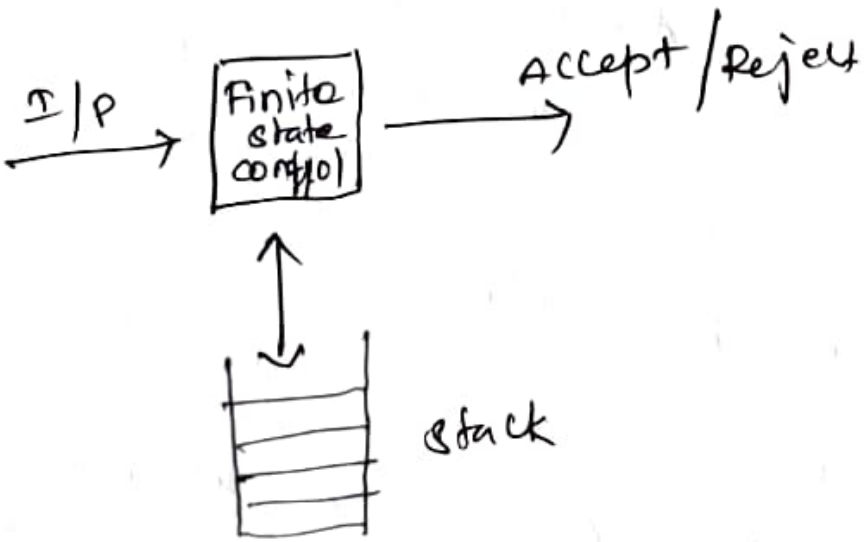
UNIT · III

PUSH DOWN AUTOMATA

- It is also one type of Automaton.
- It is a way to implement a CFG
- In a similar way we design DFA for a regular grammar.
- In DFA can remember a finite Amount of info, but a PDA can remember an infinite Amount of info.

PDA = FA + STACK



Model of PDA

- It consist of finite Set of States, finite Set of i/p symbols & a finite Set of Push down Symbols.

* The finite control of both i/p tape & the pushdown stone. - has

* In one transition of PDA
  - the control head reads a i/p symbol & goto new state.
  - Replaces the symbol at the top of the stack by any string.

Defn:- It consist of seven tuples

$$P = \{ Q, \Sigma, \Gamma, \delta, q_0, z_0, F \}$$

$Q \rightarrow$ a finite non-empty set of states

$\Sigma \rightarrow$ a finite set of i/p symbols

$\Gamma \rightarrow$ a finite non-empty set of stack symbols.

$q_0 \rightarrow$ start state

$z_0 \rightarrow$ Initial start symbol of stack

$F \rightarrow F \subseteq Q$, set of accepting states (or) final states

$\delta \rightarrow$ Transition function is given by

$$\delta : Q \times \{ \Sigma \cup \{\varepsilon\} \} \times \Gamma \rightarrow Q \times \Gamma^*$$

Moves

the interpretation of

$$\delta(q, a, Z) = (P_1, \gamma_1)\ (P_2, \gamma_2) \ldots (P_m, \gamma_m)$$

$q, P_i \rightarrow$ state

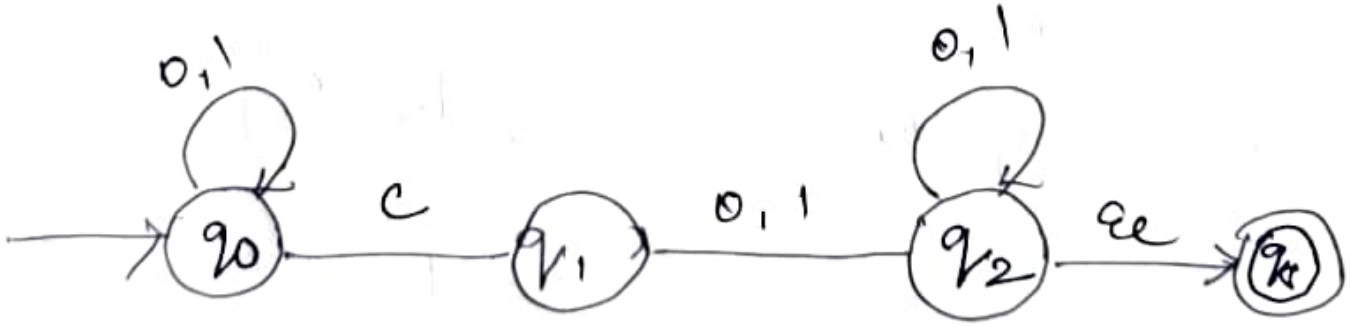
$a \rightarrow$ i/p symbol

$Z \rightarrow$ stack symbol

$\gamma_i \rightarrow$ a symbol in Γ^*

eg:

Construct a PDA that accepts $\{wcw^R / w \text{ in } (0+1)^*\}$ by an empty stack.



1. $\delta(q_0, 0, z_0) = \{q_0, 0z_0\}$
2. $\delta(q_0, 1, z_0) = \{q_0, 1z_0\}$
3. $\delta(q_0, 0, 0) = \{q_0, 00\}$
4. $\delta(q_0, 1, 0) = \{q_0, 10\}$
5. $\delta(q_0, 0, 1) = \{q_0, 01\}$
6. $\delta(q_0, 1, 1) = \{q_0, 11\}$

} PUSH

7 $\delta(q_0, c, 1) = \{q_1, 1\}$
8 $\delta(q_0, c, 0) = \{q_1, 0\}$

} Accept the Separator C

9. $\delta(q_1, 0, 0) = \{q_2, \epsilon\}$
10. $\delta(q_1, 1, 1) = \{q_2, \epsilon\}$
11. $\delta(q_1, \epsilon, z_0) = \{q_2, \epsilon\}$

} POP

For all other the i/p string is rejected

Ex:

Let 100C001 be the i/p string

$w = 100 \quad w^R = 001$

$\delta(q_0, 100C001, Z_0) \vdash (q_0, 00C001, 1Z_0)$
$\vdash (q_0, 0C001, 01Z_0)$
$\vdash (q_0, C001, 001Z_0)$
$\vdash (q_1, 001, 001Z_0)$
$\vdash (q_1, 01, 01Z_0)$
$\vdash (q_1, 1, 1Z_0)$
$\vdash (q_2, \epsilon, Z_0)$
$\vdash (q_2, \epsilon)$ accepted.

Ex:

110C10

$\delta(q_0, 110C10, Z_0) \vdash (q_0, 110C10, 1Z_0)$
$\vdash (q_0, 10C10, 11Z_0)$
$\vdash (q_0, 0C10, 011Z_0)$
$\vdash (q_0, C10, 0011Z_0)$
$\vdash (q_1, 10, 0011Z_0)$
$\vdash$ Rejected

The Language of PDA

① Acceptance by Final state
② Acceptance by Empty stack

Case 1:

$$L(M) = \{w \mid (q_0, w, Z_0) \vdash (p, \varepsilon, \gamma) \text{ for some } p \text{ in } F \text{ & } \gamma \text{ in } \Gamma^*\}$$

Case 2:

$$N(M) = \{w \mid (q_0, w, Z_0) \vdash (p, \varepsilon, \varepsilon) \text{ for some } p \text{ in } Q\}$$