true for n= k that is $2^k > k^3$

$$((k+1)^3 = k^3 + \left(3k^2 + 3k + 1\right)$$
$$< k^3 + k^3 \qquad for\ \ k \geq 10$$
$$= 2k^3$$
$$< 2\,2^k \qquad by\ \ induction\ \ hypothesis$$
$$= 2^{k+1}$$

P(k+1) : $2^{k+1} > (k+1)^3$ is true

$2^n > n^3$ is true for $n \geq 10$ by induction principle

47. Prove that $n^2 < 2^n$ for all positive integers $n > 4$.

Ans:

$5^2 = 25 < 32 = 2^5$ P(n) is true for an initial value n=5

Assume P(n) is true for n=k that is $k^2 < 2^k$

$$(k+1)^2 = k^2 + 2k + 1$$
$$< k^2 + k^2 \qquad \sin ce\ 2k+1 < k^2 \quad for\ k > 4$$
$$= 2k^2$$
$$< 2\,2^k \qquad by\ \ induction\ hypothesis$$
$$= 2^{k+1}$$

P(k+1) is true

So $n^2 < 2^n$ for all positive integers $n > 4$ is true by induction principle.

48. Use Mathematical induction to show that $\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+...\frac{1}{\sqrt{n}}>\sqrt{n},\ n\geq 2$

Solution:

For $n=2$, $P(2)=\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}=1+\frac{1}{1.14141}=1.7071>1.414=\sqrt{2}$

Assume $P(n)$ is true for $n=k$

$P(k)$: $\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+...\frac{1}{\sqrt{k}}>\sqrt{k}$.........(1)

Let us prove the result for $n=k+1$

$$P(k+1):\ \frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+...\frac{1}{\sqrt{k}}+\frac{1}{\sqrt{k+1}}$$

$$=\left(\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+...\frac{1}{\sqrt{k}}\right)+\frac{1}{\sqrt{k+1}}$$

$$>\sqrt{k}+\frac{1}{\sqrt{k+1}} \quad by\ \ induction\ \ hypothesis(1)$$

$$=\frac{\sqrt{k}\sqrt{k+1}+1}{\sqrt{k+1}}$$

$$>\frac{\sqrt{k}\sqrt{k}+1}{\sqrt{k+1}} \qquad \sin ce\ k+1\ >k \qquad \sqrt{k+1}\ >\sqrt{k}$$

$$=\frac{k+1}{\sqrt{k+1}}$$

$$=\sqrt{k+1}$$

$$\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+...\frac{1}{\sqrt{k}}+\frac{1}{\sqrt{k+1}}>\sqrt{k+1}$$

$P(k+1)$ is true. Therefore by induction principle $\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+...\frac{1}{\sqrt{n}}>\sqrt{n},\ n\geq 2$

49. Using mathematical induction show that $\sum_{k=1}^{n}k^2=\frac{n(n+1)(2n+1)}{6}$

Solution:

For $n=1$ $\quad P(1)\ =1^2=\frac{1(1+1)(2(1)+1)}{6}$

$P(n)$ n is true for $n=1$

Assume $P(n)$ is true for $n=m$

$$P(m) = \sum_{k=1}^{m} k^2 = 1^2 + 2^2 + 3^2 + ... + m^2 = \frac{m(m+1)(2m+1)}{6} ....(1)$$

Let us prove $P(n)$ is true for $n = m+1$

$$P(m+1) = \sum_{k=1}^{m+1} k^2 = 1^2 + 2^2 + 3^2 + ... + m^2 + (m+1)^2$$

$$= \left(1^2 + 2^2 + 3^2 + ... + m^2\right) + (m+1)^2$$

$$= \frac{m(m+1)(2m+1)}{6} + (m+1)^2 \quad by \quad induction\, hypothesis \; (1)$$

$$= (m+1)\left[\frac{m(2m+1)}{6} + (m+1)\right]$$

$$= (m+1)\left[\frac{\left(2m^2 + m\right) + 6m + 6}{6}\right]$$

$$= (m+1)\left[\frac{3m^2 + 7m + 6}{6}\right]$$

$$= (m+1)\frac{(m+2)(2m+3)}{6}$$

$$= \frac{(m+1)[(m+1)+1][2(m+1)+1]}{6}$$

So, $P(m+1)$ is true. By induction principle $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}, \; for\ all \;\; n$

50. Show by mathematical induction that $\left(a^n - b^n\right)$ is divisible by $(a-b)$ for all $n = 1,2,3,....$

Solution:

For $n = 1$ $P(1)$: $\left(a^1 - b^1\right)$ is divisible by $(a-b)$ is obvious

$P(n)$ n is true for $n = 1$

Assume $P(n)$ is true for $n = m$

$\left(a^m - b^m\right)$ is divisible by $(a-b)$ ……………(1)

Let us prove $P(n)$ is true for $n = m+1$

$$\left(a^{m+1} - b^{m+1}\right) = a^m a - b^m b$$
$$= a^m a - a b^m + ab^m - b^m b$$
$$= a\left(a^m - b^m\right) + (a-b)b^m$$

First term in RHS is divisible by $(a-b)$ by induction hypothesis (1)
Second term in RHS is divisible by $(a-b)$ is obvious

So RHS is divisible by $(a-b)$ and so LHS $\left(a^{m+1} - b^{m+1}\right)$ is divisible by $(a-b)$
So, $P(m+1)$ is true. By induction principle, $\left(a^n - b^n\right)$ is divisible by $(a-b)$ for all $n = 1,2,3,....$

51. Prove by mathematical induction, that for all $n \geq 1$ $n^3 + 2n$ is multiple of 3 (or divisible by 3).
Solution:

For $n = 1$ $P(1)$: $1^3 + 2(1) = 3$ is (multiple of 3)divisible by 3
$P(n)$ n is true for $n = 1$

Assume $P(n)$ is true for $n = m$
$m^3 + 2m$ is multiple of 3……………(1)

Let us prove $P(n)$ is true for $n = m+1$

$$(m+1)^3 + 2(m+1) = m^3 + 3m^2 + 3m + 1 + 2m + 2$$
$$= \left(m^3 + 2m\right) + \left(3m^2 + 3m + 3\right)$$
$$= \left(m^3 + 2m\right) + 3\left(m^2 + m + 1\right)$$

First term on RHS is multiple of 3 by induction hypothesis (1)
Second term on RHS is multiple of 3 is obvious
So RHS is multiple of 3 and so LHS $(m+1)^3 + 2(m+1)$ is multiple of 3
So, $P(m+1)$ is true. By induction principle $n^3 + 2n$ is multiple of 3 for all $n \geq 1$

52. Using mathematical induction, prove that $1^2 + 3^2 + 5^2 + ... + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$

Solution:

For $n = 1$ $P(1)$: $1^2 = \frac{1(2(1)-1)(2(1)+1)}{3}$
$P(n)$ n is true for $n = 1$

Assume $P(n)$ is true for $n = m$

$$P(m) = 1^2 + 3^2 + 5^2 + ... + (2m-1)^2 = \frac{m(2m-1)(2m+1)}{3} \quad ....(1)$$

Let us prove $P(n)$ is true for $n = m+1$

$$\begin{aligned}
P(m+1) &= 1^2 + 3^2 + 5^2 + ... + (2m-1)^2 + (2(m+1)-1)^2 \\
&= \left[1^2 + 3^2 + 5^2 + ... + (2m-1)^2\right] + (2(m+1)-1)^2 \\
&= \left[\frac{m(2m-1)(2m+1)}{3}\right] + (2m+1)^2 \\
&= (2m+1)\left[\frac{m(2m-1)}{3} + (2m+1)\right] \\
&= (2m+1)\left[\frac{(2m^2 - m) + 6m + 3}{3}\right] \\
&= (2m+1)\left[\frac{2m^2 + 5m + 3}{3}\right] \\
&= (2m+1)\left[\frac{(2m+3)(m+1)}{3}\right] \\
&= \frac{(m+1)(2m+1)(2m+3)}{3} \\
&= \frac{(m+1)(2m+1)(2(m+1)+1)}{3}
\end{aligned}$$

So, $P(m+1)$ is true. By induction principle $1^2 + 3^2 + 5^2 + ... + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$

53. Prove that $3^n > n^3$, $n \geq 4$

Solution:

Let $P(n) = 3^n > n^3$ (or) $n^3 < 3^n$

Assume $P(1)$: $1^3 \propto 3^1$; $P(k) = k^3 \propto 3^k$

Claim:

$= P(k+1) = (k+1)^3 < 3^{k+1}$,Now $(k+1)^3 = k^3 + 3k^2 + 3k + 1$

$\because k^3 < 3^k \Rightarrow 3k^2 < 3^k \Rightarrow 3k + 1 < 3^k$

$\therefore P(k+1) = k^3 + 3k^2 + 3k + 1 < 3^k + 3^k + 3^k = 3.3^k = 3^{k+1}$

$\therefore (k+1)^3 < 3^{k+1}$

$\Rightarrow$ P(k+1) is true $\Rightarrow$ P(n) is true $\forall$n by mathematical induction.

54. Prove by mathematical induction, that for all n ≥ 1 , n^3 + 2n is a multiple of 3. (N/D 2015, 2010)

Solution:

P(1) is true

Assume that P(k) is true

ie) $k^3 + 2k$ is a multiple of 3 $\forall$ k ≥ 1.

To prove P(k+1) is true $\forall$ k ≥1

Consider $(k+1)^3 + 2(k+1) = (k+1)[(k+1)^2+2]$

$= (k+1)(k^2+2k+1+2)$

$= (k+1)(k^2+2k+3) = (k^2+k)(k+1)+3(k+1)$

Clearly this is a multiple of 3 since 3(k+1) is a multiple of 3 and (k^2+2k) is a multiple of 3.

Hence P(k+1) is true $\forall$ k ϵ z.

$\Rightarrow$ P(n) is true $\forall$n.

Practice Problems:

1. Prove that by mathematical induction, that for all $n \geq 1, n^3 + 2n$ is a multiple of 3.

2. Use mathematical induction to show that $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$.

3. Use mathematical induction to show that $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \dots\dots\dots + \frac{1}{\sqrt{n}} > \sqrt{n}$, $n \geq 2$.

4. Prove by induction $1 + 2 + 2^2 + \dots\dots\dots + 2^{n-1} + 2^n = 2^{n+1} - 1$.

5. Use mathematical induction to show that $n^3 - n$ is divisible by 3, for $n \in z^+$.

6. Using induction principle, prove that $n^3 + 2n$ is divisible by 3.

7. Using mathematical induction, prove that $1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$

8. Using mathematical induction, show that for all positive integer n, $3^{2n+1} + 2^{n+2}$ is divisible by 7.

9. Prove that $8^n - 3^n$ is a multiple of 5 by using method of induction.

10. Prove by mathematical induction, that $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \dots\dots\dots + \frac{1}{n(n+1)} = \frac{n}{n+1}$.

11. Use mathematical induction to prove that $3^n + 7^n - 2$ is divisible by 8, for all $n \geq 1$.

12. Show by mathematical induction that $a^n - b^n$ is divisible by $a - b$ for all $n = 1, 2, \dots\dots$.

13. Prove by mathematical induction that

$$1.2.3 + 2.3.4 + 3.4.5 + \dots\dots\dots + n(n+1)(n+2) = \frac{1}{4} n(n+1)(n+2)(n+3).$$

14.Show that $1+2+3+........+n=\frac{n(n+1)}{2}$ by using the principle of mathematical induction.

15. Prove by mathematical induction that $6^{n+2}+7^{2n+1}$ is divisible by 43 for each positive integer n.

16. Use mathematical induction to prove the inequality $n<2^n$ for all positive integers n.

17. State the strong induction . Prove that a positive integer greater than 1 is either a prime number or it can be written as product of prime numbers.

18. Let m any odd positive integer, then prove that there exists a positive integer n such that m divides 2^n-1.

19. Prove that the number of subsets of set having n elements is 2^n.

OBJECTIVES

1.Which of the following statement is the negation of the statement " 2 is even and -3 is negative"?

a)2 is even and -3 is not negative b)2 is odd and -3 is not negative

c)2 is not odd and -3 is not negative d)2 is odd or -3 is not negative **Ans (d)**

2.The contra positive of $q\to p$ is a) $p\to q$ b) $\neg p\to\neg q$ c) $\neg q\to\neg p$ d) $p\to\neg q$ **Ans (b)**

3.What is the converse of the assertion I stay only if you go?

a) I stay if you go b)if you don't go then I don't stay

c)if I stay then you go d)if you don't stay then you go **Ans (a)**

4.$PVT\Leftrightarrow T$ is called a)identity law b)complement law c)dominant law d)idempotent law **Ans (c)**

5.The statement $PV\neg P$ is a a)contradiction b)tautology c)contrapositive d)inverse **Ans (b)**

6) Dual of $\neg(p\leftrightarrow Q)=(P\wedge\neg Q)V(\neg P\wedge Q)$

a) $\neg(P\leftrightarrow Q)\equiv(PV\neg Q)V(\neg PVQ)$ b) $(P\leftrightarrow Q)\equiv(\neg PVQ)V(PV\neg Q)$

c) $\neg(P\leftrightarrow Q)\equiv(PV\neg Q)\wedge(\neg PVQ)$ d) $\neg(P\leftrightarrow Q)\equiv(\neg PVQ)\wedge(PV\neg Q)$ **Ans (c)**

7.The rule if a formula S can be derived from another formula R and A set of premises,then the statement $R\to S$ can be derive from the set of premises is called

a)Rule CP b)Rule T c)Rule P d)Rule US

Ans (a)

8.The statement $(PVQ) \wedge (P\rightarrow R) \wedge (Q\rightarrow R)$implies a)R b)P c)Q d)$P\wedge Q$ **Ans (a)**

9.The statement $\neg (P\leftrightarrow Q)$ is equivalent to a)$P\leftrightarrow \neg Q$ b) $\neg P\leftrightarrow \neg Q$ c)$P\rightarrow \neg Q$ d) $\neg P\rightarrow \neg Q$ **Ans (a)**

10. $\neg P\rightarrow Q=$ a)$QV\neg P$ b)$Q\wedge P$ c)PVQ d) $\neg P\wedge Q$ **Ans (c)**

11. $\neg P,PVQ =>$ a) Q b) $\neg P$ c)PVQ d)$P\wedge Q$ **Ans (a)**

12.$((P\rightarrow Q)V(\neg PV(Q\rightarrow R))VT=$ a)$P\rightarrow Q$ b) $\neg P$ c)T d)F **Ans (c)**

13.A compound proposition $P=P(P_1,P_2,\ldots\ldots.P_n)$which is true for every truth values for $P_1,P_2,\ldots.,P_n$ is called

a)Contradiction b)Tautology c)Negation d)Implication **Ans (b)**

14) $(P\rightarrow \neg P)\rightarrow \neg P$ is equivalent to a)T b)F c)P d) $\neg P$ **Ans (c)**

15)The dual of $\neg P\rightarrow(P\rightarrow Q)$ is

a)$PV(\neg P\wedge Q)$ b) $\neg (\neg P) \wedge (\neg P\wedge Q)$ c)$P\rightarrow \neg (P\rightarrow Q)$ d)$(\neg P\wedge Q) \wedge \neg P$ **Ans (b)**

16.In proving that $P\rightarrow(Q\rightarrow S)$ follows from the premises $P\rightarrow(Q\rightarrow R)$ and $Q\rightarrow(R\rightarrow S)$ using CP rule, the additional premises is a)Q b)$Q\rightarrow R$ c)P d) $\neg P$
Ans (c)

17.Let P is sunny this afternoon ,Q is colder than yesterday and R is we will go for swimming. Then the statement if it is not sunny this afternoon and it is colder than yesterday, then we will go for swimming is

a)$(\neg P\wedge Q)\rightarrow R$ b)$(P\wedge \neg Q)\rightarrow \neg R$ c)$(\neg PVQ)\rightarrow R$ d)$(\neg P\wedge Q)VR$ **Ans (a)**

18.Which of the following statement is a contradiction?

a)$(P\rightarrow \neg P)\rightarrow \neg P$ b)$(P\rightarrow(P\vee Q)$ c)$(\neg Q\rightarrow P) \wedge Q$ d) $PV(P\rightarrow Q)$ **Ans (a)**

19.What is the dual of $(P\rightarrow Q)\rightarrow(\neg Q\rightarrow \neg P) \equiv T$,

a)$(\neg P\rightarrow Q)\wedge(Q\wedge \neg P)\equiv F$ b) $\neg (P\wedge Q)\wedge(Q\wedge P)\equiv T$

c)$(\neg P\wedge)\wedge(Q\wedge \neg P)\equiv F$ d) $\neg (\neg PVQ) \wedge (Q\wedge \neg P)\equiv F$ **Ans (d)**

20.$(PVQ)\wedge \neg (\neg P\wedge Q)$ is equivalent to a)P b)Q c)PVQ d)$P\wedge Q$ **Ans (a)**

21.Which one is the contra positive of $Q\rightarrow P$?

a)$P\rightarrow Q$ b) $\neg P\rightarrow \neg Q$ c) $\neg Q\rightarrow \neg P$ d) $\neg PVQ$ **Ans (b)**

22.The statement $(P\wedge Q)=>P$ is a

a)contradiction b)tautology c)inconsistent d)consistent **Ans (d)**

23.The dual of $\neg (P\wedge Q)VT$ is

a)$(PVQ)\wedge F$ b)$(PVQ)\wedge T$ c)$(P\wedge Q)VF$ d) $\neg$ $(PVQ)\wedge F$ **Ans (d)**

24.Which of the following is a statement?

(A) Open the door. (B) Do your homework. (C) Switch on the fan **(D)** Two plus two is four. **Ans (D)**

25.Which of the following is a statement in Logic?
(A) Go away (B) How beautiful! (C) $x > 5$ **(D)** $2 = 3$ **Ans (D)**

26. $\sim (p \vee q)$ is (A) $\sim p \vee q$ (B) $p \vee \sim q$ (C) $\sim p \vee \sim q$ **(D)** $\sim p \wedge \sim q$ **Ans (D)**

27.If p: The sun has set, q: The moon has raised, then symbolically the statement 'The sun has not set or the moon has not risen' is written as
(A) $p \wedge \sim q$ (B) $\sim q \vee p$ (C) $\sim p \wedge q$ **(D)** $\sim p \vee \sim q$ **Ans (D)**

28.The inverse of logical statement $p \rightarrow q$ is
(A) $\sim p \rightarrow \sim q$ (B) $p \leftrightarrow q$ (C) $q \rightarrow p$ (D) $q \leftrightarrow p$ **Ans (A)**

29.Let p: Mathematics is interesting, q: Mathematics is difficult, then the symbol $p \rightarrow q$ means
(A) Mathematics is interesting implies that Mathematics is difficult.
(B) Mathematics is interesting is implied by Mathematics is difficult.
(C) Mathematics is interesting and Mathematics is difficult.
(D) Mathematics is interesting or Mathematics is difficult. **Ans (A)**

30.Which of the following is logically equivalent to $\sim(p \wedge q)$
(A) $p \wedge q$ **(B)** $\sim p \vee \sim q$ (C) $\sim(p \vee q)$ (D) $\sim p \wedge \sim q$ **Ans (B)**

31.$\sim (p \rightarrow q)$ is equivalent to
(A)$p \wedge \sim q$ (B) $\sim p \vee q$ (C) $p \vee \sim q$ (D) $\sim p \wedge \sim q$ **Ans (A)**

32.Contrapositive of $p \rightarrow q$ is
(A) $q \rightarrow p$ (B) $\sim q \rightarrow p$ **(C)** $\sim q \rightarrow \sim p$ (D) $q \rightarrow \sim p$ **Ans (C)**

33.A compound statement $p \rightarrow q$ is false only when
(A)p is true and q is false. (B) p is false but q is true.
(C) atleast one of p or q is false. (D) both p and q are false. **Ans (A)**

34.Every conditional statement is equivalent to
(A) its contrapositive (B) its inverse (C) its converse (D)only itself **Ans (A)**

35Statement $\sim p \leftrightarrow \sim q \equiv p \leftrightarrow q$ is
(A)a tautology (B) a contradiction (C) contingency (D) proposition **Ans (A)**

36.Given that p is 'false' and q is 'true' then the statement which is 'false' is
(A) $\sim p \rightarrow \sim q$ (B) $p \rightarrow (q \wedge p)$ (C) $p \rightarrow \sim q$ (D) $q \rightarrow \sim p$ **Ans (A)**

37.Dual of the statement $(p \wedge q) \vee \sim q \equiv p \vee \sim q$ is
(A) $(p \vee q) \vee \sim q \equiv p \vee \sim q$ (B) $(p \wedge q) \wedge \sim q \equiv p \wedge \sim q$
(C) $(p \vee q) \wedge \sim q \equiv p \wedge \sim q$ (D) $(\sim p \vee \sim q) \wedge q \equiv \sim p \wedge q$ **Ans (C)**

38.$\sim[p \vee (\sim q)]$ is equal to
(A) $\sim p \vee q$ **(B)** $(\sim p) \wedge q$ (C) $\sim p \vee \sim p$ (D) $\sim p \wedge \sim q$ **Ans (B)**

39.Write Negation of 'For every natural number x, $x + 5 > 4$'.

(A) $\forall x \in N, x + 5 < 4$ (B) $\forall x \in N, x - 5 < 4$ (C) For every integer x, $x + 5 < 4$

(D) There exists a natural number x, forwhich $x + 5 \leq 4$ **Ans (D)**

40. If p is false and q is true, then

(A) $p \wedge q$ is true (B) $p \vee \sim q$ is true (C) $q \rightarrow p$ is true **(D)** $p \rightarrow q$ is true **Ans (D)**

41.If p and q have truth value 'F' then $(\sim p \vee q) \leftrightarrow \sim(p \wedge q)$ and $\sim p \leftrightarrow (p \rightarrow \sim q)$ respectively are

(A) T, T (B) F, F (C) T, F (D) F, T **Ans (A)**

42.Which of the following is logically equivalent to $\sim[p \rightarrow (p \vee \sim q)]$?

(A) $p \vee (\sim p \wedge q)$ **(B)** $p \wedge (\sim p \wedge q)$ (C) $p \wedge (p \vee \sim q)$ (D) $p \vee (p \wedge \sim q)$ **Ans (B)**

43.If $\sim q \vee p$ is F then which of the following is correct?

(A) $p \leftrightarrow q$ is T **(B)** $p \rightarrow q$ is T (C) $q \rightarrow p$ is T (D) $p \rightarrow q$ is F **Ans (B)**

44.Which of the following is true?

(A) $p \wedge \sim p \equiv T$ (B) $p \vee \sim p \equiv F$ (C) $p \rightarrow q \equiv q \rightarrow p$ **(D)** $p \rightarrow q \equiv (\sim q) \rightarrow (\sim p)$ **Ans (D)**

45.The statement $(p \wedge q) \rightarrow p$ is

(A) a contradiction. **(B)**a tautology .(C) either (A) or (B) (D) a contingency. **Ans (B)**

46.Negation of the statement: "If Dhonilooses the toss then the team wins", is

(A) Dhoni does not lose the toss and theteam does not win.

(B) Dhoni loses the toss but the team doesnot win.

(C) Either Dhoni loses the toss or the teamwins. (D) Dhoni loses the toss iff the team wins. **Ans (A)**

47.If $p \Rightarrow (\sim p \vee q)$ is false, the truth values of p and q respectively, are

(A) F, T (B) F, F (C) T, T **(D)** T, F **Ans (D)**

48.The logically equivalent statement of $p \leftrightarrow q$ is

(A) $(p \wedge q) \vee (q \rightarrow p)$ (B) $(p \wedge q) \rightarrow (p \vee q)$ **(C)** $(p \rightarrow q) \wedge (q \rightarrow p)$ (D) $(p \wedge q) \vee (p \wedge q)$ **Ans (C)**

49) By induction hypothesis, the series $1^2 + 2^2 + 3^2 + \ldots + p^2$ can be proved equivalent to ____________

a) $\frac{p^2 + 2^k}{7}$ b) $\frac{p(p+1)(2p+1)}{6}$ c) $\frac{p(p+1)}{4}$ d) $p + p^2$ **Ans: b**

50) For any positive integer m ______ is divisible by 4.

a) $5m^2 + 2$ b) $3m + 1$ c) m^2+3 d) $m^3 + 3m$ **Ans: d**

51) According to principle of mathematical induction, if $P(k+1) = m^{(k+1)} + 5$ is true then _____ must be true.

a) $P(k) = 3m^k$ b) $P(k) = m^k + 5$ c) $P(k) = m^{k+2} + 5$ d) $P(k) = m^k$ **Ans: b**

52) What is the induction hypothesis assumption for the inequality $m! > 2^m$ where m>=4?

a) $for\ m = k,\ (k+1)! > 2^k\ holds$ b) $for\ m = k,\ k! > 2^k\ holds$

c) $for\ m = k,\ k! > 3^k\ holds$ d) $for\ m = k,\ k! > 2^{k+1}\ holds$ **Ans: b**

53. For all $n \in N - \{1\}$, $7^{2n} - 48n - 1$ is divisible by ....

(a) 25 (b) 26 (c) 1234 **(d) 2304**

54. $\forall\, n \in N$, P(n):$2.7^n + 3.5^n - 5$ is divisible by ....

(a) 64 (b) 676 (c) 17 **(d) 24**

55. $\forall\, n \geq 2, n^2(n^4 - 1)$ is divisible by ....

(a) 60 (b) 50 (c) 40 (d) 70

56. For $n \in N,\ 10^{n-2} > 81n$, if….

(a) $n > 5$ **(b) $n \geq 5$** (c) $n < 5$ (d) $n > 6$

57. For each $n \in N$**,** the correct statement is …..

(a) $2^n < n$ (b) $n^2 > 2^n$ (c) $\boldsymbol{n^4 < 10^n}$ (d) $2^{3n} > 7n + 1$

58. If $a_n = 2^{2^n} + 1$, then for n>1, $n \in N$, last digit of a_n is…..

(a) 3 (b) 5 (c) 8 (d) 7

59. If $P(n): 4^n \,/\, (n+1) < (2n)! \,/\, (n!)^2$, then P(n) is true for ……

(a) $n \geq 1$ (b) $n > 0$ (c) $n < 0$ (d) $\boldsymbol{n \geq 2},\ n \in N$

60. By principle of mathematical induction,
$\forall n \in N\ cos\ 9\ cos\ 2\theta\ cos\ 4\theta \cdots cos[(2^{n-1})\theta] =$ … …

(a) $\boldsymbol{sin\ 2^n\theta \,/\, 2^n\ sin\,\theta}$ (b) $cos\ 2^n\theta \,/\, 2^n\ sin\,\theta$

(c) $sin\ 2^n\theta \,/\, 2^{n-1}\ sin\,\theta$ (d) $\sin 2^{n-1}\theta \big/ 2^n \sin\theta$

61. By principle of mathematical induction, $\forall\, n \in N$,
$1/(1.2.3) + 1/(2.3.4) + \cdots + 1\,/\,\{n(n+1)(n+2)\} =$………

(a) n(n+1) /4(n+2)(n+3) **(b) n(n+3) /4(n+l)(n+2)**

(c) n{n+2) /4(n+1){n+3) (d) None of these

62. By principle of mathematical induction, $\forall\, n \in N,\ 5^{2n+1} + 3^{n+2}.2^{n-1}$ is divisible by…..

(a) 19 (b) 18 (c) 17 (d) 14

63. The product of three consecutive natural numbers is divisible by ……

(a) 6 (b) 5 (c) 7 (d) 4

64. $\forall\, n \in N, a^n - b^n$ is always divisible by….. (a and b are distinct rational nos)

(a) 2a-b (b) a+b **(c) a-b** (d) a-2b

65. If $x^{2n-1} + y^{2n-1}$ is divisible by x+y, then n is…

(a) Positive integer (b) only for an even positive integer

(c) an odd positive integer (d) $\forall\, n \in N, n \geq 2$

66. The inequality $n! > 2^{n-1}$ is true for…..

(a) $\boldsymbol{n > 2,\ n \in N}$ (b) $n < 2$ (c) $\forall\, n \in N$ (d) n < 1

67. The smallest positive integer n for which $n! < \left\{\frac{n+1}{2}\right\}^n$ holds, is ….

(a) 1 **(b) 2** (c) 3 (d) 4

68. The greatest positive integer, which divides (n+2)(n+3) (n+4)(n+5)(n+6) $\forall\, n \in N$ is….

(a) 120 (b) 4 (c) 240 (d) 24

69. $x(x^{n-1} - n\alpha^{n-1}) + \alpha^n(n-1)$ is divisible by $(x-\alpha)^2$ for…..

(a) $n > 1$ (b) $n > 2$ **(c)** $\boldsymbol{\forall\, n \in N}$ (d) n < 2

70. For each $n \in N$, $3^{2n} - 1$ is divisible by ……

(a) **8** (b) 16 (c) 32 (d) 18

71. For each $n \in N$, $2^{3n} - 7n - 1$ is divisible by ……

(a) 64 (b) 36 **(c) 49** (d) 25

72. For each $n \in N$, $10^{2n-1} + 1$ is divisible by ……

(a) **11** (b) 13 (c) 9 (d) 15

73. For each $n \in N, 2(4^{2n+1} + 3^{n+1})$ is divisible by ……

(a) 2 (b) 9 (c) 3 (d) 11

74. Let $P(n): n^2 + n + 1$ is and odd integer. If it is assumed that P(k) is true => P(k+1) is true. Therefore, P(n) is true…

(a) for n>1 **(b)** $\forall\, n \in N$

(c) for n>2 (d) for n > 3

75. Let $P(n): 3^n < n!, n \in N$, then P(n) is true…

(a) for $n \geq 6$ **(b)** $\boldsymbol{for\ n \geq 7, n \in N}$

(c) for $n \geq 3$ (d) $\forall\ n$

76. Let $P(n): 1 + 3 + 5 + \cdots + (2n - 1) = n^2$, is…

(a) true for n>1 **(b)** $\boldsymbol{true\ \forall\ n \in N}$

(c) true for no n (d) true for n< 1

77. If $\forall\ \boldsymbol{n} \in \boldsymbol{N}$, P(n) is a statement such that, if P(k) is true => P(k+1) is true for $\boldsymbol{k} \in \boldsymbol{N}$, then P(n) is true…

(a) $\forall\ n > 1$ **(b)** $\forall\ n \in N$

(c) $\forall\ n > 2$ (d) $\forall n < 2$

78. Let $P(n): 1 + 3 + 5 + \cdots + (2^n - 1) = 3 + n^2$, then which of the following is true?

(a) P(1) is true **(b) P(k) is true=> P(k+1) is true**

(c) P(k) is true, P(k+1) is not true (d) P (2) is true

79. If matrix $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ $and\ I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then which one of the following holds $\forall\ n \in N$, (use PMI)

(a) $A^n = n.A - (n-1)I$ (b) $A^n = 2^{n-1}.A + (n-1)I$

(c) $A^n = n.A + (n-1)I$ (d) $A^n = 2^{n-1}.A - (n-1)I$

80. $S_n = 2.7^n + 3.5^n - 5$, $n \in N$ is divisible by the multiple of……

(a) 5 (b) 7 **(c) 24** (d) 25

81. $10^n + 3(4^{n+2}) + 5$, $n \in N$ is divisible by……

(a) 7 (b) 5 **(c) 9** (d) 17

82. $\forall\ \boldsymbol{n} \in \boldsymbol{N}, \left(3 + 5^{\frac{1}{2}}\right)^n + \left(3 - 5^{\frac{1}{2}}\right)^n$ is…

(a) Even natural number (b) Odd natural number

(c) Any natural number (d) Rational number

83. The remainder, when 5^{99} is divided by 13, is ……

(a) 6 **(b) 8** (c) 9 (d) 10

84. For all positive integral values of n, $n^{3n} - 2n + 1$ is divisible by ……

(a) 2 (b) 4 (c) 8 (d) 12

85. If $n \in N, then\ 11^{n+2} + 12^{2^n+1}$ is divisible by ……

(a) 113 (b) 123 **(c) 133** (d) 143

86. If $n \in N, P(n): 2^n(n-1)! < n^n$ is true, if ……

(a) n<2 **(b) n>2** (c) $n \geq 2$ (d) n > 3