18MAB 302T- DISCRETE MATHEMATICS FOR ENGINEERS

Question Bank for CT-III (CSE & ECE)

UNIT – IV

PART-A

1). $*: A \times A \to A$ is said to be a binary operation if

a) $a*b \in A$ *forsome* $a \in A$ b) $a*b \in A$ *forsome* $b \in A$

c). $a*b \in A$ *forsome* $a,b \in A$ d) $a*b \in A$ *for all* $a,b \in A$

Ans : d

2) The inverse of – i in the multiplication group {1,-1,i,-i} is

a. 1 b. i c.-1 d. –I **Ans: b**

3) Every group of prime order is ------

(a) Cyclic aod hence abelian (b) Abetian and hence cyclic

(c) Not cyclic & abelian (d) Not abelian and cyclic **Ans a**

4) What are the generators of the group (Z, +)?.

(a) 1 & 0 (b) -1 & 0 (c) 0 alone (d) 1& -1 **Ans : d**

5) The necessary oondition that a non-empty subset H of a group G to be a subgroup is

a) $a,b \in H \Rightarrow a^{-1},b^{-1} \in H$ b) $a,b \in H \Rightarrow a*b^{-1} \in H$

c) $a,b \in H \Rightarrow a*b \in H$ d) $a,b \in H \Rightarrow (a*b)^{-1} \in H$ **Ans : b**

6) Let G be a group. If $a,b \in G$ then the inverse of a*b is

(a) $a^{-1}*b^{-1}$ (b) $a*b^{-1}$ (c) $a^{-1}*b$ (d) $b^{-1}*a^{-1}$ **Ans : d**

7) The minimum order of a non abelian group is

(a) 3 (b) 6 (c) 9 (d) 4 **Ans : b**

8) Every subgroup of an abelian group is

(a) Normal (b) Abelian

(c) Cyclic (d) A permutation group **Ans : a**

9) The weight of the code word [11101] is

(a) 1 (b) 5/4 (c) 4/5 (d) 4 **Ans : d**

10. A code can connect a set of at the most k errors if and only if the minimum distance between any two code words is

a) Atmost (2k+1) b) Atmost (2k-1) c) Atleast (2k+1) d) Atleast 2k **Ans:a**

11. Any code which is group under the operation $\oplus$ is called

a) Hamming code b) Group code c) Parity check d) Symmetric code **Ans: b**

Part-B

1. Show that (i) Every cyclic group is abelian (ii) If 'a' is the generator of a cyclic group (G,*) , a^{-1} is also a generator of (G,*).
2. Let I = {1, -1, I , -i} and (I,*) is a group. Find the order of each element of a group.
3. In a group G , prove that $(a*b)^{-1} = b^{-1}*a^{-1}, \forall a,b \in G.$
4. If $a,b \in R, where (R,+,\bullet)$ is a ring show that $(a+b)^2 = a^2 + a\bullet b + b\bullet a + b^2$
5. Define a Field with example.
6. Let $H = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ be a parity check matrix. Determine the (2,5) group code function $e_H : B^2 \to B^5$.
7. Consider the (2,4) encoding function **e** as follows. How many errors will e deduct? e(00) = 0000 , e(01) =0110 , e(10) = 1011 & e(11) =1100.
8. Define an integral domain and give one example.

Part-C

9. Show that the identity element of a group (G,*) is unique.
10. Prove that a subset H of a group (G,*) is a subgroup of G if and only if $a,b \in H \Rightarrow a*b^{-1} \in H$
11. Prove that the subgroup of cyclic group is cyclic.
12. Prove that every group of prime order is cyclic.

13. Show that $(Z,\oplus,\otimes)$ is a commutative ring with identity where the operations $\oplus$ and $\otimes$ are defined for any $a,b \in Z$ as $a \oplus b = a+b-1$ and $a \otimes b = a+b-ab$.

14. If S is the set of ordered pairs (a,b) of real numbers and if the binary operations $\oplus$ and $\otimes$ are defined .by the equations

$(a,b)\oplus(c,d) = (a+c,b+d)$ and $(a,b)\otimes(c,d) = (ac-bd,bc+ad)$

Prove that $(S,\oplus,\otimes)$ is a field.

15. Given the generator matrix G = $\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$ .corresponding to the encoding function $e_G : B^4 \to B^7$. find the corresponding parity check matrix and use it to decode the following received words and hence to find the original message 110 0001 , 1110111 , 0010001 , 0011100.

16. Let $e_G : B^2 \to B^6$.is an (2,6) encoding function defined as

e(00) = 000000, e(01) = 011101

e(10) = 001110, e(11) = 111111

a) Find minimum distance.

b) How many errors can e detect?

c) How many errors can e corrects?

17. Consider the parity check matrix H given by $H = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ determine the group code function $e_H : B^2 \to B^5$.

Decode the following words relative to a maximum likelihood decoding function associated with e_H : 01110,11101,00001, 11000 .

18.If (G,*), is an abelian group,show that $(a*b)^n = a^n * b^n, \forall a,b \in G$.

UNIT – V

PART-A

1. The maximum number of edges in a simple graph with n vertices is

a.) n(n-1)/2 b) n(n+1)/2 c) (n-1)(n+1)/2 d) n/2 **Ans : a**

2. A closed directed path containing all the edges in a diagraph G is called an

a.) Closed circuit b) Hamiltonian circuit

c) Eulerian circuit d) Isomorphic circuit **Ans : c**

3. Sum of the degrees of all vertices of a group G is equal to

a)Thrice the number of edges b) Twice the number of edges

c) Number of edges d) Five times the number of the edges **Ans : b**

4. Any connected graph with n vertices and n-1 edges is

a) Graph b) Closed graph c) Tree d) Spanning tree **Ans : c**

5. Two isomorphic graphs must have

(a) Same number of vertices (b) Un equal number of edges

(c) Unequal number of vertices (d) degrees of the verties are not equal

Ans: a

6. A graph is bipartite if and only if its chromatic number is _____.

a) 1 (b) 2 (c) Odd (d) Even **Ans: b**

7. The chromatic number of the chess board is _____.

a) 1 (b) 2 (c) 3 (d) 4 **Ans: b**

8. Hamilton cycle is a cycle that contains every _____of G

a) Path (b) Cycle (c) Vertex d) Edge **Ans: C**

9. An edge with identical ends is called _____.

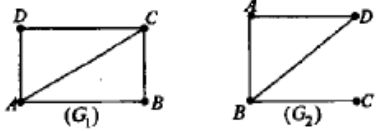
a) Distinct (b) Loop (c) Direct d) Adjacent **Ans: b**

10. Graph G is 2-colourable iff G is

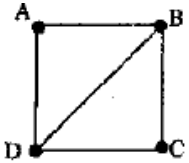
a) Bipartite (b) Simple (c) Complete (d) Trivial **Ans: a**

Part-B

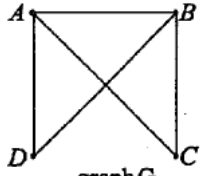
1. State and prove Handshaking theorem
2. Give an example of a graph which contains
   (i) An Eulerian circuit but not a Hamiltonian circuit
   (ii) A Hamiltonian circuit but not an Eulerian circuit
3. Which of the following graph is a Hamiltonian graph? Give reason.



4. Check whether the following graph is Eulerian and / or Hamiltonian Graph.



5. Draw all the spanning trees of the graph G shown below.

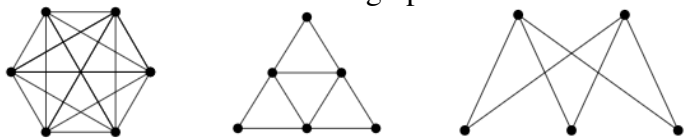


graph G

6. If G= (V,E) is an undirected graph with e edges ,prove that

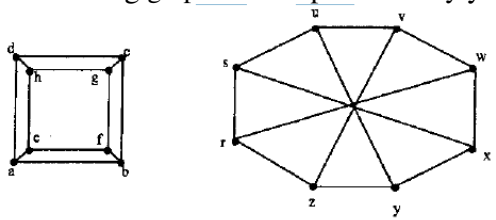
$$\sum_i \deg(V_i) = 2e$$

7. Find the chromatic number of the graphs below.

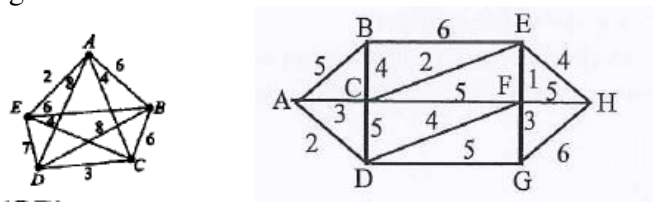


Part-C

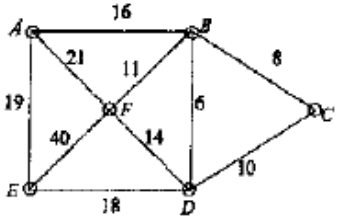
8. Prove that a tree with n vertices has n-1 edges,
9. Are the following graphs isomorphic? Justify your answer..



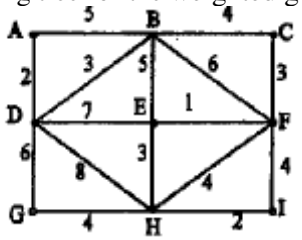
10. Find the minimum spanning tree for the weighted graph using Krushkal's algorithm.



11. Find the minimum spanning tree for the weighted gaplL using Kruskal's algorithm.

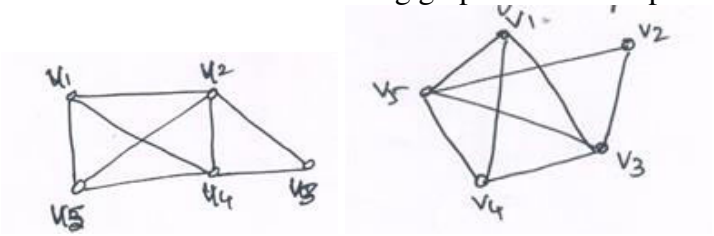


12. Give step by step procedure of Knrskal's algorithm end hence use spanning tree for the weighted graph shown below.



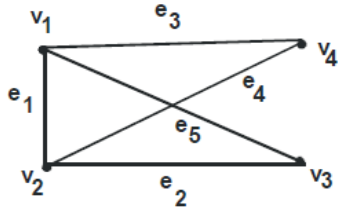
13. Prove that the number of edges in a bipartite graph with n vertices is atmost $\frac{n^2}{4}$.

14. Determine whether the following graphs are isomorphic or not.



15. Prove that the maximum number of edges in a simple disconnected graph G with n vertices and k components is $\frac{(n-k)(n-k+1)}{2}$.

16. (i) Write the incident matrix of the following graph:



(ii) Write the adjacency matrix of the following graph: