18CSE390T
Computer Vision

Fourier-Based Alignment

Fourier-Based Alignment

Fourier-based alignment relies on the fact that the Fourier transform of a shifted signal has the same magnitude as the original signal but a linearly varying phase

$$\mathcal{F}\{I_1(x+u)\} = \mathcal{F}\{I_1(x)\}e^{-ju\cdot\omega} = \mathcal{I}_1(\omega)e^{-ju\cdot\omega},$$

Fourier-Based Alignment

- where ω is the vector-valued angular frequency of the Fourier transform and we use cal-ligraphic notation $I1(\omega) = \mathcal{F}\{I1(x)\}$ to denote the Fourier transform of a signal
- Another useful property of Fourier transforms is that convolution in the spatial domain corresponds to multiplication in the Fourier domain

Fourier-Based Alignment

Fourier trans-form of the cross-correlation function ECC can be written as

$$\mathcal{F}\{E_{\mathrm{CC}}(\boldsymbol{u})\} = \mathcal{F}\left\{\sum_i I_0(\boldsymbol{x}_i) I_1(\boldsymbol{x}_i + \boldsymbol{u})\right\} = \mathcal{F}\{I_0(\boldsymbol{u}) \bar{*} I_1(\boldsymbol{u})\} = \mathcal{I}_0(\boldsymbol{\omega})\mathcal{I}_1^*(\boldsymbol{\omega}),$$

where

$$f(\boldsymbol{u}) \bar{*} g(\boldsymbol{u}) = \sum_i f(\boldsymbol{x}_i) g(\boldsymbol{x}_i + \boldsymbol{u})$$

is the correlation function, i.e., the convolution of one signal with the reverse of the other,and I∗1 (ω) is the complex conjugate of I1(ω)

Fourier-Based Alignment

- While Fourier-based convolution is often used to accelerate the computation of image correlations, it can also be used to accelerate the sum of squared differences function (and its variants)

- Its Fourier transform can be written as

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$$
\begin{aligned}
\mathcal{F}\left\{E_{\mathrm{SSD}}(\boldsymbol{u})\right\} &= \mathcal{F}\left\{\sum_i [I_1(\boldsymbol{x}_i+\boldsymbol{u}) - I_0(\boldsymbol{x}_i)]^2\right\} \\
&= \delta(\omega)\sum_i [I_0^2(\boldsymbol{x}_i) + I_1^2(\boldsymbol{x}_i)] - 2\mathcal{I}_0(\omega)\mathcal{I}_1^*(\omega).
\end{aligned}
$$

SSD function can be computed by taking twice the correlation function and sub-tracting it from the sum of the energies in the two images.