

11/03/2020

## UNIT-4

### CLASSIFICATION METHODS

→ classification can be done by different Algorithms

#### \* Statistical Based Algorithm!

→ Linear Regression

→ Naive Bayes

#### \* Distance based Algorithm:

→ k-nearest neighbour (KNN)

#### \* Decision tree based Algorithm:

\* ID<sub>3</sub>

\* C4.5

\* CART

#### \* Neural Network based Algorithm

#### \* Rule based Algorithm

#### \* Combining Techniques

\* Classification: It is a form of data analysis that extract the models describing important data classes

Ex: A class student has A, B, C, D and E classes based on the grades.

→ By using above models, we can classify the students depends upon the Grade.

$$\text{grade} \geq 90 \Rightarrow \text{class A}$$

$$\text{grade} \geq 80 \& \text{grade} \leq 90 \Rightarrow \text{class B}$$

$$\text{grade} \geq 70 \& \leq 80 \Rightarrow \text{class C}$$

$$\text{grade} \geq 60 \& \leq 70 \Rightarrow \text{class D}$$

$$\text{grade} \leq 60 \Rightarrow \text{class E}$$

### Applications:

→ Image and Pattern matching

→ Medical diagnosis

→ fraud detection

→ loan approvals

→ classifying financial market, trends etc.

### Classification Problems:

Given a database 'D',  $t_1, t_2, \dots, t_n = T$  no of tuples (items, records) and a set of classes  $C = C_1, C_2, \dots, C_m$ . The classification problem is to define the mapping  $f: D \rightarrow C$  where each item is assigned to one class. A class  $C_j$  contains precisely those tuples mapped to it.

$$C_j = \left\{ t_i \mid f(t_i), 1 \leq i \leq n \right\}$$

{GREEDY}	(S A)
(A)	

## General approach to classification problem

→ There are 2 ways to do the classification problem

- \* Learning (or) training stage
- \* Classification stage.

Learning (or) training: where the classification model is constructed

Classification: where the model is used to predict the class labels for the given data

(1) Statistical based Algorithm:

→ Refer Linear Regression topic

(2) Base classification method / Bayesian method /

Navy Bayes classification method

→ It is statistical classifier that can predict the class membership probabilities such as the

probability that given belongs to the particular

class

→ It is also called as Baye theory

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

$P(A|B)$  refers to the probability of occurrence of event A given that the event B is true

$P(A)$  and  $P(B)$  refers to the probabilities of occurrences of event A and event B

A is called preposition

B is called Evidence

$p(A)$  is called prior probability of preposition.

$p(B)$  is called prior probability of evidence

$P(A|B)$  is called posterior / posterior probability.

$P(B|A)$  is called likelihood

posterior = likelihood \* posterior probability of  
prior probability of Evidence

\* consider the given weather dataset with attributes (outlook, temperature, windy, humidity) and class label play with values {yes, No} find. the person can play or Not with given weather conditions outlook = sunny, temperature, cool, Windy = strong and humidity = high

SNO	outlook	temp	humidity	windy	play
1	sunny	Hot	Hot	weak	No
2	sunny	Hot	Hot	strong	No
3	overcast	Hot	Hot	weak	Yes
4	rainy	mild	Hot	weak	Yes
5	rainy	cool	nominal	weak	Yes
6	rainy	cool	nominal	strong	No
7	overcast	cool	nominal	strong	Yes
8	sunny	mild	Hot	weak	No
9	sunny	cool	nominal	weak	Yes
10	rainy	mild	nominal	weak	Yes
11	sunny	mild	nominal	strong	Yes
12	overcast	mild	Hot	strong	Yes
13	overcast	hot	nominal	weak	Yes
14	Rainy	mild	Hot	strong	No

SOL:  $P(Y) = \frac{9}{14}$ ,  $P(N) = \frac{5}{14}$

but you will think that probability has

no

address: as you did you go to normal sit

outlook

outlook	Y	N	P(Y)	P(N)
sunny	3	1	3/4	1/4
overcast	4	0	4/4	0
rainy	3	2	3/5	2/5

Temperature	Y	N	$P(Y)$	$P(N)$	Probability
Hot	2	1	2/9	7/9	2/5
Cold	4	2	4/9	5/9	2/5
Mild	3	1	3/9	6/9	1/5

Humidity	Y	N	$P(Y)$	$P(N)$	Probability
Hot	3	4	3/9	6/9	4/5
Nominal	6	3	6/9	3/9	1/5
Windy	Y	N	$P(Y)$	$P(N)$	Probability
Weak	6	2	6/9	3/9	2/5
Strong	3	3	3/9	6/9	1/3

probability that a player can play a game

$$P(X/\text{play} = \text{yes}) \cdot P(\text{play} = \text{yes})$$

$$= \frac{2}{9} \times \frac{3}{9} \times \frac{3}{9} \times \frac{3}{9} \times \frac{9}{14}$$

$$= 0.00529$$

$$= 0.0053$$

Probability that player cannot play the game

$$P(x | \text{play} = \text{NO}) \cdot P(\text{play} = \text{NO})$$

$$= \frac{3}{5} \times \frac{1}{5} \times \frac{4}{5} \times \frac{3}{5} \times \frac{5}{14}$$

$$= \frac{18}{875}$$

$$= 0.0205$$

$$= 0.02$$

Probability of class  $P(x) = P(x | \text{play} = \text{Yes}) \cdot P(\text{play} = \text{Yes})$

$$+ P(x | \text{play} = \text{No}) \cdot P(\text{play} = \text{No})$$

$$P(A|C) = \frac{P(C|A) \cdot P(A)}{P(C)}$$

$$P(\text{play} = \text{Yes})$$

$$P(\text{play} = \text{Yes}/x) = \frac{P(x | \text{play} = \text{Yes}) \cdot P(\text{play} = \text{Yes})}{P(x)}$$

$$= \frac{0.0053}{0.0253}$$

$$= 0.2094$$

$$P(\text{play} = \text{No}/x) = \frac{P(x | \text{play} = \text{No}) \cdot P(\text{play} = \text{No})}{P(x)}$$

$$= \frac{0.0205}{0.0253}$$

$$= 0.8102$$

since,  $P(\text{No})$  is having the maximum value when compared with Yes, Hence the person cannot play

\* find the person can go for play or not when outlook = rainy, temperature = mild, humidity = nominal, Windy = weak

$$\text{sol: } P(x/\text{play}=\text{Yes}) \cdot P(\text{play}=\text{yes}) = \frac{3}{9} \times \frac{4}{9} \times \frac{6}{9} \times \frac{6}{9} \times \frac{9}{14}$$

$$= 0.0423$$

$$P(x/\text{play}=\text{No}) \cdot P(\text{play}=\text{No}) = \frac{2}{5} \times \frac{2}{9} \times \frac{1}{5} \times \frac{2}{5} \times \frac{5}{14}$$

$$= 2.539 \times 10^{-3}$$

$$P(x) = 0.0423 + 2.539 \times 10^{-3}$$

$$= 0.0448$$

$$P(\text{play}=\text{yes}/x) = \frac{P(x/\text{play}=\text{yes}) \cdot P(\text{play}=\text{yes})}{P(x)}$$

$$= \frac{0.0423}{0.0448}$$

$$= 0.9441$$

$$P(\text{play}=\text{No}/x) = \frac{P(x/\text{play}=\text{No}) \cdot P(\text{play}=\text{No})}{P(x)}$$

$$= \frac{0.00253}{0.0448}$$

$$= 0.0558$$

Since,  $P(yes)$  is having the maximum value when compared to No. The person can play a game.

- \* find the patient having flue or No with the given conditions. cold=yes, running nose=no, headache=mild, fever=yes

SNO	cold	running nose	Headache	(fever)	flu
1	yes	NO	Mild	yes	No
2	yes	yes	no	No	yes
3	yes	NO	strong	yes	yes
4	NO	yes	mild	yes	yes
5	NO	NO	no	NO	No
6	NO	yes	strong	yes	yes
7	NO	yes	strong	NO	No
8	yes	yes	mild	yes	yes

Sol:  $P(y) = \frac{5}{8}$ ,  $P(n) = \frac{3}{8}$

↓	↓
yes	No

cold	y	n	$P(y)$	$P(n)$
yes	3	1	$3/5$	$1/3$
no	2	2	$2/5$	$2/3$

Running Nose	y	n	$P(y)$	$P(n)$
yes	4	1	$4/5$	$1/3$
no	2	2	$1/5$	$2/3$

Headache	y	n	$P(y)$	$P(n)$
hild	2	1	$2/5$	$1/3$
strong	2	1	$2/5$	$1/3$
No	1	1	$1/5$	$1/3$

fever	y	n	$P(y)$	$P(n)$
yes	4	1	$4/5$	$1/3$
no	1	2	$1/5$	$2/3$

Probability of having flu is as follows

$$\begin{aligned}
 & P(x/\text{flu} = \text{yes}) \cdot P(\text{flu} = \text{yes}) \\
 & = 3/5 \times 1/5 \times 2/5 \times 4/5 \times 5/8 \\
 & = 0.024
 \end{aligned}$$

Probability of a person not having flu is as follows.

$$P(x| \text{flu} = \text{No}) \cdot P(\text{flu} = \text{No})$$

$$= \frac{1}{3} \times \frac{2}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{3}{8}$$

$$= 9.25925 \times 10^{-3}$$

$$\text{Probability of class } P(x) = P(x| \text{flu} = \text{Yes}) \cdot P(\text{flu} = \text{Yes})$$

$$+ P(x| \text{flu} = \text{No}) \cdot P(\text{flu} = \text{No})$$

$$P(x) = 0.024 + 9.25925 \times 10^{-3}$$

$$P(x) = 0.033$$

$$P(\text{flu} = \text{Yes}|x) = \frac{P(x| \text{flu} = \text{Yes}) \cdot P(\text{flu} = \text{Yes})}{P(x)}$$

$$= \frac{0.024}{0.033}$$

$$= 0.727$$

$$P(\text{flu} = \text{No}|x) = \frac{P(x| \text{flu} = \text{No}) \cdot P(\text{flu} = \text{No})}{P(x)}$$

$$= \frac{9.25925 \times 10^{-3}}{0.033}$$

$$= 0.2805$$

- since the probability of having flu is high  
Hence the patient is having flu.
- The person is not having flu in database but is actually having flu; so, it is identified as False-positive (FP)

#### Advantages of Naive Bayes:

- It is easy & fast to predict a class of test data set.
- It performs better predictions compared to other models.
- It performs well in the place of categorical input variables compared to numerical data variable.

#### DisAdvantages of Naive Bayes:

- It is also known as bad estimator.

#### \* Distance based Algorithms:

##### k-Nearest Neighbour:

- find the value of k that must be a positive integer
- for the given datapoint, that needs to be

classified using the algorithm computes the distance b/w the given datapoint and all other data of the given dataset.

- find the k nearest neighbours based on the distance (the least distance value is the first neighbour for given datapoint)
- The given datapoint is placed in the majority of classes with respect to neighbours

Problem: find a person x belongs to which class based on the given values weight = 57 kgs.

and height = 170 cms

<u>weight</u>	<u>height</u>	<u>class</u>
51	167	under weight
62	182	normal
69	176	normal
64	173	normal
65	172	normal
56	174	under weight
58	169	normal
57	173	normal
55	170	normal

<u>sol</u>	<u>Weight</u>	<u>Height</u>	<u>Class</u>	<u>distance</u>
51	167	under weight	$\sqrt{(57-51)^2 + (170-167)^2}$ = 6.708	
62	182	Normal	$\sqrt{(62-51)^2 + (170-182)^2}$ = 13	
69	176	Normal	$\sqrt{(69-51)^2 + (170-176)^2}$ = 13.41	
64	173	Normal	$\sqrt{(64-51)^2 + (170-173)^2}$ = 7.615	
65	172	normal	$\sqrt{(65-51)^2 + (170-172)^2}$ = 8.24	
56	174	under weight	$\sqrt{(56-51)^2 + (170-174)^2}$ = 4.12	
58	169	normal	$\sqrt{(58-51)^2 + (170-169)^2}$ = 1.41	
57	173	Normal	$\sqrt{(57-51)^2 + (170-173)^2}$	
55	170	normal	$\sqrt{(55-51)^2 + (170-170)^2}$ = 2.00	

Assume  $K=3$

Then the nearest neighbours are  $d_7, d_8, d_9$ .

Hence  $x$  belongs to Normal class when

$$\underline{K=3}$$



Problem: find the person x fans following

With given values age=20 Gender=Male.

Consider male=1, female=0, assume k=3

Name	Age	Gender	class fan.
A	32	M	x1
B	40	M	x1
C	16	F	x2
D	14	F	x1
E	55	M	x2
F	40	M	x1
G	20	F	x2
H	15	M	x2
I	55	F	x1
J	15	M	x2

Name	Age	Gender	classfan	distance
A	32	M	x1	12

B	40	M	x1	20
C	16	F	x2	4.12 N2
D	14	F	x2	6.08 x
E	55	M	x2	35
F	40	M	x1	20
G	20	F	x2	1 N1
H	15	M	x2	5 N3
I	55	F	x1	35.01
J	15	M	x2	5 N3

$\therefore$  The person can follow class 6th standard.

\* Remarks of k:  
— In general, the value of k must be a positive integer.

→ K must not be a multiple of the given class parameter.

\* Fix and Hod proposed k-nearest neighbour classifier algorithm in year of 1951 for performing classifier class.

→ The simple version of this algorithm is to predict the target label by finding nearest neighbour class.

→ The class will be identified by using distance Euclidean distance or Manhattan distance.

Applications:

→ It is used in recommendation (Ex:- online shopping)

→ information storing and retrieving in the server

How to choose k value:

→ Selecting the value k in knn is the most critical problem.

- small values of  $K$  means it produce noise that will have a higher noise in research
- The value of  $K$  is large it makes computationally expensive
- The best way to choose  $K$  value in  $k$ -NN is,

$$\sqrt{n}$$

- \* Decision tree Indexing is multi-step process
- It is a flow chart like tree structure
- Where each internal node (non-leaf node) denotes test on an attribute
- Each branch represents an outcome of the test. Each leaf node represents class label
- top most node in the tree is a root node
- Internal node label with attribute  $A_j$  and leaf node labeled with class  $C_j$
- Applications: prioritize bus priority no traffic jam.
- Manufacturing and production companies
- financial Analysis
- Astronomy
- Molecular Biology

## \* Attribute Selection

- An Attribute selection is a heuristic for selecting splitting criteria that separate a given data ( $D$ ) of class label training tuples into individual classes also called as splitting rules.
- It provides the ranking for each attribute and select best score attribute as root node of a tree.
- Three popular Attribute selection methods
  - \* Information gain ( $ID_3$ )
  - \* Gain Ratio (C4.5)
  - \* Gini Index (CART)

## Information Gain ( $ID_3$ )

$$\text{Gain of Attribute } (A) = \text{Information Gain}(P, n) - \text{IG}$$

$$IG(P, n) = \frac{-P}{P+n} \log_2 \left( \frac{P}{P+n} \right) - \frac{n}{P+n} \log_2 \left( \frac{n}{P+n} \right)$$

$$\text{Entropy } (A) = \sum_{i=1}^n \frac{P_i + N_i}{P+n} I(n_i(A))$$

	→ OUTLOOK	temperature	humidity	windy
1	sunny	HOT	High	weak
2	sunny	HOT	High	strong
3	overcast	HOT	High	weak
4	Rainy	mild	High	weak
5	Rainy	COOL	nominal	weak
6	Rainy	COOL	nominal	strong
7	overcast	COOL	nominal	strong
8	sunny	mild	high	weak
9	sunny	COOL	nominal	weak
10	Rainy	mild	nominal	weak
11	sunny	mild	nominal	strong
12	overcast	mild	high	strong
13	overcast	hot	nominal	weak
14	Rainy	mild	high	strong

SOL: Information gain( $9,5$ ):

$$= \frac{-9}{9+5} \log_2 \left( \frac{9}{9+5} \right) - \frac{5}{9+5} \log_2 \left( \frac{5}{9+5} \right)$$

$$= 0.940$$

Entropy(outlook) =  $\sum_{i=1}^3 \frac{P_i + N_i}{n} I(r, A)_{(i)}$

## Entropy (outlook)

	P <sub>i</sub>	n <sub>i</sub>	I <sub>G</sub> (P <sub>i</sub> , n <sub>i</sub> )
sunny	2	3	0.97
overcast	4	0	0
rainy	3	2	0.97

$$I_G(2,3) = \frac{-2}{2+3} \log_2\left(\frac{2}{2+3}\right) - \frac{3}{2+3} \log_2\left(\frac{3}{2+3}\right)$$

$I_G(4,0) = 0$  (if any value is 0 then  $I_G=0$ )

(If both values are same then  $I_G=1$ )

$$I_G(3,2) = \frac{-3}{3+2} \log_2\left(\frac{3}{3+2}\right) - \frac{2}{3+2} \log_2\left(\frac{2}{3+2}\right)$$

$$= 0.970$$

$$\begin{aligned} \text{Entropy (outlook)} &= \frac{2+3}{14} (0.970) + 0.4 \cdot \frac{3+2}{14} (0.970) \\ &= 0.692 \end{aligned}$$

for

$$\text{Gain of Outlook} = 0.940 - 0.692$$

$$\text{Gain of outlook} = 0.248$$

## Entropy (temperature)

	P <sub>i</sub>	n <sub>i</sub>	I <sub>G</sub> (P <sub>i</sub> , n <sub>i</sub> )
Hot	2	2	0.940
mild	4	12	0.918
Cool	3	11	0.811

$$I_G(4,2) = -\frac{4}{4+2} \log_2 \left( \frac{4}{4+2} \right) - \frac{2}{4+2} \log_2 \left( \frac{2}{4+2} \right)$$

$$= 0.918$$

$$I_G(3,1) = -\frac{3}{3+1} \log_2 \left( \frac{3}{3+1} \right) - \frac{1}{3+1} \log_2 \left( \frac{1}{3+1} \right)$$

$$= 0.811$$

Entropy (temperature) =

$$\left( \frac{2+2}{14} \right) (1) + \left( \frac{4+2}{14} \right) (0.918) + \left( \frac{3+1}{14} \right) (0.811)$$

$$= 0.910 \text{ (s.p.d)} \rightarrow \left( \frac{\varepsilon}{\varepsilon+\delta} \right) \text{ s.p.d} \rightarrow \left( \frac{\varepsilon}{\varepsilon+\delta} \right) \text{ s.p.d}$$

$$\text{Gain of temperature} = 0.940 - 0.910$$

$$= 0.03$$

$$\text{Gain of temperature} = 0.03$$

Entropy (humidity)

	Pc	ni	I <sub>G</sub>
High	3	4	0.985
nominal	6	3+1=4	0.591

$$I_G(3,4) = -\frac{3}{3+4} \log_2 \left( \frac{3}{3+4} \right) - \frac{4}{3+4} \log_2 \left( \frac{4}{3+4} \right)$$

$$= 0.985$$

$$I_G(6,1) = -\frac{6}{6+1} \log_2 \left( \frac{6}{6+1} \right) - \frac{1}{6+1} \log_2 \left( \frac{1}{6+1} \right)$$

$$=$$

$$\text{Entropy (humidity)} = \frac{3+4}{14} (0.985) + \frac{6+1}{14} (0.591)$$

$$= 0.788 \text{ bits per node}$$

$$\text{Gain of humidity} = 0.940 - 0.788 \\ = 0.152$$

$$\text{Gain of humidity} = 0.152 \text{ bits per node}$$

$$\text{Entropy (windy)}$$

	$P_i$	$n_i$	$IG(P_i, n_i)$
Weak	6/11	2	0.811
Strong	10/3	3	1.091

$$IG(6,2) = \frac{-6}{6+2} \log_2 \left( \frac{6}{6+2} \right) - \frac{2}{6+2} \log_2 \left( \frac{2}{6+2} \right)$$

$$= 0.811 \text{ bits per node}$$

$$IG(3,3) = 1$$

$$\text{Entropy (windy)} = \frac{6+2}{14} (0.811) + \frac{3+3}{10} (1)$$

$$= 0.8952$$

$$\text{Gain of windy} = 0.940 - 0.8952$$

$$= 0.048$$

→ Among all the 4 attributes the Gain value of outlook is high Hence we consider this as root node.

	<u>outlook</u>	<u>temperature</u>	<u>humidity</u>	<u>windy</u>	<u>play</u>
1	sunny	Hot	High	Weak	No
2	sunny	Hot	High	Strong	No
3	sunny	mild	High	Weak	No
4	sunny	cool	nominal	Weak	Yes
5	sunny	mild	nominal	Strong	Yes
6	overcast	Hot	High	Weak	Yes
7	overcast	cool	nominal	Strong	Yes
8	overcast	mild	high	Strong	Yes
9	overcast	Hot	nominal	Weak	Yes
10	Rainy	mild	High	Weak	Yes
11	Rainy	cool	nominal	Weak	Yes
12	Rainy	cool	nominal	Strong	No
13	Rainy	mild	nominal	Weak	Yes
14	Rainy	mild	High	Strong	No

consider sunny dataset

$$Yes = 2 \quad No = 3$$

$$IG(2,3) = -\frac{2}{5} \log_2\left(\frac{2}{5}\right) - \frac{3}{5} \log_2\left(\frac{3}{5}\right)$$
$$= 0.970$$

Temperature

	$P_i$	$n_i$	$IG(P_i, n_i)$
HOT	0	2	0
mild	1	1	1
Cool	1	0	0

$$\text{Entropy (temperature)} = \frac{0+2}{5}(0) + \frac{1+1}{5}(1) + \frac{1+0}{5}(0)$$
$$= 0.4$$

$$\text{Gain (temperature)} = 0.970 - 0.4$$

$$\text{Gain (temperature)} = 0.57$$

Humidity

	$P_i$	$n_i$	$IG(P_i, n_i)$
High	0	3	0
nominal	0	2	0

$$\text{Entropy (humidity)} = 0$$

$$\text{Gain (humidity)} = 0.97 - 0$$
$$= 0.97$$

Windy

	$P_i$	$n_i$	$I_G(P_i, n_i)$
weak	0.1	2	0.918
Strong	0.1	1	1

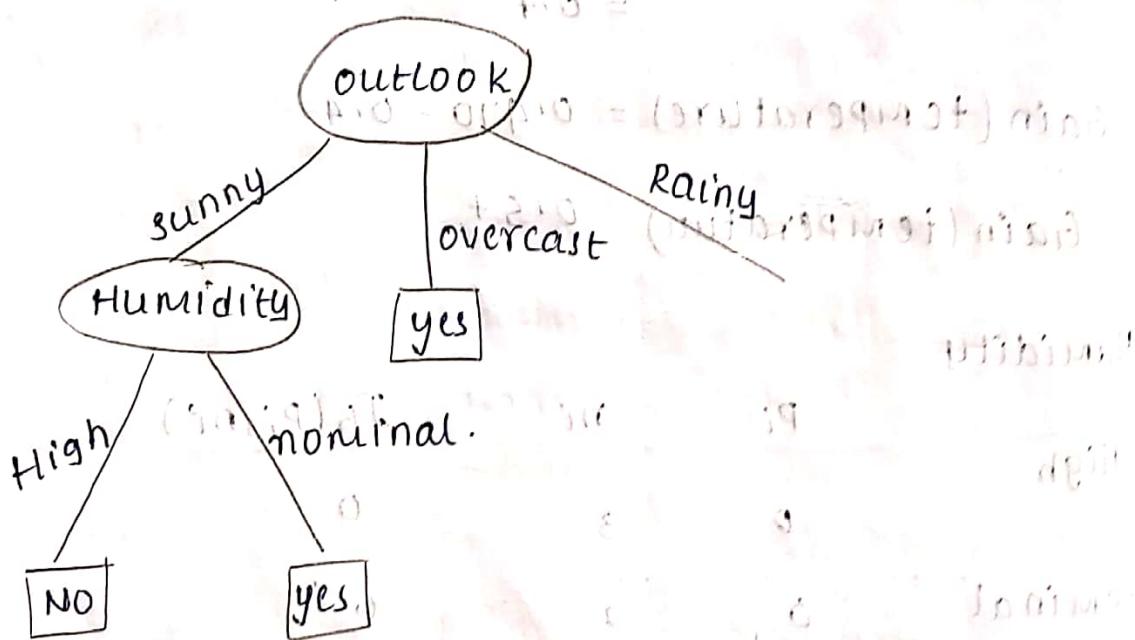
Entropy (Windy)  $\rightarrow$

$$I_G(1, 2) = \frac{-1}{1+2} \log_2\left(\frac{1}{1+2}\right) - \frac{2}{1+2} \log_2\left(\frac{2}{1+2}\right)$$
$$= 0.918$$

$$\text{Entropy (Windy)} = \frac{3}{5} (0.918) + \frac{2}{5} (1)$$
$$= 0.950$$

$$\text{Gain (Windy)} = 0.97 - 0.95$$

$$(0.97 + 0.95 + 0.95) / 3 = (0.97 + 0.95 + 0.95) / 9 = 0.9555555555555555$$
$$\text{Gain (Windy)} = 0.03$$



consider Rainy data set in our previous handout

### Temperature

		Pi	ni	Ig(Pi, ni)
cool	high	1	1	0.918
mild	medium	2	1	0.918

$$\text{Entropy}(\text{Temp}) = \frac{2}{5}(0.1) + \frac{3}{5}(0.918)$$

$$\text{Entropy}(\text{Temp}) = 0.950$$

$$\text{Gain}(\text{Temp}) = 0.970 - 0.950$$

$$G(\text{Temp}) = 0.03$$

### Humidity

		Pi	ni	Ig(Pi, ni)
High	high	1	1	0.918
nominal	medium	2	1	0.918

$$\text{Entropy}(\text{humidity}) = 0.950$$

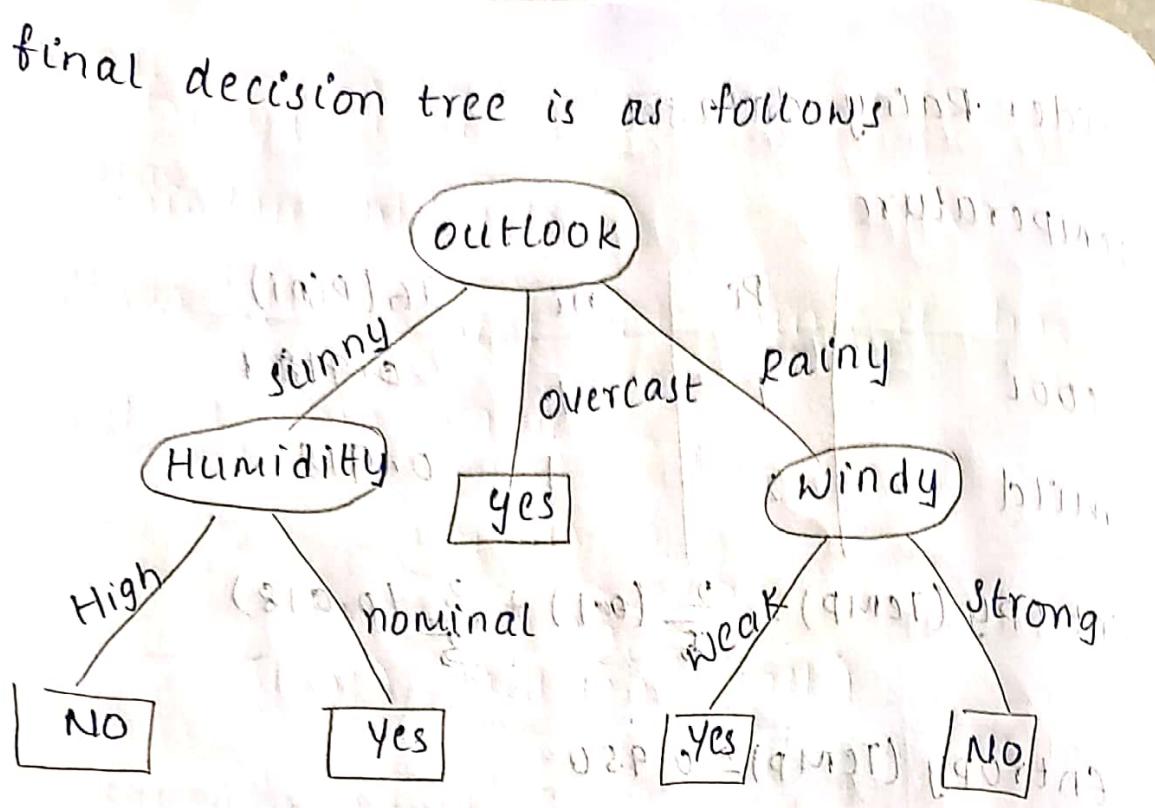
$$G(\text{Humidity}) = 0.03$$

### Windy

		Pi	ni	Ig(Pi, ni)
Weak	high	3	0	0.0
Strong	medium	0	2	0.0

$$\text{Entropy}(\text{Windy}) = 0$$

$$G(\text{Windy}) = 0.97$$



→

Gender	Height	class.
F	1.6m	short
M	2m	tall
F	1.9m	medium
F	1.88m	medium
F	1.7m	short
M	1.85m	medium
F	1.6m	short
M	1.7m	short
M	2.2m	tall
M	2.1m	tall
F	1.8m	medium
M	1.8m	medium
F	1.95m	medium
F	1.9m	medium
F	1.8m	medium

Sol: Short = 4, Tall = 3, Medium = 4

$$I_G(S_i, T_i, M_i) = \frac{4}{8+4+3} \log_2\left(\frac{4}{15}\right) - \frac{8}{15} \log_2\left(\frac{8}{15}\right) - \frac{4}{15} \log_2\left(\frac{4}{15}\right)$$

$$= 1.456$$

Gender

	$S_i$	$T_i$	$M_i$	$I_G(S_i, T_i, M_i)$
M	1	3	2	1.459
F	3	0	6	0

$$I_G(1, 3, 2) = \frac{1}{6} \log_2\left(\frac{1}{6}\right) - \frac{3}{6} \log_2\left(\frac{3}{6}\right) - \frac{2}{6} \log_2\left(\frac{2}{6}\right)$$

$$= 1.459$$

$$\text{Entropy (gender)} = \frac{1+3+2}{15} (1.459)$$

$$= 0.583$$

$$\text{Gain (gender)} = 0.873$$

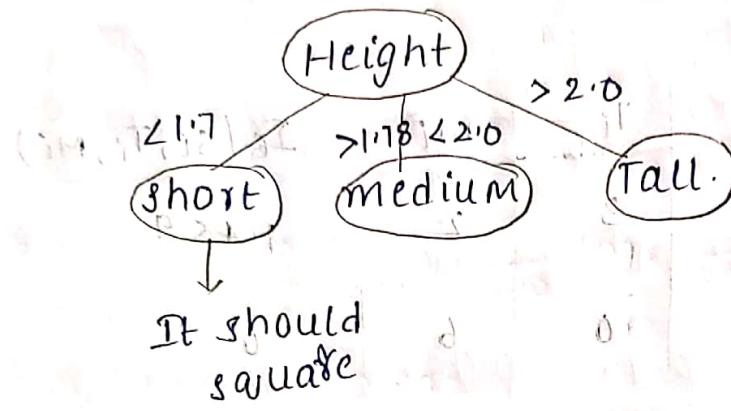
Height: consider it in ranges of  $\leq 1.7$ ,  $> 1.788 \leq 2.0$  and  $> 2.0$

	$S_i$	$T_i$	$M_i$	$I_G(S_i, T_i, M_i)$
$\leq 1.7$	4	0	0	0
$1.788 < 2.0$	0	0	8	0
$> 2.0$	0	3	0	0

Entropy (Height) = 0.11

Gain (Height) = 1.456

Height is the root node since it has highest gain value



→ Construct decision tree for below dataset.

Rid	age	income	student	credit rating	class
1	young	High	No	fair	No
2	young	High	No	Excellent	No
3	middleage	High	No	fair	yes
4	senior	medium	yes	fair	yes
5	senior	low	yes	fair	yes
6	senior	low	yes	Excellent	No
7	middle age	low	No	Excellent	yes
8	young	medium	yes	fair	No
9	young	low	yes	fair	yes
10	senior	medium	yes	fair	yes
11	young	medium	No	Excellent	yes
12	middle age	medium	yes	Excellent	yes
13	middle age	high	yes	fair	yes
14	senior	medium	No	Excellent	No

Sol: consider overall data set

$$\text{yes} = 9, \text{ No} = 5$$

$$\text{IG}(9,5) = -\frac{9}{9+5} \log_2\left(\frac{9}{9+5}\right) - \frac{5}{9+5} \log_2\left(\frac{5}{9+5}\right) = 0.940$$

Entropy (Age):

Age	Pi	ni	IG
young	2	3	0.970
middleage	4	0	0.970
senior	3	2	0.970

$$\text{IG}(2,3) = -\frac{2}{2+3} \log_2\left(\frac{2}{2+3}\right) - \frac{3}{2+3} \log_2\left(\frac{3}{2+3}\right) = 0.970$$

$$\text{Entropy (age)} = \frac{2+3}{4} (0.970) + 0 + \frac{3+2}{14} (0.970) = 0.692$$

$$\text{Gain (Age)} = 0.940 - 0.692 = 0.248$$

Entropy (Income):

Income	Pi	ni	IG
High	2	2	0.918
medium	4	2	0.918
low	3	1	0.811

$$\text{IG}(4,2) = -\frac{4}{4+2} \log_2\left(\frac{4}{4+2}\right) - \frac{2}{4+2} \log_2\left(\frac{2}{4+2}\right) = 0.918$$

$$\text{IG}(3,1) = -\frac{3}{3+1} \log_2\left(\frac{3}{3+1}\right) - \frac{1}{3+1} \log_2\left(\frac{1}{3+1}\right) = 0.811$$

$$\text{Entropy (income)} = 0.910$$

$$\text{Gain (income)} = 0.03$$

Entropy (student):

Student	P <sub>i</sub>	n <sub>i</sub>	I <sub>G</sub>
No	3	3	1
Yes	6	2	0.811

$$I_G(6,2) = -\frac{6}{6+2} \log_2\left(\frac{6}{6+2}\right) - \frac{2}{6+2} \log_2\left(\frac{2}{6+2}\right) = 0.811$$

$$\text{Entropy (student)} = 0.892$$

$$\text{Gain (student)} = 0.940 - 0.892 = 0.048$$

Entropy (credit Rating):

Credit Rating	P <sub>i</sub>	n <sub>i</sub>	I <sub>G</sub>
fair	6	2	0.811
Excellent	3	3	1

$$I_G(6,2) = -\frac{6}{6+2} \log_2\left(\frac{6}{6+2}\right) - \frac{2}{6+2} \log_2\left(\frac{2}{6+2}\right) \\ = 0.811$$

$$\text{Entropy (credit rating)} = 0.892$$

$$\text{Gain (credit rating)} = 0.048$$

→ Among all gains, Age has highest gain value.

so, Age will be root node.

Age

dataset-1:

Age	Income	Student	Credit Rating	Class
young	High	No	Fair	No
young	High	No	Excellent	No
young	medium	Yes	Fair	No
young	Low	Yes	Fair	yes
young	medium	No	Excellent	yes.

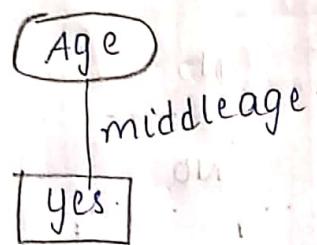
dataset-2:

Age	Income	Student	Credit Rating	Class
middleage	High	No	Fair	yes
middleage	Low	No	Excellent	yes
middleage	medium	Yes	Excellent	yes
middleage	High	Yes	Fair	yes.

dataset-3:

Age	Income	Student	Credit Rating	Class
senior	medium	Yes	Fair	yes
senior	Low	Yes	Fair	no
senior	Low	Yes	Excellent	yes No
senior	medium	Yes	Fair	yes
senior	medium	No	Excellent	No.

Since middle age have all 'yes' classes then the tree is as follows



Consider data-set -①:

$$\text{Yes} = 2, \quad \text{No} = 3$$

$$I_G(\text{young}) = -\frac{2}{2+3} \log_2\left(\frac{2}{2+3}\right) - \frac{3}{3+2} \log_2\left(\frac{3}{3+2}\right) = 0.970$$

Income:

Income	P <sub>i</sub>	n <sub>i</sub>	I <sub>G</sub>
High	0	2	0
medium	1/3	1	0.44
low	1/3	0	0.0

$$\text{Entropy } (I_{\text{income}}) = 0.40$$

$$\text{Gain}(I_{\text{income}}) = 0.57$$

student:

Student	P <sub>i</sub>	n <sub>i</sub>	I <sub>G</sub>
No	1	2	0.918
Yes	1	1	0.5

$$I_G(1,2) = 0.918$$

Entropy (student) = 0.946

Gain (student) = 0.03.

credit rating:

credit rating	P <sub>i</sub>	n <sub>c</sub>	I <sub>G</sub>
fair	1	2	0.918
Excellent	1	1	1

$$I_G(1,2) = 0.918$$

Entropy (credit Rating) = 0.946.

Gain (credit Rating) = 0.03

consider dataset-3:

$$\text{Yes} = 3, \text{No} = 2$$

$$I_G(3,2) = 0.970$$

Income:

Income	P <sub>i</sub>	n <sub>c</sub>	I <sub>G</sub>
medium	2	1	0.918
low	1	1	1

$$I_G(2,1) = 0.918$$

Entropy (income) = 0.950

Gain (income) = 0.970 - 0.950

$$= 0.03$$

## Student:

Student	Pc	ni	D <sub>G</sub>
Yes	3	1	0.811
No	0	1	0

$$D_G(s_1) = 0.811$$

$$\text{Entropy (student)} = 0.648$$

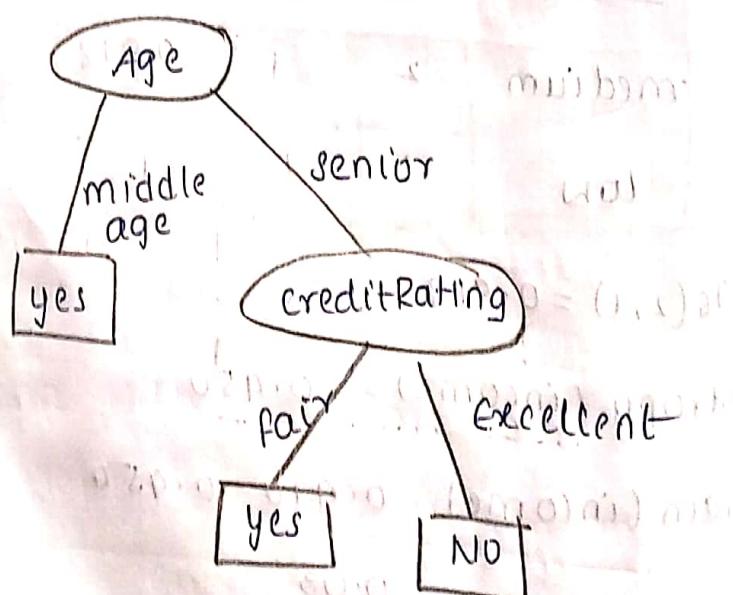
$$\text{Gain (student)} = 0.322$$

## Credit Rating:

Credit Rating	Pc	ni	D <sub>G</sub>
fair	3	0	0
Excellent	0	2	0

$$\text{Entropy (credit rating)} = 0$$

$$\text{Gain (credit rating)} = 0.970$$



from data-set-①: In the Income, we have one yes class and one No class in the middle.

so, we have to consider subdataset from data-set ①.

data-set ①:

Income	Student	credit Rating	class
medium	yes	Fair	No
medium	No	Excellent	Yes

consider Age = young

Income = middle.

Medium

yes = 1, No = 0

$$IG(1,1) = 1$$

Student:

Student	Pi	ni	IG
yes	0	1	0
No	0	0	0

$$\text{Entropy}(\text{student}) = 0$$

$$\text{Gain}(\text{student}) = 1$$

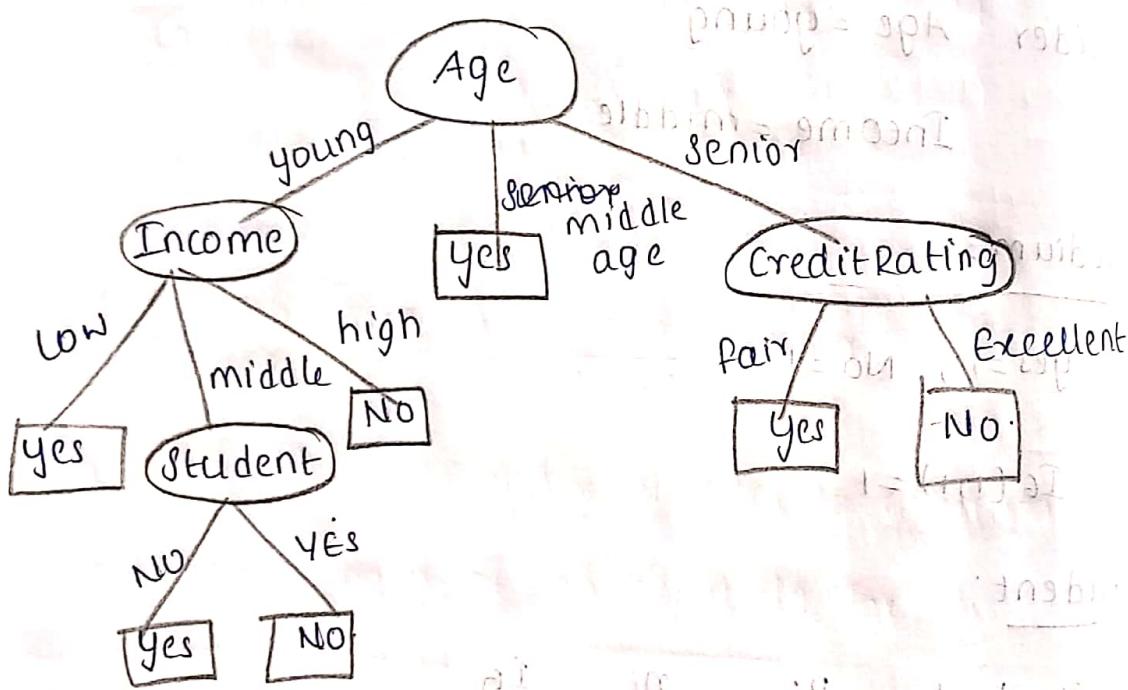
## Credit Rating:

Credit Rating	P <sub>i</sub>	n <sub>i</sub>	I <sub>G</sub>
fair	0	1	0
Excellent	0	0	0

$$\text{Entropy}(\text{Credit Rating}) = 0$$

$$\text{Gain} = 0$$

## decision tree:



→ In ID<sub>3</sub> we can construct a decision tree by considering with its class label with consideration of information gain and entropy.

→ drawback of information gain is it prefers to select attributes with having large number of values.

→ C4.5 uses an extension of information gain known as gain ratio which attempts to overcome the drawback of ID<sub>3</sub>.

→ C4.5 improves ID<sub>3</sub> with the following ways:

\* missing data.

\* continuous data

\* Tree pruning (post and pre pruning)

↓  
present overfitting  
to noise in the data  
↓  
take a fully grown decision tree  
↓  
stop growing a branch when gain becomes unreliable.  
and discard the unreliable parts.

→ Rules

→ splitting

\* C4.5 applies a kind of normalization to information gain using split information gain value defined with information gain ( $D$ ) where  $D$  indicates dataset.

$$\text{split info}(D) = \sum_{j=1}^{|D_j|} \frac{|D_j|}{|D|} \log_2 \left( \frac{|D_j|}{|D|} \right)$$

$$\text{gain ratio}(A) = \frac{\text{gain}(A)}{\text{split info}(D)}$$

Where A indicates particular attribute.

Ex: calculate split information for the attribute income and calculate gain ratio.

$$\text{split info}(D) = \frac{-4}{14} \log_2 \left( \frac{4}{14} \right) - \frac{6}{14} \log_2 \left( \frac{6}{14} \right)$$

$$= \frac{-4}{14} \log_2 \left( \frac{4}{14} \right)$$

$$= 1.557$$

$$\text{gain ratio}(\text{income}) = \frac{0.029}{1.557} = 0.019$$

CART: Here the gini index is used to calculate information gain of an attribute for the decision tree.

Gini index: It measures the impurity of the data partition (or) set of training tuples.

$$\text{Gini}(D) = 1 - \sum_{i=1}^m p_i^2$$

where P can be Yes or No gini index for a particular attribute

$$Gini_A(D) = \frac{|D_1|}{|D|} Gini(D_1) + \frac{|D_2|}{|D|} Gini(D_2)$$

where  $D_1$  and  $D_2$  are the partition data values on partition of training data.

$$\Delta Gini(A) = Gini(D) - Gini_A(D)$$

problem:

calculate Gini index for the attribute income for the given data set which consists of the class labels yes=9 and No=5

$$Gini(D) = 1 - \left(\frac{9}{14}\right)^2 - \left(\frac{5}{14}\right)^2 = 0.459$$

Gini index for the partition  $D_1$  which consists of low and medium.

$$\begin{aligned} Gini_{\text{income}}(D) &= \frac{10}{14} Gini(D_1) + \frac{4}{14} Gini(D_2) \\ &= \frac{10}{14} \left[ 1 - \left(\frac{7}{10}\right)^2 - \left(\frac{3}{10}\right)^2 \right] + \frac{4}{14} \left[ 1 - \left(\frac{2}{4}\right)^2 - \left(\frac{2}{4}\right)^2 \right] \\ &= 0.443 \end{aligned}$$

$$Gini(\text{income}) = 0.459 - 0.443$$

$$= 0.026$$

## (1) Rule based classification: (If - then)

If part - rule antecedent

Then part - rule consequent

Using if then rules for classification: A rule based classifier uses a set of if-then rules for classification. An if-then rule is an expression of the form

if condition then conclusion

Ex:

Rule R<sub>1</sub>: If (age = YOUTH AND student = yes)  
then buys-computer = yes

Rule R<sub>2</sub>: If (age = middleage AND student = No)

then buys-computer = No.

→ If part (or) left side of a rule is known as

rule antecedent (or) precondition

→ Then part (or) right side of a rule is known as

rule consequent

→ In the rule antecedent, the condition consists of one or more attribute tests (Ex: age = young or student = yes) that are logically ANDed

- It can also be written as
- If (age = youth)  $\wedge$  (student = yes)  $\Rightarrow$  buys - computer  
 = yes
- A rule R can be assessed by its coverage and accuracy. Given a tuple  $x$  from a class labelled dataset D, n<sub>correct</sub> be the no of tuples correctly classified we can define the coverage and accuracy of R(rule)

$$\text{coverage}(R) = \frac{n_{\text{covers}}}{|D|}$$

$$\text{accuracy}(R) = \frac{n_{\text{correct}}}{n_{\text{covers}}}$$

Ex: let's consider student dataset are class labelled tuples from the all electronics customer database our task is to predict whether a customer will buy a computer

→ consider rule R<sub>1</sub> which covers 2 of the 14 tuples. It can correctly classify both the tuples.

Therefore,

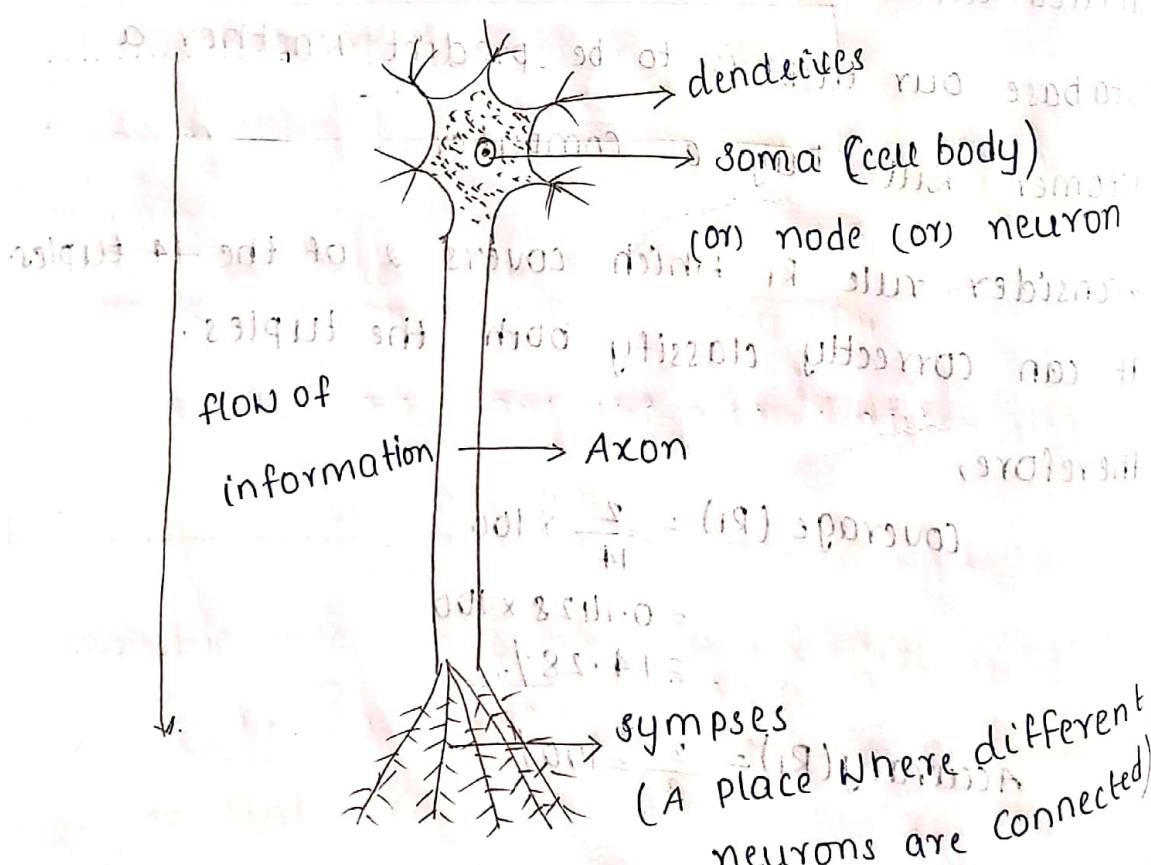
$$\begin{aligned}\text{coverage}(R_1) &= \frac{2}{14} \times 100 \\ &= 0.1428 \times 100 \\ &= 14.28\%\end{aligned}$$

$$\text{accuracy}(R_1) = \frac{2}{2} = 100\%.$$

## Neural Network:

- ANN (Artificial Neural Network) : The main objective of ANN is to develop a computer (or) to design a computer that work like a brain.
- It is also called as parallel distributing processing system
- ANN works like a large collection of units that are inter connected in some patterns to allow the communication between the units
- The units are called as Nodes (or) Neurons

## Biological Neural Network (BNN)

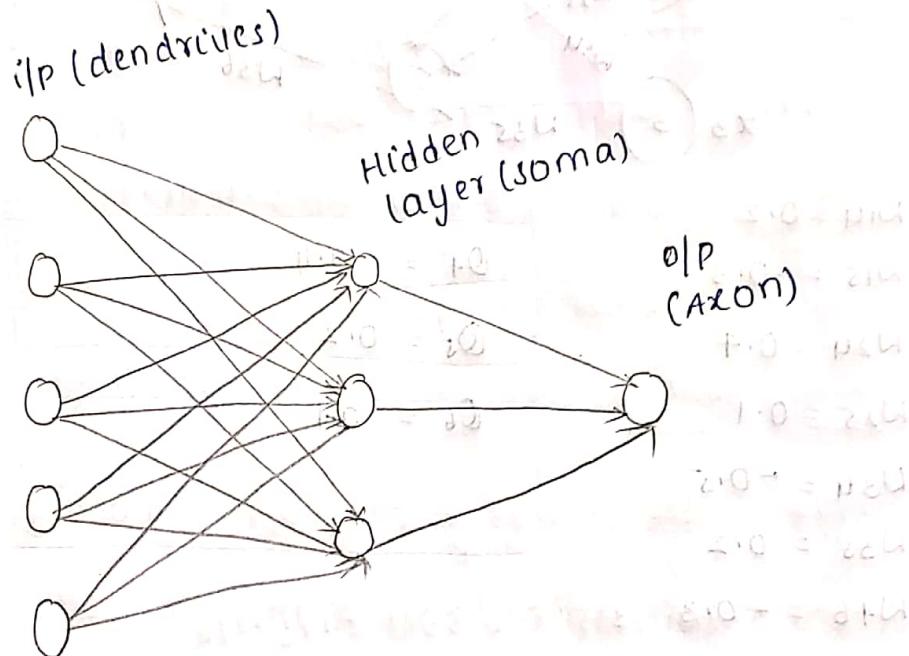


BNN

ANN

dendrites	- inputs
Synapses	- weights or inter connection
soma	- node
Axon	- o/p

\* ANN:

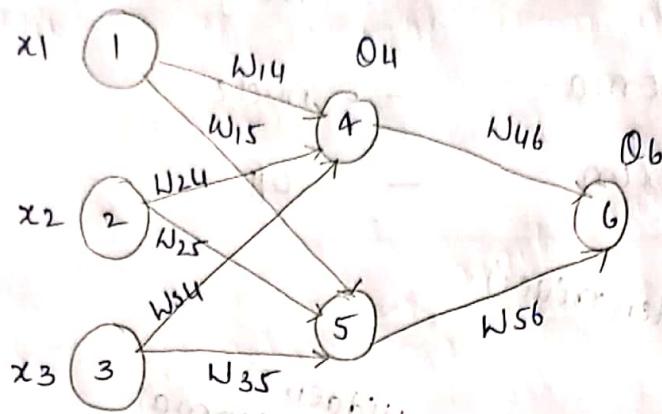


\* Differences between BNN and ANN:

function	BNN	ANN
processing	It is a parallel distributing system	It is also parallel but slower than BNN.
size	In BNN, there are $10^{11}$ neurons and $10^{15}$ inter connections	$10^2$ to $10^4$ neurons. (depending on application we can change the structure of ANN)
fault tolerance	very high & degrades the performance of object	It is not affected the performance as compared to BNN.

Problem:

$x_1, x_2, x_3 = 1, 0, 1$  target = 1, learning rate = 0.9



$$Q_4 = 0.4$$

$$Q_5 = -0.3$$

$$Q_6 = 0.2$$

$$w_{14} = 0.2$$

$$Q_4 = -0.4$$

$$Q_5 = 0.2$$

$$Q_6 = 0.1$$

$$w_{24} = -0.5$$

$$w_{34} = 0.2$$

$$w_{25} = 0.1$$

$$w_{35} = -0.2$$

$$w_{46} = -0.3$$

$$w_{56} = -0.2$$

Activation function of  $x = \frac{1}{1+e^{-x}}$

At Node 4:

$$\text{node } 4 = x_1 w_{14} + x_2 w_{24} + x_3 w_{34} + Q_4$$

$$= 1 \times 0.2 + 0 \times 1 + 1 \times (-0.5) + (-0.4)$$

$$= 0.2 - 0.5 - 0.4 = -0.7$$

$$Q_4 = \frac{1}{1+e^{-0.7}} = \frac{1}{1+e^{0.7}} = 0.333\ldots$$

At node -5:

$$\text{node } 5 = x_1 w_{15} + x_2 w_{25} + x_3 w_{35} + b_5 \\ = 1 \times (-0.3) + 0 + 1 \times (0.2) + 0.2$$

$$= 0.1$$

$$o_5 = \frac{1}{1+e^{-x}} = \frac{1}{1+e^{-0.1}} = 0.524$$

At node -6:

$$\text{node } 6 = o_4 w_{46} + o_5 w_{56} + o_6 w_{66} + b_6 \\ = 0.331(-0.3) + 0.524(-0.2) + 0$$

$$= -0.104.$$

$$o_6 = \frac{1}{1+e^{0.104}} = 0.474$$

→ The output of node 6 is and target value are not matching. Hence, adjust the weights and bias.

values to adjust the weights, the formula is as follows:

$$w_{ij \text{ new}} = w_{ij \text{ old}} + \eta \text{ Err}_j o_i$$

to calculate the error at the O/P node, the formula is as follows:

$$\text{Err}_j = o_j (1-o_j) * (T-o_j)$$

If error at Hidden nodes

$$\boxed{\text{Err}_j = o_j(1-o_j) \sum_k w_{jk} \text{Err}_k}$$

where  $w_{jk}$  is the weight from node  $j$  to node  $k$

Error at node 6:

$$\text{Err}_6 = o_6(1-o_6) * (T - o_6)$$

$$= 0.474(1-0.474)(1-0.474) = 0.008$$

$$= 0.131 * (0.224) + (0.2) * (0.880)$$

Error at node 4:

$$\text{Err}_4 = o_4(1-o_4) \sum_k w_{4k} \text{Err}_k = 0.331 * 0.331$$

~~for node 4:  $(0.331)(1-0.331) \text{Err}_6 w_{46}$  to find two sets~~

~~of  $(0.331)(1-0.331)(0.131)(-0.2)$  which is equal to~~

~~0.008 after multiplying out terms of result~~

Error at node 5:

$$\text{Err}_5 = o_5(1-o_5) \text{Err}_6 w_{56} = 0.524 * 0.524$$

$$= (0.524)(1-0.524)(0.131)(-0.2)$$

$$= -0.006$$

To adjust the bias, the formula is as follows:

$$\boxed{(0-1) * (0-1) * 0.006}$$

$$W_{j\text{new}} = W_{j\text{old}} + \lambda Err_j O_i$$

$$\begin{aligned}W_{46} &= W_{46} + (0.9) \times Err_{46} O_{46} \\&= W_{46} + (0.9) \times (0.131) (0.331) \\&= -0.260\end{aligned}$$

$$\begin{aligned}W_{56} &= W_{56} + (0.9) \times Err_{56} O_{56} \\&= (-0.2) + (0.9) \times (-0.008) \times 1 \\&= 0.1038\end{aligned}$$

$$\begin{aligned}W_{24} &= W_{24} + (0.9) \times Err_{24} \times O_1 \\&= 0.2 + (0.9) \times (-0.008) \times 1 \\&= 0.1928\end{aligned}$$

$$\begin{aligned}W_{24} &= W_{24} + (0.9) \times Err_{24} \times O_2 \\&= 0.4\end{aligned}$$

$$\begin{aligned}W_{34} &= W_{34} + (0.9) \times Err_{34} \times O_3 \\&= -0.507\end{aligned}$$

$$W_{15} = -0.305$$

$$W_{25} = 0.1$$

$$W_{35} = 0.191$$

$$O_{j\text{new}} = O_{j\text{odd}} + \lambda Err_j$$

$$O_6 = 0.218$$

$$O_5 = 0.194$$

$$O_4 = -4.08$$

Similarly, continue the process until  $P_6$  is approximately to target value.

$$0.8 \times 100 \times (P_5) + P_{6t} = P_6$$

$$0.8 \times 100 \times (P_5) + 0.6 = P_6$$

$$80 + 0.6 =$$

$$80.6 \times (P_5) + P_{6t} = P_6$$

$$80.6 \times 100 \times (P_5) + 0.6 = P_6$$

$$8060 + 0.6 =$$

$$8060.6 \times (P_5) + P_{6t} = P_6$$

$$8060.6 =$$

$$80 \times 100 \times (P_5) + P_{6t} = P_6$$

$$8000 + 0.6 =$$

$$8000.6 = P_6$$

$$P_6 = 8000.6$$

$$\boxed{P_6 = 8000.6}$$

$$8000.6 = P_6$$

$$P_6 = 8000.6$$

$$8000.6 = P_6$$