Table 1 Laws of Algebra of Propositions

Sl. No.	Name of the law	Primal form	Dual form
1.	Idempotent law	$p \vee p \equiv p$	$p \wedge p \equiv p$
2.	Identity law	$p \vee F \equiv p$	$p \wedge T \equiv p$
3.	Dominant law	$p \vee T \equiv T$	$p \wedge F \equiv F$
Negation law (or) 4.	Complement law	$p \vee \neg p \equiv T$	$p \wedge \neg p \equiv F$
5.	Commutative law	$p \vee q \equiv q \vee p$	$p \wedge q \equiv q \wedge p$
6.	Associative law	$(p \vee q) \vee r \equiv p \vee (q \vee r)$	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
7.	Distributive law	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
8.	Absorption law	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
9.	De Morgan's law	$\neg(p \vee q) \equiv \neg p \wedge \neg q$	$\neg(p \wedge q) \equiv \neg p \vee \neg q$

Double Negation Law
$\neg(\neg p) \equiv p$

Table 2 Equivalences Involving Conditionals

1. $p \rightarrow q \equiv \neg p \vee q$
2. $p \rightarrow q \equiv \neg q \rightarrow \neg p$
3. $p \vee q \equiv \neg p \rightarrow q$
4. $p \wedge q \equiv \neg(p \rightarrow \neg q)$
5. $\neg(p \rightarrow q) \equiv p \wedge \neg q$
6. $(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$
7. $(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$
8. $(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$
9. $(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$

Table 3 Equivalences Involving Biconditionals

1. $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$
2. $p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
3. $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
4. $\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

Table 4 Implications

Simplification ← 1. $p \wedge q \Rightarrow p$
Simplification ← 2. $p \wedge q \Rightarrow q$
Addition ← 3. $p \Rightarrow p \vee q$, $q \Rightarrow p \vee q$
4. $\neg p \Rightarrow p \rightarrow q$
5. $q \Rightarrow p \rightarrow q$
6. $\neg(p \rightarrow q) \Rightarrow p$
7. $\neg(p \rightarrow q) \Rightarrow \neg q$
Modus ponens ← 8. $p \wedge (p \rightarrow q) \Rightarrow q$
Modus tollens ← 9. $\neg q \wedge (p \rightarrow q) \Rightarrow \neg p$
Disjunctive syllogism ← 10. $\neg p \wedge (p \vee q) \Rightarrow q$
Hypothetical syllogism ← 11. $(p \rightarrow q) \wedge (q \rightarrow r) \Rightarrow p \rightarrow r$ → (or) Chain Rule
Dilemma ← 12. $(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r) \Rightarrow r$

Conjunction
$p, q \Rightarrow p \wedge q$

Resolution
$[(p \vee q) \wedge (\neg p \vee r)] \Rightarrow q \vee r$

Theory of Inference:

Inference Theory is concerned with getting a conclusion from certain hypothesis or basic assumptions or premises by applying principles of reasoning or rules of inference.

Premises:

Premises is a statement which is assumed to be true.

Formal proof:

The process of deriving a conclusion from a set of premises by the accepted rules of reasoning is called a formal proof.

Types:

1. Direct proof.

2. Indirect proof.

Rules of inference:

Rule P:

A premise may be introduced at any step in the derivation.

Rule T:

A formula S may be introduced in the derivation, if S is tautologically implied by one or more preceeding formulae in the derivation.

Rule CP: (Conditional proof)

If the conclusion is of the form $R \to S$, then we will take R as an additional premise and derive S using the given premises and R.

Inconsistent premises:-

A set of premises $H_1, H_2, \ldots, H_n$ is said to be inconsistent if their conjunction implies a contradiction i.e.,

$H_1 \wedge H_2 \wedge \ldots \wedge H_n \Rightarrow F$.

Consistent premises:-

A set of premises is said to be consistent if it is not inconsistent.

Problems under direct proof:

1. Show that R is a valid inference from the premises $P \to Q$, $Q \to R$ and P.

Step	Statement	Rule	Reason.
1	$P \to Q$	P	Given
2.	$Q \to R$	P	Given
3.	P	P	Given.
4	$P \to R$	T	(1,2), Hypothetical syllogism.
5.	R	T	(3,4), Modus ponens.

2. Show that $P \to S$ follows logically from the premises $\neg P \vee Q$, $\neg Q \vee R$, and $R \to S$.

Solution:

Step	Statement	Rule	Reason.
1	$\neg P \vee Q$	P	Given
2	$\neg Q \vee R$	P	Given
3	$R \to S$	P	Given.

4.	$P \rightarrow Q$	T	1, table (2.1)
5.	$Q \rightarrow R$	T	2, table (2.1)
6	$P \rightarrow R$	T	(4,5), Hypo.
7.	$P \rightarrow S$	T	(6,3), Hypo.

3. Show that $R \wedge (P \vee Q)$ is a valid conclusion from the premises $P \vee Q$, $Q \rightarrow R$, $P \rightarrow M$ and $\neg M$.

Solution:

Step	Statement	Rule	Reason.
1	$P \vee Q$	P	Given
2	$Q \rightarrow R$	P	"
3	$P \rightarrow M$	P	"
4.	$\neg M$	P	"
5.	$\neg P$	T	(4,3), Modus tollens.
6	Q	T	(5,1), Disj. syll.
7	R	T	(6,2), Modus ponens
8.	$R \wedge (P \vee Q)$	T	(7,1), conjunction

4. Show that $t \wedge s$ can be derived from the premises $p \to q$, $q \to \neg r$, r, $p \vee (t \wedge s)$

Solution:

Step	Statement	Rule	Reason.
1	$p \to q$	P	Given
2	$q \to \neg r$	P	"
3	r	P	"
4	$p \vee (t \wedge s)$	P	"
5.	$p \to \neg r$	T	(1,2), Hypothetical.
6.	$r \to \neg p$	T	5, $p \to q \equiv \neg q \to \neg p$.
7	$\neg p$	T	(3,6), Modus ponens.
8.	$t \wedge s$	T	(7,4), Disjunctive syllogism.

5. Show that $R \to S$ can be derived from the premises $P \to (Q \to S)$, $\neg R \vee P$ and Q.

Solution:

Step	Statement	Rule	Reason.
1.	$P \to (Q \to S)$	P	Given
2.	$\neg R \vee P$	P	"
3	Q	P	"
4	$R \to P$	T	2, $P \to Q \equiv \neg P \vee Q$.
5	$R \to (Q \to S)$	T	(4,1), Hypothetical
6.	$\neg R \vee (\neg Q \vee S)$	T	5, table (2.1)
7.	$(\neg R \vee \neg Q) \vee S$	T	6, Associative
8.	$(\neg Q \vee \neg R) \vee S$	T	7, Commutative.
9.	$\neg Q \vee (\neg R \vee S)$	T	8, Associative.
10.	$\neg R \vee S$	T	(3, 9), Disjunctive
11	$R \to S$	T	10, table (2.1).

Problems under conditional proof!

6. Show that $P \to S$ follows logically from the premises $\neg P \vee Q$, $\neg Q \vee R$, and $R \to S$ using CP.

Solution!

Step	Statement	Rule	Reason.
1	$\neg P \vee Q$	P	Given
2	$\neg Q \vee R$	P	"
3	$R \rightarrow S$	P	"
4	P	P	Additional
5	$P \rightarrow Q$	T	1, table (2.1)
6.	$Q \rightarrow R$	T	2, table (2.1)
7.	$P \rightarrow R$	T	(5, 6), Hypothetical.
8.	$P \rightarrow S$	T	(3, 7), "
9.	S	T	(4, 8), Modus ponens.
10.	$P \rightarrow S$	T	9, Rule CP.

7. Derive $p \rightarrow (q \rightarrow s)$ using CP rule. from the premises $p \rightarrow (q \rightarrow r)$ and $q \rightarrow (r \rightarrow s)$.

Solution:

1.	$p \to (q \to r)$	P	Given
2	$q \to (r \to s)$	P	Given.
3	p	P	Additional.
4	$q \to r$	T	(3,1), Modus ponens.
5.	$\neg q \vee r$	T	4, table (2.1)
6.	$\neg q \vee (r \to s)$	T	2, table (2.1)
7.	$\neg q \vee (r \wedge (r \to s))$	T	(5,6), Dist..
8.	$\neg q \vee s$	T	7, Modus ponens.
9.	$q \to s$	T	8, table 2.1
10	$p \to (q \to s)$	T	9, Rule cp.

problems under inconsistent!

8. Show that the following premises are inconsistent!

(i) If Jack misses many classes due to illness, then he fails in school.

(ii) If Jack fails in school, then he is uneducated.

(iii) If Jack reads a lot of books, then he is not uneducated.

(iv) Jack misses many classes due to illness and reads a lot of books.

Solution:-

Let

P: Jack misses many classes due to illness.

Q: He fails in school.

R: He reads a lot of books.

S: He is uneducated.

The premises are:

$P \to Q$, $Q \to S$, $R \to \neg S$, $P \wedge R$

Step	Statement	Rule	Reason.
1	$P \to Q$	P	Given
2.	$Q \to S$	P	"
3	$R \to \neg S$	P	"
4	$P \land R$	P	"
5.	$P \to S$	T	(1,2), Hypo/..
6.	$S \to \neg R$	T	3, $P \to Q \equiv \neg Q \to \neg P$
7.	$P \to \neg R$	T	(5, 6), Hypo/..
8.	$\neg P \lor \neg R$	T	7, table (2.1)
9.	$\neg(P \land R)$	T	8, DeMorgan's.
10	F	T	(4, 9), complement.

Q. Show that the premises are inconsistent.

(i) If Rama gets his degree, he will go for a job.

(ii) If he goes for a job, he will get Married.

(iii) If he goes for higher study, he will not get Married.

(iv) Rama gets his degree and goes for higher study.

Solution:

P : Rama gets his degree.

Q : He goes for a Job

R : He goes for higher study.

S : He will get Married.

The premises are.

$P \to Q$, $Q \to S$, $R \to \neg S$, $P \wedge R$.

The solution is same as problem number (8).

Problems Under Indirect proof:

Procedure:

1. Introduce The negation of the derived conclusion as a new premise.
2. Using The given premise and This new premise, derive a Contradiction.

problem:-

10. Use indirect method to show that

$r \to \neg q,\ r \vee s,\ s \to \neg q,\ p \to q \Rightarrow \neg p.$

solution:

Step	Statement	Rule	Reason.
1.	$r \to \neg q$	P	Given
2.	$r \vee s$	P	"
3	$s \to \neg q$	P	"
4	$p \to q$	P	"
5.	p	P	Add premise.
6.	q	T	(4,5), Modus ponens.
7.	$(r \vee s) \to \neg q$	T	(1,3), $(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$.
8.	$\neg q$	T	(2,7), Modus ponens.
9.	F	T	(6,8), Complement.

11. Use indirect Method to show that

$p \to q, \; q \to r, \; \neg(p \wedge r), \; p \vee r \Rightarrow r$.

Solution:

1.	$p \to q$	P	Given
2.	$q \to r$	P	"
3.	$\neg(p \wedge r)$	P	"
4.	$p \vee r$	P	"
5.	$\neg r$	P	Additional
6.	$p \to r$	T	(1,2), Hyp/..
7.	$\neg p$	T	(5,6), Modus tollens.
8.	r	T	(7,4), Disjunctive.
9	F	T	(8,5), complement
10	F	T	(3,9), Dominant Law.