15MA302 - DM ASSIGNMENT

K. DEPEKA
ECE-A
RA1511004040011

(1) Construct an argument using rules of inference to show that :

(a) If Arun studies well, then either Balu or Chandru will pass DM

(b) If Balu pass DM, then Arun will not study well

(c) If Dravid pass DM, then Chandru will not pass DM.

(d) Therefore, if Arun studies well, Dravid will not pass DM.

P : Arun studies well

Q : Balu will pass DM

R : Chandru will pass DM

S : Dravid will pass DM

The premisis are :

$P \to Q \vee R$, $Q \to \sim P$, $S \to \sim R$, $P \to \sim S$

SLNO	RULE	TERMS	REASON
1.	P	P	CP-Rule - Addition Premise
2.	P	$P \to Q \vee R$	
3.	T	$P \wedge P \to (Q \vee R) \equiv Q \vee R$	Using (1) and (2) Modus Ponens
4.	P	$Q \to \sim P$	
5.	T	$Q \to \sim P \equiv P \to \sim Q$	Using (4) contrapositive rule
6.	T	$P \wedge (P \to \sim Q) \equiv \sim Q$	Using (1) and (5) Modus Ponens
7.	P	$S \to \sim R$	
8.	T	$S \to \sim R \equiv R \to \sim S$	Using (7) Contrapositive rule
9.	T	$\sim Q \wedge (Q \vee R) \equiv R$	Using (3) and (6) Disjunctive
10.	T	$R \wedge (R \to \sim S) \equiv \sim S$	Using (7) and (9) Modus Ponens
11.	CP	$P \to \sim S$	

② Use mathematical induction to prove $8^n - 3^n$ is divisible by 5, for $n \geq 1$.

Let $P(n) = 8^n - 3^n$

$P(1) = 8^1 - 3^1 = 5 \Rightarrow$ Divisible by 5

$P(K) = 8^K - 3^K$

Let us assume that $P(K)$ is divisible by 5.

Hence, $P(K) = 8^K - 3^K = 5t$

$$P(K+1) = 8^{K+1} - 3^{K+1}$$
$$= (8)^K(8) - (3)^K(3)$$
$$= (5t + 3^K)(8) - (3)^K(3)$$
$$= 40t + (8)(3^K) - (3)(3^K)$$
$$= 40t + 5(3)^K$$
$$= 5[8t + (3)^K]$$

Hence $8^n - 3^n$ is divisible by 5, for $n \geq 1$

③ Show that $P \vee Q$ is a valid conclusion from the premises $C \vee D$, $(C \vee D) \rightarrow \sim H$, $\sim H \rightarrow (A \wedge \sim B)$ and $(A \wedge \sim B) \rightarrow (P \vee Q)$

SLNO	RULE	TERM	REASONS
1.	P	$C \vee D$	
2.	P	$C \vee D \rightarrow \sim H$	
3.	T	$(C \vee D) \wedge (C \vee D \rightarrow \sim H) \equiv \sim H$	Using (1) and (2) Modus Ponens
4.	P	$\sim H \rightarrow (A \wedge \sim B)$	
5.	T	$\sim H \wedge (\sim H \rightarrow (A \wedge \sim B)) \equiv A \wedge \sim B$	Using (3) and (4) Modus Ponens
6.	P	$(A \wedge \sim B) \rightarrow (P \vee Q)$	
7.	T	$(A \wedge \sim B) \wedge ((A \wedge \sim B) \rightarrow (P \vee Q)) \equiv (P \vee Q)$	Using (5) and (6) Modus Ponens

④ Determine if the compound proposition is tautology or contradiction.

$$[(P \rightarrow Q) \wedge (Q \rightarrow R)] \rightarrow [(P \rightarrow R)]$$

P	Q	R	$P \rightarrow Q$	$Q \rightarrow R$	X	$P \rightarrow R$	Y
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	F	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

X : $(P \rightarrow Q) \wedge (Q \rightarrow R)$

Y : $[(P \rightarrow Q) \wedge (Q \rightarrow R)] \rightarrow [P \rightarrow R]$

Hence, the given compound proposition is tautology

Without Truth Table :

$$[(P \rightarrow Q) \wedge (Q \rightarrow R)] \rightarrow [(P \rightarrow R)]$$

We know that $A \rightarrow B \equiv \sim A \vee B$

$[(\sim P \vee Q) \wedge (\sim Q \vee R)] \rightarrow (\sim P \vee R)$

$\sim[(\sim P \vee Q) \wedge (\sim Q \vee R)] \vee (\sim P \vee R)$

By De-morgan's Law

$\sim(\sim P \vee Q) \vee \sim(\sim Q \vee R) \vee (\sim P \vee R)$

$(P \wedge \sim Q) \vee (Q \wedge \sim R) \vee (\sim P \vee R)$

Associate law :

$[(\sim P) \vee (P \wedge \sim Q)] \vee [(R) \vee (Q \wedge \sim R)]$

$\sim P \vee R \equiv \underline{P \rightarrow R}$

⑥ Prove that $A \to B$, $\sim B \vee C$, $C \to D \Rightarrow A \to D$ by (i) Direct method, (ii) Indirect method (iii) CP Rule

(i) Direct Method:

SLNO	RULE	TERM	REASON
1.	P	$A \to B$	
2.	P	$\sim B \vee C$	
3.	P	$C \to D$	
4.	T	$B \to C$	Using (2) Implication
5.	T	$A \to C$	Using (1) and (4)
6	T	$A \to D$	Using (5) and (3)

(ii) Indirect Method:

SLNO	RULE	TERM	REASON
1.	P	$A \to B$	
2.	P	$\sim B \vee C$	
3.	P	$C \to D$	
4.	P	$A \to D$	
5.	T	$\sim(A \to D)$	Using (4) Negation - Indirect Method
6.	T	$B \to C$	Using (2) Implication
7.	T	$A \to C$	Using (1) and (6)
8.	T	$A \to D$	Using (7) and (3)
9.	T	$(A \to D) \wedge \sim(A \to D) \equiv F$	Using (8) and (5)

(iii) CP Rule:

SLNO	RULE	TERMS	REASON
1	P	$A \to B$	
2	P	$\sim B \vee C$	
3	P	$C \to D$	
4	P	A	Additional Premise - CP Rule
5	T	$B \to C$	Using (2) Implication
6	T	$A \wedge (A \to B) \equiv B$	Using (4) and (1) Modus Ponens
7	T	$B \wedge (B \to C) \equiv C$	Using (6) and (5) Modus Ponens
8	T	$C \wedge (C \to D) \equiv D$	Using (7) and (3) Modus Ponens
9	CP	$A \to D$	CP Rule

(7) Proove by indirect method:

$\forall x (P(x) \vee Q(x)) \Rightarrow \forall x P(x) \vee \exists x Q(x)$

SLNO	RULE	TERMS	REASON
1	P	$\forall x (P(x) \vee Q(x))$	
2	P	$\forall x\, P(x) \vee \exists x\, Q(x)$	
3	T	$\sim[\forall x P(x) \vee \exists x Q(x)] \equiv \exists x (\sim P(x)) \wedge \forall x (\sim Q(x))$	Using (2) Negate - Indirect Method
4	T	$\exists x (\sim P(x))$	Using (3)
5	T	$\forall x (\sim Q(x))$	Using (3)
6	T	$\sim P(a)$	Using (4) - Existential Specification
7	T	$\sim Q(a)$	Using (4) - Existential specification
8	T	$\sim P(a) \wedge \sim Q(a)$	Using (6) and (7)
9	T	$\sim [P(a) \vee Q(a)]$	Using (8) - De-morgan's Law
10	T	$P(a) \vee Q(a)$	Using (1) - Universal specification
11	T	$[P(a) \vee Q(a)] \wedge [\sim(P(a) \vee Q(a))] \equiv F$	Using (10) and (9)

(8) Show that the premises "One student in this class knows how to write programs in Java" and "Everyone who knows how to write programs in Java can get a high-paying Job" imply the conclusion "Someone in this class can get a high paying job".

$C(x) \to x$ is in the class

$J(x) \to x$ knows Java programming

$H(x) \to x$ can get a high paying Job

The premises are:

$\exists(x)[C(x) \wedge J(x)]$, $\forall x[J(x) \to H(x)]$, $\exists x[C(x) \wedge H(x)]$

SLNO	RULE	TERMS	REASON
1	P	$\exists(x)[C(x) \wedge J(x)]$	
2	P	$\forall(x)[J(x) \to H(x)]$	
3	T	$C(a) \wedge J(a)$	Using (1) - Existential specificatio
4	T	$J(a) \to H(a)$	Using (2) - Universal Specificatio
5	T	$C(a)$	Using (3)
6	T	$J(a)$	Using (3)
7	T	$J(a) \wedge [J(a) \to H(a)] \equiv H(a)$	Using (6) and (4) - Modus Ponens
8	T	$C(a) \wedge H(a)$	Using (5) and (7)
9	T	$\exists x[C(x) \wedge H(x)]$	Using (8) - Existential Generalization

(9) Show that the following premises are inconsistent

(a) If Rama gets his degree, he will go for a job

(b) If he goes for a job, he will get married soon

(c) If he goes for higher studies, he will not get married

(d) Rama gets his degree and goes for higher studies.

P: Rama gets his degree

Q: He goes for job

R: Will get married soon

S: goes for higher studies

The premises are:

$P \rightarrow Q$, $Q \rightarrow R$, $S \rightarrow \sim R$, $P \wedge S$

SLNO	RULE	TERMS	REASONS
1	P	$P \rightarrow Q$	
2	P	$Q \rightarrow R$	
3	P	$S \rightarrow \sim R$	
4	P	$P \wedge S$	
5	T	$P \rightarrow R$	Using (1) and (2)
6	T	$R \rightarrow \sim S$	Using (3) Implication
7	T	$P \rightarrow \sim S$	Using (5) and (6)
8	T	$\sim(P \wedge S)$	Using (7) Implication
9	T	$(P \wedge S) \wedge \sim(P \wedge S) \equiv F$	Using (4) and (8)

Hence the given premises are proved to be inconsistent

(10) Let $A = \{1, 2, 3, 4, 5, 6\}$ with subsets $B_1 = \{1, 3, 5\}$ and $B_2 = \{1, 2, 3\}$. Write the minsets of A and partition of A and the generated min-sets

$B_1 = \{1, 3, 5\}$ $\quad B_1^c = \{2, 4, 6\}$

$B_2 = \{1, 2, 3\}$ $\quad B_2^c = \{4, 5, 6\}$

$A_1 = B_1 \cap B_2 = \{1, 2\}$

$A_2 = B_1 \cap B_2^c = \{5\}$

$A_3 = B_1^c \cap B_2 = \{2\}$

$A_4 = B_1^c \cap B_2^c = \{4, 6\}$

Partitions of A:

$P_1 = \{\{1\}, \{2, 3, 4, 5, 6\}\}$

$P_2 = \{\{2\}, \{1, 3, 4, 5, 6\}\}$

$P_3 = \{\{3\}, \{1, 2, 4, 5, 6\}\}$

$P_4 = \{\{4\}, \{1, 2, 3, 5, 6\}\}$

$P_5 = \{\{5\}, \{1, 2, 3, 4, 6\}\}$

$P_6 = \{\{6\}, \{1, 2, 3, 4, 5\}\}$

$P_7 = \{\{1, 2\} \{3, 4, 5, 6\}\}$

$P_8 = \{\{1, 3\} \{2, 4, 5, 6\}\}$

$P_9 = \{\{1, 4\} \{2, 3, 5, 6\}\}$

$P_{10} = \{\{1, 5\} \{2, 3, 4, 6\}\}$

$P_{11} = \{\{1, 6\} \{2, 3, 4, 5\}\}$

$P_{12} = \{\{2, 3\} \{1, 4, 5, 6\}\}$

$P_{13} = \{\{2, 4\} \{1, 3, 5, 6\}\}$

$P_{14} = \{\{2, 5\} \{1, 3, 4, 6\}\}$

$P_{15} = \{\{2, 6\} \{1, 3, 4, 5\}\}$

$P_{16} = \{\{3, 4\} \{1, 2, 5, 6\}\}$

$P_{17} = \{\{3, 5\} \{1, 2, 4, 6\}\}$

$P_{18} = \{\{3, 6\} \{1, 2, 4, 5\}\}$

$P_{19} = \{\{4, 5\} \{1, 2, 3, 6\}\}$

$P_{20} = \{\{4, 6\} \{1, 2, 3, 5\}\}$

$P_{21} = \{\{5, 6\} \{1, 2, 3, 4\}\}$

$P_{22} = \{\{1, 2, 3\} \{4, 5, 6\}\}$

$P_{22} = \{\{1,2,3\}\{4,5,6\}\}$

$P_{23} = \{\{1,2,4\}\{3,5,6\}\}$

$P_{24} = \{\{1,2,5\}\{3,4,6\}\}$

$P_{25} = \{\{1,2,6\}\{3,4,5\}\}$

$P_{30} = \{\{1,3,5\}\{2,4,6\}\}$

$P_{26} = \{\{2,3,4\}\{1,5,6\}\}$

$P_{27} = \{\{2,3,5\}\{1,4,6\}\}$

$P_{28} = \{\{2,3,6\}\{1,4,5\}\}$

$P_{29} = \{\{1,3,4\}\{2,5,6\}\}$

$P_{31} = \{\{1,3,6\}\{2,4,5\}\}$

Partitions of min-set:

$P_1 = \{1,2\}$

$P_2 = \{1\}\ \{2\}$

$P_3 = \{5\}$

$P_4 = \{2\}$

$P_5 = \{4,6\}$

$P_6 = \{4\}\ \{6\}$

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(11) Define closure of a relation. Find reflexive, symmetric and transitive closure of $R = \{(1,2)(2,2)(2,3)(3,2)(4,1)(4,4\}$ defined on $A = \{1,2,3,4\}$

(a) Reflexive closure:

condition: $R_1 = R \cup \Delta$

$R = \{(1,2)\ (2,2)\ (2,3)\ (3,2)\ (4,1)\ (4,4)\}$

$\Delta = \{(1,1)\ (2,2)\ (3,3)\ (4,4)\}$

$R_1 = R \cup \Delta = \{(1,2)\ (2,2)\ (2,3)\ (3,2)\ (4,1)\ (4,4)$
$(1,1)\ (3,3)\}$

(b) Symmetric closure:

Condition : $R_1 = R \cup R^{-1}$

$R = \{(1,2)\ (2,2)\ (2,3)\ (3,2)\ (4,1)\ (4,4)\}$

$R^{-1} = \{(2,1)\ (2,2)\ (3,2)\ (2,3)\ (1,4)\ (4,4)\}$

$R_1 = R \cup R^{-1} = \{(2,2)\ (1,2)\ (2,3)\ (3,2)\ (4,1)\ (4,4)$
$(2,1)\ (1,4)\}$

(c) Transitive closure:

$R_1 = R \cup R^2 \cup R^3 \cup R^4$

$$m_R = R = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

$$R^2 = R \cdot R = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

$$R^3 = R^2 \circ R = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$R^4 = R^3 \circ R = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$R^1 = \{(1,2)$
$(2,2)\ (2,3)$
$(3,2)$
$(4,1)\ (4,4)\}$

$R^2 = \{(1,2)\ (1,3)$
$(2,2)\ (2,3)$
$(3,2)\ (3,3)$
$(4,1)\ (4,2)\ (4,4)\}$

$R^3 = \{(1,2)\ (1,3)$
$(2,2)\ (2,3)$
$(3,2)\ (3,3)$
$(4,1)\ (4,2)\ (4,3)\ (4,4)\}$

$R4 = \{(1,2)\ (1,3)$
$(2,2)\ (2,3)$
$(3,2)\ (3,3)$
$(4,1)\ (4,2)\ (4,3)\ (4,4)\}$

$R_1 = R \cup R^2 \cup R^3 \cup R4$
$= \{(1,2)\ (1,3)$
$(2,2)\ (2,3)$
$(3,2)\ (3,3)$
$(4,1)\ (4,2)\ (4,3)\ (4,4)\}$

The closure of a relation R with the respect to property P is the relation obtained by adding the minimum number of ordered pairs to R to obtain the property P.

13. If R is a relation on set of positive integers such that $(a,b) \in R$ if and only if a^2+b is even. Prove that R is equivalence relation.

Given:

$(a,b) \in R \Rightarrow a^2+b$ is even

(i) Reflexive:

$(a,a) \in R \Rightarrow a^2+a = a(a+1)$ is even

When, a is even $\Rightarrow$ (even) (odd) = even

a is odd $\Rightarrow$ (odd) (even) = even

Hence, R is reflexive

(ii) Symmetric :

$(a,b) \in R \Rightarrow a^2+b$ is even

$(b,a) \in R \Rightarrow b^2+a$ is even

The above condition is satisfied when a and b are same (even (or) odd) and each being odd and even.

Hence R is symmetric

(iii) Transitive :

When a, b, c are even, a^2+b and b^2+c are even. Also a^2+c is even, hence

$(a,c) \in R \Rightarrow a^2+c$ is even

When a, b, c are odd, a^2+b and b^2+c are even. Also a^2+c is even, hence

$(a,c) \in R \Rightarrow a^2+c$ is even

ie $(a,b) \in R$, $(b,c) \in R \Rightarrow (a,c) \in R$

Hence R is transitive

ie R is an equivalence relation.

(19) Let $A = \{1, 2, 3, 4\}$ and $R = \{(1,2)\ (2,3)\ (3,4)\ (2,1)\}$. Using Warshall's algorithm find the transitive closure.

$R = \{(1,2)\ (2,3)\ (3,4)\ (2,1)\}$

$$W_0 = M_R = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

K	W_{K-1}: Position of 1s in column K (p_i)	W_{K-1}: Position of 1s in row K (q_j)	1s in W_K (p_i, q_j)	W_K
1.	2	2	(2,2)	$W_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
2.	1, 2	1, 2, 3	(1,1) (1,2) (1,3) (2,1) (2,2) (2,3)	$W_2 = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
3.	1, 2	4	(1,4) (2,4)	$W_3 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
4.	1, 2, 3	–	–	$W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$R = \{(1,1)\ (1,2)\ (1,3)\ (2,1)\ (2,2)\ (2,3)\ (3,4)\}$