18CSE390T
Computer Vision

Incremental Refinement

Incremental Refinement

- Nearest pixel – integer pixel
- Higher accuracy is required for stabilization or stitching
- Sub-pixel estimates
  - Evaluate several values (u,v) around the best value
  - Interpolate the matching score to find the analytic minimum
- Gradient descent on SSD energy function

Incremental Refinement

- SSD energy and Taylor series expansion Lucas and Kanade (1981)

$$\begin{aligned} E_{\text{LK\_SSD}}(\boldsymbol{u}+\Delta\boldsymbol{u}) &= \sum_i [I_1(\boldsymbol{x}_i+\boldsymbol{u}+\Delta\boldsymbol{u}) - I_0(\boldsymbol{x}_i)]^2 \\ &\approx \sum_i [I_1(\boldsymbol{x}_i+\boldsymbol{u}) + \boldsymbol{J}_1(\boldsymbol{x}_i+\boldsymbol{u})\Delta\boldsymbol{u} - I_0(\boldsymbol{x}_i)]^2 \\ &= \sum_i [\boldsymbol{J}_1(\boldsymbol{x}_i+\boldsymbol{u})\Delta\boldsymbol{u} + e_i]^2, \end{aligned}$$

Image gradient or Jacobian at (x+u)

$$\boldsymbol{J}_1(\boldsymbol{x}_i+\boldsymbol{u}) = \nabla I_1(\boldsymbol{x}_i+\boldsymbol{u}) = (\frac{\partial I_1}{\partial x}, \frac{\partial I_1}{\partial y})(\boldsymbol{x}_i+\boldsymbol{u})$$

Current intensity error (residual error)

$$e_i = I_1(\boldsymbol{x}_i+\boldsymbol{u}) - I_0(\boldsymbol{x}_i)$$

Incremental Refinement

$$E_{\mathrm{LK-SSD}}(\boldsymbol{u}+\Delta\boldsymbol{u}) \approx \sum_i [\boldsymbol{J}_1(\boldsymbol{x}_i+\boldsymbol{u})\Delta\boldsymbol{u}+e_i]^2$$

$$I_x u + I_y v + I_t = 0$$

Spatial derivative

temporal derivative

Optical flow constraint or brightness constancy constraint

Incremental Refinement

$$E_{\mathrm{LK-SSD}}(\boldsymbol{u}+\Delta\boldsymbol{u}) \approx \sum_i [\boldsymbol{J}_1(\boldsymbol{x}_i+\boldsymbol{u})\Delta\boldsymbol{u} + e_i]^2$$

$$\Rightarrow \quad \boldsymbol{A}\Delta\boldsymbol{u} = \boldsymbol{b}$$

$$\boldsymbol{A} = \sum_i \boldsymbol{J}_1^T(\boldsymbol{x}_i+\boldsymbol{u})\boldsymbol{J}_1(\boldsymbol{x}_i+\boldsymbol{u}) \qquad \boldsymbol{b} = -\sum_i e_i \boldsymbol{J}_1^T(\boldsymbol{x}_i+\boldsymbol{u})$$

Gaussian-Newton approximation of the Hessian

Gradient-weighted residual vector

Incremental Refinement

- For efficiency
  - $$\boldsymbol{J}_1(\boldsymbol{x}_i + \boldsymbol{u}) \approx \boldsymbol{J}_0(\boldsymbol{x}_i)$$
  - Precompute the Hessian and Jacobian image: save significant computation
  - Precompute the inner product between the gradient field and shifted version of I1 allows the iterative re-computation of ei to be performed in constant time (independent of the number of pixels)

Incremental Refinement

- Iterations
  - The effectiveness relies on the quality of Taylor series approximation
  - When far away from the true displacement (say, 1–2 pixels), several iterations may be needed
  - It is possible to estimate a value for J_1 using a least squares fit to a series of larger displacements in order to increase the range of convergence (Jurie and Dhome 2002) or to "learn" a special-purpose recognizer for a given patch

Incremental Refinement

- Stopping criterion
  - monitor the magnitude of the displacement correction |u| and to stop when it drops below a certain threshold (say, 1/10 of a pixel)

- For larger motions
  - combine the incremental update rule with a hierarchical coarse-to-fine search strategy

Incremental Refinement

- Poorly conditioned because of lack of two-dimensional texture in the patch being aligned