13/07/2021

18MAB302T

UNIT - 1 SET THEORY

Set: It is a collection well defined objects or elements

A set is represented in two ways

(i) Roaster (ii) set builder form

Eg:- Roaster Notation of a set:

Set of all vowels in English alphabets

$V = \{a, e, i, o, u\}$

Eg:- Set Builder

$V = \{x / x$ is a vowel in English alphabets$\}$

Operations of set:-

(i) $A \cup B = \{x / x \in A \text{ or } x \in B\}$

(ii) $A \cap B = \{x / x \in A \text{ and } x \in B\}$

(iii) $A^c = \{x / x \notin A\}$

(iv) $A - B = \{x / x \in A \text{ but } x \notin B\}$

(v) $A \oplus B = \cancel{(A-B) \cup (B-A)}$ $(A \cup B) - (A \cap B)$

(vi) $A \times B = \{(a,b) / a \in A, b \in B\}$

Let $U = \{1,2,3,4,5,6\}$

$A = \{1,2,3\}$ $B = \{4,5\}$

$A \cup B = \{1,2,3,4,5\}$

$A \cap B = \phi$ (null set)

$A - B = \{1,2,3\}$

$(B - A) = \{4,5\}$

$(A \oplus B) = (A-B) \cup (B-A)$

$= \{1,2,3,4,5\}$

$(A \times B) = \{(1,4)\ (1,5)$
$(2,4)\ (2,5)$
$(3,4)\ (3,5)\}$

Duality:-

Dual for set operations for.

(i) $A \cup B$ is $A \cap B$

(ii) $A \cap B$ is $A \cup B$

(iii) U is ϕ

(iv) ϕ is U

→ Write the Dual of

(i) $(A \cup B) \cap (B \cup \phi) = B$

Sol: $(A \cap B) \cup (B \cap U) = B$

(ii) $\overline{(A \cap B \cap C)} = (\overline{A \cap C}) \cup (\overline{A \cap B})$

$\overline{(A \cup B \cup C)} = (\overline{A \cup C}) \cap (\overline{A \cup B})$

Set Identities:-

1. $(A \cup B) = (B \cup A)$ } (Commutative law)
   $(A \cap B) = (B \cap A)$
2. $A \cup (B \cup C) = (A \cup B) \cup C$ (Associative law)
3. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ } (Distributive law)
   $(A \cap B \cup C) = (A \cap B) \cup (A \cap C)$
4. $\overline{A \cup B} = \bar{A} \cap \bar{B}$ } Demorgan's law.
   $\overline{A \cap B} = \bar{A} \cup \bar{B}$
5. $A \cup U = U$ } Domination Laws
   $A \cap \phi = \phi$
6. $A \cap U = A$ } Identity laws.
   $A \cup \phi = A$
7. $(A^c) = A$ — Compliment law

* $(A\cup B)^c = A^c \cap B^c$
$(A\cap B)^c = A^c \cup B^c$ } Demorgan's law

* $S^c = \phi$
* $\phi^c = S$

Problems:

① prove that $(A\cup B)^c = A^c \cap B^c$ (or) $\overline{A\cup B} = \bar{A} \cap \bar{B}$

$LHS = (A\cup B)^c = \{x / x \notin A\cup B\}$

$= \{x / x \notin A \text{ and } x \notin B\}$

$= \{x/x \notin A\} \text{ and } \{x/x \notin B\}$

$(A\cup B)^c = A^c \cap B^c = RHS.$

$LHS = RHS$

2) Prove that $(A\cap B)^c = A^c \cup B^c$

$LHS = (A\cap B)^c = \{x / x \notin A\cap B\}$

$= \{x/x \notin A \text{ or } x \notin B\}$
$x \in B$ (or) $x \in A$

$= \{x/x \notin A\} \text{ or } \{x/x \notin B\}$

$(A\cap B)^c = A^c \cup B^c = RHS.$

$LHS = RHS$

3) Prove that $(A-C)\cap(C-B) = \phi$ analytically verify it graphically where A, B, C are any these sets.

Consider $A-C = \{x/x \in A, x \notin C\}$

$C-B = \{x/x \in C, x \notin B\}$

$(A-C)\cap(C-B) = \{x/x \in A, x \notin C \text{ and } x \in C, x \notin B\}$

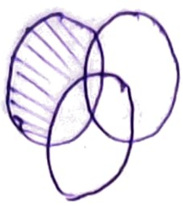
$= \{x/x \in A, x \notin C, x \in C, x \notin B\}$

$= A \cap C^c \cap C \cap B^c$

$= A \cap \phi \cap B^c$

$= \phi =$ RHS

LHS = RHS.

A - C  C - B 

$(A - C) \cap (C - B) = \phi$

Partition of a set :-

Let A be any set. The subsets $A_1, A_2 \dots A_n$ are said to be a partion of a set. If

(i) each subset is not empty for every i

$A_i \neq \phi \ \forall i$

(ii) $\bigcup_{i=1} A_i = A$

(iii) $A_i \cap A_j = \phi$ for $i \neq j$

Minsets (or) Minterms

Used to find the parition of any set

Let A be any set, B_1, B_2 be any two subsets of A. The minsets of A are $B_1 \cap B_2$, $B_1^c \cap B_2$, $B_1 \cap B_2^c$, $B_1^c \cap B_2^c$

If B_1, B_2, B_3 are any subsets of A, then the minterms are $B_1 \cap B_2 \cap B_3$, $B_1^c \cap B_2 \cap B_3$, $B_1 \cap B_2^c \cap B_3$, $B_1 \cap B_2 \cap B_3^c$, $B_1^c \cap B_2^c \cap B_3$, $B_1 \cap B_2^c \cap B_3^c$, $B_1^c \cap B_2 \cap B_3^c$, $B_1^c \cap B_2^c \cap B_3^c$

MCQ: For n subsets of A

The no. of minterms is 2^n

* Maxterm (or) Maxset :-

Maxterms (or) Maxsets :-

Let A be any set, B_1, B_2 be any two Subsets of A. The max sets of A are $B_1 \cup B_2$, $B_1^c \cup B_2$, $B_1 \cup B_2^c$, $B_1^c \cup B_2^c$

Let B_1, B_2, B_3 are any three subsets of A. Then the maxterms are

$B_1 \cup B_2 \cup B_3$, $B_1^c \cup B_2 \cup B_3$, $B_1 \cup B_2^c \cup B_3$, $B_1 \cup B_2 \cup B_3^c$
$B_1^c \cup B_2^c \cup B_3$, $B_1 \cap B_2^c \cap B_3^c$, $B_1^c \cap B_2 \cap B_3^c$, $B_1^c \cap B_2^c \cap B_3^c$

Problem Let $A = \{1,2,3,4,5,6\}$ Find minterms are generated by $\{1,3,5\}$ and $\{1,2,3\}$ and also given the partition of A

Sol:- $n=2$ $= 2^n = 2^2 = 4 =$ Subsets

$B_1 = \{1,3,5\}$
$B_2 = \{1,2,3\}$
$B_1^c = \{2,4,6\}$
$B_2^c = \{4,5,6\}$

$B_1 \cap B_2 = \{1,3\} \neq \emptyset$
$B_1 \cap B_2^c = \{5\} \neq \emptyset$
$B_1^c \cap B_2 = \{2\} \neq \emptyset$
$B_1^c \cap B_2^c = \{4,6\} \neq \emptyset$

Since each ~~ter~~ minterm is not empty $(\neq \emptyset)$

$A_i \cap A_j = \emptyset \ \forall (i \neq j)$

$\cup A_i = A$

The minterm $\{1,3\}, \{5\}, \{2\}, \{4,6\}$ form a partion of A

(5) Let $A = \{1,2,3,4,5,6,7,8,9\}$ and $B_1 = \{2,4,5,9\}$, $B_2 = \{3,4,5,6,8,9\}$

$B_3 = \{1,5,6,7\}$ Find the minterm and partition

Ans) $B_1^c = \{1,3,6,7,8\}$ $B_2^c = \{1,2,7,\}$ $B_3^c = \{2,3,4,8,9\}$

Minterms are:-

$B_1 \cap B_2 \cap B_3 = \{5\}$

$B_1^c \cap B_2 \cap B_3 = \{6\}$

$B_1 \cap B_2^c \cap B_3 = \phi$ ×

$B_1 \cap B_2 \cap B_3^c = \{4,9\}$

$B_1^c \cap B_2^c \cap B_3 = \{7\}$

$B_1 \cap B_2^c \cap B_3^c = \{2\}$

$B_1^c \cap B_2 \cap B_3^c = \{3,8\}$

$B_1^c \cap B_2^c \cap B_3^c = \{\phi\}$ ×

The partition of A are $\{4,9\}, \{6\}, \{2\}, \{3,8\}, \{1,7\}, \{5\}$

) Find the maxterms of $A = \{1,2,3,4,5,6\}$ where

$B_1 = \{1,3,5\}$ $B_2 = \{2,4,6\}$

$B_1^c = \{2,4,6\}$ $B_2^c = \{1,3,5\}$

Maxterms

$B_1 \cup B_2 = \{1,2,3,4,5,6\}$

$B_1 \cup B_2^c = \{1,3,5\}$

$B_1^c \cup B_2 = \{2,4,6\}$

$B_1^c \cup B_2^c = \{1,2,3,4,5,6\}$

paritions are, $\{1,2,3,4,5,6\}$, $\{1,3,5\}$, $\{2,4,6\}$

Relation (R):

A relation R from A to B is a subset of $A \times B$

$$\boxed{R \subseteq A \times B}$$

If R is a relation on a set A,

$$R \subseteq A \times A$$

Eg: If $A = \{1,2,3\}$ and $B = \{1,4\}$ and the relation R in $\leq$

$$A \times B = \{(1,1)\ (1,4)\ (2,1)\ (2,4)\ (3,1)\ (3,4)\}$$

$$R = \{(1,1)\ (1,4)\ (2,4)\ (3,4)\}$$

Composition of relation

If R is a relation from A to B, S is a relation from B to C then

$R \circ S$ is a relation from A to C

$R: A \to B$, $S: B \to C$ then $R \circ S$ $A \to C$

Problems:-

① If $R = \{(1,2)\ (2,4)\ (3,3)\}$
$S = \{(1,3)\ (2,4)\ (4,2)\}$ are any two relations

Find (i) $R \cup S$ (ii) $R \cap S$ (iii) $R - S$ (iv) $S - R$ (v) $R \oplus S$ (vi) $R \cdot S$.

(vii) $S \circ R$.

Sol:- i) $R \cup S = \{(1,2), (2,4), (3,3), (1,3), (4,2)\}$

ii) $R \cap S = \{2,4\}$

iii) $R - S = \{(1,2)\ (3,3)\}$

iv) $S - R = \{(1,3)\ (4,2)\}$

v) $R \oplus S = (R \cup S) - (R \cap S) = \{(1,2)\ (3,3)\ (1,3).\ (4,2)\}$

vi) $R \circ S = \{(1,4), (2,2)\}$

vii) $S \circ R = \{(1,3)\ (2,4)\}$

② If $R = \{(1,1)\ (1,2)\ (2,3)\ (2,4)\ (3,4)\ (4,1)\ (4,2)\}$

$S = \{(3,1)\ (4,4)\ (2,3)\ (2,4)\ (1,1)\ (1,4)\}$ on

$A = \{1,2,3,4\}$

Find (i) $R \circ R$ (ii) $S \circ R$

$R.R = \{(1,1)(1,2)(1,3)\ (1,4)\ (2,4)(2,1)(2,2)\ (3,1)(3,2)\ (4,1)\ (4,2)(4,3)\ (4,4)\}$

$S.R = \{(3,1)\ (3,2),\ (\cancel{2,4})\ (4,1)\ (4,2)\ (2,4)\ (2,1)(2,2)\ (1,1)(1,2)\}$

Matrix of Relation:- (Relational Matrix)

If $A = \{x,y,z\}$ $B = \{1,2,3,4\}$ R is the relation from A to B, then the matrix of relation is defined as

$$M_R = A \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline x & 1 & 0 & 1 & 1 \\ y & 0 & 1 & 0 & 0 \\ z & 0 & 0 & 1 & 1 \end{array}_{3\times 4}$$

$R = \{(x,1)\ (x,3)\ (x,4)\ (y,2)\ (z,3)(z,4)\}$

① If R is the relation on the set $A = \{1,2,3\}$ Such That $a+b =$ even iff $(a,b) \in R$. Find the relational matrix Also find

(i) $M_{\bar{R}}$ ii) $M_{\bar{R}}$ iii) M_R^2 $R \subseteq A \times A$

$A \times A = \{(1,1)(1,2)\ (1,3)\ (2,1)(2,2)\ (2,3)\ (3,1)(3,2)(3,3)\}$

$R = \{(1,1)\ (1,3)(2,2)\ (3,1)(3,3)\}$

$$M_R = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1 & 0 & 1 \\ 2 & 0 & 1 & 0 \\ 3 & 1 & 0 & 1 \end{array}$$

$$M_{R^{-1}} = M_R^T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$M_{\bar{R}} = \overline{M_R} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$M_{R^2} = M_R \cdot M_R = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

$$= \begin{bmatrix} 1+0+1 & 0+0+0 & 1+0+1 \\ 0+0+0 & 0+1+0 & 0+0+0 \\ 1+0+1 & 0+0+0 & 1+0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

② $M_R = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$ $M_S = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

Find the matrices of relation.

(i) $R \cup S$ (ii) $R \cap S$ (iii) $R - S$ (iv) $R \cdot S$ (v) $S \cdot R$ (vi) $R \oplus$

(i) $M_{R\cup S} = M_R \cup M_S = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \cup \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

$$= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

(ii) $M_{R\cap S} = M_R \cap M_S = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \cap \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

$$= \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

iii) $M_{R-S} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

$= \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(iv) $M_{R \cdot S} = M_R \cdot M_S$

$= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

$= \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

(v) $M_{S \cdot R} = M_S \cdot M_R$

$= \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

$= \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

(vi) $R \oplus S = M_{R \oplus S} = M_{R \cup S} - M_{R \cap S}$

$= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

$= \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$

2) If $R = \{(1,2)(2,4)(3,2)\}, S = \{(1,3)(2,4)(4,2)\}$ Verify.

(i) Domain $(R \cup S)$ = domain$(R) \cup$ domain(S)

(ii) Range $(R \cap S) \subseteq$ Range$(R) \cap$ Range(S)

$(a,b) \to a =$ domain

Sol:- Domain $A = \{1,2,3,4\}$

domain $R = \{1,2,3\}$

domain $S = \{1,2,4\}$

$\therefore R \cup S = \{(1,2)(2,4)(3,2)(1,3)(4,2)\}$

domain$(R \cup S) = \{1,2,3,4\}$

domain$(R) \cup$ domain$(S) = \{1,2,3,4\}$

Hence dom $(R \cup S)$ = dom$(R) \cup$ dom(S)

(ii) Range $(R) = \{2,3,4\}$ Range of $S = \{2,3,4\}$

Range $(R) \cap$ Range$(S) = \{2,3,4\}$

$R \cap S = \{2,4\}$

Range $(R \cap S) = \{4\}$

Range $(R \cap S) \subseteq$ Range $(R) \cap$ Range (S)

Types of Relations :-

(1) Reflexive (2) Symmentric (3) Irreflexive (4) Antisymmentr

(5) Asymmentric (6) Transtive

Reflexive :- $\forall a \in A, (a,a) \in R$ i.e every element is related to itself

Symmentric : $\forall (a,b) \in R, (b,a) \in R$.

Antisymmetric:
~~Irreflexive~~ : $(a,a) \notin R$, $(b,a) \notin R$ for same $(a,b) \notin R$

Asymentric : Irreflexive + Symmentric

Transitive : $(a,b) \in R, (b,c) \in R \Rightarrow (a,c) \in R$

Irreflexive : $(a,a) \notin R$, for some $a \in A$

Classification of Relation:-

Equivalence Relation	partial order Relation
Reflexive	Reflexive
Symmentric	Antisymmentric
Transitive	Transitive

Eg for Equivalence relation:-

Set of parallel Line

$\{l_1, l_2, l_3, \ldots l_n\}$

(i) Reflexive - $l_1 \parallel$ all lines

(ii) Symmentric - $l_1 \parallel l_2$, $l_2 \parallel l_1$

(iii) Transitive - $l_1 \parallel l_2$, $l_2 \parallel l_3$, $l_1 \parallel l_3$

Set of parallel lines forms an Equivalence Relation.

Eg for antisymmentric:-

$R = \{(1,2) (1,4) (4,1) (3,2) (2,3)\}$ on

$A = \{1, 2, 3, 4\}$

$(1,2) \in R$ R is Antisymmentric

$(2,1) \notin R$

(i) **Reflexive** $R = \{(1,2) (1,1) (2,2) (2,3) (3,3) (4,4)\}$

R is reflexive Reflexive - (a,a)

Same elements in set

Partially ordered set (or) Poset :-

Any set having the partial order relation is called Partial ordered set (or) poset

Conditions :- R must be Reflexive, Antisymmentric, Transitive

Problems on Equivalence (or) Partial order relation :-

① Let $S = \{1, 2, 3 \ldots 9\}$ Define R on a set S

$R = \{(x, y) / x + y = 10,\ x, y \in S\}$

Verify whether R is an Equivalence relation

For R to be an Equivalence relation

R must be

i) Reflexive ii) Symmentric iii) Transitive

$R = \{(1,9)(9,1), (2,8)(8,2), (3,7)(7,3), (4,6)(6,4), (5,5)\}$

i) Since $((1,1)(2,2) \ldots 9,9) \notin R$

$\Rightarrow$ R is Irreflexive

(ii) $\forall (a,b) \in R,\ (b,a) \in R$

$\Rightarrow$ R is a Symmentric

(iii) $(a,b)(b,c) \in R$

$\Rightarrow (a,c) \in R$

$(1,9)(9,1) \in R$ but;

$(1,1) \notin R$

R is not Transitive

Then the Relation R is not an Equivalence relation

For partial order relation,

R must be Reflexive, Antisymmentric, Transitive.

$\Rightarrow$ R is not an partial order relation.

(1) Let $S = \{1, 2, 3, 4, \dots 25\}$ and R be the relation
$R = \{(x, y) / (x - y)$ is divisible by $5\}$
Show that R is an Equivalence relation.

Sol:- $R = \{(1,6)(2,7)\}$

$R = \{(1,6)(6,1)(1,11)(11,1)(1,16)(16,1)$
$(2,7)(7,2)(2,12)(12,2) \dots$
$(5,25)(25,5) \dots\}$
$(1,1)(2,2)(3,3) \dots (25,25)\}$

(i) Reflexive:

$\forall a \in S \ (a,a) \in R$

R is reflexive

(ii) Symmentric:-

$\forall (a,b) \in R, (b,a) \in R$

R is Symmentric

(iii) Transitive:

$(1,6)(6,1)$

$(1,1)$

$\forall (a,b), (b,c) \in R$

$(a,c) \in R$

then R is Transitive

Then R is an Equivalence Relation

Hasse Diagram:-

→ Symmetric representation of poset

① If $A = \{1, 2, 3, 4, 12\}$. Consider the partial order relation of divisibility on A.
Draw the hasse diagram of the Poset $(A, |)$

$\Downarrow$ divisi.

Sol:-

$R = \{ (1,1) (2,2) (3,3) (4,4) (12,12)$
$(1,2) (1,3) (1,4) (1,12)$
$(2,4) (2,12) (3,12) (4,12) \}$

Partial order relation:- $\{ (1,2) (2,4) (3,12) (4,12) (1,3) \}$

Hase Diagram:-

② If $X = \{1, 2, 3, 6, 12\}$

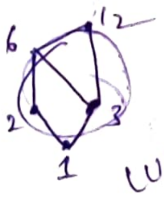
$R = \{ (x, y) / x \text{ divides } y \}$

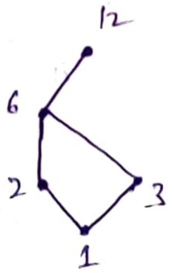
Draw Harse diagram

The partial order relation

$\{ (1,2) (1,3) (2,6) (6,12) (3,6) \}$

Harse Diagram





Least upper bound
Greatest lower bound.

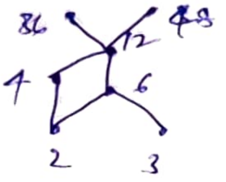
LUB $(2,3) = 6$

GLB $(2,3) = 1$

(2) Let $S = \{2, 3, 4, 6, 12, 36, 48\}$ and $R = \{x \leq y / x \text{ divides } y\}$
Draw the hasse diagram Find LUB and GLB of $\{4, 6, 12\}$

The partial order relation is

$R = \{(2,4)(4,12)(12,36)(12,48)$
$(3,6)(6,12)(2,6)\}$



upper bound of $(4,6,12)$
$\{12, 36, 48\}$

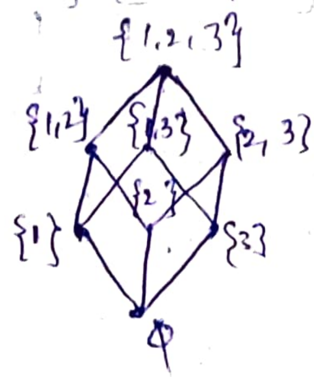
LUB = 12

Lower bound of $(4,6,12)$
$= \{2, 3\}$
$= \phi$

(3) Draw Hasse Diagram for $(P(A), \subseteq)$ where $A = \{1,2,3\}$
$P(A)$ is the power set of A

$A = \{1,2,3\}$
$P(A) = [\{\phi\}, \{1\}, \{2\}, \{3\}, \{1,2\} \{2,3\} \{1,2\} \{1,2,3\}]$.

partial order relation.
$\{ \{(\phi, \{1\})(\phi, \{2\})(\phi, \{3\})(\{1\}, \{1,2\})$
$(\{1\} \{1,3\}) (\{2\} \{2,3\}) (\{2\}, \{1,2\})$
$\{3\} \{1,3\} \quad \{3\} \{2,3\}$
$\{((1,2), A)\} \{(2,3), A\} \{(1,3), A\} \}$



Reflexive closure :-

$$R^c = R \cup \{(x, x) / x \in S\}$$

Eg. $A = \{1, 2, 3\}$ $R = \{(1,2)(2,3)(1,1)\}$

Reflexive closure: $\{(1,2)(2,3)(1,1)(2,2)(3,3)\}$

Symmentric closure :-

$$S^c = R \cup \{(x,y) / (y,x) \in R\}$$

Eg. $A = \{1,2,3\}$ $R = \{(1,2)(2,3)(1,1)\}$

Symmertric closure : $\{(1,2)(2,1)(2,3)(3,2)(1,1)\}$

Transitive closure :-

$$T^c = R \cup \{(x,y)(y,z) / (x,z) \in R\}$$

$A = (1,2,3)$

$$T^c = \{(1,2)(2,3)(1,3)(1,1)$$

Warshall's Algorithm :- (For finding Transitive closure)

① Using Warshall's Algorithm, Find Transitive closure of of the relation $R = \{(1,1)(1,2)(1,4)(2,2)(2,3)(3,4)(4,1)(3,1)\}$ Where $A = \{1,2,3,4\}$

$(1,1)(1,2)(1,3)(1,4)$ $(3,1)(3,2)(3,3)(3,4)$
$(2,1)(2,2)(2,3)(2,4)$ $(4,1)(4,2)(4,3)(4,4)$

Transitive closure : $\{(1,1)(1,2)(1,4)(2,2)(2,3)(3,4)$
$(4,1)(3,1)(1,3)(2,4)(3,3)$
$(2,1)$

Let $M_R = W_0 =$

$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 1 & 0 & 1 & 0 & 1 \\ 2 & 0 & 0 & 1 & 1 & 0 \\ 3 & 0 & 0 & 1 & 0 & 1 \\ 4 & 0 & 1 & 0 & 1 & 0 \\ 5 & 0 & 0 & 0 & 1 & 0 \end{array}$$

K	Position of 1's in Column K	Position of 1's in Row K	Relation	W_k Matrix
1	1	1, 3, 5	(1,1) (1,3) (1,5)	$\begin{pmatrix} 1&0&1&0&1 \\ 0&0&1&1&0 \\ 0&0&1&0&1 \\ 0&1&0&1&0 \\ 0&0&0&1&0 \end{pmatrix}$
2	4	3, 4	(4,3) (4,4)	$\begin{pmatrix} 1&0&1&0&1 \\ 0&0&1&1&0 \\ 0&0&1&0&1 \\ 0&1&1&1&0 \\ 0&0&0&1&0 \end{pmatrix}$
3.	1, 2, 3, 4,	(3, 5)	(1,3) (1,5) (2,3) (2,5) (3,3) (3,5) (4,3) (4,5)	$\begin{pmatrix} 1&0&1&0&1 \\ 0&0&1&0&1 \\ 0&0&1&0&1 \\ 0&1&1&1&1 \\ 0&0&0&1&0 \end{pmatrix}$
~~4.~~	~~(2,3)~~ ~~1, 2, 3, 4, 5~~	~~2, 3, 4, 5)~~		
4.	2, 4, 5	2, 3, 4, 5	(2,2) (2,3) (2,4) (2,5) (4,2) (4,3) (4,4) (4,5) (5,2) (5,3) (5,4) (5,5)	$\begin{pmatrix} 1&0&1&0&1 \\ 1&1&1&1&1 \\ 0&0&1&0&1 \\ 0&1&1&1&1 \\ 0&1&1&1&1 \end{pmatrix}$
5.	1 2 3 4 5	2, 3, 4, 5	(1,2) (1,3) (1,4) (1,5) (2,2) (2,3) (2,4) (2,5) (3,2) (3,3) (3,4) (3,5) (4,2) (4,3) (4,4) (4,5) (5,2) (5,3) (5,4) (5,5)	$\begin{pmatrix} 1&1&1&1&1 \\ 0&1&1&1&1 \\ 0&1&1&1&1 \\ 0&1&1&1&1 \\ 0&1&1&1&1 \end{pmatrix}$

(2) Using Warshall's Algorithm, Find Transitive closure of the relation

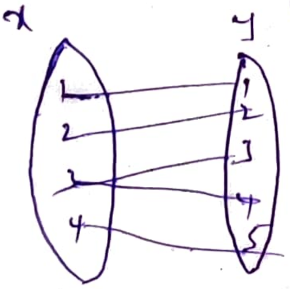
$$\begin{pmatrix} 1&1&1&1 \\ 1&1&1&1 \\ 0&0&0&1 \\ 0&0&0&0 \end{pmatrix}$$

K	Position of 1 in column K	Position of 1 in row K	Relation	W_K
1	2	2	(2,2)	$\begin{pmatrix}0&1&0&0\\1&1&1&0\\0&0&0&1\\0&0&0&0\end{pmatrix}$
2	1,2	(1,2,3)	(1,1) (1,2) (1,3) (2,1) (2,2) (2,3)	$\begin{pmatrix}1&1&1&0\\1&1&1&0\\0&0&0&1\\0&0&0&0\end{pmatrix}$
3	1,2	4	(1,4) (2,4)	$\begin{pmatrix}1&1&1&1\\1&1&0&1\\0&0&0&1\\0&0&0&0\end{pmatrix}$
4.	1,2,3	—	—	$\begin{pmatrix}1&1&1&1\\1&1&1&1\\0&0&0&1\\0&0&0&0\end{pmatrix}$

Functions :-

A relation f from a set X into Y is Called a function if for every $x \in X$, there exists $y \in Y$ such that $\boxed{f(x) = y}$

$f, x \to y$ is a function



1-1

The elements in X is image with co-domain

onto

Bijection → function f is both one-one and onto

⇒ Necessary Condition for f to be invertible

Surjection :- f is onto

Injection : f is one to one

Composite of function : If $f : A \to B$, $g : B \to C$ then $f \circ g : A \to$

$(f \circ g)(x) = f[g(x)]$

Inverse: $f(x) = y \Rightarrow y = f^{-1}(x)$

MCQ: $\rightarrow (g \circ f)^{-1}(x) = (f^{-1} \circ g^{-1})(x)$

Composition of function:-

If $f: A \rightarrow B$, $g: B \rightarrow C$ then $g \circ f: A \rightarrow C$ such that

$(g \circ f)(x) = g[f(x)] \quad \forall x \in A$

Inverse ~~Transverse~~ f^{-1}:

If $f: A \rightarrow B$, $g: B \rightarrow A$ then $f^{-1}: B \rightarrow A$, $g^{-1}: A \rightarrow B$

~~ϕ~~ (MCQ) Also $[g \circ f]^{-1} = f^{-1} \circ g^{-1}$

① If $f: R \rightarrow R$, $g: R \rightarrow R$ defined by $f(x) = 4x - 1$, $g(x) = \cos x$ Find $f \circ g$, $g \circ f$, f^{-1}, g^{-1}

Sol: $R \rightarrow$ set of all Real Numbers.

$$[f \circ g]x = f[g(x)]$$
$$= 4\, g(x) - 1$$
$$= 4 \cos x - 1$$

$$(g \circ f)x = g[f(x)]$$
$$= \cos f(x)$$
$$= \cos(4x - 1)$$

$f \circ g \neq g \circ f$ (not necessary for Equal)

(iii) $f(x) = y \Rightarrow$ ~~$y = f^{-1}(x)$~~ $x = f^{-1}(y)$

$$f(x) = 4x - 1$$
$$y = 4x - 1$$
$$\frac{y+1}{4} = x$$
$$f^{-1}(x) = \frac{x+1}{4}$$

iv) $g(x) = \cos x$

$y = \cos x$

$x = \cos^{-1} y$

$x = g^{-1}(\cos^{-1} y)$

$\boxed{g^{-1}(x) = (\cos^{-1}(x))}$

2) If $f(x) = x^2$, $g(x) = 2x-1$ find fog, gof

Sol: $f(x) = x^2$, $g(x) = 2x-1$

$fog = f[g(x)]$

$= f(2x-1)$

$= (2x-1)^2$

$gof = g(f(x))$

$= g(x^2)$

$= 2(x^2)-1$

$= 2x^2-1$

$fog \neq gof$

f^{-1} $f(x) = x^2$

$y = x^2$

$x = \pm\sqrt{y}$

$f^{-1}(x) = \pm\sqrt{x}$

$g^{-1} = g(x) = 2x-1$

$y = 2x-1$

$2x = y+1$

$x = \frac{y+1}{2}$

$g^{-1}(x) = \frac{x+1}{2}$

③ S.T $f: R \to R$ defined by $f(x) = 3x-1$ is a bijection (Invertible)

Sol:- For Bijection, f must be one-one, onto

One-one

Let $f(x_1) = f(x_2)$

$\cancel{3}x_1 - \cancel{1} = \cancel{3}x_2 - \cancel{1}$

$\boxed{x_1 = x_2}$

f is 1-1

on-to

$\forall$ $y \in R$ there exists $x \in R$ such that

$f(x) = y$

$3x - 1 = y$

$x = \frac{y+1}{3} \in R$

f is onto

f is a bijection

④ Check $f: R \to R$ defined by $f(x) = x^2$ is a bijection or not

Sol:- 1-1

Let $f(x_1) = f(x_2)$

$x_1^2 = x_2^2$

$x_1 = \pm x_2$

f is not 1-1

onto

$\forall y \in R$ there exists $x \in R$ such that

$f(x) = y$

$x^2 = y$

$y = \pm\sqrt{x}$

f is onto

f is not a bijection

⑤ Check $f: R \to R$ defined by $f(x) = \sin x$ is a bijection or not

Sol:- 1-1

Let $f(x_1) = f(x_2)$

$\sin x_1 = \sin x_2$

$x_1 \neq x_2$

f is not 1-1

onto

Range = Co-domain

Range of $\sin x = [-1, 1]$

$\neq R$ [Codomain]

f is not onto

f is not a bijection

(6) If $A = \{x \in R / x \neq 2\}$ and $f(x) = \frac{x}{x-2}$ Prove that f is 1-1 & onto Also find f^{-1}

Sol:-

1-1

$f(x_1) = f(x_2)$

$\frac{x_1}{x_1-2} = \frac{x_2}{x_2-2}$

$x_1(x_2-2) = x_2(x_1-2)$

$\cancel{x_1x_2} + \cancel{2}x_1 = \cancel{x_1x_2} + \cancel{2}x_2$

$\boxed{x_1 = x_2}$

f is 1-1

Onto

$\forall y \in R$, there exists $x \in A$ Such That $f(x) = y$

$f(x) = y$

$\frac{x}{x-2} = y$

$yx - 2y = x$

$xy - x = 2y$

$x(y-1) = 2y$

$x = \frac{2y}{y-1} \Rightarrow y \neq 1$

f is onto

So, f is a bijection, Inverse is possible

To find f^{-1}

$x = \frac{2y}{y-1}$

$f^{-1}(y) = \frac{2y}{y-1}$

$f^{-1}(x) = \frac{2x}{x-1}$

(7) If $f: A \rightarrow R$, $A = \{x \in R / x \neq \frac{1}{2}\}$ $f = \frac{4x}{2x-1}$ Find
(i) Range of f, (ii) S.T f is invertible (iii) domain (f^{-1})
(iv) Range (f^{-1}), f^{-1}.

Sol:- Range of f = Set of all images

$f(x) = \frac{4x}{2x-1} \Rightarrow y = \frac{4x}{2x-1}$

$2xy - y = 4x$

$2xy - 4x = y$

$2x[y-2] = y$

$2x = \frac{y}{2y-4}$, $y \neq 2$

Range $= \{x \in R / y \neq 2\}$

(ii) 1-1

Let $f(x) = \frac{4x}{2x-1}$

$f(x_1) = f(x_2)$

$\frac{4x_1}{2x_1 - 1} = \frac{4x_2}{2x_2 - 1}$

$8x_1x_2 - 4x_1 = 8x_1x_2 - 4x_2$

$4x_1 = 4x_2$

$\boxed{x_1 = x_2}$

onto

$\forall\, y \in R,\ y \neq 2$ there exists $x \in A$ such that

$x = \frac{y}{2y-4}$

f is onto

So, f is invertible.

(iii) $f: A \rightarrow R$ $\qquad$ $f^{-1}: R \rightarrow A$

$A = \{x \in R / x \neq \frac{1}{2}\}$ $\qquad$ Range of f $= \{x \in R / x \neq 2\}$

$dom(f^{-1}) = R = Range(f)$

$= \{x \in R / x \neq 2\}$

$Range(f^{-1}) = A = \{x \in R / x \neq \frac{1}{2}\}$

Inverse f^{-1}: $f(x) = y$

$y = \frac{4x}{2x-1}$

$\Rightarrow x = \frac{2y}{2y-4}$, $y \neq 2$

$$\boxed{f^{-1}(x) = \frac{2x}{2x-4},\ x \neq 2}$$

Properties of functions:

1. If $f: A \to B$, $g: B \to C$ are invertible $gof: A \to C$ is also Invertible

2. Inverse of a function is unique if it exsists

3. $[gof]^{-1} = f^{-1} o g^{-1}$

   f, g are invertible