Minimization of DFA

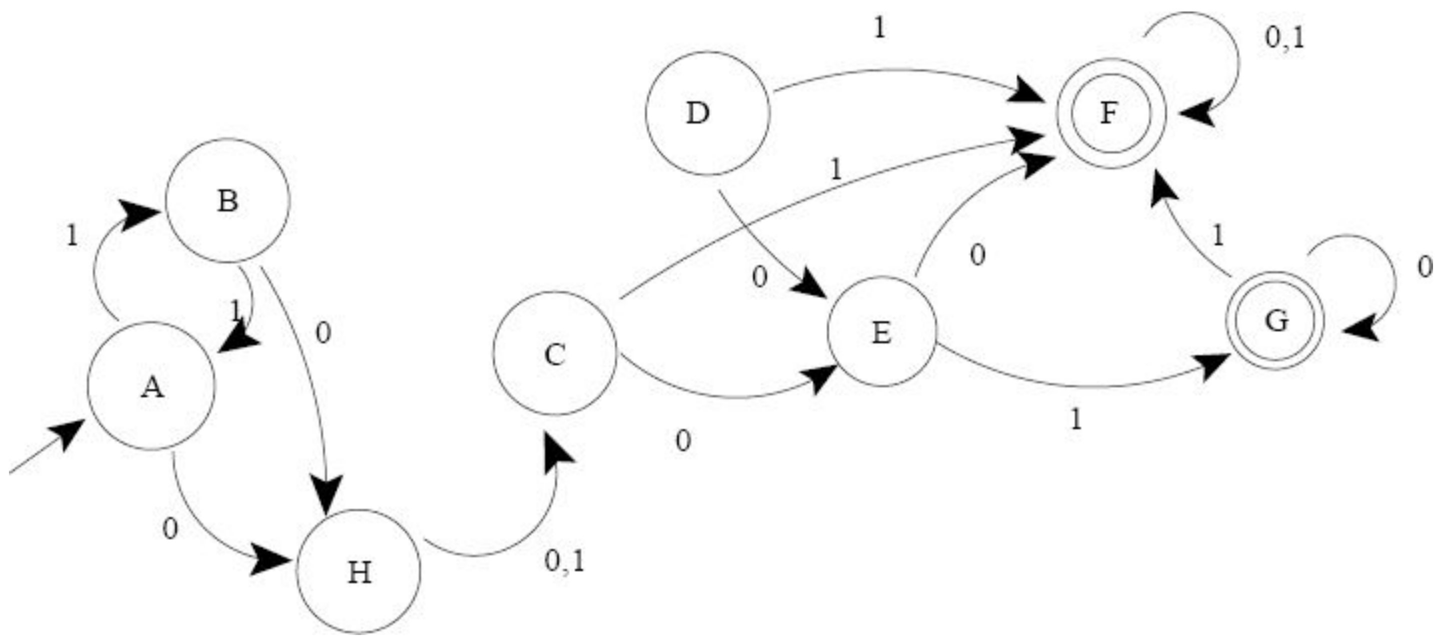
- The task of *DFA minimization*, then, is to automatically transform a given DFA into a state-minimized DFA
  - Several algorithms and variants are known
  - Note that this also in effect can minimize an NFA (since we know algorithm to convert NFA to DFA)

DFA

- Deterministic Finite Automata (DFSA)
  - (Q, Σ, δ, q_0, F)
    - Q – (finite) set of states
    - Σ – alphabet – (finite) set of input symbols
    - δ – transition function
    - q_0 – start state
    - F – set of final / accepting states

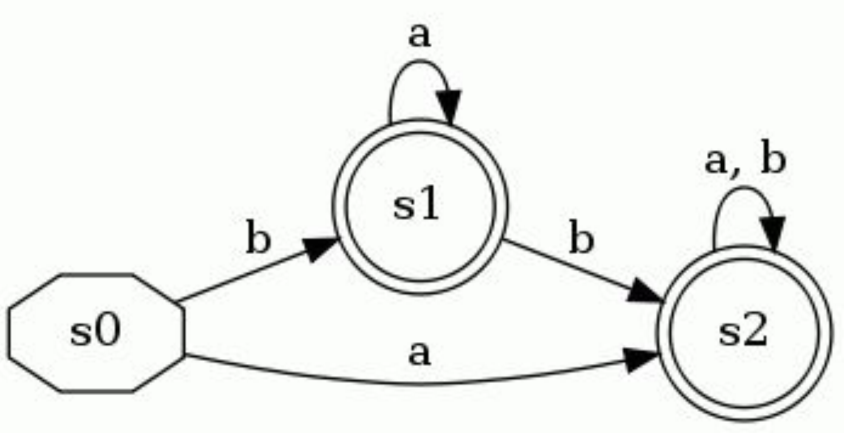
DFA

- Often representing as a diagram:



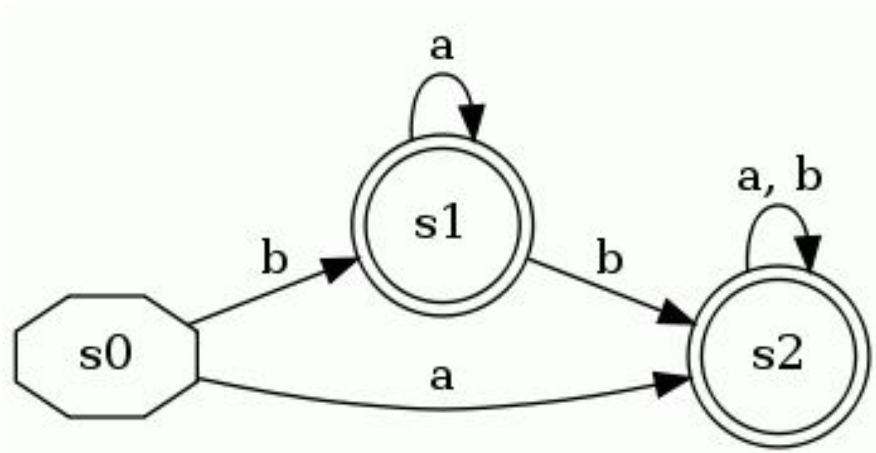
DFA Minimization

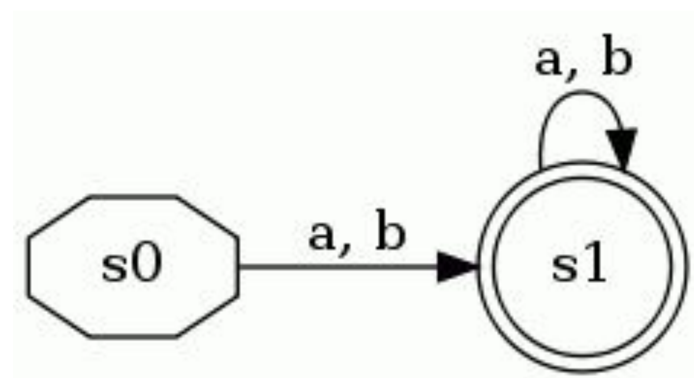
- Some states can be redundant:
  - The following DFA accepts (a|b)+
  - State s1 is not necessary



DFA Minimization

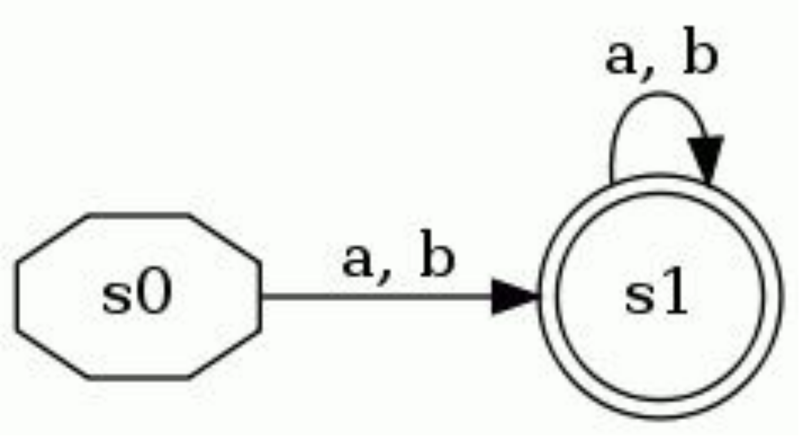
- So these two DFAs are *equivalent*:





DFA Minimization

- This is a *state-minimized* (or just *minimized*) DFA
  - Every remaining state is necessary



DFA Minimization

- The task of *DFA minimization*, then, is to automatically transform a given DFA into a state-minimized DFA
  - Several algorithms and variants are known
  - Note that this also in effect can minimize an NFA (since we know algorithm to convert NFA to DFA)

DFA Minimization Algorithm

- Recall that a DFA $M=(Q, \Sigma, \delta, q_0, F)$

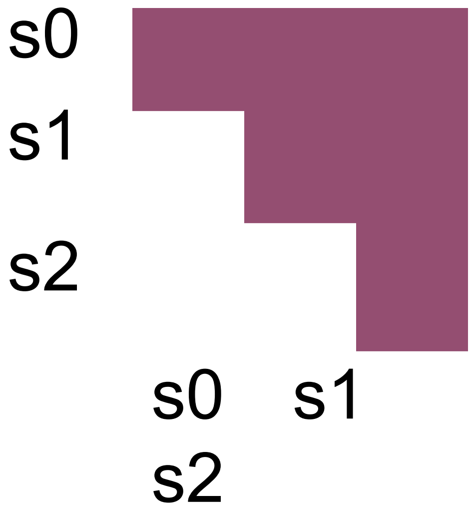
- Two states p and q are distinct if
  - p in F and q not in F or vice versa, or
  - for some α in Σ, $\delta(p, \alpha)$ and $\delta(q, \alpha)$ are distinct

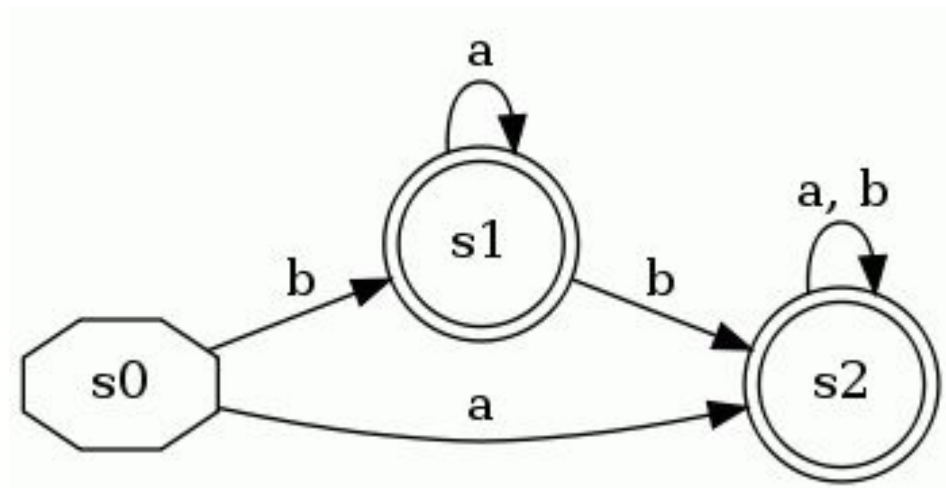
- Using this inductive definition, we can calculate which states are distinct

DFA Minimization Algorithm

- Create lower-triangular table DISTINCT, initially blank
- For every pair of states (p,q):
  - If p is final and q is not, or vice versa
    - DISTINCT$(p,q) = \varepsilon$
- Loop until no change for an iteration:
  - For every pair of states (p,q) and each symbol α
    - If DISTINCT(p,q) is blank and DISTINCT($\delta(p,\alpha)$, $\delta(q,\alpha)$) is not blank
      - DISTINCT$(p,q) = \alpha$
- Combine all states that are not distinct

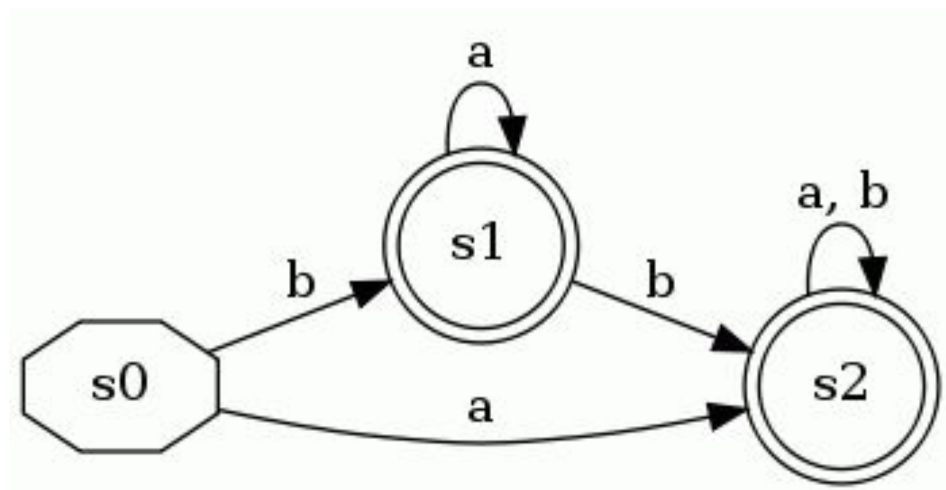
Very Simple Example





Very Simple Example

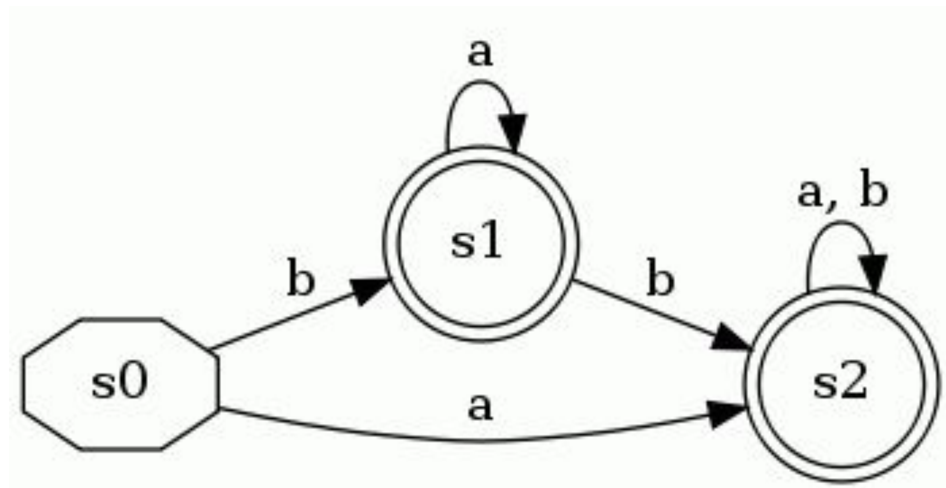
s0			
s1	ε		
s2	ε		
	s0	s1	s2



Label pairs with ε where one is a final state and the other is not

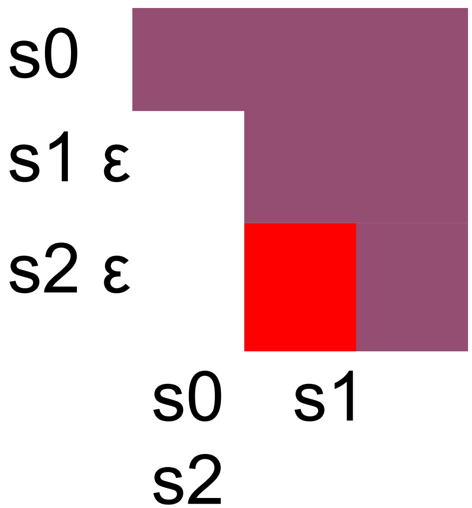
Very Simple Example

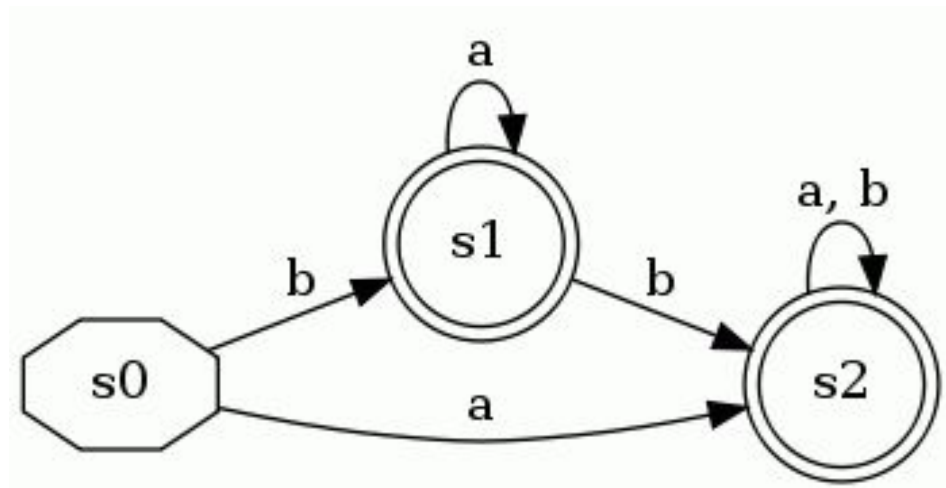
s0			
s1	ε		
s2	ε		
	s0	s1	s2



Main loop (no changes occur)

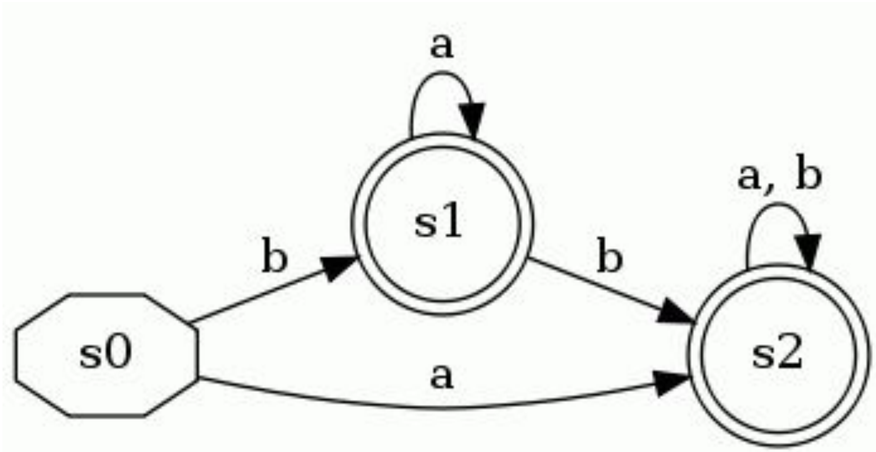
Very Simple Example



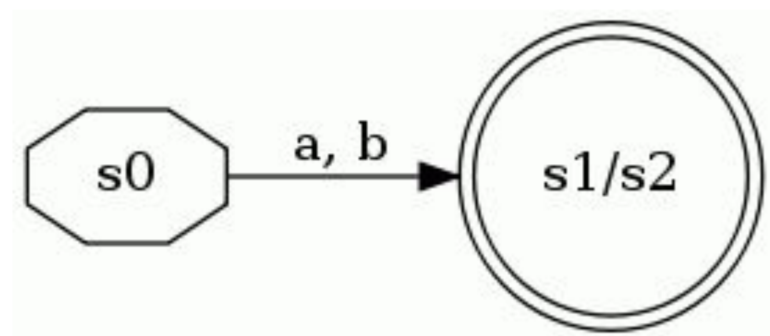


DISTINCT(s1, s2) is empty, so s1 and s2 are equivalent states

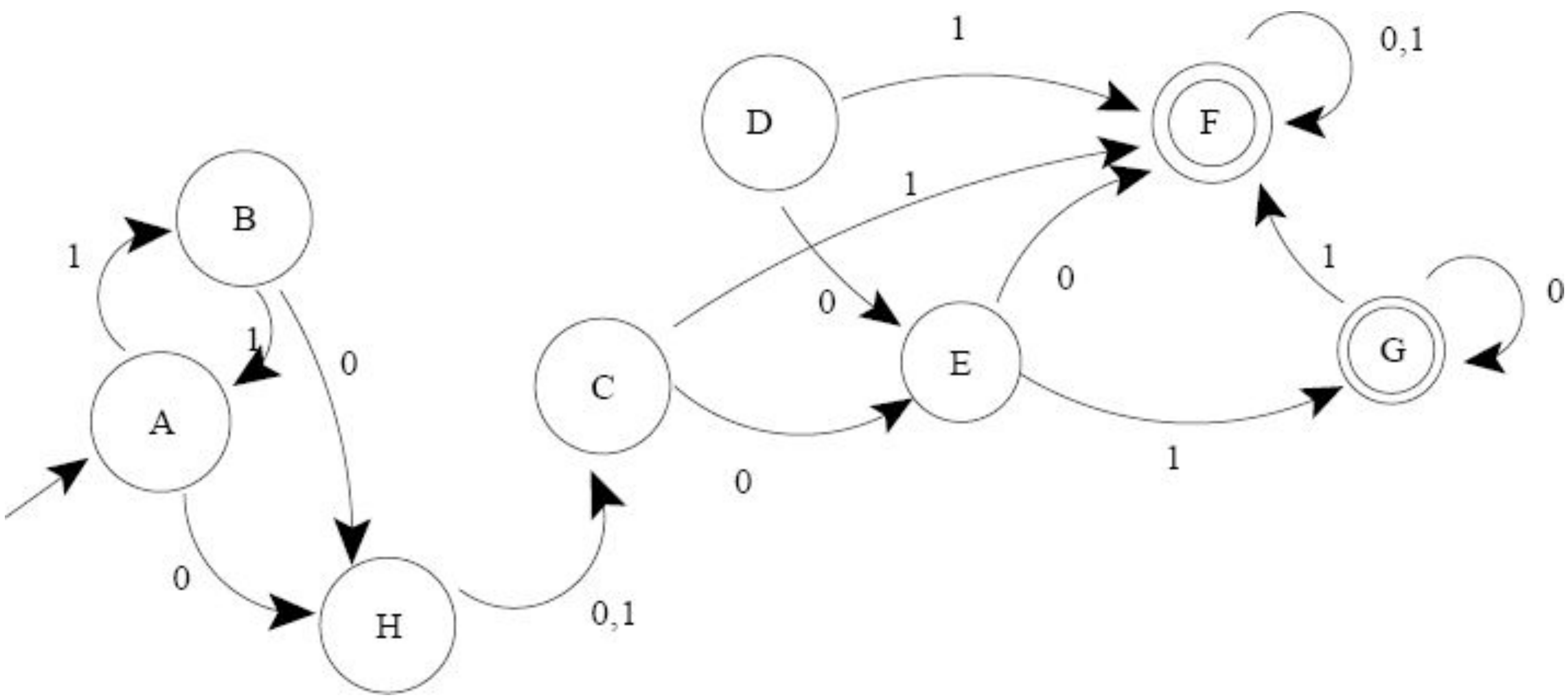
Very Simple Example



Merge s1 and s2



More Complex Example



More Complex Example

- Check for pairs with one state final and one not:

b							
c							
d							
e							
f	ϵ	ϵ	ϵ	ϵ	ϵ		
g	ϵ	ϵ	ϵ	ϵ	ϵ		
h						ϵ	ϵ
	a	b	c	d	e	f	g

More Complex Example

- First iteration of main loop:

b							
c	1	1					
d	1	1					
e	0	0	0	0			
f	ϵ	ϵ	ϵ	ϵ	ϵ		
g	ϵ	ϵ	ϵ	ϵ	ϵ		
h			1	1	0	ϵ	ϵ
	a	b	c	d	e	f	g

More Complex Example

- Second iteration of main loop:

b							
c	1	1					
d	1	1					
e	0	0	0	0			
f	ϵ	ϵ	ϵ	ϵ	ϵ		
g	ϵ	ϵ	ϵ	ϵ	ϵ		
h	1	1	1	1	0	ϵ	ϵ
	a	b	c	d	e	f	g

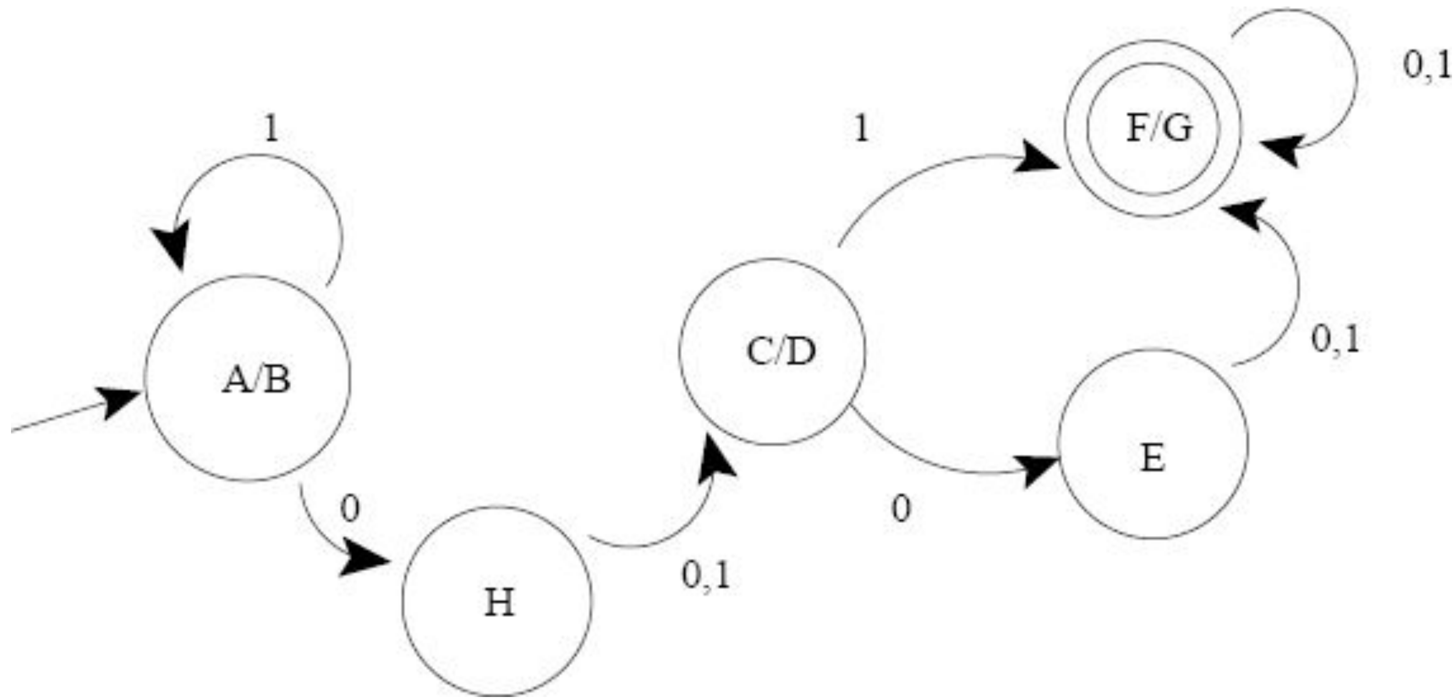
More Complex Example

- Third iteration makes no changes
  - Blank cells are equivalent pairs of states

b							
c	1	1					
d	1	1					
e	0	0	0	0			
f	ϵ	ϵ	ϵ	ϵ	ϵ		
g	ϵ	ϵ	ϵ	ϵ	ϵ		
h	1	1	1	1	0	ϵ	ϵ
	a	b	c	d	e	f	g

More Complex Example

- Combine equivalent states for minimized DFA:



Conclusion

- DFA Minimization is a fairly understandable process, and is useful in several areas
  - Regular expression matching implementation
  - Very similar algorithm is used for compiler optimization to eliminate duplicate computations
- The algorithm described is $O(kn^2)$
  - John Hopcraft describes another more complex algorithm that is $O(k\ (n \log n)\ )$