29/7/21

UNIT-II

Combinatorics

Pigeonhole principle:-

If n pigeons are assigned to m pigeonholes and $n \geq m$ then atleast one of the pigeonhole will contain two or more pigeons

Generalized pigeonhole principle:-

If n pigeons are assigned to n pigeonholes, $n \geq m$ atleast one of the pigeonhole with contains $\left[\frac{n-1}{m}\right]+1$ pigeons

Note:- $[x]$ - greatest integer $\leq x$

$[4.1] - 4$ $\quad [4.7] = 4$ $\quad [4.9] = 4$

(1) Show that if seven colours are used to paint 50 bicycles, then atleast 8 bicycles will have the same colours

$n = 50$ $\quad m = 8$.

$$\left[\frac{n-1}{m}\right]+1 = \left[\frac{50-1}{8}\right]+1$$

$$= \frac{49}{8}+1$$

$$= \frac{49+8}{8} = \frac{57}{8} = [7.1]$$

$$= 7$$

(3) S.T of 30 dictionaries in a library contain a total of 61327 pages, then one of the dictionary must have 2045 pages

$n = 61327 \quad m = 30 \quad n > m$

By P.H principle

$$\left\lceil \frac{n-1}{m} \right\rceil + 1 = \left\lceil \frac{61327-1}{m} \right\rceil + 1$$

$$= \frac{61327-1}{30} + 1$$

$$= \lceil 2044.2 \rceil + 1$$

$$= 2044 + 1$$

$= 2045$ pages.

(4) Prove that is any group of 6 people atleast three must be mutual friends (or) three must be mutual strangers

Let A be one of the 6 people and the remaining 5 people is divided into two sets.

{friends of A}, {stranger to A}

{friends of A}	{stranger to A}
1	4
2	3
3	2
4	1
5	

$n = 5 \quad m = 2$

By P.H principle $\left\lceil \frac{n-1}{m} \right\rceil + 1 = \left\lceil \frac{5-1}{2} \right\rceil + 1$

$$= \frac{4}{2} + 1$$

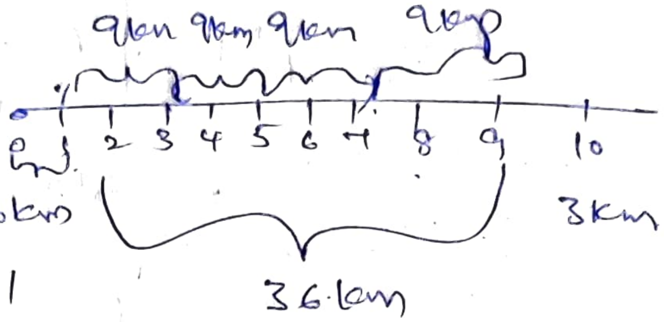
$= 3$. mutual frien[ds] or mutual st[rangers]

5) A man Rucked for 10 hrs and covered a distance of 45 KM. It is known that he hiked 6 km in the first hour, 3 km in the last hr. S.T he must have hiked 9 km within a certain period of consecutive hours.

$n = 36$

$m = 4$

By P.H principle



$$\left[\frac{n-1}{m}\right]+1 = \frac{36-1}{4}+1$$

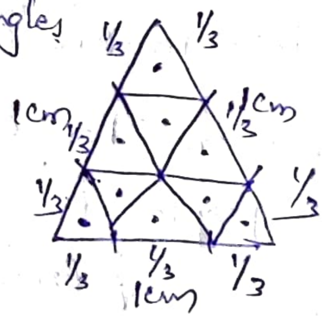
$$= \frac{35}{4}+1$$

$$= [8.7]+1$$

$$= 9.$$

3) If we select 10 interior points in the interior of a Δ^{le} of side 1 cm. S.T there will be atleast two points whose distance is $< 1/3$

:- Divide the triangle into 9 subtriangles of side 1/3 cm



$n = 10$. point $m = 9$ subtriangle

By P.H Principle

$$\left[\frac{n-1}{m}\right]+1 = \left[\frac{10-1}{9}\right]+1 = 1+1 = 2$$ points lie in with distance $< 1/3$

Inclusion - Exclusion principles :-

It is a Counting technique which generalises the familiar method of obtaining no. of element in Union of set

$|A|$ = Cardinality of A

If A & B are two sets

$$|A \cup B| = |A| + |B| - |A \cap B|$$

For Three sets.

$$|A\cup B\cup C| = |A|+|B|+|C|-|A\cap B|-|B\cap C|-|A\cap C|+|A\cap B\cap C|$$

1) There are 250 students in an Engineering clg of these 188 have taken a Course in Fortorn, 100 have taken C-Programming, 35 students taken Java, Further 88 in both Fortan & C, 23 in C & Java, 29 in Forthan & Java, 19 all of these, How many students not taken any of the sets programming Language.

Sol. Given $|F| = 188$ $|C| = 100$ $|J| = 35$, $|F\cap C| = 88$, $|C\cap J| = 23$

$|F\cap J| = 29$, $|F\cap C\cap J| = 19$.

$$|F\cup C\cup J| = |F|+|C|+|J|-|F\cap C|-|C\cap J|-|F\cap J|+|F\cap C\cap J|$$

$$= 188+100+35-86-23-29+19 = 202$$

No. of students who have not taken any of the Course $= 250-202 = 48$ Students.

2) Find the no. of integers b/w 1 & 250 that are not divisible by ~~2,3,5~~ any of the integers 2,3,5,7 ?

Sol. Let A, B, C, D be the no. of integers divisible by 2,3,5,7 respectively

$|A| = \left\lfloor\frac{250}{2}\right\rfloor = 125$ $|B| = \left\lfloor\frac{250}{3}\right\rfloor = 83$, $|C| = \left\lfloor\frac{250}{5}\right\rfloor = 50$

$|D| = \left\lfloor\frac{250}{7}\right\rfloor = 35$, $|A\cap B| = \left\lfloor\frac{250}{2\times3}\right\rfloor = 41$ $|A\cap C| = \left\lfloor\frac{250}{2\times5}\right\rfloor = 25$

$|A\cap D| = \left\lfloor\frac{250}{2\times7}\right\rfloor = 17$, $|B\cap C| = \left\lfloor\frac{250}{3\times5}\right\rfloor = 16$ $|B\cap D| = \left\lfloor\frac{250}{3\times7}\right\rfloor = 1$

$|A\cap B\cap C| = \left\lfloor\frac{250}{(2)(3\times5)}\right\rfloor = 8$ $|A\cap B\cap D| = \left\lfloor\frac{250}{2\times3\times7}\right\rfloor = 5$

$|A\cap C\cap D| = \left\lfloor\frac{250}{2\times5\times7}\right\rfloor = 3$ $|B\cap C\cap D| = \left\lfloor\frac{250}{3\times7}\right\rfloor = 2$ $|A\cap B\cap C\cap D| = \left\lfloor\frac{250}{2357}\right\rfloor = 1$

$|C\cap D| = \left\lfloor\frac{250}{5\cdot7}\right\rfloor = 7$

$|A \cup B \cup C \cup D| = |A| + |B| + |C| + |D| - |A \cap B| - |B \cap C| - |A \cap C| - |A \cap D|$
$- |B \cap D| - |C \cap D| + |A \cap B \cap C| + |A \cap B \cap D|$
$+ |B \cap C \cap D| + |A \cap C \cap D| - |A \cap B \cap C \cap D|$
$= 125 + 83 + 50 + 35 - 41 - 25 - 17 - 16 - 11 - 7$
$+ 8 + 5 + 3 + 2 - 1$
$= 193$

193 numbers are divisible by 2, 3, 5, 7 from 1 to 250

No. of integers which are not divisible by 2, 3, 5, 7 $= 250 - 193$
$= 57$

H.W

(3) Find the no. of integers which are divisible by 2, 3, 5, 6 ? Ans — 389

(4) There are 345 students at a College to have taken physics, 220 have taken maths, 175 both physics & Maths. How many have taken a course either or Maths

(A) Let A, B be the set of students taken physics & Maths respectively

$|A| = 345 \quad |B| = 220 \quad |A \cap B| = 175$

$|A \cup B| = |A| + |B| - |A \cap B|$

$= 345 + 220 - 175$

$= 565 - 175$

$= 390.$

$$\begin{array}{r} 345 \\ 220 \\ \hline 565 \\ 175 \\ \hline 390 \end{array}$$

Permutation

It is an arrangement of r individuals ~~of things~~ in a definite order

Combination

It is a selection of r elements from n individuals in any order

Formula:

(i) $^nP_r = \frac{n!}{(n-r)!}$

$^nC_r = \frac{n!}{(n-r)!\,r!}$

(ii) If repetition is allowed

$= \frac{n!}{P_1!\,P_2!\,P_3!}$

Problem: ① In how many ways can a coach choose 3 swimmers from 5 swimmers

$n = 5 \quad r = 3$

No. of ways $= {}^nC_r = \frac{5!}{(5-3)!\,3!} = \frac{5!}{2!\,3!} = \frac{5 \times \cancel{4}^{2} \times \cancel{3 \times 2}}{\cancel{2} \times \cancel{3 \times 2}}$

$= 10$ ways.

② In how many ways a committee consisting of 5 men & 3 women can be chosen from 9 men, 12 women.

Sol: No. of ways of choosing 5 men from 9 men $= {}^9C_5 = \frac{9!}{4!\,5!}$

$= \frac{9 \times \cancel{8}^{2} \times 7 \times \cancel{6} \times \cancel{5}}{\cancel{4 \times 3 \times 2}}$

$= 18 \times 7$

$= 126$

No. of ways of choosing 3 women from 12 women $= {}^{12}C_3 = \frac{12!}{9!\,3!}$

$= \frac{\cancel{12}^{2} \times 11 \times 10}{\cancel{3 \times 2}}$

$= 220$.

No. of ways of Selection 5M & 3W = $m \times n$ ways
$= 126 \times 220$
$= 27720$ ways.

③ In how many ways can you arrange the letters in the word "LOLLIPOP"

Sol.

L – 3 $\quad n = 8.$
P – 2
I – 1
O – 2

No. of permutations $= \dfrac{n!}{P_1! P_2! \cdots P_k!}$

$= \dfrac{8!}{3! 1! 2! 2!}$

$= \dfrac{8 \times 7 \times 6 \times 5 \times \cancel{4} \times \cancel{3!}}{\cancel{3!} \times \cancel{2!} \, \cancel{2!}}$

$= 56 \times 30$
$= 1680$ ways.

$\begin{array}{r} 56 \\ \times 3 \\ \hline 1680 \end{array}$

$\begin{array}{r} 56 \\ \times 3 \\ \hline 168 \end{array}$

Note. No. of ways of arranging n objects $= n!$
No. of ways of n objects in a circle $= (n-1)!$

④ In a ~~letter~~ Lottery each ticket has 5 one digit numbers from 0–9 in it

(a) You won if your ticket has the digit in any order. what are your chances of winning

(b) You would win only if your ticket has the digits in the required order what are your chances of winning

Sol: $n=10, r=5$ (Any order)

(i) $^nC_r = \frac{n!}{(n-r)!\,r!} = \frac{10!}{5!\,5!} = \frac{\cancel{10}\times9\times\cancel{8}\times7\times\cancel{6}}{\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times1}$

$= 2\times2\times9\times7$

$= 4\times9\times7$

$= 252$ ways.

(ii) $n=10, r=5$ (In required order)

$^nP_r = \frac{10!}{5!} = 10\times9\times8\times7\times6$

$= 30240$ ways.

⑤ In a dictionary. If all the permutations of the letters of the ~~word~~ word "AGAIN" are arranged in an order. What is the 49^{th} word.

Sol: No. of letters = 5

Starting with the letter A, letters are arranged from GAIN

A _ _ _ _ $4! = 24!$

Starting with the G $= \frac{4!}{2!\,1!\,1!} = \frac{4!}{2!} = 12$

Starting with the letter I, letters from AAGN $= \frac{4!}{2!\times1!\times1!} = 12$ — 48

Starting with letter N, letters from AAGI $= \frac{4!}{2!\times1!\times1!} = 12$

49^{th} word is NAAGI

~~24+12+12~~

Addition Rule:-

If the event occurs in m ways, the event e_1 occurs 'n' ways e_1, e_2 are mutually exclusive, then the no of. Chances of occuring. e_1 (or) e_2 in m+n ways.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Product Rule:-

If the event e_1, e_2 are independent. then no. of chances of happening e_1 & e_2 together m-n ways.

$$P(A \cap B) = P(A) \cdot P(B)$$

Note:- No. of different permutation of a distnict objects taken ~~of~~ are at a time every object is repeated at ~~a~~ 'n' time is given by n^r.

1) In how many ways, can we get these letters a, b, c, d, e, f, g, h, i, j arrange in a circle

∴ No. of Letters = 10

For Circular arrangement of 'n' objects

$$\text{No. of permutations} = (n-1)!$$
$$= (10-1)!$$
$$= 9!$$
$$= 3,62,880$$

How many 6 digit numbers can be formed from 0, 1, 2, 3, 4, 5, 6, 7, 8. If every number is to be start with 30 with no digits are repeated.

No. of digits = 9

6 Digits no's start with 30

We have to chose 4 digit from 7 digits

$$^7P_4 = \frac{7!}{(7-4)!} = \frac{7!}{3!} = 7 \times 6 \times 5 \times 4$$
$$= 840$$

3) A farmer purchased 3 cows, 2 pigs, 4 hens from a man who has 6 cows, 5 pigs, 8 hens. Find the chance that farmer has.

Sol: No. of ways for buying cows $= {}^6C_3 = 6\times5\times4 = 120$

pigs $= {}^5C_2 = 5\times4\times3 = 10$

Hens $= {}^8C_4 = 8\times7\times6\times5 = 70$

Total no. of chances $= \cancel{120\times60\times1680}$ $\quad 20\times10\times70$

$= \cancel{12,096,000}$ $\quad 200\times70$

$= 14000$

4) Determine the value of n if $4{}^nP_3 = {}^{(n+1)}P_3$

Sol: $4{}^nP_3 = {}^{(n+1)}P_3$

$${}^nP_r = \frac{n!}{(n-r)!} \quad \text{--- (1)}$$

$$4{}^nP_3 = 4\frac{n!}{(n-3)!} \quad \text{--- (2)}$$

$${}^{(n+1)}P_3 = \frac{(n+1)n!}{(n+1-3)!}$$

$$= \frac{(n+1)n!}{(n-2)!}$$

$$= \frac{(n+1)n!}{(n-2)(n-3)!}$$

(1) = (2)

$$\frac{4n!}{(n-3)!} = \frac{(n+1)!}{(n+1-3)!}$$

$$\frac{4n!}{(n-3)!} = \frac{(n+1)n!}{(n-2)!}$$

$$\frac{4\cancel{n!}}{\cancel{(n-3)!}} = \frac{(n+1)\cancel{n!}}{(n-2)\cancel{(n-3)!}}$$

$(n-2)4 = (n+1)$

$4n-8 = n+1$

$3n = 9$

$\boxed{n=3}$

(5) Determine the value of n if

$$^{20}C_{n+2} = {}^{20}C_{2n-1}$$

Sol:

$$\frac{20!}{(20-n-2)!(n+2)!} = \frac{20!}{(20-2n-1)!(2n-1)!}$$

$$^{n}C_{r} = \frac{n!}{(n-r)!\,r!}$$

$$\frac{\cancel{20!}}{(n+2)!\,(18-n)!} = \frac{\cancel{20!}}{(2n-1)!\,\cancel{(19-2n+1)!}\,(20-2n+1)!}$$

$$\frac{1}{(n+2)!\,(18-n)!} = \frac{1}{(2n-1)!\,(21-2n)!}$$

By formula

$$^{n}C_{x} = {}^{n}C_{y}$$

$$\boxed{x=y}$$

$$n+2 = 2n-1$$

$$\boxed{n=3}$$

MCQ Relation b/w Permutation & Combination

$$P(n,r) = {}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$C(n,r) = {}^{n}C_{r} = \frac{n!}{(n-r)!\,r!}$$

$$^{n}P_{r} = r!\,{}^{n}C_{r}$$

Divisibility: When a and b are any two integers $a \neq 0$, a is said to divide b if there exists an integer 'c' such that

$$b = ac$$

Eg: $8|4 \Rightarrow 8 = 4 \times 2$.

Prime Numbers:

A positive integer $p > 1$ is called a prime Number if the possible division are only 1 and p

Composite Numbers → Not a prime Number

Theorem:-

1. Let $a, b, c \in \mathbb{Z}$ $\mathbb{Z}$ - set of integers Then

(i) $a/b,\ a/c = a/c$

(ii) $a/b,\ b/c = a/c$

(iii) $a/b = a/mb,\ m \in \mathbb{Z}$

iv) $a/b,\ a/c \Rightarrow a/mb+nc$
$m, n \in \mathbb{Z}$

Eg: ~~(i) 12/3~~

Fundamental theorem of Arithmetic:

Every integer $n > 1$, Can be uniquely written as product of Prime factorization.

Eg: $2 = 2^1$

$14 = 2^1 \times 7$

$50 = 10 \times 5$
$= 2 \times 5 \times 5$
$= 2^1 \times 5^2$

I Find prime factorization of
i, 6647 ii) 45500 iii, 101 ~~iv) 266~~

Sol: (i) 6647 divisible by Primenumber - 2, 3, 5, 7, 11, 17, 19

$6647/17 = \frac{6647}{17} = 391,\ \frac{391}{17} = 23$

$6647 = 17 \cdot 17 \cdot 23 = 17^2 \cdot 23$

(ii) 45,500

$$\frac{45{,}500}{2} = 22750$$

$$\frac{22750}{2} = 11375$$

$$\frac{11375}{5} = 2275$$

$$\frac{2275}{5} = 455$$

$$\frac{455}{5} = 91$$

$$\frac{91}{7} = 13$$

$$45{,}500 = 2^2 \times 5^3 \times 7 \times 13$$

(iii) $10! = 3{,}628{,}800$

$$\frac{3628800}{2} = 1814400$$

$$\frac{1814400}{2} = 907{,}200$$

$$\frac{907200}{2} = 453{,}600$$

(iii) $10! = 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2$

$= 2 \times 5 \times 9 \times 2 \times 4 \times 7 \times 3 \times 2 \times 5 \times 2 \times 2 \times 3 \times 2$

$= \cancel{5^2 \times 9 \times 7 \times 3^2}$

$= 2 \times 5 \times 3 \times 3 \times 2 \times 2 \times 2 \times 7 \times 3 \times 2 \times 5 \times 2 \times 2 \times 3 \times$

$= 2^8 \times 3^4 \times 5^2 \times 7$

Division Algorithm:-

When a and b are any two integers $b > a$ and there exists $b > a$ and there exists integers q & r

Such that $\boxed{b = aq + r}$ as $r \le b$

q = quotient

r - remainder

1) Find LCM & GCD of 28, 12

The prime factorization

$28 = 4 \times 7$ $\qquad$ $12 = 3 \times 4$

$= 2^2 \times 7^1 \times 3^0$ $\qquad$ $= 3 \times 2^2$

$= 2^2 \times 3^1 \times 7^0$

$\text{lcm}(28, 12) = 2^{\max(2,2)} \times 3^{\max(0,1)} \times 7^{\max(0,1)}$

$= 2^2 \times 3^1 \times 7^1$

$= 4 \times 3 \times 7$

$= 12 \times 7$

$= 84$

HCF (or GCD$(28, 12)$) $= 2^{\min(2,2)} \times 3^{\min(0,1)} \times 7^{\min(0,1)}$

$= 2^2 \times 3^0 \times 7^0$

$= 4$

2) Find LCM & GCD of $(231, 1575)$ using Prime factorizati

Verify $\gcd(m,n) \times \text{lcm}(m,n) = m \cdot n$

Sol:

$231 = 3 \times 77$ $\qquad$ $1575 = 5 \times 315$

$= 3^1 \times 11^1 \times 7^1 \times 5^0$ $\qquad$ $= 5 \times 5 \times 63$

$= 5^2 \times 3 \times 21$

$= 5^2 \times 3^2 \times 7^1 \times$

$\text{lcm}(231, 1575) = 3^{\max(1,2)} \times 7^{\max(1,1)} \times 5^{\max(0,2)} \times 11^{\max(1,0)}$

$\Rightarrow 3^2 \times 7^1 \times 5^2 \times 11 = 6 \times 7 = 42 \times 275 = 5,1975$

HCF (or) GCD$(231, 1575) = 3^{\min(1,2)} \times 7^{\min(1,1)} \times 5^{\min(0,2)} \times 11^{\min(0,1)} = 3^2 \times 7$

$= 9 \times 1 =$

$GCD(m,n) \cdot Lcm(m,n) = m.n$

$(63) \times (51975) = (231) \times (1575)$

LHS = RHS

Verified

③ Find LCM & GCD of (337500, 21600) using P.F.

Sol: $337500 = 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5 \times 5 \times 5$

$= 2^2 \times 3^3 \times 5^5$

$21600 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5^2$

$= 2^5 \times 3^2 \times 5^2$

$Lcm(337500, 21600) = 2^{max(5,2)} \times 3^{max(3,2)} \times 5^{max(5,2)}$

$= 2^5 \times 3^3 \times 5^5$

$= 32 \times 9 \times 3125$

$= 27,00,000$

$GCD(337500, 21600) = 2^{min(5,2)} \cdot 3^{min(3,2)} \cdot 5^{min(3,2)}$

$= 2^2 \times 3^2 \times 5^2$

$= 4 \times 9 \times 25$

$= 36 \times 25$

$= 2700$

Euclid's Algorithm for finding GCD :-

When a and b are any two integers, a≥b

if r_1 is the remainder when a is divided by b, r_2 is the remainder when b is divided by r_1, r_3 is the remainder when r_1 is divided by r_2

and if $r_{k+1} = 0$, The last non-zero remainder

$\boxed{r_k \text{ is the GCD } (a, b)}$.

1) Find GCD (414, 662) using Euclids Algorithm

$a = 662, \quad b = 414$

$414 \overline{)\,662\,}(1$
$\quad 414$
$248 \overline{)\,414\,}(1$
$\quad 248$
$166 \overline{)\,248\,}(1$
$\quad 166$
$\quad 82$

$662 = 414\cdot(1) + 248$

$414 = 248\cdot(1) + 166$

$214 = 166\cdot(1) + 82$

$166 = 82\cdot(2) + 2 \rightarrow$ GCD.

$82 = 41\cdot(2) + \boxed{0}$

GCD $(414, 662) = 2$

1, Find GCD (1819, 3587) using Euclids Algorithm. Express the GCD as linear Combination of given numbers

Sol:- GCD $(1819, 3587) =$

$3857 = 1\times 1819 + 1768$

$1819 = 1\times 1768 + 51$

$1768 = 34\times 51 + 34$

$51 = 1\times 34 + 17$

$34 = 2\times 17 + 0$

GCD $(1819, 3587) = 17$

Linear Combination:- $17 = 51 - (1\times 34)$

$= 51 - 1(1768 - 34\times 51)$

$= 51 - (1\times 1768) + (34\times 51)$

$= 35\times 51 - (1\times 1768)$

$= 35\times(1819 - 1\times 1768) - (1\times 1768)$

$= 35\times 1819 - 36\times 1768$

$17 = 35\times 1819 - 36\times(3587 - (1\times 1819))$

$17 = 35\times 1819 - 36\times 3587 + (36\times 1819)$

$\Rightarrow 17 = (71 \times 1819) - (36 \times 3587)$

2) Find GCD (12345, 54321) by Euclid's algorithm and Express in Linear Combination

Sol:

Divident = quotient × divisor + Remainder

$54321 = 12345 \times 4 + 4941$

$12345 = 2 \times 4941 + 2463$

$4941 = 2 \times 2463 + 15$

$2463 = 164 \times 15 + 3$

$15 = 5 \times 3 + 0$

$$\begin{array}{r} 164 \\ \times 15 \\ \hline 820 \\ 164 \\ \hline 2460 \end{array}$$

GCD (12345, 54321) = 3

Linear Combination:

$3 = 2463 - (164 \times 15)$

$= 2463 - (164 \times (4941 - 2 \times 2463))$

$= 2463 - 164 \times 4941 + (328 \times 2463)$

$= 329 \times 2463 - 164 \times 4941$

$= 329(12345 - 2 \times 4941) - 164 \times 4941$

$= 329 \times 12345 - 658 \times 4941 - 164 \times 4941$

$$\begin{array}{r} 329 \\ \times 2 \\ \hline 658 \\ 164 \\ \hline 822 \end{array}$$

$3 = 329 \times 12345 - 822 \times 4941$

$= 329 \times 12345 - 822(54321 - 4 \times 12345)$

$= 329 \times 12345 - (822 \times 54321) + 3288 \times 12345$

$$\begin{array}{r} 3288 \\ 329 \\ \hline 3617 \end{array}$$

$\boxed{3 = 3617 \times 12345 - 822 \times 54321}$

Problems on Euclids Algorithm :-

1. Find integers m and n Such that $512m + 320n = 64$

$gcd(512, 320)$

$512 = 1 \times 320 + 192$

$320 = 1 \times 192 + 128$

$192 = 1 \times 128 + 64$

$128 = 2 \times 64 + 0$

$65 = 1 \times 63 + 3$

$63 = 21 \times 3 + 0$

$GCD(512, 320) = 64$

~~$GCD(512, 320) = 3$~~

$3 = 65 - (1 \times 63)$

$= 65 - (128 - 1 \times 65)$

$=$

$64 = 192 - 1 \times 128$

$= 192 - 1 \times (320 - 1 \times 192)$

$= 192 - 1 \times 320 + 1 \times 192$

$= 2 \times 192 - 1 \times 320$

$= 2 \times (512 - 1 \times 320) - 1 \times 320$

$64 = 2 \times 512 - 2 \times 320 - 1 \times 320$

$= 2 \times 512 - 3 \times 320$

$\boxed{m = 2, n = 3}$

2) Find the integers m & n Such that $28844m + 15712m = 4$

Sol:- $GCD(28844, 15712) =$

$28844 = 1 \times 15712 + 13132$

$15712 = 1 \times 13132 + 2580$

$13132 = 5 \times 2580 + 232$

$2582 = 11 \times 232 + 28$

$232 = 8 \times 28 + 8$

$28 = 1 \times 22 + 8$

$22 = 2 \times 8 + 6$

$8 = 1 \times 6 + 2$

$6 = 2 \times 3 + 0$

$28 = 3 \times 8 + 4$

$8 = 2 \times 4 + 0$

$4 = 28 - (3 \times 8)$

$= 28 - 3(232 - 8 \times 28)$

$= 28 - 3 \times 232 + 24 \times 28$

$= 25 \times 28 - 3 \times 232$

$= 25(2580 - 11 \times 232) - 3 \times 232$

$25 \times 11 = 275$

$= 25 \times 2580 - 275 \times 232 - 3 \times 232$

$= 25 \times 2580 - 278 \times 232$

$278 \times 5 = 1390$

$= 25 \times 2580 - 278 \times (13132 - 5 \times 2580)$

$= 25 \times 2580 - 278 \times 13132 - 1390 \times 2580$

$= 1415 \times 2580 - 278 \times 13132$

$= 1415 \times (15712 - 1 \times 13132) - 278 \times 13132$

$= 1415 \times 15712 - 1415 \times 13132 - 278 \times 13132$

$= 1415 \times 15712 - 1693 \times 13132$

$= 1415 \times 15712 - 1693 \times (28844 - 1 \times 15712)$

$\boxed{4 = 3108 \times 15712 - 1693 \times 28844}$

$m = -1693 \quad n = 3108$

Properties of GCD

1. If c/ab, a, c are co-primes then c/b
2. $\gcd(a,b)$ can be expressed as a integral prime Combination of a & b
3. $\gcd(ka, kb) = k \gcd(a,b)$
4. If $\gcd(a,b) = d$, then $\gcd\left(\frac{a}{d}, \frac{b}{d}\right) = 1$.
5. If $\gcd(a,b) = 1$ then $\gcd(ac, b) = \gcd(c, b)$

c is an integer