6/9

UNIT 2.

Combinatorics.

Combinatorics - permutation - Combination inclusion Exclusion principle - greatest Common divisor - GCD, LCM - Euclids Algorithm - pigeon hole principle, Generalised PH principle.

Permutation → Num. of ways.

Permutation:

Any finite set $S = \{x_1, x_2 \ldots x_n\}$ a permutation can be defined as a bijective mapping on the set S onto itself.

The no. of permutation on the set S is same as the total number of bijective mapping on the S.

It is denoted by nPr.

$$\boxed{{}^nP_r = \frac{n!}{(n-r)!}}$$

Q. Evaluate :

(i) $4P4$

$$^nP_r = \frac{n!}{(n-r)!}$$

$$= \frac{4!}{(4-4)!}$$

$$= 24$$

(ii) $5P3$

$$^nP_r = \frac{5!}{(5-3)!}$$

$$= 60$$

(iii) $6P_5$

$$^nP_r = \frac{6!}{(6-5)!}$$

$$720.$$

Q. if $(n+2)P_4 = 42 \times {}^nP_2$, to find n..

<u>Sol</u>

$$\frac{(n+2)P_4}{nP_2} = 42$$

$$\frac{(n+2)(n+1)n\cancel{(n-1)}}{\cancel{n(n-1)}} = 42..$$

$$(n+2)(n+1) = 42.$$

$$n^2 + n + 2n + 2 = 42.$$

$$n^2 + 3n + 2 = 42$$

$$n^2 + 3n = 40$$

$$n^2 + 8n - 5n - 40 = 0.$$

$$n(n+8) - 5(n+8) = 0$$

$$(n-5)(n+8)$$

$$-40 < \begin{matrix} 8 \\ -5 \\ \hline 3 \end{matrix}$$

$$n-5=0 \qquad n+8=0$$

$$\boxed{n=5} \qquad \boxed{n=-8}$$

$$\boxed{n=5}$$

Q. If $10 P_r = 7 P_{r+2}$ To find r.

Sol

$$10 P_r = 7 P_{r+2}$$

$$n P_r = \frac{n!}{(n-r)!}$$

$$\frac{10!}{(10-r)!} = \frac{7!}{(7-(r+2))!}$$

$720 = 6!$

$$\frac{10 \times 9 \times 8 \times \cancel{7!}}{(10-r)(9-r)(8-r)(7-r)(6-r)\cancel{(5-r)!}} = \frac{\cancel{7!}}{\cancel{(5-r)!}}$$

$$(10-r)(9-r)(8-r)(7-r)(6-1) = 6 \times 5 \times 4 \times 3 \times 2.$$

$10 - r = 6$ $r = 4$	$9 - r = 5$ $r = 4$	$7 - r = 3$ $r = 4$

Q. how Many letter String' together can be formed with the letter of the word "VOWELS" So that

(i) Total no. of possibility.

(ii) the String begin with 'E'

(iii) the String begin & end with E & W.

Q. A no. of four different digits is formed with use of digit 5, 4, 3, 2, 1 In all possible way.

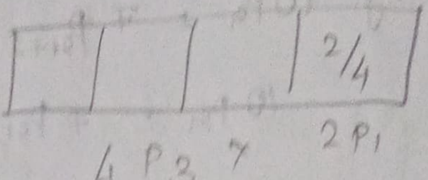
(i) how many such nos can be formed?

(ii) How many these are even?

(iii) how many of these Divisible by 4

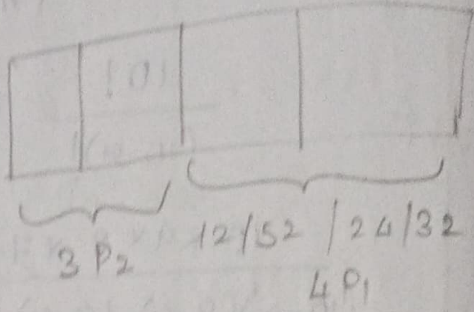
Sol Possible ways

(i) $^5P_4 = 120$



(ii) $^4P_3 \times ^2P_1 = 48$

(iii) $^3P_2 \times ^4P_1 = 24$



VOWELS

(i) Total no. of ways $= 6! = 720$

(ii) E is the 1st letter $=$ E _ _ _ _ _ $= 5!$

1 5 4 3 2 1 $= 120$

(iii) E & W $\Rightarrow$ "E _ _ _ _ W" $= 4! = 24$

1 4 3 2 1 1.

13/9 Combinations:

It is a selection of r from n individuals in any order

It is denoted by nCr.

$$^nC_r = \frac{n!}{(n-r)!\,r!}$$

Problems:

Q. In how many ways can a coach choose 3 summers from 5 summer.

$$nC_r = \frac{n!}{(n-r)!\,r!}$$

$n = 5$, $r = 3$

$$5C_3 = \frac{5!}{(5-3)!\,3!} = \frac{5!}{2!\,3!}$$

$= 10$ ways.

Q. In how many ways a committee consisting of 5 Men & 3 woman can be choosen from 9 men & 12 woman.

$$\left.\text{no. of ways of finding 5 men from 9 men.}\right\} = 9C_5$$

No. of ways of finding 12 woman $= 12C_3$.

$$^9C_5 = \frac{9!}{4!5!} = 126.$$

$$^{12}C_3 = \frac{12}{9!3!} = 220.$$

No. of ways of Selecting woman & men $= m \times m$ ways

$= 126 \times 220$

$= 27,720$ ways.

Relation between Permutation & Combination.

$$nP_r = \frac{n!}{(n-r)!}$$

$$nC_r = \frac{n!}{(n-r)!\,r!}$$

So, $nC_r = \dfrac{nP_r}{r!}$ (or) $r!\,nC_r = nP_r$.

Permutation

$$nP_r = \frac{n!}{(n-r)!}$$

Permutation with repeatation.

$$= \frac{n!}{n_1!\,n_2!\ldots n_k!}$$

Combination

$$nC_r = \frac{n!}{(n-r)!\,r!}$$

4/9 Addition rule:

Suppose there is a Certain process which have m ways & another process has n ways & both process cannot happen Simultaneously. Then total no. of ways in m+n ways.

Multiplication rule:

If the two process happens simultaneously Then the no. of ways will be m·n ways.

Q. There are 6 men & 5 woman in a interview room
To find the no. of ways of selecting 4 persons from the room
(i) They can be Male / female
(ii) Two must be men & two women. Multiply
(iii) There must be all same gender. (ADD)
$^6C_4 + {^5C_4}$

Sol:
(i) Total no. of person is 11 (6+5)
So, the no. of ways $= {^{11}C_4}$
$= \frac{11!}{7!\,4!} = 330$ ways.

(ii) No. of men (ways of selecting men) $= {^6C_2} = \frac{6!}{4!\,2!} = 15$

Woman $= {^5C_2} = \frac{5!}{3!\,2!} = 10.$

So, total no. of ways $m.n = 15 \times 10$
$= 150$ ways.

(iii) They must be same gender

No. of ways of selecting Men $= {}^6C_4 = 15$

Woman $= {}^5C_4 = 5$

Total no. of ways $m + n = 15 + 5 = 20$ ways.

Q. from a committee of 6 men & 7 women in how many way can we select a committee

(i) 3 men & 4 women

(ii) 4 member which has atleast 1 women

(iii) 4 member which has almost 1 men

(iv) 4 members of both.

Sol (i) 3 men & 4 women.

no. of ways of selecting Men $= {}^6C_3 = \frac{6!}{3!3!}$

women $= {}^7C_4 = \frac{7!}{4!3!}$

So, total no. of ways $= {}^6C_3 \times {}^7C_4$
$= 700$ ways.

(ii) 4 Members, Atleast 1 women

$1W \& 3M = {}^7C_1 \times {}^6C_3 = 140$

$2W \& 2M = {}^7C_2 \times {}^6C_2 = 315$

$3W \& 1M = {}^7C_3 \times {}^6C_1 = 210$

$4W \& = {}^7C_4 \times {}^6C_0 = 135$

Total no. of way = 140 + 315 + 210 + 35
= 700 ways.

(iii) 4 member, atmost 1 men.

1 men + 3 women = $^6C_1 \times {}^7C_3 = 210$

0 men + 4 women = $^6C_0 \times {}^7C_4 = 35$

Total no. of ways = 245 ways.

(iv) 4 member of both genders.

1 w & 3 men = $^7C_1 \times {}^6C_3 = 210$

2 w & 2 M = $^7C_2 \times {}^6C_2 = 315$

3 w & 1 M = $^7C_3 \times {}^6C_0 = 140$.

So, total no. of ways = 210 + 315 + 140
= 665 ways.

Q. In how many ways can you ~~assume~~ arrange the letter for the following words. Permutation.

(i) RADAR

(ii) ~~UNUSAL~~ UNUSUAL

(iii) LOLLIPOP

$$\text{Permutation with repeatation} = \frac{n!}{n_1!\, n_2! \ldots n_k!}$$

(i) RADAR

$n = 5$

for R as $n_1 = 2$.

for A as $n_2 = 2$.

Total no. of ways = $\frac{5!}{2!\,2!} = 30$

(ii) UNUSUAL:

$n = 7$

for U as $n_1 = 3$

for Total no. of ways $= \frac{7!}{3!} = 840$

(iii) LOLLIPOP

H.W

$n = 8$. $\quad 1680$

L as $n_1 = 3$

P as $n_2 = 2$.

O as $n_3 = 2$.

Total no. of ways $= \frac{8!}{3!\,2!\,2!} = 1680$.

Q. how much bit string of length 10 contains

a) exactly four 1's

b) atmost four 1's

c) atleast four 1's.

d) Equal no. of 0's & 1's

Sol A) $^{10}C_4 = \frac{10!}{4!(10-4)!} = 210$

B) $= 386$.

C) $= 848$.

D) $= 252$

5/9

Inclusion - Exclusion principle.

Let A and B be two sets which are not mutually exclusive and $|A|$ be the no. of elements of set A & $|B|$ be the no. of element of the set B.

Thus, there are $|A|$ ways of selecting an element & $|B|$ ways of selecting an element from B

The no. of ways of selecting an element from A or B

$$|A \cup B| = |A| + |B| - |A \cap B|$$

If we have A, B, C are sets

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |C \cap A| + |A \cap B \cap C|$$

If A, B, C, D are the set. A B C D

Inclusion - Exclusion principle for 4 sets:

$$|A \cup B \cup C \cup D| = |A| + |B| + |C| + |D| - |A \cap B| - |A \cap C| - |A \cap D| - |B \cap C| - |B \cap D| - |C \cap D| + |A \cap B \cap C| + |A \cap B \cap D| + |A \cap C \cap D| + |B \cap C \cap D| - |A \cap B \cap C \cap D|$$

A B C D → [2 elements in set]

Greatest Integers : $\leq x$

It is denoted by $[\ \ ]$

(i.e) $[2.3] = 2.$

$[5.9] = 5$

Q. How many positive integers not exceeding 1000 are divisible by 7 or 11?

A or B.

$|A| = \left[\frac{1000}{7}\right] = 142$

$|B| = \left[\frac{1000}{11}\right] = 90$

$|A \cap B| = \left[\frac{1000}{7 \times 11}\right] = 12$

The no. of ways of selecting an element from A or B

$|A \cup B| = |A| + |B| - |A \cap B|$

$= 142 + 90 - 12$

$= 220$ ways.

Q. How many positive integers not exceeding 1000 are not divisible by 3, 7 or 11?
(A = 3, B = 7, C = 11)

By inclusion- Exclusion principle:

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |C \cap A| + |A \cap B \cap C|$

$|A| = \left[\frac{1000}{3}\right] = 333$ $\quad |C| = \left[\frac{1000}{11}\right] = 90$

$|B| = \left[\frac{1000}{7}\right] = 142$

$|A \cap B| = \left[\frac{1000}{3 \times 7}\right] = 47.$ $\quad |C \cap A| = \left[\frac{1000}{11 \times 3}\right] = 30$

$|B \cap C| = \left[\frac{1000}{7 \times 11}\right] = 12$ $\quad |A \cap B \cap C| = \left[\frac{1000}{3 \times 7 \times 11}\right] = 4$

$|A \cup B \cup C| = 333 + 142 + 90 - 47 - 12 - 30 + 4$

$|A \cup B \cup C| = 480.$

non- divisible according to ques $\Rightarrow$ $1000 - 480 = 520.$

Q. Find the no. of integers b/w 1 & 250 that are not divisible by 2, 3, 5, 7?

$|A| = \left[\frac{250}{2}\right] = 125$ $\quad |C| = \left[\frac{250}{5}\right] = 50$

$|B| = \left[\frac{250}{3}\right] = 83$ $\quad |D| = \left[\frac{250}{7}\right] = 35$

$|A \cap B| = \left[\frac{250}{2 \times 3}\right] = 41$ $\quad |A \cap C| = \left[\frac{250}{2 \times 5}\right] = 25$

$|A \cap D| = \left[\frac{250}{2 \times 7}\right] = 17$ $\quad |B \cap C| = 16$; $|B \cap D| =$

$|C \cap D| = 7$ $|A \cap B \cap C| = 8$ $|A \cap B \cap D| = 5$

$|A \cap C \cap D| = 3$ $|B \cap C \cap D| = 2$ $|A \cap B \cap C \cap D| = 1$

$$|A \cup B \cup C \cup D| = 125 + 83 + 50 + 35 - 41 - 25$$
$$- 17 - 16 - 11 - 7 + 8 + 5 + 3 + 2 + 1$$
$$= 193. \text{ (Divisibility)}$$

Not- Divisibility ⇒ $250 - 193 = 57$.

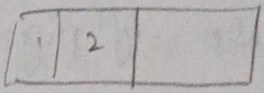
Q. There are 345 students of a college to have taken physics, 220 have taken maths, 175 take both physics & Maths.
how many have taken a course either physics or maths?

$|A| = 345$ $|B| = 220$ $|A \cap B| = 175$

$$|A \cup B| = 345 + 220 - 175$$
$$= 405 //$$

Pigeonhole principle:

If n pigeons are assigned to m pigeonholes & $n \geq m$, then atleast one of the pigeonhole will contain two or more pigeons.



generalise of Pigeonhole principle.

If n pigeons are assigned to m pigeon holes and $n > m$ atleast one of the Pigeon hole will contain

$$\left[\frac{n-1}{m}\right]+1 \text{ pigeons.}$$

Q. Show that if seven colours are used to paint 50 bicycles then atleast 8 bicycles will have the same colour.

<u>Sol</u> No. of pigeons $= n = 50$

No. of pigeon holes $= m = 8$

By pigeon hole principle $\left[\frac{n-1}{m}\right]+1$.

$$\left[\frac{50-1}{8}\right]+1$$

$$= 6+1$$

$$= 7 \text{ Colours.}$$

Q. Show that, among 100 people atleast 9 of them were both in the same Month

No. of pigeons = 100

No. of pigeon holes = 12

By generalised P.H principle.

$$\left[\frac{n-1}{n}\right] + 1 = \left[\frac{100-1}{12}\right] + 1$$

$$= 8+1$$

$$= 9.$$

Q. Show that 30 dictionaries in a library contain a total of 61,327 pages then one of the dictionary must have 2045 pgs.

Sol:

No. of pigeons $n = 61,327$

no. of pigeon holes $m = 30$

By genralised pigeon hole principle

$$\left[\frac{n-1}{m}\right] + 1 = \left[\frac{61,327-1}{30}\right]$$

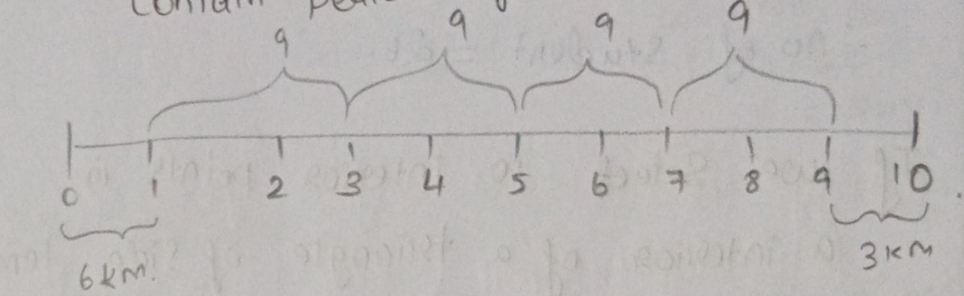
$$= 2044 + 1$$

$$= 2045 \text{ pgs.}$$

Q. A man walked for 10 hrs & covered a distance of 45 km.

It is known that he halted 6 km in the first hour, 3 km in the last hr.

Show that he must halted 9 km in a contain period of consecutive hrs.



Here No. of Pigeons $n = 36$.

No. of pigeon holes $m = 4$

By GPH principles

$$\left[\frac{36-1}{4}\right] + 1 = 8 + 1 = 9$$

Q. What is the maximum no. of students required in a DM class to be sure that six will receive the same grade of these and there are five grade A, B, C, D & F?

Sol Here the no. of student $= n$.

No. of pigeonholes $m = 5$

So by GPH principle.

$$\left[\frac{n-1}{m}\right] + 1 = 6.$$

A	B	C	D	F

$$\left[\frac{n-1}{5}\right] = 6-1$$
$$= 5$$
$$n-1 = 25$$
$$n = 26.$$

$\therefore$ no of student = 26.

H.W If we select 10 interior points in a interior of a triangle of side 1cm. S.T there will be atleast 2 pts whose distance is $< \frac{1}{3}$

Divisibility:

When a & b be any two integers a is said to divide b with $a \neq 0$, if there exists an integer 'c' such that

$a/b = c$

$8/4 = 2$. can be written as

$b = ac$

$8 = 2 \times 4$.

Prime Number:

A positive integer $p > 1$ is called prime number if the possible division are only 1 & p. [1 → Neither prime nor composite)

Composite number:

A positive integer which is not prime number is said to be composite number.

Theorem:

Let $a, b, c \in Z$ & Z be the set of integers.

(i) $a|b$, $a|c \Rightarrow a|c$

(ii) $a|b \Rightarrow a|mb$

(iii) $a|b$, $a|c \Rightarrow a/_{mb+nc}$ $m, n \in Z$.

Fundamental Theorem of Arithmetic

Every integer $n>1$ can be uniquely written as the product of primes

Eg: $14 = 2 \times 7 \times 1$

$38 = 2^2 \times 7$

$36 = 2^2 \times 3^2$

Q. Verify the theorem is fine for the following integers (using prime factorization)

(i) 6647 (ii) 45500 (iii) 10!

$$\begin{array}{r|l} 17 & 6647 \\ \hline 17 & 391 \\ \hline 23 & 23 \\ \hline & 1 \end{array}$$

$6647 = 17^2 \times 23$

$5^3 \times 2^2 \times 7 \times 13$

$\Rightarrow 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10$

$\Rightarrow 1 \times 2 \times 3 \times 2^2 \times 5^1 \times 2 \times 3 \times 7 \times 2^3 \times 3^2 \times 2 \times 5$

$= 1 \times 2^8 \times 3^4 \times 5^2 \times 7$

Q. To find the LCM & GCD of 28, 12

(ii) 36, 81 (iii) 144, 196.

Sol (i) 28, 12.

$$\begin{array}{r|l} 2 & 28 \\ \hline 2 & 14 \\ \hline & 7 \end{array} \qquad \begin{array}{r|l} 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$28 = 2^2 \times 7 \times 3^0$

$12 = 2^2 \times 3 \times 7^0$

$GCD(28,12) = 2^{\min(2,2)} \times 3^{\min(0,1)} \times 7^{\min(0,1)}$

$= 2^2 \times 3^0 \times 7^0 = 4$

$LCM(28,12) = 2^{max(2,2)} \, 3^{max(1,0)} \, 7^{max(1,0)}$

$= 84.$

(ii) $GCD(36,81) = 9$

$LCM(36,81) = 324$

(iii) $GCD(144,196) = 4$

$LCM(144,196) = 7056.$

Relation ship between $LCM(m,n)$ & $GCD(m,n)$

$LCM \times GCD = m \times n.$

Q. To find LCM & GCD & verify $GCD \times LCM = m \times n$ $(231, 1575)$. using prime factorization. $231 = 3 \times 11 \times 7$; $1575 = 5^2 \times 3^2 \times 7.$

$LCM = 51975$

$GCD = 21$

Q. To find LCM & GCD of $(337500, 21600)$ using prime factorization & verify $GCD \times LCM = m \times n.$

H.W

Division Algorithm:

When a & b are any two integer with $b > a$ and there exists integers q & r such that

$$b = aq + r \;;\; a \leq r \leq b.$$

Euclid Algorithm for finding GCD:

When a & b are any two integers $b > a$ if r_1 is a remainder when a is divided by b, r_2 is the reminder when b is divided by r_1 & so on.

Problem

Q. To find GCD (414, 662) using Euclid Algo.

Sol ⇒ $662 > 414$

By Division Algo:

$$662 = 414(1) + 248$$

⇒ $414 > 248$.

By Division Algo:

$$414 = 248(1) + 166$$

⇒ $248 > 166$.

By Division Algo:

$$248 = 166(1) + 82.$$

⇒ $166 > 82$.

By Division Alg:

$$166 = 82(2) + 2$$

⇒ $82 > 2$.

By Division Algo:

$$82 = 2(41) + 0.$$

Here the remainder is 0 & the quotient is 4!

$\therefore$ GCD(414, 662) = 2.

Q. To find GCD (1819, 3587) using Euclid. algo. Express the GCD as linear combination.

Sol ⇒ here $3587 > 1819$

By Division - Algo

$$3587 = 1819(1) + 1768.$$

⇒ $1819 > 1768$.

By Division - Algo.

$$1819 = 1768(1) + 51$$

⇒ $1768 > 51$

By Division - Algo.

$$1768 = 51(34) + 34$$

⇒ $51 > 34$

By Division. Algo:

$$51 = (34)(1) + 17.$$

⇒ $34 > 17$

By Division Algo

$$34 = 17(2) + 0.$$

$17 = 51 - 34(1)$

$34 = 1768 - 51(34)$

$51 = 1819 - 1768(1)$

$1768 = 3587 - 1819(1)$

GCD $(1819, 3587) = 17$.

Linear Combination:

$$17 = 51 - (34 \times 1)$$

$$= 51 - ((1768 - 34 \times 51) \times 1)$$

$$= 51 - ((1768 - (1768 - 51(34)) \times 51) \times 1$$

$$= 1819 - (1768 \times 1) - (1768 - (1819 - 1768)) \times (1768 - 51(34))$$

$= 1819 - (1768 \times 1) - (1768 \times 1) + (51 \times 34)$

$= 1819 - (1768 \times 1) - (1768 \times 1) + (1819 - 1768) \times 34$

$= (1819 \times 1) - (1768 \times 1) - (1768 \times 1) + (1819 \times 34) - (1768 \times 34)$

$= 1819(1+34) - 1768(1+1+34)$

$1819(35) - 1768(36$

Linear Combination of $(1819, 3587)$

$17 = 51 - 34 \times 1 \rightarrow ①$ $\quad 1768 = 3587 - 1819 \times 1 \rightarrow ④$.

$34 = 1768 - 51 \times 34 \rightarrow ②$

$51 = 1819 - 1768 \times 1 \rightarrow ③$

$17 = 51 - 34$ (By ①)

$17 = 51 - (1768 - 51 \times 34)(1)$ (By (2))

$= 51 - (1768 \times 1) + 51 \times 34$

$= (51 \times 1) - (1768 \times 1) + 51 \times 34$

$= (51 \times 35) - (1768 \times 1)$

$= (1819 - 1768(1)) \times 35 - (1768 \times 1)$ (eq (3))

$= (1819 \times 35) - 1768 \times 35 - (1768 \times 1)$

$= (1819 \times 35) - (1768 \times 36)$

$17 = (1819 \times 35) - (3587 - 1819 \times 1) \times 36$ (eq (4))

$= (1819 \times 35) - (3587 \times 36) + (1819 \times 36)$

$= (1819 \times 71) - (3587 \times 36)$

$= 1819\,m + 3587\,n$

$m = 71;\ n = 36.$

Q. To find GCD of 65 & 117 & find the linear combination (use Euclid Algo)

Sol. Here $117 > 65$, $\therefore 117 = 65(1) + 52$

$65 > 52$, $65 = 52(1) + 13$

$52 > 13$, $52 = 13(4) + 0$.

$GCD(65, 117) = 13$

Linear combination $52 = 117 - 65$

$13 = 65 - 52(1)$

$= 65 - (117 - (65 \times 1))(1)$

$= (65 \times 1) - (117 \times 1) + (65 \times 1)$

$= (65 \times 2) - (117 \times 1)$

$13 = 65m + 117n$

Here $m = 2$, $n = -1$ //

Q. To find GCD of 81 & 237 using Euclid Algo & write as linear combination.

Here $237 > 81$, so; $237 = 81 \times 2 + 75$

$81 > 75$, so: $81 = 75 \times 1 + 6$

$75 > 6$; so $75 = 6 \times 12 + 3$

$6 > 3$ $6 = 3 \times 2 + 0$

$GCD(81, 237) = 3$

$3 = 75 - 6 \times 12 \rightarrow (1)$

$6 = 81 - 75 \times 1 \rightarrow (2)$

$75 = 237 - 81 \times 2 \rightarrow (3).$

$3 = 75 - 6 \times 12$ (By (1))

$= 75 - (81 - 75) \times 12.$ (By (2))

$= 75 - (81 \times 12 - 75 \times 12)$

$= (75 \times 1) - 81 \times 12 + 75 \times 12$

$= (75 \times 13) - (81 \times 12)$

$= (237 - 81 \times 2) \times 13 - (81 \times 12)$

$= (237 \times 13) - (81 \times 26) - (81 \times 12)$

$= (237 \times 13) - (81 \times 38)$

$3 = 237m - 81n$

$m = 13, \quad n = -38$

Q. To find the GCD for 396, 504 & 636 by using Euclid Algo. (Ans = 12).

Q. To find the GCD (12345, 54321) by Euclid Algo & express in Linear Combination (Dec 2021) (CV) (Ans =

Q. To find GCD (512, 320) by Euclid Algo & express in Linear Combination)

GCD(12345, 54321) = 3.	GCD (512, 320) = 64
L.C = m = 3617; n = -822	LC = m = 2, n = -3

Relatively Prime:

Two positive integers are said to be relatively prime or co-prime if the GCD is 1.

Properties:

if $c | ab$, a, c are co-prime then $c | b$.

$GCD(ka, kb) = k\,GCD(a, b)$

if $GCD(a, b) = d$, then $GCD(a/d, b/d) = 1$

if $GCD(a, b) = 1$ then $GCD(ac, b) = gcd(c, b)$