18CSE390T COMPUTER VISION

UNIT - I

Introduction to Computer Vision

It is a field of AI that enables computers and systems to derive meaningful information from digital images / videos and other visual inputs and take actions or make recommendations based on that information.

Applications of CV

1. Optical Character recognition (OCR)
   - Reading handwritten postal codes on letters and automatic number plate recognition (ANPR)

2. Machine Inspection.
   1) Machine parts inspection for quality assurance using stereo vision → Process of extracting 3D information from digital images taken by 2 cameras displaced horizontally from one another to obtain 2 different views of the same scene.

2) looking for defects in steel castings using X-ray vision.

3. Retail: Object recognition for automated Checkout Lanes. (A place where a product or service can be purchased).

4. 3D Model Building (Photogrammetry)

• Fully automated any construction of 3D models from aerial photographs (photograph taken from the air using rockets, aircraft) used in systems such as Bing Maps geospatial mapping platform. It allows developers to create applications that layer location-relevant data, (contains photo-based images).

5. Medical Imaging.

registering pre-operative and intra-operative imagery. or performing long-term studies of people's brain morphology. (study of size, shape & structure).

6. Automotive safety.

Detecting unexpected obstacles in fully automated driving.

7. Match more.

Merging computer-generated imagery (CGI) with live action footage by tracking feature points in the source video to estimate 3D camera motion and shape of the environment.

8. Motion Capture (mocap)

Using retro-reflective markers (which refelect light back in the direction it came) viewed from multiple cameras or other vision-based techniques to capture actors for computer animation.

9. Surveillance.

monitoring for intruders, analyzing highway traffic and monitoring pools for drowning victims.

10. Finger Print recognition and Biometrics.

For automatic access authentication

11. Stitching

turning overlapping Photos into single seamless view.

12. Exposure bracketing:

Merging multiple exposures taken under challenging lighting conditions into a single perfectly exposed image.

13. Morphing:

Turning a picture of one face into another.

14. 3D Modeling

Converting one or more snapshots into a 3D model of the object

15. Face detection:

For improved camera focusing as well as more relevant image searching.

16. Visual authentication

For automatic log in into the system.

17. Video Match move & stabilization.

Inserting 2D pictures or 3D models into videos by automatically tracking nearby reference Points

Image Formation.

Geometric primitives and transformations.

- Geometric primitives form the basic building blocks used to describe 3D shapes,

Basic.
- points, Lines & planes.
- Curves, surfaces & Volumes.

2D Points.

vector → both magnitude and direction →

- Pixel co-ordinate in an image denoted by a pair of values

$x = (x, y) \in R^2$ (scalar real no).

$x = \begin{bmatrix} x \\ y \end{bmatrix}$ → Column vector.

Point at infinity or ideal point is an idealized liminting point at the end of each line.

- Also represented using homogeneous co-ordinates $\tilde{x} = (\tilde{x}, \tilde{y}, \tilde{w}) \in P^2$

↳ height/altitude

- A homogeneous vector $\tilde{x}$ can be converted back into an inhomogeneous co-ordinate $\dot{x}$ by dividing through by the last element $\tilde{w}$

$\tilde{x} = (\tilde{x}, \tilde{y}, \tilde{w}) = \tilde{w}(x, y, 1) = \tilde{w}\bar{x}$

where $\bar{x} = (x, y, 1)$ is the augmented vector

If $\tilde{w} = 0$ are called ideal points or points at infinity (pgm will crash).

2D Lines.

- represented using homogeneous co-ordinates $\tilde{l} = (a, b, c)$ and

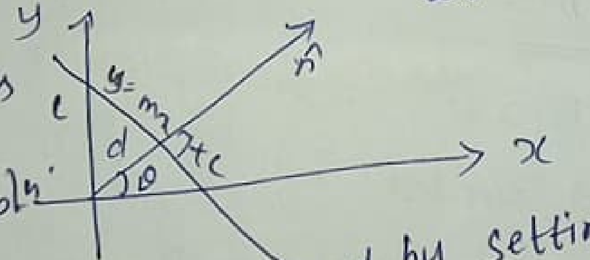
Line equation is $y = mx + c$

$\bar{x} \cdot \tilde{l} = ax + by + c = 0$

a & b should not be zero

coefficients a, b & c ~~are~~ constants

x, y are variables



x intercept is found by setting $y = 0$. ↳ point where a line crosses the x-axis

$ax + b(0) + c$

$x = c/a$.

x intercept is $(c/a, 0)$

y intercept is found by setting $x = 0$ ↳ point where a line crosses the y-axis.

$a(0) + by = c$

$y = c/b$ y intercept $(0, c/b)$.

We can normalize the line equation vector so that $l = (\hat{n}_x, \hat{n}_y, d) = (\hat{n}, d)$

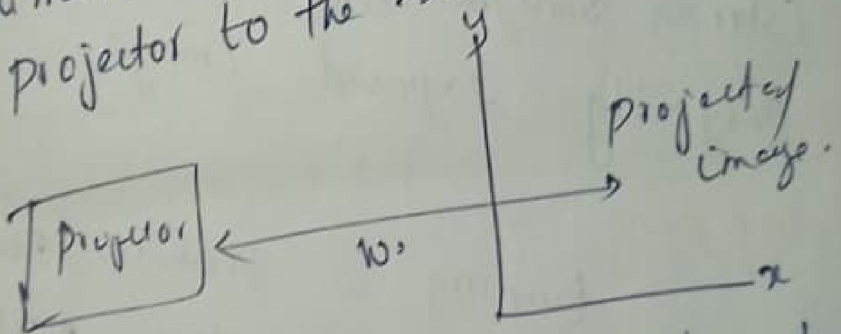
$\|\hat{n}\| = 1$. $\hat{n}$ is the normal vector perpendicular to the line and d is its distance to the origin

Face detection / visual authentication

for improved camera focusing - image searching → To allow someone inside the room by detecting the face.

Homogeneous co-ordinates.

Co-ordinates in Projective space are homogeneous co-ordinates. (extra dimension w) w dimension is the distance from the projector to the screen



If you move the projector closer to the screen, the whole 2D image becomes smaller.

If you move the projector away from the screen, the 2D image becomes larger. The value of W affects the size of the image.

For NO 3D Projector
So in 3D w is always = 1.

Operations

1. Stitching → turning overlapping photos into a single seamlessly stiched panorama (wide angle picture) (wide portion of an area)

2. Exposure bracketing.

→ merging multiple exposures taken under challenging lighting conditions. (strong sunlight and shadows) into a perfectly exposed image.

3 Morphing.

turning a picture ~~one~~ into another picture, using a seamless morph transition.

4. 3D modeling.

Converting one or more snapshots into a 3D model of the object

5. Video match move & stabilization

Inserting 2D pictures or 3D models into the videos ~~by~~ by automatically tracking nearby reference points. or using motion estimate to remore shake from the videos.

2D Conics.

- Conic Sections → intersection of a plane and a 3D cone

$$\tilde{x}^T Q \tilde{x} = 0$$

2D plane. It is a flat, 2D surface that extends indefinitely (length & width)

3D Points.

In homogeneous ~~eq~~ co-ordinates

$$x = (x, y, z) \in R^3$$

Homogeneous co-ordinates

$$\tilde{x} = (\tilde{x}, \tilde{y}, \tilde{z}, \tilde{w}) \in P^3 \rightarrow \text{projective space}$$

$$\bar{x} = (x, y, z, 1) \text{ with } \tilde{x} = \tilde{w}\bar{x}$$

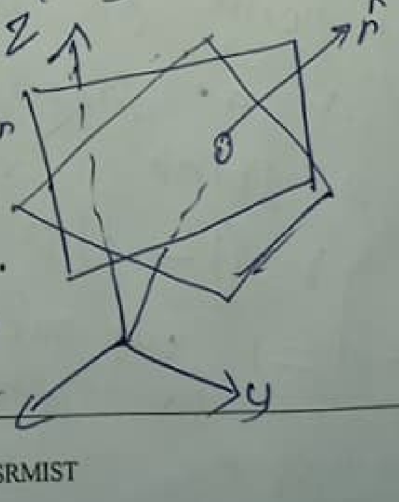
3D Planes.

Homogeneous Co-ordinate

$$\tilde{m} = (a, b, c, d)$$

Plane equation

$$\bar{x} \cdot \tilde{m} = ax + by + cz + d = 0.$$



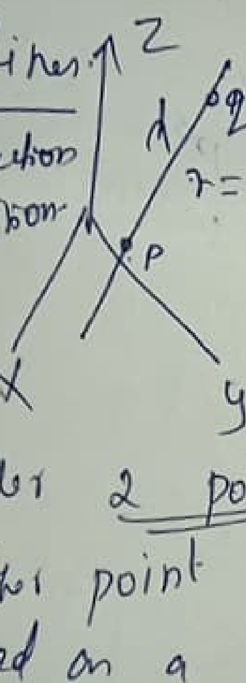
- 3 axes which intersects at the origin
- x, y, z axes are mutually perpendicular to each other
- called as hyperplane

3D plane

3 axes which intersect at the origin. x & y & z axes are mutually perpendicular each other. 3D plane is called a hyperplane.

3D Lines

- need direction
- need position



$r = (1-\lambda)p + \lambda q$.

If we use homogeneous co-ordinates, we can write line as

$\tilde{r} = \mu \tilde{p} + \lambda \tilde{q}$

A special case if q is at infinity

$\tilde{q} = (\hat{d}_x, \hat{d}_y, \hat{d}_z, 0) = (\hat{d}, 0)$

$\hat{d}$ is the direction of the line.

$\boxed{r = p + \lambda \hat{d}}$

Consider 2 points on the line (p, q)

Any other point on the line can be expressed as a linear combination of these 2 points

$$r = (1-\lambda)p + \lambda q.$$

$0 \le \lambda \le 1$ we get line segment joining p & q.

~~covers only the points~~

Cover all the points in the line

λ is called as parameter

It takes the value $-\infty$ to ∞

For one point indicates any point is in left side or right side of the given point

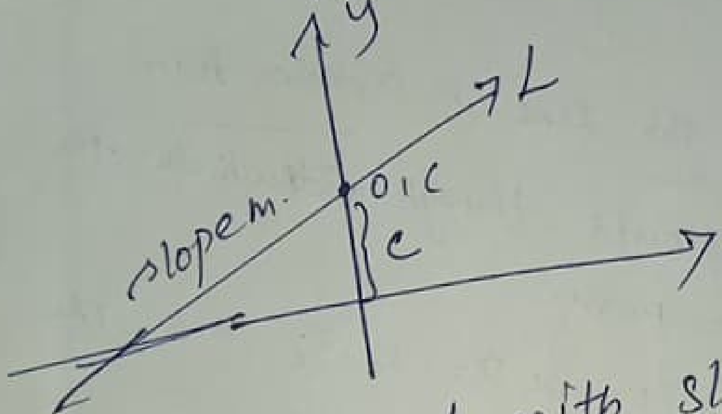
Line equation

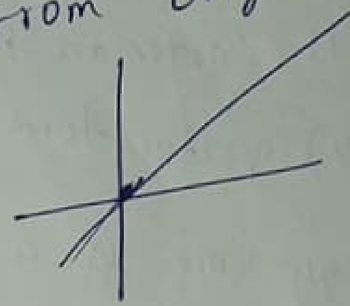
slope-intercept form

$y = mx + c$

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$\therefore y = mx$ means the lines starts from origin.



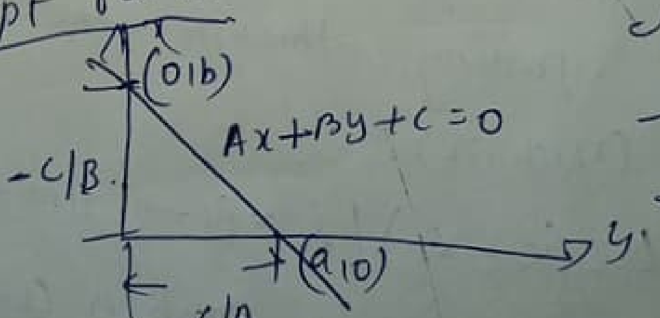


Line L with slope m cuts the y axis at a distance c from the origin where the distance c is called the y-intercept of the line L.

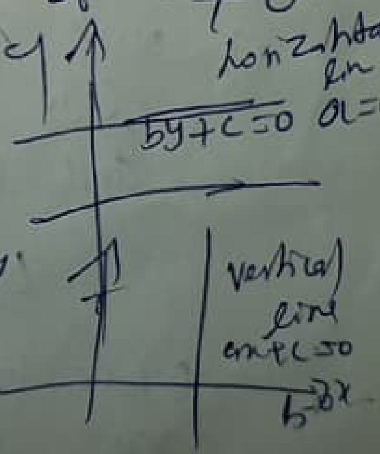
Therefore the point (x,y) on the line with slope m and y-intercept c lies on the line if and only if

$y = mx + c.$

Intercept form



if C ≠ 0.



3D Line.

3D lines are represented in 2 forms.

1) Cartesian form

2) Vector form.

Equation of a straight line in Cartesian form

- require 2 points through which the straight line passes

x_1, y_1, z_1 & x_2, y_2, z_2 are the Position Co-ordinates.

To obtain equation

1) Find the Direction Ratios (DR's) by taking the difference of the corresponding position Co-ordinates of the 2 given points

$l = (x_2 - x_1)$ $m = (y_2 - y_1)$ $n = (z_2 - z_1)$

l, m, n are the DR's.

2) Choose either of the 2 given points say we choose (x_1, y_1, z_1).

3) write the required equation of the straight line passing through the points (x_1, y_1, z_1) and (x_2, y_2, z_2)

$$L : (x - x_1)/l = (y - y_1)/m = (z - z_1)/n.$$

Where (x, y, z) are the position Co-ordinates of any variable Point lying on the straight line.

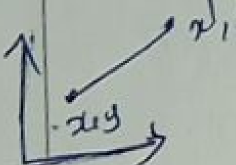
2D Transformations.

Translation → moves an object to a different position on the screen.

$$x' = x + t$$

$$x' = [I \quad t]\tilde{x}$$

where I is the (2×2) identity matrix.



$$\tilde{x}' = \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} \cdot \bar{x} \quad (or) \quad \begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$x' = x + tx$$
$$y' = y + ty.$$

Rotation + translation

* also known as 2D rigid body motion or 2D Euclidean transformation

$$x' = Rx + t$$

$$x' \; [R \quad t]\bar{x}$$

where

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Scaled rotation.

* Known as Similarity transform

$x' = SRx + t$ → where S is an arbitiary scale factor.

* scaling may be used to increase or reduce the size of object.
* If scaling factor >1, then the object size is increased.
* If scaling factor <1, then the object size is reduced.

Affine.

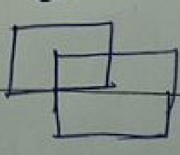
Affine Translation

$$x' = A\bar{x}$$

$$x' = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ ax & + bx & + c \\ a_{10} & a_{11} & a_{12} \\ dx & + ey & + f \end{bmatrix} \bar{x}$$

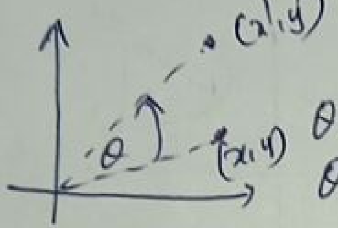
Parallel lines remain parallel under affine transformations, used to correct for geometric distortions or deformations that occur with non-ideal camera angle

Translation



Rotation → repositioning circular path.

- Rotation axis
- Rotation direction - clockwise or counter clockwise
- Rotation angle



$\theta > 0$: Rotalate counter clkwise
$\theta < 0$: Rotate cllcwise

$x' = x\cos\theta - y\sin\theta$
$y' = x\sin\theta + y\cos\theta$.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$\theta = 45°$

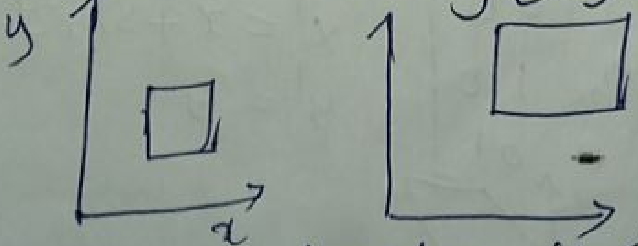
Scaling

- changes the size of the object.

$x' = x \cdot S_x$ $y' = y \cdot S_y$
↳ x direction ↳ y direction

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

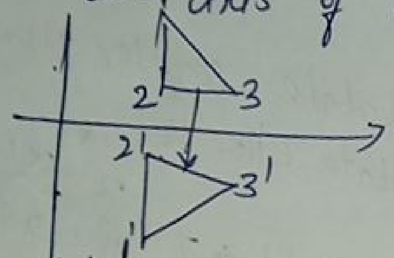
For uniform scaling
- S_x & S_y should have same value.



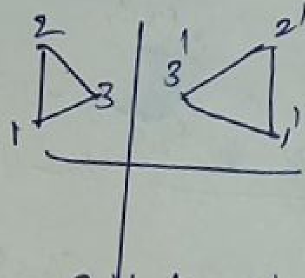
value < 1 reduce the size of object
value > 1 produce enlargement.

Reflection:

- Produces a mirror image relative to an axis of reflection.



Refelection about x axis



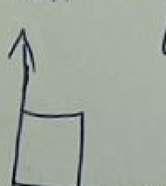
Reflection about y-axis

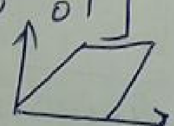
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \qquad \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Shear:

- X Shear preserves the y coordinates. Changes the x values which causes vertical lines to tilt right or left

$$X\text{-}Sh = \begin{bmatrix} 1 & 0 & 0 \\ Sh_x & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad x' = x + Sh_x \cdot y \qquad y' = y$$



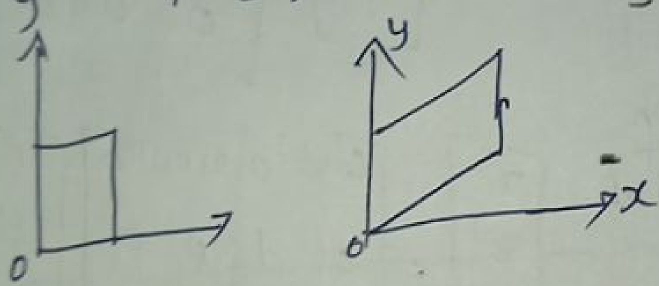


y Shear

- preserves the x-co-ordinates, but changes the y values.

$$y\,sh = \begin{bmatrix} 1 & sh_y & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$x' = x \qquad y' = y + sh_y \cdot x.$$

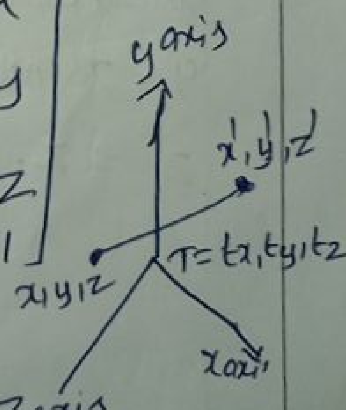


3D Transformation.

1. **3D Translation.** p → point.

$$p' = T \cdot P$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



$$x' = x + t_x \quad y' = y + t_y \quad z' = z + t_z$$

3D Rotation

x-axis Rotation

$x' = x$

$y' = y\cos\theta - z\sin\theta$

$z' = y\sin\theta + z\cos\theta$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

y-axis rotation

$x' = x\cos\theta + z\sin\theta$

$y' = y$

$z' = z\cos\theta - x\sin\theta$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

z-axis Rotation

$x' = x\cos\theta - y\sin\theta$

$y' = x\sin\theta + y\cos\theta$

$z' = z$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

3D Scaling

$$x' = x \cdot S_x$$
$$y' = y \cdot S_y$$
$$z' = z \cdot S_z$$

3D Reflection

Reflection about x-axis

$x' = x \quad y' = -y \quad z' = -z$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Reflection about y-axis

$y' = y \quad x' = -x \quad z' = -z$

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Reflection about z-axis

$x' = -x \quad y' = -y \quad z' = z$

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3D Shearing

Shearing x-direction

$x' = x$

$y' = y + s_y \cdot x$

$z' = z + s_z \cdot x$

Shearing y-direction

$x' = x + s_x \cdot y$

$y' = y$

$z' = z + s_z \cdot y$

Shearing z-direction

$x' = x + s_x \cdot z$

$y' = y + s_y \cdot z$

$z' = z$

3D to 2D Projection

Projection is a technique which is used to transform a 3D object into a 2D plane.

Projection requires

1. Projection plane
2. Projection Reference point (PRP) or center of projection (CRP)

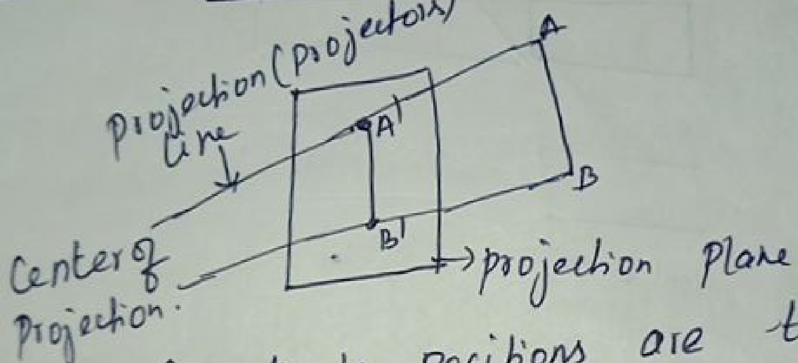
Projected View → Calculating the intersection of Projection lines with the View plane

Types

1) Parallel Projection
2) Perspective projection

transform 3D to 2D plane
Object
* projection plane
* projection Reference point
center of projection

1. Parallel Projection.

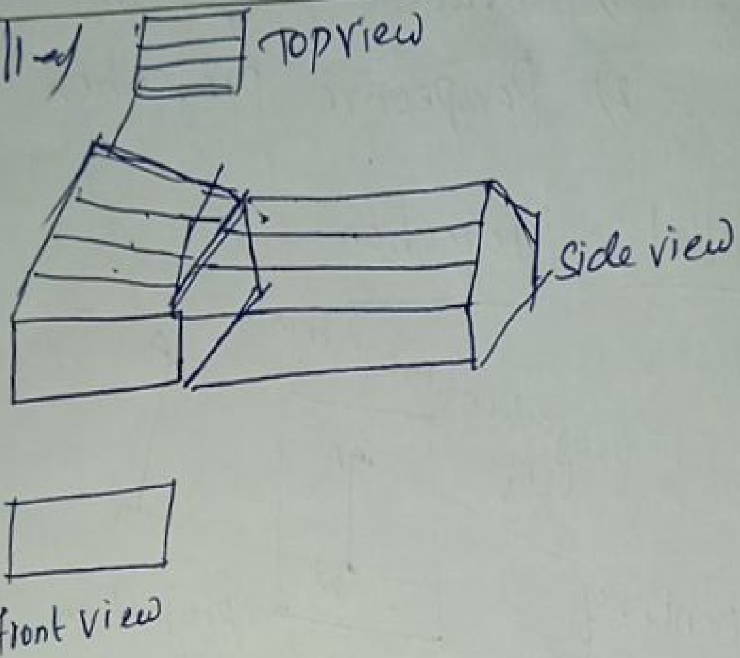


* Co-ordinate positions are transformed to the view plane along Parallel lines.
* Projection lines are parallel to each other.
* Projection lines are extended from the object & intersect the View plane.

Types.

1. Orthographic.
* Projection lines are parallel to each other & also perpendicular to the plane.
* Used to create different Views of given object. Produces front, top and side views.

Top View is called
→ Plan View



2. OBLIQU

* Projection lines are parallel to each other but not perpendicular to view plane.

CAVALIER:

In this projection lines makes angle of 30 degree with the view plane & there is no change in length of projection.

CABINET:

In this projection lines makes angle of 45 degree with the view plane & length of projected line will reduce.

orthography Projection: → simply drops z compoment of 3D co-ordinate but we keep w component

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \\ \cancel{z} \end{bmatrix}$$

$$x' = x$$
$$y' = y$$
$$w' = w.$$

$$\underset{\text{2D point}}{x} = [I_{2\times2} | 0] \underset{\text{3D point}}{P}$$

* It is an approximate model for long focal length lenses and objets whose depth is shallow relative to their distance to the camera.

Long focal length lens.

The focal length of a lens is determined when the lens is focused to infinity. Lens focal length tells us the angle of view → (i.e) how much of the scene will be captured and the magnification - how large individual elements will be.

The longer the focal length the narrower the angle of view and the higher the magnification.

(18mm)
Shorter the focal length → the wider the angle of view and the greater the area captured.

Longer the focal length (55m) the smaller the angle & the larger the subject appears to be.

In practice world co-ordinates (which may measure dimensions in meters) need to be scaled to fit onto an image sensor (measured in millimeters) (or in minimum pixels).

So scaled orthograpy is used.

$$x = \left[s I_{2\times 2} \,|\, 0 \right] p.$$

* The scaling can be the same for all parts of the scene
* or different for objects that are being modeled independently.
* The scaling can vary from frame to frame when estimation structure from motion → as the object approaches the camera

Scaled orthography is used for reconstructing the 3D shape of objects far away from the camera.

Perspective

In homogeneous co-ordinate,

$$\tilde{x} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tilde{p}$$

we drop the w component of p.

After projection, it is not possible to recover the distance of the 3D point from the image

Perspective projection is done in 2 step projection process.

1) First projects 3D co-ordinate into normalized device co-ordinates.

(i.e) 3D coordinates represent the 2D position on the screen with x & y in [-1, 1] and depth z [0, 1]

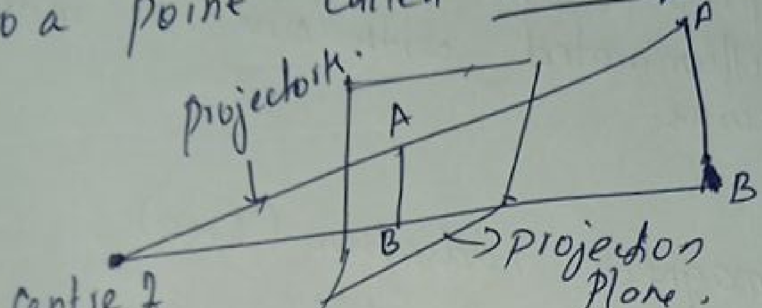
x → right
y → up
z → forward/backward.

2) use scaling factor. rescales these co-ordinates to integer pixel co-ordinates using View-port transformation

↳ area on the screen → device co-ordinates where graphics is to be displayed

Perspective Projection

* Object position are transformed to the view plane along lines that converge to a point called center of projection.



* Distant objects appear smaller than nearer objects.

Types.

1) One point perspective projection → x & y axis parallel to projection plane.
2) Two point perspective projection → z axis remain parallel to projection plane
3) Three point perspective projection
   → no any principle axis which is parallel to projection plane
   → all three axis intersects with projection plane.

* Points are projected onto the image plane by dividing them by their z component. Using Inhomogeneous

$$\bar{x} = P_z(p) = \begin{bmatrix} x/z \\ y/z \\ 1 \end{bmatrix}$$

Lighting, Reflectance & shading.

To produce an image, the scene must be illuminated with one or more light sources.

Light

Electromagnetic radiation (EMR). moving along rays in space.

* $R(\lambda)$ is EMR measured in units of power (watts) $\lambda \rightarrow$ wavelength

Light field.

* Radiation arriving at every point in space and from every direction.

Light sources can be divided into

* point
* area light sources

* A point light source originates at a single location in space. (light bulb)
* In addition to location → It has
  * intensity → amount of light.
  * color spectrum. (distribution over wavelengths $L(\lambda)$. Eg tablelamp
  * Modeled using single location $(x, y, z, 1)$

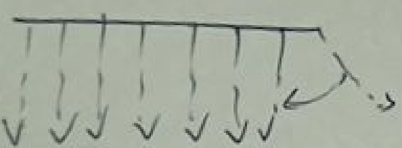
In Electromagnetic spectrum range of wavelength → human eye can detect wavelength from 380 to 700 nanometer.

Area light source.

The light source comes from a rectangular area and projects light from one side of the rectangle.

* Floresent light fixture in a ceiling panel.
  devices.

light direction



<dx,dy,dz,0>

. Incident illumination on an object sitting in an outdoor courtyard, can be represented using an environment map → also called reflection map

$L(\hat{v};\lambda)$ - $\hat{v}$ → light directions
λ - color values or wavelengths

* Light sources are at infinity → sun (all directions)

Environment map can be represented as a collection of cubical faces as a single longitude - latitude map, or as the image of a reflecting sphere.

Reflectance and Shading.

When light hits an objects surface, it is scattered and reflected. Types of Models to explain reflectance are 1) Bidirectional reflectance distribution function. 2) Diffuse, 3) Specular 4) Phong Shading Model

SRMIST,RAMAPURAM

Department of CSE

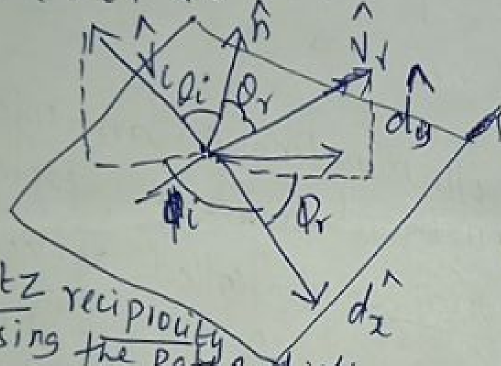
Course Code: 18CSE390T

Subject Name: COMPUTER VISION

Bidirectional Reflectance Distibution function (BRDF)

- It is a 4 dimensional function that describes how much of each wavelength arriving at an incident direction $\hat{v}_i$ is emitted in a reflected direction $\hat{v}_r$.

$$f_r(\theta_i, \phi_i, \theta_r, \phi_r; \lambda)$$



* Quantity of outgoing light $\leq$ quantity of incident light

integral of BRDF ≤ 1

Helmholtz reciprocity

- reversing the path of light produces same reflectance.

θ_i, ϕ_i, θ_r, ϕ_r

angles that the incident $\hat{v}_i$ — angles reflected $\hat{v}_r$

light ray directions make with the local surface co-ordinate frame $(\hat{d}_x, \hat{d}_y, \hat{n})$.

Amount of light exiting a surface point p in a direction $\hat{v}_r$ can be canculated using

$$L_r(\hat{v}_r; \lambda) = \int \underbrace{L_i(\hat{v}_i; \lambda)}_{\text{incoming light}} f_r(\hat{v}_i, \hat{v}_r, \hat{n}; \lambda) \cos^+\theta_i \, d\hat{v}_i$$

$$\cos^+\theta_i = \max(0, \cos\theta_i)$$

Isotropic surface — there are no preferred directions on the surface as far as light transport is concerned.

Prepared by: Dr.J.JOSPIN JEYA/AP/CSE/SRMIST

anisotropic surface → reflectance depends on the light orientation relative to the direction of scratches (brushed) scratched aluminium

Diffuse reflection

The diffuse Component (Lambertian(or) matte reflection) scatters light uniformly in all directions Eq (smooth) nonshiny surface)

surface normal $f_d(\hat{v}_i, \hat{v}_r, \hat{n}; \lambda) = f_d(\lambda)$

the amount of light depends on the angle b/w the incident light direction and the surface normal θ_i

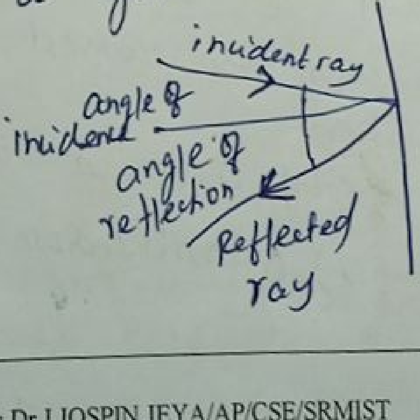
Perpendicular to a given object

* Dull, matte glass surfaces like chalk or latex Paint reflected equally in all directions.

specular reflection

* depends strongly on the direction of the outgoing light. → (ie) reflects into a single outgoing direction.
* reflection seen on smooth shiny objects. (mirror surface).
* Depends on surface orientation, viewer location.
* Reflects all light which arrives from a given direction at the same angle.

angle of incidence = angle of reflection



Phong Shading.

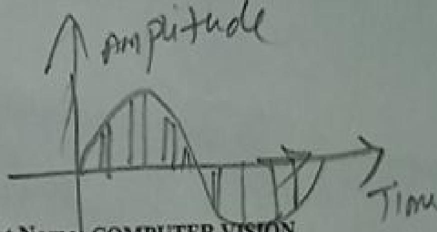
rough surface mirror

* Combine diffuse & specular component
* Called as ambient illumination.
* Objects are illuminated not only by point light source but also by a general diffuse illumination corresponding to inter-reflection (eg the walls in a room) or distant sources such as the blue sky.
* the ambient light depends on surface orientation, color of both ambient illumination $L_a(\lambda)$ and the object $k_a(\lambda)$

$$f_a(\lambda) = k_a(\lambda) L_a(\lambda)$$

Sampling and aliasing.

* Signals are analog in nature (eg. speech, weather signals);
* To process the analog signal by digital means, it is essential to convert them to discrete-time signal, and then convert them to individual a sequence of numbers.
* Analog to digital conversion steps.
  1) sampling 2) quantization 3) coding



Sampling

- A continuous-time signal is converted into discrete-time signals by taking samples of continuous-time signal at discrete time intervals.

$x[n] = x(nT_s)$

$x(nT_s) = x(t) \quad -\infty < n < \infty$

$T \rightarrow$ sampling interval

$x[n] \rightarrow$ samples of the continuous time signal $x(t)$.

$x(t)$ = Analog input signal

$T_s \rightarrow$ fixed time interval b/w samples.

- Sampling Criteria

Sampling frequency interms of sampling rate $\boxed{f_s = 1/T_s}$ $\rightarrow$ (samples / second) must be twice of the highest frequency.

$f_s = 2W$

$f_s \rightarrow$ sampling frequency

$\omega \rightarrow$ higher frequency content

2ω known as Nyquist rate

- Shannon's Sampling Theorem.

Minimum sampling rate required to reconstruct a signal from its instantaneous samples must be atleast twice the highest frequency.

$f_s \geq 2f_{max}$.

The maximum frequency in a signal is known as nyquist frequency and the inverse of the minimum sampling frequency $r_s = 1/f_s$ is Nyquist rate.

Nyquist rate.

* minimum sampling rate for the perfect reconstruction of the continuous time signals from samples.

Aliasing

* It is an effect that causes different signals to become indistinguishable when sampled. ↳ appearing to be the same.
* The distortion that results when a signal reconstructed from samples is different from the original continuous signal.

Temporal aliasing → occur in signals sampled in time. (video/audio signals)

Spatial aliasing → occur in spatially sampled signals. (more pattern) in a brick wall

If we sample at a rate which is less than the nyquist rate aliasing will occur.

Shannon's theorem tell us that if we have at least 2 samples per period of a sinusoid, we have enough information to reconstruct the sinusoid.

Image processing point operators.

~~* Image processing is an~~ application of CV.

Image processing is to preprocess the image and convert it into a form suitable for analysis. Some operations are

1) exposure correction, color balancing, reduction of image noise, increasing sharpness, straightening the image by rotating it.

* Image processing operators map pixel values from one image to another. Point operators or point process transforms image, that manipulate each pixel independently of its neighbors.

* Fourier Transform tool to analyze neighborhood operators, where each new pixels value depends on a small number of neighboring input values.

Point operators.

The simplest kind of image processing transforms are point operators, where each output pixel's value depends on only the corresponding input pixel value and brightness and contrast adjustments as well as color correction & transformation. This operation is known as point processes.

Simple point operator.

1) Brightness scaling 2) Image addition.

Pixel transforms.

A general image processing operator is a function that takes one or more i/p images.

Color

and produces an output image

$$g(x) = h(f(x)) \text{ or}$$

$$g(x) = h(f_0(x) \ldots f_n(x)).$$

x is in the D-dimensional domain of the functions (D=2 for images). and the fns f and g operate over some range, which can be either scalar or vector valued.

For discrete (sampled) images the domain consist of a finite no. of pixel locations

$$x = (i, j) \text{ and}$$

$$g(i,j) = h(f(i,j)).$$

2 Commonly used point processes are multiplication and addition with a constant.

$$g(x) = a f(x) + b.$$

$a > 0$ and b are called the gain & bias parameters used to control contrast and brightness. The bias and gain parameters can also be spatially varying $g(x) = a(x) f(x) + b(x)$

Another commonly used dyadic (2-i/p) operator is the linear blend operator

$$g(x) = (1-\alpha) f_0(x) + \alpha f_1(x)$$

$f_0, f_1 \rightarrow 2$ images

$\alpha \rightarrow$ blending factor

α 0 → 1, this operator can be used to perform a temporal cross-dissolve b/w 2 images or videos. It's a post production video editing technique in which we gently increase the opacity of one scene over the previous one.

mixing two images

→ not allowing light.

Color transforms

In an RGB model, the 3 values are red, green and blue components of the color of the pixel. In an HSI model, the 3 values are hue, saturation and intensity of the color of the pixel.

& brightness.

- Color transformations is a transformation of these values.

$$s_i = T_i(r_1, r_2, r_3) \quad i = 1,2,3.$$

r_1, r_2, r_3 represents the color component of the i/p image $f(m,n)$ and s_1, s_2, s_3 represent the color component of the output image $g(m,n)$

- Color transformation can be performed in any color space model.

Eg Simple transformation by Modifying intensity

$$g(\overline{m,n}) = k f(\overline{m,n})$$

$0 < k < 1$ is a scaling factor.

In HSI

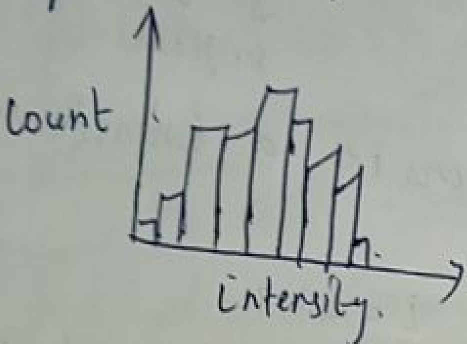
$$s_1 = r_1 \quad s_2 = r_2 \quad s_3 = k r_3.$$

In RGB

$$s_1 = k r_1 \quad s_2 = k r_2 \text{ and } s_3 = k r_3.$$

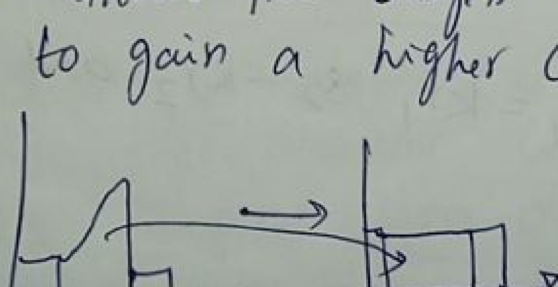
Histogram Equilization

- Used to enhance contrast. A histogra of an image is the graphical interpretation of the image's pixel intensity values - frequencie of all the pixel intensity levels in the image.



- intensity level is from 0-255.
- For RGB colored image 3 - 2D histogram are there - one for each color.
- For gray-scale only one histogr
- x-axis represents the pixel intensity level.
- y-axis indicates the no. of ~~pixels~~ pixels that have intensity values.

- Histogram equalization is an image processing technique that adjusts the contrast of an image using its histogram.
- To enhance the image's contrast, it spreads out the most frequent intensity values or stretches out the intensity range of the image.
- This allows the image's areas with lower contrast to gain a higher contrast.



Color Transforms-

HSI

Hue → Color itself. (yellow, orange & red).

saturation → how much the Color is (mixed) polluted with white color. range ^ [0, 1].

intensity → range [0, 1] 0 means black
1 means white.

Color balancing:

It is a method of shifting the colors in a photograph. We can change the color balance in the camera before taking a photo or in an editing program during post-processing.

- Performed either by multiplying each channel with a different scale factor or by changing the nominal white point and adjust the other colors, until the target object appears white. → piece of paper or a wall

Eg take a picture with a rainbow in it, and enhance the strength of the rainbow.

~~Shannon's theorem tell us that if we have at least 2 samples per period of a sinusoid, we have enough information to reconstruct the sinusoid.~~

Image noise (unwanted signal)

↳ random Variation of brightness or color information in images, produced by image sensor in digital camera

~~Point~~ In pixel transforms.

Multiplicative gain is a linear operation, since it obeys the superposition principle.

superposition principle in light states that when 2 or more waves overlap in space the resultant disturbance is equal to the algebraic sum of the individual disturbances

$$^{(50)}\quad h(f_0+f_1) = h(f_0) + h(f_1)$$

· **Blending** Pasting one image to another,

Ex $\boxed{x}$ $\boxed{y}$ $\boxed{x|y}$

we
period

Advanced Histogram equalization Techniques

1. Adaptive Histogram Equalization
2. Contrastive Limited Adaptive Equalization.

1. Adaptive Histogram Equalization (AHE)

- Utilizes the adaptive method to compute several histograms, each corresponding to a distinct section of the image. Using these histograms, this technique spread the pixel intensity values of the image to improve contrast.
- used to improve the local contrast and enhance the edges in specific regions of the image.

2. Contrastive Limited Adaptive Equalization (CLAHE)

- the contrast implication is limited by clipping the histogram at a predefined value. This clip limit depends on the normalization of the histogram or the size of the neighborhood region.
- The value b/w 3 and 4 is normally used.
- operates on small region in the image → tiles $\boxed{8 \times 8}$

Steps

1) Calculate the PMF (probability mass density function) of all pixels in the image.

* PMF is calculated by divide the count of each bar from vertical axis and divide it by Total count

2) Calculate CDF (Cumulative distributive function)

* simply keep the first value as it is, and then in the 2nd value, we will add the first one and so on. 40, 30, 60 — 40, 70, 110 ...

3) multiply the CDF value with (gray levels (minus) 1).

4) Map the new gray level values into no. of pixels.

5) Map the new values onto histogram.

Liner Filtering

Filtering is a technique used for modifying or enhancing an image like highlight certain features or remove other features.

Image filtering include

1) Smoothing 2) Sharpening 3) edge enhancement.

Linear Filtering

Linear filter is a filter which operate the pixel value in the support region in linear mamner. (ie) output pixel's value is determined as a weighted sum of input pixel values'

The support region is specified by the 'filter matrix' an be represented as H(i,j) The size of H is call 'filter region' and filter matrix has its own co-ordinate system, i is column index and j is row index. The center of it is the origin location and it is called the 'hot spot'.

* Images are stored in the form of a matrix of numbers where the number are known as pixel values
* These pixel values represent the intensity of each pixel.
* 0 represents black and 255 represents white.
* For colored three Matrices one for R a nother for G and another for B

$$g(i,j) = \sum_{k,l} f(i+k, j+l)\, h(k,l).$$

The entries in the weight kernal or mask $h(k,l)$ are often called as <u>filter co-efficients</u>

$$g = f \otimes h \rightarrow \text{correlation operator.}$$

~~The~~

A common variant on this formula is

$$g(i,j) = \sum_{k,l} f(i-k, j-l)\, h(k,l)$$

$$= \sum_{k,l} f(k,l)\, h(i-k, j-l).$$

This is called <u>Convolution operator</u>.

$g = f * h$ and h is then called the <u>impulse response function.</u>

~~Steps~~

- ~~Move the filter matrix over the image I and $H(0,0)$~~

super position (addition) of shifted impulse response function $h(i-k, j-l)$ multiplied by the input pixel values $f(k,l)$.

Histogram equalization. ~~So~~ some pictures have ~~excess of dark values~~, light values. It is better to darken some light value, & lighten some dark value.

To find an intensity mapping function $f(I)$ such that the resulting histogram is flat.

histogram → count the no. of pixels at each gray level value.

$h(I) \to$ distribution of pixels.

$I \to$ intensity value

Cumulative distribution

$$C(I) = \frac{1}{N} \sum_{i=0}^{I} h(i)$$

N is the no. of pixels in the image

<u>Linear filtering</u>

- output pixels value is determined as a weighted sum of input pixel values.

$$g(i,j) = \sum_{k,l} f(i+k, j+l)\, h(k,l)$$

~~kernel~~

~~mask~~

$h(k,l) \to$ kernel matrix.

$h(k,l) \to$ filter coefficients.

$$g = f \otimes h$$

* In image processing, a kernal / convo[illegible] matrix / mask is a small matrix us[illegible] for blurring, sharpening, embossing, edge detection. etc.

Embossing → pattern over a metal kacth, cloth,

* This is done by convolution b/w the kernel & the image.

$$\begin{array}{ll} & \text{kernel } \omega \\ \text{Identity} & \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

Sharpen the image.

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix} * \begin{bmatrix} 2 & 2 & 2 \\ 2 & 3 & 2 \\ 2 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 7 \end{bmatrix}$$

(ie) $3 \times 5 + 2 \times -1 + 2 \times -1 + 2 \times -1 + 2 \times -1$

Box blur $\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

30+ lin[illegible]

volution

Used

Both Correlation and convolution are linear Shift invariant (LSI) which obey both the super position & Shift invariance principle.

LSI is a system which has the property Linearity / superposition & Shift invariance.

1) Linearity → $f(ax+by) = af(x) + bf(y)$

2) Shift invariance → If we shift the input in time then the O/P is shifted by the same amount.

superposition → LSI operator called "behaves the same everywhere"

$$h \circ (f_0 + f_1) = h \circ f_0 + h \circ f_1$$

Shift invariance.

$$g(i,j) = f(i+k, j+l) \Longleftrightarrow (h \circ g)(i,j)$$

$$= (h \circ f)(i+k, j+l)$$

Histogram Equilization.

Algorithm

1. Convert the i/p image into a grayscale image

2. Find frequency of occurrence for each pixel value. (ie) histogram of an image values lie in the range [0, 255] for any grayscale image.

calculate PMF (probability mass function of all pixels in the img.

3. Calculate cumulative frequency of all pixel values.

4. Divide the cumulative frequencies by total no. of pixels and multiply them by maximum gray count (pixel value) in the image.

Equalize the given histogram

gray level (r)	0	1	2	3	4	5	6	7
no. of pixels.	790	1023	850	656	329	245	122	81

Gray level (r_k)	no. of pixels n_k	PDF $P_r(r_k) = n_k/n$	CDF $s_k = \sum P_k(r_k)$	$L-1(s_k) = 7 \times s_k$	Rounding off
0	790	0.19	0.19	1.33	1
1	1023	0.25	0.44	3.08	3
2	850	0.21	0.65	4.55	5
3	656	0.16	0.81	5.67	6
4	329	0.08	0.89	6.25	6
5	245	0.06	0.95	6.65	7
6	122	0.03	0.98	6.85	7
7	81	0.02	1	7	7

$n = 4096$

Equating Gray levels to no. of pixels.

$0 \rightarrow 0$

$1 \rightarrow 790$

$2 \rightarrow 0$

$3 \rightarrow 1023$

$4 \rightarrow 0$

$5 \rightarrow 850$

$6 \rightarrow 985$

$7 \rightarrow 448$

Non-linear Filtering

A non-liner combination of neighboring pixels.

Median Filtering

It selects the median value from each pixel's neighborhood.(ie) the result is the middle value after input values have been sorted.

Even order median filter, the output is the average of the central 2 samples after sorting.

Median filtering can be useful to reduce noise.

* Removing noise using the median filter does not reduce the difference in brightness of images, since the intensity values of the filtered image are taken from the original image.

* Median filter does not shift the edges of images.

Fourier Transforms.

It is a mathematical function that takes a time based pattern as input and determines the overall cycle offset, rotation speed and strength for every possible cycle in the given Pattern.

The Fourier transform is applied to waveforms which are basically a function of time, space or some other Variable. The Fourier transform decomposes a waveform into a sinusoid and thus provides another way to represent a waveform.

The result produced by the Fourier transform is a complex value function of frequency. The absolute value of the Fourier transform represents the frequency value present in the original function and its complex argument represents the phase offset of the basic sinusoidal in that frequency.

The Fourier transform of a fn $f(x)$ is given by

$$f(x) = \int_{-\infty}^{\infty} F(k) e^{2\pi i k x} dk$$

$$F(k) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i k x} dx.$$

F(u) can be obtained using inverse Fourier transform.

Properties.

1) **superposition**

It is a linear transform. The Fourier transform of a sum of signals is the sum of their Fourier transforms. Thus the Fourier transform is called as superposition. Transform signal

$f_1(x) + f_2(x) = F_1(\omega) + F_2(\omega)$

2) **Shift**

The Fourier transform of a shifted signal is the transform of the original signal multiplied by a linear phase shift.

signal $f(x - x_0)$ Transform $F(\omega) e^{-j\omega x_0}$

3) **Convolution**

The Fourier transform of a pair of convolved signal is the product of their transforms. $f(x) * h(x) = F(\omega) H(\omega)$.

4) **Correlation**

It is the product of the first transform times the complex conjugate of the second one. $F(x) \oplus h(x) = F(\omega) H^*(\omega)$.

5) **Reversal**: $f(-x) = F^*(\omega)$ Fourier Transform of the reversed signal is the complex conjugate of the signal's transform changing the sign of the imaginary part.

Two dimensional Fourier Transform, Wiener filtering

2 dimensional Fourier transforms are

$$H(\omega_x, \omega_y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(x,y)\, e^{-j(\omega_x x + \omega_y y)}\, dx\, dy$$

$\omega_x, \omega_y \rightarrow$ horizontal & Vertical frequency

In the discrete domain.

$$H(k_x, k_y) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} h(x,y)\, e^{-j2\pi \frac{k_x x + k_y y}{MN}}$$

M, N are the width and height of the image.

Wiener filtering

To ~~analyze~~ derive wiener filter, we analyze each frequency component of a signals Fourier transform independently. The noisy image formation process can be written as

$$O(x,y) = s(x,y) + n(x,y)$$

$s(x,y)$ is the image we are trying to recover, $n(x,y)$ is the additive noise signal,

$o(x,y)$ is the observed noisy image.

Because of the linearity of the Fourier transform we can write

$$O(\omega_x, \omega_y) = S(\omega_x, \omega_y) + N(\omega_x, \omega_y)$$

where each quantity in the above equation is the Fourier transform of the corresponding image.

At each frequency (ω_x, ω_y) we know from our image spectrum that the unknown transform component $S(\omega_x, \omega_y)$ has a prior distribution which is a zero mean Gaussian with Variance $P_s(\omega_x, \omega_y)$

we also have noisy measurement $O(\omega_x, \omega_y)$ whose Variance is $P_n(\omega_x, \omega_y)$ (i.e) Power spectrum of the noise which is normally assumed to be constant (white)

$$P_n(\omega_x, \omega_y) = \sigma_n^2$$

According to to Baye's Rule

the posterior estimate of s can be written as

$$P'(s|o) = \frac{P(o|s)P(s)}{P(o)}$$

where $P(o) = \int_s P(o|s)\,P(s)$ is a normalizing constant used to make the $P(s|o)$ distribution proper (integrate to 1).

The prior distribution $P(s)$ is given by

$$P(s) = e^{-\frac{(s-\mu)^2}{2P_s}}$$

μ is the expected mean at the frequency (0 everywhere except at the origin) and the measurement distribution $P(o|s)$ is given by

$$P(s) = e^{-\frac{(s-o)^2}{2P_n}}$$

$$W(\omega_x,\omega_y) = \frac{1}{1+\sigma_n^2/P_s(\omega_x,\omega_y)}$$

Power spectrum.

is the Fourier transform of optimum Wiener filter to remove the noise from an image whose power spectrum is $P_s(\omega_x,\omega_y)$

Non-linear filtering

Median Filtering

* Used to remove noise from images
* removing noise while preserving edges.
* Removing salt & pepper type noise

Salt & pepper noise

* Randomly occuring white & black pixels in the image.
* also known as impulse noise.

* Median filter works by moving through the image pixel by pixel, replacing each value with the median value of neighbouring pixels.
* The pattern of neighbours is called the window - which slides pixel by pixel over the entire image.
* The median is calculated by first sorting all the pixel value from the window into numerical order, and then replacing the pixel being considered with the middle pixel value.

Simple one dimensional signal

- window size of 3 used.

$x = 3$ | 3 | 9 | 4 | 52 | 3 | 8 | 6 | 2 | 2 | 9 | 9

$y[0] = \text{median}[3, 3, 9] = 3$

actual 3, 9, 4 → order as

$y[1] = \text{median}[3\ 4\ 9] = 4$

$y[2] = \text{"} [4\ 9\ 52] = 9$

$y[3] = \text{"} [3\ 8\ 52]\ 8$

$y[5] = M[3, 6, 8] = 6$

$y[6] = M[2\ 6\ 8] = 6$

$y[7] = M[2\ 2\ 6] = 2$

$y[8] = M[2\ 2\ 9] = 2$

$y[9] = M[2\ 9\ 9] = 9.$

$y =$ | 3 | 4 | 9 | 8 | 6 | 6 | 2 | 2 | 9 |

For $y[1]$ and $y[9]$ - extend the left most or right most value outside the boundaries of the image. Same as leaving left-most & right most value unchanged after 1-D median.

2D Median Filtering example using 3×3 Sampling window

- Keeping border Value unchanged -.

1	4	0	1	3	1
2	2	4	2	2	3
1	0	1	0	1	0
1	2	1	0	2	2
2	5	3	1	2	5
1	1	4	2	3	0

Sorted
0011 1 0
2244

0001 1 22
44

1	4	0	1	3	1
2	1	1	1	1	3
1	1	1	1	2	0
1	1	1	1	1	2
2	2	2	2	2	5
1	1	4	2	3	0

2D Median filtering using 3×3 sampling window - Extending border values outside with values at boundary

1	1	4	0	1	3	1	1
1	1	4	0	1	3	1	1
1	2	2	4	2	2	3	3
2	1	0	1	0	1	0	0
1	1	2	1	0	2	2	2
1	2	5	3	1	2	5	5
2	1	1	4	2	3	0	0
1	1	1	4	2	3	0	0

Sorted
0 0 1 1 1 2 2 4 4

2	2	2	2	2	2
1	1	1	1	1	1
1	1	1	1	2	2
1	1	1	1	1	2
1	1	1	1	2	2
1	2	2	2		
1	2	2	3	2	2

Input

output

3) Extending border values outside with 0's.

0	0	0	0	0	0	0	0
0	1	4	0	1	3	1	0
0	2	2	4	2	2	3	0
0	1	0	1	0	1	0	0
0	1	2	1	0	2	2	0
0	2	5	3	1	2	5	0
0	1	1	4	2	3	0	0
0	0	0	0	0	0	0	0

Sorted
0 0 0 0 1 2 2 4 4

0	2	1	1	1	0
0	1	1	1	1	1
0	1	1	1	2	1
0	1	1	1	1	1
1	2	2	2	2	2
0	1	1	1	1	0

(3)

The Fourier transform is simply a tabulation of the mangnitude & phase response at each frequency.

$$H(\omega) = F\{(h(x))\} = Ae^{j\phi}$$

(i.e) it is the response to a complex sinusoid of frequency ω passed through filter $h(x)$. Closed form equations for the Fourier transform exist both in continuous domain

Fourier Transform Pair is written as $h(x) \xleftrightarrow{F} H(\omega)$

$$H(\omega) = \int_{-\infty}^{\infty} h(x) e^{-j\omega x} dx$$

in discrete domain (DFT) Discrete Fourier Transform

$$H(k) = \frac{1}{N} \sum_{x=0}^{N-1} h(x) e^{-j\frac{2\pi k x}{N}}$$

N is the length of the signal or region of analysis. $K \rightarrow$ sample $\in \left[-\frac{N}{2}, \frac{N}{2}\right]$

These formulas apply both to filters such as $h(x)$ and to signals or images such as $s(x)$ or $g(x)$.

If k value is large aliasing occur for lower frequencies.

① Image Transform

Fourier Transform. →

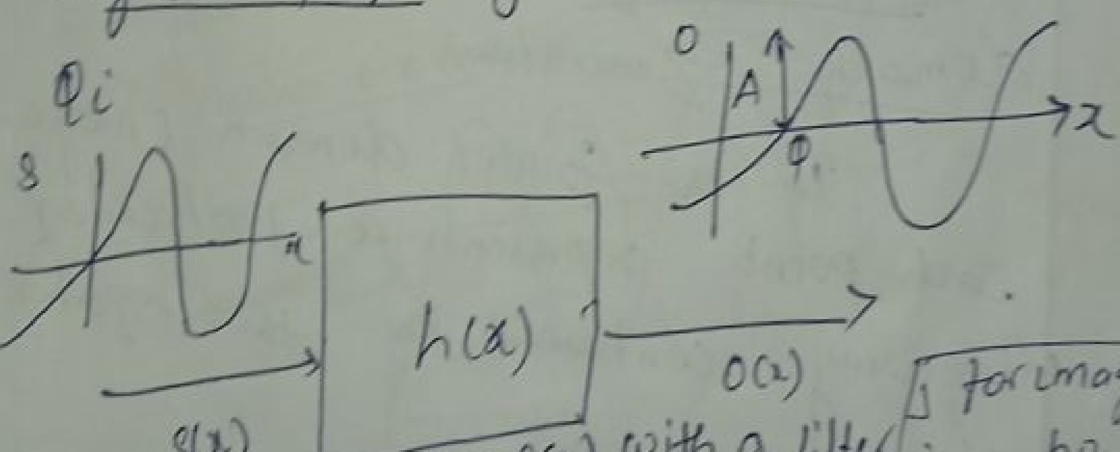
- Fourier analysis → used to analyze the frequency characteristics of known
- How can we analyze what a given filter does to high, medium and low frequencies.
- The soln is to simply pass a sinusoid of known frequency through the filter and to observe by how much it is attenuated (weaker).

Sine wave
1) amplitude
2) frequency
3) phase

$$s(x) = \sin(2\pi f x + \phi_i) = \sin(\omega x + \phi_i)$$

$s(x)$ is input sinusoid whose frequency is f, angular frequency is $\omega = 2\pi f$ and phase is

ϕ_i



for imaginary
ho

If we convolve the sine wave $s(x)$ with a filter we get another wave with same frequency but different magnitude & phase

combine

$$O(x) = h(x) * s(x)$$
$$= A \sin(\omega x + \phi_0)$$

The new magnitude A is called gain / magnitude of the filter while the phase difference $\Delta\phi = \phi_0 - \phi_i$ → shift or phase.

Fourier series:

- Periodic signals can be represented into sum of sines and cosines when multiplied with a certain weight.

Fourier Transform:

- Used to decompose an image into its sine and cosine components. The o/p of the transformation represents the image in the Fourier or frequency domain, while the i/p image is the spatial domain equivalent.

In the fourier domain image each point represents a particular frequency contained in the spatial domain image.

The compact notation to represent a sinusoid is to use the complex-value sinusoid

$$s(x) = e^{j\omega x} = \cos\omega x + j\sin\omega x.$$

$$o(x) = h(x) * s(x) = A e^{j\omega x + \phi}$$

Sine w
amplitud

Sine wave.

amplitude → the peak deviation of the fn from zero

f → no. of cycles that occur each second of time.

$\omega = 2\pi f$ → angular frequency, the rate of change of the function argument in units of radians per second

ϕ → Phase - specifies in radians where in its cycle the oscillation is at $t=0$

Sinusoidal refers to both sine waves and cosine waves with any phase offset

Eg