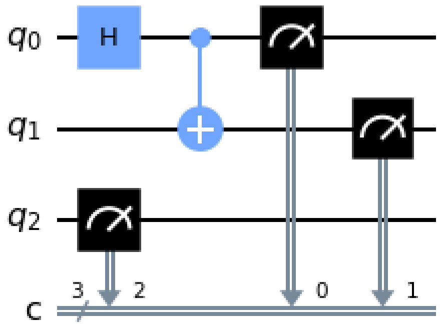
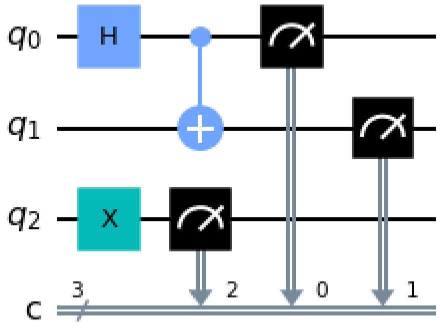
<u>Using Noisy states for good</u>

A study of following states(on ibmq_lima)-

1. (|00>+|11>) x |0>   (Bi-separable)

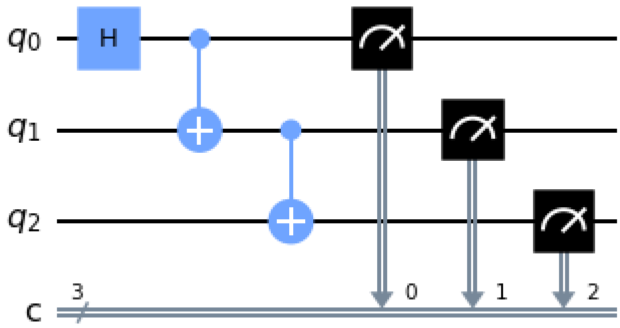


2. (|00>+|11>) x |1>   (Bi-separable2)



3. |000>   (separable) (no gates)
4. |111> (separable) (3-X gates)

5. |000>+|111>   (GHZ)



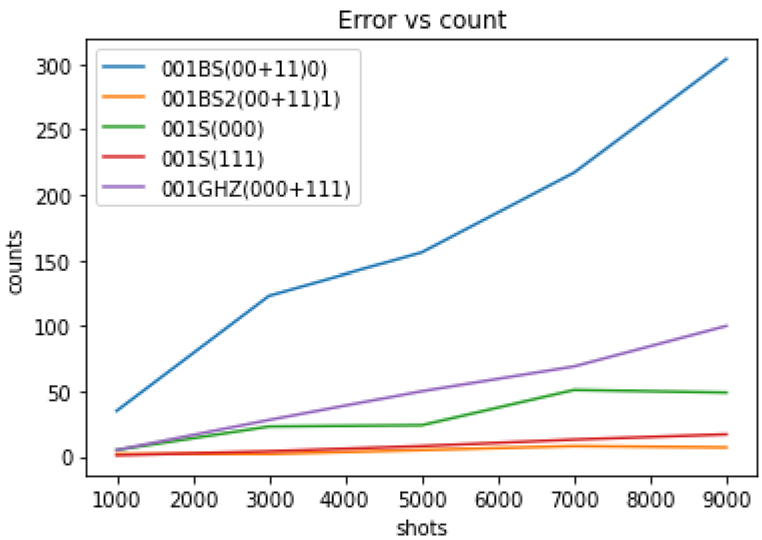
Color Coding for graph plots

1. (|00>+|11>) x |0>   (Bi-separable)
2. (|00>+|11>) x |1>   (Bi-separable2)
3. |000>   (separable)
4. |111> (separable)
5. |000>+|111>   (GHZ)

Observations (*states are coloured like their graph plots*)

Kindly ignore curves which are flat. They are intentionally made flat and are not a part of collected data . Curve for the Quantum state is plotted only if the given state is an error state for that Quantum state.

1. Check out the graph below for state **100. (Note state is written here as qubit ordering 1,2,3. In graph shows it as 001 with ordering of qubits as 3,2,1)**



Observation1 For Bi-separable state((|00>+|11>) x |0>) and GHZ state (|000>+|111>) and Separable state(|000>),

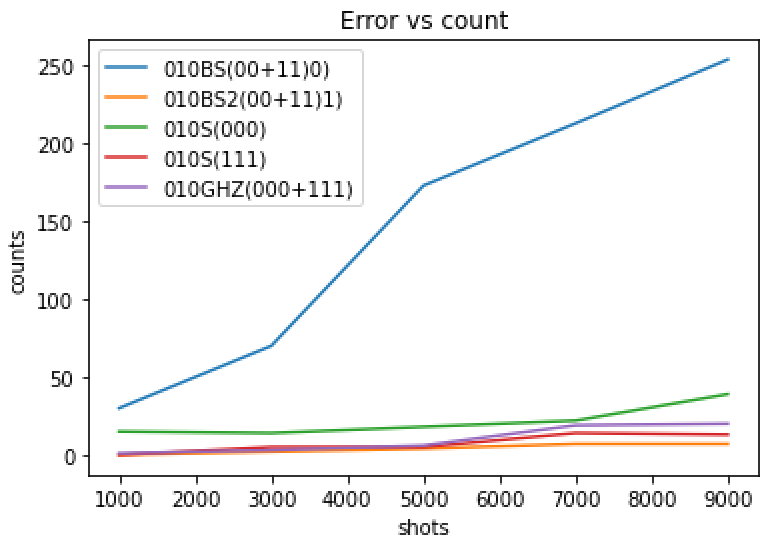
error in ‘100’ increases with the number of shots.

Explanation- Both states have equal |000> probabilities so bit flip error in first qubit can easily result to |000> → |100> .
This error can also occur in Separable state(|000>). So we observe a similar upward trend.

Observation2 Why is error count increasing more rapidly in Bi-separable state((|00>+|11>) x |0>) when compared to GHZ state (|000>+|111>) ?

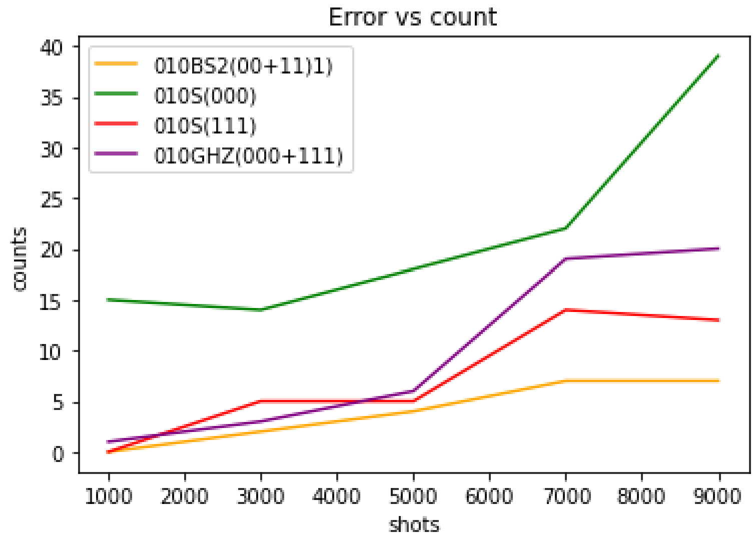
*****Explanation- Entanglement is lesser in Bi-separable state than GHZ state (|000>+|111>) .**
Although a/c to this reasoning the |000>(zero entanglement) should have even more errors than Bi-separable state but this phenomena can be attributed to comparatively high stability of |000> state as it is ground state qubits prefer to stay into.

2. Check out the graph below for state **010.**



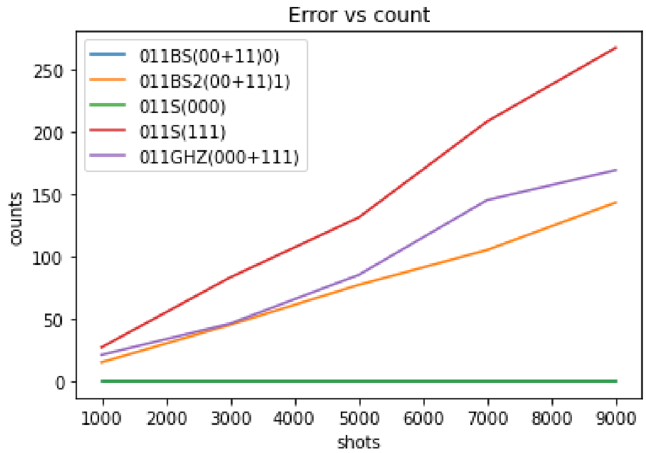
Observation1 For Bi-separable state((|00>+|11>) x |0>) error in ‘010’ increases with the number of shots more rapidly than other states.

Explanation- Bi-separable state has 50% |000> probabilities so bit flip error in second qubit can easily result to |000> → |010> . Also for state |011> a single bit flip in 3rd qubit also produces |011> → |010>. So a single bit flip error in both possible states(|000> & |011>) produces the error state |010>. That’s why the blue curve is showing much larger error counts than others.
Let’s see a more magnified view of states other than Bi-separable state



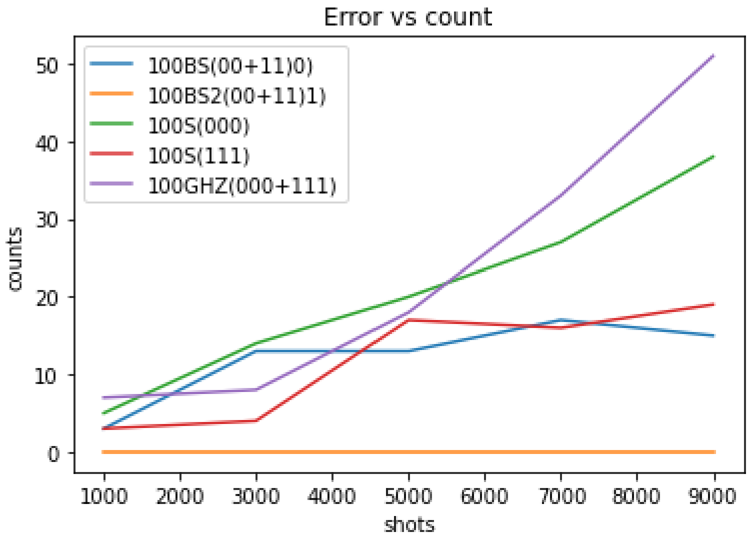
The bit-flip error in second qubit can also occur in Separable state(|000>) and GHZ state (|000>+|111>) so we observe a similar upward trend.
But there is an anomaly here.
Observation2 **The GHZ state (|000>+|111>) has 50% probability of |000> state just like the Bi-separable state((|00>+|11>) x |0>), still it has a lesser tendency to get an error state '010'.**
Explanation- The reason could be due to more entanglement in the GHZ state as compared to the Bi-separable state.

3. Check out the graph below for state '**110'.**

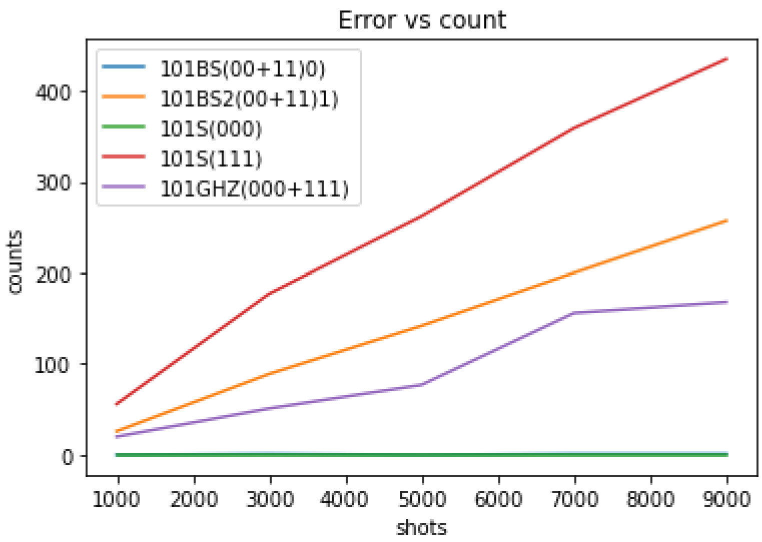


Observation1 **For Separable state(|111>) error in '110' increases with the number of shots more rapidly than other states because it requires single bit flip to get '110'. However the Bi-separable state((|00>+|11>) x |1>) is expected to show more error counts compared to GHZ state (|000>+|111>) state(*refer to Observation 2 for '010'*), but the curves are closely spaced.**
Explanation- For Bi-separable state((|00>+|11>) x |1>) to show error state the bit flip error will be in the third qubit which is in state |1> and it is not entangled with the first two qubits, hence we cannot use similar reasoning.

Support of Observation 1- Below given is the graph for error state '001'. We can compare **Bi-separable state((|00>+|11>) x |0>)** and **GHZ state (|000>+|111>).** There is no clear distinction between them when the shots performed are low (shots<5000). For a high number of shots (>5000), there is a distinct increase in the error associated with the entangled GHZ state compared to the separable state. This may be due to the fact that at higher shots, the entangled states are more prone to decoherence than the separable states. So, the error associated with the GHZ state is higher than the separable state.

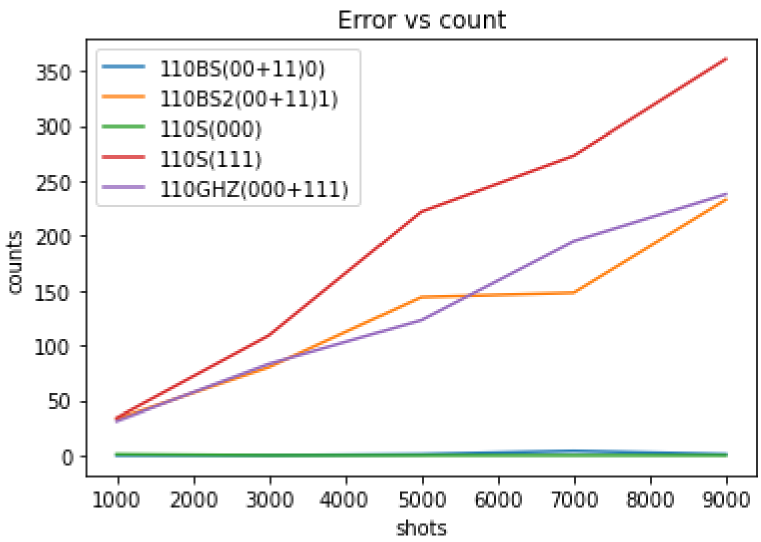


4. Check out the graph below for state '**101'.**



Observation1 **In both GHZ state (|000>+|111>) and the Bi-separable state((|00>+|11>) x |1>) have 50% probability of |111> state so a single bit flip error in second qubit can easily result to |111> → |101>.**
Still GHZ state (|000>+|111>) has a lesser tendency to get an error state '101' than Bi-separable state((|00>+|11>) x |1>) .
Explanation- The reason could be due to more entanglement in the GHZ state as compared to the Bi-separable state.

5). Check out the graph below for error state ‘**011’.**



Observation1 **In both GHZ state (|000>+|111>) and the Bi-separable state((|00>+|11>) x |1>) have 50% probability of |111> state so a single bit flip error in first qubit can easily result to |111> → |011>.**
Anomaly- Due to more entanglement in the GHZ state as compared to the Bi-separable state, we expect error count in Bi-separable state to be higher than GHZ state but this is not true in this case.

******Possible Explanation-*** The error count for GHZ state is significantly high when the ghz state makes contribution to error state due to its |111> part. The |111> with all qubits flipped to higher energy state is seemingly unstable and contributes more to single bit flip errors. This is evident in both graphs in (4) and (5). The count of GHZ is comparable to that of Bi-separable state. This means that even though entanglement seems to stabilize the GHZ state. The unstable |111> makes up for errors caused due to single bit flip in |111> state.

Final Conclusion

Looking at above experimental data we can see that entanglement in the GHZ state does seem to prevent noisy states. However the instability of |111> state leads to more error counts for states(110,011,101)(double 1’s). So a good distinction between a bi-separable state and GHZ should be to look at noisy states (100, 010, 001)(single 1’s).

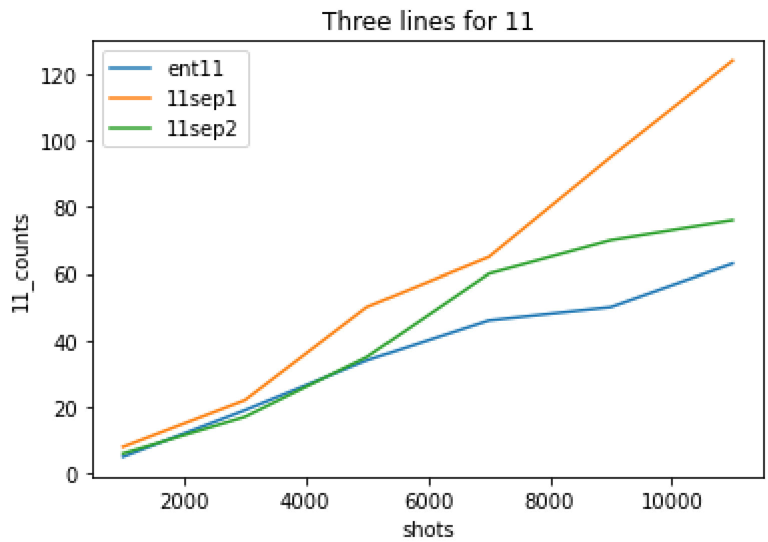
A study of following states(on ibmq_lima)-

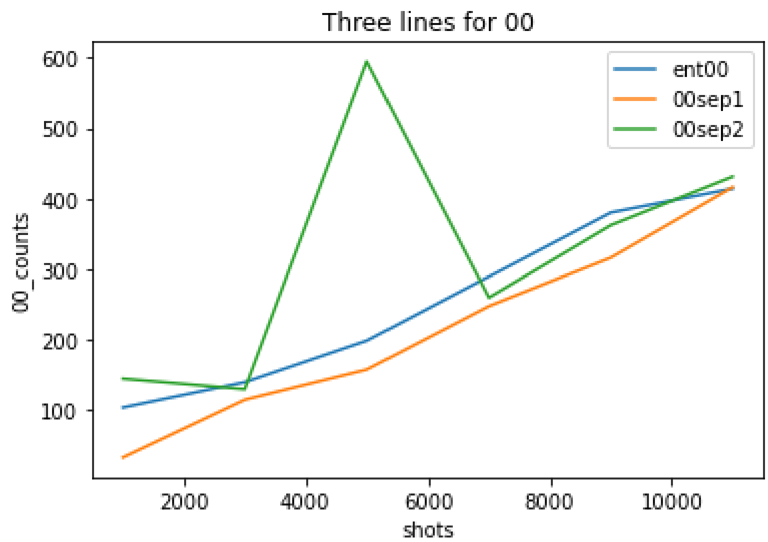
1. *(|01>+|10>)   (Bell's state)*

2. |01> (Separable state)

3. |10> (Separable state)

We have considered comparison between above mentioned states because they all have something in common. All three states have |00> and |11> as their error states.



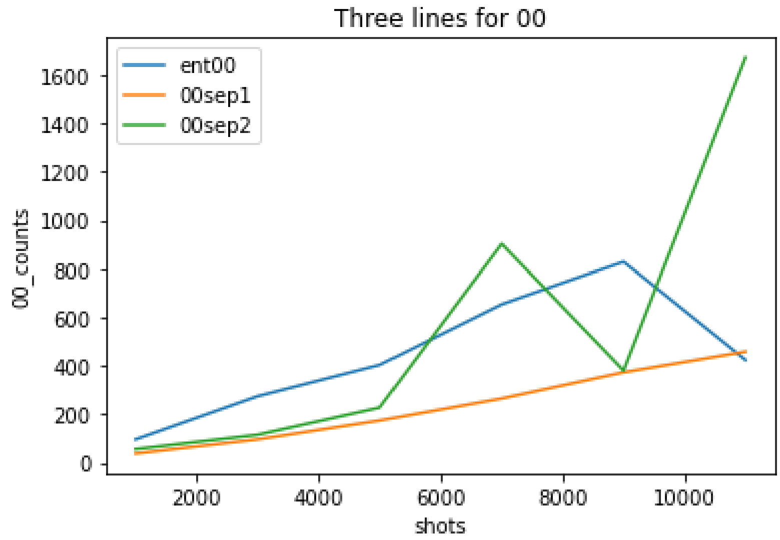


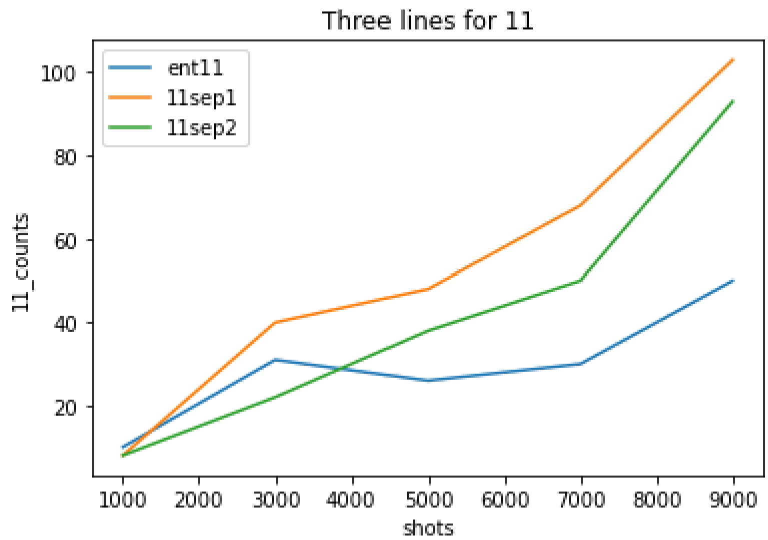
Observation1 : No conclusive predictions can be made by looking at curves(graph-2) for error state |00>, so we can assume the error state |00> has equal probabilities for all three quantum states, however

Observation2 : The error state |11>(graph-1) clearly depicts less error count for the entangled Bell's state.

Explanation- **The less error count for bell's state could be due to presence of entanglement in bell's state which is not found in other two separable states.**

Experiment Run-2





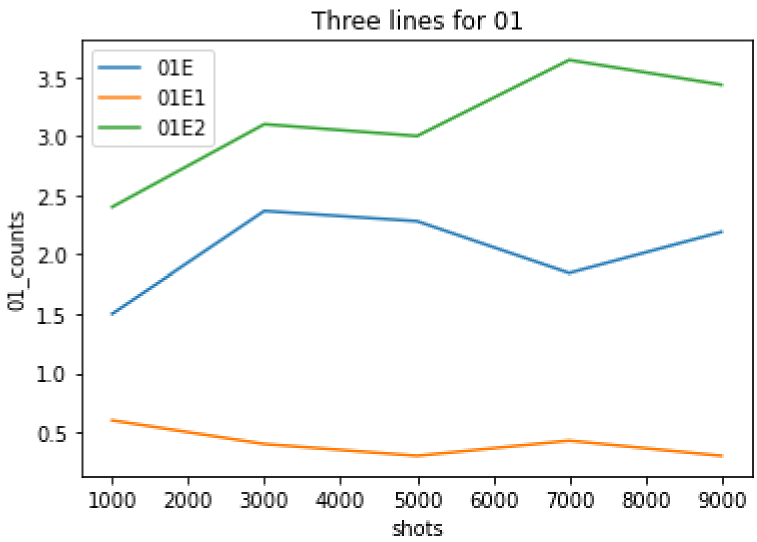
Final Conclusion- Hence we can see the plots for error state |11> provides us with definitive conclusion that entangled ***Bell's state (|01>+|10>)*** has less error counts than the two separable states. The state plots for |00> however fail to provide such concrete evidence for the role of entanglement in affecting the error counts. So to distinguish between ***Bell's state (|01>+|10>)*** other separable states we need to compare the error counts for noisy state |11>.

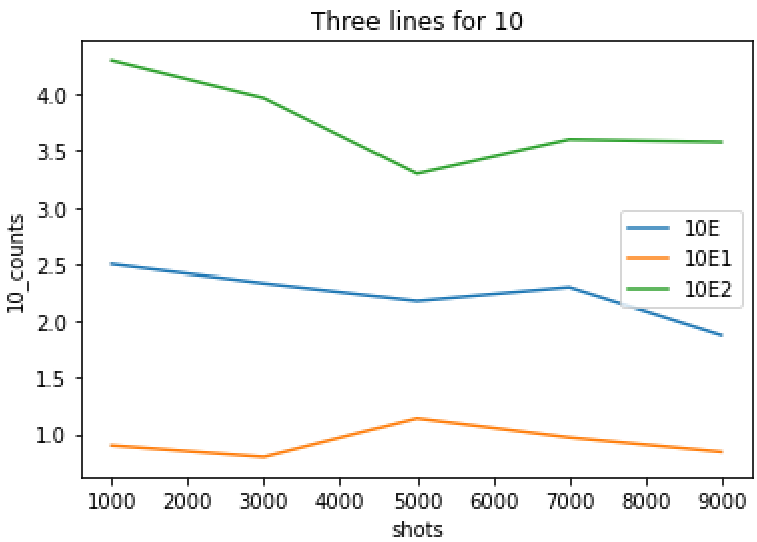
A study of following states(on ibmq_lima)-

1. *(|00>+|11>)* *(Bell's state)*

2. **a|00> +b|11> (** $|a|^2 = 0.96, |b|^2 = 0.04$ **) - Mostly |00>**

3. **c|00> +d|11> (** $|c|^2 = 0.04, |d|^2 = 0.96$ **) - Mostly |11>**





Conclusion- The above graphs depict the expected behaviour of error states i.e the state (c|00> +d|11>) has high |11> content in it. Hence this state is more prone to error than the state(a|00> +b|11>) which has |00> as more probable component. The counts for error states in the Bell's state, is in the middle since it has equal parts of both |00> and |11>. So we observe no conclusive evidence of the effect of entanglement on Noisy states.

Project Contributors

Abhishek Bhardwaj| @Bhardu_abhi29#7563| IIT Ropar,
Ranendu Adhikary| @RiemannRanendu#4372 |Jadavpur University,
Dimitra Nikolaidou| @sovy#6862| National and Kapodistrian University of Athens,
Hariprasad | @Hariprasad M#9069|Pondicherry University,
Nicole Farrier | @Nicole Farrier#7214 |Heriot-Watt University.
Soumyabrata Hazra | @Soumyabrata#7127 | S N Bose National Centre for Basic Sciences

Mentor

Lorraine T. Majiri

Appendix

Modelling Decoherence

In Addition, here we begin to model how decoherence affects entanglement of the bell states using the information we have about the quantum device and start by assuming that the qubit is found initially in the pure qubit state and interacts with its environment (bath). As the joint system evolves correlated states occur such as an entangled state of the qubit and the such as a qubit and bath entangled state.Using Open Quantum Systems to model the Relaxation and Dephasing Decay rates, assuming in the weak-coupling regime and applying the Lindblad Master equation. From the maximum to the minimum decay rates for the ibmq-lima device found here https://quantum-computing.ibm.com/services?services=systems&system=ibmq_lima.

	Maximum	Minimum	Average
Frequency (GHz)			5.19*10**9
Decay Rate Relaxation (T_1)	$134.67x10^{-6}$	$25.02x10^{-6}$	$86.99x10^{-6}$
Decay Rate Dephasing (T_2)	$170.27x10^{-6}$	$24.98x10^{-6}$	$118.61x10^{-6}$

Table 1: shows the data used to model decoherence accessed from https://quantum-computing.ibm.com/services?services=systems&system=ibmq_lima.

Using the data obtained in the Table 1 the follow system Hamiltonian is assumed and scaled for 2 Qubits is

$$H_S = \frac{\hbar\omega}{2}\sigma_z ,$$

and is inserted into the Lindblad Master Equation, assuming the systems in the weak-coupling regime. This is because the Qubits operate at low temperatures. The expectation values for the bell states and populations are plotted Figure 1 and Figure 2 respectively showing that as the rapid decay rates increase the dynamics decay rapidly.

The von Neumann entropy is a measure of a density operator's purity. Figure 3 demonstrates that the qubit becomes more mixed as it interacts with the environment, but that when it equilibrates with the environment (after a duration on the order of the relaxation time), the qubit becomes less mixed due to the low temperature. As the decay rates increase the results become more skewed. In the future, we would like to model for the states depicted in each circuit and compare the differences (however we didn't have time to complete this step).

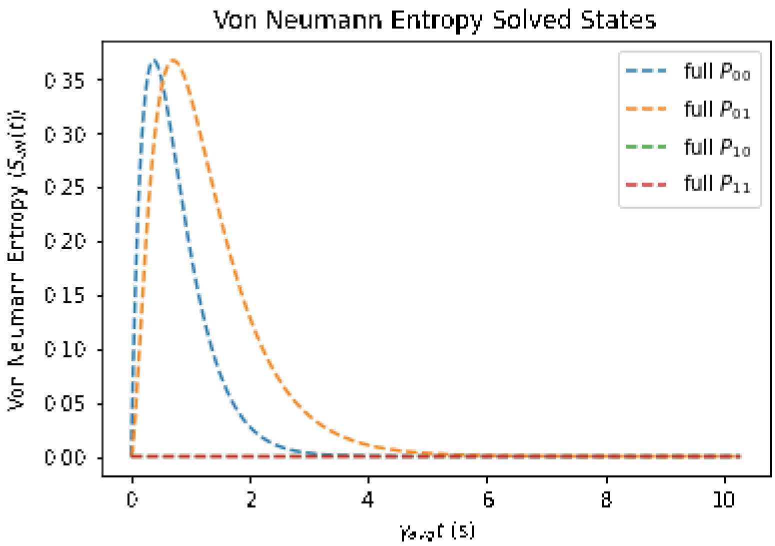


Figure 1: Shows the evolution of the Von-Neumann Entropy of the Bell States . The animation starts at the Maximum decay rates and decreases to the minimum decay rate

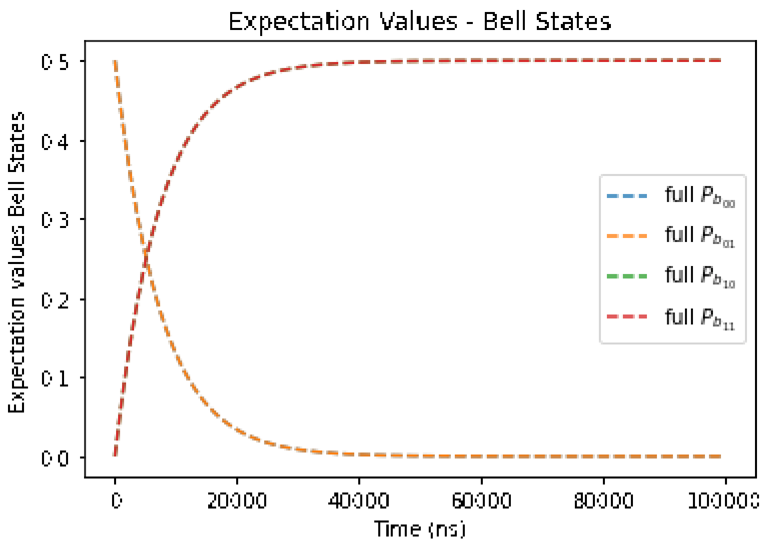


Figure 1: Shows the evolution of the expectation values of the Bell States. The animation starts at the Maximum decay rates and decreases to the minimum decay rate

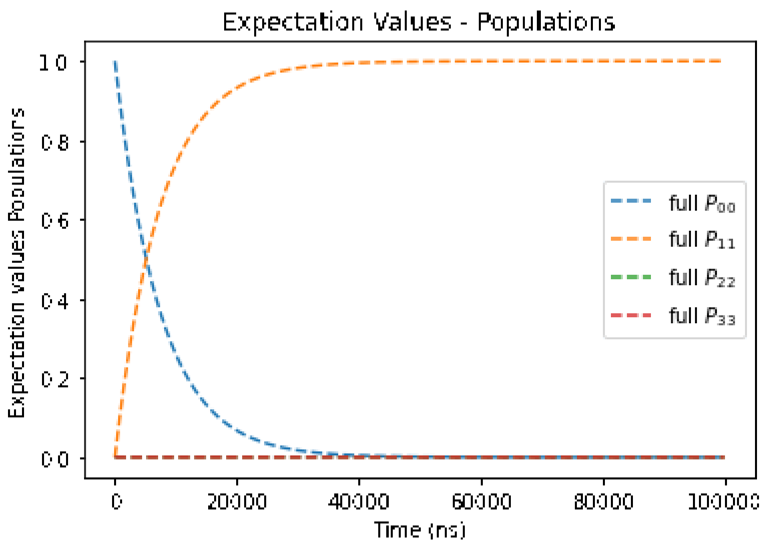


Figure 1: Shows the evolution of the expectation values of the Populations. The animation starts at the Maximum decay rates and decreases to the minimum decay rate